JOINT ECONOMIC LOT-SIZING APPROACH TO THE JUST-IN-TIME PURCHASING PROBLEM

A THESIS
SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL
ENGINEERING
AND THE INSTITUTE OF ENGINEERING AND SCIENCES
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

By
Ihsan Durusoy
February, 1993

JOINT ECONOMIC LOT-SIZING APPROACH TO THE JUST-IN-TIME PURCHASING PROBLEM
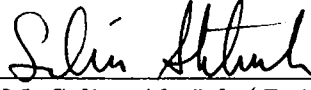
A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL

ENGINEERING

AND THE INSTITUTE OF ENGINEERING AND SCIENCES

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

İhsan Durusoy

February, 1993

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.
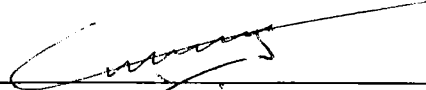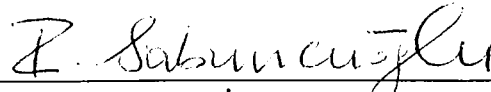
_____
Assist. Prof. M. Selim Aktürk (*Principal Advisor*)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

_____
Assoc. Prof. Cemal Dinçer

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

_____
Assist. Prof. İhsan Sabuncuoğlu

Approved for the Institute of Engineering and Sciences:

_____
Prof. Mehmet Baray
Director of Institute of Engineering and Sciences

ABSTRACT

JOINT ECONOMIC LOT-SIZING APPROACH TO THE JUST-IN-TIME PURCHASING PROBLEM

İhsan Durusoy

M.S. in Industrial Engineering

Supervisor: Assist. Prof. M. Selim Aktürk

February, 1993

One of the important concepts of JIT philosophy is the high frequency with small lots in the delivery process. However, this issue is settled between the purchaser and vendor depending on the existing balance of power. The result of such decisions could end with ordering policies, not suitable for JIT logic and place some disadvantages to one of the parties or both. Additionally, these policies have not considered the effect of transportation cost on the optimal ordering and shipment size quantities; despite the fact that purchased materials must bear transportation charges.

This paper develops joint economic lot-size model under deterministic conditions, focusing on the shipment size and its effect to the joint total cost, which also includes transportation cost. For that purpose, the joint model is arranged according to the shipment size. Then a computational analysis is made between each parties shipment size policy with the joint model. Consequently, a full factorial design is generated with four factors at three levels. By using the analysis of variance, the effects of the factors on the joint total cost are investigated.

Key words: Joint Economic Lot-size Model, Just-In-Time Purchasing

ÖZET

TAM ZAMANINDA SATINALMA PROBLEMİNE ORTAK EKONOMİK KAFİLE BÜYÜKLÜĞÜ YAKLAŞIMI

İhsan Durusoy
Endüstri Mühendisliği Bölümü Yüksek Lisans
Tez Yöneticisi: Yrd. Doç. Dr. M. Selim Aktürk
Şubat, 1993

Tam zamanında üretim felsefesinin en önemli kavramlarından biride, teslimat işleminin küçük kafilelerle yoğun olarak yapılmasıdır. Bununla birlikte bu konu satınalan ve satıcı arasındaki güç dengesine göre belirlenir. Bu tür kararların sonuçları ile ortaya çıkan sipariş politikaları çoğu zaman tam zamanında üretim mantığına uygun düşmemekte ve her iki tarafa belli kayıplar verdirmektedir. Buna ek olarak, bu politikalar taşıma maliyetinin en iyi sipariş ve taşıma büyüklüğü miktarına olan etkisini satınalınan mallara taşıma masrafının yüklenmesi gerçeğine rağmen, gözönüne almamaktadırlar.

Bu çalışmada taşıma büyüklüğü ve maliyetinin ortak toplam maliyetine etkisini belirlenmiş durumlar için görmek amacıyla ortak ekonomik kafile büyüklüğü modeli geliştirilmektedir. Bu nedenle ortak model, taşıma büyüklüğüne göre ayarlanmaktadır. Ardından her iki tarafın taşıma büyüklüğü politikaları ortak model ile maliyet açısından karşılaştırılmaktadır. Buna ek olarak, dört faktörlü üç seviyeli tam faktörel planlama yapılmaktadır. Standart sapma analizi kullanılarak bu faktörlerin ortak toplam maliyete olan etkisi incelenmektedir.

Anahtar sözcükler. Tam Zamanında Üretim Satınalması, Ortak Ekonomik Kafile Büyüklüğü.

To my mother and father

ACKNOWLEDGEMENT

Contents

List of Figures

List of Tables

Chapter 1

INTRODUCTION

Recent developments, particularly the success of Japan in the world markets and feeling that this success has been derived, to a significant degree, from superior production system have changed people's perception of the role and importance of production and production management in the industrial firm.

The firm acquires raw materials and component parts from outside suppliers and stores them until needed in the production. The function of managing purchasing and associated purchased material inventories, and procurement provides the input to the production system, which consists of production centers that process the raw materials and component parts into finished products. The capacity of a production center is determined by the manpower and facilities comprising that center. Each finished product has one or more more routings by which it can be produced. At a production center, operations are performed that utilize a certain amount of the capacity of the center for each unit of product processed. Materials being processed or waiting to be processed comprise in-process inventory. Subcontractors may supply semifinished or finished products to augment the internal capacity of the firm.

Finished products may be inventoried in regional warehouses as well as at the plant where production takes place. The management of the quantity and location of finished goods inventory is a part of the function of distribution.

1

Products leave the system to satisfy customer demand. The customer may be a consumer, a retailer, a wholesaler, or another manufacturer.

Three attributes of material flow are of primary interest to production management: quantity/time, quality, and cost. Quantity/time means the quantity of material processed in each time period at each processing time center. Quality refers to the degree of conformance of the product at established specifications. Cost is the value of all resources expended in producing the product. To regulate these attributes, formal procedures for planning and controlling their levels are established. Thus, a progressive company will have organizational units and information systems for production planning and control, inventory control, quality control, and cost control. This study focuses primarily on the two attributes of production management, quantity/time and cost. On the other hand the chosen production inventory system which is Just-In-Time (JIT), has already provides the third attribute, quality with its definition and concept. Therefore all the attributes of the material flow are tried to be satisfied.

The success of Japanese firms in the international marketplace has generated an interest among many Western companies to the JIT philosophy. Just-In-Time philosophy is a manufacturing philosophy with the goal of producing the required items, at the required quality and in the required quantities at the precise time they are required. JIT has been described by Schonberger [37] as a production system which replaces complexity with simplicity in manufacturing environment.

The JIT system arose initially in the Toyota automotive plants in Japan in the early 1960s and is currently being used in a variety of industries, including automotive, aerospace, machine tools, computer and telecommunications manufacturing.

The objective of JIT is to create a smooth and rapid flow of all products from the time materials and purchased parts are received until the time the final product is shipped to the customer. Ideally, the number of parts produced in a plant or purchased from outside suppliers at any one time should be just

enough to produce one final unit of the product. Inventories are not needed, or at least minimized. Several steps are taken to achieve the objectives of JIT. U-shaped work cells are designed to optimize material flow through the plant, one worker is assigned to multiple machines, equipment setup times are reduced, and quality control is emphasized. JIT is the integration of these techniques into an organized, focused system. JIT is simply a return to basics, attempting to use human resources and machines in a way that will eliminate waste.

There are four components of JIT systems that work together to provide important benefits for production. These components are layout and production methods, Kanban, total quality control, and suppliers.

In order for a JIT system to function effectively, fundamental changes in traditional production systems must take place. These changes require a modification of the design of the layout and material flow process. With JIT, the production layout must provide a smooth flow, in which material introduced at one end of the process moves without delay to finished product. On the production floor, careful coordination between processes must exist. Processes withdraw parts from the preceding processes at the time needed and in the necessary quantities. If such withdrawals occur in an uncontrolled atmosphere, the preceding stations will acquire large inventories in order to allow for peak demands. Therefore it is critical that a JIT system minimize fluctuation in production demand. This is accomplished by making finished product lot sizes as low as possible, ideally one.

Another factor affecting production flow is the setup time for the various production operations. In order to maintain small lot sizes, frequent changeovers between products must be made. Therefore setup times must be reduced as much as possible for JIT to work effectively. The ability to produce in small lots also increases the firm's flexibility to meet customer orders. In addition, machines will not be overworked and can be properly maintained thus avoiding unanticipated breakdowns and improving quality.

A key component of just-in-time production is an information system called Kanban. The type of units required by a process and the number required are

written on Kanbans and used to initiate withdrawal and production of items through the production process.

JIT cannot function properly if production has a high rate of defective items. Implementation of JIT requires careful attention to quality both in purchasing and in production. Since lot sizes are small and there is no safety stock to back up non-conforming items, any quality problems disrupt the flow of materials through out the plant.

The material flow cycle begins with suppliers. In the past, suppliers were considered adversaries and safety stock was maintained as insurance against poor supplier performance. JIT requires a trusting partnership between the supplier and manufacturer to deliver on time and with zero defects. To build such relationships require a reduction in the number of suppliers that are typically used. Without this reduction, JIT purchasing becomes unmanageable. A single or few sources of supply allows the manufacturer to work more closely with the suppliers, thus improving design and product quality, and reducing costs. Additionally long-term contracts encourage these suppliers loyalty and reduce the risk of an interrupted supply of parts. If the manufacturer increases the market share, then larger orders will be received by the supplier.

In true JIT environment, to maintain a smooth production flow, suppliers must make just-in-time deliveries. Instead of receiving one large shipment, that must be counted, inspected, and stored before issuance to the production floor, suppliers make smaller deliveries on a daily basis or more frequently to accommodate that day's production schedule. This is one reason why suppliers are often located in close geographical proximity to a manufacturer. In North America, where industry is frequently geographically dispersed, transportation delays often make it difficult to achieve this type of vendor support.

Since implementing a JIT system is a huge project, each of these four components should be implemented incrementally by stating from the JIT supply and purchasing system as indicated in the literature. The reason is that, buyer companies must rely on their suppliers and vendors to deliver the materials and subassemblies they need. The supplier who is the first to grasp the new

process control and management techniques that customer wants will survive. The supplier on the other hand generally does not have the expertise to make sweeping engineering changes. But does have the advantage of being small compared to the customer and it is less encumbered. That is, the supplier is able to adapt the new JIT ideas quicker because of its size. Additionally customer in the JIT environment can gain substantial benefits in inventory reduction by focusing specifically on the delivered lot size from suppliers. As a result JIT supply represents very important potential for providing both strategic and financial strength to a firm.

Considering the components of JIT philosophy, especially suppliers and purchasing component, and the attributes of production management, in this study a joint economic lot size model is developed. So by using this model, it is tried find a common lot sizing method between supplier and buyer where both can gain from such a relation. Because as it was mentioned before JIT logic expects close relations between these two parties.

In the next chapter, a literature review of Just-In-Time philosophy is presented. The JIT supply is given with its elements, characteristics and problems. Additionally general supply systems and related studies are mentioned in this chapter. Consequently in chapter 3, model formulation of the joint lot size model is presented. Here, each element of the joint model and joint total cost function is given. Furthermore, to apply the JIT purchasing logic in the model, the shipment size concept is discussed in detail. The implementation difficulties of JIT and the requirements of the logic are also covered. In chapter 4, possible extensions of the model, multiple buyer, multiple supplier cases, are analyzed. In chapter 5 the computational analysis of the model is discussed according to three criteria which are joint total cost comparison, shipment size comparison and analysis of variance. In each of these analyses, the importance and the necessity of JIT purchasing is tried to be identified. Especially in the analysis of variance, the factors that have significant effect on the joint model are investigated. Finally in chapter 6, the conclusion of the study is presented with the future recommendations.

Chapter 2

LITERATURE REVIEW

Just-In-Time (JIT) is an organizational philosophy which strives for excellence and evolved from the Japanese manufacturing environment. The term JIT is frequently used interchangeably with "Zero Inventory", "Material As Needed", continuous flow manufacturing (by IBM), stockless production, repetitive manufacturing system (by HP), or Toyota system but it represents a production strategy and not an inventory control technique.

To explain this production strategy, we start with the history of the JIT in the following section. Then the definitions of JIT philosophy and its elements are explained in section 2.2. Consequently, one of the important of component of these elements, which is the supplier relationships and programs, are studied in section 2.3. According to the literature, there are three types of JIT supply relations, which are ideal case, interposing a warehouse and a partnership approach. These types of JIT are discussed in section 2.4. The characteristics of these relations are explained and possible advantages are given in section 2.5. But implementing them into a systems approach, might create some problems. These problems are discussed with their reasons and some recommendations in section 2.6. Finally a literature review of supplier and purchasing studies other than JIT related ones is given.

6

2.1 History of Just-In-Time

JIT production system was developed by Toyota motor company by the former vice-president T. Ohno after the World War II where Japanese were suffering from deficiency of all kind of resources. In order to enter the world market and to compete with American and Western industries, they had to learn to use their scarce resources with the lowest cost possible. The JIT philosophy is emerged by taking and revising the basic ideas of American manufacturing system and shaping them in Japanese environment.

T. Ohno is affected by the supermarket logic and tried to initiate the concept to the JIT production system. In the supermarket as there is no in-between stages, customers are faced with all problems as poor quality, shortages, and perishables during the last stage for products. Additionally, the variety of products are very high. The replenishment of products are activated by the empty shelves. For the ideal case, some optional space is available for large inventories, which is adaptable for quick stock turnover and easy stock replacement [27][33]. Taking this idea as a base, T. Ohno has developed the Toyota production system, and hence JIT philosophy.

After the oil crisis in 1971, managing the scarce resources concept became popular in Japan, so the JIT philosophy. After 1980's some American companies also began to implement JIT production system in their environments.

2.2 Just-In-Time and Its Elements

Much of the literature concerning Just-In-Time (JIT) manufacturing has concentrated on a description of JIT characteristics and implementation cases. Just-In-Time can be defined as a production system designed to eliminate waste in the manufacturing environment. One way to expand this definition is as follows: "In a JIT system, the necessary material are brought to the necessary place to build the necessary products at the exact time when they are required [29]". Rhea gives another definition. "JIT is a system which is

emphasized quality in workers, materials, facilities and end products. It uses existing technology but rearranges production equipment into cells for hand to hand manufacturing. The factory is focused to bring the materials and tools close to the point of use rather than keeping them in central storage areas. Better products, greater flexibility and inventory reduction are results of this system [35]."

Karmarkar defines JIT from another point of view. " Think of JIT as a statement of objectives. It underscores the importance of lead-time management in all aspects of manufacturing. It asserts that incremental reductions in lead times are crucial indices of manufacturing improvement. JIT presumes that to achieve such reductions the system should deliver to every operator, in any conversion process, whatever he or she sends just when it is needed. It saves the money tied up in downstream inventories, protecting against long lead times. Shorter lead times mean improved responsiveness and flexibility [24]."

Also some part of the JIT literature described the Japanese development of JIT and provided case studies of Japanese success [29][37][39]. These works were technique oriented, and the major thrust was to describe success measurements, implementation techniques, and the resulting operational activities in the altered organizational structures. But all these theme depend on the following five important objectives, which are as follows:

- minimize the work-in-process inventory

- minimize fluctuations in WIP to simplify inventory controls

- minimize production instability by preventing demand fluctuations from one process to another

- provide better control through decentralized shop floor control

- reduce defects.

In order to achieve the objectives mentioned above, Japanese developed

and used two concepts: elimination of waste and respect for people [9]. The basic elements of elimination waste concept can be summarized as follows:

2.2.1 Elimination of Waste

1.Focused Factory Networks:

Instead of building a large manufacturing facility that does everything (highly vertically integrated), build small plants that are specialized. When a plant is specifically designed to do a specific thing it can be constructed and operated more economically than its universal counterpart.

2.Group Technology:

Monden [29] discusses this concept in detail which can be defined as, " GT is an engineering and manufacturing philosophy which identifies the sameness of parts, equipments or processes." Machines are grouped according to the routing required for a family of parts rather than by their function.

3.Total Quality Control:

A new inspection philosophy took its place in the quality control which is inspect to prevent the defect from occurring rather than to find the defect after it has occurred. Ultimately, a concept which Japanese call autonomation emerged. This means the autonomous control of quality and immediately stop everything when something goes wrong. This is controlling quality at source. Another interesting technique is Quality Circles. A Quality Circle is a group of employees who meet once a week on a scheduled basis to discuss their function and the problems they are encountering to try to devise solutions to those problems and to propose those solutions to their management.

4.Just-In-Time Production:

It requires the making of precisely the necessary quantities at the necessary time, with the objective of achieving plus or minus zero performance to schedule.

5.Uniform Plant Loading:

To use the Just-In-Time production concept, it is necessary that production flow as smoothly as possible in the shop. The starting point is uniform plant loading (UPL). The objective of UPL is to dampen the reaction waves that normally occur in response to schedule variations.

6.Kanban:

JIT uses an inventory/production control system which is called Kanban that is a pull type of reorder system in that authority to produce or supply comes from down-stream (assembly) operations. While work centers and vendors plan their work based on schedules, execution is based on Kanbans.

7.Reduction of Setup Time:

Reduction in set up time is critical to the JIT philosophy. Numerous setups are required to implement the uniform work load component. The savings in setup time are used to increase the number of lots produced, with corollary reduction in lot sizes.

2.2.2   Respect for People

1.Lifetime Employment:

When a worker is hired for permanent position with a company, he/she has a job with that company for life (or until retirement age) provided he/she

works diligently. This generally motivates the worker and so the productivity increases.

2.Company Unions:

As name implies each company has its own union so that this union is related to only that companies workers. The objective of both the union and management was to make the company as healthy as possible so there would be benefits accruing to the people in a secure and shared method.

