

Wireless Networks

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## Distributed Joint Flow-Radio and Channel Assignment Using Partially Overlapping Channels in Multi-Radio Wireless Mesh Networks

Online Resource 1  
Supplementary Algorithms for DFRCA

Alper Rifat Ulucinar · Ibrahim Korpeoglu

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Alper Rifat Ulucinar  
Department of Computer Engineering, Bilkent University, 06800, Ankara, Turkey  
Tel.: +90-312-2902599  
Fax: +90-312-2664047  
E-mail: ulucinar@cs.bilkent.edu.tr

Ibrahim Korpeoglu  
Department of Computer Engineering, Bilkent University, 06800, Ankara, Turkey  
E-mail: korpe@cs.bilkent.edu.tr

## Appendix A

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**Algorithm A.1** Flow Balancer on  $n_i$

**Input:**  $C$

**Output:**  $C$

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```

1: procedure FLOWBALANCER( $C$ )
2:
    $\Omega \leftarrow \left\{ (i, k) : k \in [1, D] \wedge \sum_{\forall (j,l): f_{i,j,k,l} \in F} |f_{i,j,k,l}| + \sum_{\forall (j,l): f_{j,i,l,k} \in F} |f_{j,i,l,k}| > \rho_{max} \right\}$ 
    $\triangleright \Omega$  is the set of overflown radios
3:
    $\Upsilon \leftarrow \left\{ (i, k) : k \in [1, D] \wedge \sum_{\forall (j,l): f_{i,j,k,l} \in F} |f_{i,j,k,l}| + \sum_{\forall (j,l): f_{j,i,l,k} \in F} |f_{j,i,l,k}| < \rho_{max} \right\}$ 
    $\triangleright \Upsilon$  is the set of underflown radios, i.e. radios with positive residual capacity
4: for all  $(i, k) \in \Omega$  do
5:    $E_k \leftarrow \sum_{\forall (j,l): f_{i,j,k,l} \in F} |f_{i,j,k,l}| + \sum_{\forall (j,l): f_{j,i,l,k} \in F} |f_{j,i,l,k}| - \rho_{max}$ 
6:   for all  $f_{R_k} : f_{i,j,k,l} \in F \vee f_{j,i,l,k} \in F$  do  $\triangleright$  For each flow on  $(i, k)$ 
7:     for all  $(i, k') \in \Upsilon$  do
8:       for all  $f_{R_{k'}} : f_{i,j,k',l} \in F \vee f_{j,i,l,k'} \in F$  do  $\triangleright$  For each flow on  $(i, k')$ 
9:          $\delta \leftarrow |f_{R_k}| - |f_{R_{k'}}|$ 
10:         $\Delta_{R_{k'}} \leftarrow \rho_{max} - \sum_{\forall (j,l): f_{i,j,k',l} \in F} |f_{i,j,k',l}| - \sum_{\forall (j,l): f_{j,i,l,k'} \in F} |f_{j,i,l,k'}|$ 
11:        if  $\delta > 0 \wedge E_k - \delta \geq 0 \wedge \Delta_{R_{k'}} - \delta \geq 0$  then
12:           $E_k \leftarrow E_k - \delta$ 
13:           $C[f_{R_{k'}}] \leftarrow k$ 
14:           $C[f_{R_k}] \leftarrow k'$ 
15:        end if
16:      end for
17:    end for
18:  end for
19:  if  $E_k > 0$  then
20:    exit  $\triangleright$  Heuristic failed to balance
21:  end if
22: end for
23: end procedure

```

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**Algorithm A.2** Computation of the Set of  $k$ -neighborhood Subgraphs on  $n_i$ 
**Input:**  $(N_k, F_k)$ **Output:**  $\Psi$ , the set of subgraphs

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```

1: procedure FINDSUBGRAPHS( $(N_k, F_k)$ )
2:    $\Psi \leftarrow \emptyset$  ▷  $\Psi$  is the set of subgraphs. The number of subgraphs will be  $|\Psi|$ 
3:   for all  $f_{i',j',k',l'} \in F_k$  do
4:     if  $i' \notin H_1 \wedge i' \notin H_k \wedge i' \neq i$  then
5:       continue
6:     end if
7:      $R_t \leftarrow (i', k')$ 
8:      $R_r \leftarrow (j', l')$ 
9:      $R_o \leftarrow (-1, -1)$ 
10:    for all  $\psi_1 \in \Psi$  do
11:      if  $R_t \in \psi_1$  then
12:         $R_o \leftarrow R_r$ 
13:      else if  $R_r \in \psi_1$  then
14:         $R_o \leftarrow R_t$ 
15:      end if
16:      if  $R_o \neq (-1, -1)$  then
17:         $\psi_1 \leftarrow \psi_1 \cup \{R_t, R_r\}$ 
18:        for all  $\psi_2 \in \Psi$  do
19:          if  $\psi_2 \neq \psi_1 \wedge R_o \in \psi_2$  then
20:             $\psi_1 \leftarrow \psi_1 \cup \psi_2$ 
21:             $\Psi \leftarrow \Psi \setminus \{\psi_2\}$ 
22:          end if
23:        end for
24:      end if
25:    end for
26:    if  $R_o = (-1, -1)$  then ▷ Then no other subgraph contains  $f_{i',j',k',l'}$ 
27:       $\psi_{new} \leftarrow \{R_t, R_r\}$ 
28:       $\Psi \leftarrow \Psi \cup \{\psi_{new}\}$ 
29:    end if
30:  end for
31: end procedure

