REGULAR PAPER

Funda Durupınar · Uğur Güdükbay

Visualization of crowd synchronization on footbridges

Abstract This paper proposes a framework for the visualization of crowd walking synchronization on footbridges. The bridge is modeled as a mass-spring system, which is a weakly damped and driven harmonic oscillator. Both the bridge and the pedestrians walking on the bridge are affected by the movement of each other. The crowd acts according to local behavioral rules. Each pedestrian is provided with a kinematic walking system. We extend a current mathematical model of crowd synchronization on footbridges to include human walking model and crowd simulation techniques. We run experiments to evaluate the influence of these extensions on synchronization.

1 Introduction

On its opening day, the Millennium Bridge in London started to sway synchronously from side to side when crowds of people started to flow to the bridge. This unexpected phenomenon has drawn the attention of the civil and bridge engineering society, giving rise to the construction of many theories. Unlike most of the theories, the model introduced by Strogatz et al. (2005) not only explains how the bridge starts to vibrate after the crowd reaches a critical size but also how these vibrations cause the pedestrians to synchronize their footsteps with the bridge movement. The Millennium Bridge incident and the proposed model are interesting for computer graphics and behavioral animation research as well. Many aspects of crowd simulation have been examined in the literature; however, crowd synchronization is still an unexplored area. Animation of such phenomena can help to clarify their underlying causes and replace real-world experiments, which are more difficult to conduct.

In this paper, we propose a framework for the visualization of crowd synchronization on footbridges based on the model proposed by Strogatz et al. (2005). Strogatz et al. give the mathematical foundations of crowd synchronization on foot bridges but their work does not address the visualization process. Our work

F. Durupınar · U. Güdükbay (✉)
Department of Computer Engineering, Bilkent University, 06800 Bilkent, Ankara, Turkey
E-mail: gudukbay@cs.bilkent.edu.tr
Tel.: +90-312-2901386
Fax: +90-312-2664047

F. Durupınar
E-mail: fundad@cs.bilkent.edu.tr

focuses on the visualization of crowd synchronization, incorporating a kinematic human walking model into the crowds. We animate both the bridge and the crowd walking on the bridge where they mutually influence each other.

The simulation of the crowd is handled by using the High-Density Autonomous Crowds (HiDAC) system (Pelechano et al. 2007). HiDAC addresses the simulation of local behaviors and global way-finding of crowds in a dynamically changing environment. Since way-finding is straightforward for bridges, we only utilize the local behavior modeling component. Local behaviors are governed by geometric rules, such as speed, collision avoidance and psychological rules, such as impatience, pushing and panic.

The rest of the paper is organized as follows: Sect. 2 gives a review of the related work; Sect. 3 describes the basics of the model for bridge and human simulation; Sect. 4 presents the experiments and analysis. Finally, Sect. 5 gives conclusions.

2 Related work

Crowd simulation has always attracted the interest of computer graphics researchers. The earliest crowd simulation models include rule-based flocking systems (Reynolds 1987), which specify animation as a distributed global motion with a local tendency. Later, social forces (Helbing et al. (2000) and continuum dynamics (Treuille et al. 2006) have been introduced. In addition to these methods, cognitive models that involve reasoning and planning to accomplish long-term tasks (Funge et al. 1999) and hierarchical models that organize the agents into virtual crowds, groups and individuals (Musse and Thalmann 2001) are developed.

Recently, pedestrian simulation has emerged as a new direction of research in crowd simulation (Blue and Adler 2000; Ashida et al. 2001). As well as examining crowd behavior, pedestrian simulation is also important for urban planning (Farenc et al. 2000; Schreckenber 2001). A complex pedestrian animation system, which incorporates perceptual, behavioral and cognitive control components, has been introduced recently as a combination of rule-based and cognitive models (Shao and Terzopoulos 2007).

