ROBUSTNESS OF BUFFER ALLOCATION IN MULTI-PRODUCT MULTI-BATCH DETERMINISTIC FLOW LINES

A THESIS
SUBMITTED TO THE DEPARTMENT OF MANAGEMENT
AND GRADUATE SCHOOL OF BUSINESS ADMINISTRATION
OF BILKENT UNIVERSITY
IN PARTIAL FULFILMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF BUSINESS ADMINISTRATION

By
A. AKIN KURUCU

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tarafından bağışlanmıştır

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Business Administration.

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ABSTRACT

ROBUSTNESS OF BUFFER ALLOCATION IN MULTI-PRODUCT MULTI-BATCH DETERMINISTIC FLOW LINES

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M.B.A.

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Today in industry flow lines are not just for a single end product. There is a stochasticity, such that there are various demand scenarios at hand, to be satisfied by the flow line. The performance of the flow line should not be very sensitive to demand changes. Aim of this study is to develop buffer allocation guidelines to help flow line designers.

Keywords: Buffers, Multi product flow lines, robustness, work_in process inventory (WIP).

ÖZET

ÇOK MAMÜLLÜ ÜRETİM HATLARINDA DEĞİŞKENLİKLERE KARŞI DAYANIKLI ARA STOK PLANLANMASI

A. Akın Kurucu

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Tez Yöneticisi: Dr. Selçuk Karabatı

Kasım 1993, 87 sayfa

Günümüzde üretim hatları birden fazla mamül üretmektedir. Fakat elde karşılanması gereken birden fazla üretim planı bulunması bir belirsizliğe yol açmaktadır. Üretim hattının performansı bu belirsizlik karşısında bile yüksek tutulabilmelidir. Bu çalışmanın amacı üretim hattı planlamacılarına yardımcı olmak üzere ara stok planlamsı konusunda bazı öneriler getirmektir.

Anahtar Kelimeler: Parça kapları, çok mamüllü üretim hatları, dayanıklılık, üretim zamanı stoğu.

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1. INTRODUCTION:

Mass production is the key word for the industrial revolution. The only way to achieve efficiency in production processes is mass production. The key word for the mass production is the production line. A production line, or a flow line, is a fixed sequence of production stages, each consisting of one or more machines or workstations. In designing flow lines, there are often many alternative configurations that can be considered, along with a wide variety of material handling equipment from which to choose.

A challenging problem in designing a production line involves the determination of the optimum configurations of machines and buffers in the flow line. Buffers are storage bins to keep work-in-process inventory in the production process. They are placed between production stages of the flow line and are key factors to increase efficiency when there is a variability in the production process. Function of buffers in a flow line will be explained in more detail in Chapter 2.

In calculating flow line efficiency, cycle time can be used as a performance criterion. Cycle time is the time interval between successive parts coming off the flow line. Throughput rate of the flow line is the inverse of cycle time. So throughput is the production rate of the flow line.

An important part of the flow line design is the allocation of buffers between production stages. The problem of buffer allocation in flow lines has been extensively studied in the literature. As Sarker (1984) and Smunt and Perkins (1985) stated, researchers have considered the buffer allocation problem in a variety of contexts. Major part of research studied flow lines with a single product and stochastic processing times. Researchers dealt with flow lines where station service time variability is described by normal, exponential or coaxial distributions. Models using exponential distribution are Hunt (1957), Hilier and Boling (1966,1967). Recently Hilier, Boling and So (1990) and Hilier and So (1991) have adressed buffer allocation problem in a single product flow line with stochastically identical and independent stations. Their results show the "bowl effect", whereby the center stations are given preferential treatment through more storage spaces especially when there is higher variability in the processing times. Yamashina and Okamura (1983) and Conway et al. (1988) did simulation studies and found similar results.

Above studies are for single product environments. For such environments the source of variability in processing times is the stochastic nature of the operations, and buffers between stations are present to reduce the adverse effects of this variability on the throughput rate of the flow line.

Karabatì and Kouvelis (forthcoming in Annals of O.R.) studied multi-product flow lines with deterministic processing times. In multi-product stochastic flow lines the source of variability is not the stochastic variability of each job, but the variability in processing times across the jobs. So in a multi-product environment there exists variability of processing times at each station due to the wide mix of jobs processed there.

Their study considers a flow line which produces a set of products under a cyclic scheduling policy. The cyclic scheduling approach for flow lines is based on the idea of repetitively producing a small set of items. Each set has the items to be produced in the same proportions as the production requirements of an end product. Parts are sequenced in a small production set and then simply repeat the same part sequence. Some of the advantages of cyclic scheduling policy are, smoother finished good inventory levels and implementation convenience due to the simplicity of cyclic schedules. (Karabatì and Kouvelis).

This small set of items to be produced in a repeated fashion is called a batch. So in a cyclic scheduling policy batches are produced repetitively. Karabatì and Kouvelis investigated buffer design problem in a multi-product flow line in cyclic scheduling, but for a single batch case.

Here in this study a multi-product multi-batch deterministic flow line which is operated under a cyclic scheduling policy will be considered. So our flow line has another source of variation. That is variation resulting from variation of batches besides variation of products.

Today in industry flow lines are not just for one end product. Flow lines should be able to produce a set of different end products. But the demand for these end products are not known exactly, rather there is a stochasticity present such that there are various demand scenarios on hand. One of them will be the production requirements of the flow line for over a planning horizon. But after that another demand scenario may be faced. A possible demand scenario may be 50 batches from end products A, B, and C; 20 from end products D, E, and F and so on.

In this environment, we have to consider the robustness problem of the buffer designs. Robustness means that a procedure is still able to perform its intended purpose even if the assumptions under which it was developed are slightly incorrect. Robustness in a sense is to make a product or process insensitive to variations. (Rocke, 1989)

In our case, the flow line faces a set of demand scenarios to be satisfied. That is the demand to the end products of the flow line is not known exactly. So, such a flow line facing various demand scenarios should have design robustness. Rosenblatt and Lee (1987) pointed out that robustness of the flow line in cases of demand uncertainity, is more important for the operations manager. Robustness of the flow line is an indicator of flexibility in handling demand changes. With such an approach, the designer will select a flow line design that has the highest frequency of being closest to the optimal solution under different demand scenarios.

So a buffer design that will give maximum throughput rate for batch type 1, or another that will maximize throughput rate for batch type 2, will not be the objective of the flow line designer. Flow line designer will seek a robust buffer design that will be close to optimal under all possible demand scenarios. Here of course, there is an assumption such that once the buffer design is made and buffers are placed between stations, it is impossible or very expensive to relocate them.

This study will test the robustness of the buffer design problem in a multi-product, multi-batch flow line with deterministic processing times.

2. THE FUNCTION OF WORK-IN-PROCESS INVENTORY (WIP) IN FLOW LINES

We define WIP as inventory after the first step in manufacturing and before the last. That is WIP is the inventory held in buffers. We thus exclude raw material inventory, for which there may be other considerations of delivery and price variations, and finished good inventory.

The purpose of placing buffers between stations is to give each station some degree of independent action. Two workstations in series without buffers must be perfectly synchronized to operate effectively; they must perform and be scheduled as if they were, in effect, a single machine. On the other hand, by providing buffers for some amount of WIP between the processes, each of the stations has some independence in its operation. (Conway, et al., 1988)

Without intervening WIP, unless two workstations in series finish each production cycle at precisely same instant, they will interfere with each other and production capacity will be lost. Even if they have the same average variable processing times, the first station sometimes finishes a cycle before the second. The first station must wait to dispose of its finished piece before it begins the next piece and it is said to be 'blocked'. Similarly, if the second workstation finishes a cycle before the first and must wait for input material until the first finishes, the second station is said to be 'starved'. Both blockage and starvation mean that a process is prevented from starting, and hence, potential production capacity is lost. Provision for buffers between such workstations increases capacity by reducing the frequency and severity of blockage and starvation.

A similar but more serious loss occurs if a machine is unexpectedly shut down for any reason: a breakdown, broken or missing tooling, operator unavailability etc. Again, some amount of WIP (buffers) between the stations provides a 'grace period' during which operation continues when another station is shut down (Conway, et al., 1988). Buffers also allow two workstations to work on different products, even if there is a significant 'setup time' required to change from one product to another.

Effective use of WIP is further complicated because the position of buffers are as important as their capacity. There are some locations where buffers increase cost without any commensurate benefit, and others where even a single buffer is highly productive. (Conway, et al., 1988).

But the role of WIP is to deal with short term transients; it is not capable of overcoming long term imbalance in capacity. Even so, the capacity of a facility varies importantly as a function of the amount and location of buffers. The zero WIP production capacity is related to the probability that all workstations are simultaneously in operation. At the other extreme, the infinite WIP capacity is the long term average capacity of limiting stage; this is the bottleneck in the design.

The classical investment cost of WIP, is probably the least important price one must pay for WIP. However, it is often dominated by the facilities cost of the equipment required to support and move WIP, and the cost of space it occupies. These considerations include substantial elements of 'opportunity cost' that make them harder to quantify than capital costs.

Perhaps the most important cost of WIP is the effect on manufacturing 'lead-time' or 'flow time'. Flow time is the time required to move a piece through the manufacturing process, from entry on the factory floor to completion of the last production stage. The sum of processing times for the piece is the minimum possible value of flow time, and everything above that is associated with WIP, including material handling. If for example, material handling was instantaneous and there was one unit of WIP in buffers between each pair of workstations, each piece would spend roughly as much time waiting as being processed, and the ratio of flow time to total processing time would be approximately 2 to 1. It is therefore, surprising to learn that a ratio of 10 to 1 is hard to achieve even in a modern plant and that ratios in excess of 100 to 1 are common; a piece that could be produced in a single day requires one or more months to pass through the plant. The consequences of this phenomenon is crucial. (Conway et al., 1988)

For example, consider a manufactured product for which the 'work content' (the sum of processing times) is approximately eight hours. Knowing it will take several months to actually push one of these products through the production process, we necessarily rely on some combination of the following strategies. (a) anticipate customers' needs in terms of product variations, options, and stock finished goods inventory; and (b) find customers who will tolerate several months delay in delivery.

