COMPUTING WITH CAUSAL THEORIES

A THESIS
SUBMITTED TO THE DEPARTMENT OF
COMPUTER ENGINEERING AND INFORMATION SCIENCES
AND THE INSTITUTE OF ENGINEERING AND SCIENCES
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

By
Erkan TIN

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Varol Akman
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ABSTRACT

COMPUTING WITH CAUSAL THEORIES

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M. S. in Computer Engineering and Information Sciences
Supervisor: Assoc. Prof. Varol Akman
October 1990

Formalizing commonsense knowledge for reasoning about time has long been a central issue in Artificial Intelligence (AI). It has been recognized that the existing formalisms do not provide satisfactory solutions to some fundamental problems of AI, viz. the frame problem. Moreover, it has turned out that the inferences drawn by these systems do not always coincide with those one had intended when he wrote the axioms. These issues call for a well-defined formalism and useful computational utilities for reasoning about time and change. Yoav Shoham of Stanford University introduced in his 1986 Yale doctoral thesis an appealing temporal nonmonotonic logic, the logic of chronological ignorance, and identified a class of theories, causal theories, which have computationally simple model-theoretic properties.

This thesis is a study towards building upon Shoham's work on causal theories for the latter are somewhat limited. The thesis mainly centers around improving computational aspects of causal theories while preserving their model-theoretic properties.

Keywords: Causation, causal theories, the frame problem, the qualification problem, the persistence problem, modal logics, nonmonotonic logics, temporal logics, chronological ignorance, model theory.

ÖZET

NEDENSEL TEORİLERLE HESAPLAMA

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Zaman üzerine çıkarım yapılabilmesi için sağduyu bilgisinin formel hale sokulması uzun zamandır Yapay Zekâ'nın (YZ) merkezi meselesi olmuştur. Halihazırdaki formel sistemlerin YZ'nin çerçeve sorunu gibi bazı temel problemlerine tatmin edici çözümler getirmedikleri bilinmektedir. Dahası, bu sistemlerle yapılan çıkarımlar aksiyomlarla ifade edilmek istenenlerle daima uyuşmamaktadır. Bu meseleler zaman ve değişim üzerine çıkarım yapılabilmesi için iyi tanımlanmış bir formelizmi ve yararlı hesaplama metodlarını davet etmektedir. Stanford Üniversitesi'nden Yoav Shoham doktora tezinde (Yale, 1986) kronolojik bilgisizlik adını verdiği temporel, tekdüze olmayan cazip bir mantık ortaya koymuş ve nedensel teoriler olarak adlandırılan, hesaplaması basit model teorik özellikleri bulunan bir teori sınıfı tanımlamıştır.

Bu sınıfın bazı sınırlamaları olduğu için bu tez Shoham'ın nedensel teorileri üzerine yapılan bir geliştirme çalışmasıdır. Tez özellikle bu teorilerin hesapsal yönlerinin onların model teorik özelliklerini koruyarak iyileştirilmesi etrafında yoğunlaşmaktadır.

Anahtar Kelimeler: Nedensellik, nedensel teoriler, çerçeve sorunu, kalifiye olma sorunu, kalıcılık sorunu, modal mantıklar, tekdüze olmayan mantıklar, temporel mantıklar, kronolojik bilgisizlik, model teorisi.

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CONTENTS

Chapter 1

INTRODUCTION

Reasoning about the commonsense notions of time and change is important in various areas of Artificial Intelligence (AI). There have been attempts towards formalizing common sense and various logics have been devised for commonsense reasoning. It has been recognized that reasoning about change requires temporal and nonmonotonic reasoning devices. In this direction, Shoham introduced, in his doctoral dissertation [28], [35], a temporal nonmonotonic logic that he called the logic of Chronological Ignorance (CI). CI is based on model-theoretic analysis and preference ordering among models. This, arguably, turns out to be a solution to the qualification problem. By correlating philosophical issues on causation with the notion of time, Shoham identified a class of theories, causal theories, which have computationally simple model-theoretic properties in CI. Other contributions of Shoham to temporal reasoning, and nonmonotonic reasoning include [27], [29-34], [36-37]; these works generated considerable interest [1], [4-5], [8-10], [21-23], [25-26].

In this thesis, after an examination of the preliminary notions of CI and causal theories, it is shown that computing with causal theories is time-dependent. This contradicts with the method human beings use to reason about consequences of actions and to come to conclusions in everyday life. To remove this deficiency, a new class of causal theories containing axiom schemata is introduced and computational aspects of causal theories in this class are investigated. Furthermore, an approach to remove one of the technical limitations imposed by Shoham on causal theories is proposed. A brief survey of the related literature, critiques of

Shoham's works, and a discussion on his account of causation are also included.

In Chapter 2, our notation and terminology are presented. Included are the essential notions of CI and the definition of causal theories.

Chapter 3 introduces the Yale Shooting Problem (YSP). The role of YSP in verifying formal approaches to reasoning is investigated. The weakness of causal theories in representing scenarios similar to YSP and their inefficiency in computing the consequences of these theories are demonstrated. To remove those deficiencies, a new class of causal theories, YSP-like causal theories with axiom schemata, is proposed. It is shown that computing the consequences of a theory in this class is independent of time unlike the case with the causal theories of Shoham.

Shoham did not permit simultaneous occurrence of cause and effect in his account of causation: he restricted causal theories to have causes strictly precede their effects in time. In Chapter 4, various related ideas from philosophy are mentioned. A modified definition of causal theories that permits simultaneity is given and an algorithm to compute the consequences of such theories is proposed.

Chapter 5 contains the concluding remarks together with directions for future research.

In Appendix A, after introducing two epistemological problems, a discussion on how CI suggests a solution to these problems is given.

In Appendix B, Shoham's account of causation is introduced and some issues that must be considered to obtain a more complete characterization are pointed out.

Appendix C contains omitted proofs of some theorems and propositions.

Programs listings are included in Appendix D.

Chapter 2

ESSENTIAL NOTIONS AND TERMINOLOGY

2.1 Notational conventions

Unless otherwise stated, we follow Shoham's terminology and definitions verbatim. Lower-case letters such as p and p_1 denote propositional symbols; t is used to express a time point variable, and a time point symbol (constant) when indexed (as in t_1).

The symbols $\neg$, $\wedge$, $\vee$, $\supset$, $\equiv$ are used as the standard logical connectives. $\forall$ denotes the universal quantifier. $\Box$ and $\Diamond$ are modal operators described in the following section. ■ is used to denote Q.E.D. Other notations are described when they are first used.

2.2 The logic of chronological ignorance

Nonmonotonic logics can be defined by means of a preference criterion on the interpretations of a standard logic, i.e., (classical or modal) propositional logic or first-order predicate logic. The preference criterion forms a preference relation over the models of the standard logic. Shoham [33] suggests a semantic framework in this direction. He calls such nonmonotonic logics preference logics. CI is a nonmonotonic logic obtained in this way. The standard monotonic logic on which CI is based is called the logic of Temporal Knowledge (TK). The syntax and semantics of TK are given below.

We assume the existence of the following:

P: a set of primitive propositions,

TV: a set of temporal variables,

TC: Z (integers) (This characterizes the structure of time which is discrete, linear, and unbounded in both directions),

U: $TC \cup TV$.

Well-formed formulae (wff) are defined as follows:

1. If $u_1, u_2 \in U$, then $u_1=u_2$ and $u_1 \leq u_2$ are wff.

2. If $u_1, u_2 \in U$ and $p \in P$, then $TRUE(u_1,u_2,p)$ is a wff. (From now on, without loss of generality we assume $u_1 \leq u_2$ throughout this thesis.)

3. If φ_1 and φ_2 are wff, then so are $\varphi_1 \wedge \varphi_2$, $\neg\varphi_2$, and $\Box\varphi_1$. $\Box\varphi$ reads as "φ is known." $\Diamond\varphi \equiv \neg\Box\neg\varphi$.

4. If φ is a wff and $v \in TV$, then $\forall v\ \varphi$ is also a wff.

Some abbreviations for wff are used; $\Box TRUE(t_1,t_2,p)$ is replaced by $\Box(t_1,t_2,p)$, $\Box\neg TRUE(t_1,t_2,p)$ by $\Box(t_1,t_2,\neg p)$, $\Diamond TRUE(t_1,t_2,p)$ by $\Diamond(t_1,t_2,p)$, and $\Diamond\neg TRUE(t_1,t_2,p)$ by $\Diamond(t_1,t_2,\neg p)$. $TRUE(t_1,p)$ is used as an abbreviation for $TRUE(t_1,t_1,p)$.

Definition 2.1 A *sentence* is a wff containing no free variables.

Definition 2.2 A *Kripke interpretation* (KI) is a set of infinite parallel time lines, all sharing the same interpretation of time, viz. a synchronized copy of Z. Each world describes an entire possible course of the universe, and so over the same time interval, but in different worlds, different facts are known. Formally, KI is a pair <W,M> where W is a nonempty universe of possible worlds, and M is a meaning function such that $M: P \rightarrow 2^{W \times Z \times Z}$.

Definition 2.3 A *variable assignment* is a function $VA: TV \rightarrow Z$.

Definition 2.4 A *valuation function* VAL is such that $VAL(u)=VA(u)$ if $u \in TV$ and $VAL(u)=u$ if $u \in TC$.

A KI=<W,M> and a world $w \in W$ satisfy a formula φ under VA (written $KI,w \models \varphi[VA]$) if the following hold:

1. $KI,w \models u_1=u_2[VA]$ iff $VAL(u_1)=VAL(u_2)$.

2. $KI,w \models u_1 \leq u_2[VA]$ iff $VAL(u_1) \leq VAL(u_2)$.

3. $KI,w \models TRUE(u_1,u_2,p)[VA]$ iff $<w,VAL(u_1),VAL(u_2)> \in M(p)$.

4. $KI,w \models \varphi_1 \wedge \varphi_2[VA]$ iff $KI,w \models \varphi_1[VA]$ and $KI,w \models \varphi_2[VA]$.

5. $KI,w \models \neg\varphi[VA]$ iff $KI,w \not\models \varphi[VA]$.

6. $KI,w \models \forall v\ \varphi[VA]$ iff $KI,w \models \varphi[VA']$, $\forall VA'$ that agree with VA everywhere except possibly on v.