Synthesizing realistic motion of humans has been a challenging issue in computer graphics as well as in biomechanics or robotics. Researchers have examined complex motions such as walking from different aspects. Several approaches such as kinematics or dynamics have been proposed in this area. A thorough survey on human walking is given in Multon et al. (1999). Kinematic animation techniques mostly rely on prior biomechanical information. These techniques combine forward and inverse kinematics by defining finite state machines for the walking cycle. Once a key posture is associated to each state, the parameters in the in-between frames are computed through interpolation. Dynamic approaches provide more accurate results as they consider forces with respect to the laws of physics. Forward and inverse dynamics techniques have been proposed in the literature. Examples of dynamics studies on human locomotion include Bruderlin and Calvert (1989), Ko and Badler (1996), and Hodgins et al. (1995).

Various aspects of crowd simulation have been researched as aforementioned. However, crowd synchronization is still an undiscovered area for the computer graphics society. On the other hand, several structural engineering studies investigate the results of crowd synchronization on footbridges and similar building constructs (Dallard et al. 2001; Nakamura 2004; Hauksson 2005; Caprioli et al. 2006; Eckhardt et al. 2007). Our work is inspired by the ideas of Strogatz et al. (2005), who aim to explain this incident from a different perspective; namely analyzing group behavior rather than the structural dynamics of bridges.

3 Model

Our system consists of two basic components: the crowd and the bridge, which mutually influence each other during the simulation. The proposed framework is given in Fig. 1. The crowd is composed of pedestrians, each with behavioral and motion control systems. Similarly, the bridge simulation component consists of motion control and geometric information components. Movement of the pedestrians in the crowd laterally excite the bridge and cause it to vibrate once the crowd size exceeds a critical number. This causes the agents to synchronize their steps with the oscillations of the bridge. Figure 2 presents a sequence of frames showing a highly crowded bridge swaying from right to left.
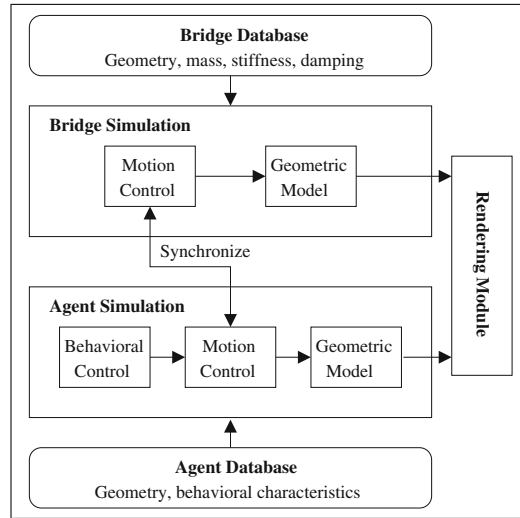
Fig. 1 The framework for the visualization of crowd synchronization on footbridges
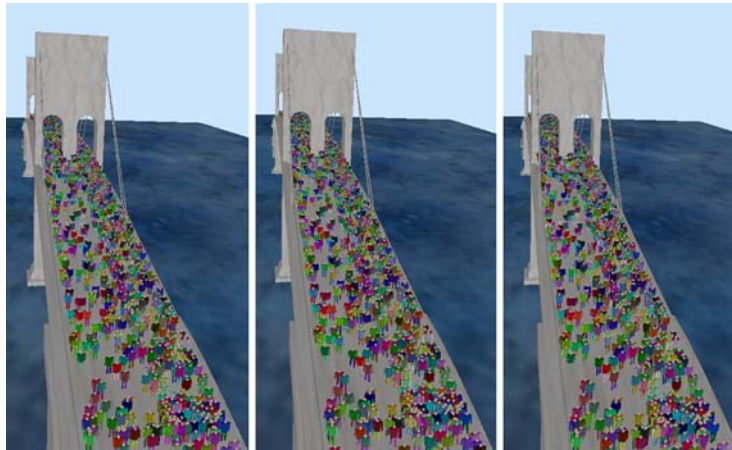

Fig. 2 Still frames from an animation of a highly crowded bridge swaying from right to left

3.1 Bridge model

The bridge is modeled as a driven harmonic oscillator with weak damping (Strogatz et al. 2005):

$$M\ddot{x}(t) + B\dot{x}(t) + Kx(t) = \sum_{i=1}^{N} f_i(t), \tag{1}$$

where $x(t)$ is the modal bridge displacement, M, B, and K are the modal bridge mass, damping coefficient, and stiffness, respectively. According to Eq. 1, each individual $i = 1, ..., N$ exerts a sideways force $f_i(t)$ on the bridge as depicted in Fig. 3.