Alternatively, a manufacturing process capable of producing one of these products to order in a week would eliminate the necessity of stocking finished products, and provide greater freedom of choice to the customer. It would also place any supplier at a distinct disadvantage whose delivery involved a long ocean voyage. Such a 'fast reaction' process has many other disadvantages to correct quality problems and implement engineering changes, and can usually be housed in a facility a fraction the size of the conventional alternative. To achieve this desirable competitive position, it is imperative to maintain a low ratio of WIP to throughput. Hence, there are abundant reasons to seek minimal WIP processes, i.e., minimal buffer size to allocate, which underscores the necessity of knowing precisely where WIP is useful and how much is valuable.
(Conway, et al., 1988)

3. PREVIOUS RESEARCH ON BUFFER DESIGN

There is an extensive literature in this field and some of these are stated at references. Here in this chapter, a small set that examplifies the nature of the previous research is covered.

3.1. HILIER, BOLING AND SO (1990)

Hilier et al.'s system consists of N single server service facilities corresponding to the N work stations of the production line, where every unit must be processed by these stations in the same fixed sequence. Their flow line is to process a single product but service times at stations have exponential distribution with identical means and all operation times are independently distributed.

There are no breakdown or down time of stations. Since the distribution is exponential, coefficient of variation of the processing times of stations are constant and equal to 1. They conclude that:

1. When the buffer capacity and number of buffer allocation spaces are equal, uniform allocation of buffers between stations is optimal.

2. When there is one more buffer after uniform allocation, place that extra at the centre of the line.

3. When there are two or more buffers after uniform allocation, the optimal pattern cannot be characterized precisely, but does tend to place the extra buffers near the center of the line.

4. Giving preference to center stations rather than stations at either end is better.

3.2. HILIER AND SO (1991)

This study extends the previous work by allowing coefficient of variation other than one. This is done by using coaxian distribution for the processing times of stations. It is again a single product flow line where stations have identical mean processing times.

They conclude that:

1. Optimal buffer allocation depends on the degree of variability in the operation times.

2. Bowl effect, that is, center stations should be given preferential treatment through more storage space is more pronounced with higher variability in the operation times.

3. Higher variability generally increases the imbalance

in the optimal allocation.

3.3. CONWAY et.al.(1988)

Conway et al. dealt with different aspects of the flow line design problem by simulation methods. For a uniformly distributed processing times of stations and with Mean/Range=1.0 they made the following suggestions:

1. Allocate buffers as nearly equally as possible through the flow line.

2. If, after equal allocation some buffers remain at hand, spread them over the line at approximately equal intervals. The first and last buffer allocation spaces should get the lowest priority in this step.

Then they simulated a flow line where stations have identical mean processing times but different standard deviations, i.e., variability.

They found that stations with higher variability of processing times should have larger number of buffers for both input and output.

3.4. KARABATI AND KOUVELIS (forthcoming in Annals of OR)

Karabatì and Kouvelis considered a multi-product flow line where stations have deterministic processing times. Variability was due to the mix of products processed at the same flow line. Their research was for a single batch case. For such an environment they made simulation studies and conclude that:

1. Sequence independent information; where sequence is the entrance sequence of jobs of the batch to the flow line; i.e., information about workload distribution and coefficient of variation of processing times at various stations is not adequate to develop buffer design rules.
2. Design rules developed for single product environments cannot be simply transferred to multi-product environments.
3. Sequence of jobs entering the flow line is an important variable in the design process.

Karabatì and Kouvelis used complete enumaration for finding optimal designs in their experiments. That is for a 6 machine flow line with 5 buffer allocation spaces, throughput rate of every combination of buffer design is calculated and the best one is chosen as the optimal design. This procedure needs C (B+(m-1), m-1) calculations where B is the total buffer capacity and m is the number of machines.

They developed two heuristic solution methods to decrease huge number of calculations that will come up for large size problems.

Greedy heuristic is an approximate solution for buffer allocation problem. This heuristic would generate an optimal solution if the objective function is convex and separable in the desicion variables. However, greedy approach can only be used as an approximate solution in this case.

Greedy heuristic places available buffers one by one between stations by finding optimal place for the buffer on hand. For a 6 station flow line there are 5 buffer allocation spaces and so 5 calculations are needed for each buffer to select the best location. Greedy heuristic needs (m-1)*B calculations for allocating B buffers between m stations.

The second approximate solution procedure is a variation of dynamic programming formulation of the resource allocation problem with integer variables. Let $Y^k(n)=(y_1^{k,n},y_2^{k,n},\ldots,y_k^{k,n})$ be an allocation of n buffers into the first k locations, that is the segment of the flow line between stations M_1 and M_{k+1}. The recursive relationship to determine $Y^k(n)$ is

$$Y^k(n)=(y_i^{k,n}=y_i^{k-1,n-L},i=1,\ldots,k-1,y_k^{k,n}=L)$$

where L is chosen such that the cycle time of the problem determined by the first k+1 stations and the buffer allocation $Y^k(n)$ is minimized. This procedure generates the optimal solution if the objective function is separable in the desicion variables, however, it can be easily shown that for the buffer allocation problem, this approach may result in non-optimal solutions.

4. METHODOLOGY

4.1. SEEKING OPTIMAL BUFFER ALLOCATION FOR MULTI-PRODUCT MULTI-BATCH DETERMINISTIC FLOW LINES

In single product flow lines with stochastic processing times, the characteristics of the flow line under consideration (information on workload distribution between stations and variability in processing times) play an important role in developing guidelines for optimal buffer design. For example, in flow lines with identical stations, that is stations with similar workloads and variability in processing times, the inverted 'bowl shaped' buffer allocation has been shown to be very effective (Hillier, Boling, So 1990). Similarly, in flow lines with balanced stations, that is stations with equal workloads, and unequal variability in processing times; the stations with larger variability in processing times should have large buffer size for both input and output. (Conway, et al. 1988)

Here we will attempt to answer the question whether similar guidelines can be developed for multi-product multi-batch flow lines which are in cyclic scheduling policy, using the information on workload distribution among stations and processing time variabilities. The workload of a station is equal to the sum of the processing times of operations that are going to be performed on this station. The processing time variability is the variability due to the mix of jobs processed at a station. The processing time variability parameter used is the coefficient of variation (C.V.). The coefficient of variation of a station is equal to the unbiased estimate of the variance of processing times divided by the square of the mean processing time in this station.

First we will examine cyclic scheduling approach as a production planning policy in flow lines:

4.2. CYCLIC SCHEDULING IN A DETERMINISTIC FLOW LINE

Let us consider a flow line with no sublines. It has m stations in series. There may be finite or infinite capacity buffers between the stations. Let r_i be the number of units of item J_i, $i=1,\ldots,L$, required to meet a production target of the line over a planning horizon, and $r= (r_1,\ldots,r_L)$ be the production requirement vector for all different items produced in the line over the same horizon. If q is the greatest common divisor of integers $r_1,\ldots,r_L$, then the vector

$$r^{*}=(r_1/q,\ldots,r_L/q)$$

is referred to as the Minimal Part Set (MPS). (Hitz, 1979). It represents the smallest part set having the same proportions as the production requirement vector. Under a cyclic scheduling policy, the flow line will repetitively produce MPSs, using the same sequence of jobs for all part sets. We may also produce an integral multiple of MPS in a repetitive manner, however, without loss of generality, we are going to confine our discussion to the production of MPSs.

Let n be the number of jobs in an MPS. We denote by p_{ij}, $i=1,\ldots n$, $j=1,\ldots m$, the processing time of job j_i on station M_j. The cyclic scheduling problem is to find the optimal sequence of jobs in a prespecified part set in order to optimize the throughput rate of the line, or equivalently its cycle time. Cycle time is the reciprocal of the throughput of the line. In our analysis by-passing of stations by jobs is not allowed. This type of a flow line is called a conventional flow line.

Parts of an MPS go through the system in a given order followed by a second MPS in the same order and so on. An MPS schedule will be represented by permutation $o=(o(1),o(2),...,o(n))$ where n is the number of jobs in the MPS and o(i) is the i-th job in the processing order. In our analysis no job passing is allowed once the MPS is released to the system. The i-th job in the r-th MPS is the r-th repetition of job o(i) and is denoted by $o_r(i)$.

In a flow line with infinite capacity buffers the completion time $C(o_r(i),j)$ of job $o_r(i)$ on station M_j can be found using this recursive formulation.

$$C(o_r(i),j)= \max \{C(o_r(i-1),j),\ C(o_r(i),j-1)\}+ p_{o(i),j} \quad (1)$$

where $C(o_r(0),j)=C(o_{r-1}(n),j)$, $r=2,3,...,$ $j=1,...,m$, and $C(o_1(0),j)=0$, $j=1,...,m$, $C(o_r(i),0)=0$, $i=1,...,n$, $r=1,2,....$ (Karabatì and Kouvelis)

Next for finite capacity buffers between stations we may formulate the problem as follows. First we may assume that all buffers have either zero capacity or infinite capacity because we can represent each unit buffer location by a station at which all processing times are equal to zero. So, in order to extend the framework, we only need to find a way to handle the case where the buffer capacity between station M_j and M_{j+1} is equal to zero. The completion time of $C(o_r(i),j)$ of the r-th repetition of job o(i) on station M_j is

$$C(o_r(i),j)=\max\{C(o_r(i-1),j+1), \max\{C(o_r(i-1),j),C(o_r(i),j-1)\}+p_{o(i),j}\}$$

(2)

where $C(o_{rPv(0)},j)=C(o^{r-1}(n),j)$, $r=2,2,\ldots,$ $j=1,\ldots,m$ and $C(o^1(0),j)=0$, $j=1,\ldots m$, $C(o^r(i),0)=0$, $i=1,\ldots,n$, $r=1,2,\ldots$

The difference between relationships (1) and (2) is due to the blocking of jobs in the presence of zero capacity buffers. (Karabatì and Kouvelis)

4.3. FORMULATION OF BUFFER DESIGN PROBLEM FOR A FIXED SEQUENCE OF JOBS

A flow line with m stations is considered. Let B be the total number of buffers to be allocated, and x^i be the number of buffers allocated between stations M^i and M^{i+1}, $i=1,\ldots,m-1$.

The throughput of the line is equal to the inverse of the cycle time, and the problem of buffer allocation can be formulated as follows:

$$(BAL): f=\min F(X)$$

$$\text{subject to} \quad \Sigma_{i=1}^{m-1} x^i = B$$

$$x^i \geq 0 \text{ and integer, } i=1,\ldots,m-1$$

BAL is a resource allocation problem with integer variables. The objective function F(x) is not separable in x^i's and it is neither convex nor concave in X. (Karabatì and Kouvelis)

4.4 SIMULATION DESIGN

Several experiments are performed to test the robustness of the buffer allocation with respect to different variables. However the simulation design is same for all these experiments. For computer simulations in this study, the pascal program developed by Karabatì and Kouvelis for their single batch study is modified and a multi-batch version is obtained.

