ROBUST FAULT DETECTION BY SIMULTANEOUS OBSERVERS

A THESIS
SUBMITTED TO THE DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING
AND THE INSTITUTE OF ENGINEERING AND SCIENCE
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

BY
NEJIB AMMAR
SEPTEMBER 2000

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September 2000

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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Abstract

This thesis addresses the problem of fault detection and isolation in linear systems based on unknown input observers.

Functional disturbance decoupled observers which estimate specified or unspecified linear functions of system states regardless of the disturbances are first studied. Necessary and sufficient condition for the existence of such observers are presented. The investigation is extended to simultaneous disturbance decoupled observers where multiple systems are observed by a single disturbance decoupled observer.

The application of disturbance decoupled observers to fault detection and diagnosis are explicitly outlined, and a new scheme that is based on simultaneous unknown input observers is proposed to enhance the already existing schemes.

Finally, a detailed simulation example is carried out to examine the utility of the proposed scheme.

Keywords: linear systems, unknown-input observers, simultaneous observers, robust fault detection, fault diagnosis, fault isolation

Özet

Bu tezde doğrusal sistemlerde oluşabilecek hataların bulunması, tanınması ve izolasyonu problemleri, bilinmeyen girişli gözleyiciler yöntemiyle incelenmektedir.

Önce bir sistemin durum vektörünün tamamını veya bir fonksiyonunu, bozucu girişlerden bağımsız olarak, gözleyebilen bilinmeyen girişli gözleyiciler incelendikten sonra, buradaki sonuçlar birden fazla sistem için tek gözleyiciden ibaret olan eşzamanlı gözleyicilere genellenmiştir.

Bilinmeyen girişli gözleyicilerin hata bulma, tanıma ve izolasyonuna nasıl uygulandığı özetlendikten sonra, orijinal bir katkı olarak, eşzamanlı bilinmeyen girişli gözleyicilerin bilinen hata bulma yöntemlerinde sağlayacağı kolaylıklar gösterilmektedir.

Bu yeni sonuçların uygulamadaki yararları bir simülasyon örneğiyle ayrıntılı bir şekilde gösterilmiştir.

Anahtar Kelimeler : Doğrusal sistemler, bilinmeyen girişli gözleyici, eşzamanlı gözleyici, hata bulma, hata tanıma, hata izolasyonu

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Chapter 1

INTRODUCTION

There has been great concern about improving the process supervision techniques in industrial control applications in order to meet the growing demands for the reliability and safety. These demands are in fact amplified by the increasing complexities and interlinkage of industrial processes as well as the growing scopes of automation and the high capital investments in such processes. The motivation is to detect and locate (isolate) the unexpected and unpermitted process deviation, the so-called fault, from the standard conditions and then take the necessary control action to stop the expansion of the fault and to avoid possible damages to the whole plant such as instabilities and degradation in performance, not to mention the possible hazards to the personnel and the economic losses[1].

The problem is approached by introducing system redundancies whether physical, which are obtained through the repeated hardware elements, or analytical, which are contained in the static and dynamic relationships among system inputs and outputs. However, because of the penalties that the hardware redundancy imposes on the system including the high cost of the extra equipments, their weight and the space needed to accommodate them, newer fault detection and isolation (FDI) approaches are emerging based on the analytical redundancy at the cost of a mathematical model of the physical plant[2, 3, 4, 5]

The basic principle of model-based (analytical redundancy-based) FDI, is the comparison of the actual behavior of the plant with an anticipated behavior generated with the help of the mathematical model of the same plant. Generally speaking, the FDI schemes of this type involve two stages: the generation of residuals and the analysis of them. Residuals are functions that are accentuated by faults. They carry relevant information exploited to extract the fault type, source, time of occurrence etc., the kind of data to isolate the fault.[2, 6]

There is a broad spectrum of model-based procedures used for FDI, which can be brought down to parity space approach [7], dedicated observer approach [14, 15, 18] fault detection filter approach [8, 9, 10, 11, 12] and parameter identification approach[3, 13]. The most effective and popular among all is observer-based approach.

In the observer-based approach, the difference between the actual and the estimated system outputs is chosen as the residual. The residual is zero as long as the system is operating fault-free and nonzero when a fault takes place. The underlying assumption is that the mathematical model is a faithful representation of the physical system. In practice, this idealized assumption is never met because of the system parameters' variation, modeling error, noises and other disturbances. As a result, there is always a mismatch between the actual and estimated outputs even in the absence of faults. Clearly, this creates a source of false alarms and corrupts the performance of the FDI system. These difficulties lead to the robustness issue recognized by [6, 19, 20, 22].

In order to ensure a robust FDI scheme, the FDI system should be designed to be sensitive to faults of interest and at the same time insensitive to other system discrepancies. These conflicting goals are achieved by employing unknown input observer (UIO), a tool to discriminate between faults and disturbances[22, 21, 15]. Unknown input observers are special kinds of Luenberger observers that continues to estimate system states even when the system input or part of it is unknown [24, 27, 28, 30, 31]. Since the FDI scheme requires the estimation of the system output, which is a function of the state vector, functional UIO seems to be more appropriate for our purpose, especially that the design problem is then formulated under more relaxed conditions[23, 33].

This thesis is devoted to various types of unknown input observers and their application to robust observer-based fault detection and isolation schemes. It is organized as follows. Chapter 1 covers the unknown input observation problem with special emphasis on the functional UIO. The results achieved by [28] and [33] are restated and alternative, more transparent proofs are provided. Chapter 2 examines simultaneous unknown input observers and provides some new sufficient conditions for their existence. The objective of Chapter 3 is to give an explicit description of the different aspects of UIO-based FDI schemes and to introduce the simultaneous UIO-based FDI. Finally, in Chapter 4 a simulation example is considered to illustrate the design procedure and highlight the applicability of simultaneous UIO for the purpose of FDI.

Chapter 2

UNKNOWN-INPUT OBSERVERS

In the classical observer theory of Luenberger, [25], the states are reconstructed or estimated from measurements of the outputs as well as the inputs. Unknown-input observers, [27], [26], reconstruct the states from the measurement of the outputs only. A main need for unknown-input observers arises whenever some disturbances unavailable for measurements influence the system. The disturbance inputs may also be superficially introduced into the model in order to summarize the effect of modeling errors or to represent unaccountable noises influencing certain state variables. In such applications, the unknown-input observers are also designated as "disturbance-decoupled observers (or estimators)", [35].

In this section, we first review the theory of (linear) functional disturbance decoupled observers. The full-state unknown-input observers are examined as a special case of functional observers.

2.1 Disturbance Decoupled Observers

Consider a linear time-invariant system in state-space representation

$$
\begin{aligned}
\tfrac{d}{dt}x &= Ax(t) + Bd(t), \\
y(t) &= Cx(t) + Dd(t), \\
z(t) &= Ex(t) + Fd(t),
\end{aligned}
\tag{1}
$$

where $x \in \mathbf{R}^{\mathbf{n}}, d \in \mathbf{R}^{\mathbf{l}}, y \in \mathbf{R}^{\mathbf{p}}$ are the state vector, the disturbance vector, and the output vector, respectively. The vector $z \in \mathbf{R}^q$ is a function of states and disturbance inputs and we are interested in estimating its value from a knowledge of y. The matrices A, B, C, D, E, and F are known matrices. The problem is to determine a system, called *observer*, of the form

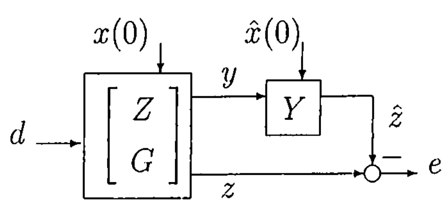
$$
\begin{aligned}
\tfrac{d}{dt}\hat{x} &= H\hat{x}(t) + Ly(t), \\
\hat{z}(t) &= M\hat{x}(t) + Jy(t),
\end{aligned}
\tag{2}
$$

such that the reconstruction error $e(t) = z(t) - \hat{z}(t)$ satisfies the following conditions:

(i) $e(t)$ is independent of $d(t)$, and

(ii) $lim_{t\to\infty}||e(t)|| = 0$ for all initial states $x(0), \hat{x}(0)$.

The observer (2) is called a *functional disturbance decoupled observer* (DDO) for (1) provided (i) and (ii) above hold. Figure 1 illustrates a functional DDO scheme in transfer matrix representation.



$$
\begin{aligned}
Z(s) =& \; C(sI - A)^{-1}B + D \\
G(s) =& \; E(sI - A)^{-1}B + F \\
Y(s) =& \; M(sI - H)^{-1}L + J
\end{aligned}
$$

Figure 1: Functional DDO

Notice that, the control input, which is assumed to be measurable, is omitted in (1) and the observer (2) does not use the control input. This causes no loss of generality since the more general case can be reduced to the above simpler case by redefining the output $y(t)$ to include the control inputs as well, see [28]. A rigorous treatment of the problem based on the frequency domain methods is presented in [28]. A necessary and sufficient condition for the existence of a functional DDO is given below. We supply an alternative proof of this result of Hautus as the proof in [28] is rather indirect.

Proposition 1.1. *There exists a functional DDO for (1) if and only if there exist stable proper rational matrices* $X(s), Y(s)$ *such that*

$$\begin{bmatrix} -X(s) & Y(s) \end{bmatrix} \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix} = \begin{bmatrix} E & F \end{bmatrix}. \tag{3}$$

Proof. We first note that the sought observer (2) can be restricted to be canonical (controllable and observable) without loss of generality. Taking the Laplace transform of $e(t)$ (with initial conditions), one gets

$$\bar{e}(s) = [E - Y(s)C](sI - A)^{-1}x(0) - M(sI - H)^{-1}z(0) + [G(s) - Y(s)Z(s)]\bar{d}(s),$$

where $\bar{e}(s), \bar{d}(\mathrm{s})$ are the Laplace transforms of $e(t), d(t)$, $Y(s) := M(sI - H)^{-1}L + J$ is the observer transfer matrix, and $Z(s) := C(sI - A)^{-1}B + D$, $G(s) := E(sI - A)^{-1}B + F$ are the transfer matrices of the system from d to y, z, respectively. The requirements (i) and (ii) are equivalent to

(*a*) $G(s) = Y(s)Z(s)$
(*b*) $E - Y(s)C = X(s)(sI - A)$,
(*c*) $X(s)$ and $M(sI - H)^{-1}$ are stable rational matrices.

Since (H, L) is controllable, it follows that $M(sI - H)^{-1}$ is stable rational if and only if $Y(s)$ is. It is now straightforward that $(a) - (c)$ are equivalent to (3) holding for some stable proper rational matrices $X(s), Y(s)$. Note that given any stable rational solution $X(s), Y(s)$ of (3) with $Y(s)$ proper, a canonical realization of $Y(s)$ is a functional DDO (2). □

The equation (3) involves the (polynomial) *system matrix*

$$S(s) := \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix},$$

and the question as to "when an appropriate solution to (3) exists" need to be answered. There are at least two alternative answers. One answer is provided by the geometric approach. The problem of *disturbance decoupled estimation* of [34], [36], [35] is precisely the functional DDO for the special case $F = 0$, the case where the outputs to be estimated are linear combinations of states only, and for $D = 0$. The condition for the existence of a functional DDO with $F = 0$ and $D = 0$ is that

$$\mathcal{D}_*^{Im\,B} \cap Ker\,C \subset Ker\,E, \tag{4}$$

where $\mathcal{D}_*^{Im\,B}$ denotes the "smallest detectability subspace containing $Im\,B$" which is the dual space of the smallest stabilizability subspace contained in $Ker\,C$, see e.g. [36]. A second approach is to transform the equation (3) to a linear matrix equation over the ring of stable proper rational functions and thereby obtain a condition for its solvability in terms of "unstable invariant zeros" of $S(s)$, see e.g. [37]. Both of these methods provide the valuable intuition that the interaction of the unstable invariant zeros of the system with the matrix on the right hand side of (3) determines whether or not a functional DDO exists. One can state more precise conditions in some special cases. One such case is considered next.

