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An intermodal multicommodity routing problem with scheduled services

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Abstract We study a multicommodity routing problem faced by an intermodal service operator that uses ground and maritime transportation. Given a planning horizon, a set of commodities to be picked up at their release times and to be delivered not later than their duedates, the problem is to decide on routes for these commodities using trucks and scheduled and capacitated maritime services at minimum cost of transportation and stocking at the seaports. Two mixed integer programming formulations and valid inequalities are proposed for this problem. The results of a computational study to evaluate the strength of the linear programming relaxations and the solution times are reported.

Keywords Intermodal routing · Multicommodity flows · Scheduled services · Valid inequalities

1 Introduction

Intermodal transportation can be broadly defined as the transportation of goods or people using more than one transportation mode, in an integrated manner from an origin to a destination (for more definitions, we refer the reader to Bontekoning et al. [6]). In this paper, we use this term to refer to the transportation of containerized goods by a same company using a mix of truck, rail, air and/or maritime transportation.

Even though some earlier examples are cited, the tremendous increase of intermodal transportation has followed the use of standardized containers (see, e.g., Slack

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[19]). The standardization in dimensions made it possible to use tools for loading and unloading at the intermodal terminals and hence decreased the complexity and the cost of these operations. The trend of increase has been amplified by the growth in international trade in the last decades. The limitations posed, and the incentives and subsidies given by governments with the aim of decreasing the ground traffic have also contributed to the increase in volumes carried using different transportation modes.

In this paper, we study a multicommodity network flow problem faced by a third party logistics company. The company uses ground and maritime transportation. We are given a set of commodities, which should be picked up from their origins at given release times and should be delivered to their destinations no later than their duedates. The commodities may be carried directly from their origins to their destinations on trucks or they may be carried on trucks to a seaport, may visit several seaports using maritime services, and then be carried to their destinations on trucks. As the company rents trucks, there is no capacity and time limitation on the use of ground transportation. However the maritime services are scheduled in advance and the company has limitations on the amounts of volume that it can carry on each service. Each maritime service has a start time of loading and a cutoff time after which loading is not permitted. For a commodity to be able to use a service, it should be available at the origin seaport of the service before the cutoff time of this service. If a commodity arrives at the origin seaport too early compared to the start of loading, then a cost is incurred for stocking at the seaport. The aim is to determine routes for commodities in order to minimize the total cost of shipments over a given time horizon, respecting the capacity and time related constraints. The total cost has two components: the transportation cost and the stocking cost at seaports. We call this problem the "*Intermodal Multicommodity Routing Problem with Scheduled Services (IMR-S)*".

Even though the study is motivated by an application in the context of freight logistics and maritime transportation, the problem applies to more general intermodal transportation settings, where some services are scheduled in advance of the routing decisions. For instance, a company selling travel packages may need to route groups of people from some origins to destinations using buses, which may be rented and thus are available at any time, together with train and plane trips, which have fixed schedules and on which the number of seats reserved to the company is limited. If the travelers need to wait for long time periods before taking a vehicle, there may be a need to provide accommodation at some additional cost. Even in cases where this is not necessary, such long waiting times may decrease customer satisfaction. Hence, it is reasonable to associate some cost for waiting times between transfers. Now, the problem of finding time and capacity feasible and minimum cost routes for groups of travelers can be modeled as *IMR-S*.

In this paper, we first investigate the computational complexity of our problem and prove that it is NP-hard. Then we seek strong and small size formulations for *IMR-S*. In the literature, similar problems are usually modeled using time-space networks. As these networks expand the underlying transport networks with a time dimension, they have large sizes. As a result, the formulations based on these networks use a large number of variables to model routing decisions. Here, we propose a first mixed integer programming formulation without resorting to a time-space network representation and strengthen it using variable fixing and valid inequalities. Next, we relax the

capacity constraints in a Lagrangian manner and show that the relaxed problem decomposes into a series of shortest path problems defined on networks augmented by time for each commodity. The corresponding Lagrangian dual yields a lower bound, which may be stronger than the one of the linear programming relaxation of our first formulation. Then we provide an extended formulation, a formulation based on a time-space network representation, whose linear programming relaxation gives the same bound as the Lagrangian dual. The time-space network that forms the basis for this formulation has a polynomial size in the size of the original service network. Finally, we report the results of our computational study where we compare the strength and solution times for these two formulations. We observe that our first formulation strengthened with the valid inequalities outperforms the time-space network based extended formulation, even though the latter formulation is stronger and the size of the time-space network grows polynomially with the size of the original network. Hence our main contribution is the derivation of this smaller size formulation and the associated valid inequalities, which enable us to solve larger instances for which the extended formulation based on the time-space network representation is computationally not efficient.

Below, we first mention some important survey papers and then we summarize the work done in the field of intermodal routing.

Bontekoning et al. [6] and Macharis and Bontekoning [15] survey the inland intermodal freight transportation literature. Macharis and Bontekoning [15] classify the studies in this field using two criteria: type of operator and time horizon of operations. They identify four types of operators, namely, drayage, terminal, network, and intermodal operators, and three planning levels, namely, strategic, tactical and operational. Our problem is an operational level planning problem faced by an intermodal service operator according to this classification and there are only a few studies in this category.

Crainic and Kim [11] survey the literature on problems encountered in operating a container-based intermodal transportation network. A survey of the studies on cargo shipping and planning problems in maritime transportation is given by Christiansen et al. [10]. Bektas and Crainic [4] give an overview of intermodal transportation with the shipper and the carrier perspectives and discuss several problems encountered in service network design, operational planning, and intermodal terminals. Pedersen [18] summarizes the development of intermodal transportation in Europe, outlines important optimization problems encountered in this domain, and studies service scheduling problems.

Barnhart and Ratliff [2] consider the problem of transporting commodities on an uncapacitated rail-truck network with the aim of minimizing the total transportation and inventory costs. When the transportation cost is charged on a per trailer basis, the problem decomposes into a set of shortest path problems, one for each commodity. However, when the charge is on a per flatcar basis, the problem is modeled as a matching problem under the assumption that each flatcar can transport at most two trailers. Extensions to incorporate duedates and restrictions on flatcar configurations are discussed. A chance constrained goal programming model to determine intermodal routes to minimize cost and risks is given by Min [16]. On time service requirements are included in this model. Boardman et al. [5] describe a decision sup-

port system to choose the least cost combination of transportation modes on an intermodal network. Bookbinder and Fox [7] study the problem of finding optimal intermodal routes for Canada-Mexico shipments under NAFTA. Intermodal alternatives on a network connecting five major Canadian cities and three major Mexican cities are evaluated and nondominated solutions with respect to total time and total cost objectives are identified. Kim et al. [14] consider the problem of determining the flow and the number of vehicles of each transportation mode on a network in order to minimize the sum of shipping and transportation costs under capacity and vehicle restrictions. They derive a mixed integer programming model and present a case study based on the container cargo data in Korea. Ziliaskopoulos and Wardell [22] present an algorithm to compute intermodal time-dependent least travel time paths. The algorithm takes into account the delays during the change of modes and the fixed schedules. Boussedjra et al. [8] develop an exact method to find the least travel time path between an origin destination pair in a time-dependent intermodal transportation network. They use the number of transshipments as a second criterion if there exists a tie between two solutions. Song and Chen [20] consider the problem of finding a least cost path between an origin destination pair with a delivery time limit on an intermodal network with scheduled services. The total cost is the sum of transportation cost, the transition cost, and the holding cost. They show that the problem can be modeled as a shortest path problem on a time-space network. Grasman [13] presents dynamic programming approaches to compute minimum cost path with on time service requirements and minimum lead time path subject to cost restrictions for intermodal networks.

