GENERATING A ROBUST MODEL FOR PRODUCTION AND INVENTORY CONTROL

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCES

OF BİLKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

BY

ASLI SENCER

FEBRUARY, 1993

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

GENERATING A ROBUST MODEL FOR PRODUCTION AND INVENTORY CONTROL

Aslı Sencer
M.S. in Industrial Engineering
Supervisor: Professor Doctor Halim Doğrusöz
February, 1993

In this study, we generate a production and inventory control model which gives 'robust' solutions against demand estimation errors. This model is applied to a real production and inventory system; however, it is a general model where the demand rate is stochastic with a known probability distribution and other parameters of the system are constant. The proposed model is a bi-objective decision making model, with two decision variables. A 'compromised' solution is found for the problem using the trade-off curve generated by a constrained sequential optimization technique, applied on a nonlinear programming model parametrically. Robustness against parameter estimation errors is tested by sensitivity analysis. Here a new dimension is added to sensitivity analysis methodology by including a sensitivity measure as a 'cost of error' of parameter estimation. By so doing, the proposed model is tested against the classical EOQ model and it is shown that the proposed model performs far better.

Key words: Production and Inventory Control, Economic Order Quantity, Sensitivity Analysis, Robustness, Cost of Error.

ÖZET

PARAMETRE TAHMİNİNDEKİ HATALARA DAYANIKLI BİR ÜRETİM VE ENVANTER KONTROL MODELİ

Aslı Sencer
Endüstri Mühendisliği Bölümü Yüksek Lisans
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Şubat, 1993

Bu çalışmada, talep tahminindeki hatalara karşı dayanıklı çözümler üreten bir üretim-stok kontrol modeli gelistirilmiştir. Bu model, gerçek bir üretim-stok kontrol sistemine uygulanmak üzere kurulmuştur. Önerilen model, çift amaçlı ve iki karar değişkenli bir karar modeli olup, talebin, yoğunluk fonksiyonu bilinen bir rastlantı değişkeni ve diğer parametrelerin sabit olduğu varsayımına dayanmaktadır. Modelde öngörülen 1-Maliyet minimizasyonu ve 2-Karşılanan talep yüzdesi maksimizasyonu amaçları arasında bir uzlaşık çözüm elde etmeye baz olacak bir değiş-tokuş eğrisi, modeli tek kısıtlı bir matematik programlama modeli gibi ve parametrik olarak işleterek elde edilmektedir. Modelin ve modelden elde edilen çözümün parametre tahminlerindeki hatalara karşı dayanıklılığı (robustness) duyarlık analizi ile ölçülüyor. Burada duyarlık ölçüsü olarak parametre tahminlerindeki 'hatanın maliyeti' kullanılmakla, duyarlık analizi metodolojisine yeni bir boyut getirilmektedir. Böylelikle, önerilen model, klasik EOQ (ekonomik sipariş miktarı) modeliyle karşılaştırılmakta ve EOQ modeline göre daha iyi sonuç verdiği gösterilmektedir.

Anahtar sözcükler. Üretim ve Stok Kontrolü, Ekonomik Sipariş Miktarı, Duyarlık Analizi, Dayanıklılık, Hata Maliyeti.

To my mother and father

ACKNOWLEDGEMENT

I am indebted to Professor Doctor Halim Doğrusöz for his supervision, and suggestions throughout this thesis study. I am grateful to Associate Professor Doctor Ömer Benli and Associate Professor Doctor Barbaros Tansel for their valuable guidance and comments. I am thankful to Associate Professor Doctor Cemal Dinçer and Associate Professor Doctor Erdal Erel for their interest in my thesis.

I would like to extend my deepest gratitude and thanks to my family and to my fiance for their morale support and encouragement, especially at times of despair and hardship. It is to them this study is dedicated, without whom it would not have been possible.

I really wish to express my sincere thanks to Levent Kandiller whose precious friendship, guidance and support turned my times of despair into enjoyable moments. I want to express my gratitude to Vedat Verter and Ceyda Oğuz for helping me with in any kind of computer work. And the last but not the least, I am grateful to my classmates İhsan Durusoy, Hakan Özaktaş and to my officemates Gülcan Yeşilkökçen, Mehmet Özkan, Pınar Keskinocak and Sibel Salman who shared my enthusiasm during the entire period of M.S. studies.

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Chapter 1

INTRODUCTION

Developing models for controlling production and inventory systems has been a major area for many researchers since the beginning of this century. As defined by Hax and Candea [14], production and inventory systems are concerned with the effective management of the total flow of goods from the acquisition of raw materials to the delivery of finished goods to the final customer.

In this thesis, we try to generate a production and inventory control model which gives insensitive solutions against demand estimation errors. The inspiration for this thesis subject came from a project conducted for production and inventory control in a process industry, a state enterprise PETKİM. Here, our aim is to design a control system which can be applied to this real system and used in the long run.

Very briefly, the problem of a plant manager in PETKİM is a specific production and inventory control problem. PETKİM is a huge complex with several factories; recently most of the managers are faced with the problem of increasing inventory levels and they can not cope with the financial burden of those inventory. What they need is a control system that provides an effective management of the elements (stocks, products, working staff, etc.,) of the system in order to deliver the final products in appropriate quantities, at the desired time, quality and at a reasonable cost.

In fact the aforementioned problem can be solved by using a general production and inventory control model; however when we look over the literature, we can see that most of the theoretical production and inventory control models were either misunderstood or not accepted by the managers in the past. The essence of implementation -especially for cases, that propose a change in ways of thinking- is to achieve *user acceptance and comfort.* No matter how a powerful system is created, it will be useless unless it is well understood and appreciated by the decision maker (DM). That's why in this thesis we will start by developing an inventory control model which will enable the DM to understand the benefits of the decision system, the decisions that are being made and results of those decisions in terms of the meaningful performance measures. This subject is analyzed by many researchers in the literature and are discussed in the next chapter [15] [19] [23] [26].

In this thesis we try to design a control system which has the basic properties discussed in the following paragraphs. Accordingly, a production and inventory control system should primarily

- *Guide the decision maker to decide.*

DM is the person who *controls* the production and inventory system. He needs an assistance while deciding on when, at what rate and how much to produce, etc. to achieve a certain service level. In this thesis, our aim is to develop a system which will give that assistance to the DM. By 'assistance', we mean that the DM should not be kept out of the decision process; quite the contrary, he should actively be involved in the decision process, rather than being replaced by an inventory control model. Thus, we should design a production and inventory control system that will generate solutions that incorporate the intuition of the DM.

The reason for searching for an information system-rather than a mathematical optimization model- is derived from the observation that,a mathematical model is only an *approximation* of the real world. An optimization model is characterized by the decision variables that optimize a *well-defined* goal (i.e., objective function) with respect to a set of constraints. The optimum solution of the model provides the 'best' vector of decisions, which means any other solution will be inferior to that of the optimum provided that the model is a 'perfect' representation of the real system. However, we know that these models involve a lot of initial assumptions and estimates. Thus any change in the initial estimates of the parameters of the model for instance may lead to incorrect decisions. The DM should be able to explain the reasons of the deviations from the expected outcome. In other words, he should be able to control the system by guessing what inputs provide what outputs and what are effects of a change. In this sense, a production and inventory control model should provide an effective assistance for the DM to help him/her learn the production system. Then we should state that another important property of our system is to

- Assist DM to learn how the system works.

As we have discussed in the above paragraphs, if a change occurs in the initial assumptions made, inventory control models may lead to misleading solutions. As we have repeated earlier, in this thesis our aim is to generate a model that gives insensitive solutions to changes in the initial estimates of the input parameters. Thus the model itself gives adaptive decisions while assisting the DM. We define these models as *robust* inventory control models and introduce a concept as the *robustness of a model.* This property is analyzed in detail in the next chapters. Thus another significant property of our model is to

- Generate adaptive decisions due to changes in the initial estimates of the parameters.

Our model is a bi-objective decision making model with two decision variables. We try to build up a control mechanism which enables the DM to make a trade-off between two conflicting objectives of minimizing the total cost (set-up cost and the inventory holding cost) and maximizing the service level, which is measured in terms of the ratio of the customers satisfied on time. The model generates a control mechanism similar to that of (s,S) policy type inventory control models. Thus, the decision variables are the maximum inventory level (produce up to level) and the reorder inventory level.

Our model originates from the classical EOQ model with a finite production rate. We know that EOQ model is based on the 'constant' demand rate assumption. When the demand rate is stochastic with a known distribution, it is likely that the solution is in error due to random fluctuations. In literature, it is shown that the optimal solution of EOQ model is insensitive to parameter estimation errors in demand rate. However, we now show that the *cost associated with a demand estimation error* may be significantly large in EOQ model when the demands are stochastic. Thus we modify the classical EOQ model by changing the decision variable from the production quantity Q to maximum inventory level I and show that this new model is insensitive to demand estimation errors when compared to the classical EOQ model. Using this model, we generate a reorder level model by incorporating the decisions related to the choice of the reorder level into the model. This optimization model is used to find the optimum values of the reorder level and maximum inventory level that minimizes the total cost function and provide a service level whose lower bound is defined by the DM. Thus our model generates an interactive control mechanism in which the intuition of the DM is actively involved.

In the next chapters, we try to explain our methodology while generating a robust and adaptive decision making model that will assist the DM while giving production and inventory control decisions. In chapter 2, we give a brief literature review of the inventory control models and then state the literature related to the difficulties encountered in implementing the theoretical models. In chapter 3, we define the properties of our production and inventory control system and discuss the main elements of our system of objectives that are considered in thesis work. Then in chapter 4, we state the steps followed in constructing the production and inventory control model. Chapter 5 includes the sensitivity analysis of our generated model to the estimation errors in the input parameters of the model and also includes the comparison of the performance of this model to that of the classical EOQ model. In chapter 6, we state the significant conclusions that we derive out of this thesis work.

Chapter 2

LITERATURE REVIEW

The earliest known analysis of inventory systems is made by Ford Whitman Harris, who first presented the 'economic order quantity' *EOQ* model in a publication in 1913 [13]. Harris's basic EOQ model became a paradigm for order quantity analysis, for at least the next 30 years. During this period, much confusion developed over the origin of the EOQ model. Most people know the EOQ formula as the *Wilson*'s lot size formula -as R.H.Wilson is claimed to be the first to use this formula in practice- while the others know it as *Green*'s formula and until thirties, for many Europeans it had been known as *Camp*'s formula. Although the original article may have been unknown for many years, the chapter version has been cited since 1931. In 1931, Raymond F.E. (see [11] for reference) gave Harris as the source of the EOQ formula; but the confusion about the formula's origin has persisted until its rediscovery in 1988 by Erlenkotter [10] [11].

During this period, the original EOQ model is developed by *Taft* (1918) (see [11] for reference) and used by many others like Green, Wilson , Alford, etc. Taft and Cooper analyzed a production and inventory system in which the production rate was finite. In 1928, Thornton C. Fry introduced the probability theory into the inventory models. He studied the cases where the demand rate is not known precisely.

Interest in the study of inventory systems has increased since World War II and numerous publications have been devoted solely to this subject. *Wagner and Whitin* [27] published an extension of the EOQ model in 1958 in which time phased dynamic demand and infinite production capacity over a finite planning horizon is considered.

With the advance of the mathematical inventory theory and easy availability of cheaper computer time, many researchers started to work on different types of inventory control models. Actually, the classical EOQ models have too many 'static' assumptions like 'constant' demand rate, constant production rate and lead time, etc. and therefore designated as a *static* model. However in the world these static assumptions hardly ever hold. Aggarwall [4] divides the inventory systems into two main categories in his 'review of current inventory theory and applications' paper (1974):

- Static inventory control models
- Dynamic inventory control models.

Dynamic models, which require a considerable amount of computation effort are obtained by varying 'constant' assumption for one or more of the variables of the static model. Other classification schemes are provided by Silver [23] and Nahmias [21]; but they all agree on such basic classifications. Dynamic models are *stochastic* if there is randomness in the process; otherwise they are *deterministic*. Thus we introduce a sub-classification:

- Stochastic vs. Deterministic dynamic inventory control models

These classifications can be further developed by considering the following properties of the inventory models:

- Single vs. Multiple items
- Single vs. Multiple echelon

- Backorders vs. Lost sales
- Zero vs Constant vs Random lead times
- Finite vs. Infinite planning horizon
- Various types of constraints and others.

More complex inventory systems are formed with several combinations of the above properties. We may give some well known examples from the literature: In 1971, Lasdon and Terjung investigated the multi-item inventory systems. In seventies, Zangwill [4] among others considered linking together of several single facilities and multiple products. Zangwill analyzed a deterministic multi-period production and inventory model that had concave production costs with backlogging allowed. Porteus [22] considered single product, periodic review, stochastic inventory model with concave cost function. Next, he advanced his previous results to prove that a generalized (s,S) policy would be optimal in a finite horizon problem, where demands were uniform.

While the deterministic inventory models were being developed by many other researchers, several others started working on stochastic inventory models. Apparently, when the parameter values vary stochastically with time, we can no longer assume that the best strategy is the one obtained from the deterministic model. This prevents us from using simple average costs over a finite or infinite planning horizon as was possible in EOQ derivation. Instead, we now have to use the information on the random parameter over a finite period, extending from the present when determining the appropriate value of current replenishment quantity. Another element of the problem that is important in selecting appropriate replenishment quantities is whether replenishments should be scheduled at specified discrete points in time or whether they can be scheduled at any point in continuous time. Thus the stochastic inventory control models can be

- Periodic review models or Continuous review models.

Still another factor that can materially influence the logic in selecting replenishment quantities is the information about the distribution of the random variable. In this thesis we specifically deal with models where the demand is random with a known distribution. We try to develop a production and inventory control model which allow for variations of the demand rate and still assure a proper control of the inventory levels. In the literature we encounter three basic types of stochastic inventory models:

A frequently used control procedure in practice is what is called an (s,Q) or two-bin system: it involves continuous review, a fixed reorder level (s) and fixed order quantity (Q). Decision rules have been developed for finding Q and s for a wide of choice of shortage costing methods and types of service constraints.

Another common control system is an (R,S) or periodic replenishment system in which an order is placed every R units of time sufficient to raise the inventory position to an order-up-to level S. Although the physical operation and costs of (R.S) and (s,Q) systems are likely to be quite different, it can be shown that the determination of S in (R,S) system is equivalent mathematically to finding the value of s in (s,Q) systems [24].

The third frequently used type of control system is (R,s,S) or periodic review, reorder level, order-up-to level system. In each R units of time an order is placed only if the stock position is below the reorder level s. Under general conditions it is shown that (R,s,S) system minimizes the expected total cost of replenishment, carrying and shortage cost [21].

However, through the years, difficulty of finding these three control parameters has made the mathematical optimality property of questionable value. Fortunately, effective heuristic procedures have been developed that permit relatively easy determination of good values of s and S [20].

We should note that these stochastic inventory control models are basicly developed for inventory systems where there is no production. In our thesis problem we have a stochastic production and inventory system with a constant production rate. Instead of incorporating the finite production rate assumption to these models, we try to generate an (s,S) type inventory control model by modifying the classical EOQ model and incorporating a service level constraint. By so doing, we try to eliminate the difficulties arising from implementation of a theoretical model in practice. In subsequent paragraphs we give some literature review related to the types and causes of problems that arise in the implementation phase of inventory control models.

Little [19] argues that a model that is to be used by a manager should be simple, robust, easy to control, adaptive, as complete as possible and easy to communicate with. By simple is meant easy to understand; by robust, hard to get absurd answers from; by easy to control, that the user knows what input data would be required to produce desired outputs; adaptive means that the model can be adjusted as new information is acquired. Completeness implies that important phenomena will be included even if they require judgemental estimates of their effect.

Similar discussion is made by Silver [23] where he makes some suggestions for bridging this gap between theory and practice. Just like Little, he suggests that more attention should be devoted to the aggregate consequences of inventory decision rules. Additionally, he states that 'exchange' curves are useful tools in this aspect, as they show the trade-offs between aggregate measures of interest for different possible decision systems.

The use of trade-off curves in generating interactive decision models is discussed by several researchers. Doğrusöz used this approach while developing an interactive decision making model for military equipment where the trade-off is made between the cost and effectiveness [9].

Wagner [26] published a paper in 1980, where the central theme was an enumeration of practical problems that needed research and analytic attention in inventory management systems. He stresses that the 'exact' assumption of parameters of demand distribution or the distribution itself often leads to misleading solutions. In order to overcome such difficulties, generally, the total cost function is optimized parametrically and the sensitivity analysis is made to test the rate of change in the optimal solution due to changes in input parameters. Note that, this way of approach requires two step calculations. Wagner suggests generating a trade-off function between the inventory investment level and required service level, which is actually our point of view in this thesis study. Commonly in literature, this service level is measured in terms of immediate stock availability; but there are many others as explained in the next chapter.

While developing a production and inventory control model, we try to consider all the ideas and suggestions given in the above paragraphs (see also [1] [2]). In light of these explanations, we try to generate a robust inventory control decision model that assists the decision maker while making decisions. However,we use the term 'robustness' in a different sense from Little. Actually we try to build-up a model which generates a robust inventory control strategy. In other words, the 'decisions' suggested by the model should be insensitive to the errors made in estimating the parameters of the model. More specifically, we define a 'robust' model as one for which the cost associated with a parameter estimation error is small. In this sense, our robustness definition differs from the 'robust estimator' definition in statistics too, i.e., according to our definition, when we have a robust model, even if the demand estimator is not robust, we still achieve an insensitive total cost function to demand fluctuations. Thus, what we are really concerned with is the robustness of the 'decisions' given by the model, rather than the estimation technique.

Huber [18] (a statistician) argues that the word 'robust' is loaded with many, sometimes inconsistent connotations. He uses this term in a relatively narrow sense as 'insensitivity against small deviations from the assumptions'. Our definition of 'robustness' is somewhat similar to this definition.

In this thesis, we measure the robustness of the generated model in terms of the costs associated with a demand rate estimation error. In the literature, we seldom come across the the term 'cost of an error' while making sensitivity analysis. In 1960's Ackoff [3] used this term to define the cost of any specified error due to search procedures. Later on in 1985, Silver and Peterson [24] used the 'PCP' (percent cost penalty) criteria as the % ratio of the cost of error to optimum total cost, while making sensitivity analysis.

In our thesis problem and in most real case problems, demands are random processes; for this reason we will stress on generating a robust model; where the cost of a demand rate estimation error is small. On the other hand, the inventory control process is maintained by generating a trade-off curve between the minimum total cost and required service level as suggested by Wagner [26] and Doğrusöz [9]. Thus we consider the problems encountered in the implementation phase and develop an inventory control model that 'assists' the decision maker, rather than one that replaces him.

Chapter 3

PRODUCTION AND INVENTORY CONTROL PROBLEM UNDER CONSIDERATION

3.1 SYSTEM ANALYSIS

Inventory systems differ from organization to organization in size and complexity, in types of items they carry, in the costs associated with the system, in the nature of the stochastic process associated with the system and the nature of information available to decision makers at any point of time. These variations have an important bearing on the type of operating doctrine that should be used in controlling the system [12]. For this reason PETKİM inventory system will be analyzed in the following basic elements:

Production and Inventory Control Activities in PETKİM.