```

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**Algorithm A.3** Computation of the  $k$ -neighborhood Conflict Graph on  $n_i$ 
**Input:**  $\Psi$ **Output:**  $G_c(\Psi, E)$ , the conflict graph of the  $k$ -neighborhood subgraphs

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```

1: procedure FINDCONFLICTGRAPH( $\Psi$ )
2:    $E \leftarrow \emptyset$  ▷ The edge set of  $G_c$ 
3:   for all  $\psi_1 \in \Psi$  do
4:     for all  $(i, k) \in \psi_1$  do
5:       for all  $f_{i,j,k,l} \in F$  do
6:         for all  $\psi_2 \in (\Psi \setminus \{\psi_1\})$  do
7:           for all  $(j', l') \in \psi_2$  do
8:             if  $(\psi_1, \psi_2) \in E$  then
9:               break
10:            end if
11:            for all  $f_{i',j',k',l'} \in F$  do
12:              if  $d(P_i, P_{j'}) \leq d_I$  then
13:                 $E \leftarrow E \cup \{(\psi_1, \psi_2)\}$ 
14:              break
15:            end if
16:          end for
17:        end for
18:      end for
19:    end for
20:  end for
21: end for
22: end procedure

```

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**Algorithm A.4** Select Radio Manager**Input:**  $(i, k)$ , radio whose channel selector is to be determined**Input:**  $M_{|\Psi|}$ **Input:**  $M_{\Psi_c}$ **Output:**  $m$ , channel selector node's id

---