7. $KI,w \models \Box\varphi[VA]$ iff $KI,w' \models \varphi[VA]$, $\forall w' \in W$.

A Kripke interpretation KI=<W,M> and a world $w \in W$ are a *model* for a formula φ (written $KI,w \models \varphi$) if $KI,w \models \varphi[VA]$ for any variable assignment VA. A wff is *satisfiable* if it has a model, and *valid* if its negation has no model. φ_1 *entails* φ_2 (written $\varphi_1 \models \varphi_2$) iff φ_2 is satisfied by all models of φ_1. It should be noted that if φ is true in $w \in W$, in KI this is written $KI,w \models \varphi$, and $KI,w \not\models \varphi$ if it is false.

Another point worth noting is that $<w,t_1,t_2> \in M(p)$ iff $<w,t_2,t_1> \in M(p)$. If a proposition holds over an interval, this does not imply that the same proposition holds over its subintervals. Below are given some more definitions.

Definition 2.5 *Base formulae* are those wff containing no occurrence of the modal operators.

Definition 2.6 The *latest time point* (ltp) of a base formula is the latest time point mentioned in it:

1. The ltp of $TRUE(t_1,t_2,p) = t_2$.
2. The ltp of $\varphi_1 \wedge \varphi_2 = \max\{$ltp of φ_1, ltp of $\varphi_2\}$.
3. The ltp of $\neg\varphi$ = the ltp of φ.

4. The ltp of $\forall v\ \varphi$ is the minimum among the ltp's of all φ' which result from substituting in φ a time point symbol for all free occurrences of v, or $-\infty$ if there is no such earliest ltp.

The preference criterion associated with TK to obtain the logic of CI is as follows.

Definition 2.7 A KI M_2 is *chronologically more ignorant* than a KI M_1 (written $M_1 \subset_{ci} M_2$) if there exists t_0 such that

1. For any base sentence φ with ltp $\leq t_0$, if $M_2 \models \Box\varphi$ then also $M_1 \models \Box\varphi$.
2. There exists a base sentence φ with ltp t_0 such that $M_1 \models \Box\varphi$ but $M_2 \not\models \Box\varphi$.

Definition 2.8 M is said to be a *chronologically maximally ignorant (cmi) model* of φ if $M \models_{\subset_{ci}} \varphi$, i.e., if $M \models \varphi$ and there is no other M' such that $M' \models \varphi$ and $M \subset_{ci} M'$.

Definition 2.9 The *logic of chronological ignorance*, CI, is the nonmonotonic logic obtained by associating the preference relation $\subset_{ci}$ with TK.

2.3 Causal theories

Definition 2.10 *Formulae* in CI are those base formulae augmented by the modal operators.

Definition 2.11 A *theory* in CI is a collection of sentences in CI.

Definition 2.12 *Base sentences* in CI are those sentences not containing any occurrence of the modal operators, i.e., sentences that refer directly to the real world and not to knowledge of it.

Definition 2.13 *Atomic base sentences* are either of the form $TRUE(t_1,t_2,p)$ or the form $\neg TRUE(t_1,t_2,p)$.

Definition 2.14 A *causal theory* Ψ is a theory in CI, in which all sentences have the form $\Phi \wedge \Theta \supset \Box\varphi$ where (in the following $[\neg]$ means that the negation sign may or may not appear)

1. φ = TRUE(t_1,t_2,$[\neg]$p). (**NB** In his original definition ([35, p. 109]), Shoham takes φ = TRUE(t_2,t_1,$[\neg]$p) which obviously leads to a contradiction, viz. overlapping cause and effect.)

2. $\Phi = \bigwedge^{n}_{i=1}\Box\varphi_i$, where φ_i is an atomic base sentence with ltp t_i such that $t_i<t_1$.

3. $\Theta = \bigwedge^{m}_{j=1}\Diamond\varphi_j$, where φ_j is an atomic base sentence with ltp t_j such that $t_j<t_1$.

4. Φ or Θ may be empty. A sentence in which Φ is empty is called a *boundary condition*. Other sentences are called *causal rules*.

5. There is a time point t_0 such that if $\Theta \supset \Box(t_1,t_2\ [\neg]p)$ is a boundary condition, then $t_0<t_1$.

6. There do not exist two sentences in Ψ such that one contains $\Diamond(t_1,t_2,p)$ on its l.h.s. and the other contains $\Diamond(t_1,t_2,\neg p)$ on its l.h.s.

7. If $\Phi_1 \wedge \Theta_1 \supset \Box(t_1,t_2,p)$ and $\Phi_2 \wedge \Theta_2 \supset \Box(t_1,t_2,\neg p)$ are two sentences in Ψ, then $\Phi_1 \wedge \Theta_1 \wedge \Phi_2 \wedge \Theta_2$ is inconsistent .

Definition 2.15 The *soundness conditions* of Ψ are the set of sentences $\Diamond(t_1,t_2,p) \supset$ TRUE(t_1,t_2,p) such that $\Diamond(t_1,t_2,p)$ appears on the l.h.s. of some sentence in Ψ.

Soundness conditions are implicitly part of the causal theories. One essential property of a causal theory is that it has cmi models, and in all of them the same set of atomic base sentences are known.

Theorem 2.1 If Ψ is a causal theory, then

1. Ψ has a cmi model.

2. If M_1 and M_2 are cmi models of Ψ, and φ is any base sentence, then $M_1 \models \Box\varphi$ iff $M_2 \models \Box\varphi$.

Proof. [35, pp. 112-113]. ■

Definition 2.16 A *time-bounded Kripke interpretation* M/t is a structure which can be viewed as an incomplete Kripke interpretation. Like a Kripke interpretation it assigns a truth value to atomic propositions, but only to those whose ltp $\leq$ t. The truth value of an arbitrary sentence whose ltp $\leq$ t is also determined in M/t, according to the usual compositional rules. It is easy to see that that this is well-defined, since, by the semantics of CI and by the definition of an ltp, the truth value of a sentence whose ltp $\leq$ t does not depend on any sentence whose ltp > t. If a sentence φ with ltp $\leq$ t is *satisfied* by M/t, this is denoted $M/t \models \varphi$.

Definition 2.17 M/t *partially satisfies* a theory Ψ if M/t satisfies all sentences of Ψ whose ltp $\leq$ t.

Chapter 3

COMPUTING SENTENCES KNOWN IN THE CMI MODELS OF CAUSAL THEORIES

Formalizing commonsense reasoning has long been (and still is) an open problem of AI. Various nonmonotonic formal systems have been proposed to facilitate it (e.g., Reiter's default logic [24] and McCarthy's circumscription [14]). *Situation calculus* [12] has initially been used to reason about the effects of actions. In the framework of situation calculus, Hanks and McDermott [7] describe what they call *temporal projection* as follows. Given a description of the current situation, some descriptions of the effects of possible actions, and a sequence of actions to be performed, how do we predict the properties of the world in the resulting situation?

Noticing that this is not a by-product of situation calculus, but is independent of the logic used, they redefine it [8, p. 385]:

> "[G]iven an initial description of the world (some facts that are true), the occurrence of some events, and some notion of causality (that an event can cause a fact to become true), what facts are true once all the events have occurred?"

Hanks and McDermott [8] applied some of the existing logics (e.g., Reiter's default logic) to scenarios to see whether the expected results are indeed produced. It turned out that these logics have some flaws [8, p. 379]:

> "Upon examining the resulting nonmonotonic theories, however, we find that the inferences permitted by the logics

> are not those we had intended when we wrote the axioms, and in fact are much weaker."

The *Yale Shooting Problem* (YSP) was posed by Hanks and McDermott [7] as a paradigm to show how the temporal projection problem arises. At some point in time, a person (Fred) is alive. A loaded gun, after waiting for a while, is fired at Fred. What are the results of this action? One expects that Fred would die and the gun would be unloaded after the firing of the gun. But Hanks and McDermott [8] demonstrate, in the framework of circumscription [15], that unintended minimal models are obtained; the gun gets unloaded during the waiting stage and firing the gun does not kill Fred.

After Hanks and McDermott showed how existing logics fail to produce the expected results for YSP, researchers proposing new formalisms applied their methods to the YSP and other similar scenarios (e.g., McCarthy's *blocks world* [15]) to show how they succeed in avoiding the unintended models.

Hanks and McDermott argue that a solution to the temporal projection problem relies on an answer to two questions [8, p. 409]:

> "(1) Given a logical theory that admits more than one model, what are the preferred models of that theory (that is, what is the preference criterion) and (2) Given a theory and a preference criterion, how do we find the theorems that are true in all 'most preferred' models?"

As they noted, Shoham's [37] preference criterion (see Definitions 2.7-8) provides a satisfactory answer to the first question. Moreover, he gives an algorithm that computes the true sentences in the models preferred under this preference criterion, thus answering the second question.

In this chapter, we argue that Shoham's computational account is not very efficient. Furthermore, since his solution, as Hanks and McDermott also point out [8, p. 410], is "very specific to the problem of temporal projection," we demonstrate how its time-dependent nature can be removed. We also show that causal theories may yield unintended models.

3.1 Time dependency in causal computations

Causal theories of Shoham contain axioms to reason about the effects of actions. Proceeding in time, knowledge about the future is obtained from what is known (and what is not known) about the past. This forms the core of the causal inference mechanism. For example, if you know that a match is struck at time t, and you don't know that it is wet at t, then you infer that the match lights at t+1. Causal theories have a nice property; all cmi models agree on what is known (see Theorem 2.1). That is, in all cmi models of a causal theory the same atomic base sentences are known. Shoham [35, p. 114] proposed an algorithm to compute the set of atomic base sentences known in all cmi models of a causal theory.