PETKİM is the only petrochemical complex of Turkey. At the present, they hold 60% of the domestic market share, which is below the current production capacity. In Turkey, the domestic market for petrochemical products is shared by the 'foreign' competitors. Domestic customers usually prefer to purchase the required products from PETKİM because of the 'quicker delivery', 'easier contact' facilities.

In the past years, the demand to PETKİM products in the domestic market has been exceeding the production as the import and export activities were limited due to government protection by means of import taxes. Bearing on the advantage of being the only producer with a high demand for petrochemical products, they were able to sell every item they could produce. Hence for this reason, they used to produce at full capacity.

However, the market share of PETKİM was deeply affected when the import taxes were lowered by the government in recent years. As a result, the *overall domestic demand for petrochemical products has decreased* and furthermore *shared by the competitors.* The entrance of the foreign competitors to the domestic market decreased the market share and forced PETKİM to determine new production and inventory control policies in this competitive market.

PETKİM tries to survive among all these negative effects, by updating the pricing policy. They try to keep the price of the items at a 'lower' level than the prices of competing imported products so as to avoid the losses in customer demands. For this reason, import prices of PETKİM products are continuously followed and recorded. Essentially, PETKİM aims to satisfy demand in the 'domestic' market and use excess production for the 'foreign' market.

Production plans are prepared annually by the plant managers. Then this plan is sent to the 'sales' department which deals with the marketing of the items. Aggregate production plan does not include the production plan for 'different types' of a specific end item. For this purpose, *detailed production plans* are prepared monthly with the consideration of the current market demand. Revisions are made according to the estimated demand rates forecasted by the 'sales department'.

Briefly, we can say that, items are produced in plants and stored in stocks under the control of plant managers. Then the stocked items are offered for sale by the sales department. Here, one should note the lack of 'coordination' between 'sales' and 'production' departments in production management.

The production and inventory control policy of PETKİM, leads to great losses at the present. The causes lie behind the following facts:

The assumption of 'every produced item is salable is an 'acceptable' one, when the demand rate is higher than the production rate. However, the changes in the market environment brought the necessity of a revision in the 'production and inventory control activities' too. Obviously the demand rate is *no more* greater than the production rate and the market share of PETKİM has decreased to 60% (which is below the production capacity). That means, it may *not* be possible to sell all that is produced at the present, if they keep producing at full capacity. Producing at a rate higher than the demand rate leads to an accumulation of excessive stocks and increases the inventory holding costs or conversely, producing below the demand rate may lead to stockouts. This means that, production activities should be controlled by establishing a balance between counteracting phenomena.

Ignoring this fact is one of the major reasons of the current production and inventory control problems of PETKİM. The production rate should have been lowered (or stopped), when a decline was detected in the demand rate. *Insisting* on producing at the maximum production rate sharply increased the inventory levels and consequently lead to bank borrowing to finance these inventories at high interest rates.

This unfavorable situation can be remedied by establishing an effective production and inventory control system. The interaction and coordination between the production and sales departments is essentially important in this sense. In the next sections, a mathematical model is constructed for designing such a system; the system so designed is presented and discussed.

Demand and ordering processes :

As already stated, demand is not seriously forecasted in PETKİM. However, orders and sales are recorded in database files which would facilitate an attempt to forecast future demand.

PETKİM products are not demanded by large number of customers; the name of the customers for each item can be listed and their demand quantities can be followed. That's why the forecasted demand do not show large deviations.

In general, orders are delivered as soon as the payment is made. In fact, this is the usual case for most of the items; as the production rate is kept higher than the demand rate and the items accumulate in the stocks. On the other hand, stockouts may occur on occasions when an *unexpectedly* high demand occurs for an item which cannot be satisfied by production or on-hand inventory. Although this does not occur very often in PETKİM, managers tend to avoid such cases, as frequent stockouts may lead to demand losses.

'Demands are *totally backordered* in stockout cases'. We can argue that, *the customers order products with the purpose of obtaining them from PETKİM.* The reasoning lies behind the fact that, importing takes some time and also it is more costly. Thus, the customers usually prefer to wait some time; instead of purchasing from an 'import company' in case of stockouts. However, we should add that, backorder period has never been more than 1 or 2 months. The customers would probably prefer to purchase the products from the importing firms, if they know that the demand can not be satisfied within 1 or 2 months. For this reason there is an essential need for plant managers to establish a measure for customer satisfaction and develop an inventory control policy accordingly.

Production properties :

Production process is usually *continuous* in petrochemical industries. Raw materials are converted into 'end items' by passing through fixed routings. Although, items flow in batches through some production facilities, the quantity of these batches are reasonably small and thus the whole process can be viewed as a 'continuous' production process. Using this property, a processing unit in PETKİM can be considered as a 'single machine, single item' production system.

Production rate : Production rate is determined by the plant managers between allowable limits. Producing below and above these limits is not allowed as the former rate lowers the quality of the products (or practically not possible) and the latter is infeasible. It is important to note that the ranges (in which the production rate is flexible) are small and it is technologically possible to keep the production rate at a chosen 'constant' level between these allowable limits. Thus in our calculations, we will assume that the production rate is 'constant'.

Obviously the production should be equal to demand rate when the demand rate is between the minimum and maximum levels of production rate. If the demand rate is greater than the maximum production rate, then every produced item is sold and no inventory build up occurs. On the other hand, periodic-shut downs and start-ups are necessary if the demand rate is less than the production rate. We should also note here that, if the demand process is *stochastic*, stockouts may occur from time to time due to the stochastic fluctuations of the demand rate. For this reason, a reorder point inventory control policy should be developed to avoid stockouts in lead time as much as possible. The amount of inventory that is left at the end of the lead time (i.e. the safety stock) is the difference between the reorder inventory level and the total demand during lead time. Note that stockouts occur if this value is less than zero.

Production lead time : We shall define the *production lead time* (or simply the lead time) for a production and inventory system as 'the interval between the time when the decision maker gives an order to start production and the time that production actually starts.

In petrochemical processes, a certain level of temperature, pressure or material balance, etc., is required in the process so as to produce the right items in the right quality. Otherwise either a different end item is obtained or the quality of the items is different than the required level. For this reason, after giving an order to start production, a certain lead time is required in each factory to make necessary preparations to establish these necessary process conditions. In PETKİM production system, lead time is not more than 10 days in most of the factories.

Generally, the production lead time is not constant; however in PETKİM production system, the variations in the lead times are small enough so that in the mathematical model lead times are assumed to be constant. That is to say, the lead times in PETKİM factories may deviate 1-2 days from the expected length of the lead time which is not very significant.

Lead time is a critical factor which should be considered in production and inventory control as ignoring it may lead to stockouts or delays in the production schedule. Together with this well known property, we should also emphasize the significance of the *ratio of the lead time to the cycle time.* Comparatively 'long' lead times to the cycle times may lead to modifications or rearrangements in the production plan. If the lead time is greater than the depletion time, then the reorder inventory level comes out to be greater than the maximum inventory level. However we initially assume that the maximum inventory level (produce up to level) is always greater than the reorder level. Thus, the production and inventory control model (which is discussed in the next chapters) probably 'fails' when the lead time is longer than the depletion time. However, it will be seen that the lead times are to be very small in PETKİM, when compared to the cycle times. For instance, the longest lead time is 10 days (in laktam factory) and the cycle time usually is 2-4 months depending on the demand rate.

We will consider these properties while analyzing the production system of PETKİM.

Cost properties :

The costs incurred in operating an inventory system play a major role in determining what the operating doctrine should be. The costs which *influence* the operating doctrine are the ones which vary as the operating doctrine varies. According to this classification, costs that are independent of the operating doctrine need not be included in any analysis as they will not affect the inventory control model.Fundamentally, there are three types of costs of the former classification:
- Production Costs
- Costs associated with the existence of inventories (when supply exceeds demand)
- Costs associated with stockouts (when demand exceeds supply) [14]

PETKİM production system can be analyzed in terms of these basic costs:

Production costs in a production cycle can be divided into two parts: Those which depend on the quantity produced in a cycle and those which are independent of the production quantity.

First is the cost of raw materials, chemicals, utilities (like electricity, water etc.) consumed while producing a unit quantity (ton) of an item. The sum of these costs simply represent the *cost of the units produced* in the planning horizon (which is a constant) assuming sales equals demand and demand is constant. Note that these costs will not be considered in modelling, as they do not affect the 'production quantity' decisions (they are independent of the decision variables) in a cycle. This fact is analyzed in detail in section 3.2.

The second type of such costs are the costs incurred in making a 'set-up'; which are independent of the quantity produced, but their value per unit time depends upon the cycle time. Set-up costs are incurred by a shut-down and start-up of the production system. We have discussed that the right quality of items are obtained when the system works at a certain state (temperature, pressure, etc.). Thus a certain lead time is necessary to prepare the production unit and start the production from a previously inactive system. Set-up costs are incurred during these lead times. At every set-up in a PETKİM factory, some amount of materials (raw materials, end items, chemicals etc.), and utilities (electricity, water, water vapor, etc.) are consumed. If we note that the lead time in some factories are about 10 days, these unavoidable losses and the incurred costs may be significantly large (may go up to 0.5 billion TL). We will simply refer such costs as the *set-up costs* while modelling the production system.

Setup costs are very high in most of the factories in PETKİM. That's why frequent start-ups and shut-downs are avoided by the plant managers. We need to emphasize once more that such a production policy leads to high inventory holding costs, which constitutes the greatest financial burden at present.

Inventory Holding Costs: When the production rate is higher than the demand rate, inventories start to accumulate. Costs associated with the existence of inventories are due to a number of causes like storage, handling, taxes, insurance, spoilage, rent, capital costs, etc. Similar to the discussion for the production costs, not all the costs of carrying inventory vary with the inventory level in the same way. Indeed those costs which are proportional with the inventory level should be brought in to analysis.

In PETKİM, inventory holding costs are expressed as the sum of *capital costs* and *corporate taxes* (analyzed in section 3.2). Other factors like storage, handling, insurance are insignificant when compared to these or invariant with the inventory level. For instance, as the whole complex belongs to PETKİM no payment is made to outside agents for rent, handling etc,. Most of the items can stay for a long time in the stocks without any damage, thus the spoilage etc., do not incur a significant cost.

Stockout costs: A stockout situation arises whenever demand occurs and the production system is out of stock. In such cases, orders are either backordered or lost. It is also possible to substitute the demanded item with another available suitable substitute in stockout cases; but this kind of a treatment is not applicable for PETKİM products. The problem of quantifying stockout costs has long been a difficult question to answer; because of the intangibles such as the loss of customer's goodwill, loss of profits due to loss of customers, etc,.

In PETKİM generally, demands can be 'totally' backordered in case of stockouts. But it should be noted that frequent stockouts probably leads to loss of goodwill and customer losses, consequently adverse effects on future demands in the long run. That's why we generate a production and inventory control model in the next chapters in which we introduce a measure for setting the expected level of customer satisfaction and minimize the total costs at the same time.

Further details of cost structures and their relations with each other are discussed in section 3.2.

3.2 OBJECTIVES

The purpose of an effective management is to determine the rules that management can use to minimize the costs associated with producing goods and maintaining inventories while satisfying the customer demand. Two fundamental questions that must be answered in controlling a production and inventory system are how long and how much to produce. Actually, production duration, production rate, when to start and when to stop production are the basic controllable variables of any type of production system.

Briefly speaking, DM's objectives in PETKİM are

1. To keep production cost and inventory holding cost as low as possible and
2. Realize a high customer satisfaction.

Customer Satisfaction:
Here, the *'customer satisfaction'* is measured in terms of the capability of the firm to deliver the products to the customers in the agreed quality, quantity and time. There may be different value measures for the customer satisfaction like the ratio of total customer demand met on time, expected number of stockouts per unit time (to be minimized), expected number of cycles in which a stockout occurs etc, [3], [23], [26]. However in our model we will introduce the most commonly used measure *SLM* as *the fraction of customer demand satisfied on time.* Here, we will assume that the produced items satisfy the quality requirements. Low customer service level leads to loss of customer goodwill and loss of sales revenues resulting from the shortage situation. Obviously, it is generally difficult to measure the service level (or degree of customer satisfaction) in monetary units. That's why, it is usually preferred to express it in nonmonetary terms.

Production Costs:
As stated in the previous section, *'production costs'* can be seen as composed

of two parts: costs which are independent of the quantity produced and costs which depend on the quantity produced. Thus the production costs in a 'cycle' are

$$ProductionCosts/Cycle = S + pQ$$

where S is the *fixed cost of production* which is independent of the production quantity Q per cycle, and p is the *marginal cost of production* which is 'constant' in our system. According to this formulation, the total production costs in the planning horizon will be

$$TotalProductionCosts = SD/Q + pD \tag{3.1}$$

where D/Q is total number of cycles per unit time.

It is important to note that the second term in 3.1 can not influence the decision on (i.e., independent of) order size as they can be defined by pD where p is the 'constant' unit purchasing (or unit manufacturing) cost and D is the total demand in the planning horizon. Since the mean demand rate can be accepted to be constant, pD constitutes a constant term in the expected total cost function. Thus it can be dropped from consideration in the analysis.

On the other hand, the cost in the first term of 3.1 is highly relevant in the decision process, since it depends upon the order size Q. If the cost of a production unit's 'shut-down and start-up' in a production system is S -called the *set-up cost-* and if Q units of items are produced in a cycle, then the set-up cost per unit item in every cycle is S/Q, meaning that it is beneficial to produce the items in large batches as 'per unit share of set-up cost' is decreased in the total cost function. In PETKİM, set-up costs are calculated as the sum of utility costs, loss of raw materials and loss of end items during a unit shut-down and start-up (as explained in section 3.1).

$$S = utility\ costs + loss\ of\ raw\ materials + loss\ of\ end\ items$$

where S:unit set-up cost

Inventory Holding Costs:

The second category of costs are the *'inventory holding costs'* that are associated with keeping inventories. As explained in section 3.1, these are due to a number of reasons among which storage and handling costs (including rent for the storing facilities), property taxes, insurance and capital costs are essential. The capital costs represents either interest on loans to finance inventory or opportunity cost. Of all the above components, only those which change proportionately as the level of inventory level changes should be brought into analysis. In PETKİM, unit inventory holding costs are calculated by the following formula:

$$h = [\frac{f - e}{1 + e} + (CTR)\,(e)\,]\ (p)$$

where

h: inventory holding cost per unit quantity, per year

f: rate of cost of capital (annual interest rate)

e: annual rate of increase in the price (or in marginal production cost) of a unit quantity of item

CTR: corporate tax rate

p: current marginal cost of a unit quantity of an item

The first term in the bracket is the cost incurred by producing and storing the items in the inventory instead of investing the capital in another area. For instance, the capital on hand can be invested in a bank with an annual interest rate of f; however, if this capital is used for producing and storing items in the inventory, then the total interest charged over the capital is lost!. This loss can be partially compensated by the increase in the price of the material stocked.

The second term of unit inventory holding cost formula is due to the 'corporate tax' charged over the capital value of the items in the stocks. The annual increase in the unit price (or marginal cost) of the produced items is actually a 'fictitious revenue'; because, if the firm is making profits, a *corporate tax* is charged over the unit price (or marginal cost) of the item; thus any increase in the marginal cost (or price) of an item increases this corporate tax. This second term is the 'extra' corporate tax paid for the items as a result of the increase in their marginal prices or costs.

Analysis of the Related System of Objectives:
Using the above explanations, main objectives of a decision maker in PETKİM can be summarized as (figure 3.1):

- O_1: Minimize set-up cost per unit time
- O_2: Minimize inventory holding cost per unit time
- O_3: Maximize SLM

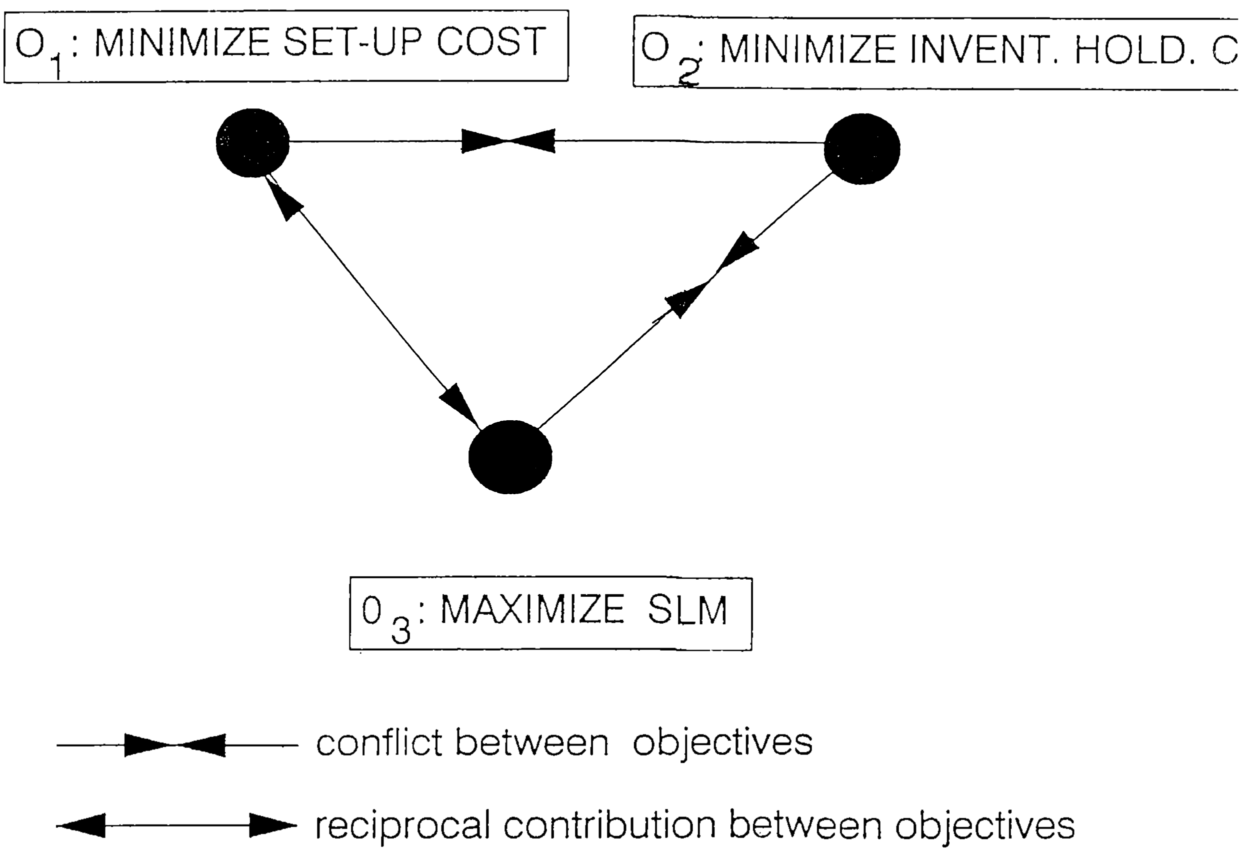

Figure 3.1: Relations between the system of objectives under consideration

These measures can be defined as a function of the maximum inventory level and the reorder level which will be stated as the decision variables in the next chapter. The relations between these objectives are given in figure 3.1.

