Int J Game Theory (2007) 36:137–146
DOI 10.1007/s00182-007-0078-1

ORIGINAL PAPER

# Social norms and choice: a weak folk theorem for repeated matching games

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Accepted: 9 March 2007 / Published online: 5 April 2007


**Abstract** A folk theorem which holds for all repeated matching games is established. The folk theorem holds any time the stage game payoffs of any two players are not affinely equivalent. The result is independent of population size and matching rule—including rules that depend on players choices or the history of play.

**Keywords** Repeated games · Matching games · Social norms · Folk theorem

**JEL Classifications** C72 · C78 · C79

## 1 Introduction

A standard economic interaction has many buyers choosing among many sellers. Hence the folk theorem, while an astonishing result, is irrelevant because it assumes a small group of players who must interact. This paper shows that the folk theorem can be applied to standard economic models if players know each others' reputation.

This paper is a contribution to a research agenda developed in Okuno-Fujiwara and Postlewaite (1995) and Kandori (1992). Okuno-Fujiwara and Postlewaite (1995) formalize the strategies used here and show that if there is a continuum of buyers and sellers, the folk theorem holds. Kandori (1992) extends these results to finite populations if the matching is random. Unfortunately choice is integral to most economic models and, as will be shown in an example, Kandori's strategy fails if even one player chooses with whom to interact, his "partner." Thus this paper establishes a folk theorem when people choose their partners.

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The equilibria will use only **local information processing**—or information players could learn from the people with whom they have interacted (Okuno-Fujiwara and Postlewaite 1995). This restriction is essential if the equilibria found here are to be applied to real matching games. If players are at geographically diffused locations then communication will be difficult. Without this restriction there would have to be a large and active central information processor dispersing information every period; such structures are both rarely observed and technically infeasible. The restriction imposed by local information processing leads to two refinements of sequential equilibrium.

Just as positing a central information processor is implausible, it is equally implausible to go to the other extreme and deny the presence of rumors, newspapers, etcetera. Thus we require information not available through local information processing to be irrelevant, in other words the strategies must be **straightforward** (Kandori 1992). The importance of this restriction is demonstrated in Hasker (2001a) which shows that even a trivial amount of central information processing can rule out trigger strategies and simple optimal penal codes.

Since the matching rule, especially with choice-driven matching, would not always be clear from local information, we must also require equilibria to be independent of the matching rule—or **universal** (Kandori 1992).[1] This requirement also includes that the strategy must work for all finite populations; a desirable and nearly trivial characteristic once the strategy works for all matching rules. Despite these requirements a folk theorem is established with the weakest sufficient condition from the standard repeated game literature (Abreu et al. 1994).

The second section of the paper presents the model. Then in Sect. 3, an example shows why Kandori's strategy does not work with choice-driven matching even though it satisfies the refinements. In that section some notation is developed that makes the folk theorem simple to prove. Section 4 contains the proof and Sect. 5 the conclusion.

## 2 The model

In a **matching game** there are $I$ **populations,** $P_i$ for all $i \in I$, each with $J$ members where $J$ and $I$ are both finite. Let $j$ denote a generic member of $P_i$. Players are matched by a **one-shot matching rule** $\mu_0 : P_i \to \times_{k \in I \backslash i} P_k$ such that the projection of $\mu_0$ on $P_k$ is one-to-one.[2] The set of $\mu_0$ is $\mathcal{M}_0$ and the set of players matched with player $j \in P_i$ is denoted $\mu(j)$. When all players are matched they play a stage game.

In the stage game, each $j \in P_i$ has a finite action set, $A_i$, and payoff function: $\pi_i : \times_{k \in I} A_k \to R$. An action profile is written $a = \{a_i, a_{-i}\}$ where $a_i \in A_i$ and $a_{-i} \in \times_{k \in I \backslash i} A_k$. Without loss of generality, players use correlated actions from the set $A = \Delta(\times_{i \in I} A_i)$. In this paper the minmax action is "not interacting," denoted $N_i$.

[1] While the terminology is new to the present paper, this refinement is used in Kandori (1992).

