SPATIAL PROPERTIES OF QUANTUM MULTIPOLE RADIATION

A THESIS
SUBMITTED TO THE DEPARTMENT OF PHYSICS
AND THE INSTITUTE OF ENGINEERING AND SCIENCE
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

BY
Muhammet Ali Can
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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

Prof. Alexander S. Shumovsky (Supervisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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Abstract

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Muhammet Ali Can
M. S. in Physics
Supervisor: Prof. Dr. Alexander S. Shumovsky
June 2000

Complete quantum mechanical treatment of multipole radiation is constructed. Vacuum noise of polarization for transversally and longitudinally polarized fields is discussed for different total angular momentum values due to the presence of quantum localized sources. It is shown that the spatial properties of the multipole vacuum noise are independent of the type of the radiation, either electric or magnetic.

New definition of polarization matrix constructed from the field-strength tensor, Ricci Tensor, is introduced. Using Jaynes-Cummings model Hamiltonian for electrical dipole atom, some statistical properties of the radiation are considered.

A new method for polarization measurement at short and intermediate distances from the source, based on the use of optical Aharonov-Bohm effect is proposed which is classified as a quantum nondemolition measurement. This proposed experiment leads to measure the longitudinal polarization and space-time correlation of polarizations of multipole radiation.

Keywords: Quantum multipole radiation, Quantum nondemolition polarization measurement, Quantum optics, Quantum entanglement.

Özet

ÇOK KUTUPLU KUVANTUM IŞINIMININ UZAYSAL ÖZELLİKLERİ

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Tez Yöneticisi: Prof. Dr. Alexander S. Shumovsky
Haziran 2000

Bu çalışmada çok kutuplu ışınımın tam kuvantum mekaniksel ifadesi ele alınmıştır. Enine ve boyuna gelişen alanlar için bu kutuplaşmanın boşluktaki gürültüsü, yerel kaynakların varlığından dolayı, farklı toplam açısal momentumlar için tartışılmıştır. Ayrıca boşluktaki çok kutuplu gürültünün uzaysal özelliklerinin elektriksel ya da manyetik ışınımdan bağımsız olduğu gösterilmiştir.

Diğer taraftan alan-etkili tensor, Ricci tensor, ile kurulan kutuplaşma matrisi tanıtılmıştır. Son olarak elektriksel iki kutuplu atom için Jaynes-Cummings model Hamiltanyen kullanılarak ışınımın bazı istatistiksel özellikleri incelenmistir.

Kaynaktan kısa yada orta mesafede kutuplaşma yada şiddet ölçümlerinde yeni bir metod, etkisiz kuvantum ölçümleri olarak adlandırılan optik Aharanov-Bohm etkisi temel alınarak kullanılmıştır. Bu yeni metod boyuna kutuplaşma ölçümlerini ve çok kutuplu ışınımın kutuplaşmasının uzay-zaman etkileşimini kapsar.

Anahtar sözcükler: Çok kutuplu kuvantum ışınımı, etkisiz kuvantum kutuplaşma ölçümleri, kuvantum bağımlılığı, kuvantum optiği .

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Chapter 1

Introduction

Measurement of polarization is one of the fundamental tools for experimental sciences such as spectroscopy and optics where it is usually considered from the classical point of view.[1] However, since the beginning of new directions such as the quantum computation and quantum information processing, deeper investigation of quantum-mechanical picture of polarization measurement has become to be important. These new topics put the entanglement of photons in the central position. Due to the photon polarization entanglement in the process of creation of a pair of photons, the detection of polarization state of one photon gives information about the state of the second photon.[2] Therefore, in recent years there has been growing interest to understand the quantum nature of polarization and polarization measurement both in theoretical and experimental ways.[3]

According to the classical point of view, the polarization phenomena is determined by given direction of oscillations of the electromagnetic field which is considered to be a transversal one. The polarization is usually calculated as though the radiation field consists of the monochromatic (or quasi-monochromatic) plane waves. In this case, the field strengths $\vec{E}$ and $\vec{B}$ are orthogonal to the direction of propagation $\vec{k}$ and have equal magnitudes, so that the polarization can be considered as the measure of transversal anisotrophy of either of the field strengths. The quantitative description of polarization is based on the use of the quadratic forms in the field strengths, forming either the

(2×2) Hermitian polarization (coherence) matrix or corresponding set of Stokes parameters[2,4] (operators, in quantum case[5]). These quadratic forms are chosen to be determined through the intensity measurements of the field..[1]

It is well known that the atomic and molecular transitions emit the multipole radiation represented by spherical electromagnetic waves.[6] In classical picture, either plane or spherical waves can be used since both of them form the complete orthogonal sets of solutions of the wave equation. However, in quantum picture, there is a fundamental difference between these two representations of electromagnetic field. First of all, the plane waves of photons correspond to the states of the field with given linear momentum. At given wave number k, they are specified by only four operators of creation and destruction with two different polarizations.[2,7] At the same time, the spherical waves of photons correspond to the states with given angular momentum. At given k, total angular momentum $j \geq 1$ and parity, they are specified by $2(2j+1) \geq 6$ different operators of creation and destruction.[7,8] Since the components of linear and angular momenta do not commute with each other, the two representations correspond to the physical observables which cannot be measured simultaneously. Therefore, in order to describe the quantum multipole radiation, we have to deal with the spherical waves of photons rather than plane waves.[7]

Quantum electrodynamics interprets the polarization as a given spin state of photons.[9] The spin of a photon, defined as the minimum angular momenta, is known to be 1.[9,10] Hence there are three different spin states of a photon. In the case of plane waves of photons, the projection of spin on the direction of propagation is forbidden and therefore there are only two allowed spin states (polarizations).[9] In this case, the polarization is the quantum number, describing the states of electromagnetic field in all space.

In contrast to the plane waves, all three projections of spin are allowed for the multipole photons, so that there are three different polarizations: two transversal with opposite helicities and one linear in the radial direction with respect to the source of radiation (e.g., see[10]). Such a three-dimensional picture of polarization should be described either by the (3×3) Hermitian polarization matrix[11] or by an equivalent set of the nine Stokes operators, forming a representation of the $SU(3)$ sub-algebra in the Weyl-Heisenberg algebra of multipole photons.[12] It should be stressed that the $SU(3)$ structure of the polarization of multipole radiation is directly connected with the spin $s = 1$ of a photon.

Existence of the third (radial) polarization of the multipole radiation is not a surprising fact. Actually, it is well known that the classical electric multipole radiation can be defined as the transverse magnetic multipole field, while the electric field strength has the radial (longitudinal) component at any point.[6] In turn, the magnetic multipole radiation is the transverse electric field with the longitudinal component of magnetic induction. It is also known that the longitudinal (radial) components of either electric or magnetic classical multipole radiation are important only in the near and intermediate zones when $kr \leq j$, while vanish at far distances. Thus, the polarization can be different at different points with respect to the source. In other words, the polarization is the local characteristics of a classical multipole radiation.

In most of the applications, the detection of the radiation occurs far away from the source of the radiation where the longitudinal component of the field vanishes and spherical waves practically considered to be expressed by plane waves. Here, the wavelength of radiation is the critical parameter. In other words if the wavelength of the radiation is also small then neither near nor intermediate field effects occur. However recent developments, such as high proton polarization and radiation from Rydberg atom, provides very long wavelength of radiation even in order of meters. That is the detector can be placed in these zones where the radiation can not be considered as classical anymore.[13–15] Moreover as a future application, the development of first generation quantum computers can use atoms as logic gates and photons as the communication tools between these atoms which can be separated in the order of wavelengths. In this case as well as the two transversal polarizations which are interpreted as logical bits, qubits, the longitudinal polarization can be used as a new degree of freedom for sending quantum information.

In the measurement proccess it is also important to figure out the noise, coming from the quantum fluctuations of polarization measurement. In other words the spatial dependence of polarization of atoms and molecules can also influence the quantum noise of polarization measurement in a different way at different distances from the source.

Chapter 2

Polarization Properties Of Quantum Multipole Radiation

2.1 Polarization Matrix Of Classical Radiation

In classical electrodynamics, the polarization is defined to be the given direction of oscillations of electromagnetic field. The different spatial components of monochromatic field, oscillating with the same frequency, may have different amplitudes and phases. In most of the conventional books on electrodynamics, the polarization is calculated as the radiation field consists of the monochromatic or quasi-monochromatic plane waves. Here both $\vec{E}$ and $\vec{B}$ have the same magnitudes. They are located in transversal planes orthogonal to the direction of the propagation $\vec{k}$. In this case, the polarization is considered as the measure of transversal anisotrophy of the field. Since a local source like an atom emits the multipole radiation represented by spherical electromagnetic waves, we have to stress the difference between the plane and spherical electromagnetic waves. Then, let us consider first the case of classical monochromatic plane wave described by the positive-frequency part of the vector potential of the form

$$\vec{A}(\vec{r}) = \gamma \sum_{\alpha=x,y} \vec{e}_\alpha e^{i(\vec{k}\cdot\vec{r}-ckt)} a_\alpha, \tag{2.1}$$

where γ is the normalization constant and a_α denotes component of the field amplitude in a plane which is transversal to the direction of propagation $\vec{e}_z = \vec{k}/k$. Then, the field strengths are defined as follows

$$\begin{aligned} \vec{E}(\vec{r}) &= ik\vec{A} = ik\gamma \sum_{\alpha=x,y} \vec{e}_\alpha e^{i(\vec{k}\cdot\vec{r}-ckt)} a_\alpha, \\ \vec{B}(\vec{r}) &= \vec{\nabla} \times \vec{A} = i\gamma \sum_{\alpha=x,y} (\vec{k} \times \vec{e}_\alpha) e^{i(\vec{k}\cdot\vec{r}-ckt)} a_\alpha. \end{aligned} \tag{2.2}$$