Our flow line has 6 stations, so there are 5 possible buffer allocation spaces.

Under the cyclic scheduling policy, the flow line will repetitively produce MPS's. The MPS is represented by a matrix in the simulation. Columns are the stations and rows are the jobs in the MPS. So element E (i,j) of the matrix represents the processing time of job i at station j for MPS E. Each column of the batch matrix has the information on the workload of that station represented by that column.

Experiments will be conducted over sets of 30 randomly generated problems. For each problem an optimal buffer design will be found by complete enumeration. A buffer design is represented by a 5 number sequence, each number representing the number of buffers allocated to that buffer space. That is, design (01111) means, there are no buffers between stations 1 and 2, and there is one buffer between every other pair of stations. Most dominant designs over 30 problems, for each category, are listed at the tables with their number of occurencies in parantheses. Here the category means a combination of C.V., number of buffers in the problem and number of jobs in the problem. So a -11111 (27)- means design (11111) is optimal in 27 problems out of 30; for the stated C.V., number of buffers and number of jobs combination.

Also greedy and dynamic programming heuristics discussed earlier, will be used for buffer designs and performances of these heuristics will be tested with respect to optimal designs.

5. EXPERIMENTS

In this section we will investigate the robustness of the buffer design problem in a multi-product, multi-batch flow line with respect to different variables by means of several experiments.

5.1. BUFFER ALLOCATION FOR STATIONS WITH IDENTICAL MEANS

In this experiment batches will be generated so that workloads of 6 stations will be equal. The coefficient of variation will be 0.25, 0.5, 0.75, and 1.00; batch sizes within an experiment set will be 3, 4, and 5. Job sizes will be 8, 10, 12; buffer sizes will be 4,5, and 6.

Tables 1,2, and 3 show results of this experiment for 3 batch, 4 batch, and 5 batch cases respectively.

5.1.1. Results:

5.1.1.1. C.V.=0.25

First of all when C.V.=0.25 uniform allocation of buffers is observed. For 4 buffers one buffer space is empty but no preference for empty buffer location has been observed. Possible designs are (11110), (11101), (11011), (10111), (01111).

For 5 buffers uniform allocation (11111) is the only way for all batch and job sizes.

For 6 buffers one extra buffer can be placed anywhere and no center preference is observed. This result is surprising that it contradicts the findings of Hillier and So (1991). They proposed storage bowl phemenon. That is extra 1 buffer should be placed at the center of the line. But this assertion was for single product flow lines. Under the randomness of a multi-product, multi-batch flow line seemingly, this rule is not valid.

For the job size variance, there is no significant difference. Job size is not a factor for determining optimal design for C.V.=0.25.

5.1.1.2. C.V.=0.50

For buffer size 4 (01111) is most popular design. But it is not dominant, other allocations like (11110) or (11011) are also present.

For buffer size 5 (11111) is again dominant for all job and batch sizes.

Coming to buffer size 6; for job sizes 8 and 10 allocation of extra buffer to any space, except last space, is seen. That is allocations (21111), (12111), (11211), (11121) has been observed. There is no difference due to batch size variance. For job size 12, for batch size 3, allocation (11211) is superior to (11121), then for batch size 4, (11121) is superior to (11211) and for batch size 5, (11121) is the only solution.

5.1.1.3. C.V.= 0.75

For 4 buffers; for 8 jobs (01111) is superior to (11011). For 10 jobs (11011) is superior to (01102) for 3 and 4 batches. (11011) is the only design for 5 batch case. For 12 jobs (10201) is superior to (11110) for 3 and 4 batches, and (10201) is the only design for 5 batch case.

For 5 buffers; for 8 jobs (11111) is superior to (01211) for all batch sizes. For 10 jobs (11102) is the only design for all batch sizes. For 12 jobs (11111) is the only design for all batch sizes.

For 6 buffers; for 8 jobs (12111) and (11211) are present for 3 batch, (11211) is superior in 4 and 5 batch cases. For 10 jobs (11112) is dominant for all batch sizes. For 12 jobs dominance of (11211) over (11121) increases with increasing batch size.

5.1.1.4. C.V.= 1.00

For 4 buffers; for 8 and 10 jobs various designs are seen but for 12 jobs center preferred designs like (01210) and (00301) are dominant for all batch sizes.

For 5 buffers; for 8 jobs, while (01121) is superior for 3 and 4 batch cases, (11111) is superior for 5 batch. For 10 jobs (12101) is the only design for 3 batch, but (11111) is the only design for 4 and 5 batches. For 12 jobs center preferred designs like (10301), (10310) and (01211) are seen.

For 6 buffers; for 8 jobs (11211) is superior for all batch sizes. For 10 jobs (12111), (11211), (21111) and (13101) are seen. For 12 jobs (10311) is the only design for all batch sizes.

5.1.2. Discussion

From all these, at first sight it is hard to conclude some rules but some points can be observed.

1. For small C.V. there are no significant design differences between different batch sizes.

2. For small C.V. uniform allocation of buffers is observed but center preference for extra buffers cannot be observed.

3. As C.V. and job size increases, deviations from the uniform design can be seen. Design (10201) is dominant to (11110) and (11101) for buffer size 4, C.V.= 0.75 and job size 12.

4. When C.V. further increases designs with 3 buffers at center is observed. (00301) for 4 buffers and (10310) for 5 buffers at C.V.=1.00 and job size 12. Hilier and So (1991)

stated that "Larger C.V. corresponds to higher variability in the operation times. With finite buffers between stations, higher variability could increase the amount of blockage (or starvation) in the line especially when buffer sizes are small. This blockage (or starvation) in the center stations might be the most critical in that they effect both preceeding and subsequent stations. Therefore, more storage space should be provided, to the center stations than to the end stations in order to protect the adverse effects due to the higher variability in the operation times".

This rule seemingly did not apply to our multi-product, multi-batch case for small C.V. But for C.V.=1.00 and especially when C.V.=1.00 and job size is 12 we saw effects of this fact clearly in our case.

5. Generally uniformity is seen. Deviations like (10201) design rather than (11110) design can be interpreted as : (1 1,5 1,5 1 0) is better than (11110) but as this is not feasible because buffer sizes are integers (10201) dominates (11110). Pay attention to 0 at both sides of 2 to compensate the dense buffer allocation at the center.

Table 1: Buffer Allocation for Stations with Identical Means - 3 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	01111 (11) 11110 (8) 10111 (6)	01111 (9) 11011 (8) 11110 (6)	11110 (7) 01111 (6) 10111 (5)
	5	11111 (30)	11111(30)	11111(30)
	6	11211 (9) 11121 (7) 11112 (6) 21111 (6)	11112 (14) 11121 (6) 11211 (5) 21111 (5)	11121 (10) 12111 (10) 11211 (5) 21111 (5)
C.V.= 0.50	4	01111 (9) 11101 (5) 11110 (4)	01111 (10) 10111 (5) 11110 (5)	01111 (12) 11101 (8) 01120 (6) 02101 (6) 10111 (6)
	5	11111 (27)	11111 (28)	11111 (30)
	6	11211 (10) 12111 (8) 11121 (7)	11121 (9) 11211 (7) 12111 (7)	11211 (15) 11121 (8) 11112 (7)
C.V.= 0.75	4	11011 (9) 01111 (8) 11110 (5)	11011 (21) 01102 (5)	10201 (18) 11110 (6) 11101 (6)
	5	11111 (17) 11201 (5) 01211 (4) 12011 (4)	11102 (30)	11111 (30)
	6	12111 (9) 11211 (8) 02211 (5)	11112 (24) 21102 (6)	11211 (18) 11121 (12)
C.V.= 1.00	4	01111 (8) 11011 (6) 10111 (4) 11101 (4)	11101 (30)	00301 (15) 01210 (15)
	5	01121 (6) 02111 (5) 11111 (5)	12101 (30)	01211 (15) 10310 (15)
	6	11211 (8) 21111 (4) 02121 (4) 21201 (4)	13101 (15) 21111 (15)	10311 (30)

Table 2: Buffer Allocation for Stations with Identical Means - 4 batches

Coefficient of Variation	Number of Buffers	Job Size: 8 jobs	Job Size: 10 jobs	Job Size: 12 jobs
C.V. =0.25	4	01111 (11) 11110 (7) 10111 (5)	11011 (10) 01111 (9) 10111 (6)	10111 (12) 11011 (6) 11101 (4) 11110 (4)
	5	11111 (30)	11111(30)	11111(30)
	6	11112 (10) 11121 (7) 11211 (7)	11112 (12) 11121 (6) 11211 (6) 21111 (6)	11211 (10) 21111 (7) 11121 (6)
C.V. =0.50	4	01111 (13) 11110 (13) 11011 (7)	01111 (11) 10111 (6) 02011 (4) 11110 (4)	01111 (9) 02101 (8) 10111 (8) 11101 (8)
	5	11111 (30)	11111 (28)	11111 (30)
	6	12111 (11) 21111 (10) 11121 (6) 11211 (6)	11211 (12) 11121 (9) 12111 (4)	11121 (20) 11211 (10)
C.V.=0.75	4	01111 (10) 11011 (6) 11110 (4)	11011 (21) 01102 (6)	10201 (18) 11110 (7) 11101 (5)
	5	11111 (21) 01211 (4) 02111 (3)	11102 (30)	11111 (30)
	6	11211 (13) 21111 (6) 11121 (5) 12111 (5)	11112 (30)	11211 (24) 11121 (6)
C.V.=1.00	4	02110 (8) 11011 (8) 11110 (8)	11110 (30)	01210 (30)
	5	01121 (8) 21101 (8) 11111 (7) 20111 (7)	11111 (30)	10310 (30)
	6	11211 (15) 21021 (8) 21111 (7)	12111 (17) 11211 (13)	10311 (30)

Table 3: Buffer Allocation for Stations with Identical Means - 5 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	01111 (13)	11011 (15)	10111 (10)
		11101 (6)	11110 (7)	11110 (8)
		11110 (6)	10111 (7)	11011 (7)
	5	11111 (30)	11111(30)	11111(30)
	6	11211 (8)	11112 (16)	11121 (9)
		21111 (8)	11211 (7)	11211 (9)
		11112 (7)	21111 (7)	21111 (8)
		11121 (7)		
C.V.= 0.50	4	01111 (13)	01111 (11)	01111 (13)
		11110 (11)	11101 (6)	11101 (7)
		11011 (5)	11110 (5)	11110 (3)
			10111 (4)	02011 (3)
	5	11111 (30)	11111 (30)	11111 (30)
	6	12111 (12)	11211 (12)	11121 (30)
		21111 (10)	11121 (7)	
		11121 (8)	12111 (5)	
C.V.=0.75	4	01111 (13)	11011 (30)	10201 (29)
		02011 (5)		
		11011 (5)		
	5	11111 (23)	11102 (30)	11111 (30)
		01211 (4)		
		02111 (3)		
	6	11211 (18)	11112 (30)	11211 (30)
		11121 (5)		
		12111 (4)		
		21111 (4)		
C.V.= 1.00	4	01111 (13)	11110 (30)	01210 (30)
		11011 (5)		00301 (14)
		11101 (5)		
		11110 (5)		
	5	11111 (15)	11111 (30)	10301 (15)
		01121 (5)		10310 (15)
	6	11211 (9)	12111 (21)	10311 (30)
		21111 (8)	11211 (9)	
		11211 (6)		

5.2. BUFFER ALLOCATION FOR STATIONS WITH INVERTED BOWL SHAPED MEANS

In this experiment batches will be generated in a way that workloads of stations will increase up to the 3-rd station, then will decrease in the same fashion from the 4-th station down to the 6-th. This indicates an inverted bowl shaped mean distribution of workloads. Same simulations are done as in the previous section. Tables 4,5 and 6 show results for 3,4 and 5 batch cases respectively.