[2] Note that the choice of $i$ is arbitrary. Two examples of common matching rules:

a. Choose one person at a time from $P_i$, and let her select one player from those not already matched in each $P_k$ $k \in I \backslash i$. This is a model of employers hiring employees.

b. Let each player from $P_k$ $k \in I \backslash i$ choose a member of $P_i$ who has not already been chosen by someone else in $P_k$. This is a model of customers choosing a supplier.

We assume that for all $a_{-i} \in \times_{k \in I \setminus i} A_k$ $\pi_i(N_i, a_{-i}) = 0$ and that if $N_{-i} = \{N_k\}_{k \in I \setminus i}$, then for all $a_i \in A_i$, $\pi_i(a_i, N_{-i}) = 0$.

In a **repeated matching game** the matching game happens ad infinitum. A **path** is a sequence of action profiles $w = \{a^t\}_{t=1}^{\infty}$, $a^t \in A$. A player's payoff from such a path is her **value**: $v_i : A^{\infty} \to R$, $v_i(w) = \Sigma_{t=1}^{\infty} \delta^{t-1} u_i(a^t)$ with $\delta \in [0, 1)$. A **matching rule**, $\mu$, is a sequence of one-shot matching rules. If it is independent of history, we call it a **random matching game**. Such a sequence is a list $\mu = \{\mu^t\}_{t=1}^{\infty}$ where $\mu^t \in \Delta(\mathcal{M}_0)$. If it depends on history, it is a **repeated matching game**. In this case, let $H_t$ be the set of possible histories of the entire game up to period $t$, then $\mu = \{\mu^t\}_{t=1}^{\infty}$ $\mu^t : H_t \to \Delta(\mathcal{M}_0)$.

The strategies we analyze are called **social norms**; a social norm is a tuple $\beta = \{Z, \sigma, \tau\}$ where:

1. $Z \equiv \times_{i \in I} Z_i$ where $Z_i$ is a finite set, the elements of $Z$ denote the **social statuses** of players in $P_i$;
2. $\sigma \equiv \{\sigma_i\}_{i \in I}$ where $\sigma_i : Z \to A_i$ determines the behavior of a player $j \in P_i$ as a function of the social status of the players with whom he is matched, and is called the **social standard of behavior**;
3. $\tau \equiv \{\tau_i\}_{i \in I}$ where $\tau_i : A_i \times A_i \times Z_i \times Z \to Z_i$ is a **transition rule** that determines $j$'s social status in period $t$. It is a composition of an **internal rule** ($\tau_i^n$) and an **external rule** ($\tau_i^x$) where;

(a) $\tau_i^n : A_i \times A_i \times Z_i \to Z_i$. The first element is the action person $j \in P_i$ took last period (denoted $a_j^{t-1}$), the second element is the action they should have taken given the outcome of the correlating device (denoted $\sigma_j^{t-1}$). The third is $j$'s social status last period ($z_j^{t-1}$). The output is the the **interim social status** of $j \in P_i$, denoted $\zeta_j^t \in Z_i$.

(b) $\tau_i^x : Z \to Z_i$. This is a funciton of $\{\zeta_l^t\}_{l \in j \cup \mu(j)} \in Z$—the interim social statuses of the people that have been matched this period. This part of the rule resolves any possible conflicts between their social statuses. In practice it will "forgive" people who ahve deviated earlier in time.

This transition rule restricts the strategy to **local information processing** since it is only affected by information about $j$ and the people with whom $j$ is matched.

We will look for sequential equilibria that satisfy the two refinements mentioned in the introduction: straightforwardness and universality. Since sequential equilibria were originally defined for finite action games, this paper uses Abreu et al. (1990) extension to repeated games. They define a strategy as a complete contingent plan for future behavior; this is given by $\beta$ in our notation. However $Z$ and $\tau$ are not under the control of players; thus what we check is if $\sigma$ is a best response given appropriate beliefs. The appropriate beliefs are any that are straightforward. Straightforwardness essentially requires that the strategy is a best response for all feasible beliefs. Denote $h_t^j$ as the part of the history of the game that $j \in P_i$ has directly observed, and let $\iota_t^j$ be her information set in period $t$—with the restrictions that $h_t^j \cup \{z_j^t, z_{\mu(j)}^t\} \subseteq \iota_t^j$ and $\iota_{t-1}^j \subseteq \iota_t^j$.

