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Hazardous waste management problem: The case for incineration

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Abstract

We define the *hazardous waste management problem* as the combined decisions of selecting the disposal method, siting the disposal plants and deciding on the waste flow structure. The *hazardous waste management problem* has additional requirements depending on the selected disposal method. In this paper we focus on incineration, for which the main additional requirement is to satisfy the air pollution standards imposed by the governmental restrictions. We propose a cost-based mathematical model in which the satisfaction of air pollution standards is also incorporated. We used the Gaussian Plume equation in measuring the air pollution concentrations at population centers. A large-scale implementation of the proposed model within Turkey is provided.

Keywords: Incinerator location; Hazardous waste; Gaussian Plume equation; Air pollution

1. Introduction

The harmful products of chemical processes produced from either industries or hospitals are called hazardous waste. Hazardous waste is also generated in recycling centers, where waste from industries is recycled (Fig. 1). Depending on the waste type, hazardous waste can be explosive, oxidizing, highly flammable, corrosive, infectious, mutagenic, irritant, toxic, or carcinogenic.

The common hazardous waste disposal methods are incineration, land disposal and new technologies like solar detoxification. Despite its high construction cost, incineration is the most popular and is the only method that offers the detoxification of waste such as combustible carcinogens and pathological waste. Incineration significantly reduces the volume of hazardous waste.

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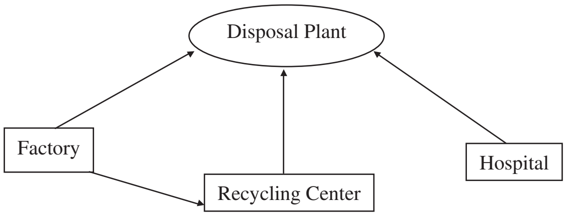

Fig. 1. Flow diagram of hazardous waste.

Each disposal method has different requirements. For land disposal, the main issue is leachate. Any land disposal facility should guarantee the safety of nearby groundwater. For incineration plants, the main requirement is the satisfaction of air pollution standards at population centers. After the incineration process, some air pollutants (e.g. SO_2, SO_3, NO, NO_2, Cl, HCl) are generated. Although the amount of air pollutants can be decreased to some extent by using filters and scrubbers, some air pollutants still remain and are emitted from the stack of the plant, causing air pollution. The ambient air concentration of the air pollutants at each population center should be less than some specified value, which is defined by legislation.

In this paper we analyze the *hazardous waste management problem*, defined as the combined decisions of selecting the disposal method, siting the selected disposal plants and deciding on the waste flow structure.

As stated before, the *hazardous waste management problem* has additional requirements depending on the selected disposal method. To the best of our knowledge, these additional requirements have not been considered much in the literature. A common synonym: “undesirable facility” is used for each type of disposal method in the literature. Most of the undesirable facility location literature focuses on selecting sites while minimizing the nuisance and the adverse effects on public and environment.

The literature can be divided into three main categories: maximin objectives, maxisum objectives and multiple objectives. In the maximin problem, the objective is to maximize the minimum distance between the undesirable facility, which is to be located and the existing facilities or population centers, which are under effect. The maximin model can be viewed as the minimization of maximum nuisance and is suitable for locating a high-risk industry such as an explosive manufacturing industry or a nuclear power plant. Dasarathy and White [1], Drezner and Wesolowsky [2], Melachrinoudis and Cullinane [3], and Melachrinoudis and Smith [4] studied the maximin problem. In the maxisum problem, the objective is to maximize the total distance between the undesirable facility and the existing facilities. The maxisum problem can be viewed as minimization of total nuisance and is suitable for locating a plant that continuously threatens the environment. Melachrinoudis and Cullinane [5], Hansen et al. [6], and Fernandez et al. [7] studied the maxisum problem. Interested readers can also refer to an extensive survey on undesirable facility location by Erkut and Neuman [8].

The multiobjective models consider different aspects of the undesirable facility location problem simultaneously. The minimization of cost, minimization of risk, and maximization of equity are the three most popular objectives used in the multiobjective literature. Ratick and White [9] were first to develop a model considering these three objectives. Later, Erkut and Neuman [10] studied the same objectives by using different risk and equity measures. There are also studies in the literature that consider the location and routing decisions together [11–14].

In the literature there are few studies that develop models specific to disposal methods. Wyman and Kuby [15] have considered new technologies such as solar detoxification and have created a model that includes technology selection. Melachrinoudis et al. [16] have studied the site selection of landfills and focused on human and non-human risks. Karkazis [17] and Karkazis and Papadimitriou [18] have focused on locating a single facility that poses air pollution. By using the Gaussian dispersion model, they minimized the total pollution concentration on existing facilities. The two main deficiencies of their research are that their model only locates a single facility and that they did not consider cost issues related to the location of the facility.

We remark here that the hazardous waste treatment industry is operated by cost-driven companies. Normally, minimizing the cost will be the leading terms in their objective rather than minimizing the adverse affects. Public and environmental safety is guaranteed through legislative restrictions imposed by the government.

In this paper, we consider the hazardous waste management problem and select incineration for the waste disposal method in our problem. We apply our model to the case of Turkey, where the Turkish Ministry of Environment plans to build three more incinerators. The model that we propose is applicable to a cost-driven sector. For public safety, we consider controlling the emitted air pollution through governmental restrictions. For this purpose the satisfaction of air pollution standards at each population center is to be incorporated into the proposed model. The Gaussian Plume model is the most applicable dispersion model in measuring the air pollution concentration on a given point. Therefore, we incorporate the Gaussian Plume equation as a constraint to our model, so that the selected sites will automatically satisfy air pollution standards at each population center.

In Section 2 we provide the basics for our model. The air pollution and the Gaussian Plume equations are also explained in this section. In Section 3 we provide the proposed mathematical model. A large-scale implementation of the proposed model within Turkey is given in Section 4. Lastly we give some concluding remarks with the future research directions in Section 5.

2. Model development

We have defined the hazardous waste management problem as establishing incinerators from a set of candidates such that all the generated waste is to be disposed of, and the air pollution standards for each pollutant type at each population center are to be satisfied. The hazardous waste management problem on general networks is NP-hard and can be shown by reducing it to the *p*-median problem.

The generated hazardous waste should be transported to and treated at the located incinerators. As stated in the introduction, the incineration process causes air pollution that affects both the public and the environment. On the other side, the incinerators, generators and transportation companies mainly consider the economical aspects of the process. Thus, the selected site should satisfy the air pollution standards and should be economically attractive. Pricing is out of the scope of this paper, and thus, we consider the travel distances as a surrogate for the cost measure. If the required pricing information were available, the proposed model would easily be applicable to the case of linear cost functions.