```

1: procedure SELECTORID( $(i, k), M_{|\Psi|}, M_{\Psi_c}$ )
2:    $m \leftarrow -1$ 
3:    $c_{max} \leftarrow 0$ 
4:    $d_{min} \leftarrow \infty$ 
5:   for all  $i' \in \text{keys}[M_{\Psi_c}]$  do
6:     for all  $\psi \in M_{\Psi_c}[i']$  do
7:       if  $(i, k) \in \psi \wedge (m = -1 \vee M_{|\Psi|}[i'] > c_{max} \vee (M_{|\Psi|}[i'] = c_{max} \wedge d(P_{i'}, P_i) > d_D \wedge d_{min} > d(P_{i'}, P_i)) \vee (M_{|\Psi|}[i'] = c_{max} \wedge (d(P_{i'}, P_i) \leq d_D \vee d(P_{i'}, P_i) = d_{min}) \wedge m > i'))$ 
8:         then
9:            $m \leftarrow i'$ 
10:           $c_{max} \leftarrow |M_{|\Psi|}[i']|$ 
11:           $d_{min} \leftarrow d(P_{i'}, P_i)$ 
12:        end if
13:      end for
14:    end for

```

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**Algorithm A.5** Prepare Delegation Map**Inputs:**  $\Psi, S_i, T$ **Output:**  $M_D$ , the dictionary that holds the master radio of a remotely managed slave radio

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```

1: procedure PREPAREDLGMAP( $\Psi, S_i, T$ )
2:   for all  $\psi_c \in S_i$  do
3:      $\psi_d \leftarrow \psi_c \setminus T$ 
4:     if  $\psi_d \neq \psi_c$  then  $\triangleright$  if this colour class is remotely managed by radio  $(m, k_m)$ 
5:       for all  $(i, k) \in \psi_d$  do
6:          $\psi \leftarrow \{(i', k') : \exists \psi \in \Psi, (i, k) \in \psi \wedge (i', k') \in \psi\}$   $\triangleright \psi$  is the subgraph of radio  $(i, k)$ 
7:          $d_{min} \leftarrow \infty$ 
8:          $(i_d, k_d) \leftarrow (-1, -1)$ 
9:         for all  $(i', k') \in \psi$  do
10:          if  $d(P_{i'}, P_m) \leq d_{min}$  then
11:             $(i_d, k_d) \leftarrow (i', k')$ 
12:             $d_{min} \leftarrow d(P_{i'}, P_m)$ 
13:          end if
14:        end for
15:        if  $d_{min} \leq d_D$  then  $\triangleright d_D$  is the delegation range
16:           $M_D[(i_d, k_d)] \leftarrow (m, k_m)$ 
17:           $\psi_d \leftarrow \psi_d \setminus \psi$ 
18:        end if
19:      end for
20:    if  $\psi_d = \emptyset$  then
21:       $S_i \leftarrow S_i \setminus \{\psi_c\}$ 
22:    else
23:       $\psi_c \leftarrow \psi_d$ 
24:    end if
25:  end for
26: end for
27: end procedure

```

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**Algorithm A.6** Computation of the Colour Classes' Weighted Conflict Graph on  $n_i$

**Input:**  $S_A$

**Output:**  $G_c(S_A, E)$ , conflict graph of the colour classes

**Output:**  $W_E$ , dictionary of edge weights of  $G_c(S_A, E)$

---

```

1: procedure FINDWEIGHTEDCONFLICTGRAPH( $S_A$ )
2:    $E \leftarrow \emptyset$  ▷ The edge set of  $G_c$ 
3:   for all  $\psi_1 \in S_A$  do
4:     for all  $(i, k) \in \psi_1$  do
5:       for all  $f_{i,j,k,l} \in F$  do
6:         for all  $\psi_2 \in (S_A \setminus \{\psi_1\})$  do
7:           for all  $(j', l') \in \psi_2$  do
8:             if  $d(P_i, P_{j'}) > d_I$  then
9:               continue
10:            end if
11:           for all  $f_{i',j',k',l'} \in F$  do
12:              $E \leftarrow E \cup \{(\psi_1, \psi_2)\}$ 
13:             if  $d(P_i, P_{j'}) < 1.0$  then
14:                $d \leftarrow 1.0$ 
15:             else
16:                $d \leftarrow d(P_i, P_{j'})$ 
17:             end if
18:              $W_E[(\psi_1, \psi_2)] \leftarrow W_E[(\psi_1, \psi_2)] + \frac{1}{d^\alpha}$  ▷  $\alpha$  is the path loss exponent
19:           end for
20:         end for
21:       end for
22:     end for
23:   end for
24: end procedure
25: end procedure

```

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**Algorithm A.7** Channel List Initialization Algorithm Running on  $n_i$

**Input:**  $|S_A|$ , cardinality of the set of colour classes of remotely and locally managed radios

**Output:**  $L$ , channel list

---

```

1: procedure CHLIST( $|S_A|$ )
2:    $\delta \leftarrow 0$ 
3:    $f \leftarrow 1$ 
4:    $C \leftarrow 11$  ▷ 11 channels for 802.11b/g
5:   if  $|S_A| = 1$  then ▷ Then randomly select a channel
6:      $f \leftarrow$  A random channel
7:   else
8:      $\delta \leftarrow \frac{C-1}{|S_A|-1}$ 
9:   end if
10:  for all  $i \in \mathbb{Z} \wedge i \in [0, |S_A|)$  do
11:     $ch \leftarrow f + i \delta$ 
12:    Round  $ch$  to the nearest integer
13:    if  $ch > C$  then
14:       $ch \leftarrow C$ 
15:    end if
16:     $L_i \leftarrow ch$ 
17:  end for
18: end procedure