**Definition 1** An equilibrium is **straightforward** if for any $j \in P_i$, $\sigma_i\left(z_j^t, z_{\mu(j)}^t\right)$ is always a best response given any beliefs consistent with any feasible $\iota_t^j$ and given that all other players use the strategy $\beta$.

The constraints on players' information—$h_t^j \cup \left\{z_j^t, z_{\mu(j)}^t\right\} \subseteq \iota_t^j$ and $\iota_{t-1}^j \subseteq \iota_t^j$—are included only because this paper does not address the issue of forgetting. It is conjectured that the equilibria in this paper require only that a player knows $\left\{z_j^t, z_{\mu(j)}^t\right\}$. Notice as well that $\iota_t^j$ can include $h_t$ and the matching rule (full information) and a player can believe that all other players also have full information.

The second refinement is universality.

**Definition 2** An equilibrium is **universal** if it is an equilibrium for all matching rules and population sizes.

When we refer to an **equilibrium**, we mean a sequential equilibrium which is straightforward and universal. Notice that the beliefs required for a sequential equilibrium are given by the straightforward refinement. A **static Nash equilibrium** is an equilibrium of the repeated matching game when the discount factor ($\delta$) is zero.

## 3 Illustrating the impact of choice

In this section we present an example of a social norm that works if matching is random but fails when matching is driven by players' preferences (or is history dependent). This example not only demonstrates the impact of choice-driven matching, but it also enables the development and explanation of some notation that will simplify the proof of the folk theorem. The stage game is the standard Prisoner's Dilemma with the option of not interacting explicitly included.

| 1 \ 2 | $C_2$ | $D_2$ | $N_2$ |
|---|---|---|---|
| $C_1$ | 2, 2 | −2, 4 | 0, 0 |
| $D_1$ | 4, −2 | 0, 0 | 0, 0 |
| $N_1$ | 0, 0 | 0, 0 | 0, 0 |

(1)

The social norm we use is the strategy used in Kandori (1992) to prove the folk theorem with random matching. Players have three possible social statuses, $Z_1 = Z_2 = \{0, 1, 2\}$; the transition rule is:

$$z_j^t = \tau\left(a_j^{t-1}, \sigma_j^{t-1}, z_j^{t-1}\right) = \begin{cases} 2 & \text{if } a_j^{t-1} \neq \sigma_j^{t-1} \\ z_j^{t-1} - 1 & \text{if } a_j^{t-1} = \sigma_j^{t-1} \text{ and } z_j^{t-1} > 0 \\ 0 & \text{else} \end{cases} \tag{2}$$

and the social standard of behavior is:

$$\sigma\left(z_1^t, z_2^t\right) = \begin{cases} \{C_1, C_2\} \text{ if } z_1^t = 0, z_2^t = 0 \\ \{C_1, D_2\} \text{ if } z_1^t > 0, z_2^t = 0 \\ \{D_1, C_2\} \text{ if } z_1^t = 0, z_2^t > 0 \\ \{D_1, D_2\} \text{ if } z_1^t > 0, z_2^t > 0 \end{cases}. \tag{3}$$

Notice the asymmetry of the strategy when a deviating player meets with a non-deviator. Players who have recently deviated "repent" by playing $C_i$ while their opponent plays $D_{-i}$. In a random matching environment, this means that a non-deviator always gets at least the payoff from $\{C_i, C_{-i}\}$; thus non-deviators have an incentive to cooperate.

However this strategy also makes players who have deviated desirable partners since $\pi_i(D_i, C_{-i}) > \pi_i(C_i, C_{-i})$. With choice-driven matching, this can cause deviators to deviate again. To illustrate this, we need to first describe a matching rule. The matching rule will reward players who have not deviated recently by giving them first choice of partner. This means that players who have not deviated will be matched with players who have deviated recently since they will get the payoff of $\pi_i(D_i, C_{-i})$. When the players are indifferent we assume first they prefer to be matched with someone who has deviated fewer times in the past, and then we assume that they prefer to interact with their partner from the last period.

To formally specify this matching rule, first define $B_j^t$ as the number of times player $j \in P_i$ has deviated in the past. Let $z_{j*}^t = \min_{l \in P_1 \times P_2} z_l^t$, and assume that $j^*$ was randomly chosen to be matched first. Then $j^*$ will be matched with someone with $z_l^t \geq 1$ if available. If there is either no one or more than one person who satisfies this criterion then the tie-breaking rule is that the player will be matched with someone who has the least $B_l^t$. If there is more than one person who satisfies all criteria above, $j^*$ will first be matched with whomever she was matched with last period and otherwise use randomization. This process is repeated among all players who have not been matched until there is no one left to be matched.