In the proposed model, public and environmental safety is considered by controlling the emitted air pollution from the incinerators. Thus, the risk “around” the incinerator location is considered. However,

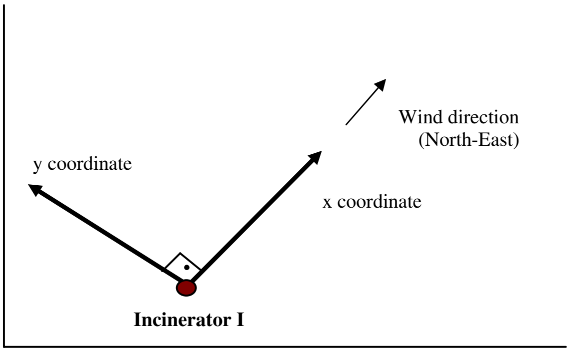

Fig. 2. New coordinate system for northeast wind direction at incinerator I.

a hazardous transportation network will be associated with the incinerator locations, and one may also consider the routing risks. As the proposed model is actually developed for a “cost-driven” sector, we select distance as our routing measure. Actually, two of the three most popular routing risk measures, population exposure and incident probability, are linear functions of distance [11]. Air pollution is the major concern in our study, and the total pollution exposed is a function of the distance traveled. Besides, for the routing risks which are linear functions of distance, the total risk decreases as the number of incinerators increases. Thus, our approach can also be considered as a risk measure that is a function of the number of incinerators. For example, for one of our application areas, the total distance traveled decreases by 35% when p is increased from 2 to 3, and decreases by 20% when p is increased from 3 to 4. We remark here that, even though we use distance as our routing measure, any of the linear risk measures can be considered and incorporated directly into our model.

To quantify the concentration of the air pollutants at any population center, we use the Gaussian Plume equation by Karkazis and Papadimitriou [18] (Eq. (1)).

$$C(x, y) = \frac{QK \exp\left[-0.5\left(y^2/(\sigma_y)^2\right) - 0.5\left(h^2/(\sigma_z)^2\right)\right]}{2\Pi u \sigma_y \sigma_z}. \tag{1}$$

Here $C(x, y)$ defines the concentration of air pollutant at the point $[x, y]$ (representing a population center). The Q value is the amount of air pollutant emitted from the stack and depends on the amount of hazardous waste incinerated. The amount of emitting air pollutants can be changed according to the technologic properties of the scrubbers used in the incineration plants. Conversion factors for finding the amount of air pollutants from the amount of incinerated hazardous waste depend on the type of hazardous waste, the technological equipment used for the scrubber, and the type of air pollutants. Such conversion factors are available in air quality books, for example, Baumbach [19]. The rest of the parameters mainly represent meteorological conditions. A detailed explanation of each term is given in Appendix A. Since the conversion factors used in finding Q are specific for each pollutant type, Eq. (1) is to be written for each pollutant type.

The x and y in the formula represent the “relative distance” between the population center under consideration and the incinerator site. For the formula, the distance is determined by using the coordinate system, which is based on the incinerator site and specified by the wind direction. The origin of the coordinate system is taken as the base of the incineration plant stack (Fig. 2). The x-axis is taken as the wind direction, and the y-axis is taken as the crosswind direction (normal to the x-axis). Since the axes are defined according to the wind direction, the x and y values of the population center change for each wind direction. In addition, as the coordinate system is based at the incineration site, each population center will have different x and y values for each candidate site.

We have developed formulas to find those x and y values. For each candidate site we first formed the coordinate system based on the wind direction. We then calculated the effect of this candidate site on every population center. Depending on the wind direction and the position of the population center relative to the candidate site, we derived 32 different formulas (eight different wind directions and four relative positions). We then automated this process by writing a simple C code. The code requires the location of population centers and candidate sites in a unique coordinate system and the prevalent wind direction of each candidate site; it outputs the (x, y) values for each candidate site and population center combination.

Other parameters in the formula depend on the meteorological conditions of the candidate site. If we know the meteorological conditions and the (x, y) values, we can incorporate the Gaussian Plume equation into a mathematical model. In the equation, the value Q will be a decision variable, and the rest will be known parameters. For the sake of representation, we used a matrix T_{jp} to denote all the known parameters for each candidate site and population center pair.

$$T_{jp} = \frac{K \exp\left[-0.5\left(y^2_{(jp)}/(\sigma_{y(jp)})^2\right) - 0.5\left(h^2/(\sigma_{z(jp)})^2\right)\right]}{2\Pi u_j \sigma_{y_{(jp)}} \sigma_{z(jp)}}. \tag{2}$$

Recall that the amount of air pollutants in Eq. (1) is expressed in terms of the amount of hazardous waste that is incinerated. Factors for converting the amount of hazardous waste to the amount of air pollutants are defined by destruction rates (DR_{lt}), which are specific to waste type t, air pollutant type l and the properties of scrubbers used [19]. Thus, the amount of air pollutant l, emitted from disposal plant j can be found by

$$(Q_j)_l = \sum_k (1 - \mathrm{DR}_{lk}) n_{jk}, \tag{3}$$

where n_{jk} is the amount of hazardous waste type k incinerated at site j.

The concentration of the air pollutant l at population center p can now be calculated as

$$[C(x, y)]_l = \sum_j (Q_j)_l T_{jp}. \tag{4}$$

Let $\mathbf{L} = \{1, \dots, l\}$ denotes the set of the air pollutants. The concentration of each air pollutant l at any population center must be less than the standard concentration of that air pollutant. Let K_l denotes the standard concentration of air pollutant l. The governmental restrictions are such that the $[C(x, y)]_l$ value should not be more than the K_l value at each population center.

Apart from the Gaussian Plume constraint, the proposed model also includes standard mass balance constraints, capacity constraints, and minimum capacity requirements.

3. The proposed model

For the mixed integer formulations the index set, parameters and decision variables are given below:

Index set:

- **J** = candidate sites $\mathbf{J} = \{1, \ldots, n\}$
- **P** = population centers $\mathbf{P} = \{1, \ldots, z\}$
- Waste generation nodes
    - **I** = factories $\mathbf{I} = \{1, \ldots, q\}$
    - **H** = hospitals $\mathbf{H} = \{1, \ldots, k\}$
    - **R** = recycling centers $\mathbf{R} = \{1, \ldots, r\}$

  Let M = **I** U **H** U **R** U **J** U **P** $\mathbf{M} = \{1, \ldots, m\}$
- Waste types
    - **W** = recyclable waste $\mathbf{W} = \{1, \ldots, w\}$
    - **U** = unrecyclable waste $\mathbf{U} = \{1, \ldots, u\}$
    - **C** = clinical waste $\mathbf{C} = \{1, \ldots, c\}$
- **L** = air pollutant type $\mathbf{L} = \{1, \ldots, l\}$

  Let T = **W** U **U** U **C**

Parameters:

$b_{iw}(b_{iu})$ = the total amount of recyclable (unrecyclable) waste type w (u) at ith factory.

b_{hc} = the total amount of clinical waste type c at hospital h.

α_{rw} = the reduction rate for waste type w at recycling center r.

T_{jp} = the fixed part of the Gaussian Plume equation for plant j and population center p.

K_l = standard ambient air concentration for air pollutant type l.

$(1 - \mathrm{DR}_{lt})$ = conversion factor from hazardous waste type t to air pollutant type l.

ct/dist = unit transportation cost.

d_{ij} = the shortest path distance between i and j.