Each individual force is defined as a periodic function:

$$f_i(t) = G \sin \theta_i(t), \tag{2}$$

where G is the maximum force and $\theta_i(t)$ is the angular frequency of stepping, which changes between 0 and 2π during a full walking cycle.

Oscillations of the bridge change the stepping frequency as (Strogatz et al. 2005):

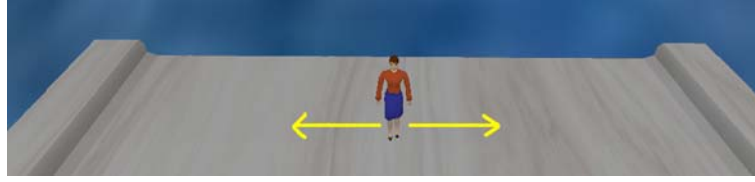$$\dot{\theta}_i(t) = \Omega_i + CA(t) \sin(\psi(t) - \theta_i(t) + \lambda), \quad \forall i = 1, \ldots, N \tag{3}$$

Fig. 3 Each individual exerts a sideways force on the bridge

where $\dot{\theta}_i$ is the frequency change caused by bridge oscillations, λ is the phase lag parameter and C is the constant for bridge oscillation force. Ω_i is the initial stepping frequency of the individual; it is computed as a Gaussian distribution with standard deviation 0.086, which is the worst case condition in terms of vibrations (Eckhardt et al. 2007). $A(t)$ is the amplitude and $\psi(t)$ is the phase of bridge vibrations, which can be derived as follows:

$$A(t) = \frac{x(t)}{\sin(\psi(t))}, \quad \psi(t) = a\tan\left(\frac{x(t)\Omega_0}{\dot{x}(t)}\right), \tag{4}$$

where $\Omega_0 = \sqrt{K/M}$ is the bridge's resonant frequency. We numerically solve these equations using the second order Euler integrator.

3.2 Crowd model

Each individual is modeled as an independent entity with behavioral and motion control components. We have integrated the local governing rules of the HiDAC system in order to implement the behavioral element. HiDAC computes the movement of each agent considering factors, such as the goal position, collisions with walls, and other agents and blends all the forces acting on an agent. Agents have traits, such as speed, interpersonal distance, and areas of influence. The details are beyond the scope of this paper and they are explained in Pelechano et al. (2007), so we will directly move on to the integration of walking into the agents.

3.3 Human walking

Human walking is a rather complicated process, which is controlled by various underlying parameters, such as angles, torques, partial derivatives, etc. Kinematic and dynamic approaches are used to simulate the walking motion. Although dynamic methods provide more accurate results, they are complex, and therefore slow. We need a fast technique since we simulate crowds of virtual humans. In addition, we just want to give the essence of walking in order to represent the synchronization of the stepping frequencies of humans. Thus, we prefer to follow the kinematic approach as it is faster and more stable.

The human skeleton is represented as an articulated figure composed of six joints, which are the two hip joints, two knees, and two shoulders. The hip joints are hinge joints and the angles they make with the upper leg are the same for both legs. We focus on the motion of legs in this study, so we define walking cycle for legs as a finite state machine (Multon et al. 1999; Chen 2003). There are six states of walking (see Fig. 4a):

1. right foot takeoff, where right knee is lifted;
2. right footstrike, where right foot hits the ground and left leg is stretched, body leaning forward;
3. swing left leg, where left leg is brought forward;
4. left foot takeoff, where left knee is lifted;
5. left footstrike, where left foot hits the ground and right leg is stretched, body leaning forward;
6. swing right leg, where right leg is brought forward.

