Stable $\mathscr{H}^\infty$ Controller Design for Systems with Time Delays

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Abstract. One of the difficult problems of robust control theory is to find strongly stabilizing controllers (i.e. stable controllers leading to stable feedback system) which satisfy a certain $\mathscr{H}^\infty$ performance objective. In this work we discuss stable $\mathscr{H}^\infty$ controller design methods for various classes of systems with time delays. We consider sensitivity minimization problem in this setting for SISO plants. We also discuss a suboptimal design method for stable $\mathscr{H}^\infty$ controllers for MIMO plants.

This paper is dedicated to Yutaka Yamamoto on the occasion of his 60th birthday.

1 Introduction

In this paper we will give an overview of recent results on design for various types of systems with time delays. The problem of finding a *stable* stabilizing controllers has been studied since 1970s, see [4, 8, 12, 18, 19] for finite dimensional systems and [1, 5, 6, 10, 16] for systems. This list is by no means complete; the reader can find various approaches and results from the references of the papers listed here.

In particular, [6] considers a class of SISO time delay systems with possibly infinitely many poles in $\mathbb{C}_+$. Under the condition that the number of zeros in $\mathbb{C}_+$ is finite, stable stabilizing controllers achieving a desired sensitivity level can be found using Nevanlinna-Pick interpolation.

Another approach for finding stable $\mathscr{H}^\infty$ controllers is to use the parameterization of all controllers achieving a desired $\mathscr{H}^\infty$ performance level, then look for a feasible free parameter which stabilizes the controller. In the context of time delay systems, this method has been studied in [5] where the suboptimal controller structure of [3, 17] is used.

By extending a result of [21], it is possible to obtain a large subset of all stable stabilizing controllers for a class of systems with time delays, [10]. Then, in this subset, we can search for controllers satisfying a desired $\mathscr{H}^\infty$ performance level.

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Definitions of various stable controller design problems are given in Section 2. In Section 3 we discuss the Nevanlinna-Pick interpolation approach from [6] for stable $\mathcal{H}^\infty$ controller design for SISO time delay systems. The result of [10] is illustrated with an example in Section 4. Concluding remarks are made in Section 5.

2 Problem Definition and Preliminary Remarks

Consider the feedback system shown in Figure 1, where C is the controller and P is the plant. We say that the system is stable if $S := (1+PC)^{-1}$, PS and CS are in $\mathcal{H}^\infty$; in this case we say that C stabilizes P and write $C \in \mathscr{C}(P)$, where $\mathscr{C}(P)$ represents the set of all controllers stabilizing P. All stable stabilizing controller are denoted by $\mathscr{C}_\infty(P) := \mathscr{C}(P) \cap \mathcal{H}^\infty$.

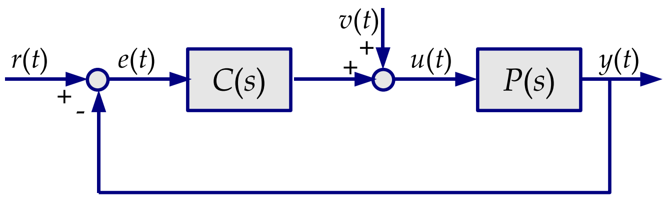

Fig. 1 Feedback System

We can define the following problems.

SS0 Given P find a controller C in $\mathscr{C}_\infty(P)$.
SS1 Given P, W_1 and $\rho > 0$, find a controller $C \in \mathscr{C}_\infty(P)$ such that $\|W_1 S\|_\infty \leq \rho$.
SS2 Given P, W_1, W_2 and $\rho > 0$, find a controller $C \in \mathscr{C}_\infty(P)$ such that

$$\left\| \begin{bmatrix} W_1 S \\ W_2(1-S) \end{bmatrix} \right\|_\infty \leq \rho.$$

SS0PD Given P find (if possible) a controller $C \in \mathscr{C}(P)$ such that

$$C(s) = K_p + K_d \frac{s}{\tau_d\ s+1}$$

for some $K_p, K_d \in \mathbb{R}$ and $\tau_d > 0$.