The two closest studies to our problem are the ones by Chang [9] and Moccia et al. [17]. Chang [9] studies the problem of determining routes of commodities on international intermodal networks. The problem is a multiobjective intermodal multicommodity flow problem with time windows and concave costs. The objectives are minimizing the total transportation cost and the total travel time. The transportation cost is a concave piecewise linear function of total flow. A time window is associated with each node to define the earliest and latest arrival times to the node in order to use the services originating from this node. Both scheduled and time-flexible services are considered. A commodity departs from a node after its service time or leaves the node at a scheduled departure time if it is using a scheduled service. The demand of a commodity may be split among several paths. This problem is converted into a single objective problem by taking a weighted sum of the objectives. Then Lagrangian relaxation is applied to decompose the problem. A heuristic to construct feasible solutions from the solutions of the relaxed problem is presented. Finally the method is tested on a small illustrative example and a larger network.

The problem studied by Moccia et al. [17] generalizes the problem in Chang [9] by considering multiple time windows for pickup and deliveries and general (not necessarily concave or convex) piecewise linear transportation costs. However, a single objective, the one of minimizing the total transportation cost, is considered and single path routing is imposed as a constraint in this study. The authors describe how to represent their problem using a directed network where multiple copies of nodes are created and some additional nodes are introduced to be able to model the multiple time windows and scheduled departure times. Then two formulations, an arc-node

and a path formulation, are given along with valid inequalities. Column generation algorithms are designed to obtain lower bounds and are combined with heuristics to get feasible solutions. The methods are tested on data obtained from an Italian freight forwarder that uses rail and road transportation.

Our problem can be seen as a particular case of the problem that is studied by Moccia et al. [17], where pickup time windows have lower and upper limits equal to the release times and the delivery time windows are equal to the intervals starting at the release times and ending at the duedates. Different from the problem of Moccia et al. [17], we include penalty costs for waiting times spent at the seaports. However, it is possible to handle these costs with the approach presented in Moccia et al. [17]. The approach we present here is tailored for our special case.

The paper is organized as follows. In Sect. 2, we give a detailed definition of our problem *IMR-S* and prove that it is NP-hard. A first mixed integer programming formulation is given in Sect. 3. In Sect. 4, we present results on variable fixing and valid inequalities. Section 5 presents a Lagrangian relaxation and an extended formulation derived using this relaxation. The results of a computational study comparing the first model and the extended model and investigating the use of valid inequalities are reported in Sect. 6. The paper is concluded with further research directions in Sect. 7.

2 Problem definition and complexity

Here, we first define our problem more formally and then establish its computational complexity. We are given a directed network $G = (N, A)$, which possibly contains parallel arcs. Each node in this network corresponds to a seaport and each arc corresponds to a scheduled maritime service. For service $a \in A$, s_a denotes the origin seaport, t_a denotes the destination seaport, $[e_a, l_a]$ denotes the time window (e_a is the start time of loading and l_a is the cutoff time), c_a denotes the cost of transporting a unit volume, τ_a denotes the travel time, and u_a denotes the capacity of service a.

We denote by p_i the cost of stocking a unit volume at seaport $i \in N$ for a unit time.

The set of commodities is denoted by K. A commodity $k \in K$ has origin o_k, destination d_k, demand volume w_k, release time r_k, and duedate q_k.

For $k \in K$, let $c_{o_k d_k}$ denote the unit cost of transporting commodity k from its origin to its destination directly using trucks and $\tau_{o_k d_k}$ denote the corresponding travel time. For $k \in K$ and $i \in N$, let $c_{o_k i}$ and $\tau_{o_k i}$ denote the unit cost and time for carrying commodity k from its origin to seaport i, and $c_{i d_k}$ and $\tau_{i d_k}$ denote the unit cost and time for carrying commodity k from seaport i to its destination directly using trucks, respectively.

For a given commodity $k \in K$, a time feasible intermodal route is a simple path that starts at the origin o_k at time r_k, visits a set of seaports using a set of maritime services respecting the cutoff times, and ends at the destination d_k not later than the duedate q_k. The cost of such a route is the sum of routing and waiting costs. For each commodity, we decide either to route the commodity directly on trucks or to route it on a time feasible intermodal route respecting the capacities of maritime services with the aim of minimizing cost.

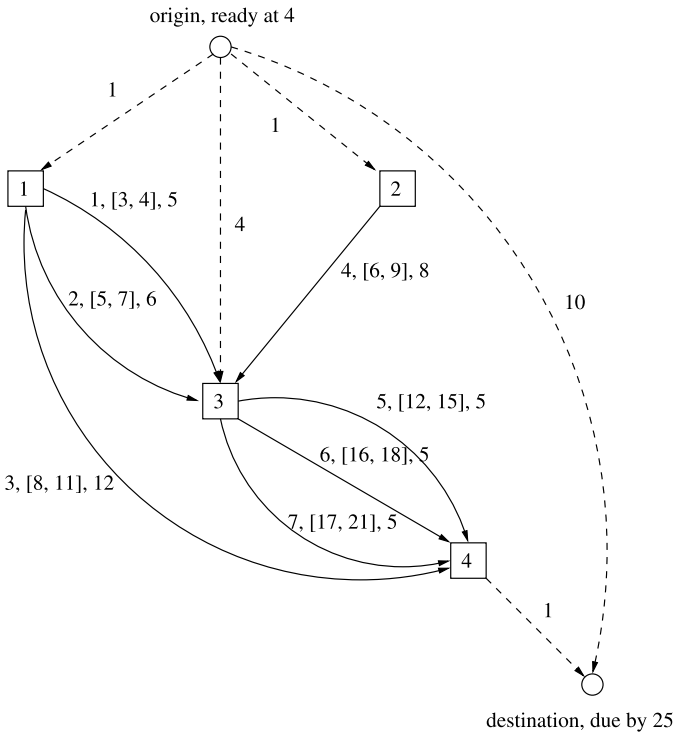

Fig. 1 An example service network

We provide an example in Fig. 1 to clarify the definitions. Here, we have four seaports and seven maritime services. Hence, $N = \{1, \ldots, 4\}$ and $A = \{1, \ldots, 7\}$. We consider a single commodity whose release time is 4 and duedate is 25. Maritime services are represented with solid lines and truck connections are represented with dashed lines. To keep the example small, we assume that truck connections from the origin to seaport 4 and from seaports 1, 2, and 3 to the destination are not available. The numbers on truck connections are the corresponding travel times. For each maritime service, we give the number of the service, the time window [the start time of loading, the cutoff time], and the travel time. In our example, it is possible to transport the commodity from its origin to its destination using trucks in 10 days. It is also possible to go to seaports 1, 2, and 3 by trucks. If the commodity is transported to seaport 1 on trucks, it arrives there at time 5. It is not possible to use maritime service 1 as its cutoff time is equal to 4, which is earlier than the arrival time of our commodity. Maritime services 2 and 3 can be used. If service 2 is used, then the commodity is loaded without waiting, leaves the seaport 1 at time 7 and arrives at seaport 3 at time 13. On the other hand, if service 3 is used, then the commodity is stocked at the seaport 1 for 3 units of time until the start of loading. Then it leaves seaport 1 at time 11 and arrives at seaport 4 at time 23. Alternatively, the commodity can be transported using trucks to seaport 2, where it waits for 1 unit of time until the start of loading, leaves seaport 2 using maritime service 4 at time 9, and arrives at seaport 3

at time 17. There are three maritime services leaving seaport 3 and arriving at seaport 4. As the cutoff time for service 5 is equal to 15, it can be used only if we arrive at seaport 3 using direct truck connection from the origin or using maritime service 2. Maritime service 7 cannot be used, as it arrives at seaport 4 at time 26, which is later than the duedate. Overall, there are seven time feasible routes for our commodity:

1. direct truck connection from origin to destination (arrival time 14)
2. truck connection from origin to seaport 1, waits at seaport 1 for 3 units of time, maritime service 3, truck connection from seaport 4 to destination (arrival time 24)
3. truck connection from origin to seaport 1, maritime service 2, maritime service 5, truck connection from seaport 4 to destination (arrival time 21)
4. truck connection from origin to seaport 1, maritime service 2, waits at seaport 3 for 3 units of time, maritime service 6, truck connection from seaport 4 to destination (arrival time 24)
5. truck connection from origin to seaport 2, waits at seaport 2 for 1 unit of time, maritime service 4, maritime service 6, truck connection from seaport 4 to destination (arrival time 24)
6. truck connection from origin to seaport 3, waits at seaport 3 for 4 units of time, maritime service 5, truck connection from seaport 4 to destination (arrival time 21)
7. truck connection from origin to seaport 3, waits at seaport 3 for 8 units of time, maritime service 6, truck connection from seaport 4 to destination (arrival time 24).