Consider the following variant of YSP. A gun, loaded at some point in time, is fired at a later time. We would like to reason about the effect of firing the gun. Shoham [35, p. 106] gives a possible axiomatization in which the gun is loaded at time 1 and fired at 5:

1. $\Box(1,\text{loaded})$.
2. $\Box(5,\text{fire})$.
3. $\Box(t,\text{loaded}) \wedge \Diamond(t,\neg\text{fire}) \wedge \Diamond(t,\neg\text{emptied-manually}) \supset \Box(t+1,\text{loaded}), \quad \forall t.$
4. $\Box(t,\text{loaded}) \wedge \Box(t,\text{fire}) \wedge \Diamond(t,\text{air}) \wedge \Diamond(t,\text{firing-pin}) \wedge \Diamond(t,\text{no-marshmallow-bullets}) \wedge \ldots \wedge \Diamond$ other mundane conditions $\supset \Box(t+1,\text{noise}), \quad \forall t.$

Axioms 1 and 2 are the boundary conditions. The third one is an axiom schema needed for persistence. It says that the gun remains loaded unless certain conditions obtain. The last one is again an axiom schema. It is a causal rule stating that firing a loaded gun causes a noise unless certain conditions obtain. In fact, causal theories can only contain axioms, not axiom schemata with time variables (see Definition 2.14). Shoham (personal communication, November 1989) explains:

> "I do assume that all boundary conditions and all causal rules contain only ground atomic sentences. If variables appear it means that this is a schema, standing for all its ground instances. I believe this restriction can be lifted, but I did impose it."

Therefore, the axiom schemata 3 and 4 above must be replicated by replacing the variable t by time points from 1 to 5. This actually corresponds to the finite causal theory below (some ◇-conditions of schema 4 are omitted):

1. ❑(1,loaded).
2. ❑(1,loaded) ∧ ◇(1,¬fire) ∧ ◇(1,¬emptied-manually) ⊃ ❑(2,loaded).
3. ❑(1,loaded) ∧ ❑(1,fire) ∧ ◇(1,air) ∧ ◇(1,firing-pin) ⊃ ❑(2,noise).
4. ❑(2,loaded) ∧ ◇(2,¬fire) ∧ ◇(2,¬emptied-manually) ⊃ ❑(3,loaded).
5. ❑(2,loaded) ∧ ❑(2,fire) ∧ ◇(2,air) ∧ ◇(1,firing-pin) ⊃ ❑(3,noise).
6. ❑(3,loaded) ∧ ◇(3,¬fire) ∧ ◇(3,¬emptied-manually) ⊃ ❑(4,loaded).
7. ❑(3,loaded) ∧ ❑(3,fire) ∧ ◇(3,air) ∧ ◇(3,firing-pin) ⊃ ❑(4,noise).
8. ❑(4,loaded) ∧ ◇(4,¬fire) ∧ ◇(4,¬emptied-manually) ⊃ ❑(5,loaded).
9. ❑(4,loaded) ∧ ❑(4,fire) ∧ ◇(4,air) ∧ ◇(4,firing-pin) ⊃ ❑(5,noise).
10. ❑(5,fire).
11. ❑(5,loaded) ∧ ◇(5,¬fire) ∧ ◇(5,¬emptied-manually) ⊃ ❑(6,loaded).
12. ❑(5,loaded) ∧ ❑(5,fire) ∧ ◇(5,air) ∧ ◇(5,firing-pin) ⊃ ❑(6,noise).

The first axiom says that "it is known that the gun is loaded at 1." The second one says that "if it is known that the gun is loaded at 1, and it is not known that it is fired at 1 and that it is emptied manually at 1, then it is known that the gun is loaded at 2." The third one says that "if it is known that the gun is loaded at 1 and that it is fired at 1, and it is not known that there is no air and that the gun has no firing pin at 1, then it is known that noise is heard at 2." The remaining axioms are analogous. Shoham's algorithm steps through each axiom and computes the base sentences known in all cmi models of this causal theory. It produces the expected atomic base sentences: TRUE(1,loaded), TRUE(2,loaded), ..., TRUE(5,loaded), TRUE(5,fire), and TRUE(6,noise).

This cmi model is computed by stepping over each axiom of the causal theory in ordered form, and checking whether the l.h.s. of the axioms are satisfied. This however is a time consuming procedure. Shoham [35, pp. 113-114] suggested improving the efficiency of the algorithm by "focus[ing] the attention on the interesting time points, those that are potentially ltp's of known atomic base sentences." In other words, "in constructing the cmi model, one can skip the time points which are not the ltp of the r.h.s. in any sentence of the causal theory: at those points no atomic base sentences are known" [35, p. 114].

Measuring the size of a causal theory in terms of the number of base sentences in the axioms, the size of the causal theory above turns out to be 47. (There exist 2 boundary conditions. Schema 3 contains 4 base sentences and schema 4 contains 5 base sentences. Axiom schemata 3 and 4 are replicated for all time points from 1 to 5, resulting in 45 base sentences.)

Now assume that the gun is loaded at time 1, and instead of 5 it is fired at 5000. The size of the causal theory describing this scenario is 45002. Consequently, the later the gun is fired, the larger the size of the corresponding causal theory becomes. Hence, more computation time and space are needed to reason about the effect of firing the gun.

However, such scenarios call for general representation mechanisms. For example, pouring water onto a dry surface will have the same effect (a wet surface) regardless of when it happens. Therefore, one should be able to say that "if the gun is fired at any time, then a loud noise is heard at the next instant." This suggests having causal theories containing axiom schemata with time variables. The theory above with two boundary conditions and two axiom schemata is such a causal theory.

Again measuring the size of a causal theory in terms of the number of base sentences in it, assume that the size of a causal theory with axiom schemata is *n*. Then, the size of the corresponding finite causal theory must be *Tmax n*, where *Tmax* denotes the number of time points (5 in this example) between the time points of the boundary conditions having the earliest (□(1,loaded)) and the latest time points (□(5,fire)), respectively. Shoham's algorithm computes the atomic base sentences known in all cmi models of a finite causal theory. Assuming that this finite causal theory corresponds to the one with axiom schemata shown above, the time complexity of his algorithm becomes O(*Tmax n* log(*Tmax n*)). This means that his approach has a deficiency when the causal theories contain axiom schemata; computation is time-dependent for the size of the corresponding finite causal theory depends on the time "span" of the theory.

3.2 YSP-like causal theories

In temporal projection scenarios, there exist two types of axiom schemata. The first takes care of the persistence of facts, permitting inferences about what remains unchanged. For example, if you load a gun, it will stay loaded unless you fire or empty it. This corresponds to axiom schema 3 in our shooting scenario. Such axiom schemata will be called *persistence axiom schemata*.

The second type of axiom schemata represent what changes occur in the environment. They will be called *causal axiom schemata*. More specifically, these schemata allow one to infer what changes actions bring about. In the shooting scenario, number 4 is a causal axiom schema. It says that firing a loaded gun causes a loud noise unless some conditions obtain (e.g., the gun lacks a firing pin).

It will be assumed in the sequel that scenarios are formalized with a persistence axiom schema and a causal axiom schema, along with two boundary conditions. The condition having the ltp generally represents an action whose consequences are to be determined. However, it need not always be an action. Instead, it can well be the knowledge that something occurred in the environment. Below is a class of causal theories to represent such scenarios.

Definition 3.1 A *YSP-like causal theory* ζ is a theory in CI containing

$\Box\varphi_s$.

$\Box\varphi_f$.

$\Box\varphi_p \wedge \Theta_p \supset \Box\varphi_p$, $\forall t$.

$\Phi_c \wedge \Theta_c \supset \Box\varphi_c$, $\forall t$.

where

1. $\Box\varphi_s$ is the *initial boundary condition* where φ_s is of the form TRUE(t_1,[¬]p).

2. $\Box\varphi_f$ is the *final boundary condition* where φ_f is of the form TRUE(t_2,[¬]p), $t_1<t_2$.

3. $\Box\varphi_p \wedge \Theta_p \supset \Box\varphi_p$ is a *persistence axiom schema* where

   (i) φ_p is of the form TRUE(t,[¬]p) (on the l.h.s.) or TRUE(t+1,[¬]p) (on the r.h.s).

   (ii) Θ_p is a (possibly empty) conjunction of sentences $\Diamond\varphi_i$, where φ_i is of the form TRUE(t,[¬]q).

4. $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ is a *causal axiom schema* where

   (i) Φ_c has two conjuncts one of which must be $\Box\varphi_p$.

   (ii) Θ_c is a (possibly empty) conjunction of sentences $\Diamond\varphi_k$, where φ_k is of the form TRUE(t,[¬]q).

   (iii) φ_c is of the form TRUE(t+1,[¬]r).

5. If $\Diamond$(t,p) (respectively $\Diamond$(t,¬p)) is a conjunct of Θ_p, then Θ_c does not contain $\Diamond$(t,¬p) (respectively $\Diamond$(t,p)).

6. If φ_p and φ_c are of the forms TRUE(t,p) (respectively TRUE(t,¬p)) and TRUE(t,¬p) (respectively TRUE(t,p)) then $\Box\varphi_p \wedge \Theta_p \wedge \Phi_c \wedge \Theta_c$ is inconsistent.

7. If φ_s (φ_f) is of the form TRUE(t_1,p) (respectively TRUE(t_2,¬p)) and φ_p is of the form TRUE(t,¬p) (respectively TRUE(t,p)) then $\Box\varphi_p \wedge \Theta_p$ is inconsistent.

8. If φ_s (φ_f) is of the form TRUE(t_1,p) (respectively TRUE(t_2,¬p)) and φ_c is of the form TRUE(t,¬p) (respectively TRUE(t,p)) then $\Phi_c \wedge \Theta_c$ is inconsistent.

Obviously, the shooting scenario with axiom schemata given in the previous section is a YSP-like causal theory.

Theorem 3.1 If ζ is a YSP-like causal theory, then ζ has cmi models and in all of these cmi models the same atomic base sentences are known.

Proof. Appendix C. ■

Proposition 3.1 The set of atomic base sentences known in any cmi model of a YSP-like causal theory ζ is exactly the same as those known in the cmi models of the causal theory Ψ corresponding to ζ (this correspondence is obtained by replacing each time variable t in axiom schemata in ζ by the time constants in the range t_1 to t_2, where t_1 and t_2 are the time points mentioned in the initial and final boundary conditions, respectively).

Proof. Ψ obtained in this way will contain the following sentences ordered with respect to their ltp's. ("Rewriting" a formula at $t=t_i$ means replacing all occurrences of t in that formula with t_i.)