I now explain how this matching rule maximizes the utilities of players who have recently deviated less often. Having players with $z_l^t \geq 1$ be matched with players who have $z_{j*}^t = 0$ clearly maximizes $j^*$'s utility as explained above. Here I illustrate beliefs that generate the tie-breaking rules. The first tie-breaking rule is that players prefer to be matched with players who have deviated fewer times. If players believe that the probability a given player $l$ will deviate again is $\lim_{\varepsilon \to 0} \varepsilon^{(B_l^t+1)^{-1}}$, then they will have these preferences. Given this, the relative likelihood of player $m$ deviating again to player $l$ deviating again is:

$$\lim_{\varepsilon \to 0} \varepsilon^{(B_l^t - B_m^t)(B_m^t+1)^{-1}(B_l^t+1)^{-1}} \in \{0, 1, \infty\} \tag{4}$$

and if $B_l^t < B_m^t$, then $m$ is infinitely more likely to deviate again than $l$, and therefore $l$ is a more desirable player to be matched with. The second tie-breaking rule is that players prefer to be matched with the same person again. These beliefs could easily be justified by having a switching cost of $\rho > 0$, then driving $\rho$ to zero quickly enough that $\frac{\rho}{\varepsilon} \to 0$.

On a priori grounds this seems a reasonable matching rule as it is designed to reward the 'less guilty.' However a problem arises because one implication of straightforwardness is that it is possible for players to always know what subgame[3] they are in. Thus they know when the tie-breaking rules will be invoked, and in a simple subgame, this matching rule actually gives players an incentive to deviate.

To show this, let $P_1$ and $P_2$ have two members; and consider the subgame where three people deviated last period and there have been no previous deviations. Assume both players in $P_1$ deviated and consider the player in $P_1$ matched with the player in $P_2$ who did not deviate. If this player cooperates today, her continuation value is:

$$\delta \left( \pi_1 (C_1, D_2) + \delta \pi_1 (C_1, C_2) + \frac{\delta^2}{1-\delta} \pi_1 (C_1, C_2) \right) . \tag{5}$$

If she deviates today, then she will not be matched with the one non-deviator, and her continuation value will be:

$$\delta \left( \pi_1 (D_1, D_2) + \delta \pi_1 (C_1, D_2) + \frac{\delta^2}{1-\delta} \pi_1 (C_1, C_2) \right) . \tag{6}$$

This means that cooperating only increases her continuation value by $\delta(-2 + \delta 4)$. The incentive to deviate in the current period is $2 = \pi_1 (D_1, D_2) - \pi_1 (C_1, D_2)$ , and since $\delta(-2 + \delta 4) < 2$, deviating is the best response. Thus this strategy is not an equilibrium. Notice that this counter-example did not actually require much choice. In the proper subgame it only requires that one person choose her partner to generate this contradiction.

How then to find an equilibrium? Given that each new matching rule can give different incentives, and the set of possible matching rules expands with the population size, when can one be sure that enough cases have been checked? In fact it is only necessary to check the worst case, where the incentive to deviate is highest. The player who cooperates in this case always will. To find this case, one finds two partial matching rules: one which gives the least possible value when a player cooperates $(a_j^t = \sigma_j^t)$ and another which gives the highest possible value when a player deviates $(a_j^t \neq \sigma_j^t)$.

These worst and best partial matching rules are equivalent to worst and best paths, and to find these, we need to define the set of possible future paths of play:

**Definition 3** Define $W_i \left( a_j^t, \sigma_j^t, z_j^t \right)$ as the set of possible future paths given $\left\{ a_j^t, \sigma_j^t, z_j^t \right\}$ where $j \in P_i$. Let $C_\beta \left( a_j^t, \sigma_j^t, z_j^t \right) \subseteq A^\infty$ be the cylinder of future action sequences consistent with all players using the social norm $\beta$ given a finite population and any initial distribution of social statuses. Then $W_i \left( a_j^t, \sigma_j^t, z_j^t \right) = C_\beta \left( a_j^t, \sigma_j^t, z_j^t \right)$.