Due to the orthogonality and symmetry relations

$$\vec{E} \cdot \vec{B} = 0,$$

$$B_x = -E_y, \quad B_y = E_x,$$

valid for the plane waves, the transversal anisotrophy of the plane wave can be specified by either of the field strengths $\vec{E}$ and $\vec{B}$. Following the conventional choice of the electric field strength, we get the Hermitian polarization (coherence) matrix of the form[2,4]

$$P_{plane} = \begin{pmatrix} E_x^* E_x & E_x^* E_y \\ E_y^* E_x & E_y^* E_y \end{pmatrix} = (k\gamma)^2 \begin{pmatrix} a_x^* a_x & a_x^* a_y \\ a_y^* a_x & a_y^* a_y \end{pmatrix}. \tag{2.3}$$

It should be stressed that, although the mode functions in Eq. 2.1 depend on $\vec{r}$, the polarization matrix Eq. 2.3 is the global object, describing the properties of the field in all space. The diagonal elements of Eq. 2.3 describe, apart from a factor of 1/2, the contribution of the two transversal components into the energy density, while the off-diagonal terms give the "phase information" about the phase difference between the transversal components.[6]

We now turn to the consideration of a classical monochromatic, pure j-pole electromagnetic radiation. To establish connection with quantum case, we shall employ the so-called helicity basis[9] ,[10]

$$\vec{\chi}_\pm = \mp \frac{\vec{e}_x \pm i\vec{e}_y}{\sqrt{2}}, \qquad \vec{\chi}_0 = \vec{e}_z. \tag{2.4}$$

These vectors formally coincide with the spin states of a photon. As usual, we assume that the origin of the reference frame coincides with the localized source (atom). In this basis, the positive-frequency part of the vector potential of the multipole field has the form[7,8,10]

$$\vec{A}_\lambda(\vec{r}) = \sum_{\mu=-1}^{1} (-1)^\mu \vec{\chi}_{-\mu} A_{\lambda\mu}(\vec{r}) e^{-ickt}. \tag{2.5}$$

In the spirit of our philosophy, we can choose to interpret $A_{\lambda\mu}(\vec{r})$ as the component of vector potential with given polarization μ at given point $\vec{r}$ where λ stands for electric or magnetic types of radiation. For the electric multipole radiation

$$\vec{A}_E(\vec{r}) \cdot \vec{r} \neq 0, \qquad [\vec{\nabla} \times \vec{A}_E(\vec{r})] \cdot \vec{r} = 0, \tag{2.6}$$

while the magnetic multipole radiation obeys the conditions

$$\vec{A}_M(\vec{r}) \cdot \vec{r} = 0, \qquad [\vec{\nabla} \times \vec{A}_M(\vec{r})] \cdot \vec{r} \neq 0. \tag{2.7}$$

The explicit form of $A_{\lambda\mu}(\vec{r})$ in Eq. 2.5 is[8,9]

$$A_{\lambda\mu}(\vec{r}) = (-1)^\mu \sum_{m=-j}^{j} V_{\lambda-\mu m}(\vec{r}) a_{\lambda m}, \tag{2.8}$$

where $a_{\lambda m}$ denotes the global field amplitude defined in all space. In the classical picture, $a_{\lambda m}$ is usually specified in terms of the source functions.[6,16] The mode functions in Eq. 2.8 are defined as follows

$$\begin{aligned} V_{M\mu m} &= \gamma_M f_j(kr) \langle 1, j, \mu, m-\mu | jm \rangle Y_{j,m-\mu}, \\ V_{E\mu m} &= \gamma_E [\sqrt{j} f_{j+1}(kr) \langle 1, j+1, \mu, m-\mu | jm \rangle Y_{j+1,m-\mu} \\ &- \sqrt{j+1} f_{j-1}(kr) \langle 1, j-1, \mu, m-\mu | jm \rangle Y_{j-1,m-\mu}]. \end{aligned}$$

Here γ_λ is the normalization factor, $\langle \cdots | \cdots \rangle$ denotes the Clebsch-Gordon coefficient and $Y_{\ell n}(\theta, \phi)$ are the spherical harmonics. The radial part of the mode functions is defined as follows

$$f_\ell(kr) = \begin{cases} h_\ell^{(1)}(kr), & \text{outgoing spherical wave} \\ h_\ell^{(2)}(kr), & \text{converging spherical wave} \\ j_\ell(kr), & \text{standing spherical wave} \end{cases}, \tag{2.9}$$

depending on the boundary conditions.[16] Here $h_\ell^{(1,2)}$ and j_ℓ denote the spherical Hankel and Bessel functions respectively.

Consider first the case of a monochromatic electric j-pole radiation. Due to the relations Eq. 2.6, the magnetic induction $\vec{B}(\vec{r})$ always oscillates in the transversal plane, while the electric-field strength has an additional (longitudinal) degree of freedom. Thus, the spatial anisotrophy of the radiation field can be specified by the components of the electric field strength. Following,[10] let us choose the polarization (coherence) matrix of the electric multipole radiation in the following form

$$P_E(\vec{r}) = k^2 \begin{pmatrix} A^*_{E+}A_{E+} & A^*_{E+}A_{E0} & A^*_{E+}A_{E-} \\ A^*_{E0}A_{E+} & A^*_{E0}A_{E0} & A^*_{E0}A_{E-} \\ A^*_{E-}A_{E+} & A^*_{E-}A_{E0} & A^*_{E-}A_{E-} \end{pmatrix}, \tag{2.10}$$

where we take into account that $\vec{E} = ik\vec{A}$ for a harmonic field. Unlike Eq. 2.3, the polarization matrix Eq. 2.10 depends on the position with respect to the source.

In the case of magnetic multipole radiation, the electric field strength is always transversal and the spatial anisotrophy of the field is defined by the magnetic induction which also dominates in the near and intermediate zones.[6] Therefore, corresponding polarization matrix should be constructed from the bilinear forms in the components of $\vec{B}(\vec{r})$. Taking into account the reciprocity relation

$$\vec{B}_M(\vec{r}) = \vec{E}_E(\vec{r}) = ik\vec{A}_E(\vec{r}) \tag{2.11}$$

we arrive at conclusion that the spatial structure of polarization of the magnetic multipole radiation is described in the same way as that of the electric multipole radiation. In both cases, the classical polarization matrix depends on the point where the polarization is measured.

2.2 Operator Polarization Matrix

The quantum counterpart of classical relations, discussed in the previous Sec., can be obtained in a standard way by substitution of the photon operators instead of the field amplitudes. For example, in the case of plane waves, we have to subject the field amplitudes in Eq. 2.1 to the Weyl-Heisenberg commutation relations

$$[a_\sigma, a^+_{\sigma'}] = \delta_{\sigma\sigma'},$$

which allows Eq. 2.3 to be cast into the normal-ordered operator polarization matrix

$$P^{(n)}_{plane} = (k\gamma)^2 \begin{pmatrix} a^+_x a_x & a^+_x a_y \\ a^+_y a_x & a^+_y a_y \end{pmatrix}. \qquad (2.12)$$

whose elements are the quadratic forms in the creation and destruction operators. To simplify the notations, hereafter we denote the photon operators by the same letters as the classical field amplitudes. Similar form can also be obtained in any other basis, for example, in the so-called circular polarization basis[5] which coincides with the helicity basis Eq. 2.5 to within the inversion of one of the vectors.

Besides Eq. 2.12, we can determine the *anti-normal* operator polarization matrix

$$P^{(an)}_{plane} = (k\gamma)^2 \begin{pmatrix} a_x a^+_x & a_y a^+_x \\ a_x a^+_y & a_y a^+_y \end{pmatrix},$$

so that the difference

$$P^{(an)}_{plane} - P^{(n)}_{plane} \equiv P^{(vac)}_{plane} = (k\gamma)^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad (2.13)$$

describes the zero point (vacuum) contribution into the polarization of plane waves. In other words, the elements of Eq. 2.13 give the vacuum fluctuations of corresponding elements of the polarization matrix of plane waves. I should be stressed that Eq. 2.13 is independent of position as well as Eq. 2.3 and Eq. 2.12. This means the homogeneity of the vacuum noise of polarization along the direction of propagation.

Taking into account that the multipole field is quantized in much the same way as the plane waves,[7] we have to subject the field amplitudes in Eq. 2.8 to the Weyl-Heisenberg commutation relations (at given k and j):

$$[a_{\lambda m}, a^{+}_{\lambda' m'}] = \delta_{\lambda\lambda'}\delta_{mm'}. \qquad (2.14)$$

Consider again the electric multipole radiation. In analogy with the case of plane waves, we can introduce the normal-ordered operator polarization matrix with the elements

$$\begin{aligned} P^{(n)}_{E\mu\mu'}(\vec{r}) &= k^2 A^{+}_{E\mu} A_{E\mu'} \\ &= k^2(-1)^{(\mu+\mu')} \sum_{m,m'=-j}^{j} V^{*}_{E-\mu m}(\vec{r}) V_{E-\mu' m'}(\vec{r}) a^{+}_{Em} a_{Em'}, \end{aligned} \qquad (2.15)$$

which is the quantum counterpart of Eq. 2.10. Then, the anti-normal operator polarization matrix has the elements

$$P^{(an)}_{E\mu\mu'}(\vec{r}) = k^2(-1)^{(\mu+\mu')} \sum_{m,m'=-j}^{j} V_{E-\mu m}(\vec{r}) V^{*}_{E-\mu' m'}(\vec{r}) a_{Em} a^{+}_{Em'}.$$

Hence, the elements of the vacuum polarization matrix of the electric multipole radiation are

$$P^{(vac)}_{\mu\mu'}(\vec{r}) = k^2(-1)^{(\mu+\mu')} \sum_{m} V^{*}_{E-\mu m} V_{E-\mu' m}. \qquad (2.16)$$

A similar analysis can be performed in the case of the magnetic multipole radiation. In view of the reciprocity relation, the spatial dependence of $P^{(n)}_{M}(\vec{r})$ is similar to that in Eq. 2.15, while the photon operators should be changed by a^{+}_{Mm} and a_{Mm}. It also follows from the commutation relations Eq. 2.14 that the vacuum polarization matrix of the magnetic multipole radiation coincides with Eq. 2.16. Hence, the vacuum noise of polarization is independent of the type of radiative transition in the source (atom). At the same time, the elements of Eq. 2.16 as well as of Eq. 2.15 depend on position in spite of the global nature of the photon operators defined in all space.