Humans can start walking from State 1 or State 4 depending which foot they prefer to step out first. This selection is performed randomly in our system. Each walking state is simulated using inverse kinematics. During the walking states, there are four angles to consider: The angle α_R between the ground normal and the upper right leg, the angle β_R between the upper right leg and the lower right leg, the angle α_L between the ground normal and the upper left leg and the angle β_L between the upper left leg and the lower left leg and. The walking parameters can be seen in Fig. 4b. As there are only four unknowns, we can solve the system
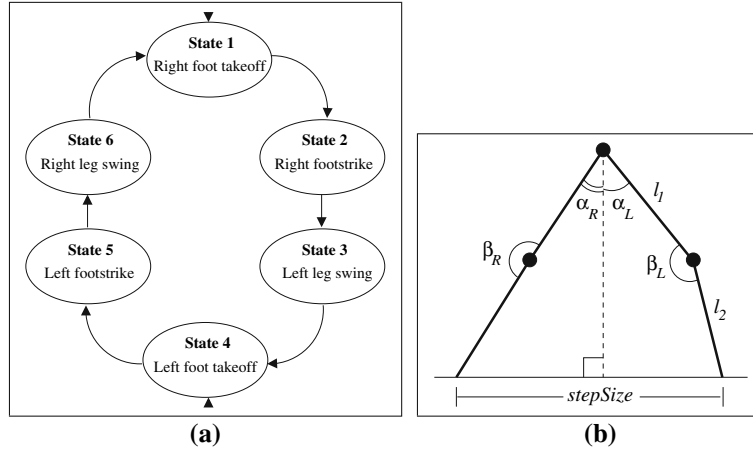
Fig. 4 a The finite state machine for human walking. **b** Human walking parameters

analytically for each state. Then, we interpolate the in-between frames using the results of each state as keyframes. Considering the sacroiliac at the origin, the state definitions are as follows:

3.3.1 State 1

In this state, the right foot is lifted, making a right angle with the ground and the left foot stays on the ground. The final angle between the upper right leg and the surface normal is α', which is a constant value. We compute the exact position (xf_R, yf_R) of the right foot as:

$$\begin{aligned} xf_R &= l_1 \sin(\alpha') \\ yf_R &= -l_1 \cos(\alpha') - l_2 \end{aligned} \tag{5}$$

where l_1 is the length of the upper leg and l_2 is the length of the lower leg. The angles can be computed as:

$$\begin{aligned} \beta_R &= a\cos\left(\frac{xf_R(t)^2 + yf_R(t)^2 - l_1{}^2 - l_2{}^2}{2l_1l_2}\right) \\ \alpha_R &= a\tan\left(\frac{-l_2\sin(\beta_R)xf_R + (l_1 + l_2\cos(\beta_R))yf_R}{-l_2\sin(\beta_R)yf_R + (l_1 + l_2\cos(\beta_R))xf_R}\right) \\ \beta_L &= \pi \\ \alpha_L &= 0 \end{aligned} \tag{6}$$

3.3.2 State 2

In this state, the body leans forward as the right foot touches the ground. Left leg is stretched and the left knee is straight. Given $\alpha_{\max}$, the exact position of the right foot can be computed as:

$$\begin{aligned} xf_R &= stepSize + (l_1 + l_2)\sin(\alpha_{\max}) \\ yf_R &= -(l_1 + l_2)\cos(\alpha_{\max}) \end{aligned} \tag{7}$$

The angles are:

$$\begin{aligned} \beta_R &= a\cos\left(\frac{(xf_R^2 + yf_R^2 - l_1{}^2 - l_2{}^2)}{(2l_1l_2)}\right) \\ \alpha_R &= a\tan\left(\frac{-l_2\sin(\beta_R)xf_R + (l_1 + l_2\cos(\beta_R))yf_R}{-l_2\sin(\beta_R)yf_R + (l_1 + l_2\cos(\beta_R))xf_R}\right) \\ \beta_L &= \pi \\ \alpha_L &= \alpha_{\max} \end{aligned} \tag{8}$$

The parameter $\alpha_{\max}$ is computed experimentally using the weight and size of the whole body considering torques and moment of inertia (Chen 2003; Bruderlin and Calvert 1989). We take $\alpha_{\max}$ as constant with slight variations among agents. *stepSize* is updated according to the speed and stepping frequency of an agent.