Cap$j$ = capacity of jth incinerator.

$\mathrm{Cap}_j^{\min}$ = minimum capacity requirement for an incinerator at site j.

p = number of incinerators to be located.

Decision variables:

$y_j = 1$ if the incinerator is opened at jth candidate site; 0 otherwise.

$u_{iju}(x_{ijw})$ = amount of unrecyclable (recyclable) waste type u (w) that goes from factory i to incinerator j.

e_{rjw} = amount of recyclable waste type w that goes from recycling center r to incinerator j.

h_{hjc} = amount of clinical waste type c that goes from hospital h to incinerator j.

q_{irw} = amount of recyclable waste type w that goes from factory i to recycling center j.

n_{jt} = amount of hazardous waste type t to be incinerated at plant j. t = recyclable, unrecyclable, clinical waste (Fig. 3).

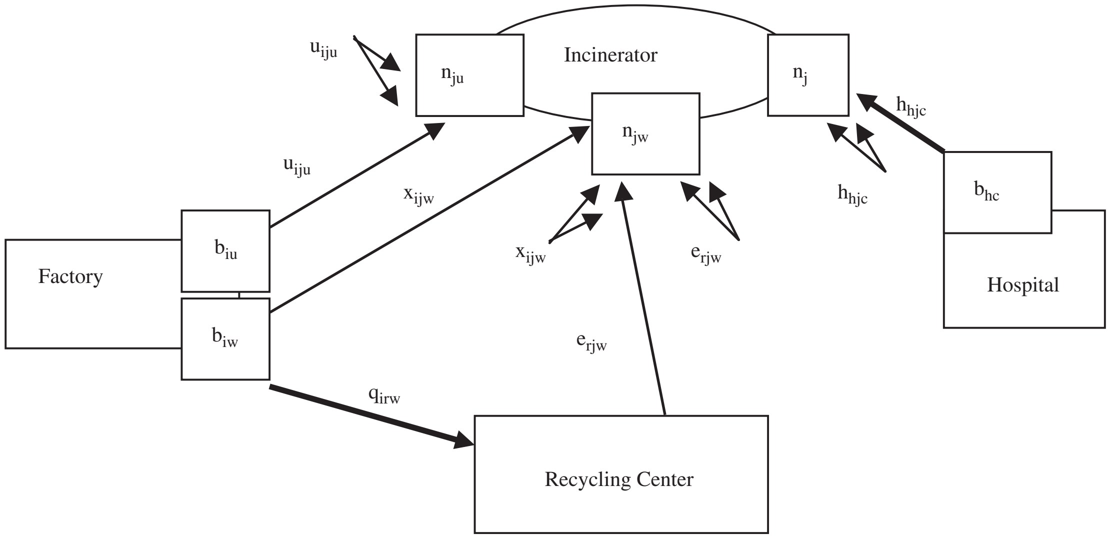

Fig. 3. Diagram representing decision variables and some parameters.

The proposed model

min

$$\sum_{i,j,w} ct/\text{dist}\,\mathrm{d}_{ij} x_{ijw} + \sum_{r,j,w} ct/\text{dist}\, d_{rj} e_{rjw} + \sum_{i,j,u} ct/\text{dist}\, d_{ij} u_{iju}$$
$$+ \sum_{h,j,c} ct/\text{dist}\, d_{hj} h_{hjc} + \sum_{i,r,w} ct/\text{dist}\, d_{ir} q_{irw}$$

s.t.

$$\sum_{i,w} x_{ijw} + \sum_{i,u} u_{iju} + \sum_{r,w} e_{rjw} + \sum_{h,c} h_{hjc} \leqslant \text{Cap}_j y_j \quad \forall j \in J \tag{5}$$

$$\sum_{j} x_{ijw} + \sum_{r} q_{irw} = b_{iw} \quad \forall i \in I, \quad w \in W \tag{6}$$

$$\sum_{j} u_{iju} = b_{iu} \quad \forall i \in I, \quad u \in U \tag{7}$$

$$n_{jw} = \sum_{i} x_{ijw} + \sum_{r} e_{rjw} \quad \forall j \in J, \quad w \in W \tag{8}$$

$$n_{ju} = \sum_{i} u_{iju} \quad \forall j \in J, \quad u \in U \tag{9}$$

$$n_{jc} = \sum_{h} h_{hjc} \quad \forall j \in J, \quad c \in C \tag{10}$$

$$\sum_j h_{hjc} = b_{hc} \quad \forall h \in H, \quad c \in C \tag{11}$$

$$\alpha_{rw} \sum_i q_{irw} = \sum_j e_{rjw} \quad \forall r \in R, \quad w \in W \tag{12}$$

$$\sum_{i,w} x_{ijw} + \sum_{i,u} u_{iju} + \sum_{r,w} e_{rjw} + \sum_{h,c} h_{hjc} \geqslant y_j \mathrm{Cap}_j^{\min} \quad \forall j \in J \tag{13}$$

$$\sum_j y_j = p \tag{14}$$

$$\sum_j \left[\sum_w (1 - \mathrm{DR}_{lw}) n_{jw} + \sum_u (1 - \mathrm{DR}_{lu}) n_{ju} + \sum_c (1 - \mathrm{DR}_{lc}) n_{jc} \right] T_{jp} \leqslant K_l \quad \forall p \in P, \quad l \in L \tag{15}$$

$$y_j \in \{0, 1\} \quad \forall j \in J \tag{16}$$

$$\text{all variables} \geqslant 0 \tag{17}$$

The objective function sums up all the transportation costs. Constraint (5) ensures that a flow to site j is only possible if there is an incinerator located at that site. The total flow into an incinerator cannot exceed its capacity, which is again satisfied by constraint (5).

Constraints (6)–(11) are the flow balance constraints for factories, disposal plants and hospitals. We need to differentiate between all waste types, since the destruction rates used in constraint (15) may differ. Constraint (12) is the mass balance constraint for the recycling centers. If waste is sent to recycling center r, it undergoes the recycling process. However, after the process, some amount of hazardous waste remains. For each recyclable waste type w and for each recycling center r, there is a conversion factor, α_{rw}, which is used to find the amount of remaining hazardous waste after the recycling process. The remaining amount is sent to the incineration plant.

Constraint (13) ensures that the flow into the plant satisfies the minimum threshold value. If the amount of hazardous waste into an incinerator j is less than a threshold value, $\mathrm{Cap}_j^{\min}$, it is not appropriate to operate that facility. The number of incinerators to be opened is fixed to p by constraint (14).

Constraint (15) is the Gaussian Plume constraint and provides the satisfaction of the ambient air concentration of air pollutant standards at each population center.

The model has n binary variables and $qnu + qnw + rnw + knw + qrw + nt$ real variables where n is the number of candidate sites, q is the number of factories, k is the number of hospitals, r is the number of recycling centers, w is the number of recyclable waste, u is the number of unrecyclable waste, and c is the number of clinical waste. Recall that T is the union of all waste types and M is the union of factories, hospitals, recycling centers and population centers. If we take t as the number of possible waste, and m as the number of all possible locations then the number of real variables will be bounded by $mt(5m + 1)$. In the same manner the number of constraints are bounded by $m(7t + l + 3)$.