2.3 Another Definition For Polarization Matrix

Properties of electromagnetic field is completely described by Maxwell equations whether we treat it as a completely classical or quantum object. Since the field-strength tensor $F^{\mu\nu}$ which is a second rank antisymmetric tensor is constructed by the components of Electric and Magnetic field variables it includes all the physical information. Taking this into account we can define a construction similar to Ricci tensor directly from the field-strength tensor as

$$R = F^{\dagger}F = \begin{pmatrix} W & \vec{S} \\ \vec{S}^{\star} & 2P \end{pmatrix} \tag{2.17}$$

where

$$F^{\mu\nu} = e^{-i\omega t}\begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}. \tag{2.18}$$

The Ricci tensor, R, is represented by the 4×4 Hermitian matrix which consists of the three blocks: scalar W, describing the energy of the field, vector $\vec{S}$, describing the energy flux of the field (Poynting vector), and sub-matrix P, describing the polarization.

Although Maxwell Stress Tensor is a Lorentz invariant object, constructed by another combination of field strength tensor, the new Ricci Tensor is not. This is expected because the polarization matrix which is a sub-matrix of Ricci tensor is a local object. In other words polarization is measured at a definite point in space where the source of the field is located at another point.

It is a straightforward manner to calculate the Ricci tensor for monochromatic plane wave propagating in the positive z direction where both E_z and B_z are zero. Using the orthogonality and symmetry relations between $\vec{E}$ and $\vec{B}$ Eq. 2.2, polarization matrix can be easily obtained as

$$P = \begin{pmatrix} |E_x|^2 & E_x^{\star}E_y & 0 \\ E_y^{\star}E_x & |E_y|^2 & 0 \\ 0 & 0 & \frac{1}{2}(|E_x|^2 + |E_y|^2) \end{pmatrix}. \tag{2.19}$$

It can be seen that this matrix consists of a conventional tensor of polarization in transversal plane and a scalar which is the half of the total intensity. Then we can conclude that this new Ricci tensor gives us compact description of polarization.

Moreover the field strength tensor can be written in terms of spin bases Eq. 2.4 by applying the unitary matrix U which rotates the Cartesian basis to the spin bases. Since the field strength tensor is a 4×4 matrix, the 4×4 unitary matrix can be constructed from U by just adding a phase rotation to the time part as

$$\tilde{U} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & U \end{pmatrix}, \qquad where \qquad U = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{-1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \end{pmatrix}. \tag{2.20}$$

Then the representation of the field strength tensor in spin bases can be found simply

$$F_\chi = \tilde{U} F \tilde{U}^\dagger \tag{2.21}$$

and the Ricci tensor can be written in new basis as follows

$$R_\chi = F_\chi^\dagger F_\chi. \tag{2.22}$$

The same procedure can be followed to obtain the polarization matrix for any type of radiation in any bases. For example for electric type radiation, it is well known that the electric field has a nonzero component along the direction of propagation unlike the magnetic field which is perpendicular to both of the electric field and the direction of propagation. That is if the direction of polarization is in z direction, $B_0 = 0$ but $E_0 \neq 0$. Then the polarization matrix is

$$P = \frac{1}{2} \begin{pmatrix} |E_+|^2 + |B_+|^2 & -E_+^* E_0 & E_+^* E_- - B_+^* B_- \\ -E_0^* E_+ & |E_0|^2 + |B_+|^2 + |B_-|^2 & -E_0^* E_- \\ E_-^* E_+ - B_-^* B_+ & -E_-^* E_0 & |E_-|^2 + |B_-|^2 \end{pmatrix}. \tag{2.23}$$

To see the contributions coming from the electric and magnetic fields, the polarization matrix can be separated into two parts P_E and P_M like,

$$2P = P_E + P_M. \tag{2.24}$$

In the above equation, the first matrix P_E coincides with Eq. 2.10.

This new method simplifies the procedures to obtain the polarization matrix. Moreover we can easily see the contributions coming from the electric and magnetic fields to the polarization matrix. Then it is a straight forward procedure to quantize and find the vacuum fluctuations. Further the new method has a mathematical elegant form which needs to be investigated using differential geometry.

Chapter 3

Spatial Properties Of The Vacuum Noise Of Polarization

It is clear that the spatial structure of the multipole vacuum state is caused by the existence of source (atom) in the origin (in fact, in the "generation zone" of the order of atomic size surrounding the origin). First of all, it is not astonishing that the presence of an atom can influence the electromagnetic vacuum state even in the absence of radiation.[17] Then, due to the spherical symmetry, the vacuum polarization matrix Eq. 2.16 should be independent of the spherical angle θ and ϕ. This intuitively clear statement can be proven in the following way.[18] First, it is easy to prove the $SU(2)$ invariance of the operator vector potential (Eq. 2.8). Then, taking into account that

$$P^{(vac)}_{\mu\mu'}(\vec{r}) = [A_{E\mu}(\vec{r}), A^{+}_{E\mu'}(\vec{r})],$$

we get the $SU(2)$ invariance of $P^{(vac)}$ as well. Hence, the polarization properties of the multipole vacuum state are determined by the distance from the source.

Taking into account the following property of spherical harmonics[19]

$$Y_{j\pm1,m-\mu}(0,\phi) = \sqrt{\frac{2(j\pm1)+1}{4\pi}}\delta_{m\mu},$$

for the mode functions in Eq. 2.8 in the "polar" direction (along $\vec{\chi}_0$) we get

$$V_{E\mu m}(r,0,\phi) \equiv \mathcal{V}_{E\mu}(r) = \frac{\gamma_E}{\sqrt{4\pi}}[\sqrt{j(2j+3)}f_{j+1}(kr)\langle 1,j+1,\mu,0|j\mu\rangle$$
$$-\sqrt{(j+1)(2j-1)}f_{j-1}(kr)\langle 1,j-1,\mu,0|j\mu\rangle]\delta_{m\mu}. \qquad (3.1)$$

Thus, only three states with $m = \mu = 0, \pm 1$ out of $2j+1$ possible multipole states can contribute into the polarization properties of the multipole vacuum in the polar direction. We now insert Eq. 3.1 into Eq. 2.16 to get

$$P^{(vac)}(r,0,\phi) \equiv \mathcal{P}^{(vac)}(r) = \begin{pmatrix} P_\perp & 0 & 0 \\ 0 & P_{\parallel} & 0 \\ 0 & 0 & P_\perp \end{pmatrix}, \qquad (3.2)$$

where

$$P_\perp(r) = k^2|\mathcal{V}_{E\pm}(r)|^2, \qquad P_{\parallel}(r) = k^2|\mathcal{V}_{E0}(r)|^2.$$

Since Eq. 2.16 is the Hermitian matrix, it can be diagonalized at any point $\vec{r}$ by a proper rotation of the reference frame. Due to the $SU(2)$ invariance of the vacuum polarization matrix Eq. 2.16, the diagonal form Eq. 3.2 represents the vacuum polarization matrix in the local frame:

$$U(\vec{r})P^{(vac)}(\vec{r})U^{+}(\vec{r}) = \mathcal{P}^{(vac)}(r), \qquad U(\vec{r})U^{+}(\vec{r}) = \mathbf{1}. \qquad (3.3)$$

It is a straightforward matter to arrive at the following explicit form of the elements of the unitary matrix $U(\vec{r})$:

$$U_{\mu\mu'} = \frac{\Delta_{\mu\mu'} + (1-\Delta_{\mu\mu})\delta_{\mu\mu'}}{\sqrt{1+\sum_{\nu\neq\mu}|\Delta_{\mu\nu}|^2}}.$$

Here $\Delta_{\mu\mu'}(\vec{r})$ is expressed in terms of the elements of matrices Eq. 2.16 and Eq. 3.2 as follows

$$\Delta_{+0} = \frac{1}{\Delta_+}[P_{--}(P_\perp - P_{++}) + |P_{+-}|^2], \qquad \Delta_{+-} = -\frac{1}{\Delta_+}[P_{0-}(P_\perp - P_{++}) + P^*_{+0}P_{+-}],$$
$$\Delta_{0+} = \frac{1}{\Delta_0}[P_{--}(P_{\parallel} - P_{00}) + |P_{0-}|^2], \qquad \Delta_{0-} = -\frac{1}{\Delta_0}[P_{+-}(P_{\parallel} - P_{00}) + P_{+0}P_{0-}],$$
$$\Delta_{-+} = \frac{1}{\Delta_-}[P^*_{+0}(P_\perp - P_{--}) + P_{0-}P^*_{+-}], \qquad \Delta_{-0} = -\frac{1}{\Delta_-}[P_{++}(P_\perp - P_{--}) + |P_{+-}|^2].$$

The following notations are used,

$$\begin{aligned}\Delta_+ &= P^*_{+0}P_{--} - P^*_{+-}P_{0-},\\ \Delta_0 &= P_{+0}P_{--} - P^*_{0-}P_{+-},\\ \Delta_- &= P_{+-}P^*_{+0} - P_{0-}P_{++}.\end{aligned}$$

The elements of the vacuum polarization matrix Eq. 3.2 describe the zero point contribution into circular polarizations $P_\perp(r)$ and linear polarization in the radial (longitudinal) direction $P_{||}(r)$ at any distance r from the source. This contribution strongly depends on the boundary conditions, defining the character of radial dependence (Eq. 2.9) of the mode functions in Eq. 2.8. For example, in the standard case of standing spherical waves, corresponding to the quantization of multipole field in a spherical volume with ideal reflecting walls,[7,10] the transversal and longitudinal zero point contributions into polarization are represented by the damped oscillating functions shown in Fig. 1 for dipole field ($j = 1$). It is seen that the vacuum fluctuations of both the transversal and longitudinal polarizations are very strong at the short distances. Even at the distance of the order of the wavelength where $kr = 2\pi$, $P_{||}$ exceeds $P_\perp$. At the same time, it is seen that $P_{||}(r)$ decays faster than $P_\perp(r)$ at the long distances. It is interesting that there are some points where the vacuum fluctuations of either $P_\perp$ or $P_{||}$ are equal to zero.