3.3.3 State 3

In this state, right foot stays on the ground while right leg is stretched and left foot swings forward. The final position of this state includes the right leg straight on the ground and the left leg brought near the right leg, bent a little. Here, we take ϵ_1 and ϵ_2 as small constant values. The angles are computed as:

$$
\begin{aligned}
\beta_L &= \pi - \epsilon_1 \\
\alpha_L &= \pi/2 - \epsilon_2 \\
\beta_R &= \pi
\end{aligned}
\tag{9}
$$

Since walking is symmetrical, the remaining states can be computed similar to the first three states, only switching legs.

Relating the kinematic walking model to the bridge movement and thus updating the velocities of humans is performed through the number of frames n_k used in interpolation between two states. We take n_k uniform for each state within a walking cycle. Thus, the total number of frames in a cycle is $6n_k$. We perform linear interpolation. An example for the angle $\alpha_R(t_{i,i+1,k})$ at frame k, between states i and $i+1$ is as follows:

$$
\alpha_R(t_{i,i+1,k}) = \alpha_R(t_i) + \frac{(\alpha_R(t_{i+1}) - \alpha_R(t_i))k}{n_k}
\tag{10}
$$

We relate the number of steps to angular frequency as

$$
n_k = c \frac{\pi}{3\dot{\theta}(t)},
\tag{11}
$$

where c is a constant, which can be approximated as the ratio of the average angular frequency and n_k for a person. The derivation of this formula comes from the fact that $2\pi \frac{dt}{d\theta}$ is proportional to $6n_k$, both of which refer to the total time for a full cycle.

In HiDAC, speeds of agents are used to perform simulation of the crowd considering collisions and individual factors. We relate our method to HiDAC by computing the step size of an agent according to the agent's angular frequency and the walking speed. The angular frequency is updated by the bridge force. The speed is computed in HiDAC using external factors, such as collisions with other agents and obstacles.

$$
stepSize(t) = \frac{speed(t)\pi}{\dot{\theta}(t)}
\tag{12}
$$

We have emphasized the motion of legs so far since leg motion is crucial to walking and consequently synchronization. However, humans also move their arms while they walk. The agents in our system are also provided with a simple arm motion model. Arms are connected to the torso via shoulder joints. Because arms move symmetrically, we only consider the angle γ between the right arm and the torso. When the right arm is rotated γ degrees, the left arm is rotated $-\gamma$ degrees. Taking c as a constant, the angle γ at time t is computed as follows:

$$
\gamma(t) = \begin{cases} c\, \alpha_R(t) & \text{if the current state} \in \{1, 2, 3\} \\ -c\, \alpha_R(t) & \text{if the current state} \in \{4, 5, 6\} \end{cases}
\tag{13}
$$

4 Experiments and discussion

In order to analyze the effectiveness of behavioral and perceptual components, we have run several experiments. Animation 1 (supplementary material) demonstrates the crowd motion while walking on the bridge. The parameters of the simulation are selected according to real-world experiments (Nakamura 2004): The modal mass (M) of the bridge is 237,000 kg. Stiffness (B) and damping (K) values are taken as 8,092,000 kg/s^2 and 22,200 kg/s respectively. The mass of each individual is approximated as 70 kg. Initial

stepping frequencies of humans are computed as a Gaussian distribution with mean 2 Hz and standard deviation 0.086 (Eckhardt et al. 2007).

The experiments test how much the pedestrians and the bridge reciprocally influence each other. For this purpose, the predicted amplitude A of the bridge's lateral vibrations with respect to time and the predicted degree of phase coherence among pedestrians as an order parameter, R are calculated as follows (Strogatz et al. 2005):

$$A = \sqrt{x^2(t) + (\Omega_0^{-1}\dot{x}(t))^2}$$
$$R = N^{-1}\left|\sum_{j=1}^{N}\exp(i\theta_j)\right| \tag{14}$$

We want to evaluate our method of relating the walking model with the proposed model. Thus, we find the standard deviation of the humans' stepping frequencies. The stepping frequency for an agent i is computed as the inverse of the time between the final and initial states as:

$$f_i = ||(t_{\text{state}_6} - t_{\text{state}_1})||^{-1} \tag{15}$$

Figure 5a shows an earlier state of the pedestrians when they have just started walking on the bridge and Fig. 5b depicts a later state. It can be seen that initially the steps of the individuals are not coherent and randomly distributed, whereas at the later stages they have synchronized steps (see Animation 1, supplementary material).