4. Computational analysis

We test the computational performance of the model in Turkey, which is divided into seven geographical regions. Hazardous waste generation is proportional to the industrialization level of the cities, and thus, we concentrate on four of the more industrialized regions. Three are coastal regions and are included in the hazardous waste management project of the Turkish Ministry of Environment. As stated in Section 1, this project is to open three more incinerators in the coastal regions (one in each region) of Turkey. The one industrialized region not included in the project is the Central Anatolian region, which also hosts the capital Ankara. We included three more cities from other regions in order to have a connected highway map (Fig. 4). For the computational analysis, we first focused on the geographically largest industrialized region Central Anatolia. In the second part of our analysis, we considered the waste generated in these four regions, but the incinerators should be located at the three coastal regions in order to align with the project of the Ministry of Environment.

We assume that industrial hazardous waste is generated at each district and the clinical hazardous waste is generated in districts with a population of more than 20,000. The amount generated is taken to be proportional to the industrialization level and to the population of the districts.

We assume that there are two categories of waste: recyclable and unrecyclable. In our computational analysis, we consider two different waste types for each category. For the clinical waste, we again assume that there are two different types. Thus, the sets **W**, **U**, and **C** all have a cardinality of two.

The locations of the recycling centers are taken from the Ministry of Environment and Forests [20]. The conversion factors (α_{rw}) for recycling centers are generated after an interview with a recycling center operator in Ankara [21]. We learned that depending on the waste type, the recycling percent could be between 0.35 and 0.95. Then, for each recycling center r and recyclable waste w pair, a random number between these limits is generated as a conversion factor.

For generating the shortest path distances and the unique coordinate system of districts, we utilize the Geographical Information System (GIS) software, Arcview 3.2 [22].

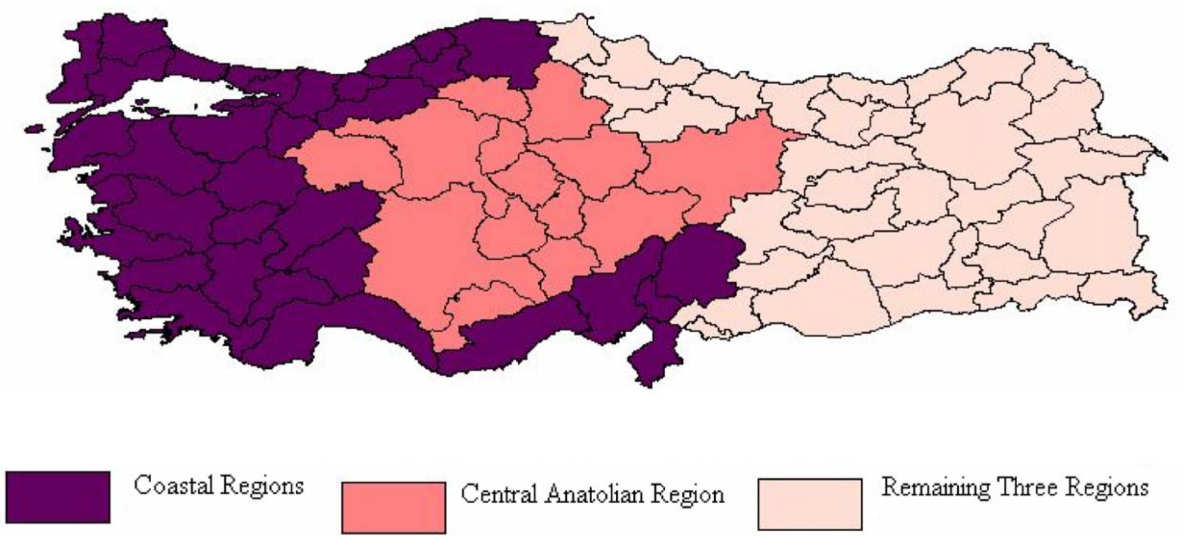

Fig. 4. Map of Turkey: Highlighting Coastal Regions and the Central Anatolian Region.

For each candidate site we have to know the wind speed and the wind direction. For some districts, this data are available from the Turkish State Meteorological Service [23]. Therefore candidate sites are determined according to their availability of meteorological data. For each candidate site, the average wind speed data of each candidate for each month between the years 1982 and 1999 are available, and we chose the smallest wind speed among these averages. The data of the prevalent wind directions of all candidates are not available from the Meteorological Service. For the unknown prevalent wind directions we considered the nearby districts with available wind direction data.

For the air pollutant type L, two main pollutants are considered throughout this study: SO_2 and NO_2. According to the legislative standards of Turkey [24] ambient air concentration of SO_2 (K_{SO_2}) at any population center is 150 ($\mu g/m^3$) and that for NO_2 is 100 ($\mu g/m^3$). The factors $(1-DR_{lw})$ used to convert the mass of hazardous waste into the mass of air pollutants are found by specifying types of hazardous waste and types of air pollutants (see [19]). In our computational analysis, we take 0.02 as the conversion factor of SO_2 and 0.13 as that of NO_2 for every waste type (these numbers are actually the conversion factors for oil; one can find the factors for many different types of waste in the stated reference [19]).

For each scenario we consider four different cases: $p = 1$, $p = 2$, $p = 3$ and $p = 4$. We remark here that the $p = 1$ case is somewhat unnecessary. A preprocessing of the candidate sites will be enough to satisfy air pollution standards. However, we included that case in our analysis in order to see the effect of the amount incinerated on constraint (15).

In order to see the effect of the Gaussian Plume constraint on the model, two different scenarios are applied: scenario I is the model without the Gaussian Plume constraint (without constraint (15)) and scenario II is the model with the Gaussian Plume constraint.

The models are solved by using CPLEX 8.1 running on a Linux server, which has 1.133 GHz speed and 256 MB memory.

4.1. Application in the Central Anatolian Region

There are 14 cities, which are divided into 183 districts in the Central Anatolian Region ($|M| = m = 183$). Thirty-seven of the 183 districts are determined as the candidate sites due to the availability of meteorological data. The number of districts with a population of more than 20,000 is 117 in the Central Anatolian Region; therefore, we have 117 districts that generate clinical waste. Due to the formulation of σ's in the Gaussian Plume equation, the candidate sites cannot be population centers. Therefore, the remaining 146 districts are considered as population centers. In the Central Anatolian region there are six districts with recycling centers. The road network, population centers and candidate sites are given in Fig. 5.

The selected sites and the required CPU times to get the solutions are given in Table 1. For $p = 1$ the selected sites in the two scenarios are completely different from each other (Fig. 6). Even though Kırşehir is the site that minimizes the total cost, meteorological conditions at that district do not satisfy the Gaussian Plume constraint. Thus, in the second scenario, another district, Kulu, which is almost 100 km away, is selected. Observe that when we open more than two incinerators, Etimesgut is chosen as an incinerator site for both scenarios.