3.Method of Compensation:

It is based on company performance bonuses. The employees have an attitude that says " if the company does well, I do well, " which is important from the standpoint of soliciting their help to improve productivity.

4.Attitude Toward Workers:

The management system must provide every worker with an opportunity to display his maximum abilities and make contributing to improve the system.

5.Automation/Robotics:

It is believed that robots free people for more important tasks. In fact workers go out of their way to figure out how to eliminate their job, if they find it dull, because they know the company will find something better and interesting for them to do.

6.Consensus Management:

It is also called bottom-round management or management by committee. The employees recognize a problem, work out a potential solution with their peers, and make recommendations to the next level of management.

7.Vendor Programs:

Suppliers for JIT system companies are considered to be part of the customer's family. Suppliers are expected to deliver high quality parts many times per day, often directly to the customer's assembly line, bypassing receiving and inspection.

One of the important elements of JIT philosophy is, as it was mentioned above, the supplier relationships and supplier programs. During this study, we studied a combination of JIT supplier relations and general supplier relations. For that purpose, it will be logical to separate the situations so that the differences can easily be identified, and interactions can be understood.

2.3 Just-In-Time Supply

A growing number of manufacturers with a large demand potential have recognized that the creation of a well-performing integrated production and scheduling network composed of the manufacturer and a selected number of appropriate suppliers is a necessary precondition for implementing Just-In-Time supply.

While it is generally acknowledged that suppliers are critical for the successful operation of JIT [21][37], very little has been offered in the literature beyond describing characteristics of JIT supply and JIT purchasing surveys such as the one contributed by Ansari and Modarress [3]. The current literature has concentrated on the types of suppliers, products supplied, and the differences between Japanese and American suppliers.

Successful application of JIT depends to a very great extent on the buying firm's suppliers. In this system suppliers must be able to provide the buyer with frequent deliveries of small lots of high quality parts, with delivery geared precisely to the buying firm's production schedule [26]. It was also stated that under the JIT concept, a supplier simply is viewed as a work station that is

located away from the buyer's manufacturing site. From a scheduling perspective the buyer's major responsibility is to coordinate the final operation at the supplier's plant with the first operation in his own production system. Dumond and Newman [14] make a similar comment that the relationship between a firm and its suppliers has been a distant "arms length" relationship. Also Gupta [20] defines such a relationship and calls it as a strategic partnership and adds that such partnerships allow manufacturers and suppliers to work together to solve common problems. Suppliers become intimately involved in the product and process design function in the early stages of the product development and in the manufacturer's scheduling and quality problems [21][37].

Today, an increasingly competitive global economy and changing production techniques are creating the need for closer, more cooperative relationships between a firm and its suppliers. While closing the gap between buyer and suppliers some problems occur in developing this type of coordination stem from things such as:

- Number of supplier utilized

- The relationship with suppliers

- Sharing information with suppliers

- Geographical dispersion

Detailed analysis of each part is discussed in the following subsections.

2.3.1 Number of Suppliers Utilized

Manoochehri [26] states that the large number of suppliers utilized by the buyer creates a number of problems, such as:

1. It is more difficult to manage the coordination of production schedules and relationships.

2. More suppliers require that more time and money be spent in developing and training them.

3. When several suppliers are utilized for the same part, it becomes less practical for each supplier to make frequent deliveries for the same part.

2.3.2 The Relationship with Suppliers

A second major difficulty experienced by many firms stems from the type of relationship developed between buyers and their suppliers. Over time major Japanese manufacturers seem to have developed a reliable network of suppliers characterized by close relationships between buyers and their counter parts in the supplier firms [26]. Newman [32] looks this relation from the supplier's view and he states that the buyer- supplier relationship should have specific concessions for the supplier to allow him to achieve economies of operation which, when realized would protect profit margins and allow for price concessions. From the buyer's perspective, these concessions should not involve any increased costs since they would dilute the gains of JIT. There are many concessions which present a low-cost concessions and a low risk for the buyer. Newman [32] lists the following concessions:

- Delivery schedule freeze

- Simple supplier networks

- Joint design and engineering change proposal reviews

- Joint value analysis programs

- Contract commitments

- Delivery process

- Contract carriers

- Packaging requirements

- Customer-supplied material

- Supplier manufacturing schedules

Dumond and Newman [14] look at this relation from another point of view and list six activities that should be logically implemented so that the gap between buyer and vendor disappears. These activities are:

- production planning system

- production/purchasing interface

- vendor base reduction

- vendor scheduling system

- vendor capacity planning

- implementation of new technology.

2.3.3 Sharing Information with Suppliers

To create desired coordination for JIT deliveries, the buyer has to share with his or her supplier a great deal of information in addition to the material specifications governing the purchase. Discussions about the specific variables involved in usage, quality, tolerance, potential production process, and production scheduling activities are essential for a reasonably complete mutual understanding of the important issues involved. Sharing this type of information requires trust and loyalty, which should be the basis for the development of a mutually buyer/supplier relationship.

2.3.4 Geographical Dispersion

Since suppliers make frequent and small deliveries when working under a JIT system, proximity to the buyer's plant is an important factor. In Japanese

setting, most suppliers are located fairly close to the buyer's plant, usually less than sixty miles away. In contrast, in U.S., suppliers may be scattered throughout the entire country, and occasionally overseas. Long transportation lines increase transportation time and cost of inventory in transit, and clearly decrease the reliability of precisely scheduled deliveries. As a practical matter, in such cases U.S. companies typically must rely to some extent on buffer stocks. To extent possible, U.S. firms experimenting with JIT are attempting to work with nearby suppliers, or trying to reduce the transportation time and cost as low as possible. Therefore geographical dispersion is an important concept in JIT supply but transportation still plays the significant role in the supply.

Dumond and Newman [14] continue that implementation of the six activities which have been discussed tends to reduce the communication gap between the buyer and vendor, and add that this gap reduction provides the organization with four primary benefits: reduction in the order cycle time, increased contribution of purchasing function, reduced costs for both companies, and increased supply assurance. As we see from both Manoochehri, Dumond, and Newman a cooperative work between supplier and the buyer will bring many benefits to the buyer and sharing information really plays an important role in the relationship between buyer and supplier. Another important point on this relation is the type of the supplier that the buyer is dealing with.

2.4   Types of JIT Supply

Fieten [15] in his paper stated that pilot studies in German industry show that there are basically two forms of integrating suppliers allowing JIT supply with minimal inventories in the logistic chain, which are the ideal case and interposing a warehouse approach. However Forbes, Jones, and Marty [16] in their paper mentioned a third type JIT supply which they called as partnership approach. The types are the following:

2.4.1 Ideal Case

The direct linkage of suppliers to the manufacturer can be regarded as an ideal solution that cannot be generally implemented. This requires JIT-continuous flow production by the suppliers and allows JIT delivery to the manufacturer [15].

2.4.2 Interposing a Warehouse

Interposing a warehouse which allows batch-type production by the suppliers and JIT delivery from the warehouse to the manufacturer as well. This warehouse can be owned by the supplier or by the manufacturer. However, it may also be a common warehouse owned by both of them which possibly may be operated by a carrier. This solution has the advantage that a specialist like a carrier company might reduce the complexity of the interface problems [15].

Also Carlson [11] in his paper discusses JIT applications to warehousing operations and explains some of these operations and systems used in. These are shipping dock operations, the down-sizing program, the daily reorder system and the short interval pick system. Also Carlson described the environment in terms of the dispatching, picking, sorting, traffic and supervisory activities associated with the daily critical dealer orders in order to demonstrate the implementation of JIT to warehousing environment.

2.4.3 Partnership Approach

Vendor relations in JIT manufacturing are based on a very different philosophy than in usual manufacturing operations. As opposed to having multiple sources and trying to select the one with lowest cost, in this approach the manufacturer looks upon the vendors as partners in a joint effort with the company and encourages constant communication to eliminate problems. Here supplier produces certain parts and assembles them for the manufacturer. After a period of development the manufacturer transfers the responsibility of ordering and supplying the components for those parts and products to the supplier.

This reduces the manufacturer's purchasing, handling and storage costs.

In this three types of supplier, JIT supply is done one way or other. But the important point is that in such a system the characteristics of the JIT supply should be setted so that the manufacturer and the supplier can both gain from this relation.

2.5 Characteristics of JIT Supply and Purchasing

Black [8] defined nine important characteristics of JIT supply and purchasing which are small lot size, single sourcing, long-term contracts, very frequent delivery, hundred percent good quality, engineering aid to the vendor, local sources, freight consolidation program, and standard packaging in fixed quantities. They are explained in detail in the following subsections.

1. Small Lot Size. A hallmark of JIT is small lot sizes. Buying or making parts in small lots, in turn, has a strong positive effect on product quality. By making parts steadily rather than in batches, most suppliers experience improvements in inventory, quality, and scrap levels. Moreover, defects are caught early, and there are fewer defective parts to discard or rework.

2. Single Sourcing. As it was explained in the above paragraphs to close the gap between the buyer and the supplier the number of suppliers should be decreased [14]. The best vendor should be selected to be the sole source for each part, component or subassembly used by the company. This reduces the the variability between parts (improves the quality), since all the parts are coming from the same manufacturing process or system.

In addition to supplier selection, supplier evaluation is an important part of the JIT supply. Schonberger and Ansari [38] in their paper discuss that suppliers must be evaluated on their ability to provide high quality products. Bernard [7] mentions the same thing but adds that results measurement does

not mean performance measurement. He also states that from the company's perspective, the vendor/company relationships can be subdivided into five areas: vendor programs, joint programs, company programs, information flow, and material flow. Good performance is then the result of managing each relationship in a manner which drives total cost down, incoming quality level up, and days late to zero.

Possible Advantages of Single Sourcing can be listed as follows:

- Divisional resources can be focused on selecting/developing /monitoring one source rather than many

- Volume buys are higher, leading to lower cost

- Vendor is more inclined to do special favors for the customer since the customer is a large account

- Easier to control and monitor for superior quality

3. Long-term contracts. The company and the supplier develop long term contracts (18-24 month) that enable the vendor to take the long range view and plan ahead. Newman [32] stated the importance in his paper under buyer concessions topic.

Possible Advantages of Long-term Contracts are listed as follows:

- Builds schedule stability.

- Better rapport. Monthly, or more often, communication between buyer and vendor.

- Better visibility. The vendor sees one year's worth of forecasted needs as soon as the company sees it, instead of a limited lead-time view.

- Less paperwork

- Inventory elimination

	A	B	C
# Parts	Low to Moderate	Low to Moderate	Very High
Volume %	Low	Low to Moderate	High
Cost %	High	Low to Moderate	Low
Receipt Frequency	Daily	Daily to weekly	Monthly or 2-4 times/yr
Strategy	Inventory Management	Space Management	Service Level

Table 2.1: ABC Analysis to JIT Supply

4. Very Frequent Delivery. The vendor will be expected to deliver materials to the company daily or weekly, depending on the type of part or subassembly. Most parts can be categorized according to an ABC analysis as shown in Table 1 [1].

5. 100 % good quality. The vendor should be taught how to to implement the JIT strategy so that the vendor can deliver the correct quantity, on time, with no incoming inspection.

6. Engineering Aid to the Vendor. The vendor and the customer work together to improve the vendor's manufacturing processes, efficiency and quality. Both must visit each other plant so that they can know how they are working. Newman [32] again mentioned about the joint design and engineering change proposals which should be a buyer concession so that especially supplier can gain from this relationship.

7. Local Sources when possible. While it is not absolutely necessary (or even possible) that all vendors be located close to the customer or the company, but it will definitely help to provide the customer with daily deliveries. Time spent during the transportation increases the cost of raw material inventory.

8. Freight Consolidation Program. Materials from the vendors can be consolidated onto trucks for transportation to the customers.

9. Standard Packaging in Fixed Quantities. This means the containers are standardized in terms of size and quantity.

These characteristics shows the effectiveness of such a system so that both buyer and supplier obtain many beneficiary results. On the other hand implementing all them is really a difficult task and especially in the case of purchasing, it creates big problems. Now we try to show these problems and recommendations to solve them.

2.6 Problems of JIT Supply and Purchasing

A move toward adoption of the JIT concept can lead the manufacturers to the attainment of higher product quality and productivity. In addition to the benefits of JIT purchasing, firms have encountered seven major problems in the implementation process [2]. In their paper Ansari and Modarress [2] uncovered these major problems and suggested some fundamental recommendations that will help to overcome these problems encountered in the implementation process. The problems with their possible reasons and their recommendations are as follows:

1. Lack of Supplier Support

Reasons:

1. Little or no incentive for suppliers to adopt JIT delivery

2. Lack of commitment from buyers

3. Considerable strain on suppliers

Recommendations:

1. Education and Training of Suppliers

   There are three different approaches for this case

   - Intensive presentation and group discussion

   - Continuing in-house training at supplier's plant

   - A periodic "vendor day" conducted at buyer's plant

2. Long-term Relationships

   Suppliers have certain expectations. These are:

   - Long-term business arrangement

   - Fair return on supplier investment

   - Adequate time for through planning

   - Accurate demand forecasts

   - Correct firm specifications

   - Parts designed to match the supplier's process capability

   - Smoothly time order releases

   - Fair profit margin

   - Minimum number of change orders

   - Fair dealings with regard to price

   - Prompt payment of invoices.

2. Lack of Top Management Support

Reasons:

1. Less concerned with long-term planning arrangements and more concerned with existing markets and short term profitability

2. Skeptical view that JIT is not suitable to their firms.

3. Frustrated by the magnitude of problems encountered sporadic results experienced during the initial phases of implementation.

Recommendations:

1. Education as a means to effect attitudinal change

2. Utilize the positive JIT results experienced by other firms.

3. Low Product Quality

Reasons:

1. Inadequate experience in supplier management

2. Past manufacturing philosophies that allowed the acceptance of an excessive percentage of defects incoming material shipments.

Recommendations:

1. Quality management program for suppliers

   - the development and utilization of a supplier certification program.
   - the utilization of supplier plant audit program

4. Lack of Employee Readiness and Support

Reasons:

1. Resistance to change

2. Fear for a job loss

3. Increased pressure and potential frustration

Recommendations:

1. Long-term continuous JIT purchasing training for employees involved with purchasing and materials activities.

2. A broader orientation of employees focusing on the company's reasons for adopting JIT, as well as the philosophy behind it.

5. Lack of Support from Carrier Companies

Reasons:

1. Not given much attention to the purchase of transportation

   • not have closely long-term relationships

   • not highly structured delivery schedules for the buying firm

Recommendations:

1. Reduction in the number of carriers used.

2. Transportation can perhaps be purchased from a contract carrier

3. Involving a computer interface with major carriers.

6. Engineering Support

Reasons:

1. Minimal interaction between design engineering and purchasing personnel

2. Purchasing people do not have enough information about design features and constraints to discuss design and quality options with suppliers

Recommendations:

1. Development of an operating climate that encourages and promotes a high level integration continuously

7. Lack of Communication

Reasons:

1. Not an effective JIT purchasing environment

2. Not enough integration of some areas and efforts to cooperate

Recommendations:

1. Continuing close cooperation and communication of purchasing personnel with personnel at all levels of the organization

These recommendations are by no means exhaustive, but they encompass the most important points revealed by the firms studied. Each organization must make appropriate modifications for its own style of manufacturing and its own unique culture and environment.

Up to this point, we studied JIT, and JIT supply and purchasing system. But there are also other studies which explain these systems under different production environments. They are discussed in the following paragraphs with their definitions, characteristics and problems. So the importance of the supply and purchasing system can be understood. Besides, JIT concept and the effectiveness of the JIT logic to such relations are determined during this discussion.

Bartholomew[6] stated the relationship between vendor and customer in a historical way. This study showed that the end of World War II created a substantially different business climate for American industry. Industry had geared up to support the war effort and, indeed, had awed the world with its ability to produce material and reduce lead times to get the material where it was needed faster.

Like so many other things, minimization of inventory in Japan was not so much a goal as a necessity. An economy devastated by a world war mandated strong controls over scarce resources, among them inventory. Not without

significance was the lack of physical space in Japan, which necessitated supplier plants in close proximity to one another and the minimization of inventory for all concerned. By that time, in America, while they were using single-sourcing policy and believed that vendors to be close at hand like Japanese do, they changed their logic by the improved modes of transportation. Speed delivery and freight charges were no longer the prohibitive factors they once were. Now the key factors were pricing, delivery performance, quality. Because of this, the number of suppliers were increased what is still vogue today known as Dual Sourcing was occurred. The weakness of the key factors is in not knowing why a problem occurred, or who caused it. Vendors cannot necessarily be expected to perform well if the company makes demands which exceed reasonable expectations.

After 1980's the importance of single-sourcing was understood because of the problems of American business especially when compared to Japanese advances during the same period of time. To solve the problems, they offered many solutions but most of them dealt with the key factor of pricing. Britney, Kuzdrall, Fartuch [10] in their paper, stated that under certain conditions the buyer can reduce policy costs by buying larger quantities if supplier makes some price discounts. These quantity discount pricing models were developed to increase the supplier profits [25][28]. The basic idea in these models are that, they provide reduction in the buyer annual inventory policy costs. By ordering an item from the vendor less frequently and in larger quantities, at a discounted unit price, the reduction in order processing and procurement costs generally offsets the resultant increase in carrying costs associated with the larger average inventory on hand which really contradicts with JIT philosophy. On the other hand, Chakavarty and Martin [12] developed a joint model to obtain desired joint savings-sharing scheme between supplier and buyer where they took the idea of Banerjee's [5] joint lot size model. Banerjee proposed a joint optimal ordering policy, which together with an appropriate price adjustment can be beneficial economically for both supplier and buyer or, at least does not place an additional cost to either party.