[3] Here "subgame" refers to a subgame of the repeated matching game with full information.

Notice that for an arbitrary social norm these sets might be quite complex—in fact they can be open—but for the social norms used in this paper these sets will be closed and have a simple structure. When these sets are not too complex, there will be a finite population size that supports all of the possible paths; and the matching rule is the one that gives the least incentive in each period for a player to cooperate. For example with Kandori's strategy these sets are:

$$\begin{array}{lll} \text{if } a_j^t = \sigma_j^t \;\; z_j^t \leq 1 & W_1\left(\sigma_j^t, \sigma_j^t, z_j^t\right) = \left\{w^1, w^2\right\} \\ \text{if } a_j^t = \sigma_j^t \;\; z_j^t = 2 & W_1\left(\sigma_j^t, \sigma_j^t, 2\right) = \left\{w^3, w^4\right\} \\ \text{if } a_j^t \neq \sigma_j^t & W_1\left(a_j^t, \sigma_j^t, z_j^t\right) = \left\{w^5, w^6\right\} \end{array} \tag{7}$$

where:

$$\begin{aligned} w^1 &= \left\{\sigma^{t+1} = \{C_1, C_2\}, \sigma^{t+s} = \{C_1, C_2\} \text{ for } s \geq 2\right\} \\ w^2 &= \left\{\sigma^{t+1} = \{D_1, C_2\}, \sigma^{t+s} = \{C_1, C_2\} \text{ for } s \geq 2\right\} \\ w^3 &= \left\{\sigma^{t+1} = \{C_1, D_2\}, \sigma^{t+s} = \{C_1, C_2\} \text{ for } s \geq 2\right\} \\ w^4 &= \left\{\sigma^{t+1} = \{D_1, D_2\}, \sigma^{t+s} = \{C_1, C_2\} \text{ for } s \geq 2\right\} \\ w^5 &= \left\{\sigma^{t+1} = \{C_1, D_2\}, \sigma^{t+2} = \{C_1, D_2\}, \sigma^{t+s} = \{C_1, C_2\} \text{ for } s \geq 3\right\} \\ w^6 &= \left\{\sigma^{t+1} = \{D_1, D_2\}, \sigma^{t+2} = \{C_1, D_2\}, \sigma^{t+s} = \{C_1, C_2\} \text{ for } s \geq 3\right\}. \end{aligned} \tag{8}$$

One can achieve any pair of paths in these sets if there are two players in each population. The value in equation 6 is $\delta\bar{v}_1\left(a_j^t, \sigma_j^t, 2\right) \equiv \max_{w \in W_1\left(a_j^t, \sigma_j^t, 2\right)} \delta v_1(w) = \delta v_1\left(w^6\right)$ and the value in equation 5 is $\delta\underline{v}_1\left(\sigma_j^t, \sigma_j^t, 2\right) \equiv \min_{w \in W_1\left(\sigma_j^t, \sigma_j^t, 2\right)} \delta v_1(w) = \delta v_1\left(w^3\right)$; thus all I have done in this example is seen if $\delta\left(v_1\left(w^3\right) - v_1\left(w^6\right)\right)$ is sufficient to ensure cooperation. Once I found it was not then it was a simple matter to backtrack and find a matching rule that supports these payoffs.

The reader can also see the difference between random matching and choice-driven matching using this example. Under random matching a player expecting the path $w^3$ will receive the path $w^5$ if she deviates, and $\delta\left(v_1\left(w^3\right) - v_1\left(w^5\right)\right) = \delta^2 4$ is sufficient when $\delta \geq 0.71$. On the other hand, with choice-driven matching it is easier to check if a strategy is an equilibrium. Under random matching, for each status profile the analyst must check two possible continuation paths—or more for more complicated strategies. Under choice-driven matching the analyst only checks one.

Clearly checking $\delta\left(\underline{v}_i\left(\sigma_j^t, \sigma_j^t, z_j^t\right) - \bar{v}_i\left(a_j^t, \sigma_j^t, z_j^t\right)\right)$ is high enough to ensure cooperation is always sufficient to prove a strategy is an equilibrium. In a working paper the author shows that when $W_i\left(a_j^t, \sigma_j^t, z_j^t\right)$ is closed and can be supported by a finite population then it is also necessary. This is what will be checked to prove the folk theorem.