In this paper we will discuss SS0 and SS1 for various classes of time delay systems. The problem SS2 is a difficult one; it can be solved by trying to find a feasible free parameter in the parameterization of all suboptimal controllers, see [5]. Due to page limitations, we will also leave SS0PD aside, but it can be solved by finding a characterization of the set of all stabilizing (K_p, K_d) pairs for each fixed $\tau_d > 0$, see e.g. [13] and its references. An alternative approach for SS0PD would be to use the results of [7, 11], where a simple but conservative design method is proposed for proportional plus derivative (PD) controller synthesis for systems with time delays.

For finite dimensional systems, it is well known that the problem SS0 is solvable if and only if P satisfies the PIP (the number of poles between every pair of blocking zeros on the extended real axis is even), [19]. This result remains valid for a large class of time delay systems, see e.g. [1].

Let us consider a plant in the form

$$P(s) = N(s)/D(s) \tag{1}$$

where $N, D \in \mathscr{H}^\infty$ are strongly coprime, [14]. Assume that N has finitely many zeros, $z_1, \ldots, z_\ell$ (assume they are distinct for simplicity) in the extended right half plane, $\mathbb{R}_{+e} = \mathbb{R}_+ \cup \{\infty\}$. A controller $C \in \mathscr{H}^\infty$ is in $\mathscr{C}(P)$ if and only if $U, U^{-1} \in \mathscr{H}^\infty$, where $U = D + NC$. Note that when $C \in \mathscr{H}^\infty$ we have $U(z_i) = D(z_i)$. The problem of finding a feasible U is solvable if and only if the set $\{D(z_1), \ldots, D(z_\ell)\}$ is sign invariant, which is equivalent to PIP.

3 Nevanlinna-Pick Interpolation for Stable $\mathscr{H}^\infty$ Controller Design

Consider the plant (1) defined in the previous section with ensuing assumptions. Besides zeros on the positive real axis, plant may have other zeros in $\mathbb{C}_+$, let us enumerate them as $z_{\ell+1}, \ldots, z_n$, and assume that they are distinct. Let $D(z_i) > 0$ for all $i = 1, \ldots, \ell$ (i.e., PIP is satisfied). In order to find a controller $C \in \mathscr{C}_\infty(P)$ we can construct a unimodular U (i.e. $U, U^{-1} \in \mathscr{H}^\infty$) such that

$$U \;:\; \mathbb{C}_+ \to \mathbb{W}_\gamma \quad \text{with} \quad U(z_i) = D(z_i) \quad i = 1, \ldots, n \tag{2}$$

where the range $\mathbb{W}_\gamma$ is defined as

$$\mathbb{W}_\gamma := \{re^{j\theta} \in \mathbb{C} \;:\; \varepsilon < r < \gamma\,, \quad -\pi < \theta < \pi\} \tag{3}$$

for some sufficiently small number $\varepsilon > 0$ and a finite number $\gamma > \varepsilon$. Note that $U(s)$ should not take negative values for $s \in \mathbb{R}_{+e}$ (otherwise U^{-1} does not exists because in that case $U(s)$ takes both positive and negative values for $s \in \mathbb{R}_+$ meaning that it has a zero in $\mathbb{R}_+$), so negative real axis is excluded from $\mathbb{W}_\gamma$. Clearly γ should be large enough so that $D(z_i) \in \mathbb{W}_\gamma$ for all $i = 1, \ldots, n$. Also note that with the above definition we guarantee the upper bounds $\|U\|_\infty < \gamma$ and $\|U^{-1}\|_\infty < \varepsilon^{-1}$. Once a feasible U is found, the controller is given by

$$C(s) = \frac{U(s) - D(s)}{N(s)}$$

which is stable by interpolation conditions, and we have $S = DU^{-1}$ and $PS = NU^{-1}$.