We define the decision version of the problem *IMR-S* as follows. Given the parameters of the problem and a scalar γ_0, does there exist a feasible solution with cost not more than γ_0? Next, we prove that this problem is NP-complete using a polynomial reduction from the 0-1 knapsack problem.

Proposition 1 *The decision version of IMR-S is NP-complete.*

Proof The decision version of *IMR-S* is in NP. Consider the decision version of the 0-1 knapsack problem. Given a set I, nonnegative integers α_i and β_i for each $i \in I$ and two positive scalars α_0 and β_0, does there exist a subset $S \subseteq I$ such that $\sum_{i \in S} \alpha_i \leq \alpha_0$ and $\sum_{i \in S} \beta_i \geq \beta_0$? This problem is NP-complete (see Garey and Johnson [12] problem [MP9]) even when $\alpha_i = \beta_i$ for all $i \in I$.

Suppose that there are only two seaports 1 and 2, and that there is a single service from 1 to 2 with capacity α_0. For each item i in set I, define a commodity from node 1 to node 2 with demand volume equal to α_i. The release time is 0 and the duedate is 1 for all commodities. Commodities can be transported at no cost and no time from their origins to seaport 1 and from seaport 2 to their destinations. The maritime service starts at time 0 and ends at time 1. The unit cost of using the service is equal to 1 and the unit cost of delivering a commodity using trucks is 2. Under this specification of parameters, the *IMR-S* reduces to the problem of finding a minimum cost partition of the set of commodities I into two sets S and $I \setminus S$ such that commodities S are transported using the maritime service and commodities $I \setminus S$ are transported using trucks. Such a partition is feasible if $\sum_{i \in S} \alpha_i \leq \alpha_0$. The cost of the associated solution is $\sum_{i \in S} \alpha_i + \sum_{i \in I \setminus S} 2\alpha_i = 2 \sum_{i \in I} \alpha_i - \sum_{i \in S} \alpha_i$. Let $\gamma_0 = 2 \sum_{i \in I} \alpha_i - \beta_0$.

Now there exists a solution to the 0-1 knapsack problem with $\beta_i = \alpha_i$ for all $i \in I$ if and only if there exists a solution to the decision version of the problem *IMR-S*. □

3 A mixed integer programming model

In this section, we present a mixed integer programming model for *IMR-S*. In this model, we use the following decision variables. For $k \in K$, y^k is 1 if commodity k is transported directly from its origin to its destination on trucks and 0 otherwise. For $k \in K$ and $i \in N$, $x^k_{o_k i}$ is 1 if commodity k is carried from its origin directly to seaport i using trucks and 0 otherwise and $x^k_{i d_k}$ is 1 if commodity k is carried from seaport i to its destination directly using trucks and 0 otherwise. For $k \in K$ and $a \in A$, x^k_a is 1 if commodity k uses maritime service a and 0 otherwise. Finally, for $k \in K$ and $i \in N$, v^k_i denotes the arrival time of commodity k at seaport i if this seaport is visited by this commodity and z^k_i is the amount of time for which commodity k waits at the stocking area at seaport i.

Using the above decision variables, we derive the following mixed integer programming formulation called *M1*.

$$\min \quad \sum_{k \in K} w_k \left(\sum_{i \in N} c_{o_k i} x^k_{o_k i} + \sum_{a \in A} c_a x^k_a + \sum_{i \in N} c_{i d_k} x^k_{i d_k} + c_{o_k d_k} y^k + \sum_{i \in N} p_i z^k_i \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{i \in N} x^k_{o_k i} + y^k = 1 \quad \forall k \in K \tag{2}$$

$$\sum_{a \in A: t_a = i} x^k_a + x^k_{o_k i} - \sum_{a \in A: s_a = i} x^k_a - x^k_{i d_k} = 0 \quad \forall k \in K,\ i \in N \tag{3}$$

$$\sum_{i \in N} x^k_{i d_k} + y^k = 1 \quad \forall k \in K \tag{4}$$

$$\sum_{a \in A: t_a = i} x^k_a + x^k_{o_k i} \le 1 - y^k \quad \forall k \in K,\ i \in N \tag{5}$$

$$\sum_{k \in K} w_k x^k_a \le u_a \quad \forall a \in A \tag{6}$$

$$v^k_i = \sum_{a \in A: t_a = i} (l_a + \tau_a) x^k_a + (r_k + \tau_{o_k i}) x^k_{o_k i} \quad \forall k \in K,\ i \in N \tag{7}$$

$$z^k_i \ge \sum_{a \in A: s_a = i} e_a x^k_a - v^k_i \quad \forall k \in K,\ i \in N \tag{8}$$

$$v^k_i \le \sum_{a \in A: s_a = i} l_a x^k_a + (q_k - \tau_{i d_k}) x^k_{i d_k} \quad \forall k \in K,\ i \in N \tag{9}$$

$$r_k + \tau_{o_k d_k} y^k \le q_k \quad \forall k \in K \tag{10}$$

$$x^k_{o_k i}, x^k_{i d_k} \in \{0, 1\} \quad \forall k \in K,\ i \in N \tag{11}$$

$$x_a^k \in \{0, 1\} \quad \forall k \in K,\ a \in A \tag{12}$$

$$y^k \in \{0, 1\} \quad \forall k \in K \tag{13}$$

$$z_i^k \geq 0 \quad \forall k \in K,\ i \in N. \tag{14}$$

Constraints (2)–(4), (12), and (13) ensure that either a commodity is transported from its origin to its destination directly using trucks or there exists a path from its origin to its destination that visits only seaports on its way. Constraints (5) eliminate cycles. First observe that there always exists an optimal solution without any cycle that does not intersect the path of a commodity, since costs are nonnegative. It is also not possible to have a cycle that starts and ends at the origin or destination of a commodity. Hence it is sufficient to ensure that each commodity can arrive at a seaport at most once to eliminate cycles.

Constraints (6) ensure that total volume to be transported on a service cannot exceed the capacity of the service.

Constraints (7) compute the arrival times of commodities to seaports. If commodity k arrives at seaport i using service a, then it leaves the origin seaport s_a at time l_a and travels for τ_a time units, and hence arrives at i at time $l_a + \tau_a$. If seaport i is the first seaport that commodity k visits, i.e., if commodity k leaves its origin and comes to i using trucks, then r_k is the time trucks leave the origin node and $\tau_{o_k i}$ is the trip time. Hence, the arrival time is $r_k + \tau_{o_k i}$. If commodity k does not visit seaport i then its arrival time is taken to be zero.

If a commodity k arrives at seaport i before loading starts for its service, then it is stocked at the seaport until the start time of loading for the service that it uses to leave seaport i. Constraints (8) and (14) compute the amount of time for which commodity k is stocked at seaport i.

Constraints (9) ensure that the commodities arrive before the cutoff time of the service they would like to use and that they arrive at their destinations no later than the duedates. Finally, constraints (10) avoid direct shipments by trucks if the commodity cannot be on time using this transportation mode. These constraints can be dropped by setting $y^k = 0$ for commodities $k \in K$ such that $r_k + \tau_{o_k d_k} > q_k$.

The objective function (1) is the sum of the cost of transportation and the cost of stocking at seaports.

Several problem variations can be modeled as follows.