```

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**Algorithm A.8** Channel Selection Estimation for the  $k$ -neighborhood of  $n_i$

**Input:**  $L$ , list of available (not yet assigned, free) channels

**Input:**  $M_C$

**Input:**  $m$ , current channel selector node's id

**Input:**  $c$ , number of channels to be selected for  $n_m$

**Output:**  $M_L$ , node id-selected channels list mappings

---

```

1: procedure CHSELECTION( $L, M_C, m, c$ )
2:   if  $c = 0$  then
3:     return
4:   end if
5:    $l \leftarrow \text{LEASTPRIORNODE}(M_C)$ 
6:   if  $l \neq -1$  then
7:      $c' \leftarrow M_C[l]$ 
8:     Del  $M_C[l]$  ▷ Remove key  $l$  from  $M_C$ 
9:     CHSELECTION( $L, M_C, l, c'$ )
10:  end if
11:  if  $m = -1$  then
12:    return
13:  end if
14:   $L_S \leftarrow \text{SELECTCH}(L, c)$ 
15:   $M_L[m] \leftarrow L_S$ 
16: end procedure

```

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**Algorithm A.9** Channel Selection from Available Channels

**Input:**  $L$ , list of available (not yet assigned, free) channels

**Input:**  $c$ , number of channels to be selected

**Output:**  $L_S$ , list of  $c$  channels selected from  $L$

---

```

1: procedure SELECTCH( $L, c$ )
2:    $\delta \leftarrow 0$ 
3:    $L_S \leftarrow \emptyset$ 
4:    $C \leftarrow 11$  ▷ 11 channels for 802.11b/g
5:   if  $c \leq 1$  then ▷ Then the median of the available channels list is selected
6:     Append median[ $L$ ] to  $L_S$ 
7:     Remove one instance of median[ $L$ ] from  $L$ 
8:   else
9:      $\delta \leftarrow \frac{c-1}{c-1}$ 
10:  end if
11:  for all  $i \in \mathbb{Z} \wedge i \in [0, c)$  do
12:     $ch \leftarrow 1 + i \delta$ 
13:    Round  $ch$  to the nearest integer
14:    if  $ch > C$  then
15:       $ch \leftarrow C$ 
16:    end if
17:    if  $ch \in L$  then
18:       $selectedCh \leftarrow ch$ 
19:    else ▷ Then select the closest channel to  $ch$  from  $L$ 
20:       $dist \leftarrow \infty$ 
21:      for all  $channel \in L$  do
22:        if  $|ch - channel| < dist$  then
23:           $selectedCh \leftarrow channel$ 
24:           $dist \leftarrow |ch - channel|$ 
25:        end if
26:      end for
27:    end if
28:    Append  $selectedCh$  to  $L_S$ 
29:    Remove  $selectedCh$  from  $L$ 
30:  end for
31: end procedure

```

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**Algorithm A.10** Least Prior Node Selection on  $n_i$ **Input:**  $M_C$ **Output:**  $l$ , least prior node's id

---

```

1: procedure LEASTPRIORNODE( $M_C$ )
2:    $c_{min} \leftarrow \infty$ 
3:    $l \leftarrow -1$ 
4:    $d_{max} \leftarrow -1$ 
5:   for all  $j \in \text{keys}[M_C]$  do
6:     if  $c_{min} > M_C[j] \vee (c_{min} = M_C[j] \wedge d(P_i, P_j) > d_D \wedge d_{max} < d(P_i, P_j)) \vee (c_{min} =$ 
        $M_C[j] \wedge (d(P_i, P_j) \leq d_D \vee d_{max} = d(P_i, P_j)) \wedge l < j)$  then
7:        $l \leftarrow j$ 
8:        $c_{min} \leftarrow M_C[j]$ 
9:        $d_{max} \leftarrow d(P_i, P_j)$ 
10:    end if
11:  end for
12: end procedure