Through these analysis, economic order quantity (EOQ), the most basic

production lot-sizing model, is frequently used. Zangwill [43] modeled Zero Inventory concept and tried to scrutinize the validity and sometimes invalidity by comparing it with EOQ. He also stated that inventory is caused by inefficiency and the more the inventory the greater the inefficiency. Most attempts reconcile the somewhat antithetical philosophies of EOQ and JIT have focused on the effect of a reduction in setup time and increase in holding costs. On the other hand, it was shown that the two strategies were consistent when the JIT policy of synchronizing production with demand is achieved [13]. Goh and Hum [17] also mentioned the same concept, and assumed that under a contract order for the supply of material to be shipped in sub-batches. In this way, a firm operating under what we term as conditions close to JIT, can service its customers more efficiently by providing high quality products at lower cost and with more frequent deliveries and consequently shorter lead times. According to this Gupta [19] made a feasibility study of JIT supply and purchasing implementation in a manufacturing facility but faced with the problems that we listed in the section of problems of JIT supply. Similar scenario was tried and similar results were obtained in the examination of a sales-oriented company's purchasing function [22]. Wehrman [40] also evaluated the total cost of a purchase decision but this time in the model, transportation cost was added. Furthermore, he stated that transportation cost in particular might be a very significant variable cost in a purchase decision. Similarly, Narasimhan and Stoynoff [31] considered a procurement allocation decision which incorporate the features of the traditional model and vendors economies of scale. Hwang, Moon, and Shinn [23] looked at the transportation cost from another point of view. They developed an EOQ model with quantity discounts for both purchasing price and freight cost. But they only consider the buyer in this situation. Also some analysis were made, and instead of quality discounts, the freight discounts were investigated. Additionally, the integration of purchasing and stock control policies were represented in reference [4].

A review of the literature indicates that the traditional approach for evaluating quantity discount offerings for purchased items has not adequately considered the effect that transportation cost may have on the optimal order quantity; despite the general fact that purchased materials must bear transportation charges. Russel and Krajewski [36], presented a simple analytical procedure for finding the order quantity that minimizes total purchase costs which reflect both transportation economies and quantity discounts. But this study did not consider the JIT philosophy. Ramasesh [34] in his research, recasted the traditional inventory model to implement JIT purchasing and gave a model to find out the optimal number of contract quantity and optimal number of shipments. Similar study was made by Golhar and Sarker [18], where they advised a economic manufacturing quantity which is suitable to JIT delivery system but again they did not consider the transportation cost.

All these studies showed that implementing the JIT philosophy to the supplier buyer relations had many problems The reasons for these problems were explained in the above subsections, but the significant ones are:

- Lack of supplier commitment

- Need for new lot-sizing policies to implement JIT philosophy, and to minimize the increase in the transportation costs due to frequent deliveries

- Considering the relations between supplier and buyer from one parties point of view, not both (joint total cost)

As a result, to solve these problems, a joint lot-size model is proposed to achieve the above expectations. It is believed that such a model is more realistic and economically beneficial for both parties, supplier and buyer, in the long-term relationships.

Chapter 3

MODEL FORMULATION

In a typical purchasing situation, the issues of price, lot sizing, etc., usually are settled through negotiations between the buyer and vendor. Depending on the existing balance of power, the outcome of such a negotiation results in near optimal or optimal policy for one party while the other party is subjected to a substantial cost penalty; in some cases undesirable policies result for both parties.

In order to solve this problem, Banerjee [5] in his paper developed a joint economic lot size model (JELS) for a special case where a vendor produces to order for purchaser on a lot for lot basis under deterministic conditions. Banerjee also stated that this JELS model can be viewed as at least an intermediate step towards the shift to JIT philosophy. JIT philosophy, as we mentioned in the literature review, states that vendors must be accepted as another work-center of the factory. This means that, they have to apply JIT philosophy to their companies in order to supply raw materials, parts, and components to the plant just in time with small lots and hence in frequent manner. However most suppliers, which are non-JIT users, believe that JIT lot-sizing and delivery system is not economically beneficial for them. Because, producing in small lots and sending them frequently will increase the supplier total cost for batch type productions. Therefore to consider all these facts we settled two main objectives for our model formulation. They are:

- Minimize the total cost (Summation of purchaser, supplier and transportation costs)

- Maximize the number of deliveries (A JIT goal)

To achieve these objectives Banerjee's JELS model is used as the first step of the formulation. The second step is to consider the behavior of the JIT lot sizing and delivery system, especially the frequent deliveries with small lots. Banerjee's model does not consider the delivery part of the product, nor the transportation cost, so that JIT philosophy seems to be very beneficial. But this is not the case. Russell and Krajewski [36], in their paper, added a new component as transportation cost which varies according to the weight carried. So frequent deliveries may decrease inventory holding part of the total cost function but will definitely increase transportation and setup part of the function.

Therefore, our aim in the model formulation, is to move away from the adversarial bargaining process and develop the concept of JELS model by adding the transportation cost part. This analysis focuses on the joint total cost function under deterministic conditions. Also JIT lot sizing and delivery system are investigated inside the model so that the cost differences of JIT logic are to be identified.

To clarify some of the important concepts, we restrict our discussion and analysis to a relatively simple purchasing scenario, and possible extensions will be discussed in Chapter 4. It is assumed that a purchaser (buyer) periodically orders some quantity (Q) of an inventory item from a vendor (supplier). With the receipt of an order, the vendor produces the required quantity of the item (i.e. the vendor follows a lot for lot policy) and, on completion of the batch ships the entire lot to the buyer. In addition to this deterministic conditions, we assume there are no other buyers for this item and the vendor in question is the sole supplier. Notation used in the mathematical formulations of this purchasing scenario is presented in Table 3.1.

As we mentioned in the above paragraphs, our first objective is to minimize

D	Annual demand or usage of the inventory item
P_s	Supplier's production rate
Q	Order or production lot size in units
C_{os}	Supplier's ordering cost per order
C_{op}	Purchaser's ordering cost per order
C_{ss}	Supplier's setup cost per setup
C_{sp}	Purchaser's setup cost per setup
I_{rs}	Supplier's inventory carrying cost of raw materials
I_{rp}	Purchaser's inventory carrying cost of raw materials
I_{fs}	Supplier's inventory carrying cost of finished goods
I_{fp}	Purchaser's inventory carrying cost of finished goods
PTC	Purchaser total cost
STC	Supplier total cost
TTC	Transportation cost

Table 3.1: Notations used in the mathematical formulations

our joint total cost which has three main parts.

- Purchaser Total Cost

- Supplier Total Cost

- Transportation Total Cost

Therefore our objective function is the summation of these parts:

$$JTC = PTC + STC + TTC \tag{3.1}$$

Each part can be defined as follows:

Purchaser Total Cost Function

Purchaser has three main costs in his cost function which are ordering cost, setup cost and inventory holding cost. We assumed that purchaser could have manufacturing facilities so that we added the setup cost as the third element to the cost function.

$$PTC = (\frac{D}{Q})(C_{op} + C_{sp}) + (\frac{Q}{2})(I_{rp} + I_{fp}) \tag{3.2}$$
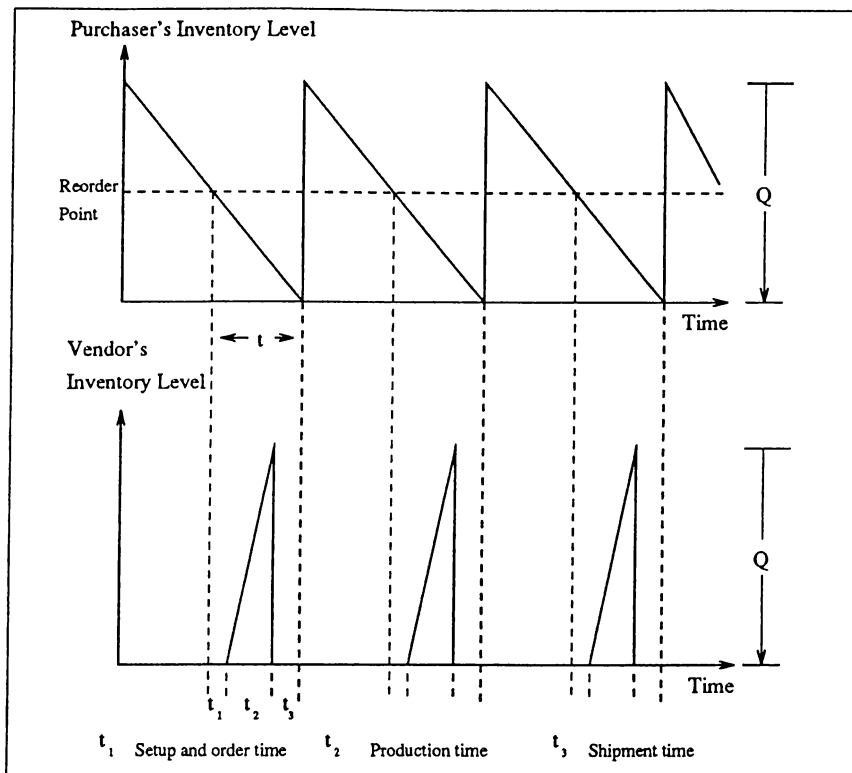
Figure 3.1: Purchaser's and Vendor's Inventory Time Plots

Supplier Total Cost Function

Similar to the purchaser, supplier has also three costs. These are setup, ordering, inventory holding costs. We assumed that supplier is buying its materials from another source so that it has also an ordering cost. In addition, the distinction between the inventory holding costs I_{rs} and I_{fs} is important when the product lot size, Q, is not equal to shipment size, X, which will be discussed later in this chapter. Also the difference between inventory holding costs of purchaser and supplier can be seen in the Figure 3.1.

$$STC = (\frac{D}{Q})(C_{os} + C_{ss}) + (\frac{D}{P_s})(\frac{Q}{2})(I_{rs} + I_{fs}) \qquad (3.3)$$

Transportation Cost Function

One of the important concept of purchasing is to arrange for the proper delivery of needed materials at the lowest total cost. It is important to recognize that all purchased materials must bear transportation costs. Therefore the

analysis must include appropriate transportation cost function with all other relevant costs to determine the lowest cost ordering policy.

Russell and Krajewski [36] in their paper analyzed the transportation cost structure in the purchaser cost function. The behavior of this cost structure is arranged according to the less than truck-load shipments which, contains quantity awarded on the basis of the number of units ordered at one time. The structure of less than truck-load (LTL) freight rates is characterized by largely fixed costs of shipping activities in which pickup/delivery and line—haul or point to point costs for the common carries, bear little relation to the quantity of freight being moved. On the other hand in our model formulation we characterize the less than truck-load logic by largely on the variable costs of the shipping activities and instead of weight, we used quantity for the freight carried. But the logic of the transportation cost is not changed.

An important aspect of this freight rate discount structure that must be considered by the shipper emanates from the temporal nature of transportation services. This temporal nature gives rise, from time to time, to the practice of over declared shipments where the shipper may find it economically advantegous to pay for a shipping quantity which is higher than the actual quantity of the shipment in order to achieve a lower freight rate and a lower total tariff. To investigate when it is economical to overdeclare a particular shipment, the shipper must determine if total freight costs would be reduced by artificially inflating the actual shipping quantity to the rate breakpoint in order that a lower marginal tariff is achieved for the entire shipment.

Over declared shipments occur when the actual shipping weight falls within a range that lies between the rate breakpoint and an indifference point which is a function of the particular freight rate schedule. The indifference point is defined as the quantity which, when multiplied by its proper rate, yields the same total tariff that is charged at the rate breakpoint. The indifference point weight (α) is expressed as:

$$\alpha_{l-1} = \frac{R_l \beta_l}{R_{l-1}} \tag{3.4}$$

where

l = rate category

R_l = transportation rate for category l , $R_l < R_{l-1}$

β_l = rate breakpoint quantity for rate category l

According to this logic, if a shipper has a quantity greater than or equal to an indifference point, but less than the next larger breakpoint quantity, the least costly option is to employ a fixed charge by overdeclaring the shipment quantity. If the shipment is greater than or equal to a breakpoint quantity but less than the next larger indifference point, it is best to use the variable cost per quantity appropriate for that quantity. The total transportation cost is given by

$$TTC(Q) = \begin{cases} \frac{D}{Q}\beta_l R_l & \beta_l \leq Q \leq \alpha_l \\ R_{l+1}D & \alpha_l \leq Q \leq \beta_{l+1} \end{cases} \tag{3.5}$$

Transportation cost function as seen from the upper formulation, has certain values between some ranges. But in that form we cannot use it in our mathematical programming model. Therefore we tried to present it in one equation. For that purpose, first we assume that it is piecewise linear function. Then by using 0-1 variables piecewise linear functions can be represented in linear form as discussed by Winston [41]. Suppose that a piecewise linear function $f(x)$ has breakpoints b_1, b_2,...b_n. For some $k(k = 1, 2, ...n - 1)$, $b_k \leq x \leq b_{k+1}$. Then for some number $z_k(0 \leq z_k \leq 1)$, x may be written as

$$x = z_k b_k + (1 - z_k)b_{k+1} \tag{3.6}$$

$$f(x) = z_k f(b_k) + (1 - z_k)f(b_{k+1}) \tag{3.7}$$

We are ready to describe the method used to express a piecewise linear function via linear constraints and 0-1 variables:

Step 1: Wherever $f(x)$ occurs in the problem, replace $f(x)$ by $z_1 f(b_1) + z_2 f(b_2) + ... + z_n f(b_n)$

Step 2: Add the following constraints to the problem:

$$z_1 \leq y_1, z_2 \leq y_1 + y_2, z_3 \leq y_2 + y_3, ..., z_{m-1} \leq y_{m-2} + y_{m-1}, z_m \leq y_{m-1} \tag{3.8}$$

$$\sum_{i=1}^{n-1} y_i = 1 \quad y_i = 0 \text{ or } 1 \qquad \sum_{i=1}^{n} z_i = 1 \quad z_i \geq 0 \tag{3.9}$$

$$x = z_1 b_1 + z_2 b_2 + \dots + z_n b_n \tag{3.10}$$

where n is determined by the summation of number of indifference points and the breakpoints in the transportation cost function and presented with the decision variables z_i values. Additionally, m, which is equal to $n - 1$, is used to present the binary variables, y_i. These binary variables were used to determine which range is selected in the model formulation by having that y_i value being one and the others are zero. Here, y_i values determine the z_i values where z_i's determine the optimum range for the transportation cost structure. So, without such a relation setted between the binary and decision variables, a linearization cannot be applied to our model.

So by using the above method we can write the transportation cost as follows:

$$TTC = z_1 TTC(\alpha_0) + z_2 TTC(\beta_1) + z_3 TTC(\alpha_1) + z_4 TTC(\beta_2) \dots \tag{3.11}$$

$$Q = z_1 \alpha_0 + z_2 \beta_1 + z_3 \alpha_1 + z_4 \beta_2 \dots \tag{3.12}$$

$$z_1 \leq y_1 \tag{3.13}$$

$$z_2 \leq y_1 + y_2 \tag{3.14}$$

$$z_3 \leq y_2 + y_3 \tag{3.15}$$

$$\vdots$$

$$z_{m-1} \leq y_{m-2} + y_{m-1} \tag{3.16}$$

$$z_m \leq y_{m-1} \tag{3.17}$$

$$\sum_{i=1}^{m} y_i = 1 \quad y_i = 0 \text{ or } 1 \tag{3.18}$$

$$\sum_{i=1}^{m+1} z_i = 1 \quad z_i \geq 0 \tag{3.19}$$

To show the logic of this method, we generated an example data so that all

calculations can be understood easily. Also we used this data for our computational analysis and experimental design in chapter 5. Let

$$TTC(Q) = \begin{cases} R_1 D & 0 \leq Q \leq 250 \\ \frac{D}{Q} R_1 250 & 250 \leq Q \leq 3000 \\ R_2 D & 3000 \leq Q \leq 4000 \\ \frac{D}{Q} R_2 4000 & 4000 \leq Q \leq 7000 \\ R_3 D & 7000 \leq Q \leq 8000 \\ \frac{D}{Q} R_3 8000 & 8000 \leq Q \end{cases} \tag{3.20}$$

$$TTC = z_1 TTC(0) + z_2 TTC(250) + z_3 TTC(3000) + z_4 TTC(4000)$$
$$+ z_5 TTC(7000) + z_6 TTC(8000) \tag{3.21}$$
$$Q = z_1(0) + z_2(250) + z_3(3000) + z_4(4000) + z_5(7000) + z_6(8000) \tag{3.22}$$
$$z_1 \leq y_1 \tag{3.23}$$
$$z_2 \leq y_1 + y_2 \tag{3.24}$$
$$z_3 \leq y_2 + y_3 \tag{3.25}$$
$$z_4 \leq y_3 + y_4 \tag{3.26}$$
$$z_5 \leq y_4 + y_5 \tag{3.27}$$
$$z_6 \leq y_5 \tag{3.28}$$
$$\sum_{i=1}^{5} y_i = 1 \quad y_i = 0 \text{ or } 1 \tag{3.29}$$
$$\sum_{i=1}^{6} z_i = 1 \quad z_i \geq 0 \tag{3.30}$$

The decision variables z_i and the binary variables y_i are related in a manner that we can determine the total cost function and quantity by using them. For instance let say that $y_2 = 1$ then all other y_i's are zero, this means that $z_2 \leq 0 + 1$ and $z_3 \leq 1 + 0$. Therefore y_2 determines the decision variables z_2 and z_3 where these two determines the range of the optimum quantity and minimum total cost. As a result, binary variables directs the decision variables and the decision variables sets the optimum decision on quantity and joint total cost.