2.2 Full-State DDO

If, in (1), one lets $E = I_n, F = 0$, then $z(t) = x(t)$ and the output $\hat{z}$ of (2) is an estimate of the states of the original system whenever the conditions (i) and (ii) hold. The observer (2) is then a full-state DDO. According to Proposition

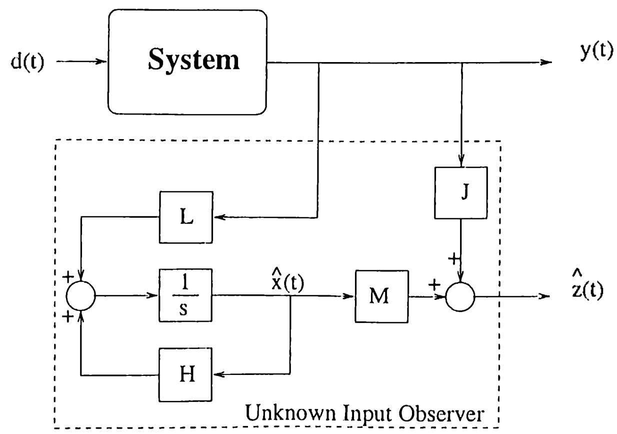


Figure 2.1: Structure of a Full-state DDO

1.1, a full-state DDO exists just in case (3) holds with $E = I, F = 0$, i.e., the equation

$$\begin{bmatrix} -X(s) & Y(s) \end{bmatrix} \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix} = \begin{bmatrix} I_n & 0 \end{bmatrix}. \tag{5}$$

has a proper stable rational solution $[-X(s)\ Y(s)]$. It is not difficult to see that if (5) is satisfied for some stable rational $X(s), Y(s)$, then

$$rank \begin{bmatrix} -sI_n + A & B \\ C & D \end{bmatrix} = n + rank \begin{bmatrix} B \\ D \end{bmatrix} \quad \forall s \in \mathbf{C}, Re(s) \geq 0. \tag{6}$$

If $X(s), Y(s)$ are moreover proper, then writing $Y(s) = Y_0 + Y_{-1}s^{-1} + ...$, one obtains from condition (a) that $Y_0 D = 0, Y_0 CB + Y_{-1} D = B$, or equivalently,

$$\begin{bmatrix} Y_0 & Y_{-1} \end{bmatrix} \begin{bmatrix} CB & D \\ D & 0 \end{bmatrix} = \begin{bmatrix} B & 0 \end{bmatrix}.$$

It follows that

$$rank \begin{bmatrix} CB & D \\ D & 0 \end{bmatrix} = rank \begin{bmatrix} CB & D \\ D & 0 \\ B & 0 \end{bmatrix} = rank D + rank \begin{bmatrix} B \\ D \end{bmatrix}. \qquad (7)$$

Hence, if (5) has a proper stable rational solution $[-X(s)\ Y(s)]$, then (6) and (7) hold. In [28], the converse is also shown. The following is Theorem 1.12 of [28].

Proposition 1.2. *There exists a full-state DDO for (1) if and only if (6) and (7) hold.*

Let us assume, without loss of generality, that the matrix $[B'\ D']'$ has full column rank. Then, the condition (6) is equivalent to the system (A, B, C, D) being left invertible (i.e., $Z(s)$ has full column rank over $\mathbf{R}(s)$) and all its invariant zeros (i.e., roots of the largest invariant factor of its system matrix $S(s)$) being in the open left half complex plane. Alternatively, (6) is equivalent to the system matrix $S(s)$ having a stable left inverse. The condition (7), on the other hand, is equivalent to $S(s)$ having a proper left inverse. Finally, the conditions (6) and (7), together, are equivalent to the existence of a proper stable left inverse for the polynomial system matrix $S(s)$.

The geometric solvability condition (4) specialized to $E = I$ (and $F = 0$, $D = 0$) becomes:

$$\mathcal{D}_*^{Im\,B} \cap Ker\, C = \{0\}.$$

An alternative condition is due to [22]: A full-state DDO for (1) with $E = I, F = 0, D = 0$ exists if and only if

(*i*) $rank\, CB = rank\, B$,
(*ii*) (C, A_1) is a detectable pair,

where $A_1 := A - B(CB)^+CA$ with $(CB)^+$ denoting a Moore-Penrose inverse of CB. Note that the first condition (*i*) is equivalent to (7) under $D = 0$. The condition implies in particular that $rank\, B \leq rank\, C$ which means that the number of disturbances that can be decoupled cannot exceed the number of independent measurements. The condition (*ii*), on the other hand, can be shown to be equivalent to (6) and means that the invariant zeros of (A, B, C) are all stable and the system is left invertible.

In the literature, considerable other work is devoted to the design of full-state unknown-input observers using procedures such as system inversion or singular value decomposition, [30], [31]. Whatever method is applied, the restrictive existence conditions for full-state observers limit their use in many applications.

2.3 Unspecified Function of States

Full-state DDO's suffer from stringent existence conditions. Fortunately, in many applications, not all states are required, rather certain states or certain combination of the states are enough to fulfill the task, [32], [33]. Moreover, in fault detection applications, it is even enough to focus on functional unknown-input observers which estimate *some* (*a priori* unspecified) function of states.

Given a system

$$\begin{aligned} &\tfrac{d}{dt}x = Ax(t) + Bd(t), \\ &y(t) = Cx(t) + Dd(t), \end{aligned} \tag{8}$$

the problem of *functional DDO with unspecified E* is to determine a function of states

$$z(t) = Ex(t), \ E \neq 0 \tag{9}$$

and a disturbance decoupled observer (2) as in the previous section. The significant difference from the problem considered in Proposition 1.1 is that $F = 0$ and the matrix E on the right hand side of equation (3) is now also an unknown and is to be determined. It is easy to see by similar reasoning that lead to Proposition 1.1 that the problem of functional DDO with unspecified E has a solution if and only if there exist stable proper rational matrices $X(s), Y(s)$ and a constant nonzero E satisfying

$$\begin{bmatrix} -X(s) & Y(s) \end{bmatrix} \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix} = \begin{bmatrix} E & 0 \end{bmatrix}. \tag{10}$$

A simple condition for solvability for the problem has been obtained in [33] using ideas from the theory of descriptor (generalized) systems. We now state this result and give an alternative proof using more elementary notions.

Proposition 1.2. *There exists a functional DDO with unspecified E for the system (8) if and only if either one of the following conditions hold:*

$$\begin{aligned} &(i)\ rank \begin{bmatrix} C & D \end{bmatrix} > rank\, D. \\ &(ii)\ S \text{ has a stable invariant zero.} \end{aligned} \tag{11}$$

Proof. Let $U(s), V(s)$ be some unimodular polynomial matrices such that $U(s)S(s)V(s) = \Lambda(s)$ is the Smith normal form of $S(s)$ over the ring of polynomials. Let $\Lambda = \Lambda_s\Lambda_a$ be a stable-antistable factorization of Λ with Λ_s square, nonsingular. Thus, $det\,\Lambda_s(s)$ includes among its zeros all zeros of Λ in the strict left half complex plane and only these.

[If] Suppose (i) in (11) holds. Then, there exists $J \in \mathbf{R}^{k\times p}$ for some $k \geq 1$, such that $JC \neq 0$ and $JD = 0$. Let $E = JC, X(s) = 0, Y(s) = J$. Then, (10) is satisfied. This shows that if (i) holds, then a constant DDO exists. If (ii) in (11) holds, then $\Lambda_s^{-1}U$ is stable rational, *non-polynomial*, and satisfies $\Lambda_s^{-1}US = \Lambda_a V^{-1}$. Partitioning $\Lambda_s^{-1}U = [\ -X \quad Y\ ]$ with X having n columns, we have

$$\begin{aligned} &X(sI - A) + YC = \Psi_1, \\ &-XB + YD = \Psi_2, \end{aligned} \tag{12}$$

for some polynomial matrices Ψ_1, Ψ_2. Let us write $X = X_+ + X_-$, where $X_- = X_{-l}s^{-l} + X_{-l-1}s^{-l-1} + ..., X_{-l} \neq 0$, and $l \geq 1$. Multiplying each term in (12) by s^{l-1} and taking the strictly proper part of each term gives

$$\begin{aligned} &(s^{l-1}X)_-(sI - A) + (s^{l-1}Y)_-C = X_{-l}, \\ &(s^{l-1}X)_-B = (s^{l-1}Y)_-D. \end{aligned}$$

Since $E := X_{-l} \neq 0$, (10) is satisfied and $(s^{l-1}Y)_-$ is a transfer matrix for a DDO. Note that the poles of $(s^{l-1}Y)_-$ are among the roots of $det\,\Lambda_s$ which are the stable invariant zeros of the system matrix $S(s)$.

[Only if] Conversely, suppose there exists a DDO so that for some $X(s), Y(s), E \neq 0$, (10) holds. We assume, without loss of generality, that in (10) $[C\ \ D]$ has full row rank. If not, by appropriately redefining $Y(s)$, a similar equation with the assumption fulfilled can be obtained. We have

$$\begin{bmatrix} -X & Y \end{bmatrix} U^{-1}\Lambda_s\Lambda_a = \begin{bmatrix} E & 0 \end{bmatrix} V$$

Let us first suppose that S, and hence Λ_a, has full row rank. Let $\begin{bmatrix} -X & Y \end{bmatrix} U^{-1} = \Theta\Gamma^{-1}$ be a right coprime polynomial factorization, where $det\,\Gamma$ is a Hurwitz stable polynomial since $\begin{bmatrix} -X & Y \end{bmatrix} U^{-1}$ is stable rational. It follows by left coprimeness of Θ and Γ and by the fact that $\Theta\Gamma^{-1}\Lambda_s\Lambda_a$ is polynomial, that $\Lambda_s\Lambda_a = \Gamma\Psi$ for some polynomial matrix Ψ. Therefore, either Γ and Λ_s have a nontrivial common left factor implying that Λ_s has a stable invariant zero, so that (ii) in (11) holds, or Γ is unimodular implying that $\begin{bmatrix} -X & Y \end{bmatrix}$ is constant. In the latter case, since $X = YC(A - sI)^{-1}$ is strictly proper, we must have $X = 0$. This gives that $E = YC \neq 0$ and $YD = 0$ which implies (i) in (11).

Suppose next that the system matrix S is not of full row rank. Let $\begin{bmatrix} -\Theta & \Psi \end{bmatrix}$ be a minimal polynomial basis, [38], for the left kernel of S, i.e.,

$$\begin{bmatrix} -\Theta & \Psi \end{bmatrix} \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix} = 0,$$

where, by $\Theta = \Psi C(sI - A)^{-1}$,

$$deg\,\Theta_{ir} < deg\,\Psi_{ir}, i = 1, ..., k, \tag{13}$$

with Θ_{ir}, Ψ_{ir} denoting the i^{th}-row of the matrix Θ, Ψ respectively. Let $\Lambda := diag\{\sigma_1, ..., \sigma_k\}$ for Hurwitz stable polynomials satisfying $deg\,\sigma_i = deg\,\Psi_i$ for $i = 1, ..., k$. Then, $X := \Lambda^{-1}\Theta, Y := (\Lambda^{-1}\Psi)_-$, and $E := -(\Lambda^{-1}\Psi)_0 C$ are such that (10) is satisfied. By the degree condition (13), X is strictly proper so that $(\Lambda^{-1}\Psi)_0 D = 0$. Further, by the definition of a minimal polynomial basis, $(\Lambda^{-1}\Psi)_0$ is of full row rank. If $E = 0$, then $(\Lambda^{-1}\Psi)_0[CD] = 0$ which, by our assumption that $[C\ D]$ has full row rank, is not possible. Hence, $E = -(\Lambda^{-1}\Psi)_0 C \neq 0$ whereas $(\Lambda^{-1}\Psi)_0 D = 0$. It follows that (i) in (11) holds. □

Note that the argument in the last paragraph of the proof also establishes the fact that "If

$$rank\, S < n + rank \begin{bmatrix} C & D \end{bmatrix}$$

(or, if $[C\ D]$ has full row rank but S has a row defect), then a constant DDO exists". A closer examination of the construction parts in the proof yields the following facts stated without proof.

Corollary 1.1. *Suppose $[C\ D]$ has full row rank. There exists a DDO with unspecified E which*

(i) is constant if and only if $rank \begin{bmatrix} C & D \end{bmatrix} > rank\, D$,

(ii) has any set of desired stable poles if and only if the system matrix S does not have full row rank.

If the system has a stable invariant zero, then there exists a DDO with unspecified E with poles a subset of the stable invariant zeros of S.

Let us now consider the following example which shows that the trivial case where $Y(s) = 0$ is a disturbance decoupled observer transfer function must be considered as an observer with fixed dynamics.

Example. Consider the system,

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = 1.$$

Then, the observer $Y(s) = 0$ and the choice $E = \begin{bmatrix} 1 & 0 \end{bmatrix}$ satisfy the requirements from a DDO with unspecified E. For this data, D is nonsingular so that, by Corollary 1.1, a constant observer does not exist. On the other hand,

$$\begin{bmatrix} A - sI & B \\ C & D \end{bmatrix}$$

is nonsingular so that, again by Corollary 1.1, an observer with assignable poles does not exist. Therefore, Y(s) = 0 must be treated as a case with "fixed dynamics" in spite of the fact that this causes an abuse of the term since 0 can be considered to have any stable denominator.

The trivial solution $Y(s) = 0$ can be avoided by assuming that (A, B) is controllable. To see this, let $M(s), N(s), \bar{N}(s), \bar{M}(s)$ be polynomial matrices such that $M(s), \bar{M}(s)$ are nonsingular,

$$\begin{aligned} &U(s) := \begin{bmatrix} M(s) & -\bar{N}(s) \\ N(s) & \bar{M}(s) \end{bmatrix} \text{ is unimodular, and} \\ &\begin{bmatrix} A - sI & B \end{bmatrix} \begin{bmatrix} M(s) & -\bar{N}(s) \\ N(s) & \bar{M}(s) \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}. \end{aligned} \tag{14}$$

Such matrices exist by controllability of (A, B), [40]. Multiplying both sides of (10) on the right by $U(s)$, and supposing $Y(s) = 0$, we obtain $-X(s) = EM(s)$. In this equality, the left hand side is a strictly proper rational matrix and the right hand side is a polynomial matrix. It follows that both sides are zero and hence $E = 0$. Therefore, $Y(s) = 0$ is not a functional DDO with unspecified E.

The assumption of controllability of (A, B) leads to a further simplification in (10).