For technical reasons, assume for the moment that the plant does not have a zero at $+\infty$, i.e. all z_i's are finite. Since $\mathbb{W}_\gamma$ is a simply connected domain there is a conformal map

$$\phi_\gamma : \mathbb{W}_\gamma \to \mathbb{D}.$$

Let φ be a conformal map from $\mathbb{C}_+$ to $\mathbb{D}$. Define

$$\alpha_i = \varphi(z_i) \in \mathbb{D}, \qquad \beta_i = \phi_\gamma(U(z_i)) \in \mathbb{D}, \quad i = 1, \ldots, n.$$

Then, finding a bounded analytic U satisfying (2) is equivalent to finding a bounded analytic function

$$\vartheta : \mathbb{D} \to \mathbb{D} \qquad \text{such that} \quad \vartheta(\alpha_i) = \beta_i, \quad i = 1, \ldots, n.$$

This is the Nevanlinna-Pick problem and it is solvable if and only if a Pick matrix is positive definite, [3, 20]. The associated Pick matrix is constructed from α_i's and β_i's, which depend on the original problem data z_i's, $D(z_i)$'s and γ. If this problem is feasible, then U can be found from ϑ as

$$U(s) = \phi_\gamma^{-1}(\vartheta(\varphi(s))).$$

Thus SS0 can be solved from the above procedure. Note that when the plant has a zero at $+\infty$, then under the φ this point is mapped to a point on the unit circle. So, we need to construct ϑ from $\overline{\mathbb{D}}$ to $\mathbb{D}$. This case requires a slight extension of the classical Nevanlinna-Pick interpolation; for a solution see Section 2.11.3 of [3].

Although γ puts a bound on $\|U^{-1}\|_\infty$, in order to find a controller for SS1 we need to have a bound for $\|W_1 S\|_\infty = \|W_1 D U^{-1}\|_\infty$. For this purpose, let us first consider an inner-outer factorization of $D = D_i D_o$ and assume D_o is invertible in $\mathscr{H}^\infty$. If the plant does not have a pole on the Im-axis then this assumption holds, and D_o^{-1} can be seen as part of N. So, we can take $D = D_i$ and under this assumption $\|W_1 S\|_\infty = \|W_1 U^{-1}\|_\infty$. Let $W_1^{-1} \in \mathscr{H}^\infty$ and define

$$F(s) := \frac{1}{\rho} W_1(s) U^{-1}(s).$$

Under the above assumptions, the problem SS1 is solvable if and only if there exists an F such that $F, F^{-1} \in \mathscr{H}^\infty$ with

$$F : \mathbb{C}_+ \to \mathbb{W}_1 \quad \text{and} \quad F(z_i) = \frac{W_1(z_i)}{\rho\, D(z_i)} \quad i = 1, \ldots, n.$$

By using the conformal maps as defined above, this problem can be transformed to a Nevanlinna-Pick problem. Once a feasible F is found a controller solving SS1 is given by

$$C = \frac{\rho^{-1} W_1 F^{-1} - D}{N},$$

which is stable by interpolation conditions and it leads to $S = \rho D W_1^{-1} F$ satisfying the $\mathscr{H}^\infty$ performance condition:

$$\|W_1 S\|_\infty = \|\rho F\|_\infty \le \rho.$$

In [6] the function F is considered to be in the form $F(s) = e^{-G(s)}$. Since $F^{-1}(s) = e^{G(s)}$ and $\|F^{-1}\|_\infty < \varepsilon^{-1}$, we are looking for a bounded analytic G such that associated interpolation conditions hold and

$$G \,:\, \mathbb{C}_+ \to \mathbb{C}_+^{\sigma_o} := \{s \in \mathbb{C}_+ \,:\, 0 < \mathrm{Re}(s) < \sigma_o = \ln(\varepsilon^{-1})\},$$

where $\varepsilon > 0$ is as in (3). Again, by a series of conformal maps construction of a feasible G can be reduced to a Nevanlinna-Pick problem, see [6] for details.