In the above problem definition, we assume that commodities are picked up at their origins at their release times. Hence a commodity that uses maritime services arrives at the first seaport i at time $r_k + \tau_{o_k i}$. If this time is earlier than the start time of the maritime service that this commodity uses, then a stocking cost is incurred. However, some clients may have depots where they can stock their goods until the pickup time at no cost. In that case, it may be cheaper to pickup these commodities later than their release times. Let K' be the set of such commodities. To model this variant, we replace constraints (8) with constraints

$$z_i^k \geq \sum_{a \in A: s(a) = i} e_a x_a^k - v_i^k \quad \forall k \in K \setminus K', i \in N \tag{15}$$

$$z_i^k \geq \sum_{a \in A: s(a)=i} e_a x_a^k - v_i^k - M x_{o(k)i} \quad \forall k \in K', i \in N, \tag{16}$$

where M is a large number. Constraints (15) are the same as constraints (8) and model the waiting times for commodities that should be picked up at their release times. Constraints (16) are for clients that have depots to stock their goods until their pickup times. These constraints ensure that the waiting time at seaport $i \in N$ is bounded below by a nonpositive number if i is the first seaport that commodity $k \in K'$ visits on its path. If i is not the first seaport, i.e., if $x_{o(k)i} = 0$, then these constraints are the same as (15).

To model situations where the departure times of maritime services are later than the cutoff times, we replace constraints (7) with constraints

$$v_i^k = \sum_{a \in A: t_a = i} (b_a + \tau_a) x_a^k + (r_k + \tau_{o_k i}) x_{o_k i}^k \quad \forall k \in K,\ i \in N$$

where b_a denotes the departure time for maritime service $a \in A$.

In our model, we assume that the stocking penalties are linear functions of the waiting time. We can easily model the situation where there is a period of stocking free of charge and there is a limit on waiting times. Suppose that at seaport $i \in N$, it is possible to stock goods at no cost for a period of length χ_i, after which a cost of π_i units is charged per unit volume and unit time and the stocking period cannot be longer than ν_i units of time. We can model these requirements by introducing the variables η_i^k for all $k \in K$ and $i \in N$, adding the constraints

$$\begin{aligned} \eta_i^k &\geq z_i^k - \chi_i && \forall k \in K,\ i \in N \\ z_i^k &\leq \nu_i && \forall k \in K,\ i \in N \\ \eta_i^k &\geq 0 && \forall k \in K,\ i \in N, \end{aligned}$$

and replacing the objective function by

$$\min \sum_{k \in K} w_k \left(\sum_{i \in N} c_{o_k i} x_{o_k i}^k + \sum_{a \in A} c_a x_a^k + \sum_{i \in N} c_{i d_k} x_{i d_k}^k + c_{o_k d_k} y^k + \sum_{i \in N} \pi_i \eta_i^k \right).$$

For those seaports $i \in N$ for which the stocking cost is positive, i.e., $\pi_i > 0$, in an optimal solution $\eta_i^k = (z_i^k - \chi_i)^+$, and hence no stocking cost is paid for the first χ_i units of the waiting time, and a stocking cost is charged for each unit of extra waiting time.

4 Variable fixing and valid inequalities

In this section, we first present a simple result, which enables us to fix the values of some of the variables. Then we derive some valid inequalities based on time restrictions.

Let F denote the set of feasible solutions to model *M1*.

Algorithm 1 Computation of earliest arrival times for a given commodity $k \in K$

for all $i \in N$ **do**
  $E_i^k \leftarrow r_k + \tau_{o_k i}$
  $temp(i) \leftarrow 1$
end for
while $\sum_{i \in N} temp(i) \neq 0$ **do**
  Let $i^* \in \arg\min\{E_i^k : i \in N, temp(i) = 1\}$(break ties arbitrarily)
  $temp(i^*) \leftarrow 0$
  for all $a \in A$ **such that** $s_a = i^*$ and $temp(t_a) = 1$ **do**
    if $E_{i^*}^k \leq l_a$ and $E_{t(a)}^k > l_a + \tau_a$ **then**
      $E_{t(a)}^k \leftarrow l_a + \tau_a$
    end if
  end for
end while

For each commodity $k \in K$ and node $i \in N$, let E_i^k be the earliest time at which commodity k can arrive at node i. These values can be computed using Algorithm 1, which is a modification of Dijkstra's algorithm (see [1]). Let $k \in K$. The algorithm starts with assigning the value $r_k + \tau_{o_k i}$ to E_i^k for all $i \in N$. At each iteration, a list of temporarily labeled nodes is kept. Initially the set of temporarily labeled nodes is equal to the set of nodes. A temporarily labeled node, say i^*, with the smallest E_i^k value is chosen as the next permanently labeled node. Then, for each service a that starts at node i^*, ends at a temporarily labeled node $t(a)$, and whose cutoff time is not earlier than $E_{i^*}^k$, we compute $l_a + \tau_a$. This is the time we arrive at the destination seaport $t(a)$ using maritime service a. If this value is less than the current value of $E_{t_a}^k$, then we update $E_{t_a}^k$ as $l_a + \tau_a$. In other words, for $a \in A$ with $s_a = i^*$, $t(a)$ temporarily labeled, and $E_{i^*}^k \leq l_a$, we update $E_{t_a}^k$ with $\min\{E_{t_a}^k, l_a + \tau_a\}$. The algorithm stops when no temporarily labeled node is left.

If the demand volume of a commodity exceeds the capacity of a service or if the earliest time at which this commodity can arrive at the origin seaport of a service is later than the cutoff time for that service, then clearly, this commodity cannot use that service. Hence, we have the following result:

Proposition 2 *Let $k \in K$ and $a \in A$. Every feasible solution satisfies $x_a^k = 0$ if $w_k > u_a$ or $E_{s_a}^k > l_a$.*

Consider commodity $k \in K$ and seaport $i \in N$. If commodity k travels directly from its origin node o_k to seaport i, then it arrives there at time $r_k + \tau_{o_k i}$. It is not possible for this commodity to use any service for which the origin seaport is node i and the cutoff time is earlier than $r_k + \tau_{o_k i}$. The set $O_i^k = \{a \in A : s_a = i, r_k + \tau_{o_k i} > l_a\}$ is the set of such services.

Similarly, if commodity k travels from seaport i directly to its destination d_k, then it should be ready to depart from i the latest at time $q_k - \tau_{i d_k}$. Hence any service a that arrives at seaport i later than this time cannot be used by commodity k. The set of such infeasible services is $D_i^k = \{a \in A : t_a = i, l_a + \tau_a + \tau_{i d_k} > q_k\}$.

Proposition 3 *For $k \in K$ and $i \in N$, the inequalities*

$$x^k_{o_k i} + \sum_{a \in O^k_i} x^k_a \leq 1 - y^k \tag{17}$$

and

$$x^k_{i d_k} + \sum_{a \in D^k_i} x^k_a \leq 1 - y^k \tag{18}$$

are valid for F.

Proof We give the proof for inequality (17). For commodity $k \in K$, if $y^k = 1$, then commodity k is transported from its origin to its destination directly on trucks and hence all associated x variables are zero. Otherwise, if $x^k_{o_k i} = 1$, then by definition of set O^k_i, commodity k cannot use any service in this set and hence $\sum_{a \in O^k_i} x^k_a = 0$. Finally, if $y^k = 0$ and $x^k_{o_k i} = 0$, then $\sum_{a \in O^k_i} x^k_a \leq 1$ as commodity k has to be carried on a simple path. The proof for inequality (18) is similar. □

For $a \in A$, define $\Delta^+_a = \{a' : s_{a'} = t_a, l_a + \tau_a > l_{a'}\}$ and $\Delta^-_a = \{a' : t_{a'} = t_a, l_{a'} + \tau_{a'} \geq l_a + \tau_a\}$. Observe that if a commodity k uses service a, then the time it reaches the destination seaport t_a is $l_a + \tau_a$. If a' is a service that starts at node t_a and if the cutoff time for this service is earlier than $l_a + \tau_a$, then the commodity cannot use this service together with service a. Hence, the set Δ^+_a is the set of services that start at node t_a and that cannot be used if service a is used to reach node t_a. The set Δ^-_a is the set of services that arrive at node t_a not earlier than service a.