```

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**Algorithm A.11** Distribute Channels Using Colour Classes' Conflict Graph on  $n_i$ **Input:**  $S_i$ , set of sets of radios  $n_i$  is responsible for selecting channels**Input:**  $G_c(S_A, E)$ , colour classes' conflict graph**Input:**  $W_E$ , dictionary of edge weights of  $G_c(S_A, E)$ **Input:**  $M_I$ , dictionary of manager node id's for the colour classes in  $S_A$ **Input:**  $M_L$ , node id-selected channels list mappings**Output:**  $\pi_i$ , candidate channel configurations for the radios of  $n_i$ **Output:**  $M_R$ , dictionary of remotely managed radios' channels

---

```

1: procedure CHDIST( $S_i, G_c(S_A, E), W_E, M_I, M_L$ )
2:   if  $|S_i| = 0$  then
3:     return
4:   end if
5:    $M_V \leftarrow \emptyset$  ▷ initialize a dictionary that holds vertex-channel mappings
6:    $v \leftarrow \text{TRAVERSENEXT}(\text{nil}, G_c(S_A, E), W_E)$ 
7:   while  $v \neq \text{nil}$  do
8:      $\forall c, I_c \leftarrow 0.0$ 
9:     for all  $c \in M_L[M_I[v]]$  do ▷ for each candidate channel  $c$ 
10:      for all  $(v, w) \in E$  do
11:        if  $w \in \text{keys}[M_V]$  then ▷ a channel has been selected for  $w$ 
12:           $I_c \leftarrow I_c + \frac{W_E[(v, w)]}{|c - M_V[w]|}$ 
13:        end if
14:      end for
15:    end for
16:     $M_V[v] \leftarrow c$ , such that  $I_c$  is minimum in  $I$  ▷ channel  $c$  is selected for colour class  $v$ 
17:     $M_L[M_I[v]] \leftarrow M_L[M_I[v]] \setminus M_V[v]$  ▷ Remove channel  $c$  from candidate channels for node
       $M_I[v]$ 
18:     $v \leftarrow \text{TRAVERSENEXT}(v, G_c(S_A, E), W_E)$ 
19:  end while
20:  for all  $v \in S_i$  do
21:    for all  $(i', k') \in v$  do
22:      if  $i' = i$  then
23:         $\pi_{i'}^{k'} \leftarrow M_V[v]$ 
24:      else
25:         $M_R[(i', k')] \leftarrow M_V[v]$ 
26:      end if
27:    end for
28:  end for
29: end procedure

```

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**Algorithm A.12** Traverse Next Vertex**Input:**  $v$ , current vertex being visited**Input:**  $G_c(S_A, E)$ **Input:**  $W_E$ **Output:**  $v$ , next vertex to be visited

---

```

1: procedure TRAVERSENEXT( $v, G_c(S_A, E), W_E$ )
2:    $\forall v \in S_A, W_V[v] \leftarrow \sum_{(v,w) \in E} W_E[(v,w)]$   $\triangleright$  Vertex weight is the sum of incident edges' weights

3:   if  $v \neq \text{nil}$  then
4:     Select  $(v,w) \in E$  such that  $W_E[(v,w)]$  is maximum in  $W_E$  and  $w$  has not been visited yet
5:     if  $w \neq \text{nil}$  then  $\triangleright$  if such a  $w$  exists
6:       return  $w$ 
7:     end if
8:   end if
9:   Select  $v \in S_A$  such that  $W_V[v]$  is maximum in  $W_V$  and  $v$  has not been visited yet
10:  return  $v$   $\triangleright v$  is nil if no such vertex exists
11: end procedure

```

---

**Algorithm A.13** Announce Channel Selections on  $n_i$ **Input:**  $S_i$ , set of sets of radios  $n_i$  is responsible for selecting channels**Input:**  $M_D$ , dictionary that holds the master radio of a remotely managed slave radio**Input:**  $\pi$ , candidate channel configurations for the radios in  $S_i$ **Input:**  $\Pi$ , list of sets of channel selection announcements**Input:**  $C$ , list of channel selection variables