Let us stress that the above qualitative dependence of the zero point contribution into polarization on the boundary conditions is not an astounding fact. The dependence of the electromagnetic vacuum on the boundary conditions is traced in the Casimir effect[20] as well.

It should be underlined that the above results were obtained under a "hidden" assumption that j is fixed. In fact, we only know that there is a source (atom) in the origin. The presence of the local source violates the symmetry properties of the possible solution of the wave equation[16] and hence leads the spatial inhomogeneity of the vacuum state. Therefore, the total zero point contribution into polarization should involve summation over all possible $j \geq 1$ in Eq. 3.2. The limit of $P^{(vac)}(r)$ at $kr \gg 1$ coincides with Eq. 2.13, while the vacuum fluctuations

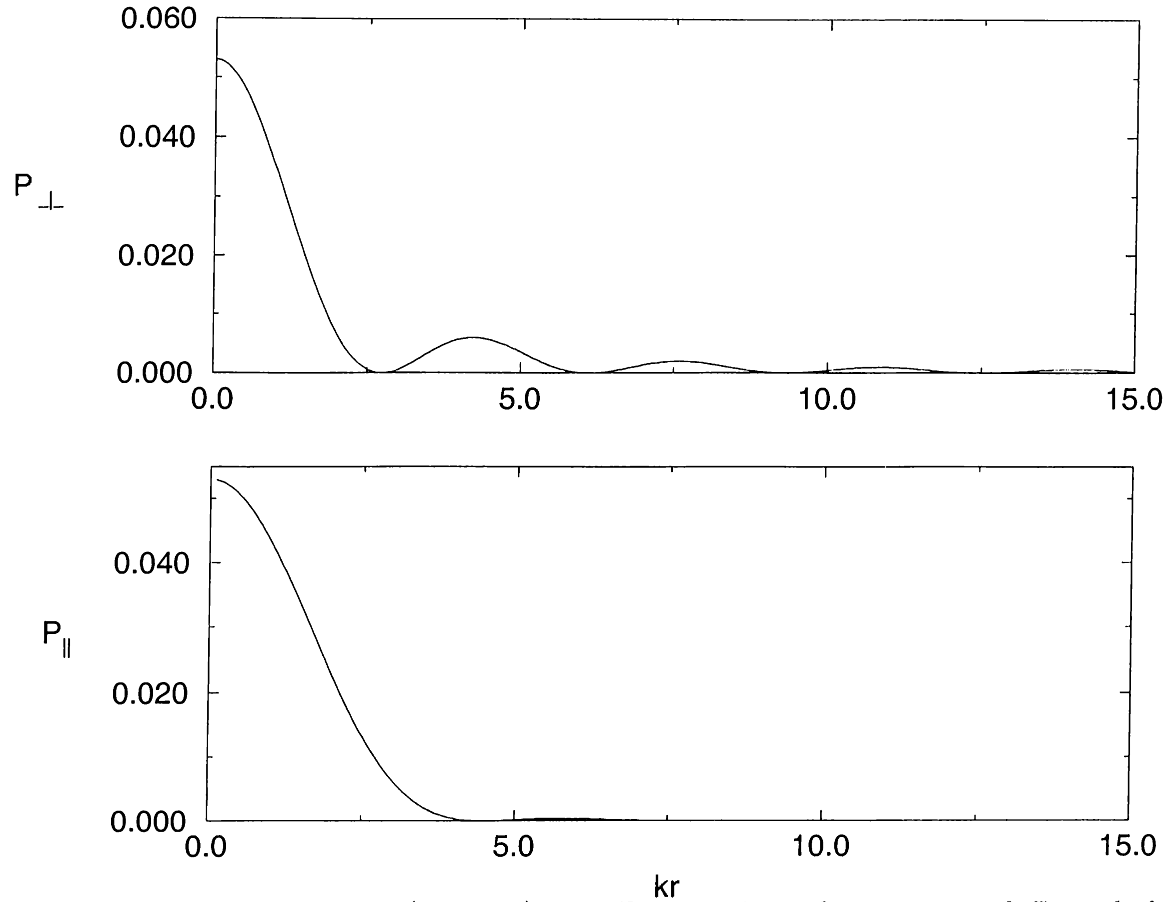

Figure 3.1: Zero point (vacuum) contributions into the transversal $P_{||}$ and the longitudinal $P_{\perp}$ polarizations versus kr for dipole field (j=1), $\theta = 0$, $\phi = 0$

of polarization are much stronger at short and intermediate distances ($kr \leq 2\pi$) than those in dipole case ($j = 1$).

In Fig.2 and Fig.3, the contributions to the vacuum fluctuations for $j = 2$ and $j = 3$ can be seen. It is understood from these figures that the magnitude of the contributions decreases very rapidly. Moreover the peak values for both transversal and longitudinal polarizations shift to the right where $kr > j$. Then for $kr \gg j$ the total fluctuation summed over j approaches to the case of plane waves where the fluctuations are homogeneous.

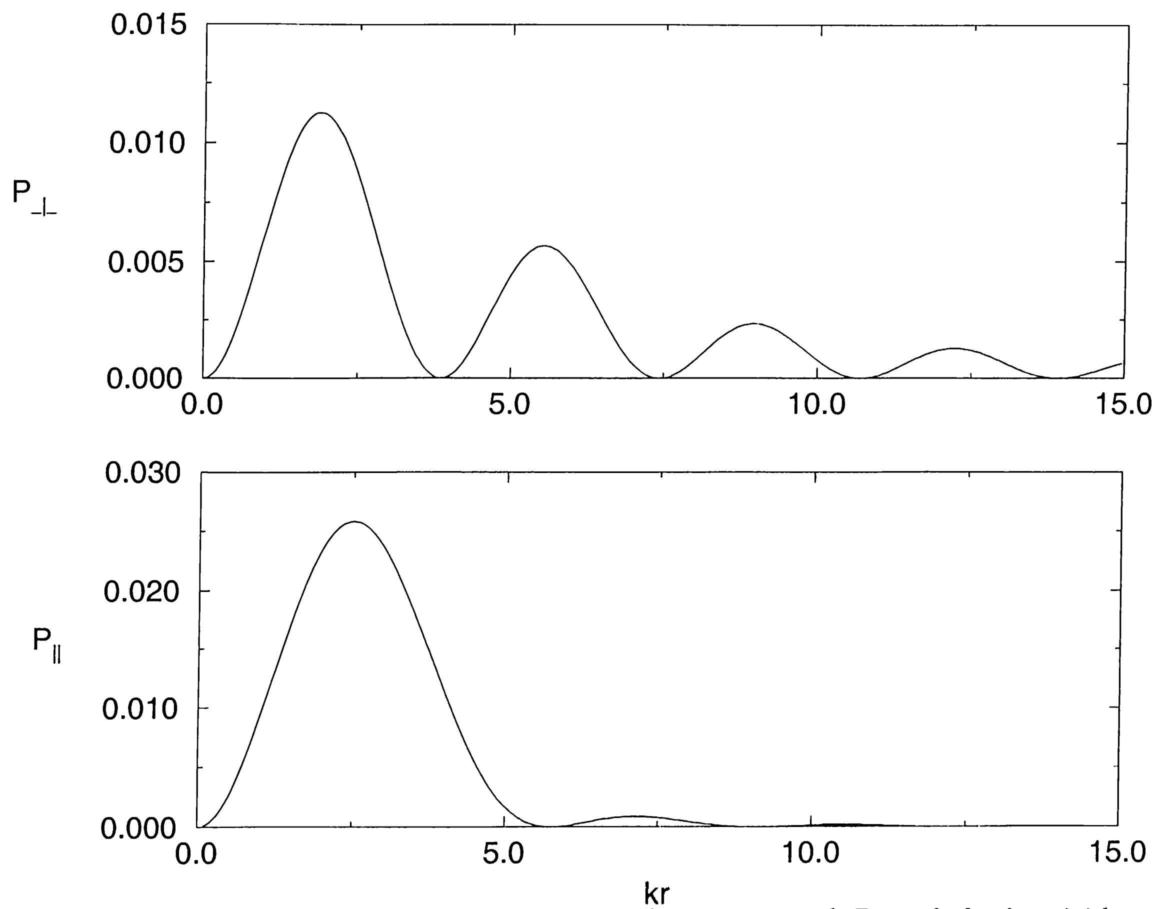

Figure 3.2: Zero point contributions into the transversal $P_{||}$ and the longitidunal $P_{\perp}$ polarizations versus kr for quadrupole (j=2)

3.1 Polarization of multipole radiation

We shall now return to the discussion of the operator polarization matrix Eq. 2.15. Consider first the polar direction when $\theta = 0$ in the mode functions in Eq. 2.8. One can get

$$P^{(n)}_{E\mu\mu'}(r,0,\phi) = k^2 \mathcal{V}^*_{E-\mu}(r)\mathcal{V}_{E-\mu'}(r) a^+_{E\mu} a_{E\mu'}. \tag{3.4}$$

Thus, the photons with $|m| \geq 2$ which may exist at $j \geq 2$ do not contribute into polarization in the polar direction.