$\Box\varphi_s$.
$\Box\varphi_p \wedge \Theta_p \supset \Box\varphi_p$ (rewrite for $t=t_1$ until $t=t_2-1$).
$\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ (rewrite for $t=t_1$ until $t=t_2-1$).
$\Box\varphi_f$.
$\Box\varphi_p \wedge \Theta_p \supset \Box\varphi_p$ (rewrite at $t=t_2$).
$\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ (rewrite at $t=t_2$).

Since this causal theory is actually a causal theory of type Ψ (see Definition 2.14) and has a unique cmi model (according to Theorem 2.1), the unique cmi model obtained for ζ in Theorem 3.1 will exactly be the same as this one. Comments on the parallelism between the construction procedures given in Theorems 2.1 and 3.1 can be found in Appendix C. ■

The specific nature of YSP-like causal theories and the construction introduced in the proof of Theorem 3.1 suggest a procedure for computing the set of atomic base sentences known in the unique cmi model of any YSP-like causal theory.

Theorem 3.2 If ζ is a YSP-like causal theory of size n, then the unique set of atomic base sentences known in any cmi model of ζ can be computed in time $O(n)$.

Proof. An $O(n)$ algorithm has been proposed in Appendix C. The steps of the model construction given in the proof of Theorem 3.1 are followed in the algorithm. A program has been implemented to test the algorithm (see Appendix D). ■

Consider the causal theory with axiom schemata given in Section 3.1. It is a YSP-like causal theory since it contains an initial boundary condition (axiom 1), a final boundary condition (axiom 2), a persistence axiom schema (schema 3) and a causal axiom schema (schema 4). Given this YSP-like causal theory (some mundane conditions are omitted), the algorithm produces the sentences: TRUE(1,loaded), TRUE(2,loaded), ..., TRUE(5,loaded), TRUE(5,fire), and TRUE(6,noise). These are exactly the sentences Shoham's algorithm yields.

Now the final boundary condition is replaced by $\Box(10^{10}$,fire). Both algorithms produce TRUE(1,loaded), TRUE(2,loaded), ..., TRUE(10^{10},loaded), TRUE(10^{10},fire), and TRUE($10^{10}+1$,noise). Since Shoham's algorithm must step through each time point between 1 and 10^{10}, it takes too long for it to jump to the conclusion that the gun will be loaded at 10^{10}, and then infer that there will be a loud noise at $10^{10}+1$. However, if one knows that the gun is loaded and that nothing has happened until the time of reasoning about the effect of firing the gun, one will immediately conclude that the gun is still loaded. Then, one will reason about the effect of firing the gun with this knowledge. In fact, this is what the $O(n)$ algorithm does; knowing that the gun is loaded at 1, and nothing interferes with the gun's being loaded, it concludes that the gun will remain loaded until it is fired at 10^{10}.

Now let the scenario change. The gun is loaded at 1 but is emptied manually at 9. Shoham's algorithm and the $O(n)$ algorithm both produce TRUE(1,loaded), TRUE(2,loaded), ..., TRUE(9,loaded), and TRUE(9,emptied-manually).

3.3 Multi-agents and a broader class of YSP-like causal theories

Restricting theories so that they contain a persistence axiom schema and a causal axiom schema does not provide the full power to represent realistic scenarios. Consider the YSP. Fred's being alive and the gun's being loaded at time 1 form the initial description. Furthermore, assume that the gun is fired at 10. An axiomatization follows:

1. $\Box(1,\text{alive})$.
2. $\Box(1,\text{loaded})$.
3. $\Box(10,\text{fire})$.
4. $\Box(t,\text{alive}) \wedge \Diamond(t,\neg\text{fire}) \wedge \Diamond(t,\text{air}) \supset \Box(t+1,\text{alive}), \quad \forall t$.
5. $\Box(t,\text{loaded}) \wedge \Box(t,\text{fire}) \wedge \Diamond(t,\text{firing-pin})$
   $\wedge \Diamond(t,\text{no-marshmallow-bullets})$
   $\supset \Box(t+1,\text{dead}), \quad \forall t$.
6. $\Box(t,\text{loaded}) \wedge \Diamond(t,\neg\text{fire}) \wedge \Diamond(t,\neg\text{emptied-manually})$
   $\supset \Box(t+1,\text{loaded}), \quad \forall t$.
7. $\Box(t,\text{loaded}) \wedge \Box(t,\text{fire}) \wedge \Diamond(t,\text{air})$
   $\wedge \Diamond(t,\text{firing-pin})$
   $\wedge \Diamond(t,\text{no-marshmallow-bullets})$
   $\supset \Box(t+1,\text{noise}), \quad \forall t$.

Axioms 1 and 2 describe the initial state. Axiom 3 indicates the occurrence of the firing action. Axiom schema 4 says that Fred remains alive unless the gun is fired at him or there is no air (and hence he suffocates). Axiom schema 5 says that firing a loaded gun causes Fred's death provided that some conditions are satisfied. Axiom schemata 6 and 7 are used in the usual sense. This theory is not a YSP-like causal theory.

Because a YSP-like causal theory must contain exactly one persistence and one causal axiom schema. Moreover, one initial and one final boundary condition are allowed. The theory above however contains two persistence and two causal axiom schemata, two initial boundary conditions, and one final boundary condition. Therefore, scenarios similar to this call for a broader class of YSP-like causal theories which will be introduced in the sequel. Before doing this, Shoham's causal theories will be examined to see whether they succeed in computing the intended models when concurrent actions are introduced.

Given an initial description of the world, one would like reason about the effects of concurrent actions. For example, turning the ignition key of a car and pressing the gas pedal at different times may not cause the car to run. But if these actions are performed simultaneously, the car starts running. Causal theories allow concurrent actions. Consider the following blocks world. There is a block initially located at a position (denoted by "at-center") on the table. There are two operations "push-left" and "push-right." Executing "push-left" moves the block to a location (denoted by "at-left"). Executing "push-right" causes the block to move to another position (denoted by "at-right"). It is assumed that the forces applied on the block are of equal magnitude when these operations are performed concurrently. Now, assume that the block is at "at-center" at time 1, and "push-left" and "push-right" are simultaneously executed at 1. One would expect that the block will not move. Let the causal theory contain the following:

1. $\Box$(1,at-center).
2. $\Box$(1,push-left).
3. $\Box$(1,push-right).
4. $\Box$(1,at-center) $\wedge$ $\Diamond$(1,$\neg$push-left) $\wedge$ $\Diamond$(1,$\neg$push-right) $\supset$ $\Box$(2,at-center).
5. $\Box$(1,at-center) $\wedge$ $\Box$(1,push-left) $\wedge$ $\Diamond$(1,$\neg$push-right) $\supset$ $\Box$(2,at-left).
6. $\Box$(1,at-center) $\wedge$ $\Box$(1,push-right) $\wedge$ $\Diamond$(1,$\neg$push-left) $\supset$ $\Box$(2,at-right).

Shoham's algorithm computes TRUE(1,at-center), TRUE(1,push-left), TRUE(1,push-right). No other base sentence is known in the cmi models of this causal theory. This is strange. Since "push-left" and "push-right" are executed concurrently, the block should remain at the center of the table. That is, the sentence TRUE(2,at-center) must be obtained.

This problem can be resolved by introducing additional axioms such as "if it is known that the block is at the center of the table, and that push-right and push-left operations are simultaneously performed, then it is known that the block remains at the center" and "if it is known that the block is at the center of the table, and that no push-right or push-left operations are performed, then it is known that the block remains at the center." Unfortunately, in more complex domains, the number of such axioms can grow quickly. There must be a way of resolving this problem with a persistence axiom.

Definition 3.2 The *set of counteractions* is the set of actions that prevent each other from being operative when performed concurrently.

For example, pushing the block to left and pushing it to right are two counteractions that prevent each other when performed simultaneously. The effect of one of these actions cannot be obtained when the other action is also performed (see Appendix B for a discussion).

Definition 3.3 Let $\Pi=\{\diamond(t_1,p_i) \mid 1 \leq i \leq n, \text{ for some } t_1\}$ where p_i's are counteractions. Letting M be the unique cmi model of a causal theory Ψ, let us write $M \models \Pi$ iff $M \models \diamond(t_1,p_i)$, $\forall \diamond(t_1,p_i) \in \Pi$, or $M \not\models \diamond(t_1,p_i)$, $\forall \diamond(t_1,p_i) \in \Pi$. Otherwise, let us write $M \not\models \Pi$.

As an illustration, the fourth axiom in the blocks world example above is replaced with the axiom below, where $\Pi=\{\diamond(1,\neg\text{push-left}), \diamond(1,\neg\text{push-right})\}$.

$\Box(1,\text{at-center}) \wedge \Pi \supset \Box(2,\text{at-center})$.

Abusing the notation, Π will be used as if it were a function over its members:

$\Box(1,\text{at-center}) \wedge \Pi(\diamond(1,\neg\text{push-left}), \diamond(1,\neg\text{push-right})) \supset \Box(2,\text{at-center})$.

Under the interpretation of Π, in all cmi models of the causal theory for the blocks world example TRUE(1,at-center), TRUE(1,push-left), TRUE(1,push-right), TRUE(2,at-center) are known.

Now a new class of causal theories with axiom schemata will be defined. It can be looked upon as a broader class of YSP-like causal theories. For this reason, any theory in this class will be called a YSP'-like causal theory.

Definition 3.4 A *YSP'-like causal theory* ζ' is a theory in CI containing

$\Box\varphi_{s_i}$, i=1,...,n.

$\Box\varphi_{f_j}$, j=1,...,m.

and sentences in one of the following forms

$\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$, $\forall t$.

$\Phi_c \wedge \Theta_c \supset \Box\varphi_c$, $\forall t$.

where

1. $\Box\varphi_{s_i}$'s form the (nonempty) set of *initial boundary conditions* where each φ_{s_i} is of the form TRUE(t_1,[$\neg$]p).

2. $\Box\varphi_{f_j}$'s form the (nonempty) set of *final boundary conditions* where each φ_{f_j} is of the form TRUE(t_2,[$\neg$]p), $t_1 < t_2$.

3. Any sentence of the form $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$ is a *persistence axiom schema* where

   (i) φ_p is of the form TRUE(t,[$\neg$]p) (on the l.h.s.) and TRUE(t+1,[$\neg$]p) (on the r.h.s.).

   (ii) $\varnothing_p$ is a (possibly empty) conjunction of Π_i, where Π_i is a set of sentences $\Diamond\varphi_j$ such that φ_j is of the form TRUE(t,[$\neg$]q).