The number of pedestrians on the bridge increases in time until there are 250 people and then it remains constant. Figure 6a shows the predicted amplitude of the bridge's lateral vibrations with respect to the number of humans, and Fig. 6c shows the predicted degree of phase coherence among pedestrians. The results are coherent with the actual experimental data (Strogatz et al. 2005). Figure 6e shows that standard deviation between the walking frequencies of humans decreases as time passes and more humans keep joining the crowd. When the crowd reaches a critical size, vibrations in the bridge start to increase and consequently, phase coherence among the pedestrians increases, creating a positive feedback loop.

Figure 6b, d, f shows the same simulation results when the footsteps of individuals are forced to remain randomly distributed during the course of the simulation. As expected, they cancel the effect of each other and the bridge oscillation is too small and random to make any observations.

We compared our animations (see Animation 1, supplementary material) with the actual video of the Millennium Bridge Incident, which can be accessed from http://www.youtube.com/watch?v=eAXVa_XWZ8. The oscillation of the bridge in the actual video because of the movement of the crowd is very similar to the one produced with our implementation. The actual video shows that the pedestrians move in synchrony with the bridge oscillations. Similar behaviors can be observed in our animations.

The simulations are run on a personal computer (Intel Core Duo Processor T2500, 2.00 GHz) with 2GB of RAM. The graphics card is NVIDIA GeForce Go 7400 with 256 MB memory size. The average frame
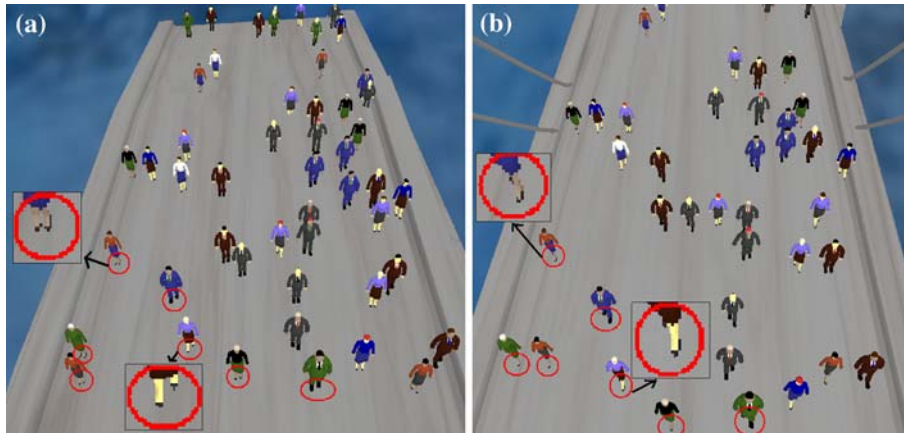