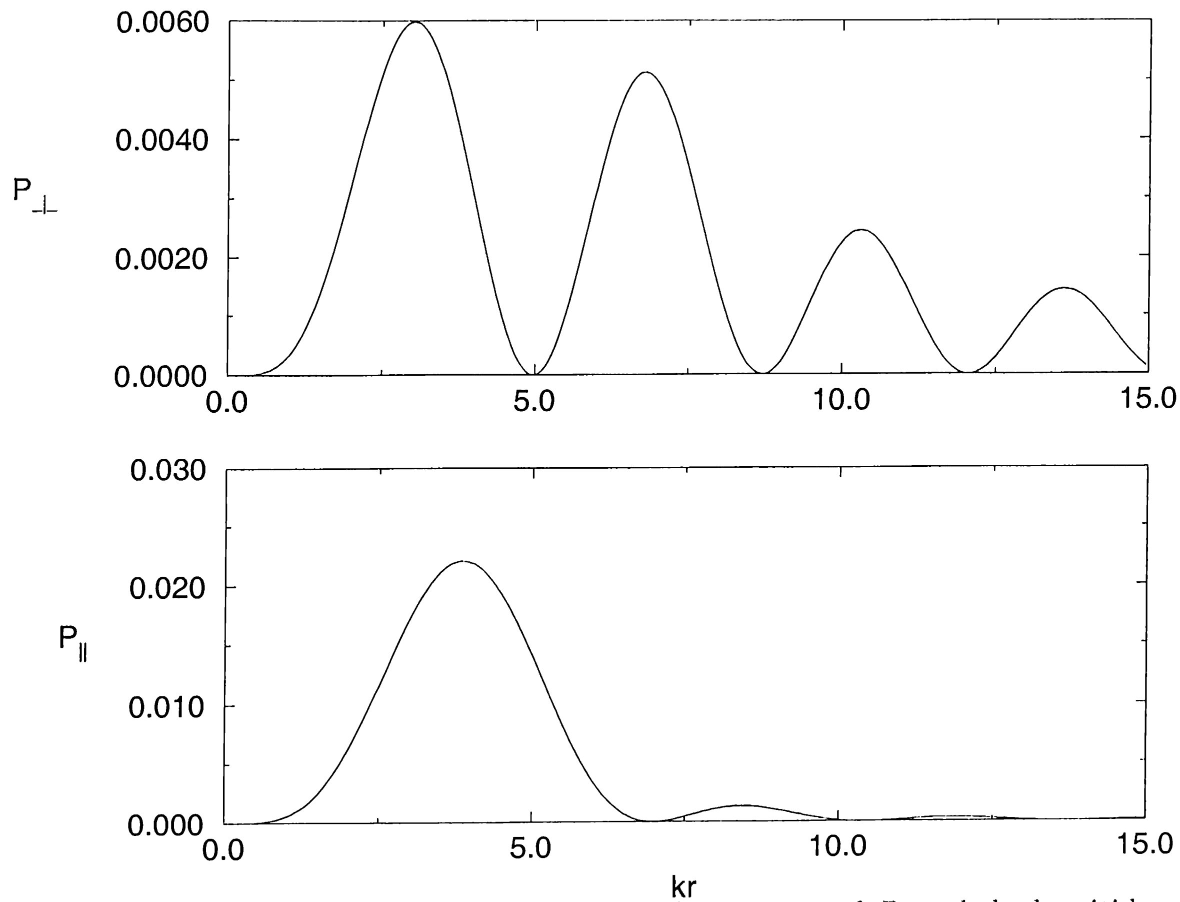

Figure 3.3: Zero point contributions into the transversal $P_{||}$ and the longitidunal $P_{\perp}$ polarizations versus kr for j=3

It should be noted that the local properties of polarization can simply be described in the proper frame which has been introduced in the previous section. Consider the operator Eq. 2.5, Eq. 2.8 as the formal vector-column

$$\vec{A}_E(\vec{r}) = \begin{pmatrix} A_{E+} \\ A_{E0} \\ A_{E-} \end{pmatrix}$$

in the three-dimensional space spanned by the basis Eq. 2.4. It is clear that the local unitary matrix Eq. 3.3 transforms $\vec{A}_E(\vec{r})$ into the operator vector

$$U(\vec{r})\vec{A}_E(\vec{r}) = \begin{pmatrix} \mathcal{A}_{E+} \\ \mathcal{A}_{E0} \\ \mathcal{A}_{E-} \end{pmatrix} \equiv \vec{\mathcal{A}}_E,$$

$$\mathcal{A}_{E\mu}(\vec{r}) = \sum_{\mu'=-1}^{1} u_{\mu\mu'}(\vec{r}) A_{E\mu'}(\vec{r}) = \sum_{\mu'=-1}^{1} u_{\mu\mu'}(\vec{r}) \sum_{m=-j}^{j} V_{E-\mu' m}(\vec{r}) a_{Em}, \quad (3.5)$$

defined in the local proper frame. Employing Eq. 2.8 and Eq. 2.14, it is a straight forward matter to arrive at the commutation relations

$$[\mathcal{A}_{E\mu}(\vec{r}), \mathcal{A}^{+}_{E\mu'}(\vec{r})] = P_{\mu}(\vec{r})\delta_{\mu\mu'}, \quad (3.6)$$

where the position dependent functions in the right-hand side are defined by the equation Eq. 3.2. Then the representation of the operator polarization matrix Eq. 2.15 in the proper frame is specified by the following elements

$$P^{(n)}_{E\mu\mu'}(\vec{r}) = k^2 \mathcal{A}^{+}_{E\mu}(\vec{r}) \mathcal{A}_{E\mu'}(\vec{r}). \quad (3.7)$$

Averaging Eq. 3.7 over the quantum state of radiation, we get the position-dependent polarization (coherence) matrix of the electric multipole radiation. As a particular example, we now consider the multipole radiation in the coherent state $|\alpha\rangle$ such that for all m

$$a_{Em}|\alpha\rangle = \alpha_m|\alpha\rangle. \quad (3.8)$$

Then, from Eq. 3.7 we get

$$\langle P^{(n)}_{E\mu\mu'}(\vec{r})\rangle = k^2 \beta^{*}_{\mu}(\vec{r})\beta_{\mu'}(\vec{r}), \quad (3.9)$$

where β_μ is defined by the condition

$$\mathcal{A}_{E\mu}(\vec{r})|\alpha\rangle = \beta_{\mu}(\vec{r})|\alpha\rangle.$$

According to Eq. 3.5, we get

$$\beta_{\mu}(\vec{r}) = \sum_{m=-j}^{j} \xi_{\mu m}(\vec{r})\alpha_m, \qquad \xi_{\mu m}(\vec{r}) = \sum_{\nu=-1}^{1} u_{\mu\nu}(\vec{r}) V_{\nu m}(\vec{r}).$$

Here $\beta_\mu(\vec{r})$ can be interpreted as the local parameter of coherent state, describing the mean amplitude of the field with given polarization μ at given point $\vec{r}$.

It is now seen that the variance of Eq. 3.7 in the coherent state Eq. 3.8 has, in view of the commutation relation Eq. 3.6, the following form

$$\langle(\Delta P^{(n)}_{E\mu\mu'}(\vec{r}))^2\rangle = k^4 P_\mu(r)\beta^*_\mu(\vec{r})\beta_{\mu'}(\vec{r})\delta_{\mu\mu'}. \quad (3.10)$$

Chapter 4

Dipole Atom As A Source Of Quantum Multipole Radiation

After the long discussion of the properties of the multipole field let us consider now the source of this radiation. It is an electron transition between two states of an atom with well-defined angular momentum and parity. Then spherical waves of photons rather than plane waves should be considered. The atom is considered to be initially in the excited state of the multipole transition and the field is in the vacuum state. In the process of transition, the atom falls into the ground state, while the photon is created.

To understand the quantum nature of this atomic transition, the Jaynes-Cummings model can be used.[21] It is well known that the Jaynes-Cummings model describes fairly well the physical picture of interaction of an atom with the cavity field and at the same time admits an exact solution.[8,22-25] In the usual treatment of the Jaynes-Cummings model, the multipole nature of the atomic transitions[26] is neglected. The process of radiation is described as though an atomic transition radiates a photon with given energy, linear momentum and polarization.[2,8,20,27] This simplified picture overlooks the fact that the multipole atomic transition can radiate and absorb corresponding multipole photon described by the quantized spherical waves.[7,8] In contrast to the case of plane photons, the multipole photons are specified by given energy and angular momentum. Let us stress that the two representations are different in principle. First, the components of linear and angular momenta do not commute and therefore the two representation correspond to the physical quantities which cannot be measured at once. Then, the two representations give different picture of polarization. In fact, polarization of plane waves is described by two vectors orthogonal to the direction of propagation $\vec{k}$. Since a monochromatic, pure j-pole spherical wave of a given type λ (either electric or magnetic) can be expanded over an infinite set of plane waves with all possible directions of $\vec{k}$ on a sphere, the polarization of multipole radiation can have any direction, depending on the choice of observation point.