   (iii) Θ_p is a (possibly empty) conjunction of $\Diamond\varphi_k$, where φ_k is of the form TRUE(t,[$\neg$]q).

4. Any sentence of the form $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ is a *causal axiom schema* where

(i) Φ_c is a nonempty conjunction of sentences $\Box\varphi_i$, where φ_i is of the form TRUE(t,[¬]p). Φ_c must contain at least one sentence of the form TRUE(t,[¬]p) which does not appear on the r.h.s. of any (persistence or causal) axiom schema (as TRUE(t+1,[¬]p)).

(ii) Θ_c is a (possibly empty) conjunction of sentences $\Diamond\varphi_j$, where φ_j is of the form TRUE(t,[¬]q).

(iii) φ_c is of the form TRUE(t+1,[¬]r).

5. TRUE(t_1,p) and TRUE(t_1,¬p) do not appear among the initial boundary conditions together.

6. TRUE(t_2,q) and TRUE(t_2,¬q) do not appear among the final boundary conditions together.

7. Let $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$ and $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ be two schemata in ζ'. If $\Diamond$(t,p) (respectively $\Diamond$(t,¬p)) is a conjunct of $\varnothing_p \wedge \Theta_p$, then Θ_c does not contain $\Diamond$(t,¬p) (respectively $\Diamond$(t,p)) as a conjunct.

8. Let $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$ and $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ be two schemata in ζ'. If φ_p and φ_c are of the forms TRUE(t,p) (respectively TRUE(t,¬p)) and TRUE(t,¬p) (respectively TRUE(t,p)) then $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \wedge \Phi_c \wedge \Theta_c$ is inconsistent,

9. Let $\Box\varphi_{s_i}$ (respectively $\Box\varphi_{f_j}$) be an initial (respectively final) boundary condition and $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$ be a persistence axiom schema. If φ_{s_i} (respectively φ_{f_j}) is of the form TRUE(t_1,p) (respectively TRUE(t_2,¬p)) and φ_p is of the form TRUE(t,¬p) (respectively TRUE(t,p)) then $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p$ is inconsistent,

10. Let $\Box\varphi_{s_i}$ (respectively $\Box\varphi_{f_j}$) be an initial (respectively final) boundary condition and $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ be a causal axiom schema. If φ_{s_i} (respectively φ_{f_j}) is of the form TRUE(t_1,p) (respectively TRUE(t_2,¬p)) and φ_c is of the form TRUE(t,¬p) (respectively TRUE(t,p)) then $\Phi_c \wedge \Theta_c$ is inconsistent.

Proposition 3.2 Any YSP'-like causal theory ζ' corresponds to a finite causal theory Ψ if each t in all axiom schemata in ζ' is replaced by constants in the range t_1 to t_2, where t_1 and t_2 are the time points mentioned in the initial and final boundary conditions of ζ', respectively.

Proof. Replacing t in axiom schemata with constants gives a finite set of axioms. These axioms together with the initial and final boundary conditions form Ψ. ■

Theorem 3.3 If ζ' is a YSP'-like causal theory, then ζ' has cmi models and in all of these cmi models the same atomic base sentences are known.

Proof. By Proposition 3.2, there exists a Ψ corresponding to ζ'. By Theorem 2.1, any Ψ has a unique cmi model. Hence, ζ' has a unique cmi model. In Appendix C a model construction procedure is given. ■

Without the notion of counteractions and the corresponding syntactic sugar Π, YSP-like causal theories are in the class of YSP'-like causal theories.

Theorem 3.4 If ζ is a YSP-like causal theory, then ζ is also a YSP'-like causal theory.

Proof. Consider the initial and final boundary conditions of ζ as the unique members of the sets of initial and final boundary conditions of a YSP'-like causal theory ζ', respectively. The causal axiom schema of ζ, being the only causal axiom schema in ζ', and the persistence axiom schema of ζ (with an empty set of $\diamond$-conditions for the set of counteractions), being the only persistence axiom schema of ζ', form a YSP'-like causal theory ζ'. ■

Theorem 3.5 If ζ' is a YSP'-like causal theory of size n, then the unique set of atomic base sentences known in any cmi model of ζ' can be computed in time $O(n \log n)$.

Proof. An algorithm is proposed in Appendix C to compute these atomic base sentences. The complexity of this algorithm is shown to be O($n$log$n$). ■

Let the following YSP'-like causal theory represent the blocks world scenario at the beginning of this section. But now assume that "push-left" and "push-right" are executed concurrently at 10.

1. $\Box$(1,at-center).
2. $\Box$(10,push-left).
3. $\Box$(10,push-right).
4. $\Box$(t,at-center) $\wedge$ Π($\Diamond$(t,$\neg$push-left), $\Diamond$(t,$\neg$push-right)) $\supset$ $\Box$(t+1,at-center), $\forall$t.
5. $\Box$(t,at-center) $\wedge$ $\Box$(t,push-left) $\wedge$ $\Diamond$(t,$\neg$push-right) $\supset$ $\Box$(t+1,at-left), $\forall$t.
6. $\Box$(t,at-center) $\wedge$ $\Box$(t,push-right) $\wedge$ $\Diamond$(t,$\neg$push-left) $\supset$ $\Box$(t+1,at-right), $\forall$t.

The O($n$log$n$) program first computes the set of base sentences that will be known at 2 from what is known (and what is not known) at 1. It finds out that TRUE(2,at-center) is known by the axiom schema 4. Then, it performs one more iteration to see what is known at 3. Again by axiom schema 4, it is seen that only TRUE(3,at-center) is known. Since the base sentences that are known at this step of the iteration are only the persistence sentences, it generates the sentences TRUE(4,at-center), TRUE(5,at-center), ..., TRUE(10,at-center). Finally, it computes the sentences that are known at 11 from the atomic base sentences known at 10. Noticing that "push-left" and "push-right" are counteractions executed simultaneously, it finds out that the l.h.s. of the axiom schema 4 is satisfied. It produces the sentence TRUE(11,at-center). Since l.h.s. of all other axiom schemata fail due to the occurrence of counteractions at 10, the atomic base sentences that are known in the cmi model of this YSP'-like causal theory are TRUE(1,at-center), TRUE(2,at-center), ..., TRUE(10,at-center), TRUE(10,push-right), TRUE(10,push-left), and TRUE(11,at-center).

Now let just one of the operations, say "push-left," be executed at time 10. The algorithm then produces TRUE(1,at-center), TRUE(2,at-center), ..., TRUE(10,at-center), TRUE(10,push-left), and TRUE(11,at-left).

To see the consequences of a more interesting YSP'-like causal theory, consider the shooting scenario. Fred is alive and that the gun is loaded at time 1. The gun is fired at Fred at time 10. The theory given for this scenario contains axiom schemata and boundary conditions. It is a typical YSP'-like causal theory. Given this theory, our $O(n \log n)$ algorithm produces the intended model. Shoham's $O(Tmax\ n \log(Tmax\ n))$ algorithm and this algorithm produce the same sentences: TRUE(1,alive), TRUE(1,loaded), TRUE(2,alive), TRUE(2,loaded), ..., TRUE(10,alive), TRUE(10,loaded), TRUE(10,fire), TRUE(11,dead), and TRUE(11,noise).

The unintended models in YSP'-like causal theories are eliminated by considering the occurrence of counteractions. This is not specific to YSP'-like causal theories. The notion of counteractions and the syntactic sugar Π can be embedded into the sentences in Shoham's causal theories as well.

3.4 When is computation time-dependent?

It is not known to what extent causal theories give a satisfactory account of the reasoning process. In the previous sections, it has been shown that computing with causal theories is inefficient in the sense that one must step through each axiom in the causal theory to compute the results of some actions. To remove this deficiency, new classes of causal theories have been introduced. Restrictions have been imposed on sentences in these classes. One may wonder whether the time-dependent nature of computations can be removed without imposing these restrictions, but still allowing axiom schemata. The answer is not in the affirmative.

For example, consider an electronic circuit which functions as a relay. The output of the relay is directly connected to its input. The output can be either "on" or "off" depending on the input. If the input is "on" (respectively "off") at some time, then the output becomes "off" (respectively "on") at the next instant of time. One can interrupt the system by the operation "interfere." When "interfere" is done, the output of the circuit is delayed. Assume that the output of the circuit is given as "on" at time 1. If "interfere" is executed at time 6, what are the consequences? Below, a causal theory is given as a formalization of this scenario. (This theory is neither a YSP-like nor a YSP'-like causal theory. For example, $\Box(t,on)$ is the unique $\Box$-condition of the axiom schema 3, but it appears on the r.h.s. of the axiom schema 4.)

1. $\Box(1,on)$.
2. $\Box(6,interfere)$.
3. $\Box(t,on) \wedge \Diamond(t,\neg interfere) \supset \Box(t+1,off), \quad \forall t.$
4. $\Box(t,off) \wedge \Diamond(t,\neg interfere) \supset \Box(t+1,on), \quad \forall t.$
5. $\Box(t,on) \wedge \Box(t,interfere) \supset \Box(t+4,on), \quad \forall t.$
6. $\Box(t,off) \wedge \Box(t,interfere) \supset \Box(t+4,off), \quad \forall t.$

TRUE(1,on), TRUE(2,off), TRUE(3,on), TRUE(4,off), TRUE(5,on), TRUE(6,off), TRUE(6,interfere), and TRUE(10,off) are obtained as the atomic base sentences known in all cmi models of the corresponding finite causal theory.

Obviously, such a scenario requires examination of each axiom schema in the theory for all time points between 1 and 6. However, by determining regularities one can jump to conclusions. Knowing that the output is initially "on" at time 1 and that the relay produces a regular sequence of "on" and "off" unless "interfere" is executed, one can directly generate the sentences TRUE(2,off), TRUE(3,on), TRUE(4,off), TRUE(5,on), and TRUE(6,off). But determining such regularities may be expensive.