Corollary 1.2. *Suppose the system (8) is controllable. There exists a* *functional DDO with unspecified E for the system (8) if and only if there exists a proper stable rational matrix $Y(s)$ and a constant $E \neq 0$ satisfying*

$$\begin{bmatrix} E & -Y(s) \end{bmatrix} \begin{bmatrix} (sI-A)^{-1}B \\ C(sI-A)^{-1}B+D \end{bmatrix} = 0. \tag{15}$$

Proof. By controllability of (8), there exists a unimodular $U(s)$ as in (14). Multiplying both sides of (3) on the right by U and using (14), we obtain $Y(s)[D\bar{M}(s) - C\bar{N}(s)] = -\bar{N}(s)E$. Since, from (14), $\bar{N}(s)\bar{M}(s)^{-1} = -(sI - A)^{-1}B$, we have $Y(s)[C(sI-A)^{-1}B + D] = E(sI-A)^{-1}B$ and (15) holds. Conversely, if (15) holds for a proper stable $Y(s)$ and constant nonzero E, then let $X(s) := Y(s)[CM(s) + DN(s)] - EM(s)$ which is stable rational. By (15), we also have $Y(s)[D\bar{M}(s) - C\bar{N}(s)] = -\bar{N}(s)E$. Combining the two, we have

$$\begin{bmatrix} -X(s) & Y(s) \end{bmatrix} \begin{bmatrix} A-sI & B \end{bmatrix} U(s) = \begin{bmatrix} E & 0 \end{bmatrix} U(s)$$

which gives (10) for a proper stable rational $Y(s)$ and a stable rational $X(s)$. However, $X(s) = [E - Y(s)C](sI-A)^{-1}$ and properness of $Y(s)$ implies that $X(s)$ is also proper (actually, strictly proper). □

Chapter 3

SIMULTANEOUS UNKNOWN-INPUT OBSERVERS

The idea of simultaneous observation can at least be traced back to [39]. It is concerned with the design of a common observer for a given set of two or more systems. Such a set of systems may result from a plant undergoing changes in its structure or in its parameters as a result of changes in operating conditions. It may also be a set of linear models matching a nonlinear system closely at various operating points. In the latter case, the simultaneous observer, whenever it exists, can be considered an approximate linear observer for the nonlinear system. Although the problem of simultaneous stabilization has been intensively examined, see e.g. [41], the dual problem seems to have attracted less attention. In [43] and [42], the problem of simultaneous observers has been investigated using coprime factorization techniques: [43] takes a similar approach to that of [41] and reduces simultaneous functional observation problem of $r+1$ plants to the same problem for r auxiliary plants. Conditions for the existence of a simultaneous observer for two plants are obtained but they are conditions on transformed data. In [42], the focus is on obtaining a parametrization of all simultaneous functional observers for a set of plants and this has been done under a rather severe restrictive assumption of the existence of a stable left inverse for a composite plant. This assumption can easily be shown to be equivalent to the assumption that the plants admit a simultaneous "full-state" observer.

In this section, we investigate the conditions for existence of functional simultaneous unknown-input or disturbance decoupled observers. Since our objective is to employ such observers in the design of fault detection and isolation, we are interested in functional observers with *unspecified function(s) of states*. Although the results below apply to an arbitrary number of systems with appropriate modifications, for simplicity, we will constrain the presentation to the synthesis of a simultaneous DDO for two systems only. We first consider the general case of simultaneous DDO where the estimation of, in general, different functions of states of the two systems is desired. The results are then specialized to the estimation of the *same function of states*.

3.1 Simultaneous DDO

Let us consider two linear, time-invariant systems Σ_1 and Σ_2 described by the following equations:

$$\Sigma_1 : \begin{array}{l} \dot{x}_1(t) = A_1 x_1(t) + B_1 d_1(t) \\ y_1(t) = C_1 x_1(t) + D_1 d_1(t) \end{array}$$

and

$$\Sigma_2 : \begin{array}{l} \dot{x}_2(t) = A_2 x_2(t) + B_2 d_2(t) \\ y_2(t) = C_2 x_2(t) + D_2 d_2(t) \end{array}$$

where $x_1(t), x_2(t) \in \mathbf{R}^n$ are the state vectors, $d_1(t) \in \mathbf{R}^{m_1}, d_2(t) \in \mathbf{R}^{m_2}$ are disturbance vectors, $y_1(t), y_2(t) \in \mathbf{R}^p$ are the measurement vectors of Σ_1 and Σ_2, respectively, and A_1, A_2, B_1, B_2, C_1, C_2, D_1, and D_2 are constant matrices with appropriate dimensions. The problem of *simultaneous DDO* is to determine matrices $T_i \neq 0$, $i = 1, 2$ and a functional DDO of the form

$$\dot{\hat{x}} = H\hat{x}(t) + Ly(t), z(t) = M\hat{x}(t) + Jy(t),$$

where $y(t)$ is a vector-input to the observer such that the errors

$$e_i(t) = T_i x_i(t) - w(t), \; i = 1, 2$$

satisfy the following conditions for $= 1, 2$: Whenever $y(t) = y_i(t)$,

(i) $e_i(t)$ is independent of $d(t)$ and

(ii) $lim_{t\to\infty}||e_i(t)|| = 0$ for all initial states $x_i(0)$ and $\hat{x}(0)$.

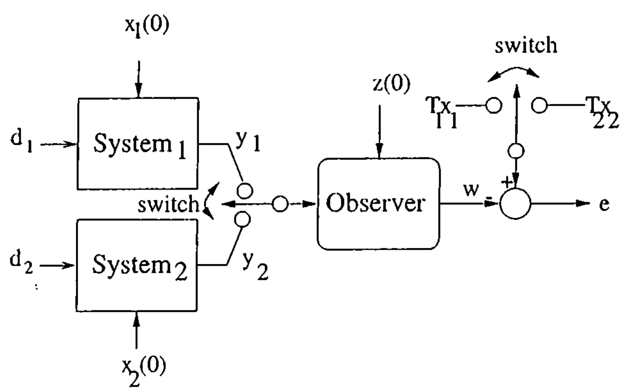


Figure 3.1: Simultaneous DDO

Proposition 2.1. *There exists a simultaneous DDO for Σ_i, $i = 1, 2$ if and only if there exist stable rational proper matrices $X_i(s)$, $i = 1, 2$, $Y(s)$ and constant $T_i \neq 0$, $i = 1, 2$ satisfying*

$$\begin{bmatrix} X_1(s) & X_2(s) & Y(s) \end{bmatrix} \begin{bmatrix} A_1 - sI & 0 & B_1 & 0 \\ 0 & A_2 - sI & 0 & B_2 \\ C_1 & C_2 & D_1 & D_2 \end{bmatrix} = \begin{bmatrix} T_1 & T_2 & 0 & 0 \end{bmatrix}. (1)$$

Proof. The result follows easily by writing (10) for both Σ_1 and Σ_2 with the same $Y(s)$ and combining the two equalities obtained in one matrix equation. □

Thus, by Proposition 2.1, a simultaneous DDO exists for Σ_i, $i = 1, 2$ just in case there is a DDO for the combined system

$$\Sigma = (\begin{bmatrix} C_1 & C_2 \end{bmatrix}, \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}, \begin{bmatrix} D_1 & D_2 \end{bmatrix})$$

with a special function of states, i.e. a function $[T_1\ T_2] \neq 0$ where T_1 and T_2 are *separately* nonzero. Theorem 1.2 applied to Σ gives that if a simultaneous observer exists, then either the system matrix associated with Σ has a stable invariant zero or $rank\,[C_1\ C_2\ D_1\ D_2] > rank\,[D_1\ D_2]$. This condition is also sufficient provided the transfer matrices $Z_i(s) := C_i + (sI - A_i)^{-1}B_i + D_i$ have full row rank for $i = 1, 2$.

Theorem 2.1. *Suppose the transfer matrices $Z_i(s)$ of Σ_i have full row rank for $i = 1, 2$. There exists a simultaneous DDO for Σ_i, $i = 1, 2$ if and only if $rank\,[C_1\ C_2\ D_1\ D_2] > rank\,[D_1\ D_2]$ or Σ has a stable invariant zero.*

Proof. The "only if" part is by Theorem 1.2. To see the "if" part, suppose either Σ has a stable invariant zero or the rank condition holds. Then, by Theorem 1.2, there exists a functional DDO for Σ for some $[T_1\ T_2] \neq 0$. We show that if $T_i = 0$ for some $i = 1, 2$, then the corresponding transfer matrix has a row rank defect. Suppose $T_1 = 0$. Then, by (1), $X_1(s)(A - sI) + X_2(s)C_1 = 0$ and $X_1(s)B_1 + X_2(s)D_1 = 0$. Now, $[X_1(s)\ X_2(s)] \neq 0$ since otherwise in (1) $T_2 = Y(s)C_2 = 0$ contradicting $[T_1\ T_2] \neq 0$. It follows that the system matrix associated with Σ_1 has a row rank defect which implies that the transfer matrix of Σ_1 has a row rank defect. □

Corollary 1.2 also gives an alternative useful condition for simultaneous DDO in terms of the existence of a left kernel of a special type.

Theorem 2.2. *Suppose Σ_i, $i = 1, 2$ are both controllable. There exists a simultaneous DDO for Σ_i, $i = 1, 2$ if and only if there exist constant $T_i \neq 0$, $i = 1, 2$, and a stable rational proper matrix $Y(s)$ satisfying*

$$\begin{bmatrix} T_1 & T_2 & -Y(s) \end{bmatrix} \begin{bmatrix} (sI - A_1)^{-1}B_1 & 0 \\ 0 & (sI - A_2)^{-1}B_2 \\ C_1(sI - A_1)^{-1}B_1 + D_1 & C_2(sI - A_2)^{-1}B_2 + D_2 \end{bmatrix} = 0. \quad (2)$$

Proof. This is a direct consequence of the problem definition and Corollary 1.2. □

3.2 Simultaneous DDO with Common Function of States

Let us now impose a further constraint that $T_1 = T_2$ in the simultaneous DDO sought for Σ_i, $i = 1, 2$. The equation (1) should now be satisfied for some $T := T_1 = T_2$. The following counterpart to Proposition 2.1 can be stated.

Proposition 2.2. *There exists a simultaneous DDO with common functions of states for Σ_i, $i = 1, 2$ if and only if there exist stable rational proper* *matrices $X_i(s)$, $i = 1, 2$, $Y(s)$ and a constant matrix $T \neq 0$ satisfying*

$$\begin{bmatrix} X_1(s) & X_2(s) & Y(s) \end{bmatrix} \begin{bmatrix} A_1 - sI & A_2 - A_1 & B_1 & B_2 \\ 0 & A_2 - sI & 0 & B_2 \\ C_1 & C_2 - C_1 & D_1 & D_2 \end{bmatrix} = \begin{bmatrix} T & 0 & 0 & 0 \end{bmatrix}. \quad (3)$$

Proof. Note that

$$\begin{bmatrix} A_1 - sI & A_2 - A_1 & B_1 & B_2 \\ 0 & A_2 - sI & 0 & B_2 \\ C_1 & C_2 - C_1 & D_1 & D_2 \end{bmatrix}$$
$$= \begin{bmatrix} I & I & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} A_1 - sI & 0 & B_1 & 0 \\ 0 & A_2 - sI & 0 & B_2 \\ C_1 & C_2 & D_1 & D_2 \end{bmatrix} \begin{bmatrix} I & -I & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

It follows that (1) has a solution X_1, X_2, Y for some $T = T_1 = T_2$ if and only if $X_1, X_2 - X_1, Y$ is a solution to (3) for that T. □

Theorem 2.3. *Suppose the systems Σ_1 and Σ_2 are both controllable. There exists a simultaneous DDO with a common function of states for Σ_i, $i = 1, 2$ if and only if there exists a proper stable rational matrix $Y(s)$ and a constant matrix $T \neq 0$ satisfying*

$$\begin{bmatrix} T & -Y(s) \end{bmatrix} \begin{bmatrix} (sI - A_1)^{-1}B_1 & (sI - A_1)^{-1}B_1 \\ C_1(sI - A_1)^{-1}B_1 + D_1 & C_2(sI - A_2)^{-1}B_2 + D_2 \end{bmatrix} = 0. \quad (4)$$

Proof. The result is an immediate consequence of the problem definition and Corollary 1.2. □