5.2.1. Results

5.2.1.1. C.V.=0.25

Design (11110) is dominant for 4 buffers, for 5 buffers (11111), (11210),(12110),(12110) are observed. For 6 buffers design (11211) is superior to (12111)

These results should be expected. Since the last station is fast (or has less workload). Blocking of the 5-th station will not be seen at all. So a buffer between 5-th and 6-th stations is less needed. So when 4 buffers are in hand, placing them at first 4 spaces is logical.

When there are 5 buffers in hand, place first 4 as explained above and put one extra in one of the front buffer allocation spaces. That is either between first and second or between second and third stations.

When there are 6 buffers now it is time to put 1 buffer between 5-th and 6-th. Otherwise imbalance will be high.

5.2.1.2. C.V.= 0.50

For 4 buffers (11110) is superior to others for all batch sizes.

For 5 buffers, designs (11111) and (12110) are present, for 8 and 10 jobs. For 12 jobs (11120) is superior to others.

For 6 buffers , for 3 and 4 batches (11211) and (12111) are seen but for 5 batch (12210) is superior.

5.2.1.3. C.V.= 0.75

For 4 buffers, optimal designs differ with varying job sizes. For 8 jobs besides (11110), now we can see designs like (02101), (02110) or (01210). For 10 jobs (02110) is the only design for all batch sizes. For 12 jobs for 3 batch and 5 batch cases (01111) is present besides (02110). For 4 batch (02110) is the only optimal design.

Here seeing 0 buffers at the first buffer space might be as the 1-st machine is faster than 2-nd, 2-nd machine will not starve at all.

For 5 buffers, for 8 jobs (11210) is superior to (11111) and (12110), for 10 jobs for 3 and 4 batch (21110) is superior to (03110) and (02120) but for 5 batch case (21110) is the only design. For 12 jobs (02111) is the only design for all batch sizes.

Coming to 6 buffers; for 8 jobs (11211) is superior to (12111) for all batch sizes. For 10 jobs, for 3 and 4 batch (22110) dominates (03120), and for 5 batch (22110) is the only design. For 12 jobs (02121) and (11211) are the optimal designs.

5.2.1.4. C.V.=1.00

For 4 buffers; for 8 jobs (02020) is the best design for all batch sizes. For 10 jobs (01210) and (02110) are both optimal for all batch sizes. For 12 jobs, for 3 and 4 batch (01210) dominates other designs and for 5 batch it is the only design.

For 5 buffers; for 8 jobs (02201) is the best design for all batch sizes. For 10 jobs (11210) is the only design for all batch sizes. For 12 jobs (11210) is the best design for all batch sizes.

For 6 buffers; for 8 and 10 jobs (12120) is the only design for all batch sizes. For 12 jobs while (11220) dominates (03111) for 3 and 4 batches, (11220) is the only design for 5 batch case.

5.2.2. Discussion:

For this kind of distribution it is hard to set a general design rule, various kinds of designs are observed for different combinations.

Table 4

Table 4: Stations with Inverted Bowl Shaped Means - 3 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	11110 (28) 11101 (3) 01111 (1)	11110 (15) 01111 (8) 01210 (3) 11200 (3)	11110 (23) 02110 (4) 01111 (3)
	5	12110 (11) 11210 (9) 11111 (7)	11111 (11) 12110 (11) 01211 (3) 12110 (3)	11210 (11) 11111 (7) 12110 (6)
	6	11211 (9) 12111 (8) 12210 (8)	11211 (12) 12210 (11) 12211 (5)	11211 (13) 12210 (9) 12111 (4)
C.V.= 0.50	4	11110 (21) 01111 (4) 02101 (3)	11110 (15) 02110 (13)	02110 (11) 11110 (6) 01120 (5) 11101 (5)
	5	11111 (16) 12110 (7) 11210 (5)	02120 (6) 03110 (6) 11111 (6) 11120 (6) 11210 (6) 12110 (6)	11120 (11) 11210 (6) 02120 (5) 11111 (5) 12110 (5)
	6	11211 (11) 12111 (11) 12210 (5)	12111 (12) 02211 (6) 02220 (6) 11211 (6) 12210 (6)	12111 (11) 12120 (6) 12210 (5) 11220 (5)
C.V.=0.75	4	11110 (9) 01210 (7) 02101 (7) 02110 (7)	02110 (30)	01111 (15) 02110 (14)
	5	11210 (15) 11111 (8) 12110 (7)	21110 (18) 03110 (12)	02111 (30)
	6	11211 (16) 11220 (8) 12111 (7) 12210 (7)	22110 (18) 03120 (12)	02121 (15) 11211 (15)
C.V.= 1.00	4	02020 (20) 01201 (8)	01210 (29) 02110 (29)	01210 (24) 11110 (6)
	5	02201 (23) 11120 (7)	11210 (30)	11210 (24) 11111 (6) 11120 (6)
	6	12120 (30)	12120 (30)	11220 (24) 03111 (6) 11211 (6) 12111 (6)

Table 5

Table 5: Stations with Inverted Bowl Shaped Means - 4 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	11110 (29) 11101 (3)	11110 (13) 01111 (11) 02110 (6)	11110 (24) 02110 (4) 01111 (2)
	5	11210 (11) 12110 (11) 11111 (8)	11111 (18) 11210 (6) 12110 (6)	11210 (11) 11111 (9) 12110 (5)
	6	12111 (11) 11211 (9) 12210 (7)	11211 (12) 12210 (12) 12111 (6)	11211 (12) 12210 (8) 12111 (6)
C.V.= 0.50	4	11110 (19) 01111 (5) 12100 (3)	11110 (18) 02110 (12)	11110 (12) 02110 (7) 01120 (6)
	5	11111 (22) 12110 (5) 11120 (3)	12110 (12) 11111 (8)	02120 (8) 11120 (8) 11210 (6)
	6	11211 (13) 12111 (11) 12120 (6)	12111 (14) 11211 (8) 02220 (6)	12210 (8) 12111 (8)
C.V.= 0.75	4	11110 (11) 02101 (10) 01210 (9)	02110 (28)	02110 (29)
	5	02111 (10) 02120 (10) 11120 (10) 11210 (10)	21110 (18) 02120 (6) 12110 (6)	02111 (30)
	6	11211 (10) 12111 (10) 12210 (10)	22110 (24) 03120 (6)	11211 (30)
C.V.= 1.00	4	02020 (28)	01210 (29) 02110 (29)	01210 (25) 02110 (6)
	5	02201 (30)	11210 (30)	11210 (24) 11120 (6)
	6	12120 (30)	12120 (30)	11220 (24) 03111 (6)

Table 6: Stations with Inverted Bowl Shaped Means - 5 batches

Coefficient of Variation	Number of Buffers	Job Size: 8 jobs	Job Size: 10 jobs	Job Size: 12 jobs
C.V.= 0.25	4	11110 (29)	11110 (24) 01111 (6)	11110 (24) 01111 (3)
	5	12110 (13) 11210 (10) 11111 (8)	11111 (16) 11210 (14)	11210 (12) 11111 (9) 12110 (6)
	6	12111 (12) 11211 (11) 12210 (6)	11211 (15) 12210 (8) 12111 (7)	11211 (13) 12210 (11)
c.v.= 0.50	4	11110 (16) 02110 (5) 11101 (5)	11110 (17) 02110 (13)	11110 (14) 01120 (9)
	5	12110 (13) 11111 (10) 11210 (7)	12110 (20) 11111 (10) 21110 (10)	11120 (10) 11111 (8) 02120 (6) 12110 (6)
	6	12210 (19) 12111 (6) 11121 (4)	11211 (10) 12111 (10) 12210 (10)	12210 (9) 12111 (8) 02220 (5)
C.V.= 0.75	4	11110 (14) 02110 (10)	02110 (30)	01111 (16) 02110 (14)
	5	11210 (15) 11111 (8) 12110 (3)	21110 (30)	02111 (30)
	6	11211 (15) 12210 (10) 12111 (5)	22110 (30)	02121 (15) 11211 (15)
C.V.=1.00	4	02020 (28)	02110 (30) 01210 (29)	01210 (30)
	5	02201 (30)	11210 (30)	11210 (30)
	6	12120 (30)	12120 (30)	11220 (30)

1. A center tendency is observed as total workload is high at center stations.

2. This center tendency is increasing with increasing C.V.

3. For small C.V.'s it is hard to find one dominant design in Tables 4,5 and 6.

4. Optimal designs are different for different job sizes, other factors being equal. This point makes it difficult to find out a robust design.

5.3. BUFFER ALLOCATION FOR STATIONS WITH BOWL SHAPED MEANS

In this experiment batches will be generated in a way that workloads of stations will decrease up to the 3-rd station, then will increase from the 4-th station to the 6-th. This is a bowl shaped mean distribution between the 6 stations of the flow line. Simulations over 30 iterations are done for finding optimal design for each iteration with varying C.V.'s, job sizes, and batch sizes. Tables 7,8,and 9 show results for 3,4 and 5 batch cases respectively.