Chapter 4

SIMULTANEITY OF CAUSE AND EFFECT

Asymmetry problem, the question of what distinguishes cause from effect, has been a crucial issue in philosophy. That "causes cannot succeed their effects in time" is accepted commonly, but not universally. Russell states that "If there are causes and effects, they must be separated by a finite time-interval" [2, p. 62]. But elsewhere he asserts that "It is not essential to a causal law that the object inferred should be later than some or all of the data. It may equally well be earlier or at the same time" [2, p. 63]. Changing his mind in a later article he states that "A causal proposition can be stated in the following way: A exists at time $t \supset B$ will exist at time $t+\Delta t$" [2, p. 63].

It should be noted that the proposition "causes cannot succeed their effects in time" does not require the precedence of the cause to its effect in time. Causes and effects may coincide in time. To quote Bunge [2, p. 39]:

> "To employ a term of which traditional philosophers are fond, the cause is existentially prior to the effect - but need not *precede* it in time."

Von Wright notes the problematic occasions in distinguishing cause and effect when there is no temporal precedence [38, p. 107]:

> "In the normal cases, the effect brought about by the operation of cause occurs later. In such cases time has already provided the distinction. More problematic is the case when cause and effect are supposed to be simultaneous. Those who think of the cause-effect distinction in terms of temporality will be at loss here."

4.1 Two philosophical accounts of simultaneous causation

Taylor [38, pp. 39-43] cites a number of examples on causal connections in everyday life where the difference between a cause and its effect cannot be based on temporality [38, p. 39]:

> "Consider, for instance, a locomotive that is pulling a caboose, and to make it simple, suppose this is all it is pulling. Now here the motion of the locomotive is sufficient for the motion of the caboose, the two being connected in such a way that the former cannot move without the latter moving with it. But so also, the motion of the caboose is sufficient for the motion of the locomotive, for given that the two are connected as they are, it would be impossible for the caboose to be moving without the locomotive moving with it. From this it logically follows that, conditions being such as they are - both objects are in motion, there are no other moves present, no obstructions to motion, and so on - the motion of each object is also necessary for the motion of the other. But is there any temporal gap between the motion of one and the motion of the other? Clearly there is not. They move together, and in no sense is the motion of one temporally followed by the motion of the other."

Taylor identifies the criterion to distinguish the cause from the effect: *the cause acts upon something else to produce some change*. For example, the locomotive acts on the caboose and pulls it whereas the caboose does not push the locomotive. Then, he notices that what is distinguished as a cause can also be an effect of other causes which are again simultaneous with their effects. He asks whether all causes are simultaneous with their effects. Noting the existence of causal chains as well as temporally separated but causally related events, Taylor concludes that causes usually precede their effects in time, but rejects the idea that causes *must* precede their effects in time. He defines cause as a given set of conditions, which is *antecedently* (but not subsequently) necessary for, or sufficient for, or both necessary and sufficient for another state of affairs.

Von Wright proposes an account of causation which would help in cause-effect asymmetry when there is no order of temporal precedence [38, pp. 95-113], [39]. The idea of causation, which he calls *manipulative* (or *experimentalist*) causation, is based on interference and action. If two simultaneous occurrences are causally connected, then the one which can be influenced by manipulating the other must be the cause of the other, except there is no *common cause* of these occurrences. Von Wright also examines the role of manipulation in functional relationships. He claims that not all factors in a functional relationship are manipulable, and the causality in these relationships is in that one has the power to change one term by manipulating the other. For example, if one can only change the volume of a gas by changing either pressure or temperature, then the changes in the volume of the gas must be effects, not causes.

In distinguishing a cause from its effect when simultaneity is of concern, Taylor and von Wright agree on how to provide the distinction: cause acts upon effect and cause can be controlled to produce the effect. But von Wright develops a formal analysis of his general account of causation and determinism, consisting of ordinary propositional logic, a tense logic, and a modal logic.

4.2 Shoham's account

From the definition of causal theories in Chapter 2, it is obvious that Shoham accepts the temporal precedence of causes over their effects. In [35, pp. 178-179] he discusses what problematic issues arise when simultaneity of causes and effects is allowed in a causal theory.

Causal theories are restricted in that causes strictly precede their effects in time. In a causal theory, for any sentence $\Phi \wedge \Theta \supset \Box(t_5,t_6,r)$, the ltp's of all conjuncts in Φ and Θ must be less than t_5 (assuming $t_5 \leq t_6$). That is, if $\Box(t_1,t_2,p) \in \Phi$ and $\Diamond(t_3,t_4,q) \in \Theta$, then it must be the case that $t_2 < t_5$ and $t_4 < t_5$ (assuming $t_1 \leq t_2$ and $t_3 \leq t_4$). One point worth mentioning is the question of whether time is viewed as discrete or dense. In causal theories, the time structure is that of Z. But Shoham admits that time should not be viewed as the integers in case of simultaneity of cause and effect, leaving the question of how to view it (dense, complete, linear, or branching) partly unanswered.

4.3 Problems with simultaneous temporal propositions

4.3.1 Self-causation and circular causation

Among the commonly agreed properties of causation three are the touchstones for a formal treatment of causation. These are its properties of being *antisymmetric*, *irreflexive*, and *transitive*. For example, Bunge [2, p. 244] proposes a relational approach where a relation, R, is supposed to hold between the cause and its effect. He specifies formal properties of R as follows (note the different logical symbols):

(a) It is a *dyadic* relation $x\mathrm{R}y$ holding among events.
(b) It is *irreflexive*, $(x) \sim (x\mathrm{R}x)$.
(c) It is *transitive*, $(x)\ (y)\ (z)\ [x\mathrm{R}y\ \&\ y\mathrm{R}z \supset x\mathrm{R}z]$.
(d) It is *asymmetrical*, $(x)\ (y)\ [x\mathrm{R}y \supset \sim y\mathrm{R}x]$.

Causation asserts that nothing is self-caused. Every change is a result of something external to the changing subject. Such a view belongs to the modern understanding of causal determinism. Causal determinism takes efficient causation for granted such that efficient cause is briefly defined as an *external actor*.

It is the irreflexivity property of causation that is absent in material implication. Given any proposition p, it *immediately implies* itself (symbolically $p \Rightarrow p$). Hence material implication cannot be regarded as a correct formalization of causal connection. The irreflexive characteristic of causation together with its transitivity property forbids *circular causation*. Causal rules in causal theories are strongly related to material implication. But causal rules are weaker in some respects and stronger in others. Shoham discusses this issue in a related section on the properties of causation [35, p. 152 and p. 160]. Bunge [2, pp. 242-243] also addresses the relation between causation and implication. The discussion is threefold. It centers around causation and the kinds of implication: *material*, *strict*, and *causal*.

Causal theories have antisymmetry and irreflexivity properties by definition since temporal precedence of causes over their effects is taken as the core principle of causal connections expressed by causal sentences. However, the transitivity characteristic is partly missing in causal theories. Temporally ordered sequences of causal relations are permitted. But this does not give a full account of the transitivity relation. A sequence of causes and effects (effects being also the causes of other effects) which are not ordered temporally, but possibly causally, and occurring simultaneously also form a transitive relation. For example, in an isolated environment an event A causes B, which in turn causes C such that there is no time difference between their occurrences and every cause is simultaneous with its effect. Then, it follows that A also (indirectly) causes C since whenever A occurs, B must be there by causally depending on A, and whenever B occurs, C must be there by causally depending on B.

It might sound confusing to talk about the conceptual inequality of *causal order* and *temporal order* of occurrences. There are situations in which two things may happen at the same time. There exists no temporal order between their occurrences. None of them occurs after the other in time. However, the occurrence of one of them can be identified as the cause of the other. In this case, it is said that there is a causal order between them; the cause is *causally* before the caused one, the *effect*.

In causal theories, causal rules can represent causation such that the $\Box$-conditions on the l.h.s. of a causal rule denote causes while the r.h.s. denotes their effects. Under this interpretation, having simultaneous temporal propositions on both sides of causal sentences may result in circular causation [35, p. 179]: $\Box(t,p_i) \supset \Box(t,p_{i+1})$, $i=1,\ldots,n-1$, $p_n = p_1$.

Simply, the causal theory may include a sentence of the form $\Box(t,p) \supset \Box(t,p)$. Then, we have *self-causation*. Our objection to this is twofold. First, causation is semantic rather than syntactic. But if circularity exists, relating causation to syntactic forms only will not be fair. Instead, causation can take the form of a mere material implication. Furthermore, sentences of the form $\Box(t_1,p) \supset \Box(t_2,p)$, where $t_1 < t_2$, are allowed in causal theories. Does this mean that p causes itself? There can be sentences in the form $\Diamond(t_1,p) \supset \Box(t_2,p)$, where $t_1 < t_2$. Is this rendered as "if $\neg p$ is not known at t_1, then p is known at t_2 *for no reason*"? Through *soundness conditions*, one can write sentences like $\Diamond(t_1,p) \supset$ TRUE(t_1,p). Shoham [35, p. 118] says "we now assume that the soundness conditions are implicitly part of the causal theory itself, and are omitted simply for reasons of economy of expression." Moreover, the boundary between $\Box$- and $\Diamond$-conditions in Shoham's account becomes hazy if $\Box$-conditions in a causal rule strictly denote the causes.

The second objection, closely connected to the first one, is that one is not supposed to look for the causes in the unique cmi model of a causal theory. If this were the case, then there would be difficulties in identifying the causes and computing possible *explanans* of the occurrences. Temporal precedence of causes over effects already provides the necessary criterion to find out the causes of a given set of effects. However, when simultaneous propositions are allowed on both sides of the causal rules, the problem becomes more complex.

As an example for cause-effect distinction, reconsider Taylor's illustration. Assume that the causal theory contains the following:

$\Box$(4,locomotive-moves) $\supset$ $\Box$(4,caboose-moves).
$\Box$(4,caboose-moves) $\supset$ $\Box$(4,locomotive-moves).

Looking only at the syntactic forms of these rules, one can say that these permit circular causation. But now add TRUE(4,locomotive-moves) to the causal theory. Then, TRUE(4,locomotive-moves) and TRUE(4,caboose-moves) will be the only sentences known in all cmi models of the causal theory. In this case, if one investigates the cause of the motion of the locomotive and the caboose, he may identify the motion of the locomotive as the cause of the motion of the caboose although $\Box$(4,caboose-moves) $\supset$ $\Box$(4,caboose-moves) implies self-causation.