Chapter 4

ROBUST OBSERVER-BASED FAULT DETECTION

4.1 Mathematical Model of The System

Recalling that the observer based FDI (and the model based FDI in general) involves a comparison between the actual system response and an anticipated system response generated using a mathematical model, the performance of the scheme depends on how faithful the model is to the underlying physical system. The better the model used to represent the dynamic behavior of the system, the better is the chance of achieving a robust fault detection and isolation system. In practice, the system is subject to different uncertainties that if omitted tend to create false alarms and corrupt the system performance. These uncertainties include modeling errors due to system parameter variations, unknown noise-type disturbances on the system and nonlinear terms in the dynamics. Therefore, the mathematical model should include a realistic description of the uncertainties in the system to be monitored. This kind of model is known as a *diagnostic system model* (in contrast to a representative model used for control purposes). In general, a dynamic system subject to faults and system uncertainties may be represented as follows [6]:

$$\begin{aligned} \dot{x} &= (A + \Delta A)x(t) + (B + \Delta B)u(t) + E_{11}d_1(t) + K_1 f(t), \\ y(t) &= (C + \Delta C)x(t) + (D + \Delta D)u(t) + E_{12}d_1(t) + K_2 f(t), \end{aligned} \tag{1}$$

where $x \in R^n, u \in \mathbf{R}^m, d_1 \in \mathbf{R}^l, f \in \mathbf{R}^g, y \in \mathbf{R}^p$ are the state vector, the control input (known), the disturbance vector, the fault vector, and the measurement vector, respectively. The matrices A, B, C, D, E_{11}, E_{12}, K_1 and K_2 are known matrices of appropriate dimensions. The matrices ΔA, ΔB, ΔC and ΔD are the parameter errors or variations representing the modeling errors. The disturbance and the fault vectors are unknown time functions whereas the fault and disturbance entry matrices , E_1, E_2, K_1 and K_2 which represent the effects of faults and disturbances on the system are known. According to [44], the modeling errors can be summarized as additive disturbances $E_{21}d_2(t)$ and $E_{22}(t)d_2(t)$ on the states and the outputs, respectively, where E_{21} and E_{22} are computed as functions of ΔA, ΔB, ΔC and ΔD, and where d_2 is an unknown function of time. We can then write (1) as

$$\begin{aligned} \dot{x} &= Ax(t) + Bu(t) + E_1 d(t) + K_1 f(t), \\ y(t) &= Cx(t) + Du(t) + E_2 d(t) + K_2 f(t), \end{aligned} \tag{2}$$

where $d(t) = [d_1(t)' \; d_2(t)']'$ is a new disturbance vector and $E_1 = [E_{11} \; E_{21}]$, $E_2 = [E_{12} \; E_{22}]$. In transfer matrix representation, (2) is

$$\bar{y}(s) = G_u(s)\bar{u}(s) + G_d(s)\bar{d}(s) + G_f(s)\bar{f}(s). \tag{3}$$

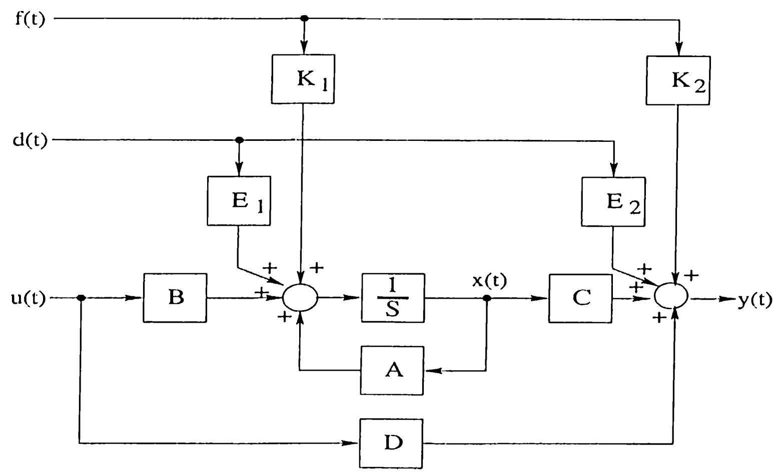


Figure 4.1: Diagnostic System Model

4.2 UIO-Based Residual Generation

A residual in the context of FDI is a scalar or vector valued signal that is accentuated by the fault vector. It carries information about the time and location of an occurrence of a fault. It is also independent of the normal operating state of the system.

To avoid false alarms that may be triggered by system uncertainties mentioned in the previous section, the residual should be insensitive to such uncertainties yet sensitive enough to faults that may occur. In other words, the residual should be chosen so that it discriminates between faults of interest and other disturbances acting on the system. Motivated by the decoupling property of the UIO, most robust residual generation is performed based on UIO. The residuals are chosen as the reconstruction errors of the states of the system which are independent of the unknown uncertainties thanks to the UIO observer. In fact, what interests us in FDI is the reconstruction of a function of states in presence of disturbances and not necessarily all the states. Recall from Chapter 2 that the existence condition for such observers are easier to satisfy.

4.2.1 Residual Generation

Let us consider a faulty system given by the equations (2) in state-space representation and by (3) in frequency domain. Let a general functional observer for this system be given by

$$\dot{\hat{x}}(t) = H\hat{x}(t) + Ly(t) + N_1u(t), z(t) = M\hat{x}(t) + Jy(t) + N_2u(t), \tag{4}$$

where the matrices H, L, M, N_1, N_2, J, and $T \neq 0$ are to be determined such that the residual

$$r(t) = Tx(t) - z(t) \tag{5}$$

in steady-state becomes zero for the fault-free case and nonzero for faulty cases. Taking Laplace transform of each term with initial conditions $x(0)$ and $\hat{x}(0)$ yields

$$\begin{aligned} \bar{r}(s) = & [T - H_y(s)C](sI - A)^{-1}x(0) - M(sI - H)^{-1}\hat{x}(0) \\ & + [T(sI - A)^{-1}E_1 - H_y(s)G_d(s)]\bar{d}(s) \\ & + [T(sI - A)^{-1}B - H_y(s)G_u(s) - H_u]\bar{u}(s) \\ & + [T(sI - A)^{-1}K_1 - H_y(s)G_f(s)]\bar{f}(s) \end{aligned} \tag{6}$$

Here, $H_y(s) = M(sI - H)^{-1}L + J$ is the observer transfer matrix from y to w and $H_u(s) = M(sI - H)^{-1}N_1 + N_2$ is the observer transfer function from u to w. We now pose the following requirements:

(i) $[T - H_y(s)C](sI - A)^{-1}$ and $M(sI - H)^{-1}$ are stable rational,

(ii) $T(sI - A)^{-1}E_1 = H_y(s)G_d(s)$,

(iii) $T(sI - A)^{-1}B = H_y(s)G_u(s) + H_u(s)$.

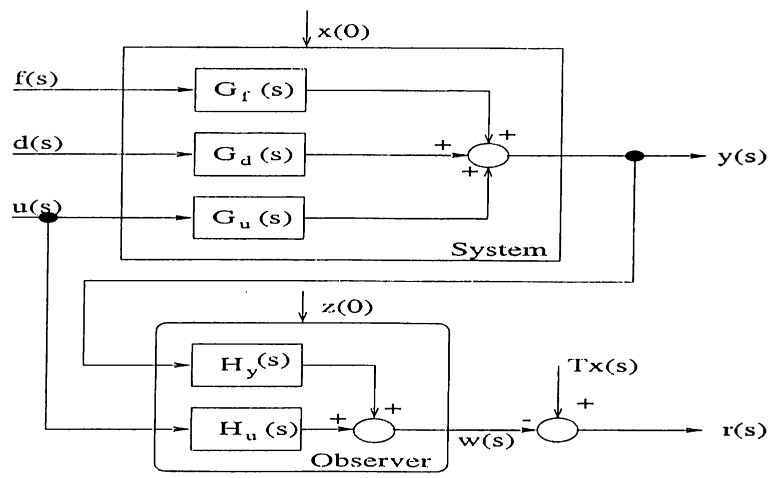


Figure 4.2: Residual Generator

Note, by (6), that (i) – (iii) hold if and only if

$$\lim_{t\to\infty} r(t) = 0 \;\forall\; x(0), \hat{x}(0);\; \forall\; u(t), d(t). \tag{7}$$

We now claim that (i) – (iii) *hold if and only if* (H, L, M, J) *is a functional DDO for the system* (A, E_1, C, E_2) *for a function* $Tx(t)$ *of the states.* To see this, first observe that the requirements $(i)-(iii)$ can be expressed equivalently by

(a) $[T - H_y(s)C] = X(s)(sI - A)$,

(b) $T(sI - A)^{-1}\begin{bmatrix} E_1 & B \end{bmatrix} = \begin{bmatrix} H_y(s) & H_u(s) \end{bmatrix}\begin{bmatrix} G_d(s) \\ I \end{bmatrix}$,

(c) $X(s), H_y(s), H_u(s)$ are stable proper rational matrices.

Since

$$\begin{bmatrix} G_d(s) \\ I \end{bmatrix} = \begin{bmatrix} C \\ 0 \end{bmatrix}(sI - A)^{-1}\begin{bmatrix} E_1 & B \end{bmatrix} + \begin{bmatrix} E_2 & D \end{bmatrix},$$

by Proposition 1.2, the requirements $(a) - (c)$ are satisfied if and only if there exist stable proper rational $X(s), H_y(s), H_u(s)$ satisfying

$$\begin{bmatrix} -X(s) & H_y(s) & H_u(s) \end{bmatrix} \begin{bmatrix} A - sI & E_1 & B \\ C & E_2 & D \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} T & 0 & 0 \end{bmatrix} \tag{8}$$

which holds if and only if

$$\begin{bmatrix} -X(s) & Y_1(s) & Y_2(s) \end{bmatrix} \begin{bmatrix} A - sI & E_1 & 0 \\ C & E_2 & 0 \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} T & 0 & 0 \end{bmatrix}$$

is satisfied for some stable proper rational $X(s), Y_1(s), Y_2(s)$. It is now clear that this last equality can be satisfied by $Y_2(s) = 0$, $Y_1(s)$ a functional DDO transfer function for (A, B, E_1, E_2), and an appropriate $X(s)$. This proves the italicized claim above. The existence conditions for a functional DDO (4) for the system (2) of this chapter and a DDO (2) for (8) of Chapter 2 thus turn out to be the same. The synthesis procedures are only slightly different. In designing a (4) for (2), one first obtains $T \neq 0$ and stable proper $X(s), Y(s)$ such that the equality (10) of Chapter 2 (with $E := T$) is satisfied. Then, $H_y(s) = Y(s), H_u(s) = X(s)B - Y(s)D$ will satisfy (8), with the same $X(s)$ and T, and hence the requirements $(a) - (c)$. A canonical realization of $[H_y(s)\ H_u(s)]$ is now a functional DDO (4) for (2).

Remark 4.1: Suppose that in our synthesis procedure we neglect the control inputs $u(t)$ altogether and choose a functional DDO (H, L, M, J), i.e., (4) with $N_i = 0$, $i = 1, 2$, for the system (2) with $T \neq 0$. We know from Section 2.1 that the requirements (i) and (ii) will be satisfied. We check the requirement (iii). The transfer function

$$T(sI - A)^{-1}B - H_y(s)G_u(s) = [T - H_y(s)C](sI - A)^{-1}B - H_y(s)D$$

is stable rational by (i) and by the stability of the observer transfer function $H_y(s)$. This implies that for all bounded control inputs $|u_i(t)| < \infty$, $i = 1, ..., m$, the effect on the residual will be bounded at all times and the presence of a fault can still be detected by a change in the steady-state residual value in the presence of a fault.

Remark 4.2: If the system (2) is stable, a common method of taking care of the control inputs $u(t)$ in the literature is canceling their effect on the outputs $y(t)$ at the outset. This requires defining

$$\hat{y}(t) := y(t) - \tilde{y}(t),$$

where $\tilde{y}(t)$ is the output of the "fault and disturbance free system", i.e., it is the output of (2) with $d(t) = 0, f(t) = 0,\ t \geq 0$. The observer (4) is then replaced by

$$\dot{\hat{x}}(t) = H\hat{x}(t) + L\hat{y}(t), z(t) = M\hat{x}(t) + J\hat{y}(t). \tag{9}$$

The residual $r(t) = Tx(t) - z(t)$ now becomes

$$\begin{aligned}\bar{r}(s) = {} & [T - H_y(s)C](sI - A)^{-1}[x(0) - \tilde{x}(0)] - M(sI - H)^{-1}\hat{x}(0) \\ & -T(sI - A)^{-1}\tilde{x}(0) + T(sI - A)^{-1}B\bar{u}(s) \\ & +[T(sI - A)^{-1}E_1 - H_y(s)G_d(s)]\bar{d}(s) \\ & +[T(sI - A)^{-1}K_1 - H_y(s)G_f(s)]\bar{f}(s),\end{aligned}$$

where the effect of the term $-T(sI - A)^{-1}\tilde{x}(0) + T(sI - A)^{-1}B\bar{u}(s)$ on $r(t)$ is a constant at the steady-state for bounded control inputs. Note that this method requires the simulation of a fault-free system model in order to obtain the output $\tilde{y}(t)$. In view of our Remark 4.1 however, this method seems rather pointless since, a functional DDO will provide constant steady-state effects on the residual for bounded inputs anyway.

A residual (6), in addition to (7), must have some further properties to detect and isolate faults:

1. The fault effect must be distinguishable from the effect of disturbances for the purpose of fault detection.

2. The effect of a fault must be distinguishable from the effects of disturbances and other faults for the purpose of fault isolation.

We now formally define detectability and isolability of faults with respect to the residual (6).

Suppose that $(i) - (iii)$ are satisfied for some $T \neq 0$ and a functional DDO (4). Then, the residual (6) becomes

$$\bar{r}(s) = \bar{e}(s) + G_{rf}(s)\bar{f}(s),$$

where

$$\bar{e}(s) := [T - H_y(s)C](sI - A)^{-1}x(0) - M(sI - H)^{-1}\hat{x}(0),$$
$$G_{rf}(s) := T(sI - A)^{-1}K_1 - H_y(s)G_f(s).$$

Let $[G_{rf}(s)]_i$ denote the i-th column of the matrix $[G_{rf}(s)]$.