Note that for $p = 1$, the Kırşehir district does not satisfy the Gaussian Plume constraint. However, when $p = 3$, Kırşehir is selected under scenario II. This is due to the fact that the amount of air pollutant emitted from the plant plays an important role in the Gaussian Plume equation. Since we open two more incinerators, the amount of hazardous waste disposed at Kırşehir is reduced. As a result, the concentrations

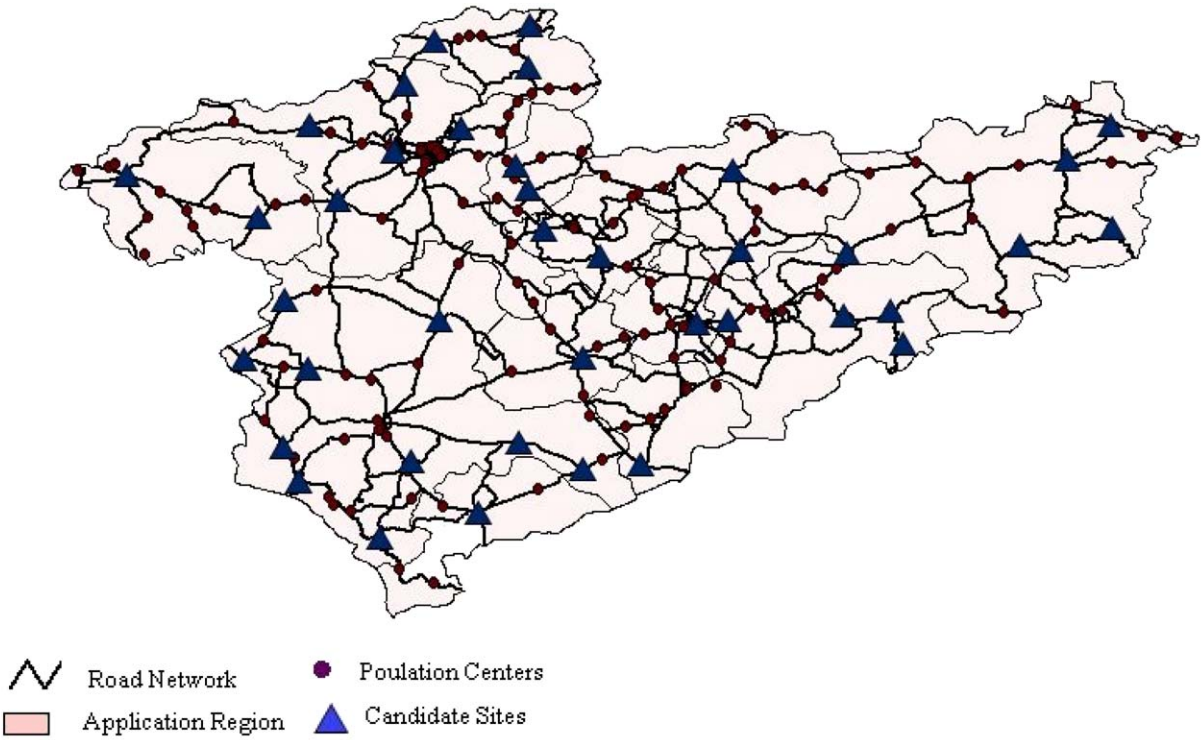

Fig. 5. The Central Anatolian Region.

Table 1
Application results for Central Anatolian Region

	Without Gaussian Plume constraints (scenario I)		With Gaussian Plume constraints (scenario II)	
	Selected site	CPU (h)	Selected site	CPU (h)
$p=1$	Kırşehir	0.26	Kulu	0.086
$p=2$	Etimesgut, Ürgüp	0.37	Etimesgut, Kulu	0.42
$p=3$	Etimesgut, Boğazlıyan, Çumra	0.57	Etimesgut, Kırşehir, Ereğli	1.27
$p=4$	Etimesgut, Sorgun, Ürgüp, Çumra	0.12	Etimesgut, Sorgun, Kırşehir, Beyşehir	0.14

of the air pollutants at population centers are changed, and Kırşehir becomes a district that satisfies air pollution standards. Figs. 7 and 8 provide the results for $p = 3$ and $p = 4$, respectively. As can be seen from the figures, each district sends its waste flows to the nearest incinerator.

When p is increased from 3 to 4, the incinerator in the southern part of the region, Ereğli, moves southwest towards Beyşehir, and the fourth incinerator mainly captures the flows of the eastern parts of the region.

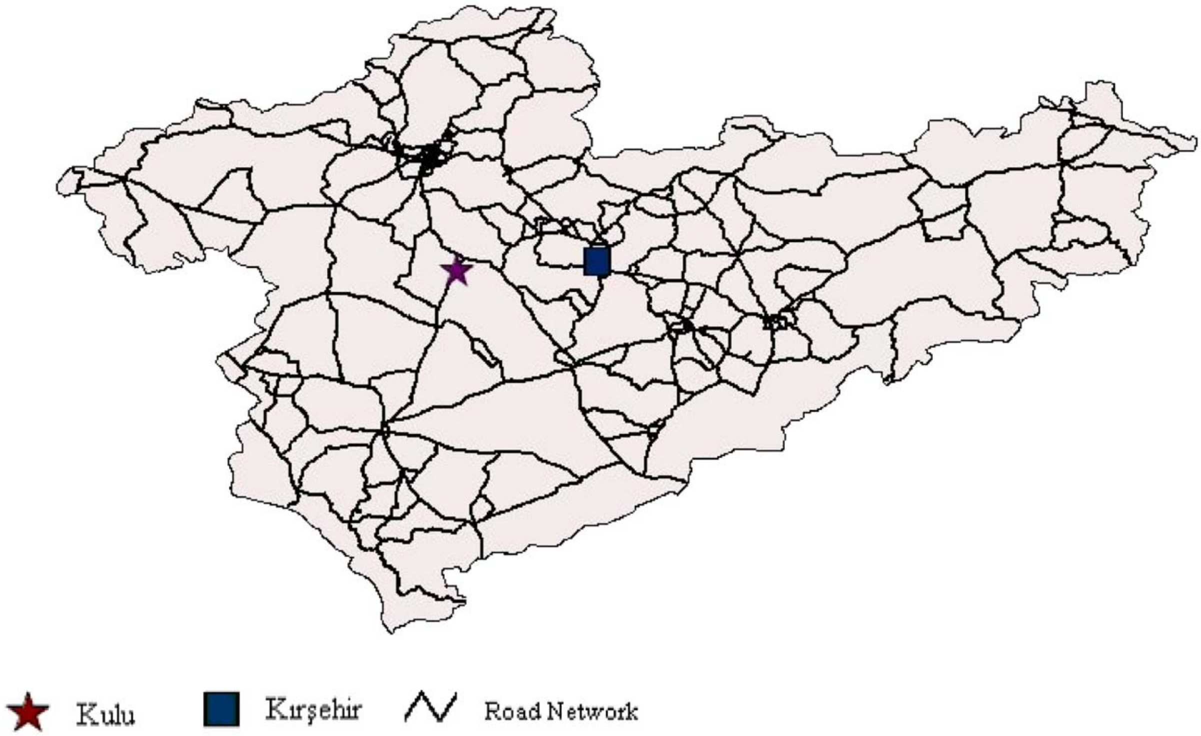

Fig. 6. Selected sites for $p = 1$ under two different scenarios.