5.3.1. Results

5.3.1.1. C.V.= 0.25

For 4 buffers (11011) is superior to other designs such as (01012) or (10111) for all batch sizes.

For 5 buffers (11012) is superior to (10112).

For 6 buffers (11112) is dominant over other designs for all batch sizes.

It is easy to see a simple design rule for this case. (11011) to (11012) and to (11112), with increasing buffer size.

5.3.1.2. C.V.=0.50

For 4 buffers; for 8 jobs designs such as (10111), (10012) and (11011) are seen in batch sizes 3,4 and 5. For 10 jobs (02011), (10111), (01012) are possible designs. For 12 jobs (10021) and (01012) are superior designs.

Here we can see as C.V. and job size increase more interesting and different designs can be seen. Designs that are further away from uniformness are observed.

For 5 buffers (10112) is superior to (11111) and (11012) for all batch sizes.

For 6 buffers; for 8 and 10 jobs (11112) is superior to other designs, but for 12 jobs (10113) and (10122) are better than others.

5.3.1.3. C.V.= 0.75

For 4 buffers; for 8 jobs (11011) is superior to (10012) for all batch sizes. For 10 jobs (02011) is dominant to other designs. For 12 jobs (30010) and (20110) are seen for all batch sizes.

For 5 buffers; for 8 jobs (11021), (10013), (10022) are possible designs. For 10 jobs while (21011) is superior to (10112) for 3 and 4 batch cases, (21011) is the only design for 5 batch. For 12 jobs (21020), (30110) and (40010) are seen for 3,4 and 5 batches.

For 6 buffers; for 8 jobs several designs are seen such as, (10122) (10023) and (11013). For 10 jobs (20112) is dominant over (10113) for 3 and 4 batches and (20112) is the only design for 5 batch case. For 12 jobs (21111) and (40110) are seen for 3 batch and (40110) is the only design for 4 and 5 batch cases.

5.3.1.4. C.V.= 1.00

For 4 buffers; (10111) is the optimal design for 8 jobs. For 10 jobs (02011) and (01111) are both seen. For 12 jobs (01111) is the optimal design.

For 5 buffers; design (11021) is superior to (02012) for 8 jobs. For 10 jobs (02111) is the only design. For 12 jobs (01112) is superior to (11111) for 3 and 4 batch and (01112) is the only design for 5 batch case.

For 6 buffers; design (12012) is superior to (11112). For 10 jobs (02112) is superior to (11211). For 12 jobs (11112) is the only design.

5.3.2. Discussion:

Allocation of buffers to buffer allocation spaces is similar to distribution of means between stations and a bowl shape is seen.

1. It is hard to find a single favorite design. Various kinds of bowl shaped designs are seen for different situations.

2. For C.V.=1.00 more uniform designs are seen than for C.V.=0.75. This can be explained as the central tendency found in Experiment I. This central tendency (because of high variation at C.V.=1.00) and, at the same time, totally contradictory bowl shaped mean distribution caused a more uniform distribution.

3. There are differences in designs for different job sizes and C.V. (though it is normal for varying C.V.) and it is hard to find one robust design for this type of mean distribution while a general tendency of bowl typed buffer allocation can be expected.

Table 7: Buffer Allocation for Stations with Bowl Shaped Means - 3 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	11011 (12) 01012 (8)	10111 (7) 11002 (7) 11011 (6)	11011 (10) 10012 (7) 10111 (6)
	5	11012 (14) 10112 (5)	10112 (13) 21002 (6) 11111 (5) 20111 (5)	11012 (15) 01112 (4) 10112 (4) 20111 (4)
	6	11112 (11) 21012 (10) 11022 (9)	11112 (13) 21102 (10) 10122 (8)	11112 (10) 11013 (5)
C.V.= 0.50	4	10111 (16) 10012 (7)	10111 (15) 02011 (6) 01012 (4)	01012 (8) 10021 (8) 01102 (5) 10102 (5)
	5	11012 (10) 20111 (7) 11111 (5)	11111 (12) 01013 (6) 10112 (6) 11012 (6)	10112 (20)
	6	11112 (8) 11013 (6) 20112 (6)	11112 (18) 01113 (6) 10113 (6)	10122 (11) 10113 (6) 11013 (5) 11112 (5)
C.V.= 0.75	4	11011 (14) 10111 (8) 10012 (7)	02011 (23) 10102 (6)	20110 (14) 30010 (14)
	5	10022 (8) 12011 (8) 11021 (7) 21011 (7)	21011 (18) 10112 (12)	21020 (15) 40010 (15)
	6	10122 (8) 11013 (8) 20022 (7) 21111 (7)	20112 (18) 10113 (6) 21012 (6)	21111 (15) 40110 (15)
C.V.= 1.00	4	10111 (21) 11011 (7)	02011 (15) 01111 (14)	01111 (30)
	5	11021 (23) 02012 (7)	02111 (30)	01112 (18) 11021 (6) 11111 (6)
	6	12012 (15) 11112 (8) 11121 (7) 12021 (7)	02112 (15) 11211 (15) 12111 (15)	11112 (30)

Table 8

Table 8: Buffer Allocation for Stations with Bowl Shaped Means - 4 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	11011 (11) 01012 (10) 00112 (5)	11011 (13) 10102 (5) 10111 (5) 11002 (5)	11011 (13) 10111 (8)
	5	11012 (15) 10112 (6)	10112 (12) 10202 (6) 11012 (6) 11102 (6) 11111 (6) 20111 (6) 21002 (6)	11012 (15) 10112 (5) 20111 (4)
	6	11112 (11) 21012 (6) 11022 (5)	11112 (18) 21102 (12)	11112 (13) 11022 (6) 20112 (4)
C.V.= 0.50	4	10111 (14) 11011 (10) 11101 (6)	10111 (19) 02011 (6) 01012 (4)	10021 (11) 01012 (7) 01102 (5)
	5	11111 (13)	10112 (18) 11012 (4) 11111 (4)	10112 (19) 10022 (8)
	6	11112 (17) 12111 (6) 20112 (5)	11112 (18) 10113 (8) 11013 (4)	10113 (9) 10122 (8) 11013 (5) 11112 (5)
C.V.= 0.75	4	10012 (10) 10111 (10) 11011 (10)	02011 (17) 01102 (6) 10102 (6)	20110 (30)
	5	10013 (10) 11021 (10) 11111 (10)	21011 (18) 10112 (12)	30110 (30)
	6	10023 (10) 21021 (10) 21111 (10)	20112 (24) 10113 (6) 21012 (6)	40110 (30)
C.V.=1.00	4	10111 (28)	01111 (29)	01111 (30)
	5	11021 (30)	02111 (30)	01112 (24) 11111 (6)
	6	12012 (30)	02112 (30)	11112 (30)

Table 9: Buffer Allocation for Stations with Bowl Shaped Means - 5 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	11011 (13) 01012 (8)	11011 (9) 10012 (8) 10102 (7) 10111 (7) 11002 (7)	11011 (14) 10111 (7)
	5	11012 (22) 01022 (3) 10112 (3)	11111 (8) 21101 (8) 10013 (7) 10103 (7) 10112 (7) 11003 (7)	11012 (14) 10112 (5) 11021 (3) 11111 (3)
	6	11112 (14) 10122 (4) 11022 (4) 21012 (4)	11112 (15)	11112 (12) 11022 (7) 21012 (4)
C.V.= 0.50	4	11011 (14) 10012 (12)	10111 (11) 02011 (8) 01012 (7)	01012 (10) 10021 (10) 01102 (5)
	5	11012 (16) 10112 (10)	10112 (10) 11012 (10) 11111 (10)	10112 (21) 10022 (6) 10121 (3)
	6	11022 (10) 12012 (7) 20022 (6) 20112 (6)	11112 (30)	10113 (12) 10122 (9) 11112 (7)
C.V.= 0.75	4	11011 (17) 10111 (9) 10012 (3)	02011 (29)	30010 (15) 20110 (14)
	5	11021 (15) 11012 (11)	21011 (30)	21020 (15) 40010 (15)
	6	11013 (8) 21021 (6) 11112 (5) 20112 (5)	20112 (30)	40110 (30)
C.V.= 1.00	4	10111 (29)	01111 (29) 02011 (15)	01111 (30)
	5	11021 (23) 02012 (7)	02111 (30)	01112 (30)
	6	12012 (15) 11112 (8) 21021 (7)	02112 (15) 11211 (15) 12111 (15)	11112 (30)

5.4. BUFFER ALLOCATION FOR STATIONS WITH UNIFORMLY DECREASING MEANS

In this experiment batches will be generated in a way that workloads of stations will decrease uniformly from the 1-st station up to the 6-th station. This is a logical way to locate the machines. As each machine is faster than the preeceding one, so no blockage will occur and system throughput will increase. We will try to find out a robust design for such a flow line. Tables 10, 11 and 12 show results for 3, 4 and 5 batch cases respectively.

5.4.1. Results

5.4.1.1 C.V.= 0.25

For 4 buffers (11110) is superior design for all batch sizes.

For 5 buffers (21110) is the superior design for all batch sizes.

For 6 buffers (22110) and (21111) are observed.

Here it is easy to see a pattern (11110) to (21110) and then to (22110) or (21111). This is the way means are distributed.

5.4.1.2. C.V.= 0.50

For 4 buffers; for 8 and 10 jobs (11110) is superior to other designs, whereas for 12 jobs (11101) can also be seen.

For 5 buffers; for 8 and 10 jobs (21110) is superior to other designs, but for 12 jobs (12101) is superior to (12110) for all batch sizes.

For 6 buffers; for 8 jobs (22110) and (21111) are best. For 10 jobs (21111) is dominant. For 12 jobs (12111) is dominant.

We know from previous experiments larger job size increases variability in the process, so causes a central tendency. Here (22110) design for 8 jobs turned to (12111) for 12 jobs because of the effect of this central tendency.

5.4.1.3. C.V.= 0.75

For 4 buffers; for 8 jobs (11110) design is superior to other forward leaned designs. For 10 jobs, for 3 batch (21100) is superior to (11110) and (21010), for 4 and 5 batches (21100) is the only design. For 12 jobs (21010) is the superior design for all batch sizes.

This can be interpreted as, variance decreases with increasing batch size, other factors being equal. In 3 batch besides (21100) more center preferred (11110) can also be seen which indicates higher variability. But for 4 and 5 batches this center preference influence weakens and (21100) is the only design.

For 5 buffers; for 8 and 10 jobs (21110) is the superior design, for 12 jobs (12110) is the superior design for all batch sizes.