Total cost function is written as the sum of production and inventory holding cost. In order to keep up a certain service level, demand can be satisfied both by the production and by the inventory held. If more frequent start-ups and shut-downs are made by decreasing the production duration (lot size), then the total set-up cost is *increased*, but the inventory holding cost is *decreased* as the 'maximum (and hence the average) inventory level' is decreased. Similarly -at a predefined service level- decreasing the number of set-ups decreases the cost of production, but increases the cost of holding inventory as the average inventory level is increased this time. That means, O_1 and O_2 are in *conflict* meaning that the attempt of achieving one of these objectives 'adversely' affects the achievement of the other.

Obviously, one of the purposes of producing goods and holding in inventory is to satisfy the customer demand on time! Higher customer service level can be obtained by *decreasing the number of stockouts in a planning horizon.* Now let's consider the ways of achieving this objective:

- One way of achieving less stockouts is to *decrease the stockout probability in lead time* by keeping a 'higher' level of inventory against stockout situations during the lead time. We know that this is achieved by increasing the reorder level. This strategy increases the total cost as inventory holding costs are increased by keeping a higher level of safety stock. As a result increasing the service level measure leads to an increase in the inventory holding cost. Thus O_3 and O_2 are in *conflict.*

- Another way of decreasing the total number of stockouts is to *decrease the number of cycles in a planning horizon.* We know that in every cycle, stockouts may occur in lead times; thus decreasing the number of cycles will decrease the total number of stockouts in the planning horizon. Here we should note that, such a strategy will lead to long production durations and increased cycle times; which will decrease the total set-up cost in return. That means, increasing the service level measure leads to a decrease in the set-up cost. As a result, we should say that there is a *reciprocal contribution* between O_1 and O_3. i.e., the pursuit of O_1

'favorably' affects the achievement of O_3 or vice versa. On the other hand, note that increasing the maximum inventory level again leads to increased inventory holding costs. Thus we show once more that O_3 and O_2 are in conflict.

An important economic problem is caused by the fact that the total cost (as the sum of production cost and inventory holding cost) can not be minimized while the customer satisfaction is maximized. These two objectives are again *in conflict*. More specifically, minimizing the total cost and maximizing the customer satisfaction can not be achieved at the same time. When the total cost is decreased (increased), then the customer satisfaction decreases (increases) too or vice versa. Hence a balance between these two objective variables should be established.

Briefly, *main* elements of our system of objectives that will be considered in throughout this thesis can be written of the form:

$$Min\ \ Total\ \ cost = Production\ \ cost + Inventory\ \ holding\ \ cost$$

$$Max\ \ SLM = Percentage\ \ of\ \ demand\ \ satisfied\ \ per\ \ unit\ \ time$$

The above discussion briefly describes the problem of a plant manager (DM) in PETKİM. More specifically, as the set-up costs are too high in PETKİM, the plant managers avoid frequent shut-downs and start-ups so as to decrease production costs. However, long production duration causes increasing stock problems. Thus decreasing the total set-up costs in a planning period increases the inventory holding costs. However, the service level is rather high for the time being as any demand for an item can be immediately satisfied from the high level of inventory held.

Decision Variables:
What is needed in PETKİM is to make a trade-off between these objectives and develop an optimum control strategy that will minimize the sum of production cost and inventory holding cost while satisfying the customer demand. Thus

a mathematical model will be developed in which the decision variable(s) can be chosen among the state variables like:

1. Production Quantity
2. Production Duration
3. Cycle Time
4. Maximum Inventory Level
5. Reorder Level
6. Safety Stock Level, etc.

Depending on the properties of the inventory system and modelling technique, decision variable(s) can be any of the state variables defined above. Actually the production and inventory control mechanism is based on the decisions about when to stop production and when to start again. In most of the inventory control models, the production duration is indirectly controlled by controlling the other state variables like the production quantity, maximum inventory level, reorder level etc. Actually, it may be easier for the decision maker to observe the incurring values of certain type of state variables. For instance in our problem, we suggest a production and inventory control model which works on the maximum inventory level and the reorder level. Production is stopped when the inventory level reaches a maximum level and it is started again when it drops to reorder level.

Note that if the decision variable is the production duration, all other state variables will stand for the consequence variables; that means the production quantity, cycle time, maximum inventory level etc. will take values as a function of the production duration. Similarly, depending on the mathematical relations between these state variables, safety stock comes out to be a consequence variable which is evaluated according to the value of the reorder level.

For the PETKİM production and inventory system, we take the maximum inventory level and the reorder level as the decision variables, instead of any other state variables. This enables us to establish a robust production and inventory control model where the losses incurred by parameter estimation errors are very low. The details of this analysis is given in the next chapters.

Chapter 4

MODEL CONSTRUCTION

In this section we deal with *sizing* and *timing* decisions for production lots and more precisely with *mathematical models* which offer an optimum production plan, namely a *lot sizing* strategy.

Our *main model* is a "bi-objective decision making' model. The first objective is to minimize the sum of production and inventory holding costs whereas the second objective is to maximize the customer satisfaction using a service level measure. In this approach, total cost can be expressed as a function of two decision variables: maximum inventory level (I) and the reorder level (g)

$$Total\ \ Cost = TC(I, g)$$

For any inventory control plan, it is possible to find a value measure for the *ratio of demand satisfied per unit time.* In other words, SLM is again a function of maximum inventory level and the reorder level.

$$Service\ \ Level\ \ Measure = SLM(I, g)$$

Then the main elements of our system of objectives comes out to be two different functions of the same decision variables I and g.

Depending on the values of the decision variables, a production and inventory control model is defined. This main model is somewhat similar to an *(S,s)* inventory control policy model, which operates on the inventory position. A regenerative process -as in the case of (S,s) systems- exists in the so-called 'main model', which operates in the following way: An order to start producing an item is given when the inventory level drops to a 'reorder' level, g (figure 4.1). Production starts after a lead time and the difference between production and demand is added to the inventory. Production continues until the inventory level reaches a 'maximum' level ($=I+ss$), then it is stopped until the inventory level drops to the reorder level (g) and a new cycle begins and so on.

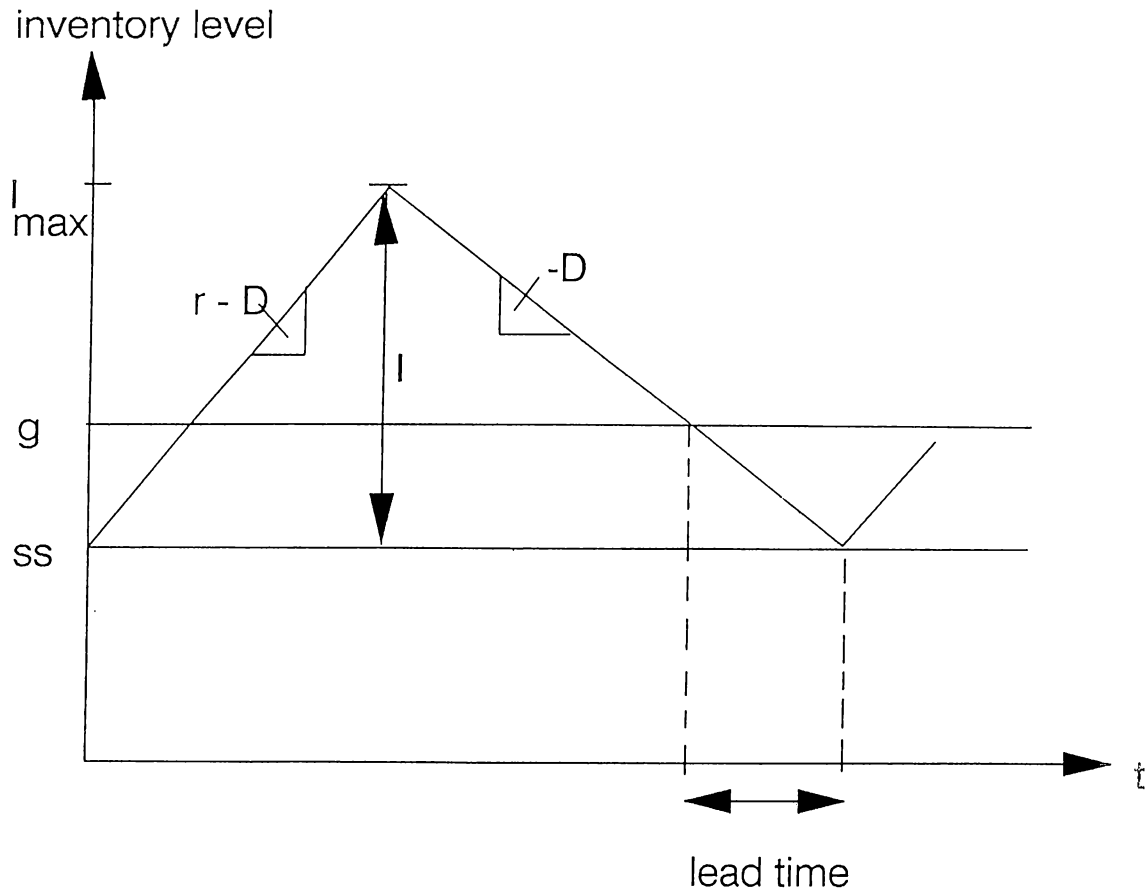

Figure 4.1: Change in the inventory level when the $SPIL$ model is applied.

In this proposed model, the inventory control mechanism is based on two decision variables: the **maximum inventory level** and the **reorder level**.

level. For this reason this proposed model will be referred to as **Stop Production Inventory Level Model** or shortly **SPIL MODEL**.

Development of the SPIL Model originates from the classical EOQ Model with finite production rate and constant demand rate. In this approach, an inventory control strategy is set which is based on the minimization of the total cost i.e., the sum of production and inventory holding costs.

However note that, it is almost never possible to predict the demand rate for certain. Actually, it is the 'expected value' of the demand rate which is considered when the classical EOQ Model is utilized. It is known that, in the classical EOQ Model, the relative change in the optimum production quantity due to change in the mean demand rate is small (explained in detail in the next chapter). However, if the *cost* associated with a change in the mean demand rate is considered, the classical EOQ model is not that much 'insensitive to the changes in demand rate'. That's why, the classical EOQ Model is modified by changing the decision variable from the 'production quantity' to the 'maximum inventory level' and a new model is obtained. Furthermore, sensitivity analysis shows that it is possible to decrease the cost of error due to a 'demand rate estimation error' by this change of the decision variable of the EOQ model.

The objective of 'maximizing the demand satisfaction' is incorporated into the model by introducing a 'reorder inventory level' which is based on the 'service level measure' defined by the decision maker. In this way, it is possible to avoid the stockout situations in lead time resulting from the probabilistic character of the demand rate. The expected 'safety-stock level' is determined as a function of the reorder level and the expected demand during lead time (i.e. expected safety stock level=reorder level - expected demand during lead time); while the stop production inventory level (I_{max} in figure 4.1) is the sum of I and safety stock level. In order to achieve a higher service level measure, the maximum inventory level and/or the reorder level should be increased. Note that the increase in the safety stock level (as a function of the reorder level) leads to an increase in the average inventory level and hence increases the total cost. However this facilitates the achievement of the service level measure, whose value is set by the plant manager (or DM). This relation enables the DM to make a trade off between the objectives and develop a '*compromised*' production and inventory control strategy.

Briefly speaking, the SPIL Model finds a *compromised* strategy. The reasoning behind a 'compromised solution' lies in the fact that, it is not theoretically possible to maximize the SLM and minimize the total cost at the same time as they are in conflict. On the other hand developing a mathematical model, that will support the DM while making a trade-off between the objectives requires a comparison of objectives with the 'same unit of measurement'. However, the SLM cannot be expressed in monetary terms. That's why, we can only seek for a 'compromising solution' by developing a trade-off curve after solving the following mathematical programming model parametrically, for varying the value of α.

$$Min\ \ Total\ \ Cost = TC(I,g)$$

$$subject\ \ to$$

$$SLM(I,g) \geq \alpha \quad \alpha = constant\ (a\ \ compromized\ \ satisficer)$$

that means,for a 'given' minimum service level measure, it is possible to find the values of I and g which minimizes the total cost function $TC(I,g)$. The minimum value of the service level measure is set by the decision maker after making a trade-off between the selected service level measure and the associated minimum total cost. An example trade-off curve is generated in figure 4.5. The steps followed in generating the trade-off curve is explained in the following sections. The trade-off between the SLM and the related total cost, i.e, the *compromise* needs the decision maker's 'judgement'. In this way the DM is actively involved in the decision process.

Finally it should be added that, we suggest an inventory and production control method which requires a *continuous review* of the inventory level. Otherwise, deviations from the control plan is unavoidable. However, in PETKİM, it is not difficult to check the inventory level continuously. There are about 20 factories in a PETKİM complex, each of which has 4-5 end items. Note that the $SPIL$ model is applied to each factory respectively and it is not too difficult to continuously follow the inventory level for the end items as the number of end items per factory is not excessive. Thus, this can not be a handicap for our system.

4.1 REVIEW OF CLASSICAL EOQ MODEL

The earliest known analysis of an inventory system was by F.W.Harris [13] in 1915. The basic model formulation assumes a continuous and infinite time axis, a constant demand rate, no backlogging possibilities, as well as constant set-up and inventory holding costs. It is also assumed that the quantity ordered is delivered to the inventory location in one lot at a specified time. Harris is assumed to be the first person who developed a lot sizing strategy, by drawing upon the *economic order quantity (EOQ)* formula,

$$Q^* = \sqrt{\frac{2SD}{h}} \tag{4.1}$$

Here, Q^* is the *optimal* lot size which minimizes the total cost function given by the below formula as the sum of set-up and inventory holding costs.

$$TC(Q,D) = \underbrace{S\frac{D}{Q}}_{Set-up Cost/Unit Time} + \underbrace{\frac{1}{2}hQ}_{Inv.Hold.Cost/Unit Time} \tag{4.2}$$

In this formula,

S : Cost of unit set-up

h : Inventory holding cost of a unit item per unit time

D : Demand rate (Quantity of an item demanded per unit time)

In the past, this formula has had more applications than any other method which gives single control strategy. Relaxations of the basic model assumptions are discussed by Hax and Candea [Hax] [14] and include models that allow for finite production rates, backlogging and quantity discounts.

The system for which this formula holds is a very special system, based on the above assumptions of the classical *EOQ* model. However, these may not actually fit to the production system of PETKİM. The initial assumptions of the theoretical and actual system can be analyzed in the following points:

While defining the properties of the PETKİM production system, it is stated that the demand rate has a stochastic pattern; that's why the assumption of constant demand rate is violated in practical applications. Instead, the 'expected value' of the demand rate is obtained by a 'forecast' based upon the demand history for the item under consideration. Hence the optimum solution of the model may not give the actual minimum total cost.

Production rate, r is also assumed to be constant. Actually, in PETKİM production system there is a short range in which the production rate can be changed, however it is possible to keep the production rate at a 'constant' level, once it is set at the beginning of the planning horizon. For our purposes, the determination of the production rate that will be valid throughout the planning horizon will be outside the scope of modelling. Thus, without loss of generality, we assume that the the production rate is 'constant'.

The assumption of 'delivery in lots' should also be relaxed so as to have a more realistic model that will fit to the production system of PETKİM. When a 'finite' production rate assumption is incorporated into the original EOQ Model, the total cost formula takes the form

$$TC(Q,D) = \underbrace{S\frac{D}{Q}}_{Set-upCost/UnitTime} + \underbrace{\frac{1}{2}hQ(1-\frac{D}{r})}_{Inv.Hold.Cost/UnitTime} \qquad (4.3)$$

where r is the production rate.

Since this is a continuous process industry, the end products outflow from the production unit *continuously* at a *constant* rate.

Here, the optimum value of the production quantity which minimizes the total cost function is:

$$Q^* = \sqrt{\frac{2SD}{h(1-\frac{D}{r})}} \quad (4.4)$$

It is assumed that the supply process is continuous and takes place at a constant rate until Q units are delivered to the stock-then it is stopped until the inventory level drops to zero again.

In the original *EOQ* model 'no shortages are allowed'. This can be achieved with certainty as the system is based on constant production and demand rate assumption. This means that, stochastic demand pattern may lead to stockouts in real applications. However, this situation is analyzed and the original EOQ model is modified so as to allow backlogging (see [14]); but it should be added that it is not easy to consider this model in real application; as the 'unit cost of a stockout' can not be easily defined for the system of PETKİM. Instead, we try to compensate for this weakness by introducing a 'service level' approach that will be a measure of the effectiveness of the model.

It is clear that, classical *EOQ* formula with constant demand rate defines a deterministic production and inventory control strategy, which is incompatible with the situation in PETKİM. However, the EOQ Model is suited to the PETKİM situation, by certain *modifications* as in section 4.2.

4.2 DERIVATION OF THE SPIL MODEL

Changing the Decision Variable of the Classical EOQ Model:

In the classical *EOQ* models , the decision variable is taken to be the production quantity per cycle, Q. The time behavior of the inventory level during the cycle is depicted in figure 4.2 for the case with a finite production rate r and demand rate D.