Definition 3.1-Fault Detectability ([21]): *A fault $f_i(t)$ is said to be* **detectable** *if* $[G_{rf}(s)]_i \neq 0$.

Thus a detectable fault signal $f_i(t)$ (from the residual $r(t)$) is such that

$$f_i(t) \neq 0 \;\Rightarrow \lim_{t\to\infty} r(t) \neq 0$$

for any $x(0), \hat{x}(0)$ and for any $d(t), u(t)$. In order for all faults to be detectable through our residual (6), in addition to $(a)-(c)$, we also need to satisfy

$$[T(sI-A)^{-1}K_1 - H_y(s)G_f(s)]_i \neq 0 \;\forall\; i = 1, ..., g \tag{10}$$

by an appropriate choice of $T \neq 0$ and M, L, J, N_1, N_2.

Definition 3.2-Fault Isolability ([21]): *A signal $f_1(t)$ is* **isolable** *from $f_2(t)$ by the residual (6) if*

$$[G_{rf}(s)]_1, [G_{rf}(s)]_2$$

are linearly independent vectors (over $\mathbf{R}(s)$). This means that the effect of fault $f_1(t)$ on the residual is different from the effect of $f_2(t)$ since $[G_{rf}(s)]_1 = [G_{rf}(s)]_2(f_1(s)/f_2(s))$ is not possible. Fault isolability is ensured together with fault detectability (by the same residual vector) if $G_{rf}(s)$ has full column rank over $\mathbf{R}(s)$, a condition that will not be satisfied in case of a large number of faults. In practice, and in our simulation example of Chapter 5, however, fault isolation is achieved by defining different residuals for capturing different faults as we discuss in the next section.

4.2.2 Structured Residuals

Fault isolation is a more difficult task than just detecting a fault. One approach to fulfill this task is to design a *structured* residual set—structured in the sense that each residual is sensitive to a certain group of faults, while insensitive to others. The design procedure consists of two steps, the first is to specify the sensitivity and insensitivity relationships between the residuals and faults, and the second is to design the residual generators that will implement these desired specifications by treating the faults that the residual is insensitive to as unknown inputs (in addition to the already existing disturbances). The advantage of the structured residuals is that the diagnostic analysis is reduced to determining which residuals are non-zero. For each residual, a threshold test is performed separately, yielding a boolean decision table that will serve to isolate the faults taking place.

Given a set a faults $f_i(t)$, (i=1,2,...,g), a set of residuals $r_i(t)$, (i=1,2,...,g) can be designed according to one the following two types of structured residuals

Dedicated Residual Set: A set of residuals obeying the following condition

$$r_i(t) = Q(f_i(t)); i \in \{1, 2, , .., g\},$$

where Q(.) denotes a functional relation. A simple threshold logic can be used to decide about the appearance of a specific fault by logic decision according to:

$$r_i(t) > T_i \Longrightarrow f_i \neq 0; i \in \{1, 2, , .., g\},$$

where T_i (i = 1,2,...,g) are threshold values. If we let g = 4, we get the following table of dependency.

	$r_1(t)$	$r_2(t)$	$r_3(t)$	$r_4(t)$
$f_1(t)$	1	0	0	0
$f_2(t)$	0	1	0	0
$f_3(t)$	0	0	1	0
$f_4(t)$	0	0	0	1

In the table above, a "1" in i_{th} row and j_{th} column denotes that the residual r_j is sensitive to the fault f_i, i.e., depends on it, whereas a "0" denotes insensitivity.

Generalized Residual Set: The residuals of this type are generated according to the following equations:

$$r_1(t) = Q(f_2(t), ..., f_g(t))$$

$$r_i(t) = Q(f_1(t), ..., f_{i-1}(t), f_{i+1}(t), ..., f_g(t))$$

$$r_g(t) = Q(f_1(t), ..., f_{g-1}(t))$$

The isolation is again performed using simple threshold testing according to the following logic:

$$\left.\begin{array}{l} r_i(t) \leq T_i \\ r_j(t) > T_j \quad \forall j \neq i \end{array}\right\} \Longrightarrow f_i(t) \neq 0; i = 1, 2, ...g.$$

Again, if we let $g = 4$, we get the following table of dependency.

	$r_1(t)$	$r_2(t)$	$r_3(t)$	$r_4(t)$
$f_1(t)$	0	1	1	1
$f_2(t)$	1	0	1	1
$f_3(t)$	1	1	0	1
$f_4(t)$	1	1	1	0

4.3 Fault Detection and Isolation Scheme

Having addressed and discussed the residual generation problem, we are now ready to introduce UIO-based FDI scheme. Given a faulty system, the basic idea is to use a bank of (functional) unknown input observers to generate a set of structured residuals. The bank of observers, known as *observer scheme* may generally consist of an arbitrary number of observers, but often equals the number of faults to be detected and isolated[1]. To be more specific, let us assume that g different faults $f_i(t)$ $(i = 1, 2, ..., g)$ may take place in the system to be monitored. A bank of g (functional) unknown input observers is driven by the control input $u(t)$ and the system output $y(t)$ to generate g residuals $r_i(t)$ $(i = 1, 2, ...g)$ which in turn drive a decision logic unit responsible for issuing the fault alarms. The system is depicted in the following diagram.

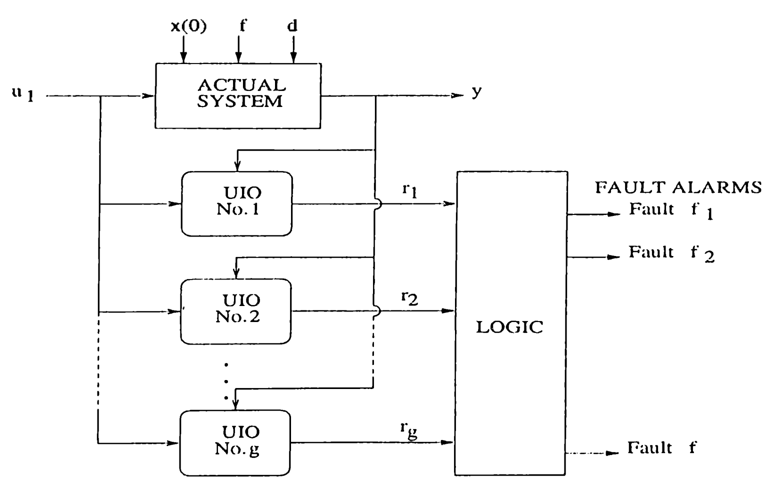


Figure 4.3: General Structure of UIO Scheme

The choice of structured residual type that the scheme relies on is determined by how many faults we want to detect and isolate at the same time. There are two main situations: either only a single fault is to be detected and isolated at a time or all faults are to be detected and isolated even if they occur simultaneously. In the first case, generalized residuals are used and the scheme is known as *generalized observer scheme*, while in the second case dedicated residuals are used and the scheme is refereed to as *dedicated observer scheme.*

4.3.1 Generalized Observer Scheme

The basic assumption underlying this approach is that only a single fault can take place at a given time, which is in practice most probable. To generate the g residuals, g different systems are formed from the state space equation of the plant (2) by treating one fault as a disturbance at a time. For each of these systems an UIO is designed to decouple the augmented disturbance vector that now includes one of the faults. A set of generalized residuals is then obtained. If a fault f_i takes place in the plant, the i^{th} residual is completely invariant to it ($r_i(t) = 0$) , whereas the remaining $g-1$ residuals $r_j(t)$, $j \neq i$, carry the fault symptoms. The appropriate evaluation of the residuals reveals the fault.

This kind of detection scheme is accurate and robust because the disturbance vector is augmented by only one fault leaving some design freedom to decouple the unknown inputs from the residual and achieve the desired robustness. However, if more than one faults simultaneously act on the system, all residuals will be affected by one fault or another and none of them is zero. In such cases, the scheme collapses.

4.3.2 Dedicated Observer Scheme

In contrast to the generalized observer scheme, the dedicated observer scheme handles simultaneous faults. Again g different systems are formed but this time all faults but one are treated as disturbances in turn. Each of the g UIO's is designed to decouple $g-1$ faults in addition to the unknown input vector. Therefore any residual $r_i(t)$ $(i=1,...,g)$ is sensitive (dedicated) to a unique fault $f_i(t)$. If multiple faults occur in the system, then the specialized residuals will be nonzero while the rest remain unaffected. By considering which residual deviates from zero the faults are detected and isolated. The problem with this scheme is that the augmented disturbance vector is overloaded by faults. In most cases it is difficult to design observers that decouple all $g-1$ faults. Even if they exist, there will be no design freedom left to reject other l true disturbances influencing the system resulting in a nonrobust FDI system.

4.4 Introducing Simultaneous UIO in FDI

The observer schemes, generalized and dedicated alike suffer from limitations that make it sometimes difficult to design a reliable detection and isolation system. In the previous section, we have seen that the former fails to reveal faults that act simultaneously on the system and the latter is not robust enough and is difficult to design. Nevertheless, each of them has its nice features; while the generalized observer scheme is relatively immune against system discrepancies, the dedicated observer scheme is capable of coping with simultaneous faults affecting the system. We want to combine both schemes so that the limitation of one of them is compensated by the other resulting in a robust scheme against disturbances as well as simultaneous faults. The approach is based on simultaneous functional UI observers of both types,the one that estimates the same function T of states and the other that estimates different functions T_1 and T_2 of states.

As a first step, we form g augmented systems using the plant equations (2) by augmenting one fault at a time to the disturbance vector as in the case of the generalized observer scheme. We then partition the overall fault vector into non-disjoint q sets Q_i $(i = 1, ..., q)$ with each set containing at least two isolable faults. They also should obey the following rule: For any set Q_i containing j faults, the corresponding j augmented systems formed in the first step should have a simultaneous UIO with the same function of states T. This requirement allows a single residual from the set

$$\mathcal{R}_d = \{R_i(t),\ i = 1, ..., g\}$$

to be sensitive only to the faults belonging to the complementary set of Q_i denoted by $\bar{Q}_i$. In such a case, we say that $R_i(t)$ is dedicated to $\bar{Q}_i$ which is slightly different from the usual dedicated residual scheme in the sense that a residual is dedicated to a group of faults instead of a single one. This kind of residuals allows the designer to exploit the decoupling properties of the system to the maximum without overloading the disturbance vector, a major problem that most of the time hinders the implementation of a dedicated observer scheme. Another advantage is that the designer can dictate the desirable robustness to the unknown inputs affecting the system, just like the case of generalized residuals. Of course, most likely, this will be at the cost of increasing the set of faults the residual is sensitive to and which in fact we want to bring as close to a single fault as possible.

To complete the picture, a regular set of generalized residuals

$$\mathcal{R}_g = \{r_i(t),\ i = 1, ..., g\}$$

is also designed. This time, simultaneous UIO's with different functions of states are used. Given the g augmented systems, an UIO is designed to observe different state functions of two or more systems. So more than one residual can be generated based on a single observer. Therefore, the number of observers deployed is reduced. The appeal to generating the residual set $\mathcal{R}_d$ in addition to the generalized set $\mathcal{R}_g$ is mainly to allow some sort of isolation in the case of multiple faults acting simultaneously. In fact, by partitioning the fault vector into smaller sets, the faults are squeezed into smaller groups and the isolation task is narrowed down to these groups. If the number of sets is high enough and/or well portioned, the redundancies allow an exact isolation of faults. Even if we fail to tell exactly which faults are affecting the system, we can at least limit them to a few possibilities. There is also the strategy of using these residuals in conjunction with some knowledge about the fault. For instance in the simulation example in the next chapter, the assumption that faults occur abruptly helps to exactly isolate them. Note that single faults are guaranteed to be detected and isolated thanks to $\mathcal{R}_g$, in this case $\mathcal{R}_d$ can be used for validation, to confirm the decision reached based on $\mathcal{R}_g$.

Algorithm

There are two possible algorithms that can be followed in obtaining a simultaneous DDO for a given number of two or more systems. The first algorithm is based on Propositions 1.2, 2.1, and 2.2 and Theorem 2.1. The second is based on Theorems 2.2 and 2.3.

Given Σ_i, $i = 1, 2$, of Section 3.1, the first algorithm that is used to design a functional simultaneous DDO is based on the fact that "either $rank\,[C_1\ C_2\ D_1\ D_2] > rank\,[D_1\ D_2]$ or Σ has a stable invariant zero" is a necessary condition for its existence. The algorithm that is described in the "if" part of Proposition 1.2 can hence be applied to the system matrix

$$S_{12}(s) := \begin{bmatrix} A_1 - sI & 0 & B_1 & 0 \\ 0 & A_2 - sI & 0 & B_2 \\ C_1 & C_2 & D_1 & D_2 \end{bmatrix}$$

Algorithm 1:

Step 1: Determine if a constant matrix J exists such that $J[D_1\ D_2] = 0$ but $T_1 := JC_1 \neq 0$ and $T_2 := JC_2 \neq 0$. If "yes" J is a constant simultaneous observer. If "no" check for a dynamic observer through steps 2-4.

Step 2: Determine the Smith Normal Form (SNF) and the associated unimodular matrices of $S_{12}(s)$. Let $\Lambda := US_{12}V$, where U and V are unimodular matrices, be the SNF.