---

```

1: procedure ANNOUNCESELECTIONS( $S_i, M_D, \pi, \Pi, C$ )
2:   for all  $\psi \in S_i$  do
3:     for all  $(i', k') \in \psi$  do
4:       if  $i' = i$  then
5:          $C_{k'} \leftarrow \pi_{i'}^{k'}$   $\triangleright$  Initialize channel selection variables
6:          $\Pi_{k'} \leftarrow \Pi_{k'} \cup \{(|\Psi_i|, X_i, i, k', \pi_{i'}^{k'})\}$ 
7:         Send CS message,  $(|\Psi_i|, X_i, i, k', \pi_{i'}^{k'})$ , to one-hop neighbors on the subgraph of  $(i', k')$ 
8:       else
9:         Send CS message,  $(|\Psi_i|, X_i, i, k', M_R[(i', k')])$ , to  $(i', k')$ 
10:      end if
11:    end for
12:  end for
13:  for all  $(i', k') \in \text{keys}[M_D]$  do  $\triangleright$  Send delegation requests
14:    Send DR message,  $((i', k'))$ , to  $M_D[(i', k')]$ 
15:  end for
16: end procedure

```

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**Algorithm A.14** Handle CS Announcement on  $n_i$ 
**Input:**  $\Pi$ , list of sets of channel selection announcements**Input:**  $M_P$ , proxy dictionary**Input:**  $k$ , radio id for which the announcement has been received

---

```

1: procedure HANDLECSANNOUNCEMENT( $\Pi, M_P$ )
2:    $\Psi_{max_k} \leftarrow \max \left\{ |\Psi_z| : \text{for } \exists (X_z, z, l, \pi_z^l), (|\Psi_z|, X_z, z, l, \pi_z^l) \in \Pi_k \right\}$ 

3:    $X_{max_k} \leftarrow \max \left\{ X_z : \text{for } \exists (\Psi_{max_k}, z, l, \pi_z^l), (\Psi_{max_k}, X_z, z, l, \pi_z^l) \in \Pi_k \right\}$ 

4:    $N_{min_k} \leftarrow \min \left\{ z : \text{for } \exists (\Psi_{max_k}, X_{max_k}, l, \pi_z^l), (\Psi_{max_k}, X_{max_k}, z, l, \pi_z^l) \in \Pi_k \right\}$ 

5:    $D_{min_k} \leftarrow \min \left\{ l : \text{for } \exists (\Psi_{max_k}, X_{max_k}, N_{min_k}, l, \pi_z^l), \right.$ 
       $\left. (\Psi_{max_k}, X_{max_k}, N_{min_k}, l, \pi_z^l) \in \Pi_k \right\}$ 

6:    $C_k \leftarrow \pi_z^l$  ▷ Where  $(\Psi_{max_k}, X_{max_k}, N_{min_k}, D_{min_k}, \pi_z^l) \in \Pi_k$ 
7:   if  $|\Psi_j| > \Psi_{max_k} \vee (|\Psi_j| = \Psi_{max_k} \wedge X_j > X_{max_k}) \vee$ 
       $(|\Psi_j| = \Psi_{max_k} \wedge X_j = X_{max_k} \wedge N_{min_k} > j) \vee (N_{min_k} = j \wedge D_{min_k} > k')$  then
8:     Send CS message,  $(|\Psi_j|, X_j, j, k', \pi_j^{k'})$ , to one-hop neighbors on the subgraph of  $(i, k)$ 
9:     for all  $(z, l) \in M_P[(i, k)]$  do ▷ Announce to delegated radios
10:      Send CS message,  $(|\Psi_j|, X_j, j, k', \pi_j^{k'})$ , to  $(z, l)$ 
11:     end for
12:   end if
13:    $\Pi_k \leftarrow \Pi_k \cup \left\{ (|\Psi_j|, X_j, j, k', \pi_j^{k'}) \right\}$ 
14: end procedure

```

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