Joint Total Cost Function

Up to this point we analyzed three cost functions. These cost functions are supplier, buyer and transportation costs. To find out the best result for both supplier and buyer including the shipment costs the following model is generated by adding all three functions.

$$STC = (\tfrac{D}{Q})(C_{os} + C_{ss}) + (\tfrac{D}{P_s})(\tfrac{Q}{2})(I_{rs} + I_{fs}) \qquad (3.31)$$

$$PTC = (\tfrac{D}{Q})(C_{op} + C_{sp}) + (\tfrac{Q}{2})(I_{rp} + I_{fp}) \qquad (3.32)$$

$$TTC(Q) = \begin{cases} \tfrac{D}{Q}\beta_l R_l & \beta_l \leq Q \leq \alpha_l \\ R_{l+1}D & \alpha_l \leq Q \leq \beta_{l+1} \end{cases} \qquad (3.33)$$

$$\text{Minimize } JTC = STC + PTC + TTC \qquad (3.34)$$

Subject to:

$$TTC = z_1 TTC(\alpha_0) + z_2 TTC(\beta_1) + z_3 TTC(\alpha_1) + z_4 TTC(\beta_2) \ldots \ (3.35)$$

$$Q = z_1\alpha_0 + z_2\beta_1 + z_3\alpha_1 + z_4\beta_2 \ldots \qquad (3.36)$$

$$z_1 \leq y_1 \qquad (3.37)$$

$$z_2 \leq y_1 + y_2 \qquad (3.38)$$

$$z_3 \leq y_2 + y_3 \qquad (3.39)$$

$$\vdots$$

$$z_{m-1} \leq y_{m-2} + y_{m-1} \qquad (3.40)$$

$$z_m \leq y_{m-1} \qquad (3.41)$$

$$\textstyle\sum_{i=1}^{m} y_i = 1 \quad y_i = 0 \text{ or } 1 \qquad (3.42)$$

$$\textstyle\sum_{i=1}^{m+1} z_i = 1 \quad z_i \geq 0 \qquad (3.43)$$

We explained all these three functions in the previous sections. But to remember them briefly, supplier and purchaser cost functions have three main
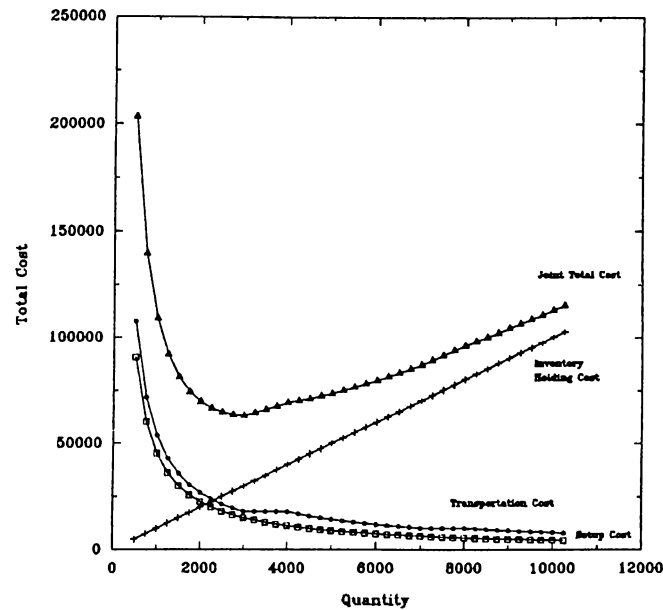
Figure 3.2: Total Cost versus Individual Cost Components

parts; ordering, setup, and inventory holding costs. The third element, transportation cost is a piecewise-linear function. So in order to make it linear we used the method and formalization technique given by Winston [41]. The result of our model formulation gave the above nonlinear mathematical programming model. Therefore, by solving this model we can obtain our first objective which is minimizing joint total cost. The general behaviors of each cost function in the joint total cost are presented in Figure 3.2. To achieve the second objective, which is maximizing the number of deliveries, we have to deal with shipment size concept where a new formulation is given in the following subsection.

Shipment Size Concept

One of the important concepts of JIT philosophy is the high frequency with small lots in the delivery process. Because of this, we try to see the performance of JIT delivery logic in our model. For that purpose, we arranged supplier, purchaser and transportation cost according to the shipment size. Basic idea

is to divide the optimum quantity, which we found from our model formulation, to n number of shipments and take a look at the behavior of each cost function. This analysis is very helpful for the decision maker when they can reduce the transportation cost with an additional investment. Otherwise frequent deliveries which is a JIT goal, will definitely increase the transportation cost. In order to maximize the number of deliveries, a decrease in the transportation charges should be obtained so that JIT philosophy and delivery system can be implemented.

Our model formulation for the shipment size concept is as follows:

$$PTC = \frac{D}{Q}(C_{op} + C_{sp}) + (1 + \frac{D}{P_s})(\frac{Q}{2})(I_{rp} + I_{fp}) \qquad (3.44)$$

$$STC = \frac{D}{Q}(C_{os} + C_{ss}) + (\frac{QD}{2P_s})I_{rs} + (\frac{XD}{2P_s})I_{fs} \qquad (3.45)$$

$$TTC(X) = \begin{cases} \frac{D}{X}\beta_l R_l & \beta_l \leq X \leq \alpha_l \\ R_{l+1}D & \alpha_l \leq X \leq \beta_{l+1} \end{cases} \qquad (3.46)$$

$$\text{Minimize } JTC = STC + PTC + n.TTC \qquad (3.47)$$

Subject to:

$$TTC = z_1 TTC(\alpha_0) + z_2 TTC(\beta_1) + z_3 TTC(\alpha_1) + z_4 TTC(\beta_2) \ldots (3.48)$$

$$Q = z_1\alpha_0 + z_2\beta_1 + z_3\alpha_1 + z_4\beta_2 \ldots \qquad (3.49)$$

$$Q = n.X \quad n \text{ is an integer} \qquad (3.50)$$

$$z_1 \leq y_1 \qquad (3.51)$$

$$z_2 \leq y_1 + y_2 \qquad (3.52)$$

$$z_3 \leq y_2 + y_3 \qquad (3.53)$$

$$\vdots$$

$$z_{m-1} \leq y_{m-2} + y_{m-1} \qquad (3.54)$$

$$z_m \leq y_{m-1} \qquad (3.55)$$

$$\sum_{i=1}^{m} y_i = 1 \quad y_i = 0 \text{ or } 1 \qquad (3.56)$$

$$\sum_{i=1}^{m+1} z_i = 1 \quad z_i \geq 0 \qquad (3.57)$$
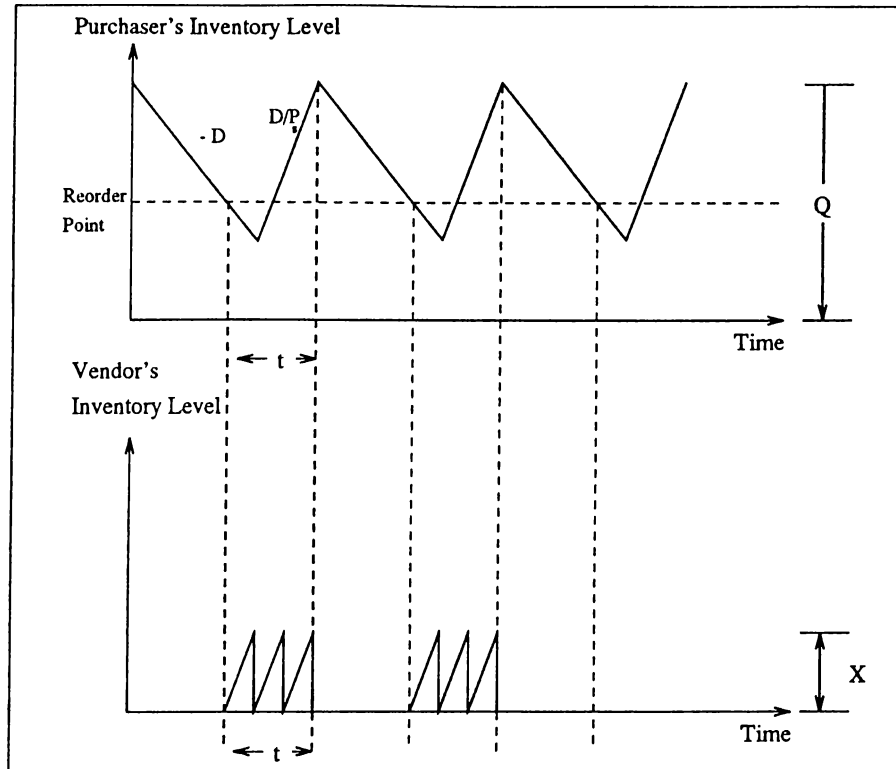
Figure 3.3: Purchaser's and Vendor's Inventory Time Plots for Shipment Size

At this point, in order to write the above equations, we assumed that production lot size, Q, is equal to n times shipment size, X, lead time is zero, and supplier production rate is greater or equal to the demand value. Under these circumstances, the inventory holding cost part of both supplier and the purchaser changed. The other parts, ordering and setup costs, are still same. For the supplier, raw material handling cost is the same but for the finished goods inventory part, as the supplier sends the product in X units, the cost of holding is the average inventory carrying cost of X units. For the purchaser, the scenario is different. It is simply average inventory holding cost when the demand is D and the production rate is P_s. Finally the third difference occurs in the transportation cost. As production lot size is equal to n times shipment size, the optimum quantity must be carried in n times, which is n times the shipment cost of X units. Here we assumed that n must be an integer. Additionally the presentation of the concept can be seen from the Figure 3.3.

As a result, in this analysis, we saw that supplier's finished goods inventory holding cost decreased by an amount of $\frac{(Q-X)D}{2P_s}I_{fs}$ and purchaser's holding cost increased by an amount of $\frac{DQ}{2P_s}(I_{rp}+I_{fp})$. Also transportation cost increased, as the decreasing shipment size caused frequent deliveries. All these results indicated that we should try to seek different ways to minimize the transportation cost in order to implement the JIT philosophy. Because the increase in the transportation cost cannot be easily offsetted with the decrease in the supplier cost function. To show the cost differences, the situation is analyzed with some numerical examples in chapter 5.

During the model formulation, we dealt with single supplier single purchaser purchasing scenario. In the following chapter, the possible extensions, multiple supplier, multiple buyer, of this scenarios are discussed.

Chapter 4

THE MODEL EXTENSION

In the model formulation of our joint total cost function, we restricted our analysis to single buyer, single supplier purchasing scenario. In this section we try to extend this scenario into two dimensions; multiple buyer and multiple supplier cases. All the model parameters are given in Table 4.1.

4.1 Possible Extensions

4.1.1 Single Supplier Multiple Buyer

Suppose that a number of purchasers order different quantities Q_i of an inventory item from a vendor. With the receipt of the orders, the vendor produces the required quantities of the item. At this point, it is assumed that sum of each buyers processing time of the required quantity must be less than or equal to the total processing time of the supplier so that suppliers capacity is enough to produce the required quantity. In addition to this deterministic condition, the vendor in question is the sole supplier and delivery requirements are made by a common carrier where it is assumed that the carriers capacity must be greater than or equal to the sum of the wanted quantities by the buyers. Common carrier concept is explained in detail in case 2.

When there is a single supplier and multiple buyers, supplier's capacity must be enough to produce all the required quantities. At this point, supplier is controlling all those buyers as it is the sole supplier. This makes the supplier more powerful on decision judgements about production and shipment. Another important point, is that, it is a disadvantage for the buyers as they depend on the supplier's production schedule. In addition, they need to reschedule their production for late arrivals from the supplier. Cost penalties will not work under this situation as the supplier is more powerful than the buyers. Also JIT system is really difficult to implement if the supplier is not using this philosophy. The reason is that if supplier is working in batch type production then the buyer who are working according to JIT will have some problems in the production area and in the shipment. Therefore in order to implement JIT into this situation, first, supplier must implement JIT logic. Finally, the geographical location plays an important role because of the common carrier concept. If the distance between buyers and the supplier are not close, the carrier's travelling time increases which makes the frequent deliveries really costly alternative with the common carrier. This problem can be formulated as follows:

$$STC = \frac{D}{Q}C_{os} + \frac{D}{Q}C_{ss} + (\frac{D}{P_s})(\frac{Q}{2})(I_{rs} + I_{fs}) \tag{4.1}$$

$$PTC_j = (\frac{D_j}{Q_j})C_{opj} + (\frac{D_j}{Q_j})C_{spj} + (\frac{Q_j}{2})(I_{rpj} + I_{fpj}) \tag{4.2}$$

$$TTC(Q) = \begin{cases} \frac{D}{Q}\beta_l R_l & \beta_l \leq Q \leq \alpha_l \\ R_{l+1}D & \alpha_l \leq Q \leq \beta_{l+1} \end{cases} \tag{4.3}$$

$$\text{Minimize} \quad JTC = STC + \sum_{j=1}^{m} PTC_j + TTC \tag{4.4}$$

$$\text{Subject to:} \quad TTC = z_1 TTC(\alpha_0) + z_2 TTC(\beta_1) + z_3 TTC(\alpha_1)$$

$$+ z_4 TTC(\beta_2) \ldots \tag{4.5}$$

$$Q = z_1\alpha_0 + z_2\beta_1 + z_3\alpha_1 + z_4\beta_2 \ldots \tag{4.6}$$

$$z_1 \leq y_1 \tag{4.7}$$

$$z_2 \leq y_1 + y_2 \tag{4.8}$$

$$z_3 \leq y_2 + y_3 \tag{4.9}$$

D	Annual demand or usage of the inventory item
P_s	Supplier's production rate
Q	Order or production lot size in units
C_{os}	Supplier's ordering cost per order
C_{op}	Purchaser's ordering cost per order
C_{ss}	Supplier's setup cost per setup
C_{sp}	Purchaser's setup cost per setup
I_{rs}	Supplier's inventory carrying cost of raw materials
I_{rp}	Purchaser's inventory carrying cost of raw materials
I_{fs}	Supplier's inventory carrying cost of finished goods
I_{fp}	Purchaser's inventory carrying cost of finished goods
a_i	Processing time of the ith supplier
b	Suppliers total time available
b_i	Available time of the ith supplier
JTC	Joint total cost
STC	Supplier's total cost
PTC	Purchaser's total cost
TTC	Transportation cost
WTC	Transportation cost from warehouse to buyer

Table 4.1: Notations used in the mathematical formulations

$$\vdots$$

$$z_{r-1} \leq y_{r-2} + y_{r-1} \qquad (4.10)$$

$$z_r \leq y_{r-1} \qquad (4.11)$$

$$\sum_{k=1}^{r} y_k = 1 \qquad y_k = 0 \text{ or } 1 \qquad (4.12)$$

$$\sum_{k=1}^{r+1} z_k = 1 \qquad z_k \geq 0 \qquad (4.13)$$

$$\sum_{j=1}^{m} aQ_j \leq b \qquad (4.14)$$

$$\sum_{j=1}^{m} Q_j = Q \qquad (4.15)$$

In this model, different from the JELS model, there are multiple buyers. Because of this, purchaser cost function is defined as the jth purchaser's cost function in equation (4.2) and is placed as a sum of purchasers cost function in equation (4.4) in the joint total cost function. Multiple buyer case also adds new constraints which are equations (4.14) and (4.15) to the mathematical

programming model. First one is the capacity constraint of supplier in which the total processing time should not exceed the supplier's total time available for production. Second one ensures the consistency between the supplier's lot-size with the purchaser's lot-sizes.

4.1.2   Single Buyer Multiple Supplier With Capacity Constraints

In this purchasing scenario, there is a number of suppliers producing the required item and a single purchaser is buying them (but multiple buyer case can easily be adapted by using the assumptions in case 1). Buyer periodically orders total quantity of Q of an inventory item to the suppliers. Each vendor produces his required quantity Q_i under certain capacity limits. Therefore, suppliers total production capacity must be greater than or equal to the buyer's required quantity. Also the shipment of the items to the buyer brings the question of transportation cost as there are multiple number of suppliers. There are three alternatives for this problem, which are as follows:

1. Each supplier has different transportation cost

2. Single transportation cost or common carrier case

3. Warehouse approach

Each supplier has different transportation cost

Each supplier carries the product by its own sources. Therefore transportation cost differs from supplier to supplier. So total transportation cost is equal to the the summation of the all suppliers transportation costs. But in order to use this alternative under JIT environment, suppliers must understand the JIT logic or use the JIT production system itself and behave accordingly. Because as all suppliers send their products by their own resources, be on time they

must deliver it according to JIT delivery system which is high frequency in number of deliveries with small high quality lots. Therefore without understanding this point this choice will be very costly for both buyer and suppliers as we are considering the joint cost in this analysis.