Proposition 4 *Let $k \in K$ and $a \in A$.*

If $r_k + \tau_{o_k t_a} \leq l_a + \tau_a$ and $l_a + \tau_a + \tau_{t_a d_k} \leq q_k$, the inequality

$$\sum_{a' \in \Delta^-_a} x^k_{a'} + \sum_{a' \in \Delta^+_a} x^k_{a'} \leq 1 - y^k \tag{19}$$

is valid for F.

If $r_k + \tau_{o_k t_a} > l_a + \tau_a$ and $l_a + \tau_a + \tau_{t_a d_k} \leq q_k$, the inequality

$$x^k_{o_k t_a} + \sum_{a' \in \Delta^-_a} x^k_{a'} + \sum_{a' \in \Delta^+_a} x^k_{a'} \leq 1 - y^k \tag{20}$$

is valid for F.

If $r_k + \tau_{o_k t_a} \leq l_a + \tau_a$ and $l_a + \tau_a + \tau_{t_a d_k} > q_k$, the inequality

$$\sum_{a' \in \Delta^-_a} x^k_{a'} + \sum_{a' \in \Delta^+_a} x^k_{a'} + x^k_{t_a d_k} \leq 1 - y^k \tag{21}$$

is valid for F.

If $r_k + \tau_{o_k t_a} > l_a + \tau_a$ *and* $l_a + \tau_a + \tau_{t_a d_k} > q_k$, *the inequality*

$$x^k_{o_k t_a} + \sum_{a' \in \Delta_a^-} x^k_{a'} + \sum_{a' \in \Delta_a^+} x^k_{a'} + x^k_{t_a d_k} \leq 1 - y^k \tag{22}$$

is valid for F.

Proof We give the proof for inequality (22). If $y^k = 1$, then the vector x^k is a zero vector. If $y^k = 0$ and $x^k_a = 1$, then $\sum_{a' \in \Delta_a^+} x^k_{a'} = 0$ since service a arrives at seaport t_a later than the cutoff time for these services, $x^k_{t_a d_k} = 0$ since it is not possible to meet the duedate as $l_a + \tau_a + \tau_{t_a d_k} > q_k$, $x^k_{o_k t_a} + \sum_{a' \in \Delta_a^- \setminus \{a\}} x^k_{a'} = 0$ since commodity k arrives at seaport t_a using service a and cannot use any other service that ends at the same seaport and cannot be carried to this seaport from its origin. Finally, if $y^k = 0$ and $x^k_a = 0$, then we know that $x^k_{o_k t_a} + \sum_{a' \in \Delta_a^-} x^k_{a'} \leq 1$ and $\sum_{a' \in \Delta_a^+} x^k_{a'} + x^k_{t_a d_k} \leq 1$ since commodity k can arrive at and leave seaport t_a at most once. Moreover, all services in set Δ_a^- arrive at t_a too late to be able to use any service from set Δ_a^+ or for the commodity to be delivered to its destination on time. Hence at most one of the sums $x^k_{o_k t_a} + \sum_{a' \in \Delta_a^-} x^k_{a'}$ and $\sum_{a' \in \Delta_a^+} x^k_{a'} + x^k_{t_a d_k}$ can be 1. □

Before concluding this section, we note that an alternative formulation can be obtained by dropping constraints (9) and using inequalities (17)–(22). But these inequalities do not necessarily imply (9) in the linear programming relaxation. Hence, we keep (9) in our model when we add valid inequalities (17)–(22) to obtain a stronger formulation.

5 Lagrangian relaxation and an extended formulation

In the first model given above, the only constraints that link different commodities together are the capacity constraints (6). Next, we investigate a Lagrangian relaxation where these constraints are dualized. Let α_a be the Lagrange multiplier associated with constraint (6) for service $a \in A$. For a given vector $\alpha \geq 0$, the relaxed problem disaggregates into $|K|$ problems as follows.

$$LR(\alpha) = -\sum_{a \in A} \alpha_a u_a + \sum_{k \in K : r_k + \tau_{o_k d_k} \leq q_k} w_k \min\{LR^k(\alpha), c_{o_k d_k}\} + \sum_{k \in K : r_k + \tau_{o_k d_k} > q_k} w_k LR^k(\alpha)$$

where

$$LR^k(\alpha) = \min \quad \sum_{i \in N} c_{o_k i} x^k_{o_k i} + \sum_{a \in A} (c_a + \alpha_a) x^k_a + \sum_{i \in N} c_{i d_k} x^k_{i d_k} + \sum_{i \in N} p_i z^k_i \tag{23}$$

$$\text{s.t.} \quad \sum_{i \in N} x^k_{o_k i} = 1 \tag{24}$$

$$\sum_{a\in A:t_a=i} x_a^k + x_{o_k i}^k - \sum_{a\in A:s_a=i} x_a^k - x_{id_k}^k = 0 \quad \forall i \in N \tag{25}$$

$$\sum_{i\in N} x_{id_k}^k = 1 \tag{26}$$

$$\sum_{a\in A:t_a=i} x_a^k + x_{o_k i}^k \leq 1 \quad \forall i \in N \tag{27}$$

$$v_i^k = \sum_{a\in A:t_a=i} (l_a + \tau_a) x_a^k + (r_k + \tau_{o_k i}) x_{o_k i}^k \quad \forall i \in N \tag{28}$$

$$z_i^k \geq \sum_{a\in A:s_a=i} e_a x_a^k - v_i^k \quad \forall i \in N \tag{29}$$

$$v_i^k \leq \sum_{a\in A:s_a=i} l_a x_a^k + (q_k - \tau_{id_k}) x_{id_k}^k \quad \forall i \in N \tag{30}$$

$$x_{o_k i}^k, x_{id_k}^k \in \{0, 1\} \quad \forall i \in N \tag{31}$$

$$x_a^k \in \{0, 1\} \quad \forall a \in A \tag{32}$$

$$z_i^k \geq 0 \quad \forall i \in N. \tag{33}$$

For a given commodity $k \in K$, the aim of the above problem is to find a time feasible intermodal path from the origin of commodity k to its destination that minimizes the sum of transportation and stocking costs using scheduled services for a unit flow. The cost of using service $a \in A$ is equal to $c_a + \alpha_a$ in this problem.

If it is feasible to route commodity k directly from its origin to its destination, i.e., if $r_k + \tau_{o_k d_k} \leq q_k$, then to compute the contribution of commodity k to the objective function value of the relaxed problem, we take the minimum cost choice among a best intermodal route and the direct shipment. If $r_k + \tau_{o_k d_k} > q_k$, then the contribution of commodity k to the objective function value of the relaxed problem is equal to the routing and stocking costs on a best intermodal route computed solving the related subproblem.