Step 3: Factorize $\Lambda = \Lambda_s \Lambda_a$ into stable-antistable matrices, with the stable matrix Λ_s square and nonsingular. Then, partition the stable rational and non-polynomial matrix $\Lambda_s^{-1}U = [\ -X \quad Y\ ]$ with X having $2n$ columns.

Step 4: Write $X = X_+ + X_-$, where X_- is the strictly proper part of X. Expand the power series $X_- = X_{-l}s^{-l} + X_{-l-1}s^{-l-1} + ...$, $X_{-l} \neq 0$ and $l \geq 1$.

Step 5: Check if there exists a constant T_0 such that $T_0 X_{-l} =: [T_1\ T_2]$ (with each T_i having n columns) satisfies $T_i \neq 0$ for $i = 1, 2$. If "yes", then The transfer function of a simultaneous DDO is $H_y(s) = T_0(s^{l-1}Y)_-$. If a T_0 exists further satisfying $T_1 = T_2$, then the constructed observer is a simultaneous DDO with the same state function.

The second algorithm can be applied if Σ_i, $i = 1, 2$, are both controllable.

Let $Z_{12}(s) := \begin{bmatrix} (sI - A_1)^{-1}B_1 & 0 \\ 0 & (sI - A_2)^{-1}B_2 \\ C_1(sI - A_1)^{-1}B_1 + D_1 & C_2(sI - A_2)^{-1}B_2 + D_2 \end{bmatrix}$.

Algorithm 2:

Step 1: Let $[\Theta(s)\ \Psi(s)]$ be a minimal polynomial basis for the left kernel of $Z_{12}(s)$, where Θ has n and Ψ has p columns. Let the number of rows of the minimal polynomial basis be k and its row degrees be n_i; $i = 1, ..., k$. Ley

$$[\Theta_p(s)\ \Psi_p(s)] := diag\,\{s^{-n_1}, ..., s^{-n_k}\}[\Theta(s)\ \Psi(s)].$$

Note that, by properties of a minimal polynomial basis, $[\Theta_p(s)\ \Psi_p(s)]$ is a proper rational matrix and its constant coefficient matrix $[\Theta_p(s)\ \Psi_p(s)]_0$ is of full row rank.

Step 2: Determine a proper rational row vector $\Phi_p(s) \in \mathbf{R}(s)^{1\times k}$ such that $T_0 := \Phi_p(s)\Theta_p(s) \in \mathbf{R}^{1\times n+p}$-$\{0\}$, i.e., it is constant and nonzero.

Step 3: Partition T_0 as $[T_1\ T_2]$, where T_1 consists of the first n entries of T_0. If $T_1 \neq 0$ and $T_2 \neq 0$, then a DDO transfer matrix is given by $Y(s) := -\Phi_p(s)\Psi_p(s)$. If not, go back to step 2 and determine a different $\Phi_p(s)$. Note that whenever $T_1 = T_2$, this algorithm produces a simultaneous DDO with the same T. Alternatively, Algorithm 2 can be directly applied to the transfer matrix $\begin{bmatrix} I_n & I_n \\ 0 & I_p \end{bmatrix} Z_{12}(s)$ which appears in Theorem 2.3, instead of to $Z_{12}(s)$.

Both Algorithm 1 and 2 have the drawback of being based on "sufficient but not necessary conditions". In some cases where a simultaneous DDO exist, the algorithms may fail to produce one. However both sufficient conditions, especially the one that Algorithm 1 is based on, are "weak" so that they are close to being necessary. The chances of success of the algorithms is thus high.

Chapter 5

SIMULATION EXAMPLE: A FOUR-TANK SYSTEM

5.1 System description

In this chapter, detection and isolation of faults using simultaneous unknown input observers is illustrated. The example, a pilot plant, was originally studied by [46] then by [14] and [48]. Figure (5.1) shows a four-tank water flow system, with water levels x_1, x_2, x_3, and x_4. The water level of the second tank, x_2 is assumed unavailable for measurement. The tank is driven by the water flow input u_1. The linear model of the system in state space is given by

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

There are eight possible faults $f_i(t)$, $i = 1, 2, ..., 8$, that can affect the system. The first four $f_i(t)$, $i = 1, 2, 3, 4$ are leakages in the tank i and the remaining $f_{i+4}(t)$, $i = 1, 2, 3, 4$ are cloggings in pipe i. Table (5.1) gives the expressions

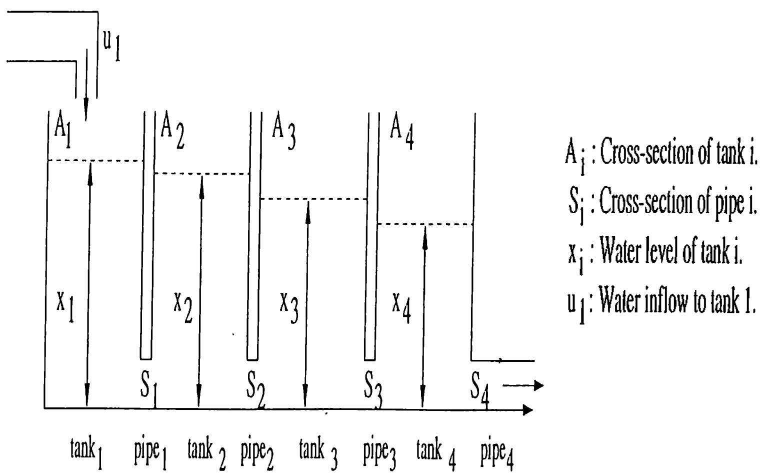


Figure 5.1: Four-Tank System

$f_i(t) = \frac{\tilde{S}_i}{A_i}[2g(x_i - h_i)]^{\frac{1}{2}}, i = 1, 2, 3, 4.$
$f_{i+4}(t) = -S_i^*[2g(x_i - x_{i+1})]^{\frac{1}{2}}$, i=1,2,3.
$f_8(t) = -S_4^*(2gx_4)^{\frac{1}{2}}.$

Table 5.1: Failure Functions of the Pilot Plant

of these fault signals, where $\tilde{S}_i$ and h_i are the cross-section and the height of the leak in tank i, respectively, and S^* is the reduction of cross-section of pipe i due to clogging, $i = 1, 2, 3, 4$. The fault entry matrices are depicted in Table (5.1).

Leakage				Clogging			
1	0	0	0	$\frac{1}{A_1}$	0	0	0
0	1	0	0	$-\frac{1}{A_2}$	$\frac{1}{A_2}$	0	0
0	0	1	0	0	$-\frac{1}{A_3}$	$\frac{1}{A_3}$	0
0	0	0	1	0	0	$-\frac{1}{A_4}$	$\frac{1}{A_4}$

Table 5.2: Failure Signature Matrices of the Pilot Plant

In addition, it is assumed that the system is under the effect of some disturbance $d(t)$ (this was not considered in the works of [46],[14] and [48]) acting on the rate of change of the unmeasurable water level $x_2(t)$ and directly influencing the water levels $x_1(t), x_4(t)$. Since the "entry vectors" of faults f_4 and f_8 are linearly dependent, these two faults are not isolable (see the discussion in Section 4.2). Hence they will be treated as a single fault. This isolation limitation is in fact the result of assuming that the fault signals are unknown. However, if a prior knowledge about these two faults is available, they may be isolated from each other. Letting the cross sections of the four tanks to be equal to 1, the overall system description is given in the fashion of (2) by the following equations

$$\dot{x}(t) = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u_1(t) + \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} f(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} d(t), \qquad (1)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} d(t).$$

5.2 FDI System Design

5.2.1 System Configuration

The proposed fault detection and isolation system to monitor the tank system is based on six unknown input observers, five of them simultaneous. Their role is to generate the two residual sets $\mathcal{R}_d$ and $\mathcal{R}_g$ of Chapter 4. Given that the system tank is under the influence of seven different isolable faults, seven generalized residual signals $\mathcal{R}_g = \{r_j(t),\ j = 1, ..., 7\}$ are produced. The number of the other residual signals are set to three $\mathcal{R}_d = \{R_i(t),\ i = 1, 2, 3\}$ insensitive to the fault sets $\mathcal{Q}_1 = \{f_1, f_2, f_3, f_5, f_6\}$, $\mathcal{Q}_2 = \{f_5, f_6\}$ and $\mathcal{Q}_3 = \{f_3, f_4, f_7\}$. The decision concerning the number of the last set of residuals as well as the partition of the faults in the three sets was reached after considering different choices and checking the existence conditions of the UIOs.

Figure 5.2 depicts the overall system. It includes a fault free duplicate of the plant which serves the purpose of canceling the effect of the control input u_1 on the residual signals. The output difference between the faulty and fault free tank systems is fed to the six observers. The residual array generated drive a decision logic unit which makes use of the sensitivity relation of the residuals to issue an alarm signal stating the fault number(s) and the time of occurrence.

5.2.2 Observers Design

To illustrate the process of observers design, consider the first unknown input observer which generates the residual signals $r_1(t)$ and $r_3(t)$ insensitive to the faults f_1 and f_3 respectively. Two systems, $\Sigma_1 = (A, B1, C, D1)$ and $\Sigma_3 = (A, B3, C, D3)$ are formed by throwing fault f_1 in the first case and fault f_3 in

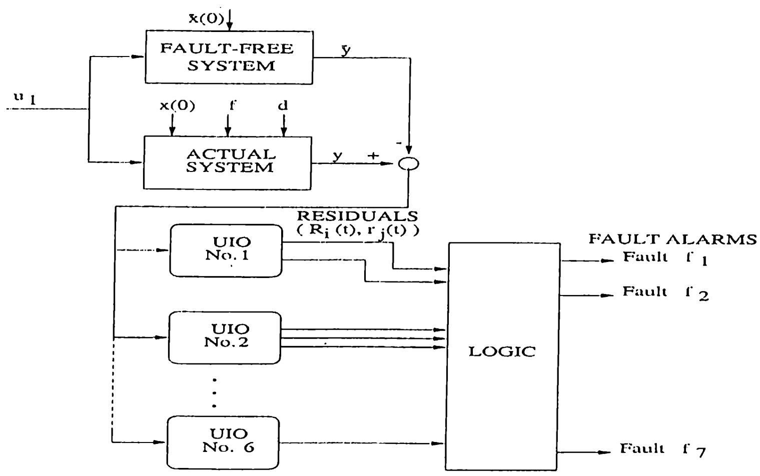


Figure 5.2: Four Tank System

the second case to the disturbance vector. Here, the control input and the rest of the fault vector are omitted because they do not play a role in the observer design. The augmented disturbance entry matrices, B_1, D_1, $B_3$and D_3 are given by

$$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, D_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, D_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Since the transfer functions of Σ_1 and Σ_3,

$$Z_1(s) = \frac{1}{s^4+7s^3+15s^2+10s+1} \begin{bmatrix} s^3+6s^2+10s+4 & (s^3+6s^2+10s+4)(s+1) \\ s+2 & (s+2)(s+1) \\ 1 & (s^3+6s^2+9s+2)(s+1) \end{bmatrix}$$

$$Z_3(s) = \frac{1}{s^4+7s^3+15s^2+10s+1} \begin{bmatrix} s+2 & (s^3+6s^2+10s+4)(s+1) \\ s^3+5s^2+7s+2 & (s+2)(s+1) \\ s^2+3s+1 & (s^3+6s^2+9s+2)(s+1) \end{bmatrix}$$

do not have full row rank, Theorem 3.1 cannot be applied to check for the existence of a common UIO for the two systems. Nevertheless, Algorithm1 was given a try and applied to the composite system matrix

$$S_{13} = \begin{bmatrix} -1-s & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & -2-s & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -2-s & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 1 & -2-s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1-s & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2-s & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2-s & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2-s & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

According to Step 1, a constant observer should be targeted first, however since C and D_i, (i=1,2,...,7) are common to all formed systems Σ_i, it follows that if such observer exists, it will be insensitive to all faults. (indeed such an observer exists). Consequently, throughout the design Step 1 is skipped.