Observe that, even the $p = 3$ case (the case taking longest CPU time) can be solved optimally in less than 2 h. This result proves that besides being realistic, our model is also efficient in terms of CPU time requirements.

4.2. Application in the four industrialized regions

For this part we consider the four most industrialized regions of Turkey. There are 47 cities, which are divided into 551 districts in this application area. Population-wise, these four regions constitute 69% of Turkey. Twenty-two of these 551 districts have recycling centers. In this application we force the incinerator locations to the three coastal regions, and within those regions the meteorological data is available for 56 districts.

However, we could not solve the model with 551 districts (leading to 551 waste generation nodes) and 56 candidate sites within a reasonable time limit (26 h). We decreased the number of districts (which determine the number of waste generation nodes and population centers) by considering the ones with a population of more than 40,000. This resulted in 166 districts. For the candidate sites, we eliminated eight districts, which have a population of less than 10,000. The candidate sites and population centers are visualized in Fig. 9.

The application results for both scenarios and for $p = 1$, 2, 3 and 4 are given in Table 2. Observe that Kocaeli is always selected and satisfies air pollution standards. This selection is quite interesting, since the only incinerator currently operating in Turkey is located in Kocaeli.

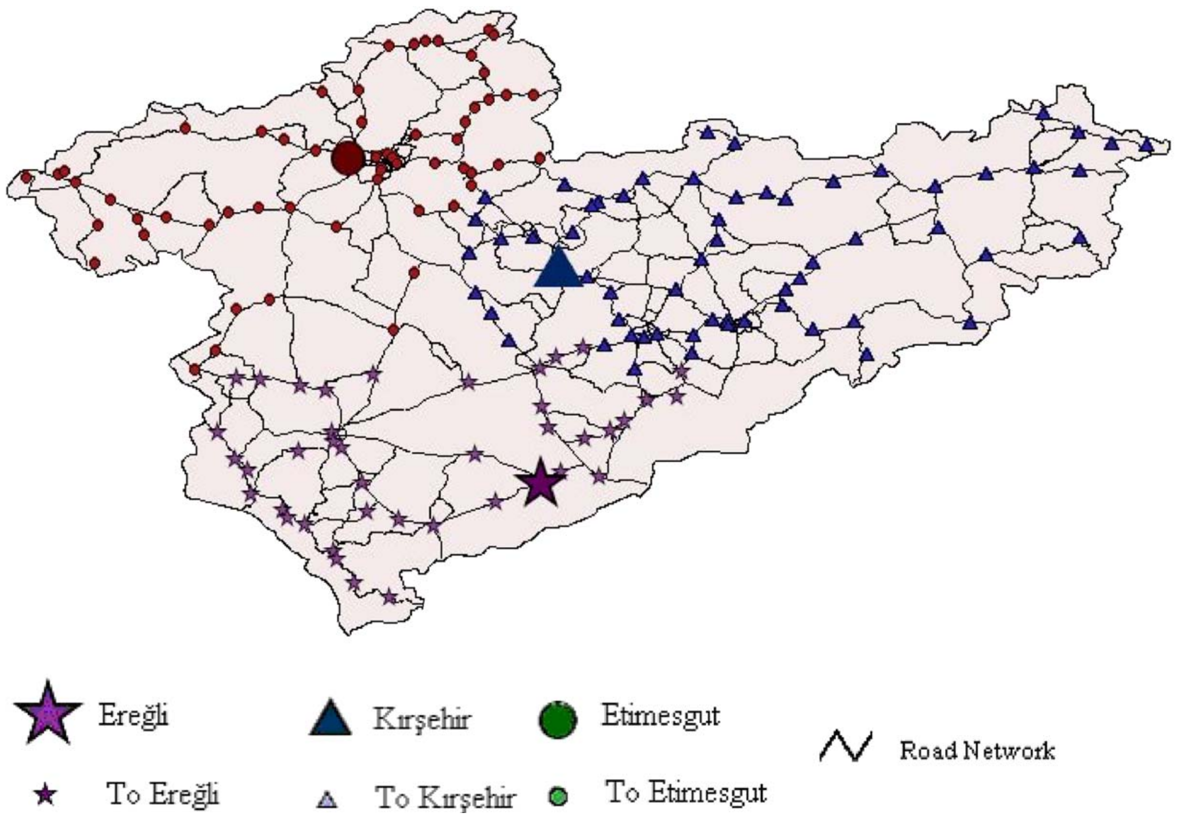

Fig. 7. Waste flows for $p = 3$ with the Gaussian Plume model.

The behavior of the CPU times seems inconsistent. As can be seen from the table, the CPU times increase gradually until $p = 3$, and then suddenly, we face a sharp decrease. Actually a similar decrease can be observed in Table 1. The only reason that we could think of for this result is that $p = 4$ is more than enough for the application areas. In both of the application areas, the $p = 3$ cases required the maximum CPU times. The CPU times in Table 2 signal that the solution time of the model is dependent on the number of candidate sites. To visualize the selected sites and the corresponding waste flow structure, the case for $p = 3$ is given in Fig. 10.

We also wanted to test the performance of our model with a different number of districts, m. We fixed the number of incinerators to be opened as 2 and the number of candidate sites as 48. For the number of districts we selected those with a population of more than 15,000 (263 districts), with a population of more than 25,000 (213 districts), and with a population of more than 40,000 (166 districts). The results are given in Table 3.

As can be seen from the table, even the $m = 263$ case can be solved in reasonable time (within 17 h). When the number of incinerators is fixed to 2, one incinerator is located near Istanbul, in the district of Kocaeli, and the other incinerator is located in the southern part of Turkey. We want to remark here that currently, Kocaeli hosts the only incinerator in Turkey.