Now we want to give an example from [6] for the class of plants which can be handled in the above framework. Consider

$$P(s) = \frac{(s+1)+4e^{-3s}}{(s+1)+2(s-1)e^{-2s}} = \frac{1e^{-0s} + \left(\frac{4}{s+1}\right)e^{-3s}}{1e^{-0s} + 2\left(\frac{s-1}{s+1}\right)e^{-2s}} =: \frac{R(s)}{T(s)}$$

where $R(s)$ has four zeros in $\mathbb{C}_+$: $z_{1,2} \approx 0.31 \pm j0.85$ and $z_{3,4} \approx 0.1 \pm j2.7$, so define

$$N_i(s) = \prod_{i=1}^{4} \frac{s - z_i}{s + z_i}.$$

Note that relative degree of the plant is zero hence $+\infty$ is not a zero of P, so we do not have to deal with interpolation conditions at the boundary. Also, the plant has infinitely many poles in $\mathbb{C}_+$; in this situation we define

$$\bar{T}(s) := e^{-2s}T(-s)\left(\frac{s-1}{s+1}\right) = 2 + \left(\frac{s-1}{s+1}\right)e^{-2s}$$

and check that $\bar{T}(s)$ is stable and it does not have zeros in $\mathbb{C}_+$. Thus the plant admits the following coprime factorization

$$P(s) = \frac{N_i(s)N_o(s)}{D_i(s)} \quad \text{with} \quad D_i(s) = \frac{T(s)}{\bar{T}(s)}, \quad N_o(s) = \frac{R(s)}{N_i(s)}\,\frac{1}{\bar{T}(s)}.$$

If we choose $\sigma_o = \ln(\varepsilon^{-1}) = 3$, i.e. $\varepsilon = e^{-3} \approx 0.05$, and $W_1(s) = (1+0.1s)/(s+1)$, then we can find a solution for SS1 with $\rho = 1.0815$, and the resulting F is given as

$$F(s) = \exp\left(-\frac{\sigma_o}{2} - j\frac{\sigma_o}{\pi}\ln\left(\frac{1+\widetilde{G}(s)}{1-\widetilde{G}(s)}\right)\right) \quad \text{where}$$

$$\widetilde{G}(s) \approx j\frac{-0.99(s-3.473)(s+1)(s^2-0.03s+7.56)}{(s+3.415)(s+1.007)(s^2+0.034s+7.57)}.$$

As $\varepsilon \to 0$ we see that the smallest ρ for which SS1 is solvable decreases to 1.0726.

At this point we should mention that the zeros $z_{3,4}$ have not been taken into account in [6], so the numerical example given there is not correct (it is correct only for a plant with two zeros $z_{1,2}$ in $\mathbb{C}_+$ with same interpolation conditions). It is interesting that $z_{1,2}$ are the dominant zeros in the sense that when interpolation conditions due to $z_{3,4}$ are ignored the smallest ρ for which SS1 is solvable can be computed to be 1.0704 as $\varepsilon \to 0$.

4 Suboptimal Stable $\mathscr{H}^\infty$ Controllers

In this section we first consider SS0 for MIMO plants in the form $P = D^{-1}N$, where all entries of $N(s)$ and $D(s)$ are in $\mathscr{H}^\infty$. A controller C is in $\mathscr{C}_\infty(P)$ if all entries of C are in $\mathscr{H}^\infty$, and $U = D + NC$ is unimodular, i.e. U and U^{-1} have all its entries in $\mathscr{H}^\infty$. In this setting N, D, C, U are appropriate size matrices whose entries are in $\mathscr{H}^\infty$. For notational convenience, without specifying the matrix size we write $D, N, C, U \in \mathscr{H}^\infty$.