Additionally, the delivery schedule of the suppliers must be arranged according to JIT philosophy that no extra part will wait in the production area of the buyer. Another important point in this option is the geographical location of the buyer and suppliers. If the location of the buyer and its suppliers are in the same or almost same location, this choice can be adapted to JIT environment, otherwise the transportation part of the joint cost will increase in a way that to control and manage it becomes impossible. As a result in this alternative both suppliers and buyer understand JIT production and delivery system so that they can benefit from this relation. This alternative can be formulated as follows:

$$STC_i = (\tfrac{D}{Q_i})C_{osi} + (\tfrac{D}{Q_i})C_{ssi} + (\tfrac{D}{P_s})(\tfrac{Q_i}{2})(I_{rsi} + I_{fsi}) \qquad (4.16)$$

$$PTC = \tfrac{D}{Q}C_{op} + \tfrac{D}{Q}C_{sp} + (\tfrac{Q}{2})(I_{rp} + I_{fp}) \qquad (4.17)$$

$$TTC_i(Q_i) = \begin{cases} \tfrac{D}{Q_i}\beta_l R_{(l)i} & \beta_l \leq Q_i \leq \alpha_l \\ R_{(l+1)i}D & \alpha_l \leq Q_i \leq \beta_{l+1} \end{cases} \qquad (4.18)$$

$$\text{Minimize} \quad JTC = \sum_{i=1}^{n} STC_i + PTC + \sum_{i=1}^{n} TTC_i \qquad (4.19)$$

Subject to:

$$TTC_i = z_{1,i}TTC_i(\alpha_0) + z_{2,i}TTC_i(\beta_1) + z_{3,i}TTC_i(\alpha_1)$$

$$+z_{4,i}TTC_i(\beta_2) \ldots \quad \forall i \qquad (4.20)$$

$$Q_i = z_{1,i}\alpha_0 + z_{2,i}\beta_1 + z_{3,i}\alpha_1 + z_{4,i}\beta_2 \ldots \quad \forall i \qquad (4.21)$$

$$z_{1,i} \leq y_{1,i} \qquad \forall i \qquad (4.22)$$

$$z_{2,i} \leq y_{1,i} + y_{2,i} \qquad \forall i \qquad (4.23)$$

$$z_{3,i} \leq y_{2,i} + y_{3,i} \qquad \forall i \qquad (4.24)$$

$$z_{r-1,i} \leq y_{r-2,i} + y_{r-1,i} \qquad \forall i \qquad (4.25)$$

$$z_{r,i} \leq y_{r-1,i} \qquad \forall i \qquad (4.26)$$

$$\sum_{k=1}^{r} y_{k,i} = 1 \quad y_{k,i} = 0 \ \text{or} \ 1 \quad \forall i \qquad (4.27)$$

$$\sum_{k=1}^{r+1} z_{k,i} = 1 \quad z_{k,i} \geq 0 \qquad \forall i \qquad (4.28)$$

$$a_i Q_i \leq b_i \qquad \forall i \qquad (4.29)$$

In this model constraint (4.16) and (4.18) differ from the original model. The reason is that, there are multiple number of suppliers. So each supplier's cost function must be mentioned. And in the joint total cost function, suppliers total cost is equal to the summation of all suppliers cost values. Additionally, each supplier sends their product by their own resources, therefore transportation cost differs from supplier to supplier. So it is also presented the summation of all transportation cost values in the joint cost function.

As it was mentioned before, this model has three alternatives. For all of them, the above statement is true. Constraint (4.29) ensures that the production lot-sizes for each supplier is less than or equal to the suppliers available capacity.

Single transportation cost or common carrier case

As it was briefly mentioned in case 1, buyer sends a carrier to the suppliers, carrier travels all the suppliers in any order and pick up all the items that they produced. The assumption is that carriers load capacity is greater than or equal to the sum of produced quantity by suppliers. By making this assumption, vehicle routing problem is eliminated.

As in the first alternative in this second option, the geographical location of the suppliers are important. The reason is that, as the buyer sends carrier to the suppliers, they must be close enough to travel by the carrier. Furthermore, the pickup and delivery must be done on a scheduled basis. Additionally to use a single carrier to travel the suppliers the amount to pickup must be arranged in a way that the carrier capacity will be enough for them. For that purpose buyer must adjust the quantity that each supplier must give to the carrier so

that the carrier will not be over loaded.

This type of delivery can easily be adapted to JIT environment if the location of the suppliers are close to each other and to the buyer. Additionally it can be used more than one day in a week so that the high frequency in delivery is obtained. Ansari and Heckel [1], in their paper recommended ABC analysis for determination of the frequency of the deliveries. They advised the high frequency delivery for A and B type inventory items. They also suggested to purchase these type of inventory items in daily basis. To stock this kind of materials increase in the inventory holding cost as cost per part is very high. Therefore, under this type of shipment in daily basis, the JIT delivery system can be easily implemented. But the geographical location is so important that the buyer which wants to use such a pickup and delivery system, must recheck the locations of his suppliers. A good example to this situation happened in United States. Hoover moved one of its factories to the plant of Nissan which is one of the important buyer of Hoover [26]. This example shows the importance of a close relationship between suppliers and the buyer. This case can be formulated as follows:

$$STC_i = (\tfrac{D}{Q_i})C_{osi} + (\tfrac{D}{Q_i})C_{ssi} + (\tfrac{D}{P})(\tfrac{Q_i}{2})(I_{rsi} + I_{fsi}) \qquad (4.30)$$

$$PTC = \tfrac{D}{Q}C_{op} + \tfrac{D}{Q}C_{sp} + (\tfrac{D}{P_s})(\tfrac{Q}{2})(I_{rp} + I_{fp}) \qquad (4.31)$$

$$TTC(Q) = \begin{cases} \tfrac{D}{Q}\beta_l R_l & \beta_l \leq Q \leq \alpha_l \\ R_{l+1}D & \alpha_l \leq Q \leq \beta_{l+1} \end{cases} \qquad (4.32)$$

$$\text{Minimize} \quad JTC = \sum_{i=1}^{n} STC_i + PTC + TTC \qquad (4.33)$$

Subject to:

$$TTC = z_1 TTC(\alpha_0) + z_2 TTC(\beta_1) + z_3 TTC(\alpha_1) + z_4 TTC(\beta_2) \ldots (4.34)$$

$$Q = z_1\alpha_0 + z_2\beta_1 + z_3\alpha_1 + z_4\beta_2 \ldots \qquad (4.35)$$

$$z_1 \leq y_1 \qquad (4.36)$$

$$z_2 \leq y_1 + y_2 \tag{4.37}$$

$$z_3 \leq y_2 + y_3 \tag{4.38}$$

$$\vdots$$

$$z_{r-1} \leq y_{r-2} + y_{r-1} \tag{4.39}$$

$$z_r \leq y_{r-1} \tag{4.40}$$

$$\sum_{k=1}^{r} y_k = 1 \quad y_k = 0 \text{ or } 1 \tag{4.41}$$

$$\sum_{k=1}^{r+1} z_k = 1 \quad z_k \geq 0 \tag{4.42}$$

$$\sum_{i=1}^{n} Q_i = Q \tag{4.43}$$

$$a_i Q_i \leq b_i \qquad \forall i \tag{4.44}$$

In this alternative, as in the first one, the same variables and constraints were used. Only difference came from the equation (4.43) which means, buyer's total required quantity must be equal to the sum of each supplier's produced quantity so that buyer do not have extra product waiting in the production area or do not have enough product to produce his required quantity.

Warehousing Approach

This situation is a combination of the alternatives one and two. The reason is that all suppliers send their products to the common warehouse by their own sources. From the warehouse, buyer sends his own carriers to the warehouse and picks up the items. So the first part consists of different transportation costs and the second part is an example of single transportation cost. Both of them are added to the joint total cost function.

JIT delivery can also be achieved from a warehouse interposed between suppliers and the buyer. This can be owned by the supplier or by one of the buyers. However, it may also be a common warehouse owned by both of them. JIT supply of the manufacturer from an interposed warehouse is a good alternative for certain conditions such as; suppliers situated at a considerable distance away from the buyer, batch type production by the supplier and therefore low

flexibility in production, average transportation requirements, etc. For materials and components to which these conditions apply, JIT delivery to the buyers would be uneconomical. However, this does not mean that an intensive coordination between buyer and the suppliers would not be advantageous for both. If in that common warehouse certain processes are done both suppliers and buyer can gain from this warehouse. For example if, other than warehousing, this place do the receiving and checking, administrative activities, inventory control and expediting, both suppliers and buyer can benefit from time and certain costs such as transportation costs. Warehousing approach is presented in Figure 4.1. For instance, if the products are checked and controlled in the warehouse, the defective items could be separated from the normal ones and sent back to the supplier to produce them again. Advantage of this process is that, after the control is done in the warehouse, there will be no need to check these items before entering the production area and these parts can simply enter the assembly area. As a result, this approach could be one of the best implementation of JIT purchasing and delivery system, and can be formulated as follows:

$$STC_i = (\tfrac{D}{Q_i})C_{osi} + (\tfrac{D}{Q_i})C_{ssi} + (\tfrac{D}{P_j})(\tfrac{Q_i}{2})(I_{rsi} + I_{fsi}) \qquad (4.45)$$

$$PTC = \tfrac{D}{Q}C_{op} + \tfrac{D}{Q}C_{sp} + (\tfrac{Q}{2})(I_{rp} + I_{fp}) \qquad (4.46)$$

$$TTC_i(Q) = \begin{cases} \tfrac{D}{Q_i}\beta_l R_l & \beta_l \leq Q_i \leq \alpha_l \\ R_{(l+1)i}D & \alpha_l \leq Q_i \leq \beta_{l+1} \end{cases} \qquad (4.47)$$

$$WTC(Q) = \begin{cases} \tfrac{D}{Q}\beta_l R_l & \beta_l \leq Q \leq \alpha_l \\ R_{l+1}D & \alpha_l \leq Q \leq \beta_{l+1} \end{cases} \qquad (4.48)$$

$$\text{Minimize} \quad JTC = \sum_{i=1}^{n} STC_i + PTC + \sum_{i=1}^{n} TTC_i + WTC \qquad (4.49)$$

Subject to:

$$TTC_i = z_{1,i}TTC_i(\alpha_0) + z_{2,i}TTC_i(\beta_1) + z_{3,i}TTC_i(\alpha_1)$$
$$+z_{4,i}TTC_i(\beta_2)\ldots \forall i \qquad (4.50)$$

$$Q_i = z_{1,i}\alpha_0 + z_{2,i}\beta_1 + z_{3,i}\alpha_1 + z_{4,i}\beta_2 \dots \forall i \tag{4.51}$$

$$z_{1,i} \leq y_{1,i} \qquad \forall i \tag{4.52}$$

$$z_{2,i} \leq y_{1,i} + y_{2,i} \qquad \forall i \tag{4.53}$$

$$z_{3,i} \leq y_{2,i} + y_{3,i} \qquad \forall i \tag{4.54}$$

$$z_{r-1,i} \leq y_{r-2,i} + y_{r-1,i} \qquad \forall i \tag{4.55}$$

$$z_{r,i} \leq y_{r-1,i} \qquad \forall i \tag{4.56}$$

$$\sum_{k=1}^{r} y_{k,i} = 1 \quad y_{k,i} = 0 \text{ or } 1 \quad \forall i \tag{4.57}$$

$$\sum_{k=1}^{r+1} z_{k,i} = 1 \quad z_{k,i} \geq 0 \qquad \forall i \tag{4.58}$$

$$a_i Q_i \leq b_i \qquad \forall i \tag{4.59}$$

$$\sum_{i=1}^{n} Q_i = Q \qquad \forall i \tag{4.60}$$

$$WTC = v_1 WTC(\alpha_0) + v_2 WTC(\beta_1) + v_3 WTC(\alpha_1)$$
$$+ v_4 WTC(\beta_2) \dots \tag{4.61}$$

$$Q = v_1 \alpha_0 + v_2 \beta_1 + v_3 \alpha_1 + v_4 \beta_2 \dots \tag{4.62}$$

$$v_1 \leq t_1 \tag{4.63}$$

$$v_2 \leq t_1 + t_2 \tag{4.64}$$

$$v_3 \leq t_2 + t_3 \tag{4.65}$$

$$\tag{4.66}$$

$$\vdots$$

$$v_{w-1} \leq t_{w-2} + t_{w-1} \tag{4.67}$$

$$v_w \leq t_{w-1} \tag{4.68}$$

$$\sum_{l=1}^{w} t_l = 1 \quad t_l = 0 \text{ or } 1 \tag{4.69}$$

$$\sum_{l=1}^{w+1} v_l = 1 \quad v_l \geq 0 \tag{4.70}$$

In this last alternative, another transportation cost is added to the joint total cost function. It is the shipment cost of carrying products from the warehouse to the buyer's plant and presented as WTC in equation (4.48).
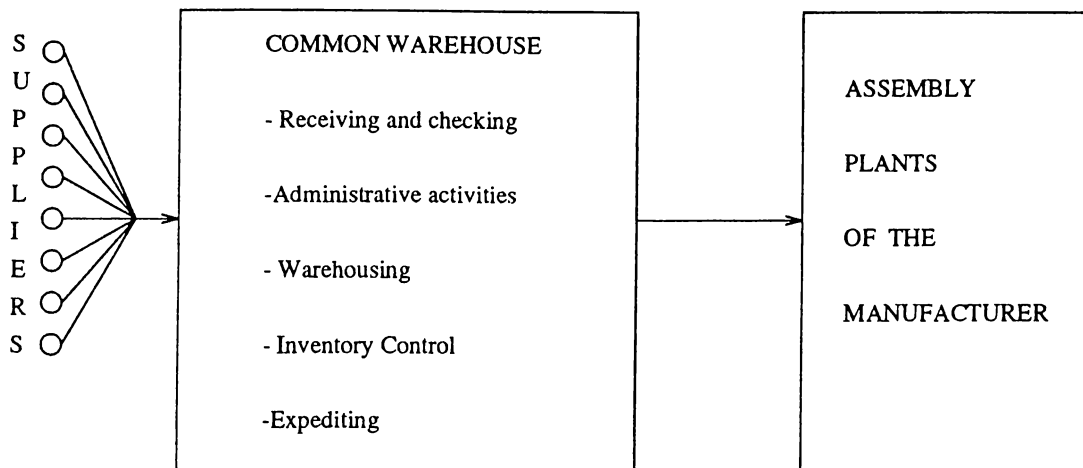
Figure 4.1: Warehousing approach for JIT supply

4.2 Chapter Summary

Through the model extensions, two dimensions of our model were discussed. These were multiple buyer and multiple supplier cases. Taking our single buyer single supplier model as a base, by these extensions, it was tried to show the different type purchasing scenarios which make the situation closer to real world circumstances. As a consequence, these model extensions added some extra constraints, and variables. This will increase the completion time to solve these formulations, but does not effect the proposed solution procedure.

Another important point during this extension is the JIT implementation. These cases showed that JIT can be implemented to all of them. But the JIT logic must be understood by both parties. In addition, it must be added that JIT implementation to multiple supplier case is lot easier than multiple buyer case. Because nowadays suppliers are becoming a part of the buyers so that working with a supplier which is producing to several buyers lost its advantages under this economical conditions. From the multiple supplier case, the first alternative offered, each supplier has different transportation cost, can only be used when both suppliers and the buyer are JIT users and close geographical locations. Otherwise batch production may not work effectively in a JIT philosophy. Second alternative, single transportation cost or common carrier case, is more easy to implement but again the capacity limits of the carrier and

production schedules must be arranged in a way that no extra part will wait in the production area. Finally the third alternative, warehouse approach, is the best one for suppliers that are not using JIT production system or geographically apart from each other. In this alternative suppliers can produce in batches and from the warehouse buyer takes the quantity that he will use in the production.

All these analysis in the model extensions, multiple supplier case was to be considered as the preferable one because of its logical approach to JIT. In addition to this fact, the first alternative of the multiple supplier option, each supplier has different transportation cost, could be the best choice if both supplier and buyer are using JIT production system. However; the third alternative, which is the warehouse approach could also be advantegous for the non-JIT users in the production as it provides batch type production for both parties.

After these extensions, the computational analysis of the solution procedure for single buyer and single supplier is discussed in the next chapter. Also the experimental design procedure and the results are given.

Chapter 5

COMPUTATIONAL ANALYSIS

Model formulation of the joint total cost function showed that there were three main parts in this cost function; supplier, purchaser, and transportation costs. In addition, various extensions can be generated from the joint function such as multiple supplier and multiple buyer cases. These were discussed in chapters 3 and 4.

In this chapter, we have two main objectives, which are:

- Computational analysis of the model

- Analysis of variance

To verify our model and its extensions, an experimental plan is developed to generate a set of test problems. Experiments are carried out by investigators in all fields of study either to discover something about a particular process or to compare the effect of several factors on some phenomena. In industrial research, the experiment is almost always an intervention or change in the routine operation of a system which is made with the objective of measuring the effect of intervention.

5.1 Experimental Design

Statistical design of experiments is the process of planning the experiment so that appropriate data will be collected which may be analyzed by statistical methods resulting in valid and objective conclusions. The statistical approach to experimental design is necessary if we wish to draw meaningful conclusions from the data. When the problem involves data that are subject to experimental errors, statistical methodology is the only objective approach to analysis. Thus there are two aspects of any experimental problem: the design of the experiment and the statistical analysis of the data.

To use the statistical approach in designing and analyzing an experiment, Montgomery [30] gives an outline of the recommended procedure. We use this procedure to explain our experimental problem. The procedure is as follows:

1. Recognition of and statement of the problem

2. Choice of factors and levels

3. Selection of a response variable

4. Choice of experimental design

5. Performing the experiment

6. Data Analysis

7. Conclusions and Recommendations

Detailed analysis of each step is discussed in the following subsections.

5.1.1 Recognition of and statement of the problem

The experimental design problem in this study is the joint economic lot size model(JELS). The model combines both suppliers and the buyers cost function in a single function. In addition to that, transportation cost function is also

added to identify the importance of it. JELS model tries to obtain middle way between purchaser and vendor so that both can minimize their cost function and gain from the relationship. During the analysis we also dealt with the shipment size, since high frequency with small lots is one of the important concepts in JIT philosophy. Also some recommendations are given to use the proposed JELS model in such a philosophy.