Recall that the hazardous waste management project of the Turkish Ministry of Environment was to open one incinerator in each of the three coastal regions of Turkey. As a last application, we include a

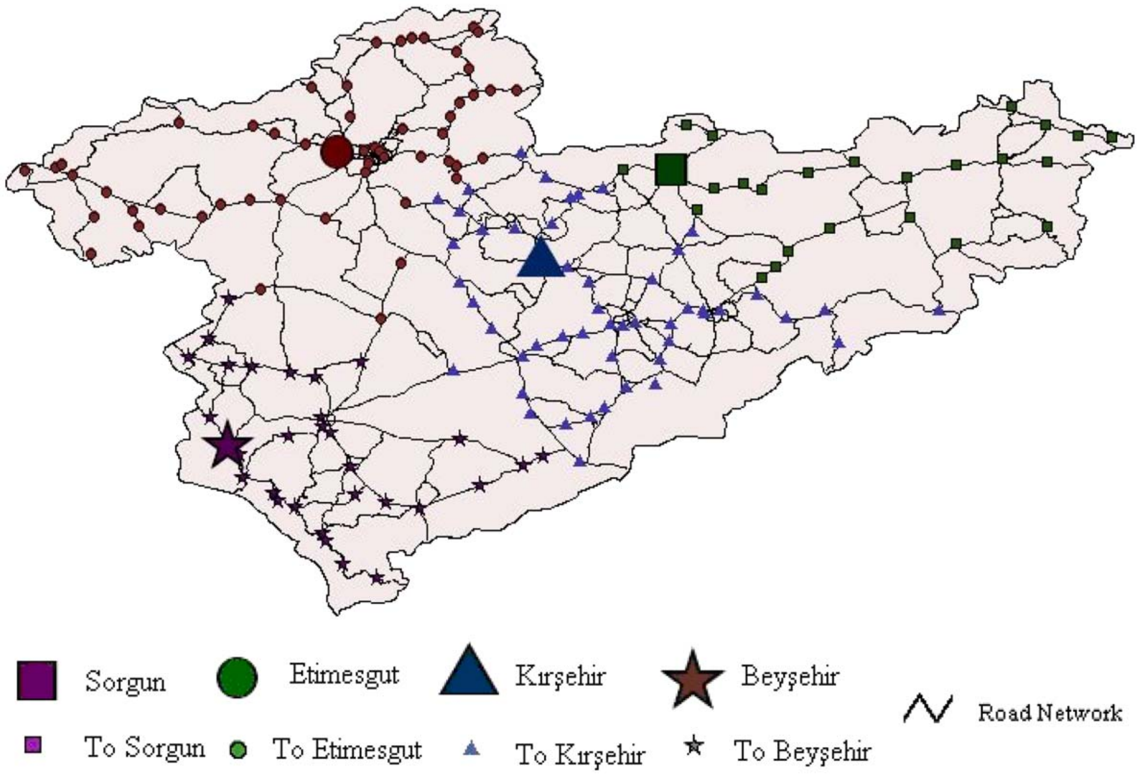

Fig. 8. Waste flows for $p = 4$ with Gaussian Plume model.

set of constraints so that every region contains one incinerator. Since there is one incinerator currently located in Kocaeli, we also fix an incinerator at that location. Note that the problem cannot be three sub-problems, one for each region, because we might have waste flows between the regions (which happened in the resulting solution given in Table 4). In this instance, since we restricted the model, we solved it with the largest data we have: 551 districts and 56 candidate sites. The results are given in Table 4.

As can be seen from the table, two of the locations would not get governmental approval, if selected under scenario I. Even though the model under scenario II requires a lot more CPU time, we believe that the extra time pays for itself, since the resulting sites will get approval automatically.

5. Conclusions and future directions

In this paper we have analyzed the hazardous waste management problem. We first observed that the method of hazardous waste disposal plays an important role in the model development phase of this problem. We then focused on incineration and analyzed the criteria specific to siting incinerators. We observed that the satisfaction of the ambient air concentration of air pollutants is the most important criterion. The air pollution can be qualified by the Gaussian Plume equation.

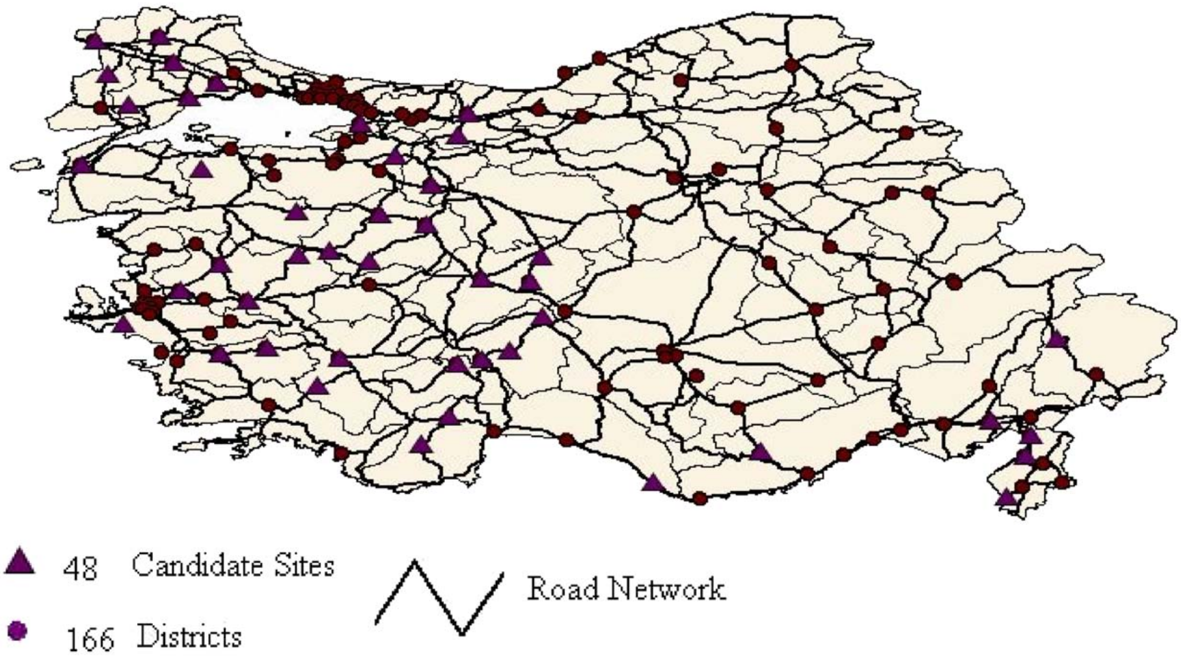

Fig. 9. The four regions with candidate sites and population centers.

Table 2
Application results for four industrialized regions

	Without Gaussian Plume constraints (scenario I)		With Gaussian Plume constraints (scenario II)	
	Selected site	CPU (h)	Selected site	CPU (h)
$p = 1$	Kocaeli	0.36	Kocaeli	0.83
$p = 2$	Kocaeli, Erdemli	4.8	Kocaeli, Kozan	8.04
$p = 3$	Kocaeli, Bornova, Erdemli	21.29	Kocaeli, Bornova, Kozan	25.63
$p = 4$	Kocaeli, Emirdağ, Bornova, Ceyhan	0.55	Kocaeli, Emirdağ, Bornova, Kozan	0.22

We developed a methodology to include the Gaussian Plume equation in our mathematical model. We then proposed a mathematical model for the hazardous waste management problem, which also includes the satisfaction of the ambient air concentrations of the air pollutants at each population center. Even though the proposed model is applicable for a fixed number of incinerators, if the fixed setup costs for incinerators are known, it can be easily modified to determine the number of required incinerators.