4.1 Model Hamiltonian and Coupling Constant

Next, let's consider[28,29] the Jaynes-Cummings model, describing an electric dipole transition between the states $|j = 1; m = 0, \pm 1\rangle \equiv ||m\rangle$ and $|j' = 0; m' = 0\rangle \equiv ||g\rangle$ of an atom located at the center of an ideal spherical cavity. Here the former wave function corresponds to the triple-degenerated with respect to the projection of angular momentum excited atomic state, while the latter describes the ground atomic state . The coupling constant of the atom-field interaction can be found from the matrix element[8,30]

$$-\frac{e}{2m_e c}\langle m|\vec{p}\cdot\vec{A}+\vec{A}\cdot\vec{p}|g\rangle = ik_0\langle m|\vec{d}\cdot\vec{A}|g\rangle = ig_m \tag{4.1}$$

where e and m_e denote the charge and mass of electron, respectively $k_0 = \omega_0/c$ is the wave number, corresponding to the transition frequency, $\vec{d} = e\vec{r}$ is the dipole moment, and $\vec{A}(\vec{r})$ denotes the vector potential of the electromagnetic field and $ik\vec{A} \equiv \vec{E}$ is the electric field strength. In the usual treatment of the atom-field interaction,[8,30] the matrix elements in the above equation are calculated as though the cavity field is represented by the plane wave with the operator vector potential

$$\vec{A}(\vec{r}) = \gamma \sum_{\mu=\pm 1} \vec{e}_\mu e^{i\vec{k}\cdot\vec{r}} a_{k\mu} + H.c. \tag{4.2}$$

It should be stressed that the operator vector potential Eq. 4.2 is defined in the so-called circular polarization basis[2]

$$\vec{e}_\pm = (\vec{e}_x \pm i\vec{e}_y)/\sqrt{2}, \qquad \vec{e}_0 = \vec{k}/k. \tag{4.3}$$

In Eq. 4.2, γ denotes the normalization constant and $a_{k\mu}$ is the destruction operator of a photon with energy $\hbar kc$ and polarization μ, propagating along $\vec{k}$. Since the matrix element in Eq. 4.1 is not equal to zero only inside the generation zone of the order of atomic size where $kr \ll 1$, the exponential position-dependent term in Eq. 4.2 is approximated by unit. Such a calculation leads to the same value of the atom-field coupling constant, defined by Eq. 4.1 for all $m = 0, \pm 1$.[10,29]

In contrast to Eq. 4.2, the monochromatic electric dipole filed is described by the operator vector potential[7,10]

$$\vec{A}(\vec{r}) = \sum_{\mu=-1}^{1} (-1)^\mu \vec{\chi}_{-\mu} A_\mu(\vec{r}) \tag{4.4}$$

where the vectors

$$\vec{\chi}_\pm = \mp \frac{\vec{e}_x \pm i\vec{e}_y}{\sqrt{2}}, \qquad \vec{\chi}_0 = \vec{e}_z, \qquad \vec{\chi}_\mu \cdot (-1)^\mu \vec{\chi}_{-\mu'} = \delta_{\mu\mu'}, \tag{4.5}$$

are spin states of spin 1 of a photon and $A_\mu(\vec{r})$ denotes the spherical component of the vector potential

$$A_\pm(\vec{r}) = \mp \frac{1}{\sqrt{2}}[A_x(\vec{r}) \pm iA_y(\vec{r})], \qquad A_0(\vec{r}) = A_z(\vec{r}).$$

Unlike Eq. 4.2, the equation Eq. 4.4 describes the three possible directions of polarization in any point. In fact, it is well known that the electric dipole radiation always has the radial (longitudinal) component of the electric field strength together with the two transversal components.[6,7,10] Let us emphasize that here in contrast to Eq. 4.3 $\vec{\chi}_0$ does not coincide with the direction of $\vec{k}$ but shows an arbitrary radial direction. Moreover, k is the scalar quantity in the case of multipole radiation.[6] Let us point that the use of the base vectors Eq. 4.5 lead to well-known interpretation of the polarization in terms of spin states of photons, forming the radiation field.[9,10] The spherical components of the operator vector potential of the electric dipole radiation are defined at equation Eq. 2.8 with $\lambda = E$ and $j = 1$.

Taking into account that the spin part of the atomic state does not change in the electric dipole transition, we can represent the atomic states under consideration as follows[26]

$$\begin{aligned} ||m\rangle &= \mathcal{R}_{exc}(r)Y_{1m}(\theta,\phi), \\ ||g\rangle &= \mathcal{R}_{ground}(r)Y_{00}(\theta,\phi) = \frac{\mathcal{R}_g(r)}{\sqrt{4\pi}}. \end{aligned}$$

Then, representing $\vec{d}$ in the basis $\{\vec{\chi}_\mu\}$ as,

$$\vec{r} = \frac{r}{\sqrt{2}} sin\theta e^{i\phi}\vec{\chi}_{-1} + rcos\theta\vec{\chi}_0 - \frac{r}{\sqrt{2}} sin\theta e^{-i\phi}\vec{\chi}_1 \tag{4.6}$$

and performing the scalar product $\vec{r}.\vec{A}$ in terms of their spherical components by means of the equation

$$\vec{A}.\vec{B} = \sum_{\mu=-1}^{1} A_\mu B_{-\mu} \tag{4.7}$$

the coupling constant of the atom-field interaction can be found after simple calculations

$$g_m \equiv k_0\langle m|\vec{d}\cdot\vec{A}|g\rangle = \begin{cases} \frac{-\gamma k_0}{\sqrt{6\pi}}(\frac{7}{10}D_2 + D_0) & \text{if } m = \pm 1 \\ \frac{\gamma k_0}{\sqrt{6\pi}}(\frac{1}{5}D_2 - D_0) & \text{if } m = 0. \end{cases} \tag{4.8}$$

where

$$D_\ell = \int_0^{r_a} r^3 R_{ex}^* R_{gr} f_\ell dr \tag{4.9}$$

The radial dependence in Eq. 2.8 and Eq. 2.9, corresponding to the standing waves in the cavity, is given by spherical Bessel functions with half-integer index[6,7,10]

$$f_\ell(kr) = \sqrt{\frac{\pi}{2kr}} J_{\ell+1/2}(kr). \tag{4.10}$$

Within the generation zone where $kr \ll 1$, they can be approximated as follows

$$f_0(kr) \approx 1, \qquad f_2(kr) \approx 0.$$

Then the coupling constant in this limit takes the following form,

$$g_m \equiv k_0 \langle m|\vec{d}\cdot\vec{A}|g\rangle = \begin{cases} -\frac{k_0}{\sqrt{6\pi}} D\gamma & \text{if } m = \pm 1 \\ -\frac{k_0}{\sqrt{6\pi}} D\gamma & \text{if } m = 0. \end{cases} \tag{4.11}$$

This means that although the coupling constants are generally different for $m = 0$ and $m = \pm 1$, they have the same values for the transitions $||\pm\rangle \rightarrow ||g\rangle$ and $||0\rangle \rightarrow ||g\rangle$ for the electric dipole case of $kr \ll 1$.

Finally, the model Hamiltonian of the two-level electric dipole atom, interacting with the cavity field, can be represented in the rotating-wave approximation as follows

$$\begin{aligned} H &= H_0 + H_{int}, \\ H_0 &= \hbar \sum_{m=-1}^{1} (\omega a_m^+ a_m + \omega_0 R_{mm}), \\ H_{int} &= \sum_{m=-1}^{1} i g_m R_{mg} a_m + H.c., \end{aligned} \tag{4.12}$$

where the atomic operators are defined in the standard way[30]

$$R_{mg} = ||m\rangle\langle g||, \qquad R_{mm'} = ||m\rangle\langle m'||$$

and ω_0 and ω are the transition and cavity frequencies respectively.

Chapter 5

Dynamics Of Multipole Single-Atom Radiation

At the end of the last chapter the model Hamiltonian for the atom-field system, which is the well known Hamiltonian for Jaynes-Cummings model for electric dipole transition in rotating wave approximation (Eq. 4.12), was defined. To investigate the steady-state dynamics of the system for this Hamiltonian, we have to clarify the wave function. Then let us first assume that the atom is initially in the excited state $||m\rangle$ with given m, while the cavity field is in the vacuum state. The most general wave function can be written as

$$\psi = \lambda_0 \psi_0 + \lambda_1 \psi_1 \tag{5.1}$$

where

$$\psi_0 = ||m\rangle |vac\rangle \quad and \quad \psi_1 = ||g\rangle |1km\rangle. \tag{5.2}$$

After writing the time independent Schrdinger equation $H\psi = E\psi$ and equating the coefficients of ψ_0 and ψ_1, the energy eigenvalues E can be found easily

$$E_\ell = \hbar\omega_0 - \frac{\hbar\Delta}{2} + \ell\Omega_m, \qquad \ell = \pm 1 \tag{5.3}$$

where

$$\Omega_m = \sqrt{g_m^2 + (\hbar\Delta/2)^2}. \tag{5.4}$$

Here $\Delta = \omega_0 - \omega$ is the detuning parameter and $|1km\rangle$ denotes the state of the cavity field with one dipole photon with the projection of total angular momentum m. So the steady-state wavefunction can be written, using the normalization condition $\lambda_0^2 + \lambda_1^2 = 1$, as

$$|\psi_\ell\rangle = [(E_\ell - \hbar\omega_0)^2 - g_m^2]^{-1/2}[ig_m||m\rangle|vac\rangle + (E_\ell - \hbar\omega_0)||g\rangle|1km\rangle]. \tag{5.5}$$

In order to investigate the dynamics of the atom-field system we should consider the time evolution of the wave function. Then let us describe the wave function as

$$|\Psi(t)\rangle = \sum_{\ell=\pm 1} \Lambda_\ell e^{-iE_\ell t}|\psi_\ell\rangle. \tag{5.6}$$

Taking into account the initial conditions, i.e. the atom is in excited and the field is in vacuum state, $|\Psi(0)\rangle = \psi_0$, the constants Λ_ℓ in equation Eq. 5.6 are given by,

$$\Lambda_\ell \;=\; \frac{\Omega_m + \ell\hbar\Delta/2}{ig_m 2\Omega_m}\sqrt{\hbar\Delta(\hbar\Delta/2 - \ell\Omega_m)}. \tag{5.7}$$

Averaging over the state given in Eq. 5.6, we get

$$\langle a_m^+ a_{m'}\rangle_t \;=\; \frac{1}{4\Omega_m^2}[\sqrt{\hbar\Delta(\hbar\Delta/2 - \Omega_m)} + \sqrt{\hbar\Delta(\hbar\Delta/2 + \Omega_m)} \tag{5.8}$$

$$-\; 2\cos(2\Omega_m t)]\delta_{mm'}. \tag{5.9}$$

We now note that the modes of the cavity field for different m values normally have different period of oscillations. In fact, according to Eq. 4.8, the Rabi frequency in Eq. 5.9 have the form

$$\Omega_m = \begin{cases} \sqrt{g_{\pm 1}^2 + (\hbar\Delta/2)^2} & \text{if } m = \pm 1 \\ \sqrt{g_0^2 + (\hbar\Delta/2)^2} & \text{if } m = 0. \end{cases} \tag{5.10}$$

If we look at the behaviour of the field in the generation zone, i.e. $kr \ll 1$, the Rabi frequency is same too, since the coupling constants g_m for different m's are the same (Eq. 4.11).