In the sequel, we assume that cycles do not occur due to the cost structure. For a given commodity $k \in K$, the above subproblem can be solved using a shortest path algorithm on an acyclic graph. For $i \in N$, let $I_i = \{a \in A : t_a = i\}$ be the set of services for which i is the destination seaport and $\Gamma_i^k = \bigcup_{a\in I_i} \{l_a + \tau_a\} \cup \{r_k + \tau_{o_k i}\}$ be the set of possible arrival times at node i. Observe that the size of set Γ_i^k is at most the number of incoming arcs of node i in the original graph G plus 1. Define an auxiliary graph $G_k = (N_k, A_k)$ where $N_k = \{o_k, d_k\} \cup \{(i, j) : i \in N, j \in \Gamma_i^k\}$, $O_k = \{(o_k, (i, r_k + \tau_{o_k i})) : i \in N\}$, $D_k = \{((i, j), d_k) : (i, j) \in N_k, j + \tau_{id_k} \leq q_k\}$, $S_k = \{((i_1, j_1), (i_2, j_2)) : (i_1, j_1), (i_2, j_2) \in N_k, \exists a \in A : s_a = i_1, t_a = i_2, j_1 \leq l_a, l_a + \tau_a = j_2\}$, and $A_k = O_k \cup D_k \cup S_k$. The node set N_k includes the origin and destination of commodity k and one node for each seaport i and a possible arrival time at i. The arc set O_k consists of all arcs from the origin to nodes $(i, r_k + \tau_{o_k i})$, which represent the seaports and the associated arrival times from the origin using trucks. The arc set D_k consists of all arcs from nodes (i, j) to the destination d_k such that it is possible

to arrive at the destination by the duedate if we leave seaport i using trucks at time j. Finally, the arc set S_k consists of arcs from node (i_1, j_1) to node (i_2, j_2) such that there exists a maritime service from seaport i_1 to seaport i_2 whose cutoff time is not earlier than j_1 and whose arrival time at seaport i_2 is equal to j_2. In other words, arc $((i_1, j_1), (i_2, j_2))$ is in set S_k if and only if it is possible to travel from seaport i_1 to seaport i_2 using a maritime service that arrives at seaport i_2 at time j_2 if we arrive at seaport i_1 at time j_1.

First remark that graph G_k does not contain parallel arcs. Second, the number of nodes in this graph is in the order of the number of maritime services, i.e., $O(|A|)$. Third, if we assume that all travel times are positive, then the graph G_k is acyclic and can be lexicographically ordered by sorting the nodes (i, j) in nondecreasing order of j.

Consider a path from node o_k to node d_k in this graph. If this path uses an arc $((i_1, j_1), (i_2, j_2))$, then the commodity arrives at seaport i_1 at time j_1 and then travels directly to seaport i_2 and arrives there at time j_2. As this arc exists in graph G_k, we know that there exists at least one maritime service that departs from seaport i_1 and arrives at the destination seaport i_2 at time j_2 and for which the cutoff time is not earlier than j_1. Hence a path from node o_k to node d_k in graph G_k is a path from node o_k to node d_k that satisfies all time restrictions.

Next, we define the lengths of the arcs. The length of arc $(o_k, (i, r_k + \tau_{o_k i})) \in O_k$ is equal to $c_{o_k i}$ and the length of arc $((i, j), d_k) \in D_k$ is equal to $c_{i d_k}$. Finally, the length of an arc $((i_1, j_1), (i_2, j_2)) \in S_k$ is equal to $\min_{a \in A: s_a = i_1, t_a = i_2, j_1 \le l_a, l_a + \tau_a = j_2} (c_a + \alpha_a + p_{i_1}(e_a - j_1)^+)$. Here we look at all maritime services that travel from seaport i_1 to seaport i_2, whose cutoff times are not earlier than j_1 and whose arrival times are j_2. If a is such a service and if the commodity uses this service, then it waits at seaport i_1 until the start time of loading, i.e., for $(e_a - j_1)^+$ units of time.

Song and Chen [20] use a graph generation procedure similar to ours. The difference is that they use both arrival and departure times in defining their nodes, whereas we only use arrival times.

Now $LR^k(\alpha)$ is equal to the length of a shortest path from the origin o_k to the destination d_k in graph G_k. As the size of the graph G_k grows polynomially with the size of the problem, $LR(\alpha)$ can be computed efficiently.

The Lagrangian dual bound is $LD = \max_{\alpha \ge 0} LR(\alpha)$. This bound is at least as good as the linear programming bound of model *M1*. As the subproblems are shortest path problems, we can derive an extended formulation whose linear programming relaxation yields the same bound as the Lagrangian dual.

For each commodity $k \in K$, we define a graph where a simple path from origin o_k to destination d_k defines a trip that starts at o_k, ends at d_k, and respects all time restrictions. We define the costs of arcs on the new graph in such a way that the sum of costs of arcs on a path is equal to the transportation and stocking cost for the corresponding trip. The problem *IMR-S*, then, is equivalent to the problem of finding a path for each commodity k such that the capacity constraints are satisfied.

For each $k \in K$, we define the following graph $G'_k = (N_k, A'_k)$. The set A'_k consists of arcs of O_k and D_k and a set of service arcs S'_k defined as follows. For each service $a \in A$ and nodes (i_1, j_1) and (i_2, j_2) in N_k such that $s_a = i_1$, $t_a = i_2$, $j_1 \le l_a$ and $l_a + \tau_a = j_2$, we add an arc ω from node (i_1, j_1) to node (i_2, j_2) with cost $\sigma_\omega =$

$c_a + p_{i_1}(e_a - j_1)^+$. In other words, if it is possible to go from seaport i_1 to seaport i_2 using maritime service a that leaves i_1 at a time not earlier than j_1 and arrives at i_2 at time j_2, then we add arc ω. The unit cost associated with arc ω is equal to the unit transportation cost using maritime service a plus the cost of waiting at seaport i_1 from time j_1 until time e_a, the start time of loading for service a.

We define $\rho(\omega) = a$. For arc $\omega = (o_k, (i, r_k + \tau_{o_k i})) \in O_k$, we define $\sigma_\omega = c_{o_k i}$ and for arc $\omega = ((i, j), d_k) \in D_k$, we define $\sigma_\omega = c_{i d_k}$.

Note that different from graph G_k, in graph G'_k we may have parallel arcs with different costs and/or capacities. Still, the number of arcs remains polynomial in the number of maritime services.

For the extended formulation, we define the following additional variables. Let $k \in K$. For arc $\omega = (o_k, (i, r_k + \tau_{o_k i})) \in O_k$, let f^k_ω be 1 if commodity k travels directly from its origin to seaport i and arrives there at time $r_k + \tau_{o_k i}$ and 0 otherwise. For arc $\omega = ((i, j), d_k) \in D_k$, let f^k_ω be 1 if commodity k arrives at seaport i at time j and travels from i directly to its destination and 0 otherwise. Finally, for arc $\omega \in S'_k$ from node (i_1, j_1) to (i_2, j_2), we define f^k_ω to be 1 if commodity k arrives at seaport i_1 at time j_1 and uses service $\rho(\omega)$, which departs from seaport i_1 and arrives at seaport i_2 at time j_2 and 0 otherwise.

Let $\delta^-(k, i1, j1)$ and $\delta^+(k, i1, j1)$ be the sets of incoming and outgoing arcs of node $(i_1, j_1) \in N_k$ in graph G'_k.

The extended formulation, *M2* is as follows.

$$\min \quad \sum_{k \in K} w_k \left(\sum_{\omega \in A_k} \sigma_\omega f^k_\omega + c_{o_k d_k} y^k \right) \tag{34}$$

$$\text{s.t.} \quad \sum_{\omega = (o_k, (i, r_k + \tau_{o_k i})) \in O_k} f^k_\omega + y^k = 1 \quad \forall k \in K \tag{35}$$

$$\sum_{\omega \in \delta^+(k, i1, j1)} f^k_\omega - \sum_{\omega \in \delta^-(k, i1, j1)} f^k_\omega = 0 \quad \forall k \in K,\ (i_1, j_1) \in N_k \tag{36}$$

$$\sum_{\omega = ((i, j), d_k) \in D_k} f^k_{(i, j), d_k} + y^k = 1 \quad \forall k \in K \tag{37}$$

$$\sum_{k \in K} w_k \sum_{\omega \in S'_k : \rho(\omega) = a} f^k_\omega \le u_a \quad \forall a \in A \tag{38}$$

$$r_k + \tau_{o_k d_k} y^k \le q_k \quad \forall k \in K \tag{39}$$

$$f^k_\omega \in \{0, 1\} \quad \forall k \in K,\ \omega \in A'_k \tag{40}$$

$$y^k \in \{0, 1\} \quad \forall k \in K. \tag{41}$$

Constraints (35)–(37) and (39)–(41) ensure that each commodity is routed on a path that satisfies all time related constraints. Constraints (38) are the capacity constraints for the maritime services.