5.1.2 Choice of factors and levels

While choosing the factors of the experiment, JELS's model cost function is used. It is made of three main parts. First part is the supplier cost, secondly the buyer cost part and finally transportation cost part. As it was mentioned in the literature review part, transportation cost function was added to this cost model in very few studies. To understand whether it has an effect on the cost function and lot sizing, transportation rate value is chosen as the first factor of the experiment. Then economic order quantity of the JELS model is found without considering the transportation cost part. Furthermore this formula showed that demand must be the second factor as it is one of the important determiners of the economic order quantity. Third factor is also found from EOQ formula which is setup plus order cost(from now on setup plus order cost is mentioned as setup cost only), to inventory holding cost ratio. The reason is square root of S/I ratio times demand gives the economic order quantity (EOQ). Therefore this ratio plays an important role in lot-sizing policies. In addition, these costs are the main parts of the joint total cost function. For that purpose S/I ratio should be considered in the experimental plan. But in order to identify the supplier's and buyer's role in the joint model, two additional factors are defined which are purchaser S/I ratio and supplier S/I ratio. As a result it is decided to have four factors in this experiment.

Determination of the levels is another concept. After choosing the factors, the behavior of the factors in the lot sizing model were checked. All factors do not behave in linear terms, sometimes they act like quadratic functions. For that reason three levels are setted for each of the factors so that it could be easy to understand the significance of each factor in the model. These levels

Factors and levels	LOW	MEDIUM	HIGH
Demand	U(500,700)	U(2000,4000)	U(7000,10000)
Supplier S/I Ratio	U(1,5)	U(40,50)	U(250,350)
Purchaser S/I Ratio	U(1,5)	U(40,50)	U(250,350)
Transportation Rate	U(1,2)	U(5,10)	U(20,30)

U stands for the uniform distribution

Table 5.1: Generated Data Summary

are Low, Medium, High. For each of the factors these levels are given in some ranges, as shown in Table 5.1.

5.1.3   Selection of a response variable

All the factors chosen are the elements of the joint cost function. To see their behavior and importance in the model generated, the cost function is used and the model itself tries to minimize this cost function. Therefore joint total cost function is dependent on the factors mentioned. As a result it becomes the dependent variable which means the response variable. For different values of factors the cost is calculated, and the results and conclusions are drawn according to these values.

5.1.4   Choice of experimental design

Since we have four factors, full factorial design is the most efficient choice for this type of experiment. By a factorial design, we mean that in each complete trial or replication of the experiment all possible combinations of the levels of the factors investigated. Note that factorial designs have several advantages. They are more efficient than one-factor-at-a-time experiments. Furthermore, a factorial design is necessary when interactions may be present, to avoid misleading conclusions. Finally factorial designs allow effect of a factor to be estimated at several levels of the other factors, yielding conclusions that are valid over a range of experimental conditions. As a result factorial design is chosen for our experiment. As it was stated we have four factors with

$C_{os} + C_{ss}$	$\leq$	3000
$C_{os} + C_{ss}$	$\leq$	3000
Supplier Inventory holding cost $= \frac{1}{3}.I_{rs} + \frac{2}{3}.I_{fs}$		
Purchaser Inventory holding cost $= \frac{1}{3}.I_{rp} + \frac{2}{3}.I_{fp}$		
$I_{rs} + I_{fs} = 0.75 \times R_1$		
$I_{rp} + I_{fp} = 0.75 \times R_1$		

Table 5.2: Assumptions of the Experimental Plan

three levels so that our design is 3^4 factorial design which means eighty one treatment combinations and sample size of the experiment is chosen to be 243 which means three replicates of the factor levels are setted in ranges.

5.1.5 Performing the experiment

In the last three steps, the factors, their levels, the response variable and the design of the experiment was chosen. To perform the experiment using all these decisions, a data set is generated. For formation of 243 numbers a PASCAL program was written and used. All the numbers were generated uniformly and real numbers are rounded to its nearest integer so that further calculations can be made more easily. Then by using supplier's and buyer's S/I ratios, the values of setup, ordering, and inventory holding costs were calculated. But during these calculations, three assumptions were made so that the numbers are fit into our model and experimental design. Also these assumptions make our purchasing scenario more realistic. These assumptions and fixed system parameters are given in Table 5.2 and Table 5.3. The definition of fixed system parameters were given in chapter 3.

After calculating all these terms which are the parameters of the joint total cost function, GINO, which is a nonlinear programming software, was used to solve the JELS model. However, GINO cannot deal with the mixed-integer nonlinear programming problems. Therefore we used enumerative method in order to solve the mixed-integer nonlinear problem. Generation of the problems are done by a Pascal program. It took approximately 20 seconds on a PC-486 to solve our model, which is a reasonable computation time for such a decision

Production rate	10000
Freight Rate 1 (β_1)	250
Freight Rate 2 (β_2)	4000
Freight Rate 3 (β_3)	8000
Indifference point 1 (α_0)	0
Indifference point 2 (α_1)	3000
Indifference point 3 (α_2)	7000

Table 5.3: Fixed system parameters

problem. The minimum cost of these six models was chosen to be the optimum joint total cost and the quantity to be the optimum quantity.

The Yates Algorithm which will be discussed in detail in the next section is applied to construct the analysis of variance table. The experimental plan of this algorithm is given in Table 5.4.

5.1.6 Data Analysis

Up to this point, all the necessary operations were done to achieve the main objectives which we were as follows:

- Computational Analysis of the data

    - Joint Total Cost Comparison

    - Shipment Size Cost Comparison

- Analysis of Variance (ANOVA)

In the joint cost comparison, we analyzed the situation from the point of buyer and supplier separately. For that purpose, supplier's and buyer's cost function were discussed and economic order quantities (EOQ) were found. These cost functions and EOQs are as follows:

For the supplier;

$$STC = \frac{D}{Q}(C_{os} + C_{ss}) + (\frac{D}{P_s})(\frac{Q}{2})(I_{rs} + I_{fs}) \qquad (5.1)$$

Tr. Rate	S. S/I Ratio	P. S/I Ratio	Demand	Plan	Div	Div	$2^r.3^t.n$
A	B	C	D	Effect	r	t	
0	0	0	0				
1	0	0	0	A_L	1	3	162
2	0	0	0	A_Q	1	4	486
0	1	0	0	B_L	1	3	162
1	1	0	0	$AB_{L \times L}$	2	2	108
2	1	0	0	$AB_{Q \times L}$	2	3	324
0	2	0	0	B_Q	1	4	486
1	2	0	0	$AB_{L \times Q}$	2	3	324
2	2	0	0	$AB_{Q \times Q}$	2	4	972
0	0	1	0	C_L	1	3	162
1	0	1	0	$AC_{L \times L}$	2	2	108
2	0	1	0	$AC_{Q \times L}$	2	3	324
0	1	1	0	$BC_{L \times L}$	2	2	108
1	1	1	0	$ABC_{L \times L \times L}$	3	1	48
2	1	1	0	$ABC_{Q \times L \times L}$	3	2	216
0	2	1	0	$BC_{Q \times L}$	2	3	324
1	2	1	0	$ABC_{L \times Q \times L}$	3	2	216
2	2	1	0	$ABC_{Q \times Q \times L}$	3	3	648
0	0	2	0	C_Q	1	4	486
1	0	2	0	$AC_{L \times Q}$	2	3	324
2	0	2	0	$AC_{Q \times Q}$	2	4	972
0	1	2	0	$BC_{L \times Q}$	2	3	324
1	1	2	0	$ABC_{L \times L \times Q}$	3	2	108
2	1	2	0	$ABC_{Q \times L \times Q}$	3	3	648
0	2	2	0	$BC_{Q \times Q}$	2	4	972
1	2	2	0	$ABC_{L \times Q \times Q}$	3	3	648
2	2	2	0	$ABC_{Q \times Q \times Q}$	3	4	1944
0	0	0	1	D_L	1	3	162
1	0	0	1	$AD_{L \times L}$	2	2	108
2	0	0	1	$AD_{Q \times L}$	2	3	324
0	1	0	1	$BD_{L \times L}$	2	2	108
1	1	0	1	$ABD_{L \times L \times L}$	3	1	72
2	1	0	1	$ABD_{Q \times L \times L}$	3	2	216
0	2	0	1	$BD_{Q \times L}$	2	3	324
1	2	0	1	$ABD_{L \times Q \times L}$	3	2	216
2	2	0	1	$ABD_{Q \times Q \times L}$	3	3	648
0	0	1	1	$CD_{L \times L}$	2	2	108
1	0	1	1	$ACD_{L \times L \times L}$	3	1	72
2	0	1	1	$ACD_{Q \times L \times L}$	3	2	216
0	1	1	1	$BCD_{L \times L \times L}$	3	1	72
1	1	1	1	$ABCD_{L \times L \times L \times L}$	4	0	48

Table 5.4: Continued on next page

Tr. Rate	S. S/I Ratio	P. S/I Ratio	Demand	Plan	Div	Div	$2^r.3^t.n$
A	B	C	D	Effect	r	t	
2	1	1	1	$ABCD_{Q\times L\times L\times L}$	4	1	144
0	2	1	1	$BCD_{Q\times L\times L}$	3	2	216
1	2	1	1	$ABCD_{L\times Q\times L\times L}$	4	1	144
2	2	1	1	$ABCD_{Q\times Q\times L\times L}$	4	2	432
0	0	2	1	$CD_{Q\times L}$	2	3	324
1	0	2	1	$ACD_{L\times Q\times L}$	3	2	216
2	0	2	1	$ACD_{Q\times Q\times L}$	3	3	648
0	1	2	1	$BCD_{L\times Q\times L}$	3	2	216
1	1	2	1	$ABCD_{L\times L\times Q\times L}$	4	1	144
2	1	2	1	$ABCD_{Q\times L\times Q\times L}$	4	2	432
0	2	2	1	$BCD_{Q\times Q\times L}$	3	3	648
1	2	2	1	$ABCD_{L\times Q\times Q\times L}$	4	2	432
2	2	2	1	$ABCD_{Q\times Q\times Q\times L}$	4	3	1296
0	0	0	2	D_Q	1	4	486
1	0	0	2	$AD_{L\times Q}$	2	3	324
2	0	0	2	$AD_{Q\times Q}$	2	4	972
0	1	0	2	$BD_{L\times Q}$	2	3	324
1	1	0	2	$ABD_{L\times L\times Q}$	3	2	216
2	1	0	2	$ABD_{Q\times L\times Q}$	3	3	648
0	2	0	2	$BD_{Q\times Q}$	2	4	972
1	2	0	2	$ABD_{L\times Q\times Q}$	3	3	648
2	2	0	2	$ABD_{Q\times Q\times Q}$	3	4	1944
0	0	1	2	$CD_{L\times Q}$	2	3	324
1	0	1	2	$ACD_{L\times L\times Q}$	3	2	216
2	0	1	2	$ACD_{Q\times L\times Q}$	3	3	648
0	1	1	2	$BCD_{L\times L\times Q}$	3	2	216
1	1	1	2	$ABCD_{L\times L\times L\times Q}$	4	1	144
2	1	1	2	$ABCD_{Q\times Q\times L\times Q}$	4	2	432
0	2	1	2	$BCD_{Q\times L\times Q}$	3	3	648
1	2	1	2	$ABCD_{L\times Q\times L\times Q}$	4	2	432
2	2	1	2	$ABCD_{Q\times Q\times L\times Q}$	4	3	1296
0	0	2	2	$CD_{Q\times Q}$	2	4	972
1	0	2	2	$ACD_{L\times Q\times Q}$	3	3	648
2	0	2	2	$ACD_{Q\times Q\times Q}$	3	4	1944
0	1	2	2	$BCD_{L\times Q\times Q}$	3	3	648
1	1	2	2	$ABCD_{L\times L\times Q\times Q}$	4	2	432
2	1	2	2	$ABCD_{Q\times L\times Q\times Q}$	4	3	1296
0	2	2	2	$BCD_{Q\times Q\times Q}$	3	4	1944
1	2	2	2	$ABCD_{L\times Q\times Q\times Q}$	4	3	1296
2	2	2	2	$ABCD_{Q\times Q\times Q\times Q}$	4	4	3888

0 stands for LOW, 1 stands for MEDIUM, 2 stands for HIGH

Table 5.4: The Plan of the Yates Algorithm for 3^4 Design

$$EOQ_s = \sqrt{\frac{2P_s(C_{os}+C_{ss})}{I_{rs}+I_{fs}}} \tag{5.2}$$

For the purchaser;

$$PTC = \frac{D}{Q}(C_{op} + C_{sp}) + \frac{Q}{2}(I_{rp} + I_{fp}) \tag{5.3}$$

$$EOQ_p = \sqrt{\frac{2D(C_{op}+C_{sp})}{I_{rp}+I_{fp}}} \tag{5.4}$$

Next step was to apply these EOQs to our joint total cost function. Then we compared the results with our optimal solution values. The differences between these values were presented in percentages. The calculation is as follows:

Percentage Difference for Supplier;

$$PD_s = \frac{JELS(Q^*) - JTC(EOQ_s)}{JELS(Q^*)}.100 \tag{5.5}$$

Percentage Difference for Purchaser;

$$PD_p = \frac{JELS(Q^*) - JTC(EOQ_p)}{JELS(Q^*)}.100 \tag{5.6}$$

where Q^* is the optimum quantity of our model.

The minimum and the maximum values for percentage differences for supplier and buyer were given in Table 5.5. These results show that our JELS model at the worst case is 3.33% better than supplier EOQ and 4.35% better than purchaser EOQ. This situation occurs when demand and S/I ratios are high for both parties and transportation rate low for the supplier and transportation rate medium for the purchaser. The reason for is that as the setup cost and demand is high, the EOQs appear very close to the joint model. The importance of setup and inventory holding cost increase and transportation cost decreases. But still our joint model performs better than the separate supplier and purchaser EOQ models.

For the best case of our model, the difference for supplier reaches to a point of 436.22% when supplier S/I ratio is low and other factors are high. For the purchaser it becomes 486.84% when purchaser S/I ratio is low and other factors are high. Since S/I ratios are low, each party tries to send the products in small

	MIN	AVG	MAX
PD_s	- 3.33%	- 86.09%	- 436.22%
PD_p	- 4.35%	- 100.56%	- 486.84%

Table 5.5: Min-Max values in Percentage Differences

batches which increases, the transportation cost. As you can see from the Table 5.5, our results on the average is 86.09% better the supplier's PD value and 100.56% better than the purchaser's PD value. Consequently, as seen from the above calculations, percentage differences were calculated according to the EOQ formula. In both, supplier and buyer the only common thing in EOQs are the S/I ratios. In order to analyze the PD value calculations in detail, it is decided to examine them according to each parties S/I ratio levels. This generates nine different cases, which are studied in the following paragraphs.

The first case is where both S/I ratios are low. It was understood that when supplier and purchaser S/I ratios were low, the percentage differences (PD) of both parties appeared nearly same to each other but they changed in a range of; for the supplier, (-38.95%, -325.18%), for the purchaser, (-32.13%, -311.67%). These PD values increased when the demand level increased. The values given in the ranges as min and max values had appeared when demand is low and high respectively. The reason for the increase is that, as the S/I ratios are low, the EOQ values arises as small numbers, and this increases the transportation costs. On the other hand, when demand is high from D/P_s ratio in the inventory holding cost that part increases. Therefore the increase in transportation and inventory holding costs will definitely increase the overall solution.

The second case appears, when purchaser S/I ratio is low and supplier S/I ratio is medium. In this situation, the PD differed and supplier difference is much less than the purchaser's PD. Again when demand level increases, the PDs increase. The range for the supplier is (-12.94%, -40.76%), and for the purchaser is (-96.70%, -323.01%). The reason for such a difference occurs between supplier and buyer, and the joint cost is, when purchaser S/I ratio is

low, the EOQ_p appears a small quantity compared to demand so the setup cost and inventory cost decreases but the transportation cost increases drastically that such a difference occurs. For the supplier, as the ratio is medium, EOQ_s is found to be a very close to the joint quantity value. Setup cost decreases as the quantity increased, also the transportation cost decreases as shipment size increased, on the other hand inventory holding cost increases as the quantity increased. But this time, the decreases offset the increase in inventory cost that the range appeared 10 percent close the optimum value.

The third case is the situation when the purchaser S/I ratio is low, and supplier S/I ratio is high. This is similar to the above discussion. Only difference comes from the the quantity. The EOQ_p is again a low quantity because of the low S/I ratio. Compared with the supplier PD values, the purchaser's PD values in this case is much greater. The reason is that for the supplier, as the D/P ratio becomes closer to one when the demand level increases, the percentage difference becomes closer to the optimal value. The ranges for the supplier is (-5.81%, -93.34%), and for the purchaser, (-179.49%, -486.84%). The maximum PD value arised in this category where purchaser wants small lots as his setup cost and inventory cost are low and the supplier wants to send the lots in big batches to minimize those costs. But decrease in the lot size will increase the transportation cost that no other cost can offset such increase.

The fourth case appears when purchaser S/I ratio is medium and supplier S/I ratio is low. When demand is low the PD values appeared to be nearly same where both EOQ value is very small compared to the optimal quantity. As the S/I ratio is small the EOQ_s is also small, and for the purchaser, demand is low the EOQ_p is also small. When the quantities are small the increase in the shipment cost can not be offsetted by the decrease in setup and inventory costs. When demand level increases the value of the EOQ_p becomes closer to the optimal quantity, however the supplier PD increases in a very high trend because the EOQ_s do not depend on the demand value. The ranges for the supplier is, (-39.30%, -374.36%) and for the purchaser (-44.60%, -65.20%).

The fifth case is the situation of supplier and purchaser have medium S/I

ratio. The formulation showed that other than high demand values, there is a significant difference between PD values of both parties. The reason again arised from the economic order quantities. Furthermore this quantities effects the transportation cost as the shipment size which is EOQs are small. The range for the supplier is (-8.66%, -40.67%) and for the purchaser (-48.37%, -68.08%).