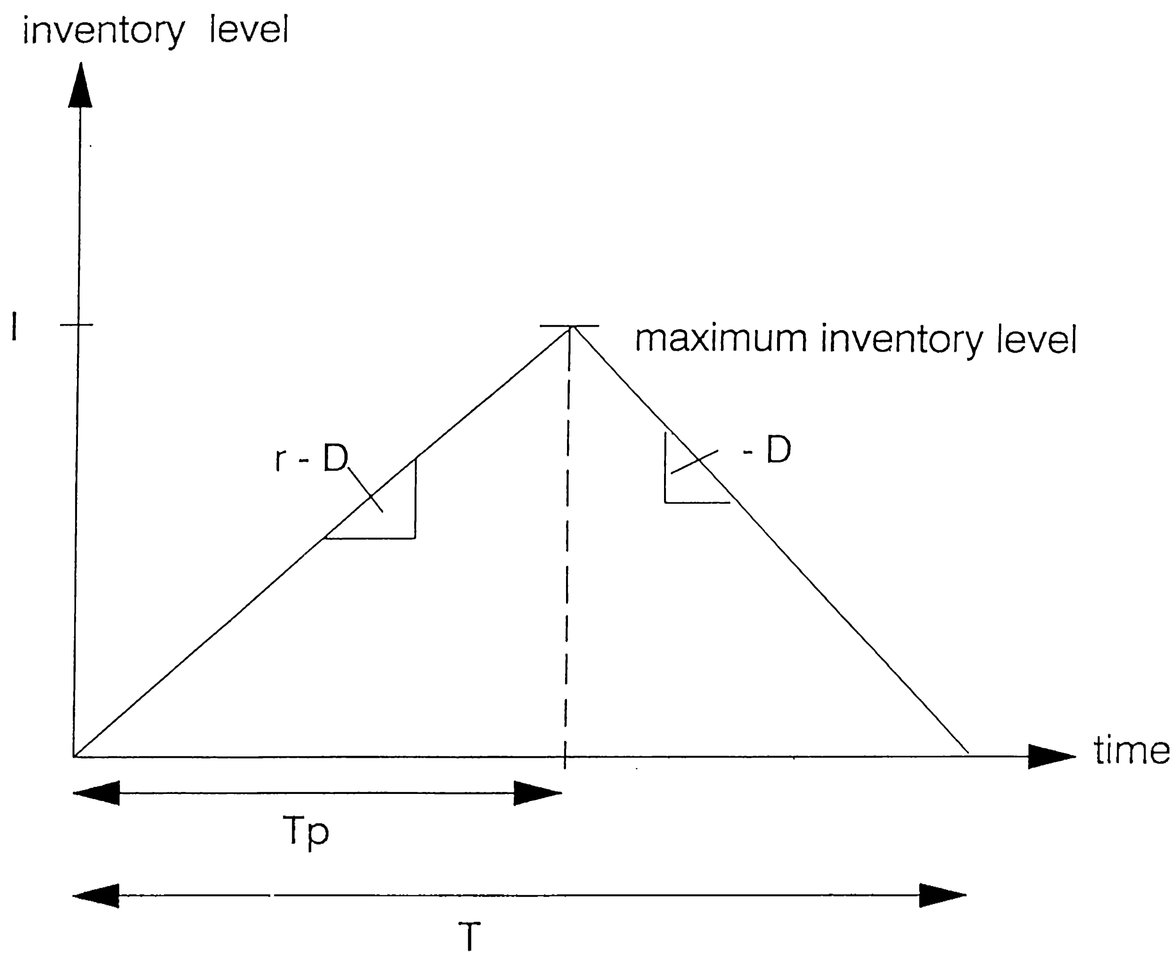

Figure 4.2: Change in the inventory level of the classical *EOQ* model with fixed production rate

In this figure T represents the 'cycle time' and T_p is the period of time over which the production takes place. Production starts with zero inventory level and continues at a constant rate of r, until Q units are produced in T_p time units. During this production period, T_p, inventory level increases at a rate of $r - D$ as items are input to the inventory at a rate of r and removed at a rate of D simultaneously. The inventory level reaches at its maximum I at the end of production period, then starts to decrease at a rate of D and reaches to zero level at which the production starts once again.

Here the relation between the maximum inventory level, I, and production quantity Q is given by

$$I = Q(1 - \frac{D}{r}) \tag{4.5}$$

The 'proposed' model is based on the hypothesis that a *robust* model can be constructed by taking the *maximum inventory level, I,* instead of the *production quantity, Q,* as the decision variable. The total cost function of the proposed model is formed by expressing Q in terms of I (as in 4.5) and substituting in 4.3,

$$TC(I, D) = S\frac{D}{I}(1 - \frac{D}{r}) + \frac{1}{2}hI \tag{4.6}$$

where, I is the maximum inventory level

The optimum value, I^*, of I, which minimizes this total cost function is given by the formula

$$I^* = \sqrt{\frac{2SD(1 - \frac{D}{r})}{h}} \tag{4.7}$$

Interpretation of this 'proposed' strategy is the following: Production starts with zero inventory level and continues at a rate of r until the inventory level reaches to I^*, then stops until the inventory level drops back to zero level again. The cycles continue in the same manner.

Generally speaking, it would be possible at this stage, to determine a production and inventory control strategy based on the 'maximum inventory level' (i.e., by using the formulation in 4.6 and 4.7), if demand were deterministic. Thus, taking the randomness of the demand into account, a control mechanism is required to take care of the 'customer demand satisfaction' by setting a *'reorder inventory level'* and a *'safety stock level'* against demand rate fluctuations. In the next section, the SPIL Model will be developed to incorporate the **Reorder Level** Model.

SPIL Model with a Reorder Point:

In a production and inventory system, whenever the inventory drops down to a certain level called *the reorder level*, an order to start production is given. This reorder inventory level should be sufficient to meet demand through the lead time. When demand is uncertain as in our case, it is obvious that the reorder point should be set to a reasonably high level to take precautions against stockouts. This reasonable high level is obtained by adding an 'allowance' to the mean demand during the lead time for protection against the uncertainty inherent in any forecast. This allowance is called the *safety stock* [14]. It is stated in section 3.2 that the quantity of this allowance should be increased in order to increase the customer satisfaction during the lead time.

In this section, we introduce a 'service level measure' for developing a reorder level model. As stated previously, our aim is to balance the 'service level' of the system against the increased production and inventory holding costs. The DM, drawing upon the graphical and tabular representation of trade-offs between those two objectives, determines a service level and applies the related plan to the production system on hand. For this purpose the service level measure is defined as:

- *SLM : Expected fraction of all demand that is met on time.*

For any fixed value of SLM, there may be different inventory control strategies in terms of the maximum inventory level and the safety stock level. Each of these reorder level strategies can be evaluated according to their associated total cost. Note that the objectives of 'increasing the service level measure' and 'decreasing the total costs' are in conflict with each other. Besides, it is not possible to express SLM in monetary terms (as previously stated). For this reason the trade-off between these objectives is subjected to the decision maker's judgement of choice.

Information on 'manufacturing lead time' and 'distribution of demand during the lead time' are essentially required for such an evaluation.

Demand rate estimation: The number of units demanded in lead time depends on the number of orders and size of each order. If we assume that the number of orders in lead time (N) is a *Poisson process*, $N \sim Po(\lambda)$ and the size of (number of units demanded in) each order (Y_i) is *normally distributed*, $Y_i \sim N(\mu', s^2)$ then the total number of units demanded during the lead time will be a *compound Poisson process* with mean $= \lambda\mu'$ and variance $= \lambda s^2 + \lambda\mu'^2$.

It is important however to recognize that when the demand is treated as continuous, the most frequently used distribution to describe the quantity demanded in a given time interval is the *normal distribution* [12]. Empirical studies have shown that, quite often the normal distribution seems to approximate the demand distributions very well over the relevant time intervals which are encountered in practice. Besides for large means, the Poisson distribution can be approximated by the normal distribution [17]. So without loss of generality, it is not wrong to assume that the demand rate during lead time is approximately 'normally' distributed with mean μ $(=\lambda\mu')$ and variance σ^2 $(=\lambda s^2 + \lambda\mu'^2)$.

Mathematical formulation of the reorder level model:

- For a given value of the reorder level (g), the probability of satisfying demand in lead time (PSD) is

$$PSD = P(x \leq g) = \int_{-\infty}^{g} f(x)d(x) \qquad where\ \ x \sim N(\mu, \sigma^2) \qquad (4.8)$$

  here,

  x is a r.v. showing the total demand during lead time and $f(x)$ is the

normal pdf.
μ is the expected value of the total demand in lead time and
σ^2 is the variance of the total demand in lead time.

- $B(g)$ is the expected quantity of stockouts in a cycle, when the reorder level is g

$$B(g) = \int_g^{\infty} (x - g) f(x) d(x) \tag{4.9}$$

Note that this function is not easy to evaluate, thus it is necessary to make use of the *normal loss function* (i.e., NL(t) tables where $t = (g - \mu)/\sigma$). This is valid under the 'normality' assumption of the lead time demand, where reorder level is defined by $g = \mu + t\sigma$.

Thus 4.9 can be written as

$$B(g) = \sigma NL(t) \quad and \quad t = \frac{g - \mu}{\sigma} \tag{4.10}$$

- '*Safety stock* is the expected inventory level at the end of the lead time'. We have discussed in section 3.1 that whole demand is backordered in case of stockouts. Assuming no loss of customer demand, the safety stock level can be calculated by

$$ss = \int_{-\infty}^{\infty} (g - x) f(x) d(x) \tag{4.11}$$

by simplification ss can be expressed as

$$ss = g - \mu \tag{4.12}$$

- The service level measure (SLM) is the expected ratio of demand met on time. Assuming a planning horizon of one year and expected demand rate per year D, we can write

$$SLM = \frac{Exp.\ Tot.\ demand/year - Exp.\ quantity\ of\ stockouts/year}{Exp.\ Total\ demand/year} \tag{4.13}$$

If Q is the production quantity/cycle, then the *Expected quantity of stockouts/year* will be

$$E(stockouts/year) = B(g)\frac{D}{Q} \tag{4.14}$$

where $B(g)$ is the expected quantity of stockouts/cycle and D/Q is the expected number of cycles/year.

Substituting 4.14 in 4.13 gives

$$SLM = 1 - \frac{B(g)}{Q} \tag{4.15}$$

Using the relation 4.5 and 4.9, SLM can be written as a function of the decision variables I and g

$$SLM(I, g) = 1 - \frac{\int_g^\infty (x - g) f(x) d(x)}{I}(1 - \frac{D}{r}) \tag{4.16}$$

Note that here

$$I = I_{max} - ss \tag{4.17}$$

where ss is defined by 4.12.

In SPIL Model, total cost is expressed by 4.6 as the sum of production and inventory holding costs. However, when it takes the form of a reorder level model with a reorder level included, the expected cost of holding safety stock should be added; then the expected value of the total cost takes the following form:

$$TC(I, ss) = \frac{SD}{I}(1 - \frac{D}{r}) + h\frac{I}{2} + h\ ss \tag{4.18}$$

where, D is the mean demand rate in the planning horizon.

Here, we should note once more that, SPIL Model is a bi-objective mathematical model with two decision variables I and g. When both objectives are expressed in terms of the decision variables, SPIL model will be

$$MIN \;\; TC(I,g) = \frac{SD}{I}(1-\frac{D}{r}) + h(\frac{I}{2}+g-\mu) \qquad (4.19)$$

$$MAX \;\; SLM(I,g) = 1 - \frac{\int_g^\infty (x-g)f(x)d(x)}{I} \; (1-\frac{D}{r}) \qquad (4.20)$$

In this expression, it is assumed that the demand rate and production rate are held constant. The value of expected safety stock level ss and stop production inventory level I_{max} are determined as a function of the decision variables I and g, by using the equalities 4.12 and 4.17.

As we have emphasized several times, it is not possible to find an optimum solution for the cost minimization problem when the SLM is *maximized.* However, if we are able to fix (or at least give a lower bound) for the value of SLM, then it is possible to find the optimum values of I and g, which minimizes total cost for that given value of SLM. The solution will be compromised, as the second objective is bounded -rather than optimized- by the value of SLM determined by the DM. Thus the SPIL Model will be

$$Min \;\; TC(I,g) = \frac{SD}{I}(1-\frac{D}{r}) + h\,(\frac{I}{2}+g-\mu)$$

$$subject \;\; to$$

$$SLM(I,g) \geq \alpha \;\; , \quad (0 < \alpha < 1)$$

4.3 SOLUTION TECHNIQUE

We use the *sequential optimization* technique suggested by Doğrusöz [8] to solve the SPIL Model stated above. That means, the total cost function is 'sequentially' minimized over I and g, subject to a service level measure constraint.

$$\min_{I,g} \; TC(I,g) = \min_{I} \; [\; \min_{g} \; TC(I,g) \;]$$

$$subject \; to$$

$$SLM(I,g) \geq \alpha \quad , \qquad (0 < \alpha < 1) \tag{4.21}$$

In other words, SPIL Model is optimized by solving the following system

$$\min_{I} \; [\; \min_{g} \; \frac{SD}{I}(1 - \frac{D}{r}) + h(\frac{I}{2} + g - \mu) \;]$$

$$subject \; to$$

$$[1 - \frac{\int_g^\infty (x-g)f(x)d(x)}{I} \; (1 - \frac{D}{r})] \geq \alpha \; , \quad 0 < \alpha < 1 \tag{4.22}$$

Substituting $B(g) = \int_g^\infty (x-g)f(x)d(x)$, 4.22 simplifies into

$$\frac{(1-\alpha)rI}{(r-D)} \geq B(g) \tag{4.23}$$

Note that the LHS of this inequality is a function of I; but constant for a given value of I. Let,

$$k(I) = \frac{(1-\alpha)rI}{(r-D)}$$

Hence, 4.23 can be written as

$$k(I) \geq B(g) \tag{4.24}$$

$B(g)$ is a decreasing function of g. Thus 4.24 is satisfied for those values of g which are greater than $B^{-1}(k(I))$, i.e.,

$$g \geq B^{-1}(k(I))$$

should be satisfied. This fact can be visualized in figure (4.3). For every fixed value of I, $k(I)$ defines an upper bound for $B(g)$. Our aim is to find the optimum value of g for a given value of I, under the SLM constraint. It is possible to minimize the total cost function over I, after expressing the optimum value of g as a function of I.

Thus the SPIL Model defined in 4.21 takes the simplified form as equation 4.25 as follows:

$$\min_{I,g} \; TC(I,g) = \min_{I \geq 0} \; [\min_{g \geq B^{-1}(k(I))} \; TC(I,g) \;] \tag{4.25}$$

$$= \min_{I \geq 0} \; [\min_{g \geq B^{-1}(k(I))} \frac{SD}{I}(1 - \frac{D}{r}) + h(\frac{I}{2} + g - \mu) \;] \tag{4.26}$$

Total cost function is an increasing function of g; thus it takes its minimum value at the lower bound $B^{-1}(k(I))$ of g. That means in the optimum solution g should be equal to $B^{-1}(k(I))$, i.e., 4.26 will be

$$\rightarrow \qquad \min_{I \geq 0} \; [\frac{SD}{I}(1 - \frac{D}{r}) + h(\frac{I}{2} + B^{-1}(\frac{(1-\alpha)rI}{(r-D)}) - \mu) \;] \tag{4.27}$$

Remember that, the unconstrained case of the total cost minimization problem has an unbounded solution; the above discussion shows that the constraint $SLM \geq 0$ is a *binding* constraint; hence g is not allowed to take values less than $B^{-1}(k(I))$ (see figure 4.3).

The last step of sequential optimization is to minimize 4.27 over I. Note that the function $B(g)$ is not invertible; thus the optimum value of I which minimizes the total cost function is found numerically by evaluating 4.27 for varying values of I. The solution found by this method gives the unique optimum of the SPIL model as the total cost function in 4.27 is *convex*. Convexity of the total cost function is obvious as it is the sum of the convex total cost function of the classical EOQ model and another convex term $hB^{-1}(k(I)) - h\mu$. Here $h\mu$ is a constant term; thus it may not be considered in calculations.

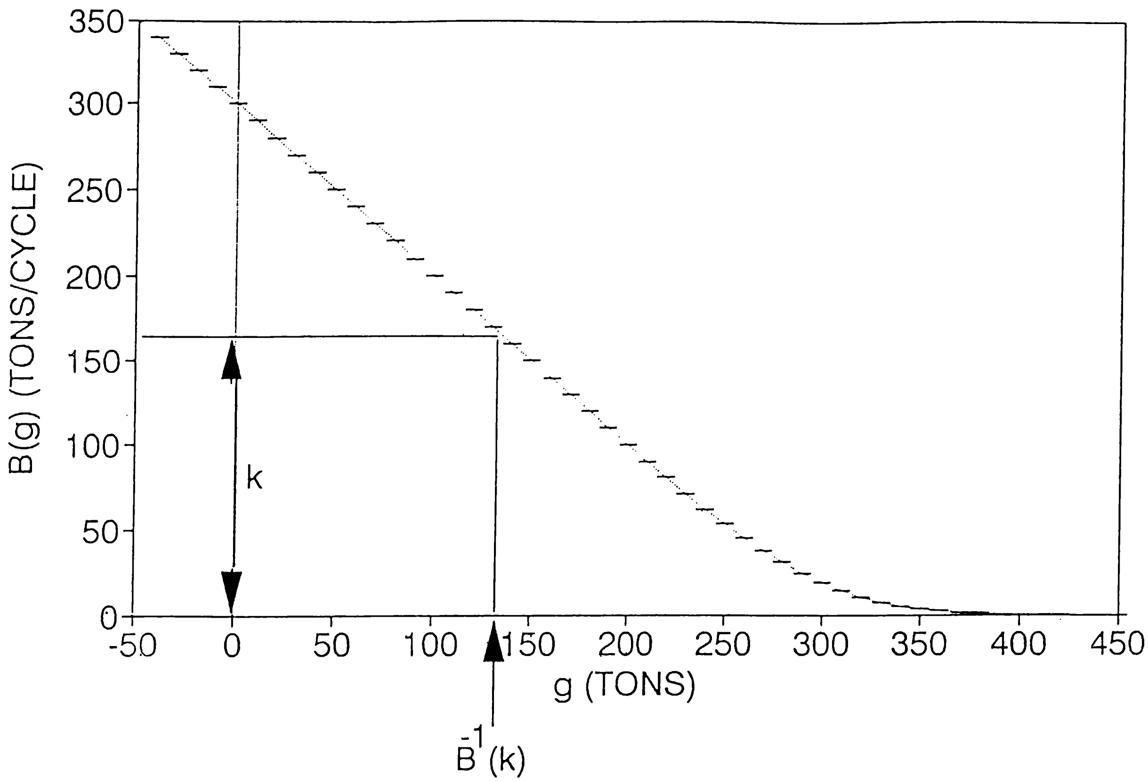

Figure 4.3: $B(g)$ versus g (α=0.80)

Actually we expect the optimum value of I in the $SPIL$ model to be greater than the optimum solution of the classical EOQ model. It can also be followed from the figure (4.4) that, if a decreasing function of I ($hB^{-1}(k(I)) - h\mu = h\ ss$) is added, the cost function of the classical EOQ model shifts rightward. Thus the optimum value of I which minimizes the total cost function in 4.27 is greater than the optimum solution found from the classical EOQ model (equation 4.7).

That means for economy of computations the iterations should start with

$$I_0 = \sqrt{\frac{2SD(1 - \frac{D}{r})}{h}}$$

and the value of I should be iteratively increased until an increase is detected in the total cost function.