The system given below illustrates one possible class of plants which can be studied in this framework:

$$P(s) = \frac{(s-4)e^{-3hs}}{(s+1-2e^{-0.4s})} \begin{bmatrix} \frac{1}{s+2} & \frac{-1}{s+4} & \frac{1}{s+3} \\ 0 & 0 & \frac{e^{-hs}}{s+1+e^{-s}} \end{bmatrix}, \quad h > 0 \tag{4}$$

which can be factored as $P(s) = D(s)^{-1} N_i(s) N_o(s) N_1(s)$ where N_i is inner, N_o is finite dimensional outer and N_1 is right invertible infinite dimensional outer matrix:

$$N_i(s) = \frac{s-4}{s+4}\, e^{-3hs} \begin{bmatrix} 1 & 0 \\ 0 & e^{-hs} \end{bmatrix}, \qquad N_o(s) = \frac{1}{s+1} I,$$

$$N_1(s) = \frac{s-p}{s+1-2e^{-0.4s}} \begin{bmatrix} \frac{s+4}{s+2} & -1 & \frac{s+4}{s+3} \\ 0 & 0 & \frac{s+4}{s+1+e^{-s}} \end{bmatrix}$$

and $D(s) = \dfrac{s-p}{s+1} I$ with $p > 0$ being the only root of $s + 1 - 2e^{-0.4s} = 0$ in $\mathbb{C}_+$ (note that $p \approx 0.5838$). For this plant, a controller $C \in \mathscr{H}^\infty$ is in $\mathscr{C}_\infty(P)$ if and only if

$$U = D + N_i N_o N_1 C$$

is unimodular. Note that N_1 admits a right inverse

$$N_1^\dagger(s) = \frac{s+1-2e^{-0.4s}}{s-p} \begin{bmatrix} 2\frac{s+2}{s+4} & 0 \\ 1 & \frac{s+1+e^{-s}}{s+3} \\ 0 & \frac{s+1+e^{-s}}{s+4} \end{bmatrix} \in \mathscr{H}^\infty.$$

If we define $C = N_1^\dagger C_1$ where $C_1 \in \mathscr{H}^\infty$ is free, then this controller is in $\mathscr{C}_\infty(P)$ if $U = D + N_i N_o C_1$ is unimodular.

Let $R := (D - I)$, then $C \in \mathscr{C}_\infty(P)$ if $C_1 \in \mathscr{H}^\infty$ satisfies

$$\|R + N_i N_o C_1\|_\infty < 1. \tag{5}$$

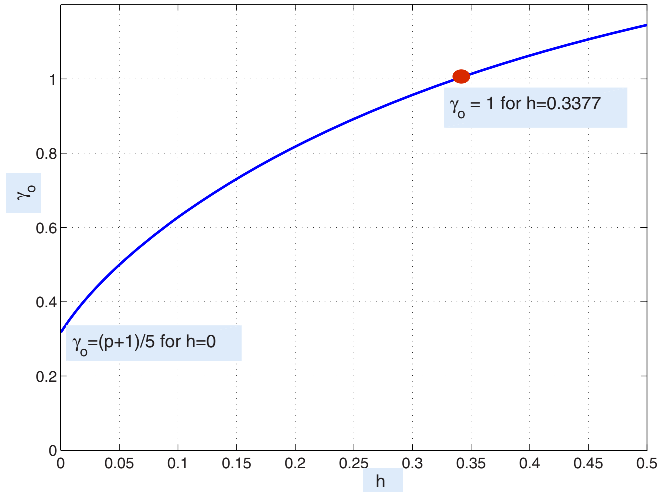

Fig. 2 γ_o versus h

The problem of finding a suitable C_1 is an $\mathscr{H}^\infty$ control problem and can be solved using one of many alternative techniques from the literature, see e.g. [9]. For the numerical example given above, the problem (5) has a solution if and only if

$$\gamma_o := \inf_{Q \in \mathscr{H}^\infty} \left\| \frac{p+1}{s+1} - \frac{(s-4)}{(s+4)(s+1)} e^{-4hs} Q \right\|_\infty < 1. \tag{6}$$