The sixth case appears when purchaser S/I ratio is medium and supplier S/I ratio is high. The EOQs play an important role again. The PD value for the purchaser stays in a very narrow range as the EOQ_p depends on demand values. On the other hand, supplier's S/I ratio is high and its EOQ_s depends on the production rate which is a constant. For that reason, EOQ_s appears to be very close number to optimal values. Also PD values of supplier decreases when the level of demand increases because of the D/P_s of the inventory holding cost. The ranges are; for the supplier (-5.06%, -82.92%) and for the purchaser (-57.82% , -100.53%).

The seventh case is the situation of purchaser S/I ratio is high, supplier S/I ratio is low. Not surprisingly purchaser PD values appeared to be smaller which means the cost values become closer. For the supplier same situation continues as in the fifth case. The effect of transportation cost and the demand values can easily be seen in this situation. The range for the supplier (-69.52%, -436.22%) and for the purchaser (-6.37%, -18.00%). The maximum PD value for the supplier appears in this case. In this situation purchaser requires the products in big lots so that the setup and transportation cost can be minimized. On the other hand, supplier's setup cost is very low that he/she tries to send in small lots. But again as the number of shipments increase, transportation cost increases. This increase will effect the overall solution.

The eight case appears when the purchaser S/I ratio is high and the supplier S/I ratio is medium. Here again, there is a difference between quantities so that the PD values differ from each other. Supplier EOQ is smaller than purchaser EOQ so that the transportation cost of supplier is much more greater than purchaser's. Such differences occur when demand level is high. When the

demand level increases, the PD values of purchaser decreases and PD values of supplier increases. The ranges are; for the supplier (-4.43%, -64.37%) and for the purchaser (-5.38%, -26.11%).

The last case is the situation where both S/I ratios are high. Similar results had obtained with the eighth case. But the PD values become more closer to zero which means close values in the cost functions. Also for both purchaser and supplier the minimum PD values arised under this case. The ranges are; for the supplier (-3.33%, -45.05%) and for the purchaser (-4.35%, -31.04%).

All these calculation show that during the level changes in the S/I ratios, the PD values changed according to the demand and transportation rate. Especially, transportation cost is so important that any decrease or increase effected the overall solution. Therefore in Figures 5.1, 5.2, 5.3, the joint total comparison is drawn according the changing levels of transportation charge. As seen from these figures, the black dotted lines, which represent the total cost of the JELS model, are below the supplier's and buyer's joint total cost values. These results show again the effectiveness and the superiority of the JELS model with respect to other models. Additionally, in joint total cost, each cost depends on the demand values. For that reason demand played the second important role in these analysis.

The second part of our first objective was the shipment size comparison. In this part of the analysis, to see the effects of JIT delivery system as in Golhar and Sarker [18] did, we divided the optimum quantity to smaller lots. Division operation was done according to the integer divisor of the optimum quantity, which was found from the GINO output. The mathematical formulation of this problem, which was already discussed in Chapter 3, can be given as follows

$$PTC = \frac{D}{Q}(C_{op} + C_{sp}) + (1 + \frac{D}{P_s})(\frac{Q}{2})(I_{rp} + I_{fp}) \qquad (5.7)$$

$$STC = \frac{D}{Q}(C_{os} + C_{ss}) + (\frac{QD}{2P_s})I_{rs} + (\frac{XD}{2P_s})I_{fs} \qquad (5.8)$$

$$TTC(X) = \begin{cases} \frac{D}{X}\beta_l R_l & \beta_l \leq X \leq \alpha_l \\ R_{l+1}D & \alpha_l \leq X \leq \beta_{l+1} \end{cases} \qquad (5.9)$$
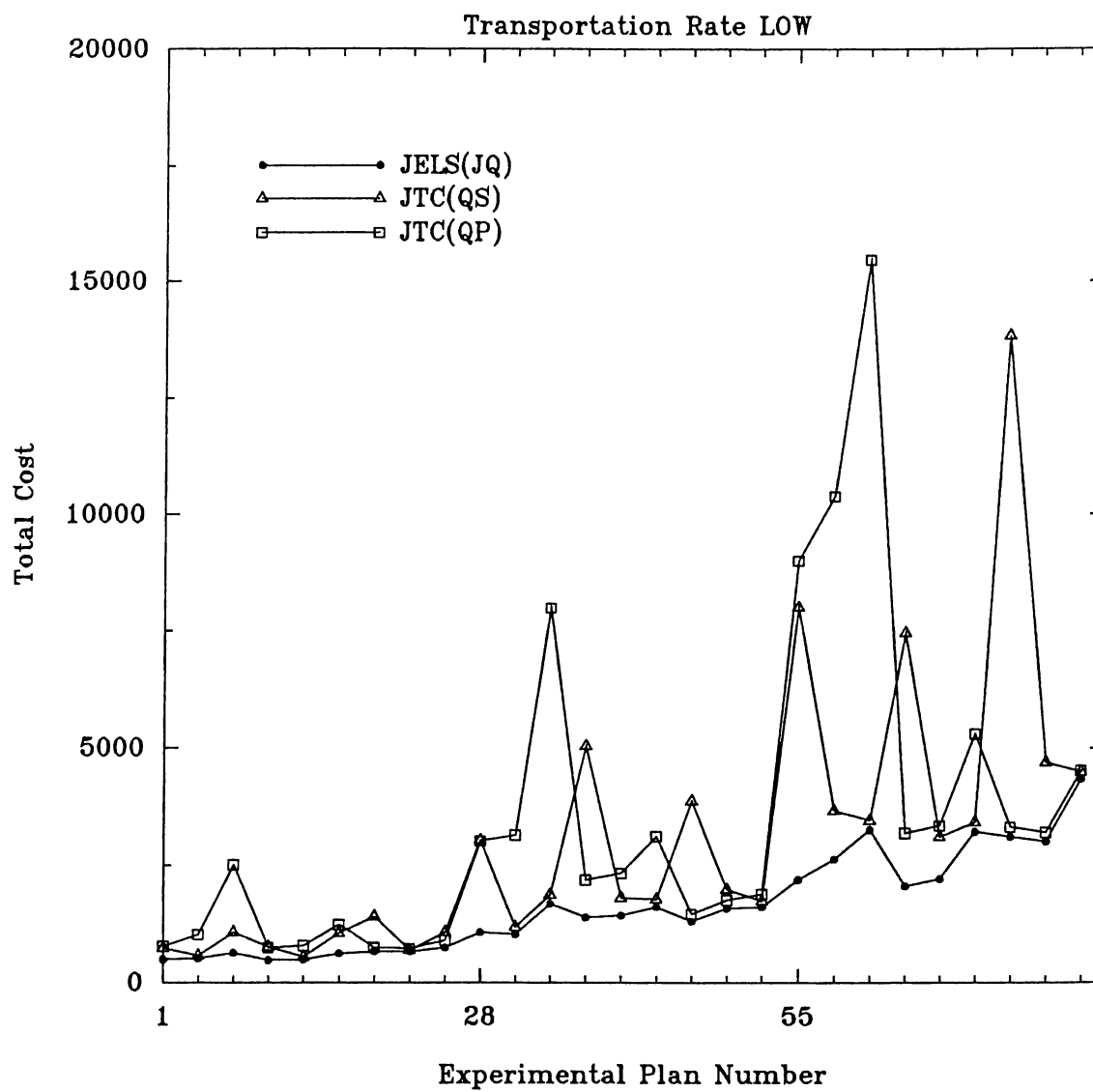
$$Q = nX \qquad (5.10)$$

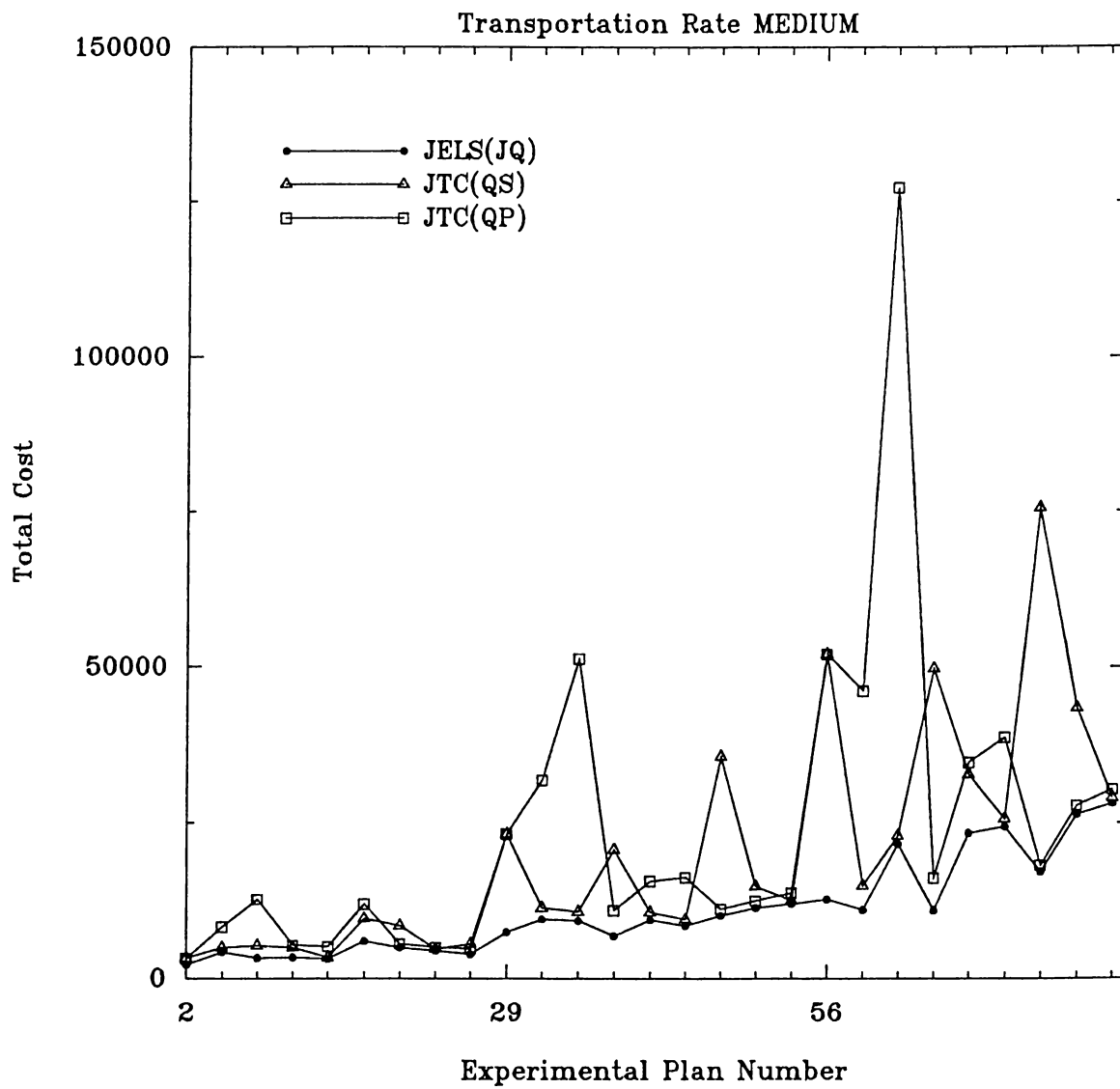Figure 5.1: Joint Total Cost Comparison for Low Transportation Rate

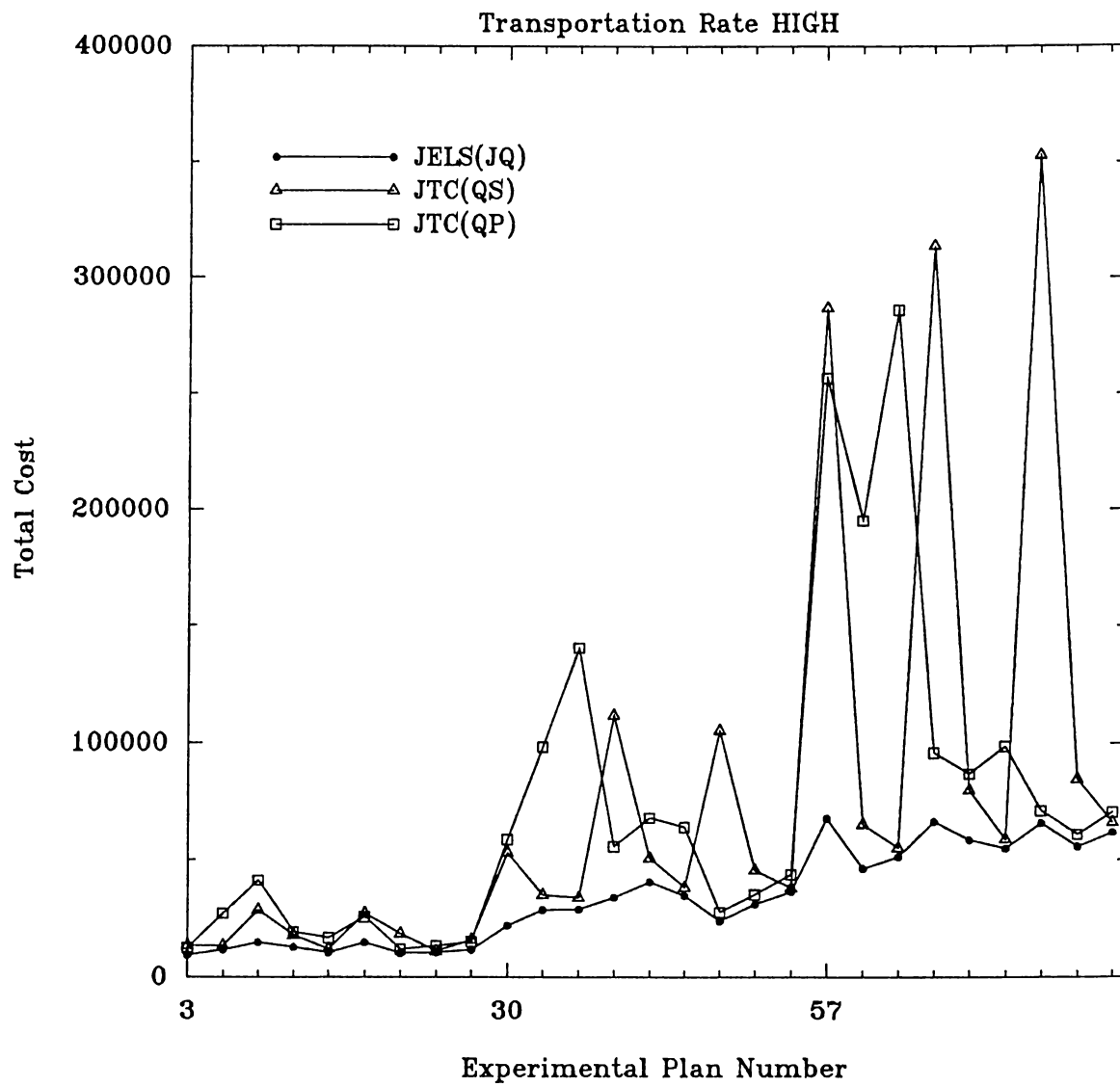Figure 5.2: Joint Total Cost Comparison for Medium Transportation Rate

Figure 5.3: Joint Total Cost Comparison for High Transportation Rate

$$\text{where} \quad JTC = STC + PTC + n.TTC \tag{5.11}$$

We applied the above formulation and the results are summarized in Table 5.6. The results showed that as the number of shipments increase, transportation cost increases so the joint total cost increases. This does not mean that high frequency with small lots, simply, JIT delivery logic cannot be implemented as it is costly. To implement such a delivery system, transportation cost must be reduced with different approaches such as warehousing etc. For this decision making problem, Table 5.6 becomes very helpful for the decision makers when they can reduce the transportation cost with an additional investment. Percentage differences can be used to cost-justify these new investments, so that the system can benefit from such implementations.

Our second objective was the analysis of variance (ANOVA). Here we used Yates Algorithm to determine which of the factors have significant effect on the experiment and the results are going to be presented in ANOVA table. First we explain the algorithm and then discuss the results.

Yates Algorithm

Yates's [42] algorithm is applied to estimate the effects and determining the sum of squares in a 2^k and 3^k factorial design. The response column contains the total of all observations taken under the the corresponding treatment combination which is the joint total cost value in our experiment. Other than response column, to calculate the sum of squares, there are k number of columns. Here k is the number of factors used in the experiment. The entries in column (1) are computed as follows: The first third of the column consists of the sums of each of the three sets of three values in the Response column. The second third of the column is the third minus the first observation in the same set of three. this operation computes the linear component of the effect. The last third of the column is obtained by taking the sum of the first and third minus twice the second in each set of three observations. This computes the quadratic component. The other columns are calculated in the same manner by using the previous column as the response column.

n	Shipment Size	Total Cost	Percentage Difference
2	720	65828.700	130.83%
3	480	119372.698	318.59%
4	360	194552.510	582.22%
5	288	291295.397	921.46%
6	240	394086.613	1281.90%
8	180	517949.998	1716.24%
9	160	579891.793	1933.45%
10	144	641837.629	2150.67%
12	120	765737.383	2585.13%
15	96	951599.137	3236.88%
16	90	1013555.076	3454.13%
18	80	1137468.973	3888.65%
20	72	1261384.891	4323.17%
24	60	1509220.768	5192.23%
30	48	1880980.645	6495.84%
32	45	2004900.604	6930.38%
36	40	2252744.563	7799.47%
40	36	2500588.522	8668.56%
45	32	2810394.481	9754.92%
48	30	2996278.461	10406.74%
60	24	3739816.399	13014.03%
72	20	4483356.358	15621.33%
80	18	4979050.338	17359.53%
90	16	5598668.317	19532.28%
96	15	5970438.297	20835.93%
120	12	7457524.276	26050.54%
144	10	8944610.256	31265.15%
160	9	9936000.235	34741.56%
180	8	11175240.235	39087.07%
240	6	14892958.215	52123.62%
288	5	17867132.194	62552.85%
360	4	22328396.194	78196.71%
480	3	29763834.174	104269.80%
720	2	44634714.174	156416.00%
1440	1	89247352.153	312854.58%

Table 5.6: The Shipment Size Comparison for Run 36

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A = Transportation Rate	41759429517.000	2	20879714758.500	57.23^a
B = Supplier S/I Ratio	170806845.682	2	85403422.841	0.23
C = Purchaser S/I Ratio	162676270.639	2	81338135.319	0.22
D = Demand	19832105469.938	2	9916052734.969	27.18^a
AB	113827770.927	4	28456942.732	0.08
AC	189492072.238	4	47373018.060	0.13
AD	13931215990.422	4	3482803997.605	9.55^a
BC	43455056.056	4	10863764.014	0.03
BD	116683695.243	4	29170923.811	0.08
CD	213211613	4	53302903.405	0.15
ABC	93879956.154	8	11734994.519	0.03
ABD	555462985.704	8	69432873.213	0.19
ACD	160165147.215	8	20020643.402	0.05
BCD	74107211.588	8	9263401.449	0.03
ABCD	258063044.140	16	16128940.259	0.04
Error	59107601449.000	162	364861737.339	
Total	136782184095.875	242		

a Significant at 1 percent

Table 5.7: Analysis of Variance for the 3^4 Design

The plan effect column (see Table 5.4) is determined by converting the treatment combinations at the left of the row into corresponding effects. That is, 10 represents the linear effect of A,A_L, and 11 represents the AB_{LxL} component of the AB interaction. The entries in the divisor column are found from $2^r.3^t.n$ where r is the number of factors in the effect considered, t is the number of factors in the experiment minus the number of linear terms in this effect, and, n is the number of replicates.