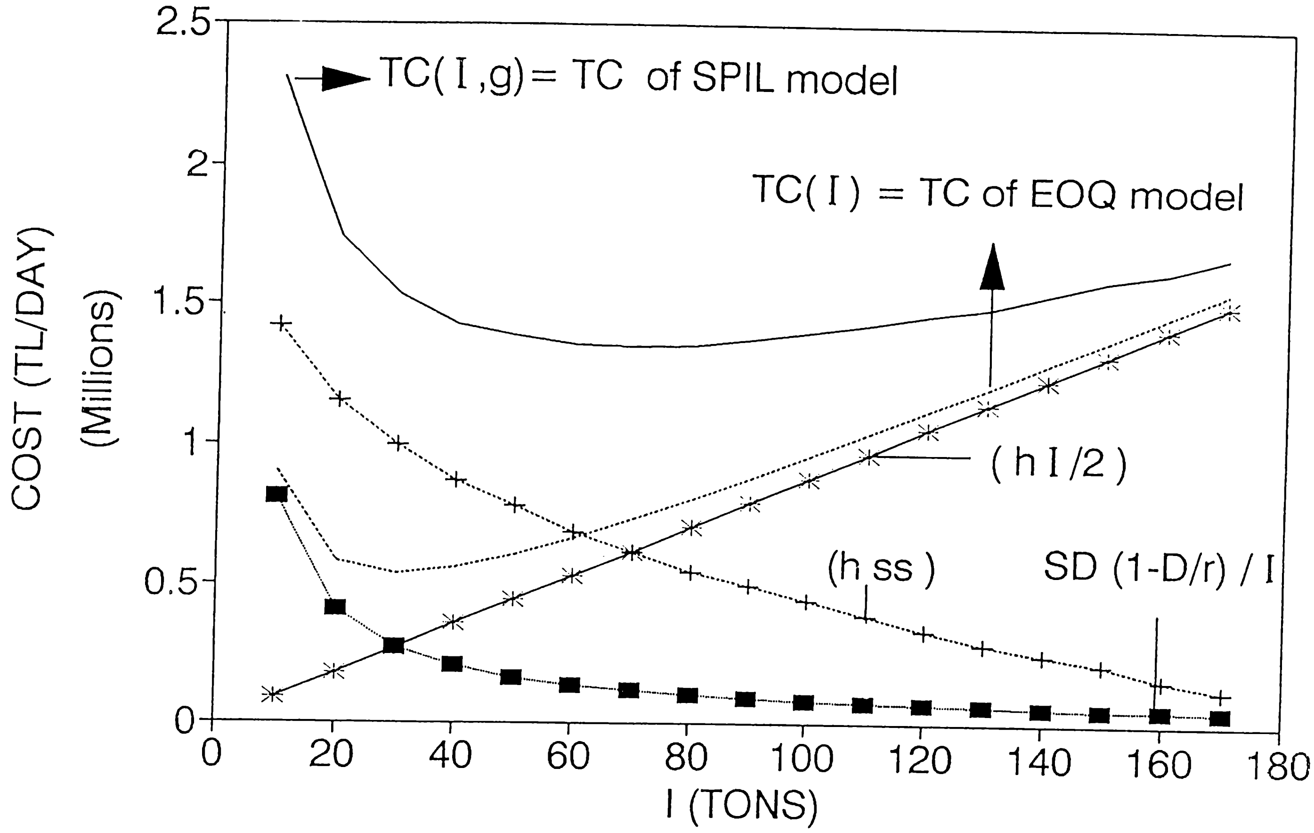

Figure 4.4: Total cost function versus I (α=0.95)

Selection of α:

The SPIL Model is actually based on the judgement of the α value by the decision maker. This value is determined by making a trade-off between different values of α and the minimum total cost associated with each. As we have repeated earlier, in this way the decision maker is actively involved in the decision process [9] (see table 4.1). In order to facilitate setting an α value and give an insight to the DM, an example problem is solved for different values of α and the results are summarized in the form of a trade-off curve in figure 4.5.

Parameters:

COSTS: S= 514,000,000 TL/set-up h=17,678 TL/unit/day

STATISTICS:D=30 tons/day μ=300 tons/TL

r=65.15 tons/day σ=47.4 tons/TL

SLM	MIN TC (TL/DAY)	g (TONS)	I_{max} (TONS)
0.90	13,605,309	74	1519
0.91	14,000,332	102	1486
0.92	14,384,556	129	1453
0.93	14,758,868	154	1425
0.94	15,124,870	178	1397
0.95	15,485,454	201	1371
0.96	15,846,462	223	1352
0.97	16,222,136	246	1325
0.98	16,636,746	270	1313
0.99	17,173,112	301	1293

Table 4.1: Optimum solution of the $SPIL$ model for different values of α

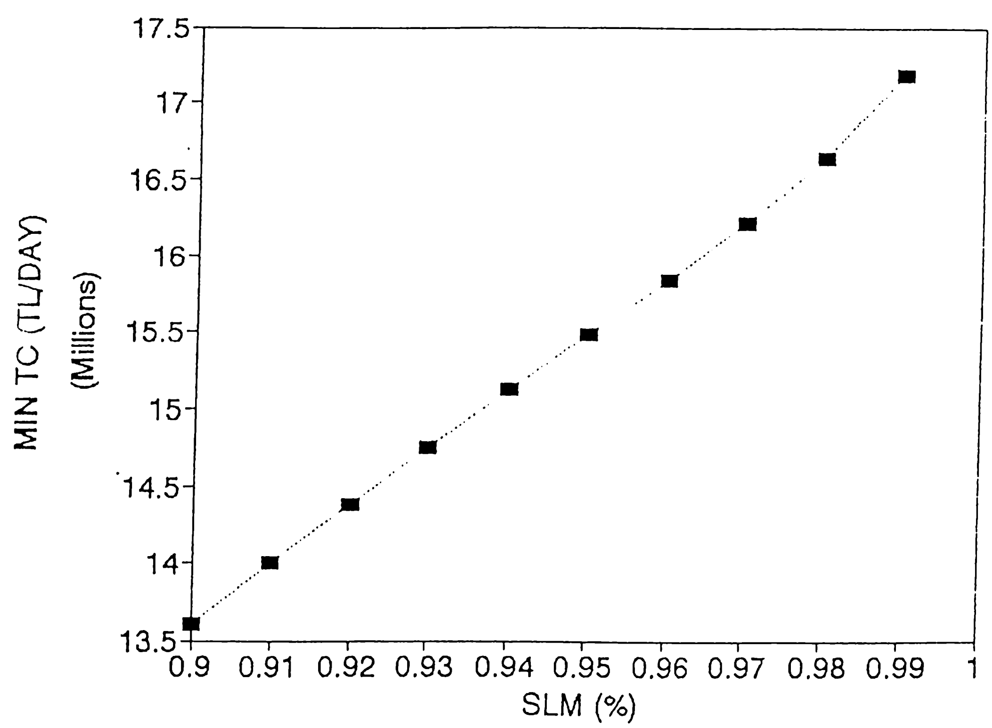

Figure 4.5: The trade-off curve showing the min total cost versus α

Chapter 5

SENSITIVITY ANALYSIS ON THE EOQ AND SPIL MODELS TO FACILITATE LEARNING

In this chapter we discuss, why there is a need to generate the SPIL model, instead of using the EOQ model with finite production rate for PETKİM production system. For this reason, we evaluate them according to some performance criteria and show that the SPIL Model is superior to the EOQ Model (-with finite production rate-) in all types of sensitivity analysis.

In the literature, the meaning of 'sensitivity analysis' is usually conceived as:

- How does the optimal solution vary as the parameter varies?
- How does the optimum value of the objective function vary when the decision variable is non-optimal?

  However in this thesis, we introduce another point of view for the sensitivity analysis. In other words, sensitivity analysis is conceived as

- What is lost by the error in parameter estimation?

Actually, our concern in making a sensitivity analysis is to understand whether the losses (or the costs) due to a parameter estimation error is great or small. Note that, almost all kinds of inventory control models are based on the 'estimated' values of some parameters. That means, the optimal values of the objective function and the decision variables found by using these 'estimated' parameters are in fact the 'expected optimum' values. In other words, an inventory control model can be *validated* (i.e., the model represents the real system on hand), provided that the 'true' values of the parameters of the model are identical with (if not closer to) the 'estimated' ones. This is of course an idealistic statement as real life is generally undeterministic. As a matter of fact, if one can show that the costs associated with an estimation error is relatively 'small' than, the model can be still 'validated'. In other words, even if the true values of the parameters are different than the expected ones, this may not cause a trouble, unless the cost associated with such errors is very large.

'Cost of an estimation error' is one of the properties that characterize the *adaptability* of the decisions given by the model. If the cost of the estimation errors is reasonably small, then we can say that 'the model is adaptive to the changing conditions of the environment'. Note that, as a result of this characteristic, it will be possible for the DM to operate the inventory system in a confident manner; as the resulting total cost (at the end of the planning horizon) will not be too far from the expected optimum. This will be achieved by generating a model that adapts itself to the changes in the environment. Little [19] defines the adaptability as a 'model that is capable of being updated as new information becomes available. This is especially true of the parameters; but to some extend of structure too.

More specifically, we want to show in this chapter that the cost of a parameter estimation error in $SPIL$ model is 'not' very 'significant'. For this purpose, we compare the performance of the $SPIL$ model with that of the classical EOQ model. Our mathematical and numerical analysis show that

The cost of an error in parameter estimations in $SPIL$ model is always less than (or at most equal to) the classical EOQ model.

We know that in literature, classical EOQ model is recognized as a 'robust' model in the sense that the optimum solution and optimum value of the objective function is 'insensitive' to the parameter estimation errors. Although this is the case, we show in this chapter that, the *cost associated with a demand rate estimation error* is 'not' that much insignificant in the classical EOQ models.

Solomon (see [25] for reference) showed that the total cost in the neighborhood of the optimum lotsize is relatively insensitive to the small variations in the quantity ordered. Brown [5] argues that, if the lotsize is within the range 70-140 % of the true optimum, the total annual costs rise less than 6% above the true optimum. By sensitivity tests, Zimmerman and Sovereign [29] conclude that the sensitivity of the total cost with respect to errors in set-up and inventory holding costs is very small in classical EOQ models, if the errors are made in the same direction. Sensitivity of the extensions EOQ models are analyzed by several researchers in the recent years and they all found that the sensitivity of the EOQ to the forecast errors is negligible small [7], [16]. If we incorporate the 'finite production rate' assumption to their analysis we have

$$TC(Q) = \frac{SD}{Q} + \frac{1}{2}hQ(1 - \frac{D}{r}) \tag{5.1}$$

and the optimum value of Q which minimizes 5.1 is

$$Q^* = \sqrt{\frac{2SD}{h(1 - \frac{D}{r})}} \tag{5.2}$$

However, in the $SPIL$ model total cost function is

$$TC(I) = \frac{SD}{I}(1 - \frac{D}{r}) + \frac{1}{2}hI \tag{5.3}$$

where the optimum value of I which minimizes 5.3 is

$$I^* = \sqrt{\frac{2SD(1 - \frac{D}{r})}{h}} \tag{5.4}$$

Note that if we substitute 5.4 in 5.3 or 5.2 in 5.1, then we find the expression for the optimum total cost TC^* as

$$TC^* = TC(Q^*, D) = TC(I^*, D) = \sqrt{2SDh(1 - \frac{D}{r})} \tag{5.5}$$

In the following two sections classical EOQ model and the $SPIL$ model is analyzed and compared by the traditional way of sensitivity analysis, i.e., according to the sensitivity of the optimum solution and the objective function to the changes in the parameters.

These analysis show that,

- The optimum solution of the $SPIL$ model is *less* sensitive to the changes in the demand rate than the classical EOQ model.
- They are equivalently insensitive to the changes in other parameters of the system.

In the last section we introduce a different way of sensitivity analysis, i.e., we compare both models according to the cost associated by a parameter estimation error.

5.1 SENSITIVITY OF THE OPTIMUM SOLUTION Q^* OR I^* TO THE CHANGES IN PARAMETERS

5.1.1 Sensitivity of the optimal solution to the changes in D:

In order to find out how the optimal value of production quantity and the maximum inventory level changes when the 'true' value of D, (D^*) comes out to be different than the 'estimated' value of D, $(\hat{D})$, let Q^* and I^* be the 'true' optimum value of Q and I, while $\hat{Q}$ and $\hat{I}$ be the 'estimated' optimum value of Q and I, then by using 5.2 and 5.4 we can write

$$\frac{Q^*}{\hat{Q}} = \frac{\sqrt{\frac{2SD^*}{h(1-\frac{D^*}{r})}}}{\sqrt{\frac{2S\hat{D}}{h(1-\frac{\hat{D}}{r})}}} = \sqrt{\frac{D^*(r-\hat{D})}{\hat{D}(r-D^*)}} \tag{5.6}$$

$$\frac{I^*}{\hat{I}} = \frac{\sqrt{\frac{2SD^*(1-\frac{D^*}{r})}{h}}}{\sqrt{\frac{2S\hat{D}(1-\frac{\hat{D}}{r})}{h}}} = \sqrt{\frac{D^*(r-D^*)}{\hat{D}(r-\hat{D})}} \tag{5.7}$$

- Using 5.6, we can see that the rate of increase in the optimum value of Q is an increasing function of D^*. On the other hand, using 5.7, we see that the rate of increase in the optimum value of I is a concave function of D^*. Thus, the optimum value of I is *less* sensitive than Q to the changes in D.

5.1.2 Sensitivity of the optimal solution to the changes in r:

By using the above argument, the rate of change in the expected optimum solution due to a change in the production rate r will be :
In the classical EOQ model,

$$\frac{Q^*}{\hat{Q}} = \frac{\sqrt{\frac{2SD}{h(1-\frac{D}{r^*})}}}{\sqrt{\frac{2SD}{h(1-\frac{D}{\hat{r}})}}} = \sqrt{\frac{(1-\frac{D}{\hat{r}})}{(1-\frac{D}{r^*})}} \tag{5.8}$$

In the $SPIL$ model,

$$\frac{I^*}{\hat{I}} = \frac{\sqrt{\frac{2SD(1-\frac{D}{r^*})}{h}}}{\sqrt{\frac{2SD(1-\frac{D}{\hat{r}})}{h}}} = \sqrt{\frac{(1-\frac{D}{r^*})}{(1-\frac{D}{\hat{r}})}} \tag{5.9}$$

Using 5.8 and 5.9, we can see that

- The rate of change in the optimum solution Q and r are in opposite directions; in other words Q decreases by any increase in the value of r. However the rate of change in the optimum Q value is smaller than that of r.
- The rate of change in the optimum solution I and r are in the same direction, i.e., the optimum value of I increases by any increase in r.
- Both in $SPIL$ model and the EOQ model, the rate of change in the optimum solution is *less* than the rate of change in r.

5.1.3 Sensitivity of the optimal solution to the changes in S and h :

Using the above procedure, the rate of change in the optimal solution due to a change in S is given as follows

$$\frac{Q^\star}{\hat{Q}} = \frac{\sqrt{\frac{2S^\star D}{h(1-\frac{D}{r})}}}{\sqrt{\frac{2\hat{S}D}{h(1-\frac{D}{r})}}} = \sqrt{\frac{S^\star}{\hat{S}}} \tag{5.10}$$

$$\frac{I^\star}{\hat{I}} = \frac{\sqrt{\frac{2S^\star D(1-\frac{D}{r})}{h}}}{\sqrt{\frac{2\hat{S}D(1-\frac{D}{r})}{h}}} = \sqrt{\frac{S^\star}{\hat{S}}} \tag{5.11}$$

Similarly it can be shown that

$$\frac{Q^\star}{\hat{Q}} = \sqrt{\frac{\hat{h}}{h^\star}} = \frac{I^\star}{\hat{I}} \tag{5.12}$$

where $Q^\star$, $I^\star$, $S^\star$, $h^\star$ are the 'true' values and $\hat{Q}$, $\hat{I}$, $\hat{S}$, $\hat{h}$ are the 'estimated' values of the production quantity, production rate, set-up cost and unit inventory holding cost respectively.

Note that

- The relative change in the optimum value of the production quantity is identical with that of maximum inventory level due to a change in S or h.
- Both Q and I change in the same direction with S, but in the opposite direction with h.
- The relative changes in the optimum value of Q and I are 'less' than the rate of change in S and h respectively.

5.2 CHANGE IN THE TOTAL COST FUNCTION WHEN Q OR I IS NONOPTIMAL

If Q^*, $\hat{Q}$ denote the 'true' and 'estimated' values of the optimum production quantity and TC^* (as given in equation 5.5) , $\hat{TC}$ (using 5.1) denote their respective total costs, then

$$\frac{\hat{TC}}{TC^*} = \frac{\frac{SD}{\hat{Q}} + \frac{1}{2}h\hat{Q}}{\frac{SD}{Q^*} + \frac{1}{2}hQ^*} = \frac{\frac{SD}{\hat{Q}} + \frac{1}{2}h\hat{Q}}{\sqrt{2SDh(1-\frac{D}{r})}}$$

where all the parameters other than $\hat{Q}$ and $\hat{TC}$ are the 'true' values.

By simplification one can show that

$$\frac{\hat{TC}}{TC^*} = \frac{1}{2}[\frac{Q^*}{\hat{Q}} + \frac{\hat{Q}}{Q^*}]$$

Similarly, if we use the $SPIL$ model instead of the classical EOQ model

$$\frac{\hat{TC}}{TC^*} = \frac{\frac{SD}{\hat{I}}(1-\frac{D}{r}) + \frac{1}{2}h\hat{I}}{\frac{SD}{I^*}(1-\frac{D}{r}) + \frac{1}{2}hI^*} = \frac{1}{2}[\frac{I^*}{\hat{I}} + \frac{\hat{I}}{I^*}]$$

Thus,

- The effect of a change in the optimum value of the decision variable on total cost function is identical in both the classical EOQ model and the $SPIL$ model. However, considering I^* being less sensitive to D than Q^*, we should conclude that the $SPIL$ model is *less* sensitive to the changes in demand rate than the classical EOQ model.

- Total cost function increases as Q (or I) gets far from the true optimum value of the production quantity, Q^* (or maximum inventory level I^*). In other words, total cost 'increases' with an estimation error in 'both' directions.

- The relative increase in the total cost function is 'less' than the rate of change in Q or I for the EOQ and $SPIL$ models respectively.

The above discussion shows that, in the classical EOQ model, a great deal of accuracy is unnecessary in estimating the parameters involved in the calculations. Such characteristics of the EOQ lotsize formula have made it widely used in practice. However we have also showed by comparing the results of the sensitivity analysis that, the performance of the $SPIL$ model is equivalent to (or even better than) that of the classical EOQ model.

After stating the performance criteria of sensitivity analysis in the literature, we compare the $SPIL$ model and the EOQ model in terms of the cost of a parameter estimation error in the following sections. More specifically, we find out what is lost by a demand rate, production rate, set-up cost, unit inventory holding cost estimation error? As a result we conclude that, the $SPIL$ model is 'superior' to classical EOQ models in the following senses:

- In $SPIL$ model, the cost of an error in demand rate (D) estimation is 'significantly' smaller than the classical EOQ model.

- In $SPIL$ model, the cost of an error in other parameters of the system, (namely set-up cost, unit inventory holding cost and production-rate) is exactly the same as the classical EOQ model.

As we have discussed at the beginning of this chapter, these characteristics enables us to define the *SPIL* model as more "adaptive' than the classical *EOQ*; because in spite of the changes in the inputs, the model gives robust solutions and in a way, *adapts itself* to the changes in the system.

5.3 COST OF AN ESTIMATION ERROR

5.3.1 Cost of an Estimation Error in Demand Rate

Both the SPIL model and the classical EOQ models are based on deterministic demand rate assumption. However, it is the *'estimate'* of the future demand rate that is considered, when the models are applied to the actual system. That is to say, any deviation from the forecasted demand rate will usually lead to an incorrect decision rule. Thus, it is reasonable to compare both models in terms of the 'cost associated with an error in demand estimation'.

Cost of Error due to an error in Estimated Demand Rate in EOQ Model

Imputed production plan: Let us say, for a given planning period, the DM estimates the demand rate as $\hat{D}$. Based on this value, the optimum production quantity and production duration are found to be $\hat{Q}$ (equation 5.2) and $\hat{T}_p$ respectively. Inventory build up and decline in the imputed production plan is represented in figure 5.1. The rate of increase in the inventory level is estimated to be $r - \hat{D}$ with the same assumption. It is planned to stop production after $\hat{Q}$ amount of an item are produced until the inventory level drops to zero. Production continues with the similar cycles.