Using the results of [3, 9] we can compute $\gamma_o < (p+1)$ from the smallest root ω_o of

$$\tan^{-1}\omega_o + 2\tan^{-1}\frac{\omega_o}{4} + 4h\omega_o = \pi, \quad \text{where} \quad \omega_o = \sqrt{\frac{(p+1)^2}{\gamma_o^2} - 1}.$$

Figure 2 shows γ_o as a function of h. It implies that for the given plant we can find a controller $C \in \mathscr{C}_\infty(P)$ using this method if and only if $h < 0.3377$.

Let us now study SS1 for the SISO version of the plants considered in this section, $P = N/D$. A controller $C = Q \in \mathscr{H}^\infty$ solves SS1 if $U = D + NQ$ is unimodular and $\|\rho^{-1}W_1DU^{-1}\|_\infty \leq 1$, equivalently

$$|\rho^{-1}W_1(j\omega)D(j\omega)| \leq |D(j\omega) + N(j\omega)Q(j\omega)|, \qquad \omega \in \mathbb{R}.$$

Using $R := D - 1$ we see that a sufficient condition for the above is

$$|\rho^{-1}W_1(j\omega)D(j\omega)|^2 + |R(j\omega) + N(j\omega)Q(j\omega)|^2 \leq 1/2 \qquad \omega \in \mathbb{R}.$$

Assume that $\rho > \sqrt{2}\|W_1 D\|_\infty$, then we can find $V_\rho \in \mathcal{H}^\infty$ such that $V_\rho^{-1} \in \mathcal{H}^\infty$ and

$$|V_\rho(j\omega)|^2 = \frac{1}{2} - |\rho^{-1} W_1(j\omega) D(j\omega)|^2 \qquad \omega \in \mathbb{R}.$$

With this spectral factorization, SS1 is solvable if

$$\gamma_1 := \inf_{Q_1 \in \mathcal{H}^\infty} \|V_\rho^{-1} R + N Q_1\|_\infty < 1. \tag{7}$$

If (7) holds, then $C = V_\rho Q_1$ is an admissible solution of SS1 for all $Q_1 \in \mathcal{H}^\infty$ satisfying $\|V_\rho^{-1} R + N Q_1\|_\infty < 1$.

Let us now consider this problem for the plant $P = N/D$

$$D(s) = \frac{s-p}{s+1}, \qquad N(s) = \frac{s-4}{(s+4)(s+1)} e^{-4hs},$$

with $p = 0.5838$ and $h > 0$. Take $\rho = 2$ and $W_1(s) = \frac{s+1}{10s+1}$, and check that $\rho > \sqrt{2}\|W_1 D\|_\infty = \sqrt{2}p$. Below table shows the values of γ_1 for varying h. We see that the largest h for which we can find a solution to SS1 using this method is 0.1354.

h	0	0.01	0.05	0.10	0.13	0.1354	0.14	0.15	0.2
γ_1	0.45	0.52	0.71	0.89	0.98	0.9991	1.013	1.041	1.165

It is interesting to compare the results of this table with Figure 2. For each fixed h we have $\gamma_1 > \gamma_o$. This is expected since SS1 is more stringent than SS0. In fact, due to added conservatism in our approach to SS1, for each fixed h we have that $\gamma_1 \to \sqrt{2}\gamma_o$ as $\rho \to \infty$.

5 Conclusions

Stable $\mathcal{H}^\infty$ controller design problems are discussed and two alternative methods are illustrated for two different classes of plants with time delays. Here we considered the sensitivity minimization problem only. Generalization of the proposed methods to mixed sensitivity minimization is a non-trivial problem which remains unsolved.

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