Here we remark that model *M2* uses a time-space network to model time related constraints. However, unlike most models based on time-space networks, the size of

model *M2* does not depend on the length of the planning horizon and is polynomial in the number of commodities and number of services. This is due to the fact that the services are scheduled and it is sufficient to consider only the arrival times at seaports in generating the time-space network.

In the above formulation, if we define the direct shipments using arcs (o_k, d_k), then we obtain an origin-destination integer multicommodity flow problem (see, e.g., Barnhart et al. [3]).

To conclude this section, we compare the strength of bounds we can obtain from the linear programming relaxations of the models with and without variable fixing and valid inequalities and the Lagrangian dual bound.

Let *M1T* be model *M1* together with the constraints $x_a^k = 0$ for all $k \in K$ and $a \in A$ with $E_{s_a}^k > l_a$ and inequalities (17)–(22). In other words, *M1T* is model *M1* strengthened with variable fixing and valid inequalities based on time restrictions. Let *M1TC* be model *M1T* together with the constraints $x_a^k = 0$ for all $k \in K$ and $a \in A$ with $w_k > u_a$, i.e., the model *M1* strengthened with variable fixing and valid inequalities based on time restrictions and capacity constraints.

Let *M2C* be model *M2* together with the constraints $\sum_{\omega \in S'_k : \rho(\omega) = a} f_\omega^k = 0$ for all $k \in K$ and $a \in A$ with $w_k > u_a$. Finally let *LDC* be the Lagrangian dual bound when the same Lagrangian relaxation is applied to model *M1* together with the constraints $x_a^k = 0$ for all $k \in K$ and $a \in A$ with $w_k > u_a$. Note that these additional constraints can be handled by ignoring the corresponding services in constructing graph G_k.

For a given model M, let $LP(M)$ be the optimal value of its linear programming relaxation. The following theorem gives a comparison of the linear programming bounds of several models and the Lagrangian Dual.

Theorem 1 *$LD = LP(M2) \geq LP(M1T)$ and $LDC = LP(M2C) \geq LP(M1TC)$.*

Proof We give the proof for $LD = LP(M2) \geq LP(M1T)$.

We first prove that the Lagrangian dual bound *LD* is equal to the linear programming bound of *M2*.

First observe that $LR(\alpha) = -\sum_{a \in A} \alpha_a u_a + \sum_{k \in K} w_k \overline{LR}^k(\alpha)$ where

$$\begin{aligned}
\overline{LR}^k(\alpha) = \min \quad & \sum_{\omega = (o_k, (i, r_k + \tau_{o_k i})) \in O_k} \sigma_\omega f_\omega^k + \sum_{\omega \in S'_k} (\sigma_\omega + \alpha_{\rho(\omega)}) f_\omega^k \\
& + \sum_{\omega = ((i,j), d_k) \in D_k} \sigma_\omega f_\omega^k + c_{o_k d_k} y^k \\
\text{s.t.} \quad & \sum_{\omega = (o_k, (i, r_k + \tau_{o_k i})) \in O_k} f_\omega^k + y^k = 1 && (42) \\
& \sum_{\omega \in \delta^+(k, i1, j1)} f_\omega^k - \sum_{\omega \in \delta^-(k, i1, j1)} f_\omega^k = 0 \quad \forall (i_1, j_1) \in N_k && (43) \\
& \sum_{\omega = ((i,j), d_k) \in D_k} f_\omega^k + y^k = 1 && (44)
\end{aligned}$$

$$f_\omega^k \in \{0, 1\} \quad \forall \omega \in A'_k$$
$$y^k \in \{0, 1\}.$$

As the above problem is a shortest path problem, the convex hull of its feasible solutions is described by constraints (42)–(44) and nonnegativity constraints on variables. Hence the Lagrangian dual bound *LD* is equal to the linear programming bound of *M2* (see, e.g., Wolsey [21]).

Now suppose that we apply the Lagrangian relaxation to the model *M1T*. As the constraints $x_a^k = 0$ for all $k \in K$ and $a \in A$ with $E_{s_a}^k > l_a$ and inequalities (17)–(22) are redundant in the relaxed problem, we have that $LD \geq LP(M1T)$.

The second statement can be proved in a similar way. □

6 Computational results

In this chapter, we first describe the test data and then present the results obtained by solving our integer models.

We use two service networks in our computational study. The first one is the network of the third party logistics company mentioned in the Introduction. This network includes 34 important seaports in Europe, Russia, China, Hong Kong, Singapore, and Malaysia and 167 services of the world's major sea transportation operators. The schedules of services are obtained from the company. These schedules are for the services provided in March 2008. The other parameters we obtained from the company are the costs and capacities.

We named the company's network as O1. We also generated a random network O2 with 66 nodes and 1200 arcs. For the instances on network O1, we randomly generate the demands. For the other instances, we generate all data randomly.

For each service network, we generate 4 core instances with different number of commodities, namely, $|K|$ is equal to 400, 600, 800, and 1000.

For each core instance, we multiply the arc capacities by a capacity factor to create three different instances with tight, medium, and loose capacities. The capacity factor takes the values $2/3$, 1, and 2. As a result, we have 24 problem instances. An instance is named as follows: name of the service network-capacity factor-number of commodities.

We solve our models using CPLEX 11.0. All runs are taken on a 2.67 GHz 2×Quadcore Xeon Processor with 8 GB Ram.

To compare the performances of the two models and investigate the effect of variable fixing and valid inequalities, we solve each problem instance using models *M1*, *M1v* (the first model with valid inequalities), *M1f* (the first model with variable fixing), *M1TC* (the first model with variable fixing and valid inequalities), *M2C* (the extended model with variable fixing based on capacity restrictions). For each instance and model, we report the duality gap, i.e., the percentage gap between the optimal value of the integer model and the optimal value of its linear programming relaxation, the number of nodes in the branch and cut tree and the cpu time in seconds. We impose a time limit of one hour. For the instances that are not solved to optimality

Table 1 The duality gaps for O1 instances

Instance	*M1*	*M1v*	*M1f*	*M1TC*	*M2C*
O1-2/3-400	22.52	18.12	5.64	1.72	0.53
O1-2/3-600	16.35	13.63	4.92	1.76	0.40
O1-2/3-800	13.40	11.23	5.11	2.41	1.00
O1-2/3-1000	11.39	9.96	4.79	2.74	1.74
O1-1-400	19.56	12.69	12.38	2.59	1.71
O1-1-600	14.73	10.27	10.45	3.90	1.68
O1-1-800	12.36	8.73	9.53	4.77	1.71
O1-1-1000	10.44	7.81	8.18	4.82	1.73
O1-2-400	16.91	4.08	16.87	4.06	1.04
O1-2-600	13.52	4.24	13.50	4.22	0.75
O1-2-800	12.59	5.10	12.55	5.08	0.81
O1-2-1000	11.08	5.35	11.02	5.30	0.86

Table 2 The number of nodes for O1 instances

Instance	*M1*	*M1v*	*M1f*	*M1TC*	*M2C*
O1-2/3-400	830	0	786	0	0
O1-2/3-600	–	0	–	0	1252
O1-2/3-800	–	0	–	0	5235
O1-2/3-1000	–	67	–	67	–
O1-1-400	1054	0	992	0	1520
O1-1-600	–	1	–	1	10195
O1-1-800	–	1	–	1	–
O1-1-1000	–	409	–	409	–
O1-2-400	2887	10	2887	10	2729
O1-2-600	–	96	–	96	–
O1-2-800	–	221	–	201	–
O1-2-1000	–	1259	–	1252	–

in one hour, we report the remaining percentage gap, as reported by the solver, in parenthesis in tables reporting cpu times.

In Tables 1, 2 and 3, we report the duality gaps, the number of nodes, and the cpu times for the instances on O1 network, respectively. We report the number of nodes for the instances and models for which optimality could be proved in one hour.