True optimum production plan: Our concern at this point is the effect of an error in the demand rate estimation on this imputed scenario of figure 5.1. If the true demand rate comes out to be $D^\star$ -instead of $\hat{D}$-, then the optimum value of production quantity in a cycle will be $Q^\star$ -instead of $\hat{Q}$- which is found by substituting the true demand rate $D^\star$ in formula 5.2. Therefore the total cost associated with $\hat{Q}$ may not be the 'minimum attainable cost. Inventory build up and decline for the true optimum plan is again given in figure 5.1.

Parameters (from the PETKİM production system):
S=415,249,640 TL
h=16,720 TL/unit/day
r=65.15 tons/day
$\hat{D}$=30 tons/day (estimated demand rate)
D^*=40 tons/day (true demand rate)

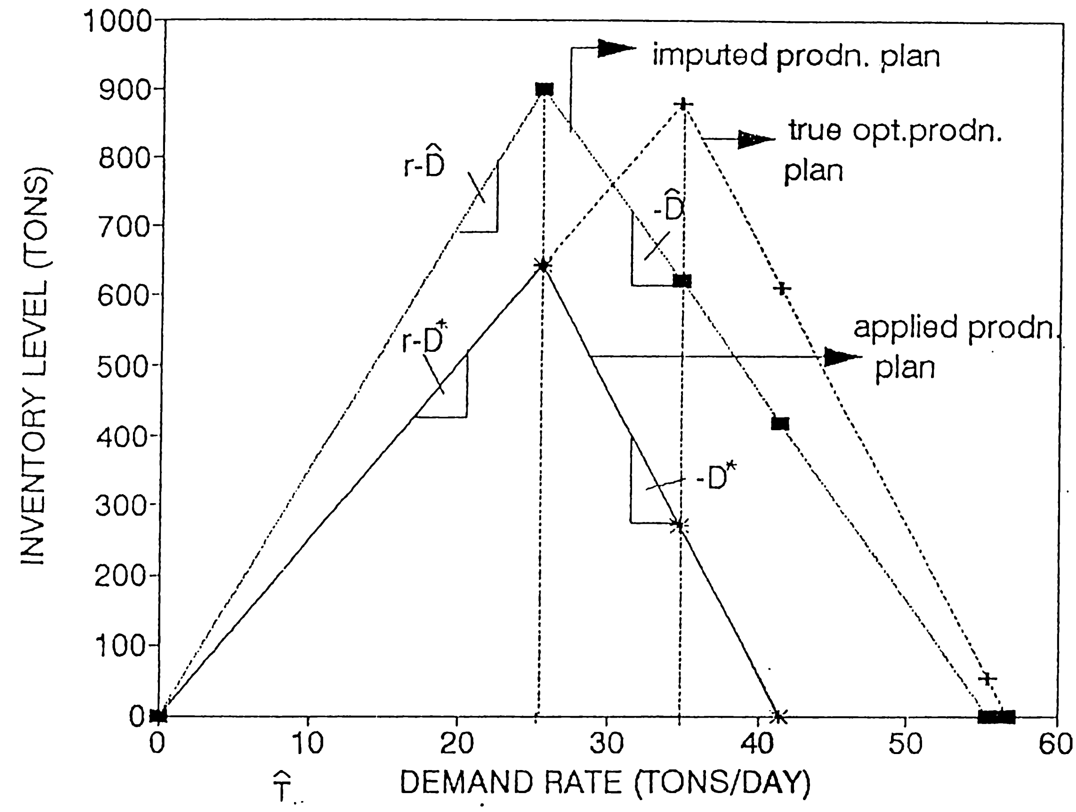

Figure 5.1: Change in the production plan due to an error in estimating D, when the decision variable is Q

It follows from this figure that, if the true demand rate is greater than the estimated value ($D^* > \hat{D}$), then the true optimum production quantity is greater than the imputed optimum production quantity ($Q^* > \hat{Q}$). However, we should add that due to the given values of the estimated and true values of the demand rates, true value of the production quantity is less than the imputed value if $D^\star < \hat{D}$.

Applied production plan : However, the DM, being unaware of this true demand rate $D^\star$ will try to apply the imputed scenario and produce $\hat{Q}$ instead of $Q^\star$ (note the difference between the applied and true optimum plan in terms of the changes of inventory level in figure 5.1).

Let us briefly repeat what we have discussed so far: The DM estimates the demand rate to be $\hat{D}$ in the planning horizon. Using this value he computes the optimum value of the production quantity to be $\hat{Q}$ and applies this plan to the production system, i.e., he produces $\hat{Q}$ then stops until the inventory level drops to zero... However, the true demand rate turns out to be $D^\star$, which is different from the estimated value $\hat{D}$; thus the true optimum value of the production quantity/cycle is $Q^\star$ is different from $\hat{Q}$ with the same methodology. This means that, the DM applies the 'nonoptimal' production policy by producing $\hat{Q}$ instead of the true optimum quantity $Q^\star$ and this certainly leads to a higher total cost in the classical EOQ model.

The difference between the total costs of applied and true optimum production plan is the *cost of error due to a demand rate estimation error.* If the EOQ model is utilized, this cost of error will be referred to as **Classical Model Cost of Error** or shortly, *CMCE*, which is found by using the below formula:

$$CMCE = TC(\hat{Q}, D^\star) - TC(Q^\star, D^\star) \tag{5.13}$$

where,

$$\hat{Q} = \sqrt{\frac{2S\hat{D}}{h(1-\frac{\hat{D}}{r})}} \tag{5.14}$$

and

$$Q^\star = \sqrt{\frac{2SD^\star}{h(1-\frac{D^\star}{r})}} \tag{5.15}$$

The second term in 5.13 is the true optimum value of the total cost defined by 5.5 and by using the total cost formula of the EOQ model in equation 5.1 we can write

$$CMCE = [\frac{SD^\star}{\hat{Q}} + \frac{1}{2}h\hat{Q}(1 - \frac{D^\star}{r})] - \sqrt{2SD^\star h(1 - \frac{D^\star}{r})} \qquad (5.16)$$

The $CMCE$ can be expressed as function of the estimated and true values of demand rate, when 5.14 is substituted in 5.16.

$$\rightarrow \quad CMCE = [\frac{SD^\star}{\sqrt{\frac{2S\hat{D}}{h(1-\frac{\hat{D}}{r})}}} + \frac{1}{2}h\sqrt{\frac{2S\hat{D}}{h(1-\frac{\hat{D}}{r})}}\,(1 - \frac{D^\star}{r})] - \sqrt{2SD^\star h(1 - \frac{D^\star}{r})}$$

Algebraic operations and manipulations simplify into

$$CMCE = \sqrt{\frac{Sh}{2r}}\,[\frac{D^\star(r - D^\star) + \hat{D}(r - \hat{D})}{\sqrt{\hat{D}(r - \hat{D})}} - 2\sqrt{D^\star(r - D^\star)}] \qquad (5.17)$$

The graphical representation of $CMCE$ for different values of estimated demand rate $\hat{D}$ and true demand rate $D^\star$ for a PETKİM product is given in figure 5.2. It is apparent from these curves that, $CMCE$ strictly increases if the true demand rate $D^\star$ is different than the expected demand rate $\hat{D}$.

Same values of the cost parameters are used with the previous figure.
$\hat{D}$=30, 45, 50 tons/day

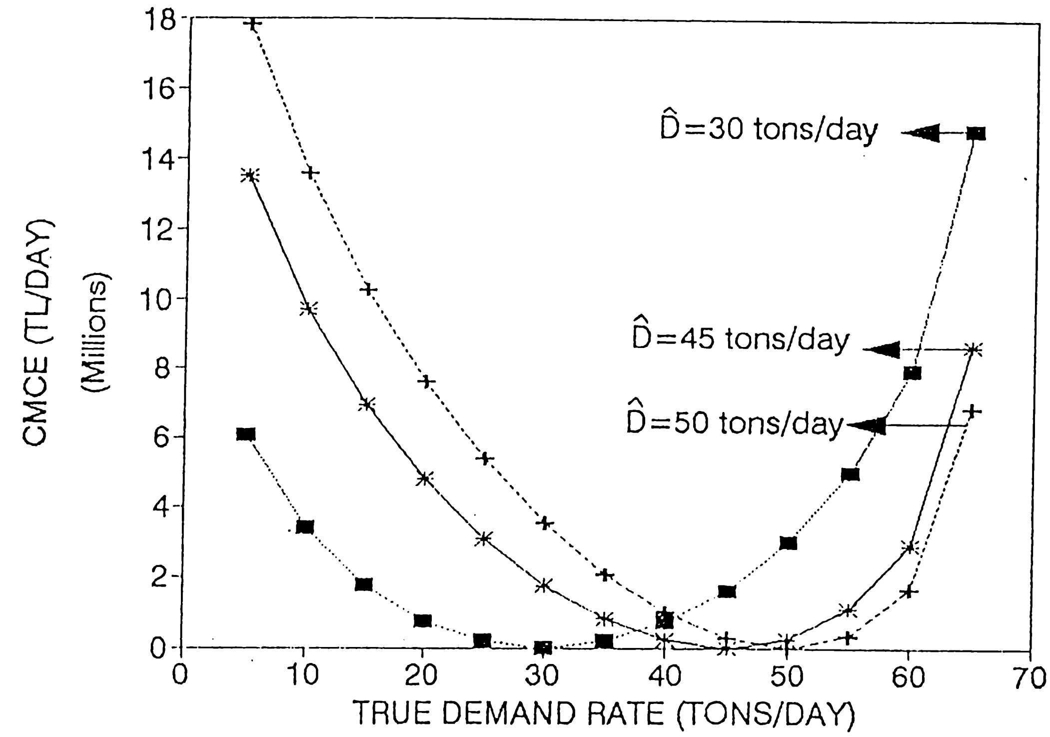

Figure 5.2: *CMCE* versus true demand rate (D^*)

Cost of Error due to a Change in Estimated Demand Rate in SPIL Model

We apply the same argument to the SPIL model this time. Remember that total cost of SPIL Model is expressed as a function of the maximum inventory level I and demand rate D.

Imputed production plan: The DM estimates the demand rate to be $\hat{D}$ and finds the corresponding optimum value of maximum inventory level $\hat{I}$ (equation 5.19), which minimizes the total cost function. The production starts with zero inventory level and continues until the inventory level reaches $\hat{I}$. During this production period, inventory level should increase at a rate of $r - \hat{D}$ and decrease at a rate of $-\hat{D}$ after the production is stopped. Each cycle begins when the inventory level is zero and continues in this manner during the planning horizon (figure 5.3 represents the variation of the inventory level for the imputed production plan).

Using the same values of the parameters used in figure 5.1

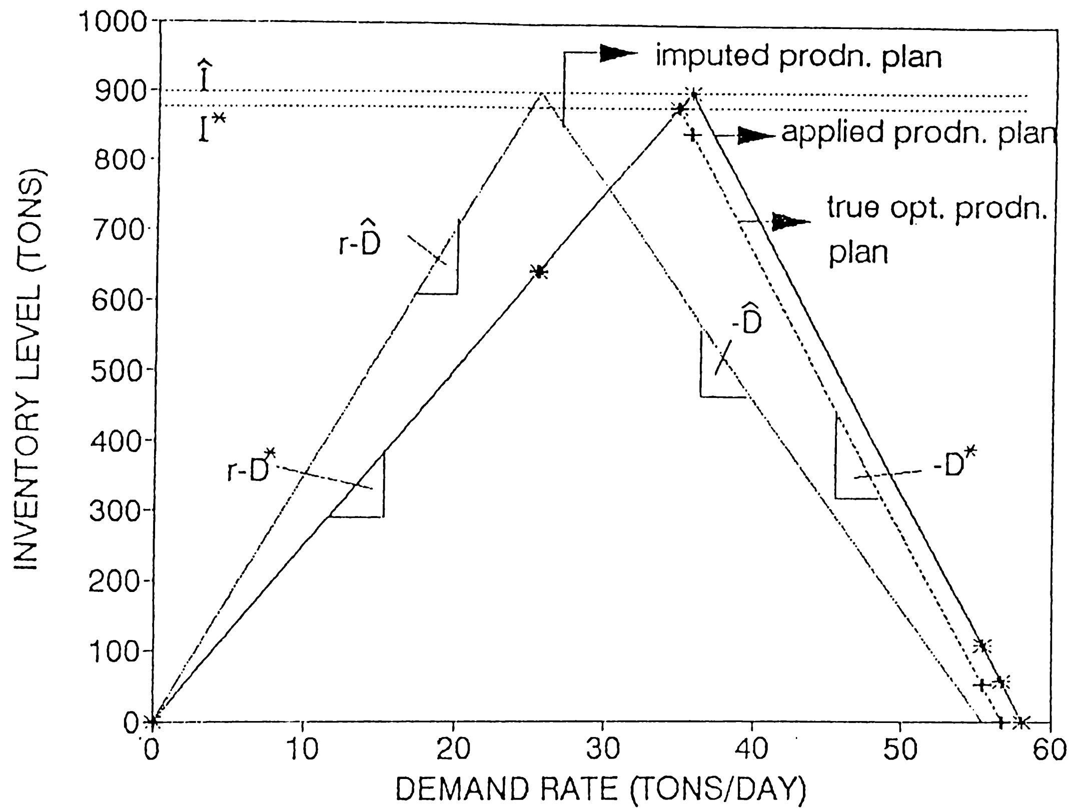

Figure 5.3: Change in the production plan due to an error in estimating D, when the decision variable is I

True optimum plan: What if the true demand rate D^* turns out to be different from the estimated value $\hat{D}$? Briefly in this case, optimum value of 'maximum inventory level' should be I^* (using 5.20), where the actual demand rate is D^*. The variation of the inventory level in true optimum plan is given in figure 5.3. Using the same parameters as in the case of classical *EOQ* model, it follows from this figure that if the estimated demand rate is less than the true demand rate, the maximum inventory level of the true optimum plan is less than the imputed plan. However we should note that this is not always the case: The optimum value of the maximum inventory level is a concave function of the demand rate (equation 5.20); thus depending on the values of the estimated and true demand rates, the true optimum value of the maximum inventory level may be less than or greater than the imputed optimum value.

Applied plan: Because of the error in demand rate estimation, the DM will consider $\hat{I}$ in the production plan instead of I^* which is the 'true' optimum value of maximum inventory level. The production will continue until $\hat{I}$ then will be stopped as shown in figure 5.3. (unlike the classical EOQ model (figure 5.1), when the $SPIL$ model is used, note the similarity between the applied and true optimum plan in terms of the changes of inventory level in figure 5.3).

Similar to the case in the classical EOQ model, what actually happens is the following: The DM estimates the demand rate to be $\hat{D}$ and finds the related optimum value of the maximum inventory level to be $\hat{I}$ (using 5.19). He builds up a production and inventory control strategy using these values and applies this plan to the system on hand. However, the true value of the demand rate turns out to be D^* (which is different from $\hat{D}$) and in this case the true optimum value of the maximum inventory level should be I^* (using 5.20). Note that as a result of an error in the demand rate estimation, the DM applies the 'nonoptimal' maximum inventory level $\hat{I}$, instead of the true optimum maximum inventory level I^*. Obviously, we should expect an increase in the total cost by applying the nonoptimal plan.

When we use the $SPIL$ model, the difference between the total cost functions of the applied and true optimum production plans is referred to as the **SPIL Model Cost of Error or** $SMCE$. In other words, $SMCE$ is the cost incurred by producing up to $\hat{I}$ instead of I^* (the true optimum value), when the true demand rate is D^* (equation 5.18).

$$SMCE = TC(\hat{I}, D^*) - TC(I^*, D^*) \tag{5.18}$$

where,

$$\hat{I} = \sqrt{\frac{2S\hat{D}(1 - \frac{\hat{D}}{r})}{h}} \tag{5.19}$$

and

$$I^* = \sqrt{\frac{2SD^*(1 - \frac{D^*}{r})}{h}} \quad (5.20)$$

Actually the second term in 5.18 ($TC(I^*, D^*)$) is the true optimum value of the total cost function which is simply defined in 5.5 and by using the total cost formula of the $SPIL$ model in equation 5.3 we can write

$$SMCE = [S\frac{D^*}{\hat{I}}(1 - \frac{D^*}{r}) + \frac{1}{2}h\hat{I}] - \sqrt{2SD^*h(1 - \frac{D^*}{r})} \quad (5.21)$$

The $SMCE$ can be expressed a function of the estimated and true values of the demand rate, when 5.19 is substituted in 5.21.

$$\rightarrow \quad SMCE = [\frac{SD^*}{\sqrt{\frac{2S\hat{D}(1-\frac{\hat{D}}{r})}{h}}}(1-\frac{D^*}{r})+\frac{1}{2}h\sqrt{\frac{2S\hat{D}(1-\frac{\hat{D}}{r})}{h}}]-\sqrt{2SD^*h(1-\frac{D^*}{r})}$$

Algebraic operations and manipulations simplify into

$$\rightarrow \quad SMCE = \sqrt{\frac{Sh}{2r}}\ [\frac{D^*(r - \hat{D}) + \hat{D}(r - D^*)}{\sqrt{\hat{D}(r - \hat{D})}} - 2\sqrt{D^*(r - D^*)}] \quad (5.22)$$

The SPIL Model is applied on the production of a PETKİM product and the cost of error due to a demand rate estimation error is calculated for different values of $\hat{D}$. Graphical representation is given in figure 5.4. The result is charming in the sense that, unlike $CMCE$, $SMCE$ is not strictly increasing. It is minimized for more than one value of the true demand rate D^* and is rather 'flat' between these roots.