Step 2:

The Smith normal form of S_{13} and the associated left unimodular matrix are given by

$$\Lambda_{13} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s+2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & s^2+3s+2 & 0 \end{bmatrix}$$

$U_{13} =$

$[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$

$[s+2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]$

$[s^2+4s+3, s+2, 1, 0, 0, 0, 0, 0, 0, 0,]$

$[0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$

$[0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, s+2, 1, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, s^2+4s+3, s+2, 1, 0, 0, 0, 0]$

$[s^2+4s+3, s+2, 1, 0, s^2+4s+3, s+2, 1, 0, s^3+5s^2+6s+1, 0, -1]$

$[0, -1, -s-2, 0, s^3+6s^2+10s+4, s^2+4s+2, 0, 1, 1, -s^2-4s-3, s+2]$

$[-s-2, -s-3, -s^2-4s-4, 0, s^4+8s^3+22s^2+23s+6,\ s^3+6s^2+10s+3, 0, s+2, -s^2-2s+1, -s^3-6s^2 11s-5, s^2+4s+4]$

$[s^2+3s+2, s^4+7s^3+16s^2+14s+3,\ s^5+9s^4+30s^3+45s^2+28s+4, -1, -s^7-13s^6-68s^5-183s^4-268s^3-205s^2-69s-6, -s^6-11s^5-64s^4-91s^3-86s^2-33s-3, 0, -s^4-7s^3-16s^2-13s-3, -s^4-6s^3-12s^2-9s-1, s^6+11s^5+47s^4+98s^3+102s^2+46s+6, -s^5-9s^4-30s^3-45s^2-29s-6]$

Step 3:

$\Lambda_{13s} = \Lambda_{13}$

$X =$

$[1, 0, 0, 0, 0, 0, 0, 0]$

$[(s+2), 1, 0, 0, 0, 0, 0, 0]$

$[(s^2+4s+3), (s+2), 1, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 1, 0, 0, 0]$

$[0, 0, 0, 0, s+2, 1, 0, 0]$

$[0, 0, 0, 0, s^2+4s+3, s+2, 1, 0]$

$[s^2+4s+3, s+2, 1, 0, s^2+4s+3, s+2, 1, 0]$

$[0, -1, -s-2, 0, s^3+6s^2+10s+4, s^2+4s+2, 0, 1]$

$\left[-1, \frac{-s-3}{s+2}, -s-2, 0, \frac{s^4+8s^3+22s^2+23s+6}{s+2}, \frac{s^3+6s^2+10s+3}{s+2}, 0, 1\right]$

$\Big[1, \frac{s^4+7s^3+16s^2+14s+3}{s^2+3s+2}, \frac{s^5+9s^4+30s^3+45s^2+28s+4}{s^2+3s+2},$

$\frac{-1}{s^2+3s+2}, \frac{-s^7-13s^6-68s^5-183s^4-268s^3-205s^2-69s-6}{s^2+3s+2}, \frac{-s^6-11s^5-64s^4-91s^3-86s^2-33s-3}{s^2+3s+2}, 0, \frac{-s^4-7s^3-16s^2-13s-}{s^2+3s+2}$

$$Y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ s^3+5s^2+6s+1 & 0 & -1 \\ 1 & -s^2-4s-3 & s+2 \\ \frac{-s^2-2s+1}{s+2} & \frac{-s^3-6s^2 11s-5}{s+2} & s+2 \\ \frac{-s^4-6s^3-12s^2-9s-1}{s^2+3s+2} & \frac{s^6+11s^5+47s^4+98s^3+102s^2+46s+6}{s^2+3s+2} & \frac{-s^5-9s^4-30s^3-45s^2-29s-6}{s^2+3s+2} \end{bmatrix}$$

Step 4:

$X_- =$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$[0, 0, 0, 0, 0, 0, 0, 0]$

$\left[0, \frac{1}{s+2}, 0, 0, 0, \frac{1}{s+2}, 0, 0\right]$

$\left[0, \frac{1}{s^2+3s+2}, \frac{s+2}{s^2+3s+2}, \frac{1}{s^2+3s+2}, \frac{s+2}{s^2+3s+2}, \frac{1}{s^2+3s+2}, 0, \frac{s+1}{s^2+3s+2}\right]$

$$X_{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}$$

Step 5:

Let

$$T_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

then,

$$t_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$t_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 \end{bmatrix}$$

and the observer transfer matrix is

$$h_{13} = T_0(s^{1-1}Y)_- = \begin{bmatrix} \frac{1}{s+2} & \frac{1}{s+2} & 0 \\ \frac{1}{s^2+3s+2} & \frac{-s}{s^2+3s+2} & 0 \end{bmatrix}$$

Since the second state is not available for measurements, the first rows of t_1 and t_3 are chosen zeros.

The remaining five observers are designed in a similar fashion based on Algorithm 1 when the observer is common to two systems and a generalized version of it when more than two systems share one observer as in the case of $H_1(s)$ where five system are observed simultaneously. The summary below gives the transfer functions of the observers, the residuals being generated by each of them, and the corresponding T.

1. $H_1(s) = \begin{bmatrix} 0 & \frac{s+1}{s^2+3s+2} & 0 \end{bmatrix}$
   used to generate$R_1(t)$ and $r_2(t)$ with
   $T_1 \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$ and $t_2 = \begin{bmatrix} -1 & 0 & 1 & 2 \end{bmatrix}$

2. $H_2(s) = \begin{bmatrix} \frac{-s-2}{s^2+3s+2} & \frac{-1}{s^2+3s+2} & \frac{s+1}{s^2+3s+2} \end{bmatrix}$
   used to generate $R_2(t)$ and $r_4(t)$, with
   $T_2 = \begin{bmatrix} -1 & 0 & 0 & -1 \end{bmatrix}$ and $t_4 = \begin{bmatrix} -2 & 0 & 1 & 0 \end{bmatrix}$.

3. $H_3 = \begin{bmatrix} \frac{2}{s^2+3s+2} & \frac{s+2}{s^2+3s+2} & 0 \\ \frac{1}{s^2+3s+2} & \frac{1}{s^2+3s+2} & 0 \end{bmatrix}$
   to generate $R_3(t)$ with
   $T_{347} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$

4. $h_{56}(s) = \begin{bmatrix} -\frac{s+2}{s^2+3s+2} & -\frac{3s+4}{s^2+3s+2} & 0 \end{bmatrix}$

to generate $r_5(t)$ and $r_6(t)$ with
$t_5 = \begin{bmatrix} -1 & -1 & 0 & -2 \end{bmatrix}$, and $t_6 = \begin{bmatrix} 0 & -1 & -1 & -3 \end{bmatrix}$

5. $h_7(s) = \begin{bmatrix} \frac{-s}{s^2+3s+2} & \frac{2s^2-s+2}{s^2+3s+2} & \frac{s+1}{s^2+3s+2} \end{bmatrix}$, to generate $r_7(t)$ with $t_7 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$

Tables 5.3 and 5.4 give the sensitivity relations of the obtained residual vectors R(t) and r(t) respectively, to the faults, where "0" represents insensitivity.

	$R_1(t)$	$R_2(t)$	$R_3(t)$
$f_1(t)$	0	1	1
$f_2(t)$	0	1	1
$f_3(t)$	0	1	0
$f_4(t)$	1	1	0
$f_5(t)$	0	0	1
$f_6(t)$	0	0	1
$f_7(t)$	1	1	0

Table 5.3: Sensitivity Relations of Residual Vector R(t)

	$r_1(t)$	$r_2(t)$	$r_3(t)$	$r_4(t)$	$r_5(t)$	$r_6(t)$	$r_7(t)$
$f_1(t)$	0	1	1	1	1	1	1
$f_2(t)$	1	0	1	1	1	1	1
$f_3(t)$	1	1	0	1	1	1	1
$f_4(t)$	1	1	1	0	1	1	1
$f_5(t)$	1	1	1	1	0	1	1
$f_6(t)$	1	1	1	1	1	0	1
$f_7(t)$	1	1	1	1	1	1	0

Table 5.4: Sensitivity Relations of Residual Vector r(t)

5.3 Simulation Results

The fault detection and isolation system to monitor the pilot plant is simulated using SIMULINK. Figure 5.3 depicts the simulink model of the overall system. It contains nine main blocks:

ACTUAL PLANT: This block is the representation of the four tank system as described by the state equations (5.1). See Figure 5.4.

FAULT-FREE PLANT: This is the model of the same plant but with the fault and disturbance vectors. It is driven only by the control input as depicted in Figure 5.5.

UIO1,....,UIO6: These blocks represent the six unknown input observers we have designed. They are implemented using state space equations. An example of UIO is depicted in Figure 5.6.

FAULT SIGNAL GENERATOR: This block is used to generate the fault signals described in Table (5.1) and to program the timing of start for a particular fault. See Figure 5.7.

The control input applied to the actual and fault-free plants is a unit step function. The disturbance d(t) is a random signal uniformly distributed in the interval [-.05 0.5] which is 50% of the control input. The leak fault signal is generated with the leak cross-section $\tilde{S}$ equals the tenth of the corresponding tank cross-section and hight h = .01. Similarly, the clogging fault signal is

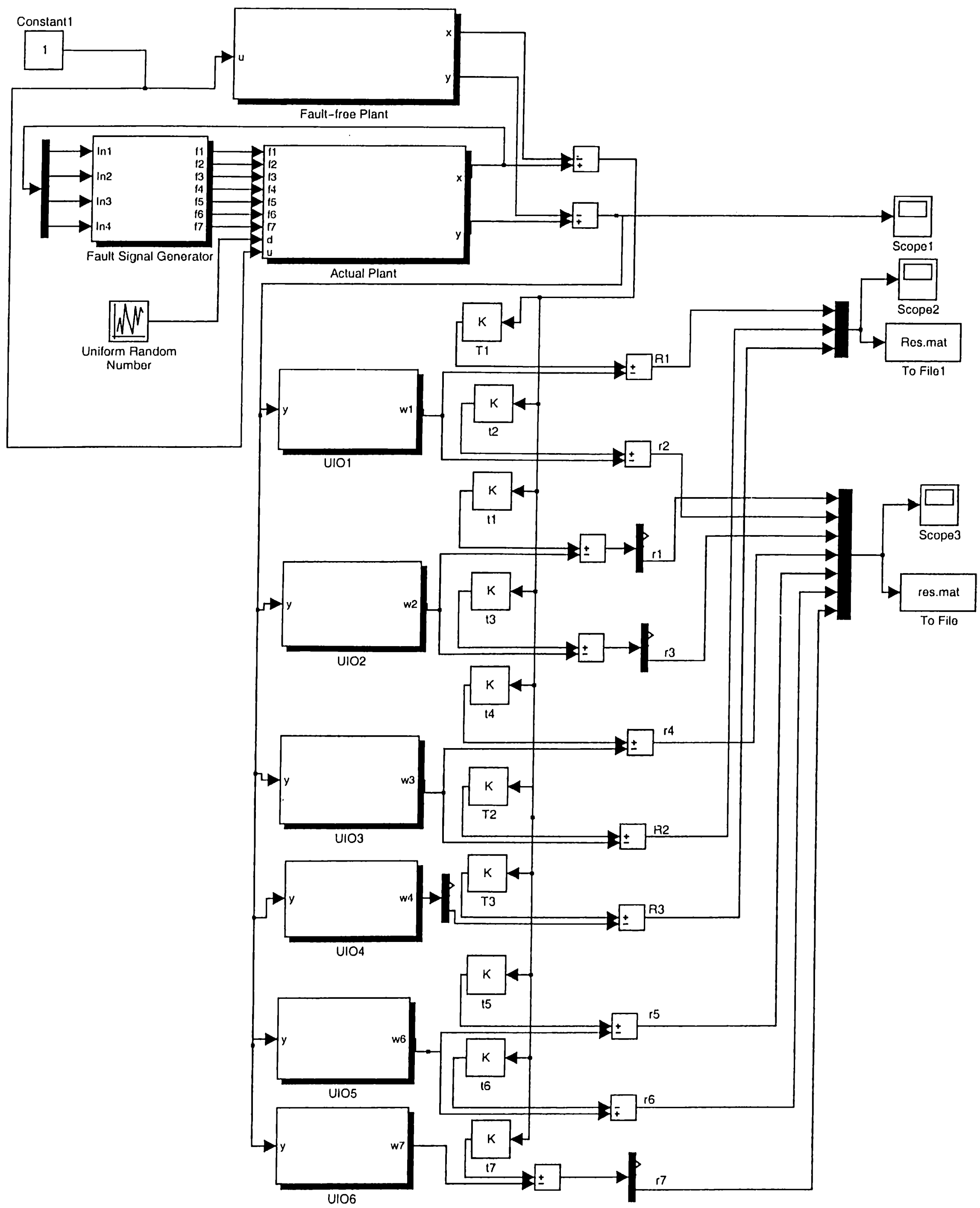


Figure 5.3: Overall Simulink Model

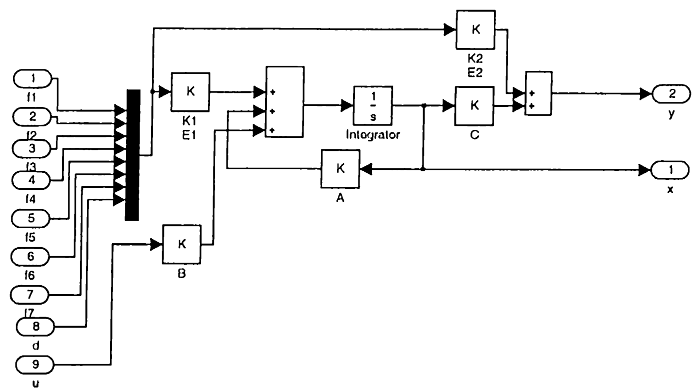


Figure 5.4: Actual Plant Model

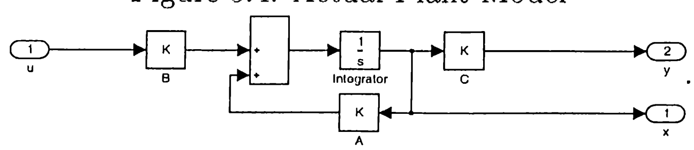


Figure 5.5: Fault-free Plant Model

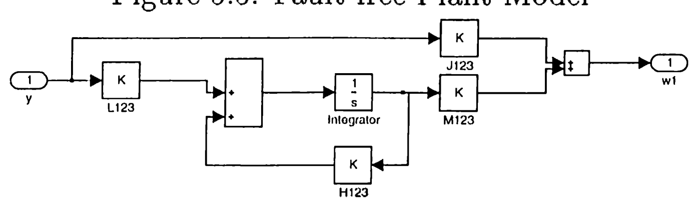


Figure 5.6: Simultaneous UIO No.1 Model

generated by considering the reduction of the pipe cross-section by the tenth of its normal value. All faults are programmed to occur suddenly without building up. In all experiments, the simulation time was 60 seconds and the initial conditions were chosen randomly including those of the each of the plants and observers.