The averages in Eq. 5.9 defines the structure of the polarization matrix

$$\mathcal{P}^{(n)}_{\mu\mu'}(\vec{r}) = \langle P^{(n)}_{\mu\mu'}(\vec{r})\rangle_t = (k\gamma)^2(-1)^{(\mu+\mu')}\sum_{m=-1}^{1} V^*_{-\mu m}(\vec{r})V_{-\mu' m}(\vec{r})\langle a^+_m a_m\rangle_t \quad (5.11)$$

obtained from Eq. 2.16 by averaging over Eq. 5.6. The matrix in Eq. 5.11 is the generalization of the so-called coherence matrix[9] in the case of electric dipole radiation.

It can be easily seen from Eq. 5.11 that even if the atom emits the photon of a given type m, all three polarizations $\mu = 0, \pm 1$ can be observed in the radiation field. It should be noted here that due to the choice of basis Eq. 4.5, the component with $\mu = 0$ corresponds to the radial (longitudinal) polarization in the direction of $\vec{\chi}_0$, while the other two components $\mu = \pm 1$ correspond to the transversal circular polarizations with positive and negative helicities, respectively.

5.1 Measurement Of Quantum Multipole Polarization

The standard polarization measurement, including the measurement of variances, is based on the intensity measurement in conjunction with a linear polarizer, quarter-wave plates or equivalents and a beam splitter.[1] The intensity measurement via transformation of photons into electronic signals in a photodetector is, in principle, called the *local measurement.*[2,20,31] The rigorous description of a local measurement of quantum electromagnetic field needs the picture of localizing photons.[2,20,31,32]

It should be noted that the problem of photon localization has attracted a great deal of interest.[2,20,32] The point is that the photon operators, creation and destruction, in any representation (plane photons, multipole photons etc.) are the global objects defined in all space. It has been shown that the position operator cannot be defined for the photon[33] and that the maximum precise description of localization is provided by the notion of wavefront.[34] At the same time, the transformation of a photon into an electric signal in a photodetector can be interpreted as a manifestation of strong localization.[2,20] Another example of strong localization is provided by generation of a photon by an atom, when the photon (at the time preceding the emission) is assumed to be confined in the generation region of the order of atomic size.

An important step forward in the understanding of the problem of localization has been taken by Mandel.[31] He defined the photon localization in operational way (in terms of what is measured) via the 'configurational' number operator, represented by the integral of intensity over the 'volume of detection' (a cylinder whose base is the sensitive surface of the photodetector and whose height is proportional to the detection time).

It should be stressed that the objects, considered in the previously such as the operator polarization matrix Eq. 2.15 or Eq. 3.7 are local by construction, however the photon operators are global in nature. Therefore, it looks tempting to 'renormalize' the operators $\mathcal{A}_{E\mu}(\vec{r})$ in Eq. 3.7 in the following way[18]

$$b_{E\mu}(\vec{r}) = \frac{1}{\sqrt{P_\mu(r)}}\mathcal{A}_{E\mu}(\vec{r}),$$

in order to introduce the 'local' representation of multipole photons with given polarization, specified by the Weyl-Heisenberg commutation relations

$$[b_{E\mu}(\vec{r}), b^+_{E\mu'}(\vec{r})] = \delta_{\mu\mu'}, \tag{5.12}$$

which directly follows from Eq. 3.6.

Consider now the complete scheme of two identical atoms, including both the generation of multipole photons and their detection. One of the atoms (source), located at the point $\vec{r}_s$ is prepared initially in the excited state of some multipole transition. The second atom (detector), located at the point $\vec{r}_d$, is initially in the ground state. Then the process of generation and detection can be described in terms of interaction of atomic transitions with the photons. In taking into account the geometry of the system under consideration, we have to assume that the multipole field in the superposed state of the outgoing and converging spherical waves focuses on the source and on the detector, respectively. The boundary condition, describing this superposition, is for the real radiation field. Then the spatial components of the operator vector potential of the superposed field can be expressed as

$$A_{\lambda\mu}(\vec{r}) = \sum_{j=1}^{\infty} \sum_{m=-j}^{j} [V_{\lambda jm\mu}^{(out)}(\vec{r}) a_{\lambda jm}^{(out)} + V_{\lambda jm\mu}^{(conv)}(\vec{r}) a_{\lambda jm}^{(conv)}], \tag{5.13}$$

where

$$[a_{\lambda jm}^{(out)}, a_{\lambda' j' m'}^{(conv)+}] = 0,$$

while the photon operators of the same kind (either outgoing or converging) obey the standard Weyl-Heisenberg commutation relations Eq. 2.14. Hence, the vacuum noise of polarization is specified by the matrix with the following elements

$$P_{\mu\mu'}^{(vac)}(\vec{r}) = 2k^2[A_{E\mu}(\vec{r}), A_{E\mu'}^{+}(\vec{r})]$$
$$= 2k^2 \sum_{j=1}^{\infty} \sum_{m=-j}^{j} [V_{Ejm\mu}^{(out)*}(\vec{r}) V_{Ejm\mu'}^{(out)}(\vec{r}) + V_{Ejm\mu}^{(conv)*}(\vec{r}) V_{Ejm\mu'}^{(conv)}(\vec{r})]. \tag{5.14}$$

Thus, the spatial properties of the multipole vacuum noise of polarization in the system under consideration are caused by both atoms. In the spirit of our philosophy, this seems to be natural. Both atoms should influence the surrounding space in the same way. In particular, this means that the vacuum noise of polarization and field amplitude influence the process of measurement even if the distance between the two atoms strongly exceeds the wavelength. If the distance between the atoms is of the order of the wavelength or even shorter, than the vacuum noise, arising from the presence of source, strongly influences the zero-point fluctuations in the location of detecting atom.

We now note that the space-time properties in the source-detector system of two identical atoms can be described through the use of special form of Bethe ansatz[35] potential.

The measurement of intensity or polarization with the aid of a local photodetector presupposes detection of a photon by absorption and, hence the change of its state. Besides the local measurement, the quantum detection of the topological properties of the vector potential is allowed through the use of the

Aharonov-Bohm effect.[36] In this case, the magnetic flux through a macroscopic closed loop produces this effect rather than the local value of the field strength. If the magnetic flux passes through a conducting loop, used as a quantum interferometer, the change of the phase of electron state in the loop is directly proportional to the magnitude of the flux.

Usually the quantum Aharonov-Bohm interferometry is applied to the static or slowly varying fields.[36] It has been shown recently[37] that the longitudinal optical frequency fields can also produce a measurable effect. As an example, the TE_{01} mode of the fiber field,[40] inducing the oscillations of conductance in the loop, surrounding the fiber, was considered.[37] Definitely, such a topological measurement of electromagnetic field neither leads to the absorption of a photon nor changes its quantum state. In other words, this is an example of *nondemolition* measurement of the photon propagation.

It should noted that this topological measurement can be applied to the detection of magnetic multipole radiation. In fact there is a radial (longitudinal) magnetic field in the case of magnetic multipole radiation, which can be strong enough for measurement in the near and intermediate zones when $kr \leq 2\pi$. Surrounding the radial direction, corresponding to the maximum intensity in the angular distribution of the magnetic multipole field,[6,10] by a conducting loop of proper radius, we can measure the propagation of magnetic multipole photons via the resistance oscillations in the loop as in the case of optical fiber.[37] It should be stressed that such a measurement reveals only the linearly polarized radial component and not the transversal components. In principal, combining the topological and local measurements of polarization at different distances from the source, it is possible to measure the spatial correlations of different polarizations.

It is clear that topological nondemolition measurement of polarization of visible light encounters a number of technical obstacles. First of all, the wave length is very short which implies that the measurement should be made at very short distances from the source (shorter than $100nm$) and conducting loop should have smaller radius. Moreover, the magnetic multipole radiation of atoms is much (in about$(137)^2$ times) weaker than the electric multipole radiation with the same

j.[26]

Nevertheless, a powerful and at the same time local, source of quantum magnetic dipole radiation exists.[41] It is represented by a system of protons (usually, solids or liquids, containing a lot of hydrogen atoms per unit volume). The spin of photons are polarized by an external static magnetic field. After that, the system is culled to increase the spin relaxation time. Then, the external static magnetic field is inverted to populate the upper sub-level in the Zeeman splitting. Such a system can amplify the thermal noise resonant with the Zeeman splitting like conventional paramagnetic maser.[38] Under some conditions, the linear amplification stage is transformed into the Dicke superradiance,[39] when the energy of inverted spin system is emitted in the form of very short and powerful pulse of coherent radiation (e.g., see[30]). The frequency of radiation in such a system coincides with the Zeeman splitting

$$\omega = g\mu_B B_{ext},$$

where g is the Lande factor, μ_B is the Bohr magneton and B_{ext} is the magnetic induction of static external field, responsible for the Zeeman splitting of proton spin. Depending on the magnitude of B_{ext}, the wavelength of this radiation can vary from ten centimeters to even hundred meters, while the source occupies the region with the linear size of the order of few centimeters.[41,39] Taking into account the sharp radiation pattern of the superradiance which can be narrowed by a proper choice of the shape of source,[30] it seems to be quite realistic to perform the nondemolition polarization measurements of the longitudinal component of radiation at short or intermediate distances or even to measure the correlation between the linear longitudinal polarization at short distances and transversal polarization at far distances from the source.