Permitting causal sentences of the form $\Box(t,p_i) \supset \Box(t,p_{i+1})$, i=1,...,n-1, $p_n=p_1$ introduces no peculiarity at all in constructing cmi models. However, the original algorithm must be revised to obtain the known atomic base sentences. This will be studied in Section 4.5.

4.3.2 Self-change

Shoham says [35, p. 179]:

> "[O]ne might have a set of sentences $\Box(t,p_i) \supset \Box(t,p_{i+1})$, i=1,...,n-1, $p_n = \neg p_1$; This would destroy the independence of the past from the future in general, and the 'unique'-model property in particular. Or, as another example, one might have sentences of the form $\Diamond(t,p) \supset \Box(t,\neg p)$, which would have a similarly detrimental effect on the properties of causal and inertial theories."

However, by placing some restrictions on the sentences in the definition of causal theories these problems can be eliminated. In the former case, it is possible to impose some restrictions on the sentences similar to the one in the definition of the original causal theories. Recall that consistency of the causal theories is maintained by the following:

> If $\Phi_1 \wedge \Theta_1 \supset \Box(t_1,t_2,p)$ and $\Phi_2 \wedge \Theta_2 \supset \Box(t_1,t_2,\neg p)$ are two sentences in Ψ, then $\Phi_1 \wedge \Theta_1 \wedge \Phi_2 \wedge \Theta_2$ is inconsistent.

To see what kind of situations yield inconsistency, two possibilities are examined below.

(a) Causal connections can be unidirectional:

$\Box(t_1,p_1) \supset \Box(t_1,p_2)$.
$\Box(t_1,p_2) \supset \Box(t_1,p_3)$.

... ...
$\Box(t_1,p_n) \supset \Box(t_1,\neg p_1)$.

If there exists another sentence with $\Box(t_1,p_1)$ on its r.h.s., then this may result in inconsistent inferences.

(b) The causal connections can be bidirectional:

$\Box(t_1,p_1) \supset \Box(t_1,p_2)$.
$\Box(t_1,p_2) \supset \Box(t_1,p_1)$.
$\Box(t_1,p_2) \supset \Box(t_1,p_3)$.
$\Box(t_1,p_3) \supset \Box(t_1,p_2)$.
$\Box(t_1,p_3) \supset \Box(t_1,\neg p_1)$, where there exists at least one causal chain from $\Box(t_1,p_1)$ to $\Box(t_1,\neg p_1)$.

In this case, if the r.h.s. of any of these rules is satisfied, due to the existence of causal chain the sentences TRUE(t_1,p_1) and TRUE$(t_1,\neg p_1)$ will be obtained.

Instead of prohibiting sentences of the form $\Box(t,p_i) \supset \Box(t,p_{i+1})$, $i=1,\ldots,n-1$, $p_n = \neg p_1$, more special forms of sentences can be permitted. In this way, the possibility for a proposition to be true and false at the same time can be eliminated.

4.4 Should simultaneity be treated by causal theories?

As can be seen from the preceding sections, simultaneous causation is possible and there may be situations in which cause and effect occur at the same time.

Consider the following illustration attributed to von Wright [38, p. 108], [39]. There is a horizontally positioned pipe whose two ends are controlled with two valves. These are connected in such a way that if one is opened at one time, then the other valve is closed at the same time, and vice versa. Opening of one valve and closing of the other occur simultaneously. The pipe has also a top-inlet continuously supplying high pressure water into it. Hence, the pipe, with the valves directing the water flow to only one direction at a time, functions as a two-way watering system. Now let the state of the first valve's being open be represented by p while that of the second by q. Then, at any time either $p \wedge \neg q$ or $\neg p \wedge q$. Additionally, let the flow of the water through the first (respectively second) valve be represented by r (respectively s).

Obviously, the cause of r is p and the cause of s is q. Then, a causal theory will contain the sentences:

$$\Box(t,p) \wedge \Theta_1 \supset \Box(t+1,r), \quad \forall t, \text{ where } \Theta_1=\bigwedge^{m}_{i=1}\Diamond(t_{i1},t_{i2},a_i).$$

$$\Box(t,q) \wedge \Theta_2 \supset \Box(t+1,s), \quad \forall t, \text{ where } \Theta_2=\bigwedge^{n}_{j=1}\Diamond(t_{j1},t_{j2},b_j).$$

Assume that the state of the valves are causally related to a *common cause* (e.g., if there is a possibility for an agent pushing only the first valve and closing it, this action causes the first valve to close and the second valve to open). In this case, the causal theory above might contain the following sentences where the pushing of the first valve is represented by u:

$$\Box(t,u) \wedge \Theta_3 \supset \Box(t+1,\neg p), \quad \forall t, \text{ where } \Theta_3=\bigwedge^{m}_{i=1}\Diamond(t_{i1},t_{i2},a_i).$$

$$\Box(t,u) \wedge \Theta_4 \supset \Box(t+1,q), \quad \forall t, \text{ where } \Theta_4=\bigwedge^{n}_{j=1}\Diamond(t_{j1},t_{j2},b_j).$$

It is noted that if $\Theta_3=\Theta_4$, one cause produces more than one effect. This suggests that causal rules can represent multiple effects. This issue is addressed in Appendix B.

If the two changes have separate causes, the situation is easy. For example, let the first valve be open and the second closed. If there is an agent pushing the first valve to close it, there may be another agent pulling the second valve to open it. Then, closing of the first valve can be attributed to the pushing of it, and opening of the second valve to the pulling of it. Or, it may well be the case that one agent pushes the first valve while another pushes the second valve. In this case, there are two causes, namely pushing of the first and the second valves, that intervene with each other. Although each cause separately has efficacy to produce its effect(s), now they prevent the changes that they will bring about. Since each one prevents the other from being operative, these two can be termed *counteracting causes*, following von Wright [39, pp. 75-77]. These two causes must be involved in the related causal sentences either in the form $\Box(t,u) \wedge \Diamond(t,\neg v)$ or in the form $\Box(t,v) \wedge \Diamond(t,\neg u)$ where u and v denote pushing of the first and the second valves, respectively.

So far, everything is on the side of the temporal precedence of causes over their effects. But, what happens if there is no cause of these two changes? This is quite possible because causal theories allow atomic sentences in the form of boundary conditions to be asserted for no reason. That is, things can come into being for no reason. In fact, either p (the first valve's being open) or $\neg q$ (the second valve's being closed) can be asserted at any time into the causal theory. Then, how does one assure that, when only one of them is known, he will know the other occurred at the very same time? A set of sentences in the following form might be helpful:

$$\Box(t,p) \supset \Box(t,\neg q), \quad \forall t.$$
$$\Box(t,\neg q) \supset \Box(t,p), \quad \forall t.$$

Note that they do not contain a $\Diamond$-condition. This means that occurrence of p (respectively $\neg q$) *unconditionally* necessitates occurrence of $\neg q$ (respectively p). However, there may be cases in which some qualifications must hold for occurrences to be simultaneous.

Ginsberg suggests that distinguishing between the qualifications of actions and their effects by imposing a temporal order on them is a weak approach [6, p. 233]:

> "Suppose I am attempting to take a photograph inside a cave. At great expense, I bring in a power supply and a battery of lights. I also bring a camera.
>
> Now it turns out that my camera has twelve on-board computers and must also be connected to the power supply.
>
> Unfortunately, my power supply cannot both operate the camera and run the lights. Thus, when I press the

shutter release, two things will happen: the shutter will open, and the lights will dim.

I need the lights on to take the photograph. And furthermore, I need them on at the precise instant that the shutter opens. It is imperative that the qualification to taking the photograph (that the lights are on) be simultaneous with the *effect* of the action (that the shutter drop) in this example."

For example, the following can represent the situation:

$$\Box(t,\text{press-shutter-release}) \wedge \Diamond(t+1,\text{lights-on}) \supset \Box(t+1,\text{shutter-drops}), \quad \forall t.$$

4.5 Causal theories: An extended definition

Definition 4.1 An *extended causal theory* Ω is a theory in CI, in which all sentences have the form $\Phi \wedge \Theta \supset \Box\varphi$, where

1. $\varphi = \text{TRUE}(t_1,t_2,p)$.

2. $\Phi = \bigwedge^{n}_{i=1}\Box\varphi_i$, where φ_i is an atomic base sentence with ltp t_i, $t_i \leq t_1$.

3. $\Theta = \bigwedge^{m}_{j=1}\Diamond\varphi_j$, where φ_j is an atomic base sentence with ltp t_j, $t_j \leq t_1$.

4. Φ or Θ may be empty.

5. It is assumed that there exists t_0 such that if $\Theta \supset \Box(t_1,t_2,p)$ is in Ω, then $t_0 < t_1$.

6. There do not exist two sentences in Ω such that one contains $\Diamond(t_1,t_2,p)$ on its l.h.s. while the other contains $\Diamond(t_1,t_2,\neg p)$ on its l.h.s.

7. If $\Phi_1 \wedge \Theta_1 \supset \Box(t_1,t_2,p)$ and $\Phi_2 \wedge \Theta_2 \supset \Box(t_1,t_2,\neg p)$ are in Ω, then $\Phi_1 \wedge \Theta_1 \wedge \Phi_2 \wedge \Theta_2$ is inconsistent.

8. There do not exists sentences in Ω of the form $\Phi \wedge \Diamond(t_1,t_2,\neg p) \wedge \Theta \supset \Box(t_2,t_3,p)$ or $\Phi \wedge \Diamond(t_1,t_2,p) \wedge \Theta \supset \Box(t_2,t_3,\neg p)$.

Informally, this definition says that causes can occur simultaneously with their effects (they can only coincide at a time point where the cause ceases while its effect starts). If cause and effect overlap for a period of time, the direction of prediction changes: either the past determines the future or the future determines the past.

Definition 4.2 The *earliest time point* (etp) of a base formula is the earliest time point mentioned in it.

1. The etp of $TRUE(t_1,t_2,p) = t_1$.
2. The etp of $\varphi_1 \wedge \varphi_2 = \min\{$etp of φ_1, etp of $\varphi_2\}$.
3. The etp of $\neg\varphi$ = the etp of φ.
4. The etp of $\forall v\ \varphi$ is the minimum among the etp's of all φ' which result from substituting in φ a time point symbol for all free occurrences of v, or $-\infty$ if there is no such minimum etp.