First consider the case of a single fault acting on the pilot plant. The fault f_4 is programmed to occur at time t = 10 sec. By looking at the signal $e(t) = y(t) - y_o(t)$ (Figure 5.8) driving the observers, where $y(t)$ and $y_o(t)$ are the actual and fault-free plants outputs respectively, the presence of the random disturbance is clearly noticed. Based on $e(t)$ it is very difficult to

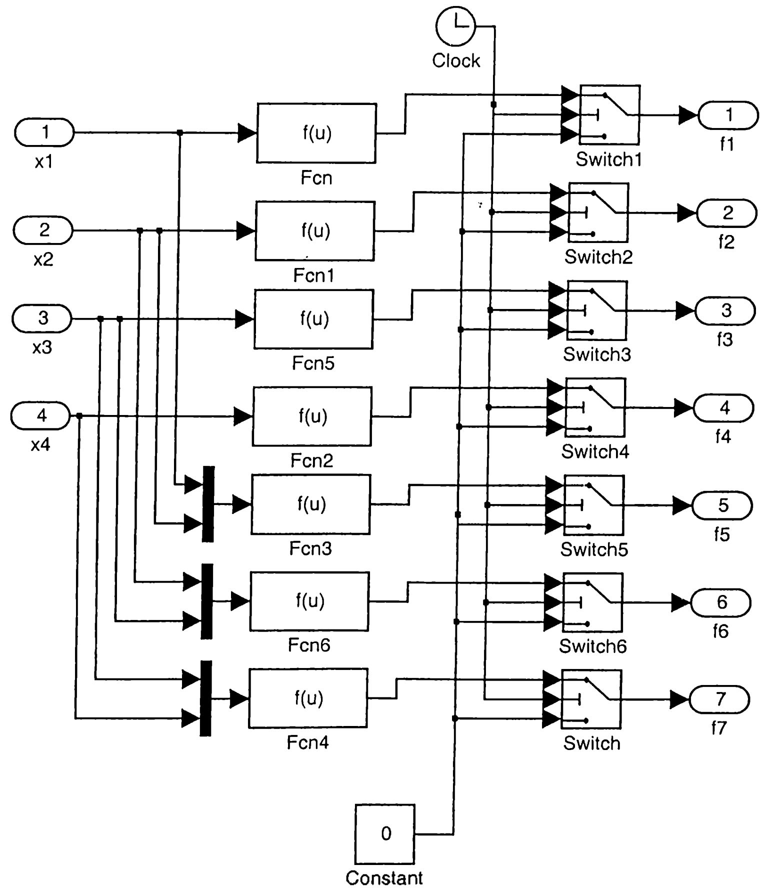


Figure 5.7: Fault Signal Generator

decide if a fault took place at all.

Based on the residual signals $r(t)$ of Figure 5.9, it is observed that fault f_4 took place at $t = 10$ sec. In fact, prior to this time instant and after the transients died out, all the residuals were identically zero indicating that the system is running fault free. At $t = 10$ sec, the abrupt change of the residual signals reveal that a fault just took place. Since all residuals but $r_4(t)$ are non zero, f_4

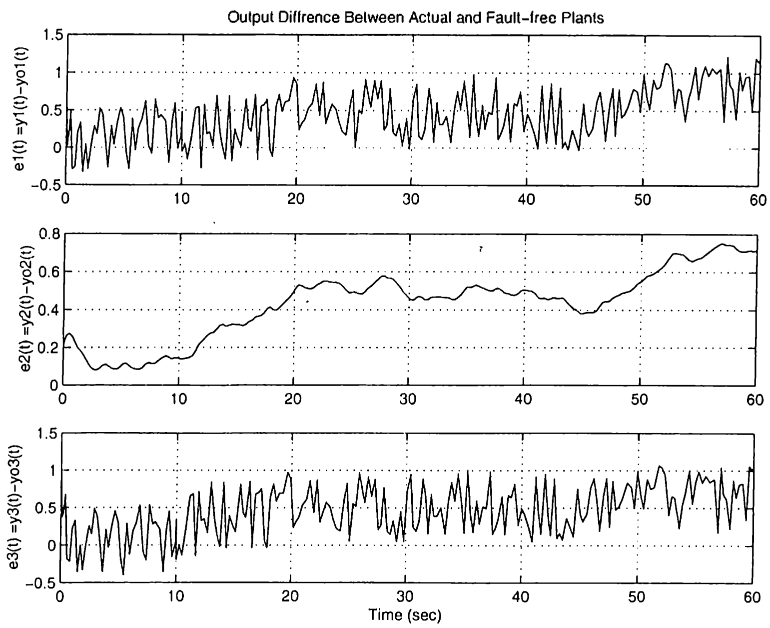


Figure 5.8: Output Difference of Actual and Fault-free Plants

must have taken place. Note that there is no trace of the random disturbance signal in any of the residuals. This is noticed especially when the system was operating fault-free.

Next, consider the pilot plant under the influence of multiple faults. It is assumed that multiple faults do not start to act at the same instant, instead they are separated by some time interval, which is practically a valid assumption. Four among the seven faults, f_2, f_7, f_6 and f_4 are programmed to take place in the system starting at time instants 10 sec 20 sec 30 sec and 40 sec respectively. Figure 5.10 depicts the residual vector r(t) as a function of time. At t = 10 sec, all the residuals but $r_2(t)$ start to deviate remarkably from zero which was their common value after the transients vanished. Hence fault f_2 took place at that instant.

Ten seconds later, an abrupt change in the signals' behavior including $r_2(t)$ (but not $r_7(t)$) is observed. This means that a new fault just took place and induced the new residual behavior. However, the logic decision for generalized

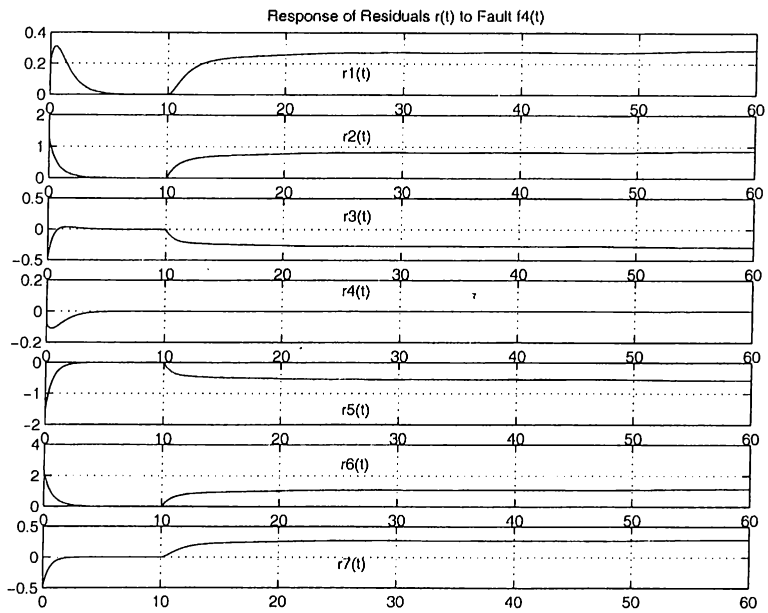


Figure 5.9: Generalized Residual Response to Single Fault

residuals cannot be applied anymore. This time the second set of residuals, $R(t)$ come into picture. Figure 5.11 shows that in addition to the change taking place at t = 10 sec as a result of f_2 and in which $R_2(t)$ and $R_3(t)$ switch to nonzero signals, the residual $R_1(t)$ became nonzero starting at t = 20 sec. Since $R_1(t)$ is sensitive only to f_4 and f_7 it is concluded that the new fault

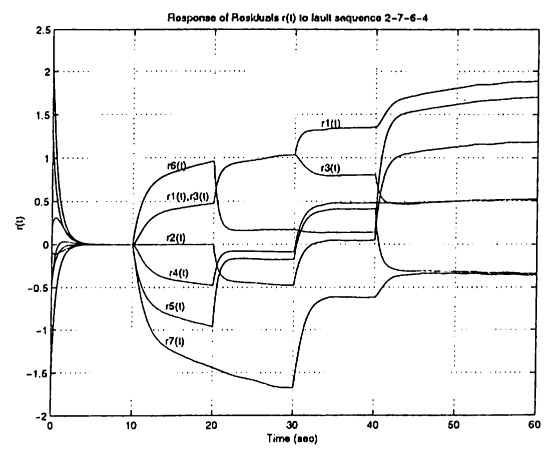


Figure 5.10: Generalized Residual Response to Multiple Faults

must be one of them. Furthermore, by taking into consideration that faults

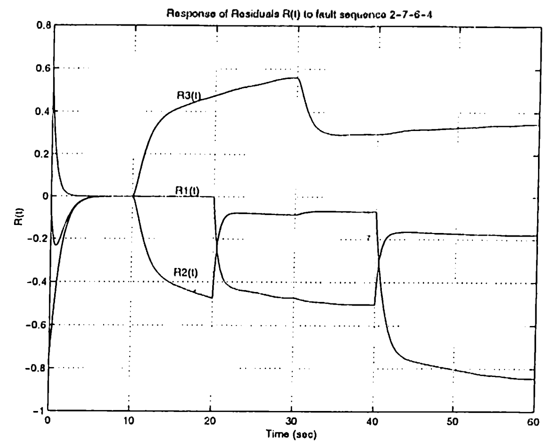


Figure 5.11: Residual Vector R(t) Response to Multiple Faults

occur suddenly and that $r_7(t)$ did not show any abrupt change while $r_4(t)$ did, it can be inferred that the fault is f_7.

Again the abrupt behavior of the residual vectors $r(t)$ and $R(t)$ at $t = 30$ sec indicates a new additional fault. By looking R(t), it is noticed that $R_1(t)$ and $R_2(t)$ continue to run smoothly whereas $R_3(t)$ changes abruptly suggesting that f_5 or f_6 took place. From the observation of the abrupt change i n $r_5(t)$ but not $r_6(t)$ it is deduced that f_6 is the fault.

Finally at $t = 40$ sec the new sudden change of $R_1(t)$ and $R_2(t)$ but not $R_3(t)$ indicates the presence of f_4.

This example demonstrates that assuming the abrupt occurrence of faults, the use of the generalized residual vector $r(t)$ in conjunction with $R(t)$ enables and enhances the detection and localization of multiple faults acting on the system, a goal that cannot be achieved with $r(t)$ only. Still, without the mentioned assumption the first fault can be detected and located and the second can be detected and isolated to two possibilities: f_4 and f_7.

If the number of residuals $R(t)$ are increased, there is certainly a better chance of detecting and isolating multiple faults. Of course, the method is restricted by the existence conditions of the corresponding simultaneous UIO with common T.

Chapter 6

CONCLUSION

Robustness is a decisive criterion for a practical fault detection system. The failure to address this problem in the design of FDI scheme could jeopardize its ability to distinguish between faults and disturbances stemming from model inaccuracies and different noise sources, thereby causing false alarms or missed targets. Unknown input observer based residuals discriminate between fault signals and other disturbances, thus revealing reliably any occurrence of a fault provided that the fault is detectable.

FDI schemes, whether generalized or dedicated, rely on structured residuals in achieving the tasks of fault detection and localization simultaneously. The generalized scheme is robust as most of the design freedom is invested in disturbance decoupling. However, it fails when more than one fault simultaneously act on the system. In contrast, the dedicated scheme could handle more than one fault since each residual is specialized in monitoring one particular fault. The price to pay for this scheme if it could be implemented at all is the lack of robustness as a result of exhausting the decoupling capabilities of the system.

We have proposed an alternative scheme that detects and isolates simultaneous faults while conserving the robustness thereby combining the advantages of the dedicated and the generalized schemes. Maximum exploitation of the decoupling capabilities of the system is sought by designing residuals dedicated to groups of faults rather than to a single fault. The redundancies among these groups either allow the isolation of a fault or at worst narrow down the isolation to small groups of faults. A regular set of generalized residuals is also employed in the scheme to guarantee the isolation of single faults and to help validating the decisions reached by the group dedicated residuals in the case of multiple faults. Generation of group dedicated residuals as well as a reduction in the total number of observers deployed in the proposed scheme are achieved thanks to simultaneous unknown input observers.

Since our main interest in simultaneous unknown input observers is their application in FDI, functional observers with unspecified function of states are examined. Two classes are distinguished. The first is the class of observers with different functions of states used to generate two or more regular residuals per observer. The second is the class of observers with common functions of states used to design group dedicated residuals. Based on some sufficient conditions for the existence of such observers, two algorithms are proposed in order to assist the design of regular and simultaneous unknown input observers,

A simulation example of a four tank system reveals the potential of the proposed scheme in detecting and isolating simultaneous faults without sacrificing from robustness. However, its performance depends heavily on how fine the fault vector is portioned into smaller sub-vectors and how many groups of dedicated residuals are deployed. Therefore, there is a great need to further study the existence conditions for simultaneous unknown input observers and to parameterize all such observers for a given fault-monitored system.

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