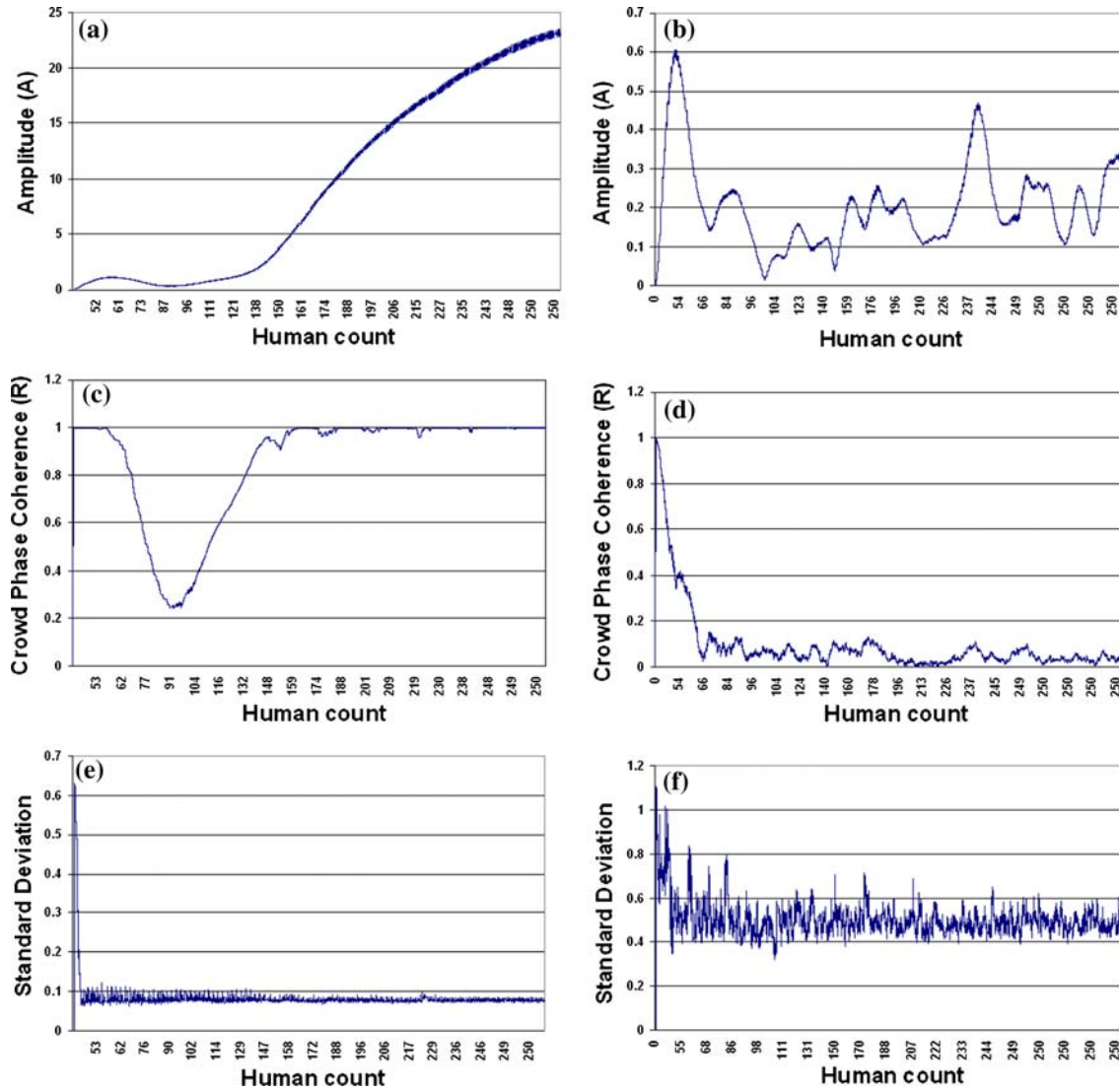**Fig. 5** (**a**) Early and (**b**) later stages in simulation showing synchronization in time

Fig. 6 a Bridge oscillation amplitude, **c** crowd phase coherence, and **e** standard deviation among humans' walking frequencies versus number of humans on the bridge. **b, d, f** are the corresponding graphs where the steps of the individuals remain random

rates for the simulation of a crowd population of 250 people is 63 frames per second (fps). The frame rates decrease to 25 fps for a crowd population of 500 people.

5 Conclusions

In this study, we introduce a system to visualize the walking synchronization of crowds on pedestrian footbridges. Understanding the basics of crowd synchronization is important for engineering concerns as well as behavioral studies. Visualization of the existing theories of synchronization provides engineers with a convenient way to run experiments. Despite our current focus on bridges, the system can be extended to include other engineering constructs such as stadiums or ships.