The sums of squares are obtained by squaring the element in the last column and dividing by the corresponding entry in the divisor column. The sum of squares column contains all of the required quantities to construct the analysis of variance table. The analysis of variance is presented in Table 5.4.

As seen from the ANOVA table three source of variation variables are significant at 1 percent. These are transportation rate, demand and finally the

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A_L	39882483135.188	1	39882483135.188	109.31[a]
A_Q	1876946381.791	1	11876946381.791	5.14[b]
D_L	19366363599.156	1	19366363599.156	53.08[a]
D_Q	465741870.794	1	465741870.794	1.28
$AD_{L \times L}$	13186164887.938	1	1131186164887.938	36.14[a]
$AD_{Q \times L}$	525866780.301	1	525866870.301	1.44[c]
$AD_{L \times Q}$	216968039.128	1	216968039.128	0.59
$AD_{Q \times Q}$	2216193.055	1	2216193.055	0.01
Error	59107601449.000	162	364861737.339	
Total	136782184095.875	242		

a Significant at 1 percent
b Significant at 2.5 percent
c Significant at 25 percent

Table 5.8: Analysis of Variance for the Significant Factors of 3^4 Design

interaction of these two factors. This means that any change in these factors from one level to another level, the cost function will change.

In this second ANOVA table (Table 5.8), we analyzed the significant factors appeared from the first ANOVA table according to their linear and quadratic components. First significant factor was transportation rate. Linear part of this factor (A_L) is again significant at 1 percent. On the other hand, its quadratic component, A_Q, is significant at 2.5 percent. Therefore the linear part of the transportation rate is more effective on the behavior of the factor. Second significant factor was demand. The linear term of the demand factor is again significant at 1 percent, but the quadratic component is not significant. As a result, demand is a linear function under these conditions. Finally the third significant factor was the transportation rate demand interaction. This interaction has four components which are $AD_{L \times L}$, $AD_{Q \times L}$, $AD_{L \times Q}$, $AD_{Q \times Q}$. $AD_{L \times L}$ is significant at 1 percent and $AD_{Q \times L}$ is significant at 25 percent. But the other two components of this interaction are not significant. The reason is that, demand's quadratic component is not significant so it effects these two interaction components, which again indicates the consistency of our results.

5.1.7   Conclusions and Recommendations

During the analysis of the test problems, we have two main objectives which were computational analysis and analysis of variance. The first objective was discussed under two concepts which were joint total cost comparison and shipment size comparison. First part which is the joint total cost comparison, showed that the transportation cost and demand are the effective elements of the joint model. In the second part, we discussed the shipment size comparison. Table 5.6 becomes very helpful for the decision makers when they can reduce the transportation cost with an additional investment. Percentage differences can be used to cost justify these investments. Shipment size concept is also a passing way of our model to JIT philosophy.

For the second objective was the analysis of variance. By using Yates algorithm, analysis of variance(ANOVA) was made for 3^4 factorial design. ANOVA showed that transportation rate has the biggest effect to overall system. Secondly, demand has an important effect but it was not as significant as the transportation rate. Thirdly, the ANOVA table (Table 5.7) showed that the interaction between demand and transportation rate has also an effect on the system. These three source of variation variables are significant at one percent. This again shows the importance of including the transportation cost to our model.

Furthermore Table 5.8 was presented to see the effects of linear and quadratic components of the significant factors. According to this table, transportation rate is nearly a linear function but quadratic component has also an effect on the factor. On the other hand, demand appeared to be a linear factor as the quadratic component is not significant. And for the interaction, the transportation rate effect had seen more clearly that the terms $AD_{L \times L}$ and $AD_{Q \times L}$ appeared to be significant.

As a final point, we could recommend to supplier and buyer to have their demand values close to the production rate so that percentage difference could be minimized if they do not follow the joint model as suggested.

Chapter 6

CONCLUSION

In this study, a joint economic lot-size model for the Just-In-Time purchasing problem was discussed. The joint model has three main cost functions; supplier, purchaser, and transportation costs. One of the important reasons to develop such a joint model was, to find a common lot-sizing policy between supplier and purchaser that both can gain from this relationship. Additionally it was tried to minimize the overall system cost. For that purpose three analysis criteria was used which were joint total cost comparison, shipment size comparison and analysis of variance.

In the joint total cost comparison, the JELS model was compared with the economic order quantities of supplier and purchaser according to the percentage differences in the cost functions and presented in Table 5.5. This analysis showed that JELS model at the worst case is still 3.3% better than the supplier EOQ and 4.35% better than the purchaser EOQ. The reason for that is, in that run, as the setup cost and demand was high, the EOQs appear to be very close to the joint model. The importance of setup and inventory holding cost increase while transportation cost decreases. But while one party has the minimum percentage difference value in a given purchasing scenario, the other party could not. Therefore the one with the greater percentage difference value should compensate the other for using the a common lo-sizing policy. On the other hand, JIT philosophy is based on the close relationship between the both

parties as discussed in section 2.5. This relationship can be characterized as long-term contracts, engineering aid to the vendor, education and training of suppliers which is more important than price compensation.

For the best case of JELS model, the difference for supplier reaches to a point of 436.22% where supplier S/I ratio is low and other factors are high. For the purchaser it becomes 486.84% where purchaser S/I ratio is low and the factors are high. Since the low S/I ratios are low, each party tries to send the products in small batches. This would definitely increased the transportation cost. All these calculations showed that during the level changes in the S/I ratios, the PD values changed according to the demand and transportation rate. Especially, transportation cost was so important that any decrease or increase effected the overall solution. So making this type of analysis, it is understood that JELS model dominated the individual economic order quantities and the cost functions and it will bring real benefits if both parties apply the JELS model.

How we can apply the JELS model to the JIT philosophy was another discussion point in this study. One of the important concepts of JIT philosophy is the high frequency with small lots in the delivery process. Therefore in order to include this concept, the JELS model was arranged according to the shipment size and a comparison was made between the original model and frequent deliveries results. As a result, in this analysis, we saw that supplier's finished goods inventory holding cost decreased by an amount of $\frac{(Q-X)D}{2P_s}I_{fs}$ and purchaser's holding cost increased by an amount of $\frac{DQ}{2P_s}(I_{rp} + I_{fp})$. Also transportation cost increased, as the decreasing shipment size caused frequent deliveries. All these results indicated that we should try to seek different approaches such as warehousing, to minimize the transportation cost in order to implement the JIT philosophy. Because the increase in the transportation cost cannot be easily offsetted with the decrease in the supplier cost function. Table 5.6 becomes very helpful for the decision makers when they can reduce the transportation cost with an additional investment. Percentage differences can be used to cost-justify these new investments, so that the system can benefit from such implementations.

Finally by using Yates algorithm, analysis of variance was made for 3^4 full factorial design. The first ANOVA table (Table 5.7) showed that transportation rate has the most significant effect on the joint model, which is significant at 1 percent. These results again showed the importance of including the transportation cost to the JELS model. Also as it was presented in the second ANOVA table (Table 5.8), the transportation rate's linear component is significant at 1 percent. But the quadratic component is significant at 2.5 percent. This means that transportation rate is almost linear. Another important result obtained from Table 5.8 was the linearity of the demand factor.

Through the model extensions, two dimensions of our model were discussed. These were multiple buyer and multiple supplier cases. Taking our single buyer single supplier model as a base, by these extensions, it was tried to show the different type purchasing scenarios which makes the situation closer to real world circumstances. As a consequence, these model extensions added some extra constraints, and variables. This will increase the completion time to solve these formulations, but does not effect the proposed solution procedure. All these analysis in the model extensions showed that multiple supplier case is to be considered as the preferable one because of its logical approach to JIT. In addition to this fact, the first alternative of the multiple supplier option, each supplier has different transportation cost, could be the best choice if both supplier and buyer are using JIT production system. However; the third alternative, which is the warehousing approach could also be advantegous for the non-JIT users in the production as it provides batch type production for both parties.

For the further research, approaches to minimize the transportation cost can be studied in the JELS model in the way that Black explained the RETAD (The rapid exchange of tooling and dies) in his book in chapter 5 which is simply the reduction of setups [8]. Similar approach was also discussed by Shingo [39] and he called it as SMED (Single-minute exchange of dies). Also a more detailed analysis of variance can be made by adding other factors that may affect the behavior of the JELS model, such as separate setup and inventory holding costs components instead of an aggregated S/I term.

Bibliography

[1] Ansari, A. and Heckel, J., "JIT Purchasing: Impact of Freight and Inventory Costs", *Journal of Purchasing and Materials Management*, Vol. 23, No. 2, 1987, pp. 24-28.

[2] Ansari, A. and Modarress, B., "Just-In-Time Purchasing: Problems and Solutions", *Journal of Purchasing and Materials Management*, Vol. 22, No. 2, 1986, pp. 11-15.

[3] Ansari, A. and Modarress, B., "The Potential Benefits of Just-In-Time Purchasing for U.S. Manufacturing", *Production and Inventory Management Journal*, Second Quarter, 1987, pp. 30-35.

[4] Arcelus, F.J., and Rowcroft J.E., "Inventory Policies with Freight Discounts and Disposals", *International Journal of Operation and Production Management*, Vol. 11, No. 4, 1991, pp. 89-93.

[5] Banerjee. A., "A Joint Economic Lot-size Model for Purchaser and Vendor", *Decision Sciences*, Vol. 17, 1986, pp. 292-311.

[6] Bartholomew, D., "The Vendor-Customer Relationship Today", *Production and Inventory Management Journal*, Second Quarter, 1984, pp. 106-121.

[7] Bernard, P., "Managing Vendor Performance", *Production and Inventory Management Journal*, Vol. 30, No. 1, 1989, pp. 1-9.

[8] Black, JT., *The Design of the Factory with a Future*, McGraw-Hill,Inc., New York, 1991.

[9] Black, JT., "The Just-In-Time Manufacturing/Production System", WP #84001, Advanced Manufacturing Technology Center, Auburn University, 1984.

[10] Britney, R.R., Kuzdrall, P.J., and Fartuch, N., "Note on Total Setup Lotsizing with Quantity Discounts", *Decision Sciences*, Vol. 14, 1984, pp. 282-291.

[11] Carlson, J.G., "JIT Applications to Warehousing Operations", *Engineering Costs and Production Economics*, Vol. 17, 1989, pp. 315-322.

[12] Chakravarty, A.K. and Martin, G.E., "An Optimal Joint Buyer-Seller Discount Pricing Model", *Computers and Operations Research*, Vol. 15, No. 3, 1988, pp. 271-281.

[13] D'Ouville, E., Willis, T.H., and Huston, C.R., "A Note on the EOQ-JIT Relationship", *Production Planning and Control*, Vol. 3, No. 1, 1992, pp. 57-60.

[14] Dumond, E.J., and Newman W., "Closing the Gap Between the Buyer and Vendor", *Production and Inventory Management Journal*, Fourth Quarter, 1990, pp. 13-17.

[15] Fieten, R., "Integrating Key Suppliers - Essential Part of a Just-In-Time Concept", *Engineering Costs and Production Economics*, Vol. 15, 1988, pp. 185-189.

[16] Forbes, R.S., Jones, D.F. and Marty S.T., "Managerial Accounting and Vendor Relations for JIT: A Case Study", *Production and Inventory Management Journal*, First Quarter, 1989, pp. 76-81.

[17] Goh, M., and Hum, S.H., "Costs Bounds for Inventory Systems Approaching JIT", *Journal of Operations and Production Management*, Vol. 11, No. 8, 1991, pp. 59-63.

[18] Golhar, D.Y. and Sarker, B.R., "Economic Manufacturing Quantity in a JIT Delivery System", *International Journal of Production Research*, Vol. 30, No. 5, 1992, pp. 961-972.

[19] Gupta, Y.P., "A Feasibility Study of JIT Purchasing Implementation in a Manufacturing Facility", *International Jornal of Operations and Production Management*, Vol. 10, No. 1, 1990, pp. 31-41.

[20] Gupta, Y.P., Mangold, W.G., and Lonial, S.C., "An Empirical Examination of the Characteristics of JIT Manufacturers versus Non-JIT Manufacturers", *Manufacturing Review*, Vol. 4, No. 2 1991, pp. 78-86.

[21] Hall, R.W., *Zero Inventories* Dow Jones-Irwin, Homewood, Illinois, 1983.

[22] Honson, T.G., and Dale B.G., "An Examination of the Purchasing Function in a Sales-Oriented Company", *International Jornal of Operations and Production Management*, Vol. 11, No. 5, 1991, pp. 71-82.

[23] Hwang, H., Moon, D.L. and Shinn, S.W., "An EOQ Model with Quantity Discounts for Purchasing Price and Freight Cost", *Computers and Operations Research*, Vol. 17, No. 1, 1990, pp. 73-78.

[24] Karmarkar, U., "Getting Control of Just-In-Time", *Harvard Business Review*, September-October 1986, pp. 122-131.

[25] Lee, H.L. and Rosenblatt, M.J., "A Generalized Quantity Discount Pricing Model to Increase Supplier's Profits", *Management Science*, Vol. 32, No. 9, 1986, pp. 1177-1185.

[26] Manoochehri, G.H., "Suppliers and Just-In-Time Concept", *Journal of Purchasing and Materials Management*, Vol. 20, No. 4, 1984, pp. 16-21.

[27] McMillan, C.J., "Production Planning in Japan", *Journal of General Management*, Vol. 8, No. 4, 1983, pp. 44-72.

[28] Monahan, J.P., "A Quantity Discount Pricing Model to Increase Vendor Profits", *Management Science*, Vol. 30, No. 6, 1984, pp. 720-726.

[29] Monden, Y., *Toyota Production System*, Industrial Engineering and Management Press, Norcross, Georgia, 1983.

[30] Montgomery, D.C, *Design and Analysis of Experiments*, John Wiley and Sons, Toronto, 1991.

[31] Narasimhan, R., and Stoynoff, L.K., "Optimizing Aggregate Procurement Allocation Decisions", *Journal of Purchasing and Materials Management*, Vol. 22, No. 1, 1986, pp. 23-30.

[32] Newman, R.G., "The Buyer-Supplier Relationship Under Just-In-Time", *Production and Inventory Management Journal*, Third Quarter, 1988, pp. 45-49.

[33] Oğuz, C., *Design and Analysis of Just-In-Time Production Systems*, Unpublished M.S. Thesis, Industrial Engineering Department, Bilkent University, October 1988.

[34] Ramasesh, R.W., "Recasting the Traditional Inventory Model to Implement Just-In-Time Purchasing", *Production and Inventory Management Journal*, First Quarter, 1990, pp. 71-75.

[35] Rhea, N.W., "Material Management", *Material Handling Engineering*, December 1985.

[36] Russel R.M., and Krajewski, L.J., "Optimal Purchase and Transportation Cost Lot-sizing for a Single Item", *Decision Sciences*, Vol. 22, 1991, pp. 940-952.

[37] Schonberger, R.J., *Japanese Manufacturing Techniques: Nine Hidden Lessons in Simplicity*, The Free Press, New York, 1982.

[38] Schonberger, R.J., and Ansari, A., "Just-In-Time Purchasing Can Improve Quality", *Journal of Purchasing and Materials Management*, Vol. 20, No. 1, 1984, pp. 2-7.

[39] Shingo, S., *Study of Toyota, Production System from Industrial Engineering Viewpoint*, Japan Management Assn., 1981.

[40] Wehrman, J.C., "Evaluating the Total Cost of a Purchase Decision", *Production and Inventory Management Journal*, Fourth Quarter, 1984, pp. 86-90.

[41] Winston, W.L., *Operation Research: Applications and Algorithms*, PWS-KENT Publishing Company, Boston, 1987.

[42] Yates, F., "Design Analysis of Factorial Experiments", Tech Comm No. 35, *Imperial Bureau of Soil Sciences*, London, 1937.

[43] Zangwill, W.I., "From EOQ Toward ZI", *Management Science*, Vol. 33, No. 10, 1987, pp. 1209-1223.