For 6 buffers; for 8 jobs, for 3 and 4 batches (21120) and (31110) are superior, for 5 batch (21120) is dominant over (21111). For 10 and 12 jobs (22110) is optimal.

5.4.1.4. C.V.= 1.00

For 4 buffers; for 8 jobs, for 3 batch (12010) is dominant over (11110) whereas for 4 and 5 batches (12010) is the only design. For 10 jobs (03010) is the only design. For 12 jobs (11110) is the only design. For 12 jobs (11110) is the only design.

For 5 buffers; for 8 jobs, for 3 batch (12020) is dominant over other designs, whereas for 4 and 5 batches (12020) is the only design. For 10 jobs (12110) is the only design. For 12 jobs, for 3 batch (11210) is superior to (12020), is dominant over (12020) for 4 batch and is the only design for 5 batch case.

For 6 buffers; for 8 jobs, for 3 batch (12120) is dominant over (12111), for 4 and 5 batches it is the only design. For 10 jobs (12120) and (13110) are both optimal. For 12 jobs, for 3 and 4 batch (12111) is superior to (21210), but for 5 batch (12111) is the only design.

5.4.2. Discussion:

1. Here a buffer distribution consistent with the workload distribution is observed.

Table 10: Stations with Uniformly Decreasing Means - 3 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	11110 (14) 21100 (14) 11101 (6)	11110 (14) 11101 (8) 21100 (6)	11110 (21) 21100 (7)
	5	21110 (28) 21101 (5)	21110 (25) 21101 (6)	21110 (22) 12110 (4) 11111 (4)
	6	22110 (14) 21111 (12) 31110 (6)	21111 (15) 22110 (15) 22101 (9)	21210 (10) 21111 (9) 22110 (8)
C.V.= 0.50	4	11110 (17) 21100 (9) 21010 (4)	11110 (17) 21100 (6) 21010 (5)	11110 (11) 11101 (8) 12100 (6)
	5	21110 (22) 12110 (5) 21101 (3)	21110 (18) 11111 (6) 12110 (6)	12101 (14) 12110 (8) 21110 (6)
	6	22110 (10) 21111 (5) 22101 (5)	21111 (18) 12111 (6) 12120 (6) 22110 (6)	12111 (11) 21210 (6) 31200 (6)
C.V.= 0.75	4	11110 (9) 12010 (7) 21010 (7) 21100 (7)	21100 (18) 11110 (6) 21010 (6)	21010 (29)
	5	21110 (22) 12110 (8)	21110 (24) 22010 (6)	12110 (15) 22010 (15)
	6	21120 (8) 31110 (8) 21210 (7) 22110 (7)	22110 (30) 21210 (6)	22110 (30)
C.V.= 1.00	4	12010 (20) 11110 (7)	03010 (29)	11110 (30)
	5	12020 (23) 11111 (7) 11120 (7)	12110 (30)	11210 (12) 12020 (6) 12101 (6) 21110 (6)
	6	12120 (23) 12111 (8) 12210 (7)	12120 (15) 13110 (15)	12111 (24) 12120 (12) 21210 (6)

Table 11: Stations with Uniformly Decreasing Means - 4 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	11110 (17) 21100 (13) 11101 (4)	11110 (18) 21100 (10)	11110 (25) 21100 (3)
	5	21110 (30)	21110 (18) 21101 (6) 22100 (6)	21110 (24) 12110 (4)
	6	22110 (16) 21111 (8)	21111 (12) 22110 (12) 22101 (6)	21111 (12) 21120 (6) 22110 (5)
C.V.= 0.50	4	11110 (24) 21100 (4)	11110 (18) 11101 (6) 21010 (4)	11101 (11) 12100 (8) 11110 (6)
	5	21101 (9) 21110 (9) 12110 (5)	21110 (22) 11111 (6)	12101 (13) 12110 (10)
	6	21111 (18) 22110 (12)	21111 (20) 21120 (4)	12111 (13) 31200 (9)
C.V.= 0.75	4	11110 (19) 12010 (10)	21100 (29)	21010 (30)
	5	21110 (20) 22010 (10)	21110 (30)	12110 (30)
	6	21120 (10) 31110 (10) 32010 (10)	22110 (30)	22110 (30)
C.V.= 1.00	4	12010 (28)	03010 (29)	11110 (30)
	5	12020 (30) 12120 (30)	12110 (30)	11210 (24) 12020 (6)
	6	12120 (30)	12120 (30)	12111 (18) 21210 (12)

Table 12: Stations with Uniformly Decreasing Means - 5 batches

Coefficient of Variation	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
C.V.= 0.25	4	11110 (19) 21100 (11) 11101 (4)	11110 (16) 21100 (8) 11101 (6)	11110 (27) 21010 (2)
	5	21110 (30)	21110 (30)	21110 (24) 11111 (3) 12110 (3)
	6	22110 (15) 21111 (14)	21111 (22) 22110 (16)	21111 (10) 22110 (9) 21210 (5)
C.V.= 0.50	4	11110 (15) 11101 (9) 21100 (7)	11110 (11) 11101 (9) 21010 (7)	11101 (13) 11110 (10) 12100 (4)
	5	21110 (19) 12110 (7) 21101 (4)	21110 (30)	12101 (11) 12110 (8) 21110 (6)
	6	22110 (12) 21111 (7) 21210 (6)	21111 (30)	12111 (13) 31200 (6) 21210 (5)
C.V.= 0.75	4	11110 (11) 21100 (8) 12010 (7)	21100 (30)	21010 (29)
	5	21110 (27) 22010 (3)	21110 (30)	12110 (30)
	6	21120 (17) 21111 (5)	22110 (30)	22110 (30)
C.V.= 1.00	4	12010 (28)	03010 (29)	11110 (30)
	5	12020 (30)	12110 (30)	11210 (30)
	6	12120 (30)	12120 (15) 13110 (15)	12111 (30)

2. Increasing C.V. brings center preference influence to buffer design. For example, for job size 12 (21110) is best for C.V.=0.25, where (11210) is best for C.V.=1.00, for 3 batch.

3. Increasing job size brings center preference influence. For example (21110) is best for C.V.= 0.75, job 8, where (12110) is optimal for 12 jobs.

5.5. BUFFER ALLOCATION FOR STATIONS WITH IDENTICAL MEANS AND DECREASING C.V.'S

In this experiment batches will be generated in a way that workloads of stations will be constant over the entire flow line but coefficient of variation will decrease through the line from C.V.=1 from the first station, to 0.85 at second, 0.70 at 3-rd, 0.55 at 4-th, 0.40 at 5-th, and 0.25 at the 6-th station. Results are at Table 13.

5.4.1. Results

For 4 buffers (11110) is dominant for all batch sizes.

For 5 buffers; for 8 jobs (11111) and (12110) is superior, for 10 jobs (21110) is dominant over (12110) for all batch sizes, for 12 jobs (21110) is the only design for all batch sizes.

For 6 buffers; for 8 jobs, for 3 batch (11211) is superior to (12111), for 4 batch (11211), (12111) and (21210) are all equal. For 5 batch (12111) and (21210) are equal. In this case, we can see clearly that, with increasing batch size central tendency influence decreases. For 3 batch (11211) is superior to (12111), but for 5 batch there is no more (11211), but rather there is (21210) besides (12111).

For 10 jobs, for 3 batch (22110) is superior to (21111), for 4 batch they are equal, for 5 batch (22110) is again superior to (21111). For 12 jobs (12210) and (21111) are both observed.

5.5.2. Discussion

In this experiment it is easy to see, more variance attracts more buffer space. Buffer allocation is in a decreasing fashion through the spaces as C.V. decreases throughout the flow line.

Table 13: Stations with Identical Means and Decreasing C.V.'s

Number of Batches	Number of Buffers	Job Size		
		8 jobs	10 jobs	12 jobs
5	4	11110 (30)	11110 (30)	11110 (30)
	5	11111 (15) 12110 (15)	21110 (21) 12110 (9)	21110 (30)
	6	12111 (15) 21210 (15)	22110 (22) 21111 (8)	12210 (15) 21111 (15)
4	4	11110 (30)	11110 (30)	11110 (30)
	5	11111 (12) 12110 (12) 21110 (6)	21110 (22) 12110 (8)	21110 (30)
	6	11211 (6) 12111 (6) 12120 (6) 21210 (6) 22110 (6)	21111 (13) 22110 (13) 11211 (4)	21111 (30)
3	4	11110 (27) 11011 (3)	11110 (22) 21100 (8)	11110 (30)
	5	11111 (9) 12110 (9) 21110 (6)	21110 (17) 12110 (9) 11120 (4)	21110 (30)
	6	11211 (9) 12111 (6) 22110 (6)	22110 (18) 21111 (8) 12111 (4)	12210 (15) 21111 (15)

Another point can be detected. For 5 buffers, 4 batches (11111) and (12110) are possible for 8 jobs, but (21110) is the only design for 12 jobs. So more jobs cause more forward leaned designs. Another example is, (11111) and (12110) are seen for 8 jobs for 5 batch but (21110) is the only design for 12 jobs.

Findings in this experiment are consistent with the one's Conway et al. (1988) found for single product flow lines. They proposed that workstations with large variability of processing times should have larger buffers for both input and output, increasing the variability of one station increase the total number of buffer spaces required to attain a target output rate.

5.6. BUFFER ALLOCATION FOR STATIONS FACING VARYING SCHEDULES IN THE CYCLIC PRODUCTION POLICY

These 3 experiments are designed for a 4 batch, 10 job flow line. This time no new matrices will be generated for each iteration, rather the rows of the matrix are interchanged at each iteration. That is the order of the jobs entering the flow line is changed at every iteration. So at every iteration a new sequence of jobs is formed and an optimal design for this sequence of jobs is found. C.V. again varies fom 0.25 to 1.00, buffer size varies from 4 to 6, and 30 such iterations are performed for every combination.

Aim of this experiment is to find whether it is possible to find robust designs that will not be affected from sequence changes.

5.6.1. STATIONS WITH IDENTICAL WORKLOADS

This experiment will be performed on a flow line with stations having identical workloads. That is identical means for columns of the batch matrix. Table 14 shows results for this experiment.

5.6.1.1. Results

5.6.1.1.1. C.V.= 0.25

For 4 buffers (11110), (01111), (11101) are all seen. For 5 buffers (11111) is the only design. For 6 buffers (11121), (11211), (11112) are seen.