## 4 A folk theorem allowing for choice

Only one modification of Kandori's social norm is required to make it an equilibrium with choice-driven matching. If the social status found above was an interim social status—$\zeta_j^t$—and it was further required that only the latest deviators in any interaction be punished (by an external transition rule) then this would be an equilibrium. However this would still leave us with a folk theorem that works only for interactions where punishments hurt one person without hurting others; excluding common and intuitive punishments like not interacting. Quite surprisingly a simple modification of the strategy in Fudenberg and Maskin (1986) using technics from Abreu et al. (1994) proves a folk theorem with the weakest sufficient condition in the folk theorem literature:

**Definition 4** A stage game satisfies **non-equivalent utilities** (NEU) if for every $i$ and $k \neq i$ ($\{i, k\} \subseteq I$) there is no $\alpha \geq 0$ and $\beta \in R$ such that for all $a \in A$ $\pi_i(a) = \alpha\pi_k(a) + \beta$.

Abreu et al. (1994) show that this is equivalent to the existence of asymmetric payoff points. These are action profiles $\{b^i\}_{i\in I}$ such that $\pi_i(b^i) < \pi_i(b^k)$ for all $i$ and $k \in I\backslash i$.

Using these payoffs we will support constant initial paths, specifically any $a^0$ such that $\pi_i(a^0) > 0$ for all $i$. Given $\{b^i\}_{i\in I}$ and $a^0$ we can construct $\{a^i\}_{i\in I}$ such that $i$ gets a lower payoff from the action profile $a^i$ than $a^0$ or $a^k$ for $k \in I\backslash i$. This is done by using the public randomizing device to correlate over $a^0$, $b^i$, and the worst payoff of population $i$ as is shown in Abreu et al. (1994).

Given the initial profile $a^0$ and $\{a^i\}_{i\in I}$ define:

$$a\left(z_j^t, z_{\mu(j)}^t\right) = \begin{cases} a^0 \text{ if } \forall l \in P_k \cap (j \cup \mu(j))\ z_l^t = 0 \\ a^k \text{ if } \exists l \in P_k \cap (j \cup \mu(j))\ z_l^t > 0 \end{cases} \tag{9}$$

the social standard behavior is:

$$\sigma\left(z_j^t, z_{\mu(j)}^t\right) = \begin{cases} \{N_i\}_{i\in I} & \text{if } \exists l \in P_k \cap (j \cup \mu(j))\ z_k^t > 1 \\ a\left(z_j^t, z_{\mu(j)}^t\right) & \text{if } \forall l \in P_k \cap (j \cup \mu(j))\ z_k^t \leq 1 \end{cases}. \tag{10}$$

The transition rule is a two step function $\tau = \{\tau^n, \tau^x\}$. The internal updating rule is:

$$\zeta_j^t = \tau_i^n\left(a_j^{t-1}, \sigma_j^{t-1}, z_j^{t-1}\right) = \begin{cases} T+1 & \text{if } a_j^{t-1} \neq \sigma_j^{t-1} \\ z_j^{t-1} - 1 & \text{if } a_j^{t-1} = \sigma_j^{t-1} \text{ and } z_j^{t-1} > 1 \\ z_j^{t-1} & \text{else} \end{cases} \tag{11}$$

and the external updating rule is:

$$z_j^t = \tau_i^x = \begin{cases} 0 & \exists l \in \mu(j)\ \zeta_j^t \leq \zeta_l^t \\ \zeta_j^t & \forall l \in \mu(j)\ \zeta_j^t > \zeta_l^t \end{cases}. \tag{12}$$

In this strategy a deviator faces a two tiered punishment. First she is minmaxed for $T$ periods and then always plays $a^i$. The critical difference between this and Kandori's strategy is that players are "forgiven" ($z_j^t = 0$) if they interact with someone who deviated in a later period. Define $A^+ = \{a | a \in A, \forall i, \pi_i(a) > 0\}$.