By using the same parameters in figure 5.1
$\hat{D}$=30, 45, 50 tons/day respectively

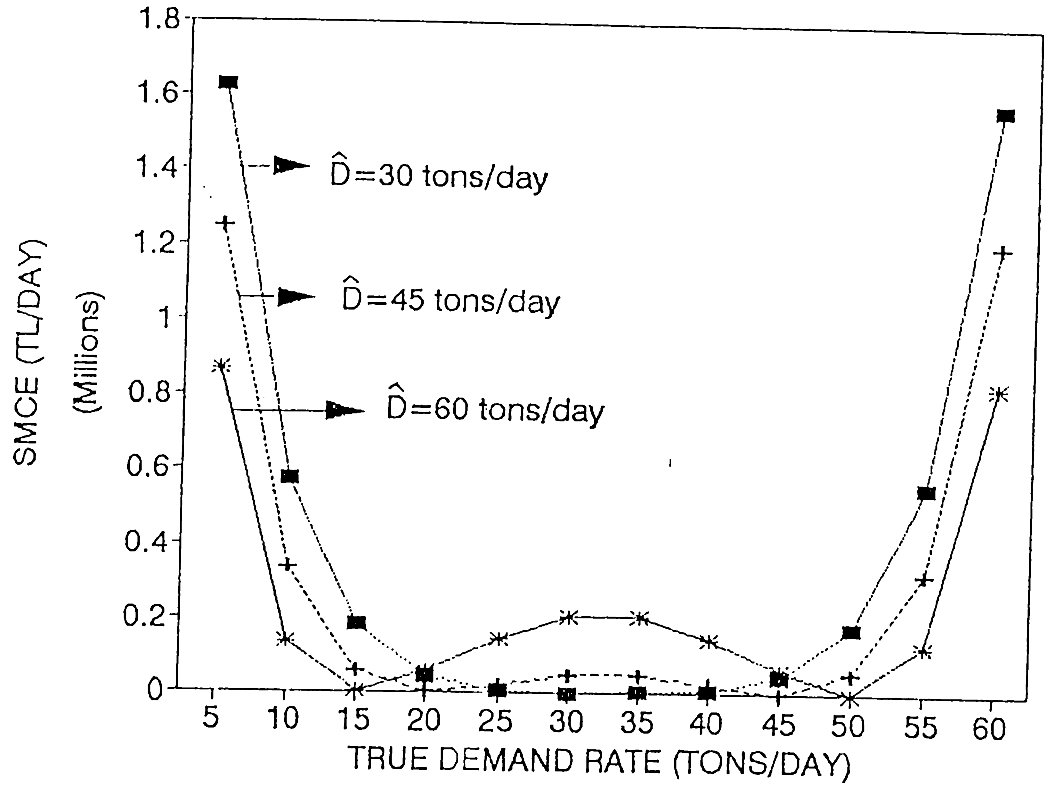

Figure 5.4: $SMCE$ versus true demand rate (D^*)

Basic properties of the SMCE Function

The roots of the $SMCE$ function is analyzed by taking the first derivative of equation 5.22 with respect to true demand rate D^* and equating to zero. As a result it is found that the function has three roots:

1- $D^* = \hat{D}$
2- $D^* = r - \hat{D}$
3- $D^* = \frac{r}{2}$

Cost of error is minimized (actually it takes its zero value) at the first and second roots. The first root is obvious since when the estimated demand rate ($\hat{D}$) and the true demand rate (D^*) are the same , the imputed and the true optimum plans as well as associated costs are the same and therefore the cost of error is zero. However the second root brings the flexibility that it is possible to loose nothing by an estimation error, if the sum of true and estimated demand rates is equal to the production rate r (i.e..$D^{\star}+\hat{D}$) . Even if the DM makes a production plan based on a wrong estimate $\hat{D}$, he/she may be able to catch the true optimum total cost. Mathematical analysis shows that, the optimum value of the maximum inventory level I is a *concave* function of the demand rate D. For this reason, when the $SPIL$ model is used it is possible to have the same optimal solution for I, for different values of D. On the other hand note that, the optimal value of the order quantity Q in the classical EOQ model is an increasing function of D. For this reason, unless $D^{\star} = \hat{D}$, $Q^{\star}$ is always different from $\hat{Q}$. Thus, we show that, starting with the classical EOQ Model, a robust model against demand estimation errors can be constructed. In fact, this property is the *core point* of the SPIL Model as well as this thesis work.

Another important property of the $SMCE$ is the 'maximum' cost of error between the given 'minimizing' roots of the $SMCE$ function which occurs at the third root $D^{\star} = \frac{r}{2}$.
The third root does not depend on the value of the estimated demand rate $\hat{D}$. A bump up of the function is observed when the true demand is half of the production rate (figure 5.4). The height of this bump (i.e., the maximum cost of error between the roots $\hat{D}$ and $r-\hat{D}$) can be found by substituting $D^{\star} = r/2$ in 5.22 as follows:

$$SMCE_{D^{\star}=\frac{r}{2}} = \sqrt{\frac{Sh}{2r}}\ [\frac{\frac{r}{2}\,(r-\hat{D})+\hat{D}(r-\frac{r}{2})}{\sqrt{\hat{D}(r-\hat{D})}} - 2\sqrt{\frac{r}{2}(r-\frac{r}{2})}\ ]$$

$$\rightarrow \qquad SMCE_{D^{\star}=\frac{r}{2}} = \sqrt{\frac{Sh}{2r}}[\frac{r^2}{2\sqrt{\hat{D}(r-\hat{D})}} - r]$$

It is also obvious from the curves of figure 5.4 that the height of this bump-up between the roots increases as the distance between them $(r-2\hat{D})$ increases toward the limits of the domain (0,r). However we will show in this section that this global maximum value of SMCE at D^*=r/2 can never exceed CMCE.

At this stage, we need to emphasize a special case of the $SPIL$ model i.e., when $\hat{D} = \frac{r}{2}$: According to our initial assumption, demand rate is always less than the production rate. For this reason, while making sensitivity analysis we consider only those values of D^* where $0 < D^* < r$. In figure 5.5, the three roots of the $SPIL$ model is located as a function of $\hat{D}$.

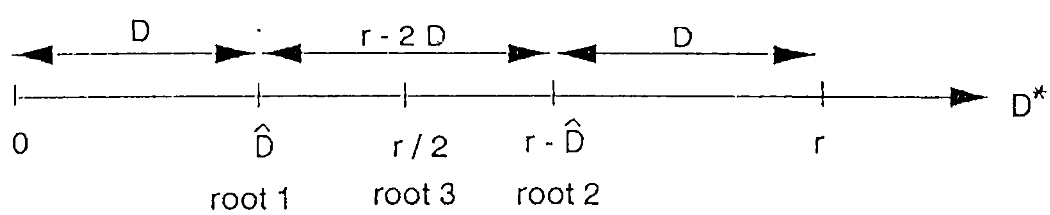

Figure 5.5: Roots of the Cost of Error function in $SPIL$ model

Note that the first root $D^* = \hat{D}$ and the second root $D^* = r - \hat{D}$ are equidistant ($= \hat{D}$) from the limits of the domain $(0, r)$. The third root, $D^* = \frac{r}{2}$ is just in the middle of this range. It follows from the discussion in the above paragraph that, these three roots are equal to each other when the estimated demand rate is $r/2$, i.e.,

$$\hat{D} = \frac{r}{2} \quad \rightarrow \quad root1 = \hat{D} = \frac{r}{2} \ , \ root2 = r - \hat{D} = \frac{r}{2} \ , \ root3 = \frac{r}{2}$$

Thus the shape of the $SPIL$ model is a convex function if $\hat{D} = \frac{r}{2}$.

Comparison of the Cost of an Estimation Error in the Classical EOQ Model and SPIL Model

- The above analysis of both models shows that the cost of an error in demand estimation is zero when the estimated demand rate is the same as the true demand rate (figure 5.6).

By using the same values of the cost parameters in the previous figures
r = 65.15 tons/day,
$\hat{D}$=45 tons/day,

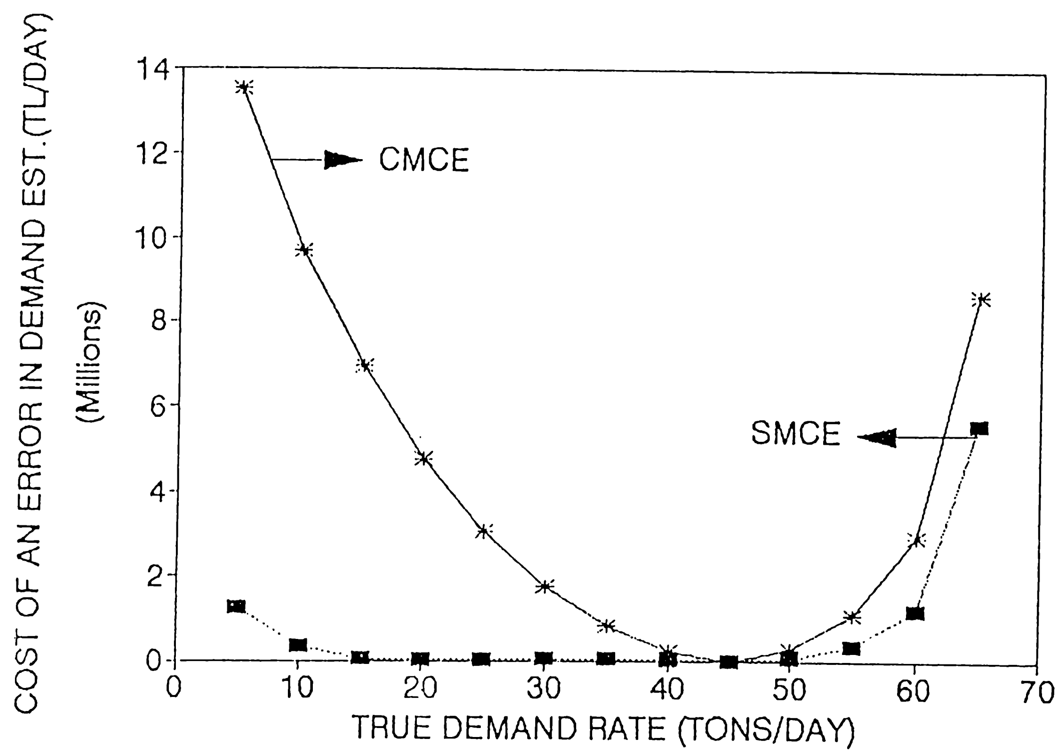

Figure 5.6: Comparison of the cost of a demand rate estimation error in the classical *EOQ* model and the *SPIL* model

However there exists a second value of actual demand rate (where true demand rate = production rate - estimated demand rate) in SPIL Model for which the cost of an estimation error is still zero! This brings a high flexibility and adaptation capability to the SPIL Model. The graphical comparison of both models in terms of the cost of a demand estimation error is given in figure 5.6.

- Cost of an estimation error in classical EOQ Model is always greater than (or equal to) the SPIL Model. This fact is shown by the analysis of the difference between costs of error associated with these models. Subtracting SMCE from CMCE given in 5.22 and 5.17 respectively, we obtain

$$CMCE - SMCE = \sqrt{\frac{Sh}{2r}}\ [\ (\frac{D^*(r-D^*)+\hat{D}(r-\hat{D})}{\sqrt{\hat{D}(r-\hat{D})}} - 2\sqrt{D^*(r-D^*)}$$

$$- \frac{D^*(r-\hat{D})+\hat{D}(r-D^*)}{\sqrt{\hat{D}(r-\hat{D})}} + 2\sqrt{D^*(r-D^*)}\ ] \quad (5.23)$$

$$\rightarrow \quad CMCE - SMCE = \sqrt{\frac{Sh}{2r\hat{D}(r-\hat{D})}}\ (D^*-\hat{D})^2 \quad (5.24)$$

By using the same values of the cost parameters in the previous figures $\hat{D}$=30, 45, 60 tons/day respectively

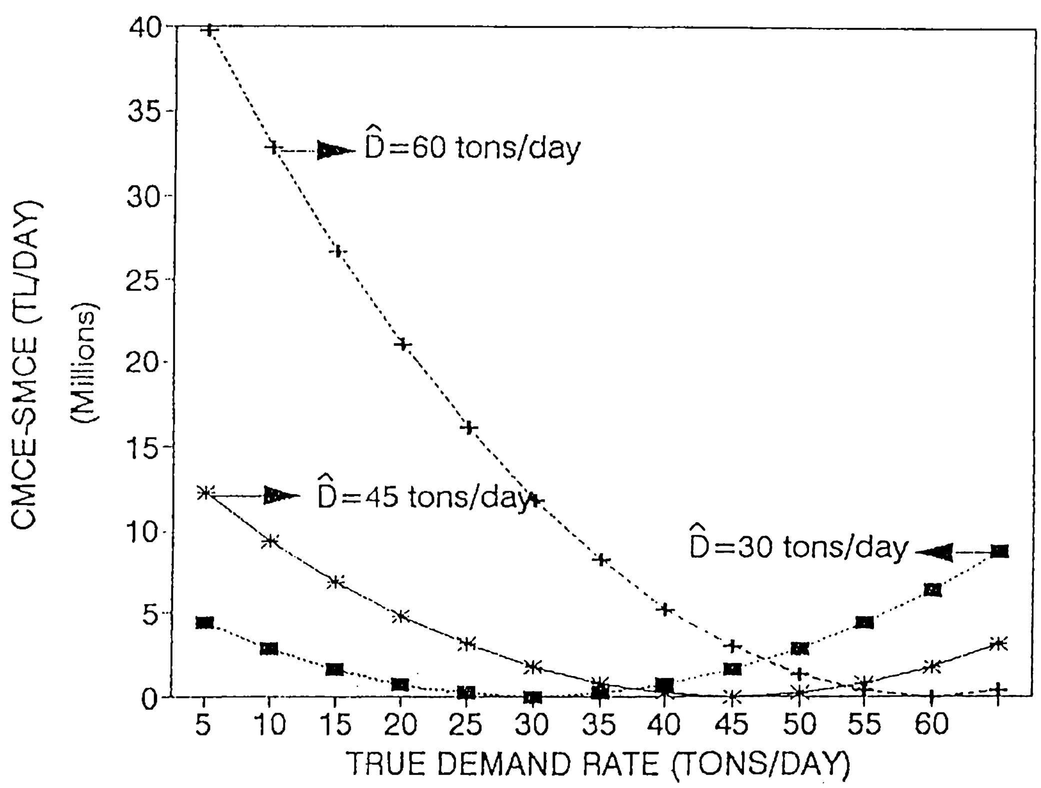

Figure 5.7: Difference of 'the cost associated due to a demand rate estimation error' in the classical EOQ and $SPIL$ model versus the true demand rate, (i.e, $CMCE - SMCE$ versus D^*)

The first multiplier of 5.24 in the square-root is always positive, since we assume that production rate is greater than the demand rate (i.e., $r > D$).

The second multiplier of the expression is the 'square' of the demand rate estimation error, which is always nonnegative. Thus the difference of the $CMCE$ and $SMCE$ turns out to be nonnegative, meaning that $CMCE \geq SMCE$. Graphical representation of this fact is given for different values of $\hat{D}$ and $D^\star$ in figure 5.7.

- Another aspect that should be considered is the ratio of 'cost of an estimation error' and the 'true optimum total cost'. We need to determine 'how significant the cost of error' is when compared to the 'true optimum total cost'. Although the SPIL Model gives more 'robust' solutions against demand estimation errors than the classical EOQ Model, this fact will be of *less* value if they are not significantly different when compared to the true optimum total cost.

  The ratio of cost of error to true optimum total cost is stated in the following way for both models using 5.21 and 5.5

$$\frac{SMCE}{TC(I^\star, D^\star)} = \frac{TC(\hat{I}, D^\star) - TC(I^\star, D^\star)}{TC(I^\star, D^\star)}$$

$$\rightarrow \quad = \frac{[S\frac{D^\star}{\hat{I}}(1-\frac{D^\star}{r}) + \frac{1}{2}h\hat{I}] - \sqrt{2SD^\star h(1-\frac{D^\star}{r})}}{\sqrt{2SD^\star h(1-\frac{D^\star}{r})}}$$

by substituting 5.19 in place of $\hat{I}$, the above equation will be

$$\frac{SMCE}{TC(I^\star, D^\star)} = \frac{(\sqrt{D^\star(r-\hat{D})} - \sqrt{\hat{D}(r-D^\star)}\ )^2}{2\sqrt{D^\star\hat{D}(r-D^\star)(r-\hat{D})}} \qquad (5.25)$$

Similarly, by using 5.16 and 5.5 same ratio is calculated for the classical EOQ models as

$$\frac{CMCE}{TC(Q^\star, D^\star)} = \frac{TC(\hat{Q}, D^\star) - TC(Q^\star, D^\star)}{TC(Q^\star, D^\star)}$$

$$\rightarrow \quad = \frac{[\frac{SD^\star}{\hat{Q}} + \frac{1}{2}h\hat{Q}(1-\frac{D^\star}{r})] - \sqrt{2SD^\star h(1-\frac{D^\star}{r})}}{\sqrt{2SD^\star h(1-\frac{D^\star}{r})}}$$

By substituting 5.14 in place of $\hat{Q}$, the above equation will be

$$\frac{CMCE}{TC(Q^\star, D^\star)} = \frac{(\sqrt{D^\star(r - D^\star)} - \sqrt{\hat{D}(r - \hat{D})}\ )^2}{2\sqrt{D^\star \hat{D}(r - D^\star)(r - \hat{D})}} \tag{5.26}$$

When these ratios are plotted against the % error in demand estimation as in fig 5.8, we see that the *ratio of the cost of a demand estimation error to true optimum total cost* is significantly *smaller* when the SPIL Model is utilized instead of the classical EOQ Model.

Same values of the cost parameters are used with the previous figure.
$\hat{D}$=45 tons/day

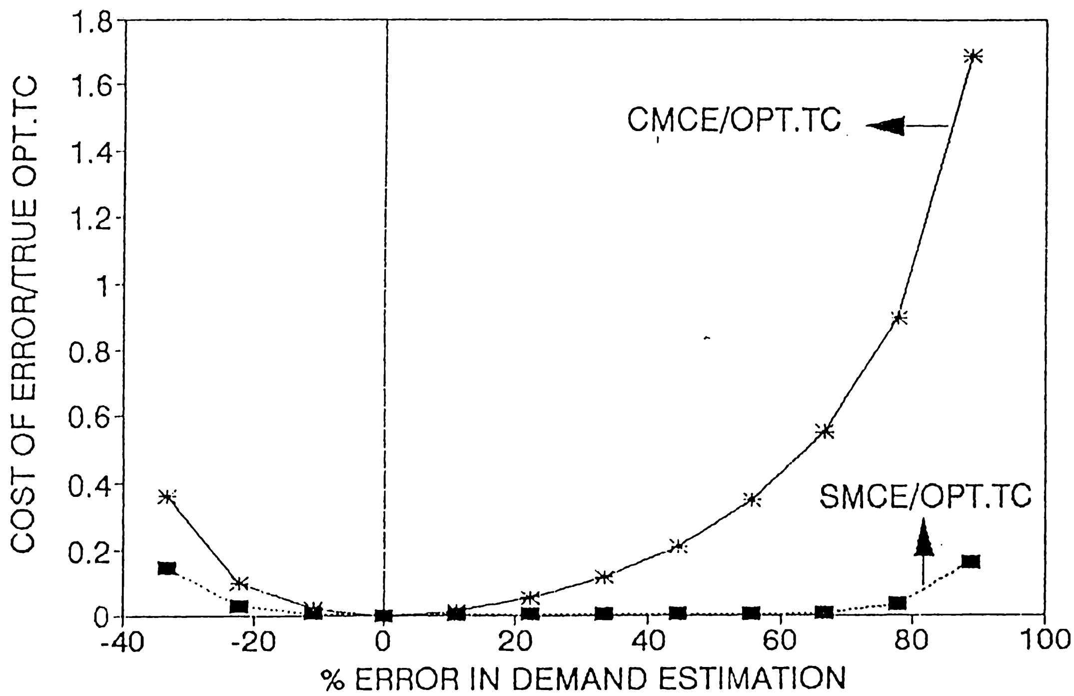

Figure 5.8: Ratio of the cost of a demand rate estimation error versus the % error in demand rate estimation in the classical *EOQ* and *SPIL* model.

5.3.2 Cost of an Error in Estimating the Other Parameters of the System

We have already discussed the cost associated with a demand rate (D) estimation error for both models. However, it is also necessary to perform a similar analysis for the other parameters of the system like *set-up cost S,unit inventory holding cost (h), and the production rate (r).* Note that the sensitivity analysis is again based on the 'cost associated with an estimation error' in one of these parameters of the production system.

The value of the cost parameters S and h are inferred by the management by deriving on some statistical techniques and the production rate r is determined by the plant managers in the technical limits (as explained in section(3.1)).