5.6.1.1.2. C.V.=0.50

For 4 buffers (01111) (11110) are seen dominant. For 5 buffers (11111) is the dominant design. For 6 buffers (11211), (11121), (12111) are seen.

5.6.1.1.3. C.V.= 0.75

For 4 buffers (01111), (10111), (11011), (11110) are seen. For 5 buffers (11111) is the dominant design. For 6 buffers (11211) is superior to other designs.

5.6.1.1.4. C.V.=1.00

For 4 buffers (01111), (11110), (02020) are seen. For 5 buffers (11111) is no more dominant, other designs like (02111) are also seen. For 6 buffers (12111), (11121), (11211) are seen.

5.6.1.2. Discussion:

In this experiment a robust pattern can be seen.

1. For 4 buffers (01111) is superior.

Table 14: Buffer Allocation for Stations Facing Varying Schedules - Identical Workloads

Coefficient of Variation	Number of Buffers	Observed Designs
C.V.= 0.25	4	11110 (9) 01111 (7) 11101 (7)
	5	11111 (30)
	6	11121 (9) 11211 (7) 11112 (6)
C.V.= 0.50	4	01111 (9) 11110 (8)
	5	11111 (26)
	6	11211 (8) 11121 (7) 12111 (7)
C.V.= 0.75	4	01111 (5) 10111 (5) 11011 (5) 11110 (5)
	5	11111 (24)
	6	11211 (14) 12111 (6)
C.V.= 1.00	4	01111 (9) 11110 (8) 02020 (4)
	5	11111 (16) 02111 (4)
	6	12111 (9) 11121 (8) 11211 (5)

2. For 5 buffers (11111) is superior.

3. For 6 buffers (11211) is superior.

4. For 5 buffers (11111) is the only design for C.V.=0.25, but for C.V.=1.00 is optimal in 16 iterations over 30. Here center tendency influence can also be seen.

5.6.2. STATIONS WITH DECREASING WORKLOADS

This experiment is again for a 4 batch 10 job flow line. This time workloads of stations are decreasing unifomly (means of the columns of the batch matrix are decreasing uniformly) through the flow line. Table 15 shows results for this experiment.

5.6.2.1. Results

5.6.2.1.1. C.V.= 0.25

For 4 buffers (11110) and (21100) are superior. For 5 buffers (21110) is superior. For 6 buffers (22110) is superior to (31110).

5.6.2.1.2. C.V.= 0.50

For 4 buffers (11110) is superior. For 5 buffers (21110) is superior to (12110). For 6 buffers (21111) and (22110) are superior.

5.6.2.1.3. C.V.=0.75

For 4 buffers (11110) is superior. For 5 buffers (21110) is superior to (12110). For 6 buffers (22110) and (12111) are both observed.

5.6.2.1.4. C.V.=1.00

For 4 buffers (11110) is superior. For 5 buffers (21110) and (12110) are seen. For 6 buffers (21120) is superior to (12120) and (21111).

5.6.2.2. Discussion:

For 4 buffers (11110), for 5 (21110) are superior. For 6 buffers (22110) is robust for small C.V. (21120) is best for C.V.=1.00.

Table 15: Buffer Allocation for Stations Facing Varying Schedules - Decreasing Workloads

Coefficient of Variation	Number of Buffers	Observed Designs
C.V.= 0.25	4	11110 (11) 21100 (11) 11101 (5)
	5	21110 (20) 11111 (4) 21101 (4)
	6	22110 (19) 31110 (4) 32100 (3)
C.V.= 0.50	4	11110 (25) 11101 (5)
	5	21110 (21) 12110 (6)
	6	21111 (10) 22110 (8) 12111 (4)
C.V.= 0.75	4	11110 (20)
	5	21110 (13) 12110 (6) 11120 (3)
	6	22110 (7) 12111 (5) 12120 (4) 21111 (4)
C.V.= 1.00	4	11110 (22)
	5	21110 (10) 12110 (7) 11111 (4)
	6	21120 (8) 12120 (5) 21111 (5)

Here we can see from table 15 that as C.V. increases things get complicated and a center tendency influence appears. For example (22110) is dominant at C.V.=0.25 and (21120) at C.V.=1.00.

5.6.3. STATIONS WITH INVERTED BOWL SHAPED WORKLOADS

This experiment is again for a 4 batch 10 job flow line. This time workloads of stations are in an inverted bowl pattern through the flow line. Table 16 shows results for this experiment.

5.6.3.1. Results

5.6.3.1.1. C.V.=0.25

For 4 buffers (02110) is superior to (01111). For 5 buffers (11210) is superior to (12110). For 6 buffers (12210) is dominant.

5.6.3.1.2. C.V.= 0.50

For 4 buffers (11110) is dominant. For 5 buffers (12110) is superior to (11210). For 6 buffers (11211) is superior to

(12111).

5.6.3.1.3. C.V.= 0.75

For 4 buffers (11110) is superior to (01111). For 5 buffers (11111), (11210), and (02111) are seen. For 6 buffers (11211) and (12111) are both observed.

5.6.3.1.4. C.V.= 1.00

For 4 buffers (02110) is superior to (11110). For 5 buffers (11111), (02111) and (02120) are seen. For 6 buffers (11211) is superior to (12120).

5.6.3.2. Discussion

Here we see no design is robust to schedule changes for this type of mean distribution. One design that is best for one C.V. perform inferior for another C.V. But a general center tendency that suits the mean distribution can be seen.

Table 16: Buffer Allocation for Stations Facing Varying Schedules - Inverted Bowl Shaped Workloads

Coefficient of Variation	Number of Buffers	Observed Designs
C.V.= 0.25	4	02110 (15) 01111 (7) 11110 (5)
	5	11210 (13) 12110 (11) 11111 (4)
	6	12210 (21)
C.V.= 0.50	4	11110 (25)
	5	12110 (12) 11210 (8) 11111 (5)
	6	11211 (9) 12111 (7) 12120 (6) 12210 (6)
C.V.= 0.75	4	11110 (11) 01111 (6) 01210 (4)
	5	11111 (6) 11210 (5) 02111 (5)
	6	11211 (8) 12111 (8) 12120 (4) 12210 (4)
C.V.= 1.00	4	02110 (14) 11110 (8)
	5	11111 (5) 02111 (5) 02120 (5)
	6	11211 (12) 12120 (8)

5.6.4. Overall Discussion For This Experiment

In this experiment robustness of designs to schedule changes are tested. There are no robust designs to varying schedules. Various designs that suit the workload distribution of stations are seen but no one is dominant against others, especially when C.V. is high.

5.7. BUFFER ALLOCATION FOR MULTI-BATCH FLOW LINES WHERE BATCHES HAVE UNEQUAL WEIGHTS

So far our experiments considered flow lines producing 3,4 or 5 batches in cyclic production policy. For allocating buffers between stations the importance of these batches was assumed to be equal. So no one batch was more important than others.

This phenomenon can be interpreted in the following way too. Consider a flow line that will produce 3 sets of jobs (3 batches) continuously. But the probability of producing a set may not be equal to others. For example one set is going to be produced 10 times while other will be produced 5, and the last 2 times. Then the first set is 2 times more important than the second in buffer allocating. Buffer allocation that will maximize the throughput of set 1 will be more important for the factory manager, than another design that will maximize the throughput of set 2 or 3.

Therefore, there will be a compromise, giving weights to the batches in calculating the objective function to allocate buffers. So far these weights were equal in our experiments.

In this experiment a 3 batch, 10 job flow line is considered. 3 batches will be generated and different weights will be given to each batch at each iteration. These weights will be from (1-1-1) up to (3-2-1),covering all combinations. There are 19 such combinations, so this experiment will be over 19 iterations.

This experiment is done for 4 times to get 4 different -3 batch- sets. Results are presented at Table 17.

5.7.1. Results

5.7.1.1. C.V.= 0.25

For 4 buffers; in two of the 4 experiments (11110) is the only design with 19 over 19 iterations. In one experiment (01111) is the only design and in the last one (11110) is dominant over others.

For 5 buffers (11111) is the only design for all four experiments.

For 6 buffers (11121) is optimal for one experiment, (12111) is optimal for another, (11211) is optimal for one other and for the last one (11211) is dominant over (12111).

5.7.1.2. C.V.= 0.50

For 4 buffers, in one experiment (11011) is optimal, in one (11011) is dominant over (02011), in one (11101) is optimal, in the last one (01111), (01120), (10120) are all seen.

For 5 buffers (11111) is the only design for 2 experiments, (02111) is the only design for one experiment, (01121), (01211) are both optimal for the last one.

For 6 buffers (12111) is dominant in one experiment, (21111) in another and in the remaining two (11211) and

(11121) are superior.

5.7.1.3. C.V.= 0.75

For 4 buffers (10201) is optimal in one experiment, (01111) in another, (02020) is superior in the 3-rd, and (02011) and (11020) are superior in the 4-th.

For 5 buffers (11111) is the only design in experiments 1 and 2; (11201) is the only design for experiment 3, (11102) is superior to others in experiment 4.

For 6 buffers (11211) is the only design with 19 over 19 in two experiments, (21102) is superior in one experiment and (12111) is superior in the other.

5.7.1.4. C.V.= 1.00

For 4 buffers (01201) is superior in 2 experiments (10120) is superior in other in one experiment and (02020) is the only design in one experiment.

For 5 buffers (02111) is optimal with 19 over 19 in two experiments. (11111) is optimal in one experiment and (11111) is superior in the last experiment.

For 6 buffers (12111) is dominant in two experiments, (11211) is optimal in one experiment. (11211) and (12111) are both optimal in one experiment.

5.7.2. Discussion:

Looking at table 17, it is easily seen that most solutions rank 17, 18 or 19 over 19. That is they are close to optimal or just optimal. Weak designs ranking 5,6 7 over 19 are few, so we can say that designs are robust to weight changes.

But this has a simple explanation. As three batches are similar (although means are different, mean distributions are uniform for all these), giving more weight to one other batch does not make so much difference. So we get more or less same results for this experiment, that we got for experiment set 1, that is stations with identical workloads and batches have identical weights.

So here is a valuable conclusion, when there is ambiguity in production probabilities in a multi-batch flow line, buffer allocation is robust and there is no need to determine the exact weights of the batches,if batches have the same workload distribution. A robust design is possible only with some information on production scenarios.