**Theorem 5** *If the stage game satisfies the NEU conditions, then as* $\delta \to 1$, *every* $a^0 \in A^+$ *can be supported as the equilibrium initial path of a social norm.*

*Proof* There is only one path if a player deviates, therefore $\overline{v}_i\left(a_j^t, \sigma_j^t, z_j^t\right) = \frac{\delta^{T-1}}{1-\delta}\pi_i\left(a^i\right)$ for all $z_j^t$ and $a_j^t \neq \sigma_j^t$. Define $\underline{a}^{-i} \in \arg\min_{k \in I \setminus i} \pi_i\left(a^k\right)$; then if $z_j^t = 0$ we have $\underline{v}_i\left(\sigma_j^t, \sigma_j^t, 0\right) = \frac{\delta^{T-2}}{1-\delta}\pi_i\left(\underline{a}^{-i}\right)$. If $z_j^t = 1$, the worst possible future depends on the discount factor; it is either $\underline{v}_i\left(\sigma_j^t, \sigma_j^t, 0\right)$ or $\frac{1}{1-\delta}\pi_i\left(a^i\right)$ but for sufficiently high $\delta$

$$\underline{v}_i\left(\sigma_j^t, \sigma_j^t, 1\right) = \frac{1}{1-\delta}\pi_i\left(a^i\right) < \frac{\delta^{T-2}}{1-\delta}\pi_i\left(\underline{a}^{-i}\right).$$

Likewise if $z_j^t = \hat{z}_j^t > 1$ the worst future depends on the discount factor; but for sufficiently high $\delta$ $\underline{v}_i\left(\sigma_j^t, \sigma_j^t, \hat{z}_j^t\right) = \frac{\delta^{\hat{z}_j^t - 2}}{1-\delta}\pi_i\left(a^i\right)$.

Now $\lim_{T\to\infty}\lim_{\delta\to 1}\underline{v}_i\left(\sigma_j^t, \sigma_j^t, 1\right) - \overline{v}_i\left(a_j^t, \sigma_j^t, 1\right) = \infty$ thus choose $T$ so that a persons with $z_j^t = 1$ will cooperate. Since $\lim_{\delta\to 1}\underline{v}_i\left(\sigma_j^t, \sigma_j^t, 0\right) - \overline{v}_i\left(a_j^t, \sigma_j^t, 0\right) = \infty$ if $z_j^t = 0$ the player will cooperate for sufficiently high $\delta$. And since $\{N_i\}_{i\in I}$ is a static Nash equilibrium and for $\hat{z}_j^t > 1$ $\underline{v}_i\left(\sigma_j^t, \sigma_j^t, \hat{z}_j^t\right) \geq \overline{v}_i\left(a_j^t, \sigma_j^t, \hat{z}_j^t\right)$ if $z_j^t = \hat{z}_j^t$ the player will cooperate. □

If players do not have the option of not interacting, this folk theorem can easily be extended if the minmax is in pure strategies or if mixed strategies are observable. As well the correlating device can be removed like in Fudenberg and Maskin (1991), provided that all cycles depend on the period and not the time since deviation.

## 5 Conclusion

We can now see that the folk theorem does apply to ordinary market interactions, the type that are standard in economic analysis. We understand how Consumers and Suppliers, Laborers and Employers, and people involved in other standard competitive matching games can reap both the benefits and the costs of the folk theorem. Furthermore this folk theorem has simple existence conditions, compensating for the complex nature of these interactions. The results depend only on the payoffs of the stage game and the frequency of interaction.

There are two potential research extensions. The author has addressed one of these in a working paper that deals with how to remove the assumption that players know each other's reputations. In Hasker (2001b) it is shown that this can be replaced with formalized references; analyzing what can be done with letters and other verifiable messages make it possible to remove this assumption. A second desirable extension would be the study of accommodating heterogeneity—since this is a primary motivation for matching. Clearly a reasonable amount of heterogeneity would be easily overcome, and the author conjectures that even significant heterogeneity, as in Dutta (1995), could be overcome if the social norm is independent of this heterogeneity. Dealing with this more difficult case would be an interesting topic for future research.

Even in these more complex environments, social norms are simple to follow. A concern of many theorists is the compositional complexity of equilibria. Social norms are simple; as Okuno-Fujiwara and Postlewaite (1995) point out, it is easier to follow a social norm than calculate best responses in many situations. No computation is required to determine the optimal choice, one simply follows the rules. Players can blithely live their lives knowing that whatever happens they should just do as they are told.

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