Let us stress that recent success in high proton polarization at reasonably high temperatures[13] can lead to technical simplification of Dicke superradiance by a system of polarized proton spins.

Chapter 6

Conclusion

Let us briefly summarize our results. We have studied the vacuum or zero-point noise caused by the presence of quantum localized sources (atoms). It was shown that the vacuum fluctuations of the field amplitude (operator vector potential) of multipole photons differ essentially from those in the case of quantized plane waves. Although the latter are spatially invariant, the former are local in principle because of the strong dependence on position with respect to the source (atom). The spatial structure of the multipole vacuum noise is described by the Hermitian (3×3) vacuum polarization matrix whose elements correspond to the commutators of conjugated components of the operator vector potential. The spatial properties of the multipole vacuum noise are independent of the type of radiation, being electric or magnetic. At short and intermediate distances from the source, the multipole vacuum noise is much stronger than that predicted within the model of plane photons. Also an alternative way to define the polarization matrix from the field-strength tensor, Ricci Tensor, that gives mathematical simplifications to the theory of polarization, has been showed.

In spite of the fact that, at far distances, the multipole waves can be well approximated by plane waves, the multipole vacuum noise can strongly influence the process of detection even in the far zone. For example, in a special case when an atom is used as a detector of photons, it also influences the surrounding space[42] and produces spatial inhomogeneity of the electromagnetic vacuum state which leads to a strong increase of zero-point fluctuations in the polarization or intensity measurements in comparison with conventional case of plane waves of photons. This fact can be important for the polarization entanglement processing and atomic entanglement engineering in the system of Ridberg atoms.[43] Similar noise effect also takes place in the case of local measurement by a photodetector with finite sensitive area, measuring the plane waves of photons.

A new method of polarization or intensity measurement at short and intermediate distances from the source, was proposed based on the use of optical Aharonov-Bohm effect,[37] to detect the propagation of the linearly-polarized longitudinal component of magnetic induction, generated by a magnetic dipole transition. Since the Aharonov-Bohm effect is caused by the topological properties of the field and therefore does not influence the state of photons, this is a quantum nondemolition measurement of polarization (intensity). In combination with conventional photodetection at far distances, it permits to measure the space-time correlation of polarizations of multipole radiation. Since the ratio between the transversal and longitudinal polarizations of multipole radiation is determined by distance from the source, the different polarizations can be considered as the entangled quantum quantities. Hence, the above proposed nondemolition measurement of correlation of polarizations opens a tempting possibility in the quantum entanglement processing.

Bibliography

[1] R.M.A. Azzam and N.M. Bashara , *Ellipsometry and Polarized Light* (North-Holland, Amsterdam, 1987); D.S. Kliger, J.W. Lewis, and C.E. Randal, *Polarized Light in Optics and Spectroscopy* (Academic Press, New York, 1990); S. Huard, *Polarization of Light* (Wiley, New York, 1996); C. Brosseau, *Fundamentals of Polarized Light* (Wiley, New York, 1998).

[2] L. Mandel, E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, New York, 1995).

[3] A.K Ekert, Phys. Rev. Lett. **67**, 661 (1991); C.H. Benett and S.J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992); C.H. Bennet, G. Brassard, C. Crepeau, R. Jozza, A. Peres, and W.K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993); D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfürter, and A. Zeilinger, Nature **390**, 575 (1997); D. Boschi, S. Branca, F. De Martini, L. Hardy, and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998); J.-W.Pan, D. Bowmeester, H. Weinfürter, A. and Zeilinger, Phys. Rev. Lett. **80**, 3891 (1998).

[4] M. Born and E. Wolf, *Principles of Optics* (Pergamon Press, Oxford, 1970).

[5] J.M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesly, Reading MA, 1959); A. Luis, L.L. Sánchez-Soto, Phys. Rev. A **48**, 4702 (1993).

[6] J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1978).

[7] W. Heitler, *The Quantum Theory of Radiation* (Dover, New York, 1984).

[8] C. Cohen-Tannouji, J. Dupolnt-Roc, and G. Grinberg, *Atom-Photon Interaction* (Wiley, New York, 1992).

[9] V.B. Berestetskii, E.M. Lifshitz, and L.P. Pitaevskii, *Quantum Electrodynamics* (Pergamon Press, Oxford, 1982).

[10] A.S. Davydov, *Quantum Mechanics* (Pergamon Press, Oxford, 1976).

[11] A.S. Shumovsky and Ö.E. Müstecaplıoğlu, Phys. Rev. Lett. **80**, 1202 (1998).

[12] A.S. Shumovsky, J. Phys. A **32**, 6589 (1999).

[13] M.Iinuma *et al.*, Phys. Rev. Lett. **80**, 171, (2000).

[14] Serge Haroche, AIP Conf. Proc. **461**, 144 (1999).

[15] Serge Haroche, AIP Conf. Proc. **464**, 45 (1999).

[16] H. Bateman, *The Mathematical Analysis of Electric and Optical Wave-Motion* (Dover, New York, 1955).

[17] P.W. Milony , *The Quantum Vacuum* (Academic Press, San Diego, 1994); G. Compagno , R. Pasante, and F. Persico, *Atom-Field Interaction and Dressed Atoms* (Cambridge University Press, Cambridge, 1995).

[18] A.S. Shumovsky and A.A. Klyachko, *Los-Alamos e-print* quant-ph/9911087.

[19] I.S. Gradshtein and I.M. Ryzhik, *Table of Integrals, Series, and Products* (Academic Press, Boston, 1994).

[20] M.O. Scully and M.S. Zubairy, *Quantum Optics* (Cambridge Univ. Press, New York, 1997).

[21] E.B. Jaynes, F.W. Cummings, Proc. IEEE **51**, 89 (1963).

[22] Yoo, H.I. and Eberly, J.H. 1985 *Phys. Rep.* **118**, 239.

[23] Kien, F.L. and Shumovsky, A.S. 1991 *Int. J. Mod. Phys.* B **5**, 2287.

[24] Shore, B.W. and Knight, P.L. 1993 *J. Mod. Optics* **40**, 1195.

[25] Aliskenderov, E.I., Dung, H.T., and Shumovsky, A.S. 1993 Phys. Part. Nucl. **24** 177.

[26] Condon, E.U. and Shortley, G.H. 1987 *The Theory of Atomic Spectra* (New York: Cambridge University Press).

[27] Loudon, R. 1983 *The Quantum Theory of Light* (Oxford: Clarendon Press).

[28] Shumovsky, A.S. 1997 *Optics Commun.* **136**, 219.

[29] Shumovsky, A.S. and Müstecaplıoğlu, Ö.E. 1997 *Phys. Lett.* A **235**, 438.

[30] Allen, A. and Eberly, J.H. 1987 *Optical Resonance and Two-Level Atoms* (New York: Dover).

[31] L. Mandel, Phys. Rev. **144**, 1071 (1966).

[32] W.E. Lamb Jr., Appl. Phys. B **60**, 77 (1995).

[33] T.D. Newton and E.P. Wigner, *Rev. Mod. Phys.* **21**, 400 (1949).

[34] R. Acharya and E.C.G. Sudarshan, *J. Math. Phys.* **1**, 532 (1960).

[35] V.I. Rupasov and V.I. Yudson, Sov. Phys. JETP **60**, 927 (1984); D. Kaup and V.I. Rupasov, J. Phys. A **29**, 6911 (1996); V.I. Rupasov and M. Singh, Phys. Rev. A **77**, 338 (1996).

[36] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959); I.O. Kulik, JETP Lett. **11**, 275 (1970); M. Buttiker, Y. Imry, and R. Landauer, Phys. Lett. A **96**, 365 (1983); M. Peshkin and A. Tonomura, *The Aharonov-Bohm Effect* (Springer, Berlin, 1989); B.L. Altshuler, P.A. Lee, and R.A. Webb (editors), *Mesoscopic Phenomena in Solids* (North-Holland, Amsterdam, 1991).

[37] I.O. Kulik and A.S. Shumovsky, Appl. Phys. Lett. **69**, 2779 (1996).

[38] W.H. Louisell, *Radiation and Noise in Quantum Electronics* (Mc-Grow Hill, New York, 1964); R.H. Pantell and H.E. Puthoff, *Fundamentals of Quantum Electronics* (Wiley, New York, 1969).

[39] Yu. F. Kiselev, A.S. Shumovsky, and V.I. Yukalov, Mod. Phys. Lett. B **3**, 1149 (1989).

[40] G. Agrawal, *Nonlinear Fiber Optics* (Academic Press, Boston, 1989).

[41] Yu. Kiselev, A. Prutkoglyad, A. Shumovsky, and V. Yukalov, Sov. Phys. JETP **94**, 344 (1988).

[42] P.W. Milony, *The Quantum Vacuum* (Academic Press, San Diego, 1994); G. Compagno, R. Pasante, and F. Persico,

[43] S. Haroche, *Cavity Quantum Electrodynamics: a Review of Ridberg Atom-microwave experiments*, in AIP Conf. Proc. Vol. **464**, Issue 1, p. 45 (1999); J.M. Raimond, E. Hagley, X. Maitre, G. Nogues, C. Wunderlich, M. Brune, and S. Haroshe, in AIP Conf. Proc. Vol. **477**, Issue 1, p. 209 (1999).