Table 17: Buffer Allocation for Multi Batch Flow Lines Where Batches Have Unequal Weights

Coefficient of Variation	Number of Buffers	Experiment no I	II	III	IV
C.V.= 0.25	4	11110 (19)	01111 (19)	11110 (12) 02101 (6) 11101 (6)	11110 (19)
	5	11111 (19)	11111(19)	11111(19)	11111 (19)
	6	11121 (19)	11211 (14) 12111 (4)	11211 (19)	12111 (19)
C.V.= 0.50	4	01111 (7) 01120 (7) 10111 (5) 10120 (5)	11011 (18)	11101 (17)	11011 (12) 02011 (7)
	5	02111 (19)	11111 (19)	11111 (19)	01121 (19) 01211 (19)
	6	21111 (18)	12111 (18)	11121 (14) 12111 (5) 21111 (5)	11211 (12) 11112 (7)
C.V.= 0.75	4	01111 (19)	02011 (14) 11011 (14)	10201 (18)	02020 (12) 01021 (7)
	5	11111 (19)	11111 (19)	11201 (19)	11102 (11) 11021 (6) 11120 (2)
	6	11211 (19)	12111 (13) 21111 (6)	11211 (19)	21102 (10) 11112 (7) 21021 (2)
C.V.= 1.00	4	01201 (12) 02110 (7)	01201 (13) 02110 (6)	02020 (19)	10120 (12) 00121 (7)
	5	02111 (19)	02111 (19)	11111 (19)	11111 (13) 21110 (6)
	6	12111 (15) 02211 (4)	12111 (15) 02211 (4)	11121 (19) 11211 (19)	11211 (19)

6. CONCLUSION

The design of production line systems has been studied in the research literature with the primary focus on how to improve their efficiency. Considering large costs associated with production lines, a slight improvement in efficiency can lead to very significant savings over the life of the flow line. Division of work among stations and allocation of buffers between stations are critical design factors that have attracted the attention of many researchers.

Most researchers considered flow lines producing single products but in our time flow lines have to be able to produce multiple products.

Aim of this study is to find robust designs for flow lines facing multiple demand scenarios. Findings of this study can be stated as design guidelines for a flow line designer.

Our major findings are:

1. Workload distribution of stations are extremely important. It is impossible to find a design that is robust

to workload distribution.

There is one to one correspondence between, buffer distribution between stations and workload distribution of stations. Uniform workload distribution brings uniform buffer allocation, bowl shaped workload distribution brings bowl shaped buffer allocation and so on.

So the first thing a designer must do is to determine the workload distribution of stations with respect to demand scenarios.

If a designer determines the workloads of stations or arrange them into a pattern like bowl shape, uniform, or decreasing, buffer design will be obvious. It will follow the same pattern as the workload distribution. For fine tuning of the design he/she should follow the following steps.

2. After step 1, another point is the robustness of the buffer designs to the variations of workloads of stations. Buffer designs do not seem to be robust to changes in the coefficient of variation. Increase in variation brings center preference to designs.

So a designer should determine the variation of processing times of stations before deciding on design.

3. Then comes the robustness of designs to varying job sizes. Designs are not robust to job size changes. Larger job sizes within a production batch seem to increase variability in the system and cause center preference influence.

So a designer will fine tune his buffer design according to the number of jobs within batches of demanded end products.

4. Here is a good news for the designer. Buffer allocation seem to be totally robust to batch size variance. Whether there are 3, 4, or 5 different batches to be processed does not make a differrence, if batches have similar workload distributions.

5. If the variation of processing times of stations are not equal, more variant stations attract more buffers. So a designer should determine variations of processing times of stations and make his design accordingly.

6. Coming to robustness to scheduling desicions of cyclic scheduling policy, unfortunately buffer allocation is not robust to varying schedules. Different schedules result in different designs.

7. Buffer allocation is robust to changes in weights of batches on objective function. That is, having the same type of workload distribution and variance, different weights of batches resulted in the same designs.

This fact is obviously related to the robustness to batch size variance. That is a batch having more weight than others, behaves like there are more batches in the system. For example, if weights of batches are W^1=2, W^2=1 and W^3=1 then this system behaves like a 4 batch system because batches are identical in workload distribution and variance.

So, finally the point is that buffer allocation depends totally on workload distribution of stations. The two are in the same pattern. Other variables in the system like C.V., job size are just for fine tuning of the design.

To help buffer designers, performances of the two heuristics developed by Karabatì and Kouvelis, are tested. It is found that in most cases their performances lie within 1 % error margin. So for large size problems buffer designer need not to find optimal solutions, rather greedy and dynamic heuristics can be utilized. Sample results are exhibited in Table 18.

In conclusion, buffer design, although may sound trivial at first, is a complex problem and finding good solutions to this problem may increase efficiency and so decrease costs, that is vital to be able to achieve a competitive position in the industry.

Table 18 : Average percentage deviations of performances of two heuristics from the performance of optimal designs over 30 problems

- Stations with Identical Means - 3 batch case

Coefficient of Variation	Number of Buffers	Job Size	Heuristic Greedy	Heuristic Dynamic
C.V.= 0.25	4	8	0.552	0.195
		10	0.643	0.077
		12	0.413	0.055
	5	8	0.424	0
		10	0.127	0
		12	0	0
	6	8	0.061	0.158
		10	0.028	0.104
		12	0	0.257
C.V.= 0.50	4	8	1.016	0.258
		10	1.094	0.487
		12	0.848	0.491
	5	8	0.5	0.085
		10	0.27	0.231
		12	3.778	0
	6	8	0	0.238
		10	0.197	0.403
		12	0.173	0.07
C.V.= 0.75	4	8	0.972	0.195
		10	1.847	2.526
		12	0.997	1.296
	5	8	0.675	0
		10	1.919	0
		12	1.588	0
	6	8	0.061	0.048
		10	0.031	0.289
		12	0.089	0.293
C.V.= 1.00	4	8	0.666	0.356
		10	3.165	0
		12	1.707	1.933
	5	8	0.528	0.635
		10	1.671	2.696
		12	0.415	1.517
	6	8	0.151	0.551
		10	0	0.832
		12	0.547	0.739

REFERENCES:

1. Karabatì, S., and Kouvelis, P. "The Interface of Buffer Design and Cyclic Scheduling Desicions in Deterministic Flow Lines", forthcoming, Annals of Operations Research.

2. Conway, R., Maxwell, W., McClain, J.O., Thomas, L.J. "The Role of Work-in-Process Inventory in Serial Production Lines. Operations Research, Vol. 36, No. 2, pp. 229-241, 1988.

3. Hillier, F.S., and So, K.C. "The Effect of the Coefficient of Variation of Operation Times on the Allocation of Storage Space in Production Line Systems". IIE Transactions, Vol. 23, No. 2, pp 198-206, June 1991.

4. Hillier, F.S., Boling , R.W., and So, K.C. "Toward Characterizing the Optimal Allocation of Storage Space in Production Line Systems with Variable Operation Times. Working paper, Stanford University Department of Operations Research, March 1990.

5. Hitz, K.L. "Scheduling of Flexible Workshops".

Technical Report, MIT, Cambridge, MA, 1979.

6. Kouvelis, P., Kurawarwala, A., and Gutierrez G. "Algorithms for Robust Single and Multiple Period Layout Planning for Manufacturing Systems". June 1991.

7. Yamashina, H. and Okamura, K. "Analysis of in-Process Buffers for Multi-Stage Transfer Line Systems". International Journal of Production Research, Vol. 21, No. 2, pp. 183-195, 1983.

8. Rocke, D. M. "Robust Control Charts". Technometrics, Vol. 31, No. 2, pp. 173-184, May 1989.

9. So, K. C. "The Impact of Buffering Straregies on the Performance of Production Line Systems". International Journal of Production Research, Vol. 28, No. 12, pp. 2293-2307, 1990.

10. Iskander, W. H. and Chou, J. "Unbalanced Production Line Scheduling With Partial Job Specialization". Naval Research Logistics, Vol. 37, pp. 789-805, 1990.

11. Posner, M. J. M. and Berg, M. "Analysis of a Production-Inventory System with Unreliable Production Facility". Operations Research Letters, Vol. 8, pp. 339-345, 1989.

12. Rosenblatt, M. J. and Carlson, R. C. "Designing a Production Line to Maximize Profit". IIE Transactions, Vol. 17, No. 2, pp. 117-122, 1985.

13. Wemmerlöv, U. and Vakharia, A. J. "Job and Family Scheduling of a Flow-Line Manufacturing Cell: A Simulation Study". IIE Transactions, Vol. 23, No. 4, pp. 383-393, 1991.

14. Cao, X. R. and Ho, Y. C. "Sensitivity Analysis and Optimization of Throughput in a Production Line with Blocking". IEEE Transactions on Automatic Control, Vol 32, No 11, pp 959-967, 1987.

15. Villa, A., Fassino, B. and Rossetto, S. "Buffer Size Planning Versus Transfer Line Efficiency". Journal of Engineering for Industry, Vol. 108, No. 2, pp. 105-112, 1986.

16. Smith, J. M. and Daskalaki S. "Buffer Space Allocation in Automated Assembly Lines". Operations Research,

Vol. 36, No. 2, pp 343-358, 1988.

17. Kackar, R. N. and Shoemaker, A. C. "Robust Design: A Cost-Effective Method For Improving Manufacturing Process". AT&T Technical Journal, Vol. 65, No. 2, pp. 39-50, 1986.

18. Lishchinskii, L. Y. "Planning a Cyclic Production Process in a Flexible Manufacturing System". Soviet Engineering Research, Vol. 6, No. 7, pp. 44-49, 1986.

19. Philipoom, P. R. and Fry, T.D. "The Robustness of Selected Job-Shop Dispatching Rules with Respect to Load Balance and Work-Flow Structure". Journal of the Operational Research Society, Vol. 41, No. 10, pp. 897-906, 1990.

20. Le, D. "A Fast and Robust Unconstrained Optimization Method Requiring Minimum Storage". Mathematical Programming, Vol 32, No 1, pp 41-68, 1985.

21. Fanti, M. P., Maione, B., Semeraro, Q., and Turchiano, B. "Assessing the Operational Analysis Robustness of Flexible Manufacturing Systems Modeling". International Journal of Systems Science, Vol. 19, No. 11, pp. 2381-2394,

1988.

22. Zheng, Q. "Robust Analysis and Global Optimization". Computers & Mathematics with Applications, Vol.21, No. 6-7, pp 17-24, 1991.

23. Rosenblatt, M.J., and H.L. Lee. "A Robustness Approach to Facility Design". International Journal of Production Research, Vol. 25, No. 4, pp 474-486, 1987.