The current mathematical models of synchronization formulate only the frequency of human walking cycle. The main parameter that determines the walking synchronization is the stepping frequency, which is calculated by the number of frames interpolated between the keyframes. Our walking model simulates the gait cycle as a simple kinematic model that is solved analytically. Alternatively, motion capture techniques can also be used to determine the keyframes. However, we preferred to incorporate the phases of walking using inverse kinematics. Nevertheless, we are aware that human walking is a rather complicated process, which is most

accurately defined by dynamic approaches. However, the dynamic approaches are computationally demanding and stability issue still needs to be solved for human motion. Providing agents with a dynamic walking model that considers the external forces exerted by the bridge can be investigated as a future work.

References

Ashida K, Lee S, Allbeck J, Sun H, Badler N, Metaxas D (2001) Pedestrians: creating agent behaviors through statistical analysis of observation data. In: Proceedings of IEEE conference on computer animation, pp 84–92

Blue V, Adler J (2000) Cellular automata model of emergent collective bi-directional pedestrian dynamics. In: Proceedings of artificial life VII, pp 437–445

Bruderlin A, Calvert T (1989) Goal-directed, dynamic animation of human walking. In: Proceedings of ACM SIGGRAPH, pp 233–242

Caprioli A, Reynolds P, Vanali M, Zappa E (2006) Comparison of the effects of a moving crowd on different grandstands during similar events. In: 24th International modal analysis conference (IMAC XXIV)

Chen Sl (2003) Animating human walking. Available at http://www.ugcs.caltech.edu/shang/cs174/walking_report.pdf. Accessed April 2009

Dallard P, Fitzpatrick T, Flint A, Low A, Smith RR, Willford M, Roche M (2001) London millennium bridge: pedestrian-induced lateral vibration. J Bridge Eng 6(6):412–417

Eckhardt B, Ott E, McRobie A (2007) Modeling walker synchronization on the millennium bridge. Phys Rev E 75(2):021110

Farenc N, Musse SR, Schweiss E (2000) A paradigm for controlling virtual humans in urban environment simulations. Appl Artif Intell 14:69–91

Funge J, Tu X, Terzopoulos D (1999) Cognitive modeling: knowledge, reasoning and planning for intelligent characters. ACM Comp Graph (Proceedings of SIGGRAPH'99), pp 29–38

Hauksson F (2005) Dynamic behaviour of footbridges subjected to pedestrian-induced vibrations. Master's thesis, Lund University, Sweden

Helbing D, Farkas I, Vicsek T (2000) Simulating dynamical features of escape panic. Nature 407:487–480

Hodgins JK, Wooten WL, Brogan DC, O'Brien JF (1995) Animating human athletes. In: Proceedings of ACM SIGGRAPH, pp 71–78

Ko H, Badler NI (1996) Animating human locomotion with inverse dynamics. IEEE Comput Graph Appl 16(2):50–59

Multon F, France L, Cani-Gasguel P, Debunne G (1999) Computer animation of human walking: a survey. J Vis Comput Animat 10:39–54

Musse SR, Thalmann D (2001) Hierarchical model for real time simulation of virtual human crowds. IEEE Trans Vis Comput Graph 7(2):152–164

Nakamura S (2004) Model for lateral excitation of footbridges by synchronous walking. J Struct Eng 131(7):1150–1151

Pelechano N, Allbeck J, Badler N (2007) Controlling individual agents in high-density crowd simulation. In: Proceedings of the Eurographics/ACM SIGGRAPH symposium on computer animation, pp 99–108

Reynolds CW (1987) Flocks, herds and schools: a distributed behavioral model. ACM Comput Graph (Proceedings of SIGGRAPH'87) 21(4):25–34

Schreckenber M (2001) Pedestrian and evacuation dynamics. Springer, New York

Shao W, Terzopoulos D (2007) Autonomous pedestrians. Graph Models 69(5–6):246–274

Strogatz S, Abrams D, McRobie A, Eckhardt B, Ott E (2005) Crowd synchrony on the millennium bridge. Nature 438(3):43–44

Treuille A, Cooper S, Popovic Z (2006) Continuum crowds. ACM Trans Graph (SIGGRAPH '06) 25(3):1160–1168