In Table 1, we observe that with model *M1*, the duality gaps decrease as the number of commodities and the capacity factor increase. The effect of variable fixing is stronger for the problems with tight capacities. This is expected as one of the variable fixing rules is based on capacity constraints. The addition of valid inequalities results in higher improvements for problems with larger capacity factors. After variable fixing and the addition of valid inequalities, the duality gaps are improved significantly but are still much higher compared to the duality gaps of model *M2C*.

Table 3 The cpu times for O1 instances

Instance	*M1*	*M1v*	*M1f*	*M1TC*	*M2C*
O1-2/3-400	480.6	38.7	441.5	37.9	4.4
O1-2/3-600	1h(0.18)	58.0	1h(0.16)	57.9	42.8
O1-2/3-800	1h(0.44)	76.0	1h(0.44)	67.4	197.9
O1-2/3-1000	1h(0.39)	109.3	1h(0.38)	108.6	1h(0.02)
O1-1-400	812.4	47.5	779.3	38.7	18.1
O1-1-600	1h(0.58)	80.1	1h(0.58)	64.6	334.2
O1-1-800	1h(18.13)	108.7	1h(18.13)	105.6	1h(0.05)
O1-1-1000	1h(21.12)	155.8	1h(21.10)	194.4	1h(0.11)
O1-2-400	1131.4	47.4	1022.2	46.2	31.5
O1-2-600	1h(1.74)	94.7	1h(1.74)	75.5	1h(0.13)
O1-2-800	1h(2.44)	137.4	1h(2.44)	138.3	1h(0.21)
O1-2-1000	1h(4.28)	303.3	1h(4.28)	240.9	1h(0.24)

In Tables 2 and 3, we see that only instance with 400 commodities are solved to optimality within one hour of CPU time with model *M1*. Variable fixing does not change this situation but results in a decrease in the number of nodes evaluated and the solution times for the instances solved to optimality. For a few of the unsolved instances, there is a slight decrease in the final gaps due to variable fixing. However, with both models *M1* and *M1f*, final gaps are very high for medium capacity factor problems with 800 and 1000 commodities. With formulation *M2C*, we can solve larger instances to optimality for small capacity factors. However, for the large capacity factor, only the instance with 400 commodities has been solved to optimality within the timelimit. With this formulation, the remaining gaps is less than 0.25% for the unsolved problems.

All instances are solved to optimality in less than six minutes when the valid inequalities are added to model *M1*. For the instances that are solved to optimality with both *M1v* and *M2C*, we see that the solver enumerated a smaller number of nodes with *M1v* even though this model results in larger duality gaps. This is probably due to the set packing structure that is created by the valid inequalities and that enables the solver to generate further valid inequalities.

Finally, we observe that even though variable fixing has a significant effect on the duality gaps for some instances, it does not result in an improvement in solution times for those instances.

Next, we report the results obtained for the instances on the randomly generated network O2. This network contains about seven times more arcs than the original graph O1. The results are given in Tables 4, 5 and 6.

Comparing Tables 1 and 4, we see that the duality gaps are much higher for the random instances with model *M1* for the small capacity factor. As in the case of O1 instances, we see that the effect of valid inequalities is higher for problems with larger capacity factors and the effect of variables fixing is higher for problems with smaller capacity factors. Compared to the gaps with the O1 instances, with the O2 instances,

Table 4 The duality gaps for O2 instances

Instance	*M1*	*M1v*	*M1f*	*M1TC*	*M2C*
O2-2/3-400	43.48	40.47	8.78	4.08	0.84
O2-2/3-600	40.22	37.27	9.72	5.07	1.28
O2-2/3-800	35.55	32.02	10.31	8.26	1.82
O2-2/3-1000	33.61	28.66	10.79	9.20	2.36
O2-1-400	15.97	10.69	14.85	10.38	0.26
O2-1-600	14.91	7.59	14.22	7.41	0.31
O2-1-800	14.86	8.67	14.37	8.53	0.53
O2-1-1000	14.85	13.01	14.48	12.91	0.50
O2-2-400	15.84	10.55	14.59	10.13	0.03
O2-2-600	14.54	7.17	13.77	6.91	0.01
O2-2-800	14.35	7.80	13.79	7.61	0.01
O2-2-1000	14.68	8.79	14.26	8.65	0.03

Table 5 The number of nodes for O2 instances

Instance	*M1*	*M1v*	*M1f*	*M1TC*	*M2C*
O2-2/3-400	1640	0	1589	0	0
O2-2/3-600	–	0	–	0	0
O2-2/3-800	–	0	–	0	0
O2-2/3-1000	–	0	–	0	0
O2-1-400	69	0	69	0	0
O2-1-600	495	0	487	0	0
O2-1-800	–	33	–	33	0
O2-1-1000	–	228	–	228	0
O2-2-400	0	0	0	0	0
O2-2-600	10	0	10	0	0
O2-2-800	–	28	–	28	0
O2-2-1000	–	111	–	111	0

the duality gaps with the first model *M1TC* are higher and the ones with the second model *M2C* are usually smaller.

We could solve larger instances to optimality with model *M1* for the O2 network. Still the instances with 800 and 1000 commodities are not solved to optimality in one hour. The final gaps remain the same after variable fixing. With the addition of valid inequalities, all instances are solved to optimality in maximum six minutes. Model *M2C* could also solve all the instances optimally. From Table 5, we see that these instances are solved at the root node without branching. Even though more nodes are evaluated with model *M1v*, the solution times are shorter compared to the ones with *M2C*.

As a summary, we could not solve large instances to optimality with the first model without the valid inequalities. With the inclusion of the valid inequalities, model 1

Table 6 The CPU times for O2 instances

Instance	*M1*	*M1v*	*M1f*	*M1TC*	*M2C*
O2-2/3-400	240.9	111.5	232.8	101.9	272.1
O2-2/3-600	1h(0.78)	166.4	1h(0.78)	183.7	298.2
O2-2/3-800	1h(2.58)	231.1	1h(2.58)	247.1	503.9
O2-2/3-1000	1h(4.04)	335.0	1h(4.04)	346.2	552.4
O2-1-400	209.1	110.9	198.1	99.3	269.1
O2-1-600	3211.9	151.2	3054.3	136.4	378.5
O2-1-800	1h(1.12)	227.0	1h(1.12)	219.2	468.2
O2-1-1000	1h(2.74)	240.6	1h(2.74)	231.5	548.1
O2-2-400	232.3	103.6	229.1	101.2	254.0
O2-2-600	667.1	162.4	663.0	160.8	362.7
O2-2-800	1h(0.60)	188.2	1h(0.60)	185.3	476.7
O2-2-1000	1h(0.84)	418.7	1h(0.84)	414.2	572.3

outperformed the extended model in terms of computation times and could solve all our instances to optimality in not more than six minutes.

7 Conclusion

In this paper, we considered an intermodal multicommodity routing problem with scheduled and capacitated services. We established the computational complexity of the problem and proposed two mixed integer programming formulations. We compared our formulations using Lagrangian relaxation. We strengthened the first formulation with valid inequalities that modeled conflicts due to time related restrictions. Even though the first model strengthened with valid inequalities is still weaker than the second model, it performed better in terms of solution times. We managed to solve instances with 1000 commodities to optimality in maximum six minutes for the two networks considered.

As a further research, we are considering to devise heuristic algorithms based on Lagrangian relaxation. We tested some basic versions of this heuristic on the instances used in the computational study, but we could not obtain acceptable gaps in shorter times than the computation times of the first model with valid inequalities. This was mainly because the subgradient algorithm converged very slowly. We investigate ways to make the convergence faster. We also believe that devising alternative heuristic approaches such as tabu search or variable neighborhood search may be a nice contribution.

Another area of research is to extend the problem proposed in this paper to model more general cost structures and to include higher level decisions, such as the choice of maritime service arcs and their schedules.

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