The model proposed in this paper has a cost-driven objective. Risk exposed to the public is considered by controlling the air pollution through rules defined by the government. However, one may also want to consider the transportation risk associated with the transportation of waste from districts to incinerator sites. The most popular transport risk measures are linear functions of distance, and our proposed model can easily be adapted to any linear risk measure, such as population exposure or traditional risk. In its

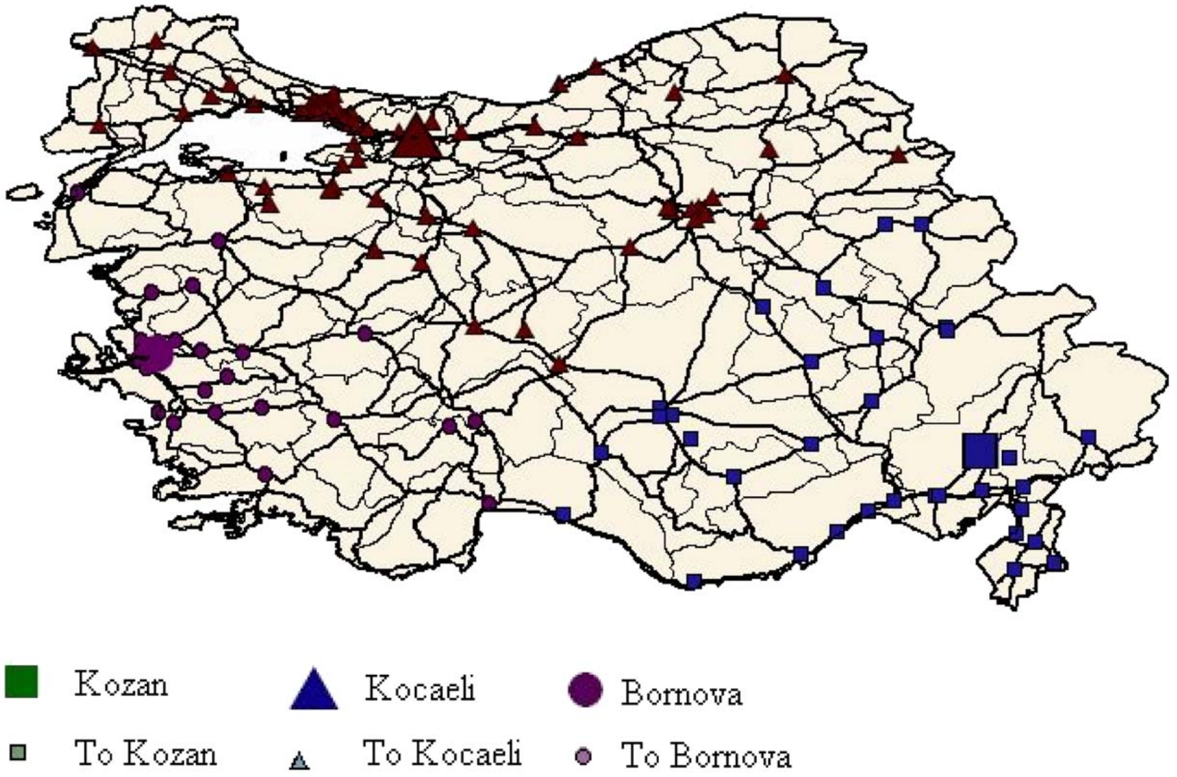

Fig. 10. Waste flows for $p = 3$ with Gaussian Plume model.

Table 3
Application results for four industrialized regions for $p = 2$

	Without Gaussian Plume constraints (scenario I)		With Gaussian Plume constraints (scenario II)	
	Selected site	CPU (h)	Selected site	CPU (h)
$m = 263$	Kocaeli, Erdemli	11.29	Kocaeli, İskenderun	16.16
$m = 213$	Kocaeli, Erdemli	8.15	Kocaeli, Kozan	11.65
$m = 166$	Kocaeli, Erdemli	4.8	Kocaeli, Kozan	8.04

Table 4
Application results for the restricted case

Without Gaussian Plume constraints (scenario I)		With Gaussian Plume constraints (scenario II)	
Selected site	CPU (h)	Selected site	CPU (h)
Bozüyük, Bornova, Erdemli	6.08	Keleş, Bornova, Karaisalı	24.7

current form, the proposed model minimizes the total distance traveled while considering public safety by controlling air pollution.

As can be seen from our computational analysis, the inclusion of the Gaussian Plume constraint in the model may change the selected site. Since our mathematical model still aims to minimize the total cost, the site selected by the model will be the location with the least cost, thus satisfying air pollution standards. We remark here that incinerator location is a strategic decision, and the data used should be accurate and should incorporate possible growth of the districts for at least 10 years.

In addition to its applicability, the proposed model is also easily solvable with commercial LP solvers like CPLEX. For example, the problem with a 183 node network was solved in less than 2 h for $p = 3$. However, we observe that the increase in the number of candidate sites increases the CPU times significantly. For such instances, one should utilize heuristic solution approaches, which is an open research area for the problem proposed in this paper.

We remark here that we did not include any preprocessing techniques in our computational analysis. As a future research direction, we plan to develop heuristics to provide bounds to CPLEX and may develop certain preprocessing strategies to speed up the CPU times. Another future direction may be to calculate upper bounds on the amount of hazardous waste that can be incinerated at each candidate site and then to decide on the locations and capacities of the incinerators.

Since our proposed model satisfies air pollution standards at population centers using the Gaussian Plume constraint, it is also applicable for the location of any facility that causes air pollution, for example, a cement plant.

We stated that the wind direction of a candidate site is not constant throughout the year. Since we aim to satisfy air pollution standards, it is sufficient to select the wind direction of a candidate site as the prevalent one. However, the frequency of each wind direction for each candidate site can be included in the model easily. The main obstacle for this inclusion is finding accurate information related to the frequencies of the wind directions for each candidate site. As future research, we plan to collect such data and to revise the model such that it includes the frequencies of wind directions.

Appendix A

Gaussian Plume equation

$$C(x, y) = \frac{QK \exp\left[-0.5(y^2/(\sigma_y)^2) - 0.5(h^2/(\sigma_z)^2)\right]}{2\Pi u \sigma_y \sigma_z}, \tag{A.1}$$

where $C(x, y)$ is the concentration of the air pollutants at the given point x, y (mg/m^3), Q the amount of air pollutant emitting from the stack (kg/h), K the scaling factor ($10^6/3600$), u the wind speed in the given region (m/s), h the stack height (m), σ_z, σ_y the dispersion factors (m), x, y the distance between the incineration plant and the given population center, but in a different coordinate system (m).

Dispersion factors (σ_z and σ_y) depend on the atmospheric stability, stack height and the value of x. There are three different types of atmospheric conditions: stable, unstable, and neutral. For the air pollutant dispersion, the worst condition is the stable condition. In this atmospheric condition, the air pollutants do not disperse within the air, but stay in concentrated amounts, which cause more damage to the public and environment. The formulas for σ_z and σ_y for the stable atmospheric condition and 150 m stack height

(150 m is the most common stack height for an incinerator) are given below

$$\sigma_y = 0.31 \times x^{0.71}, \quad \sigma_z = 0.06 \times x^{0.71}.$$

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