Actually, we show in this section that, the cost of error due to estimation errors in other parameters of the system (i.e., S, h, r) are 'exactly' the same as the classical EOQ model.

Cost of an error in estimating the set-up cost parameter, S:

Using the same methodology as in the previous section, cost of error due to an estimation error in the set-up cost, S can be written for both models as

$$CMCE = TC(\hat{Q}, S^*) - TC(Q^*, S^*) \tag{5.27}$$

where

$$\hat{Q} = \sqrt{\frac{2\hat{S}D}{h(1-\frac{D}{r})}} \tag{5.28}$$

and

$$Q^* = \sqrt{\frac{2S^*D}{h(1-\frac{D}{r})}} \tag{5.29}$$

and similarly,

$$SMCE = TC(\hat{I}, S^*) - TC(I^*, S^*) \tag{5.30}$$

where

$$\hat{I} = \sqrt{\frac{2\hat{S}D(1-\frac{D}{r})}{h}} \tag{5.31}$$

and

$$I^{\star} = \sqrt{\frac{2S^{\star}D(1-\frac{D}{r})}{h}} \tag{5.32}$$

These formulas are similar to those of the costs associated with demand rate estimation errors. However the estimated parameter is changed from D to S. Note that in 5.27 and 5.30 the second terms are the same as both define the optimal value of the total cost with the *true* value of the parameter S given in equation (5.5).

Thus it will be enough to find the *difference* between $TC(\hat{Q}, S^{\star})$ and $TC(\hat{I}, S^{\star})$ in order to compare the cost of error due to the *set-up cost*, S estimation error. Now we show that this difference (5.33) is always zero.

$$CMCE - SMCE = TC(\hat{Q}, S^{\star}) - TC(\hat{I}, S^{\star}) \tag{5.33}$$

$$\rightarrow \quad = [\frac{S^{\star}D}{\hat{Q}} + \frac{1}{2}h\hat{Q}(1-\frac{D}{r})] - [\frac{S^{\star}D}{\hat{I}}(1-\frac{D}{r}) + \frac{1}{2}h\hat{I}] \tag{5.34}$$

Note that we can define $\hat{Q}$ as

$$\hat{Q} = \frac{\hat{I}}{(1-\frac{D}{r})} \tag{5.35}$$

where $\hat{Q}$ and $\hat{I}$ are given as 5.28 and 5.31.

Substituting 5.35 in 5.34 gives

$$[\frac{S^{\star}D}{\hat{I}}(1-\frac{D}{r}) + \frac{1}{2}h\frac{\hat{I}}{(1-\frac{D}{r})}(1-\frac{D}{r})] - [\frac{S^{\star}D}{\hat{I}}(1-\frac{D}{r}) + \frac{1}{2}h\hat{I}] = 0$$

Thus we conclude that,

- The cost of error due to an estimation error in S is the same for both the classical EOQ model and $SPIL$ model.

Using the same example (from the PETKİM production system) in the previous section, the cost of error associated with a set-up cost estimation error is given in figure 5.9 .

$\hat{S}$=415,249,640 TL

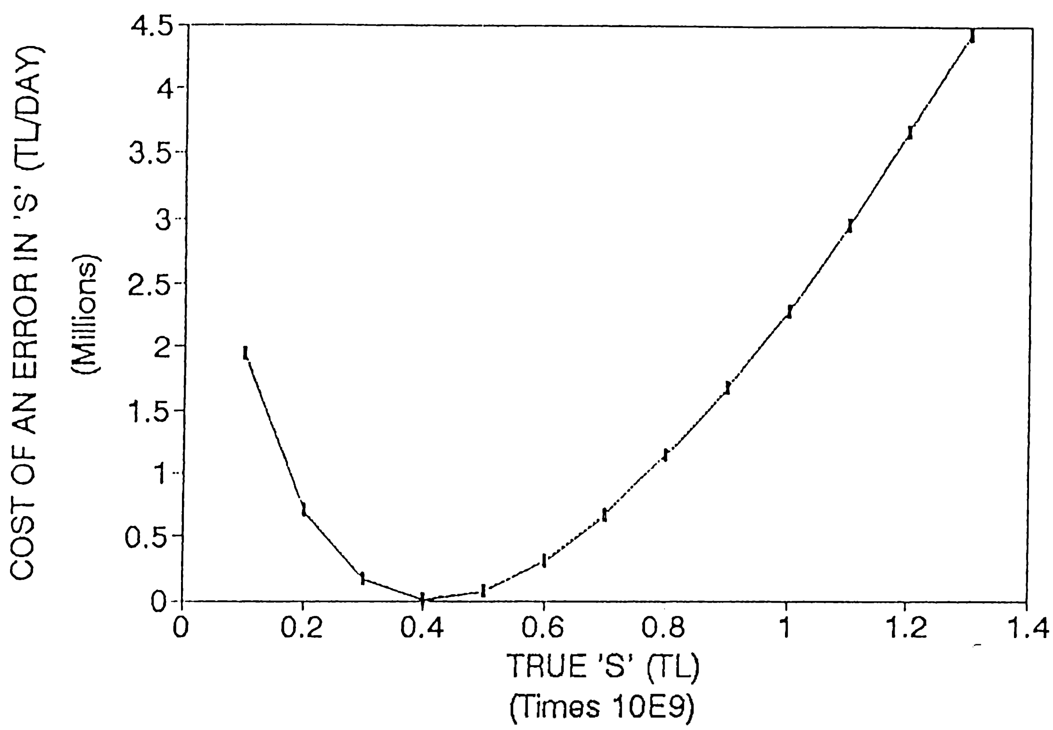

Figure 5.9: Cost of an error due to a set-up cost estimation error vs the true value of set-up cost (S^{*}) in the classical EOQ model and $SPIL$ model

Cost of an error in estimating the unit inventory holding cost, h:

While defining the cost due to an estimation error in h, we carry out the similar discussion for S. By changing the estimation parameter from S to h in 5.33, we can write

$$CMCE - SMCE = TC(\hat{Q}, h^*) - TC(\hat{I}, h^*) \tag{5.36}$$

$$\rightarrow \quad = [\frac{SD}{\hat{Q}} + \frac{1}{2}h^*\hat{Q}(1 - \frac{D}{r})] - [\frac{SD}{\hat{I}}(1 - \frac{D}{r}) + \frac{1}{2}h^*\hat{I}] \tag{5.37}$$

$\hat{Q}$ and $\hat{I}$ are again defined as in 5.28 and 5.31 by changing the estimated parameter from S to h.

$$\hat{Q} = \sqrt{\frac{2SD}{\hat{h}(1 - \frac{D}{r})}} \tag{5.38}$$

and

$$\hat{I} = \sqrt{\frac{2SD(1 - \frac{D}{r})}{\hat{h}}} \tag{5.39}$$

We can define $\hat{Q}$ as a function of $\hat{I}$ using equation 5.35, thus 5.37 can be written as

$$\hat{Q} = \frac{\hat{I}}{(1 - \frac{D}{r})} \tag{5.40}$$

where $\hat{Q}$ and $\hat{I}$ are given as 5.38 and 5.39.

Substituting 5.40 in 5.37 gives

$$[\frac{SD}{\hat{I}}(1 - \frac{D}{r}) + \frac{1}{2}h^*\frac{\hat{I}}{(1 - \frac{D}{r})}(1 - \frac{D}{r})] - [\frac{SD}{\hat{I}}(1 - \frac{D}{r}) + \frac{1}{2}h^*\hat{I}] \quad = \quad 0$$

Thus we make the same conclusion that

- The cost of error due to an estimation error in h is the same for both the classical EOQ model and $SPIL$ model.

This result can be seen graphically in figure 5.10 , where the values of the parameters are identical with the previous figure.

$\hat{h}$=16,720 TL/unit/day

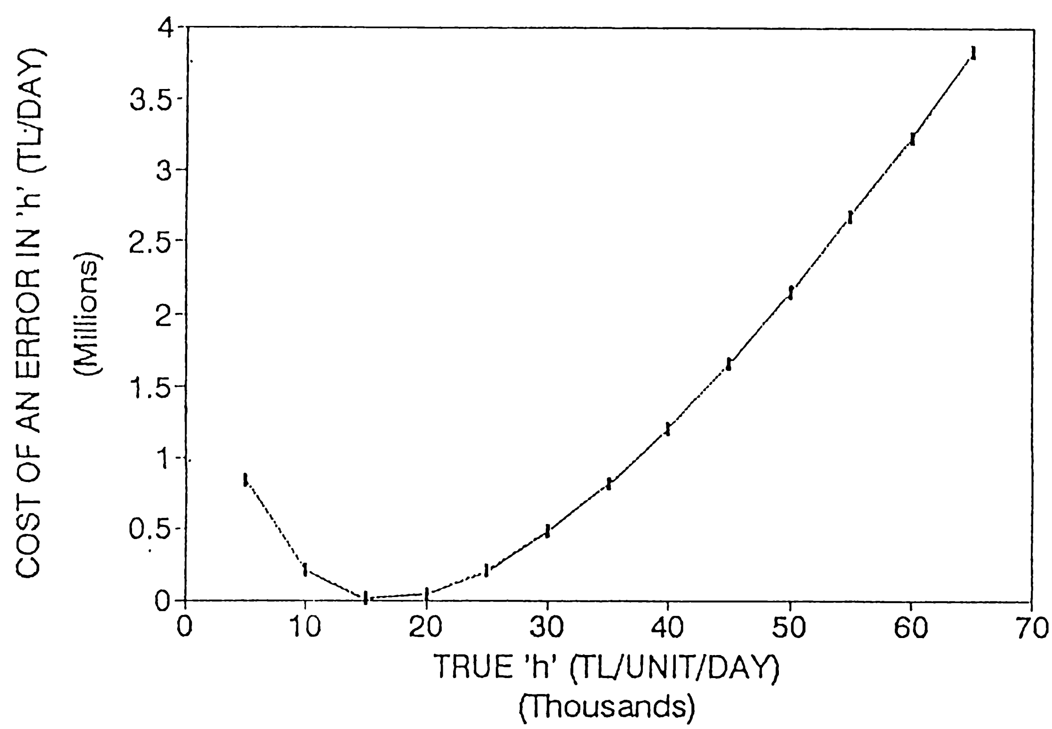

Figure 5.10: Cost of an error due to a unit inventory holding cost estimation error vs the true value of unit inventory holding cost (h^*) in the classical EOQ model and $SPIL$ model

Cost of an error in estimating the value of the production rate, r:

In section 3.1, we have discussed that it is technologically possible in PETKİM to keep the production rate at a 'constant' value. That's why, once the value of r is set by the decision maker, it will be valid all through the planning horizon.

However, we still need to test the performance of the SPIL model against the changes in the production rate in order to generate a better understanding of how the system behaves when a change occurs in r. Besides, such analysis is essentially needed to learn about the 'applicability' of the SPIL Model to other type of production systems, where production rate is not constant.

Using the same argument in the previous parts, we can write

$$CMCE - SMCE = TC(\hat{Q}, r^*) - TC(\hat{I}, r^*) \tag{5.41}$$

Now we show that the difference of the cost of estimation errors in r between the classical EOQ model and the $SPIL$ model is zero. Actually, it is enough to show that

$$CMCE = SMCE$$

$$\rightarrow \quad TC(\hat{Q}, r^*) = TC(\hat{I}, r^*)$$

$$\rightarrow \quad [\frac{SD}{\hat{Q}} + \frac{1}{2}h\hat{Q}(1 - \frac{D}{r^*})] = [\frac{SD}{\hat{I}}(1 - \frac{D}{r^*}) + \frac{1}{2}h\hat{I}] \tag{5.42}$$

Here, $\hat{Q}$ and $\hat{I}$ are defined as

$$\hat{Q} = \sqrt{\frac{2SD}{h(1 - \frac{D}{\hat{r}})}} \tag{5.43}$$

and

$$\hat{I} = \sqrt{\frac{2SD(1 - \frac{D}{\hat{r}})}{h}} \tag{5.44}$$

We define $\hat{Q}$ in terms of $\hat{I}$ as

$$\hat{Q} = \frac{\hat{I}}{(1 - \frac{D}{\hat{r}})} \tag{5.45}$$

Substituting 5.45 in 5.42 gives

$$[\frac{SD}{\hat{I}}(1 - \frac{D}{\hat{r}}) + \frac{1}{2}h\frac{\hat{I}}{(1 - \frac{D}{\hat{r}})}(1 - \frac{D}{r^*})] = [\frac{SD}{\hat{I}}(1 - \frac{D}{r^*}) + \frac{1}{2}h\hat{I}]$$

$$\rightarrow \quad \frac{SD}{\hat{I}}(1 - \frac{D}{\hat{r}}) - \frac{1}{2}h\hat{I} = (1 - \frac{D}{r^*})\ [\frac{SD}{\hat{I}} - \frac{1}{2}h\frac{\hat{I}}{(1 - \frac{D}{\hat{r}})}]$$

$$\rightarrow \quad \frac{h}{2\hat{I}} \underbrace{\left[\frac{2SD(1-\frac{D}{\hat{r}})}{h} - \hat{I}^2\right]}_{=0} = \frac{(1-Dr^*)}{(1-\frac{D}{\hat{r}})} \underbrace{\left[\frac{2SD(1-\frac{D}{\hat{r}})}{h} - \hat{I}^2\right]}_{=0}$$

In the RHS and LHS of this equality, two multipliers are identical and equal to zero as $\hat{I}$ is defined by 5.44. Thus we show that

- The cost of an error due to an estimation error in r is the same for both the classical EOQ model and the $SPIL$ model.

The cost associated with an error in estimating the production rate, r is given in figure 5.11. using the same values of the parameters as in the previous figures.

$\hat{r}$=65.15 TL/day

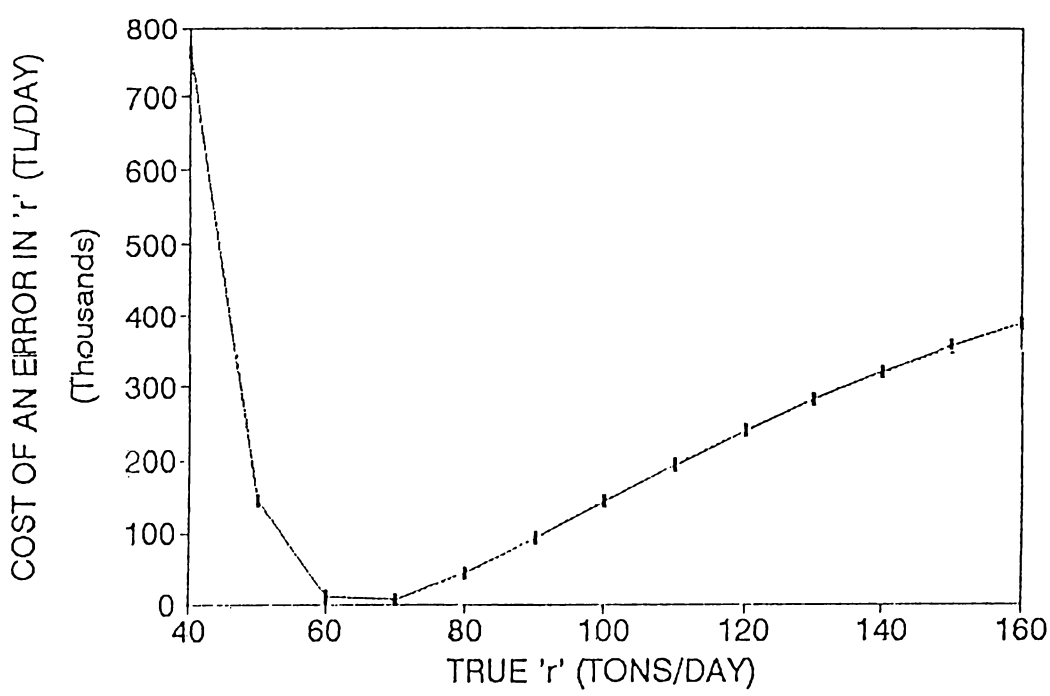

Figure 5.11: Cost of an error due to a production rate estimation error vs the true value of production rate (r^*) in the classical EOQ model and $SPIL$ model

Chapter 6

SUMMARY AND CONCLUSION

In this study, we propose a 'robust' production and inventory control model. The SPIL model is implemented in a real system PETKİM; however, it is a general model that can be applied to any production and inventory system where there is a constant production rate with random demand pattern. Cost parameters related to set-up and inventory holding costs are constant. Backorder is allowed; but the related cost is intuitively evaluated by the DM.

SPIL model originates from the classical EOQ model with finite production rate. By certain modifications on the EOQ model, we derive a bi-objective model with two decision variables. Decisions are subject to a service level constraint which requires user involvement. A 'compromised' solution is obtained by using the sequential optimization technique. Solution is in the form of a trade-off curve, which shows the relation between the service level measure (SLM) and the associated minimum total cost. The DM selects a production and inventory control strategy using the trade-off curve. The optimal values of the decision variables are the 'maximum inventory level' (produce up to level) and the reorder inventory level that minimize the total cost (as the sum of set-up and inventory holding costs) for the required level of SLM.

The 'robustness' of the SPIL model is measured by testing its sensitivity against demand estimation errors and comparing with those of classical EOQ model. We show in this study that, by changing the decision variable of the classical EOQ model from 'production quantity' to 'maximum inventory level', we obtain a 'robust' model against demand estimation errors. Thus SPIL model is more insensitive to the demand estimation errors than the classical EOQ model. In order to enhance the 'generality' of the SPIL model, sensitivity analysis is made due to changes in other parameters of the model i.e., production rate, unit inventory holding cost and set-up cost. The results are again compared with those of the classical EOQ model and it is concluded that the sensitivity of the SPIL model to other parameters is 'identical' with the classical EOQ model.

Initially, sensitivity analysis of both models are based on the criteria of 'cost of a parameter estimation error', which is very seldom used in literature. We emphasize in this thesis work that, this way of approach to sensitivity analysis makes more sense than the classical approach which is based on the 'rate of change in the optimal solution due to a change in one of the input parameters'. We need to stress that, 'even if the rate of change of the optimal solution' due to a parameter estimation error is large, a robust model can still generate robust decisions where the cost of error is small.

Additionally, we also compare both models by testing their sensitivity due to parameter estimation errors using the classical way of sensitivity analysis. The results obtained from this analysis are 'consistent' with ones obtained by our new approach.

One of the core points of this thesis study is the introduction of a 'robust model' concept. Robustness is measured in terms of the 'cost of a parameter estimation error'. SPIL model is a robust model against demand estimation errors, meaning that it generates 'robust decisions' due to errors made in estimating demand rate. From this point of view, SPIL model is an 'adaptive' control model. Actually, it adapts itself to the changing conditions by generating a robust inventory control strategy.

Finally, SPIL model is an interactive decision making model where the DM is actively involved in the decision process, by making a trade-off between the SLM and the minimum total cost.

We conclude this chapter by restating that, robust inventory control models can be derived by using the theoretical models in literature. Eventually, the aim of the researchers involved in this area should be generating adaptive models that assist the DM while giving decisions related to production and inventory control. This is the central theme of this study.

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