Definition 4.3 The *temporally meeting set of sentences at time t*, TMS_t, are those sentences in the causal theory Ω such that for any sentence, the etp and ltp of the base sentence on its r.h.s. is the same and equal to *t*, and the ltp of at least one of the base sentences on its l.h.s. is equal to the etp (ltp) of the base sentence on its r.h.s.

For example, if Ω contains $\Box(t_1,t_2,p) \wedge \Diamond(t_3,t_4,q) \supset \Box(t_4,r)$ and $\Box(t_1,t_4,u) \supset \Box(t_4,w)$, then they are in TMS_{t_4} (assuming that $t_1 \leq t_2 \leq t_4$ and $t_3 \leq t_4$).

Definition 4.4 The *bounded set of sentences at time t*, BS_t, are those sentences with ltp *t*.

If $\Box(t_1,t_2,p) \wedge \Diamond(t_3,t_4,q) \supset \Box(t_4,t_6,r)$ and $\Box(t_3,t_6,u) \supset \Box(t_6,w)$ are in Ω, then they are in BS_{t_6}. But note that only $\Box(t_3,t_6,u) \supset \Box(t_6,w)$ is in TMS_{t_6}. Then, a sentence in Ω is always in BS_t for some time t. But it may or may not be in any TMS.

Definition 4.5 The *temporally dependent set of sentences at time t*, TDS_t, are the sentences in TMS_t of Ω such that if φ is on the r.h.s. of a sentence in TMS_t, then it should be the case that either $\Box\varphi$ or $\Diamond\neg\varphi$ appears on the l.h.s. of other sentences in TMS_t.

For example, let TMS_{t_6} for an extended causal theory Ω be as follows:

$$TMS_{t_6} = \{\Box(t_1,t_2,p) \wedge \Diamond(t_3,t_6,q) \supset \Box(t_6,r),\\ \Box(t_6,r) \supset \Box(t_6,s),\\ \Box(t_1,t_2,p) \wedge \Diamond(t_5,t_6,u) \supset \Box(t_6,v)\}.$$

Then, $TDS_{t_6}=\{\Box(t_1,t_2,p) \wedge \Diamond(t_3,t_6,q) \supset \Box(t_6,r), \Box(t_6,r) \supset \Box(t_6,s)\}$.

Extended causal theories are consistent and satisfiable. Moreover, in all cmi models of an extended causal theory the same atomic base sentences are known.

Theorem 4.1 If Ω is an extended causal theory, then

1. Ω has a cmi model.
2. If M_1 and M_2 are both cmi models of Ω, and φ is any base sentence, then $M_1 \models \Box\varphi$ iff $M_2 \models \Box\varphi$.

Proof. Appendix C. ■

Theorem 4.2 If Ω is a finite extended causal theory of size n , then the set of the atomic base sentences known in the cmi models of Ω has size $O(n)$ and can be computed in time $O(n^2)$.

Proof. Appendix C. ■

Chapter 5

CONCLUSION

Shoham's causal theories have computationally simple model-theoretic properties. However, it turns out that computing with causal theories is not very efficient. Axiom schemata are not directly allowed in causal theories. If there is any axiom schema, it stands for all its ground instances. New classes of causal theories have been introduced to capture generality with axiom schemata as well as to efficiently compute the atomic base sentences known in all cmi models of causal theories. This has been done without destroying the unique-model property of causal theories. It has been shown that computing with these causal theories is not time-dependent. It turned out that causal theories, in general, call for a syntactic sugar to obtain intended models. Such a syntactic sugar has been embedded in our YSP'-like causal theories. A model construction procedure has been proposed to compute the atomic base sentences in all cmi models of YSP'-like causal theories.

There are still some technical problems. One is prohibiting simultaneity of cause and effect. More generally, temporal propositions are not allowed on both sides of sentences in causal theories. In the second part of the thesis, it has been emphasized that permitting such propositions provides more expressive power and an extended definition has been given. The intervals of the propositions on both sides of sentences in these theories are not allowed to overlap, but meet at certain points in time. Provided that some assumptions hold, it has been shown that extended causal theories have unique cmi models. In order to compute the atomic base sentences known in cmi models of causal theories, an $O(n^2)$ algorithm has been proposed.

It is not known whether there exist realistic domains to which Shoham's causal theories and the ones proposed in this thesis can be applied. Moreover, the points mentioned in Appendix B for a full account of causation (and also for reasoning) suggest new research directions.

Also open are some technical questions that have to do with efficiency. Perhaps one can devise $O(n)$ algorithms for causal theories containing axiom schemata with weaker restrictions.

Appendix A

TWO EPISTEMOLOGICAL PROBLEMS IN AI

AI programs suffer from a lack of *generality* [16] and the key for a solution is an expressive language to represent *general commonsense knowledge* [11]. *Situation calculus*, introduced by McCarthy and Hayes [12], is a formalism to express the consequences of actions. The problem with the situation calculus is of epistemological nature [13]. It is a problem centering around the question of what can formally be represented in a computer. In the following, the *frame* and the *qualification problems* are examined.

A.1 The frame problem

Assume that presently there are two blocks on a table. One of them is red and the other is blue. Also assume that a robot in the present situation moves the red block onto the blue one, resulting in a new situation. What are the colors of the blocks in this new situation? The answer is that the colors remain unchanged. But this must be embedded into the formalism by adding the rule that moving a block in a situation does not change the color of any block. Rules for each action asserting what features of the situation do not change by performing that action are called *frame axioms* [12]. As McCarthy and Hayes [12] argue if one had a number of actions to be performed in sequence, one would have quite a number of conditions to write down that certain actions do not change the values of certain fluents. In fact, with n actions and m fluents, one must write down mn such conditions.

So far the literature has come up with a variety of definitions of the frame problem. McCarthy states [17, p. 3]:

> "The frame problem is that of specifying what does not change when an event occurs."

What makes the frame problem difficult is the word "specifying" in this definition. Why should one specify for each individual event and for each individual property, that the occurrence of the event does not affect that property? If the problem is to determine what remains unchanged after the occurrence of some event, then the problem can be softened by transforming the question into "Does property p of a situation preserve its existence in the next situation?" This is equivalent to writing down axioms asserting that a property p persists if some conditions hold in that situation.

The coincidence of frame problem and persistence problem has been noticed by Shoham who describes the *persistence problem* in [35, p. 18] as the problem of predicting (on the basis of the past) that a fact will remain unchanged throughout a lengthy future interval. He gives the example of a billiard ball placed on a chosen spot on the table. One would like to predict that it will remain in that spot until it gets hit. The general problem, which he calls in [35, p. 17] the *extended-prediction problem*, is that of predicting *arbitrary things* about a lengthy future interval. That is, it is the problem of predicting not only what will not change but also what changes will occur throughout that future time interval.

Shoham and McDermott identify the extended-prediction problem as one of the formalism-independent problems of the prediction task and their claim is that it is a general problem in nature [35, pp. 3-18], [36-37]. They admit that the extended-prediction problem arises in classical mechanics since it is a hard task to formalize the process of making predictions over extended periods of time when the axioms of a temporal theory are expressed as differential equations. Moreover, they argue that this problem also occurs in the "histories" framework of Hayes. In contrast, Rayner claims that "there is actually nothing mysterious about the process of making predictions about continuous-time processes, and that these can readily be formalized with no more theoretical apparatus than is afforded by classical logic, together with the differential and integral calculus" [22, p. 382] and the extended-prediction problem has been solved [22, p. 385]. Additionally, he shows how one can predict collisions using classical logic. Shoham's objection there is that the approach taken by Rayner is less efficient than using CI. Obviously, the efficiency of both approaches can be compared if one can propose algorithms in CI for continuous time. It should also be noted that Naur voices a strong objection and considers that the problem has been solved long ago by Newtonian mechanics [19].

A.2 The qualification problem

The second fundamental epistemological problem is the *qualification problem*. McCarthy defines the problem [14, p. 27]:

> "The 'qualification problem' immediately arose in representing general common sense knowledge. It seemed that in order to fully represent the conditions for the successful performance of an action, an impractical and implausible number of qualifications would have to be included in the sentences expressing them. For example, the successful use of a boat to cross a river requires, if the boat is a rowboat, that the oars and rowlocks be present and unbroken, and that they fit each other. Many other qualifications can be added, making the rules for using a rowboat almost impossible to apply, and yet anyone will still be able to think of additional requirements not yet stated."

The problem has been emphasized by Shoham [35, p. 16]:

> "[It] is the problem of trading off the amount of knowledge that is required in order to make inferences on the one hand, and the accuracy of those inferences on the other

> hand. In the particular context of predicting the future, it is the problem of making sound predictions about the future without taking into account everything about the past. Notice that the problem would disappear if we were willing to dramatically idealize the world: we could take it as a fact that noise always follows the firing of a loaded gun, simply assume that guns always have firing pins, that there are never vacuum conditions, and so on. The premise of this discussion, however, is that such an overidealization is a nonsolution, since the whole point is for our robots to be able to function in a realistically complex environment."

The qualification problem is twofold. First, there is the problem of fully specifying the characteristics of the qualifications in a realistically complex environment. For example, if one would like to predict noise when a loaded gun is fired, it is more appropriate to talk about the existence of a general environment carrying sound rather than a specific environment, namely air. This calls for categorization and close analysis of the properties of the related entities in a particular context of reasoning.

The second aspect of the qualification problem is computational. Since the number of qualifications can be very large, devising a formalism that will allow correct reasoning efficiently becomes crucial. Nonmonotonic logic has been widely accepted as a standard tool towards a solution to this problem.

CI is claimed to be a solution to the qualification problem since it "allows one to omit 'obvious facts', and still be able to deduce the desired facts about the future" [35, p. 174]. Shoham, however, admits that computing the consequences of any theory in CI is hard. Yet, *causal theories* have some nice properties that yield a good time bound.

It has been thought that qualifications may become infinite showing the impossibility of an efficient computational model for intelligent robots. Naur sees the formalism introduced by Shoham as a nonsolution to the qualification problem. His objection centers around the number of $\Diamond$-conditions in an axiom [20]:

> "$\Box(t,loaded) \wedge \Box(t,fire) \wedge \Diamond(t,air) \wedge \Diamond(t,firing\text{-}pin) \wedge$ $\Diamond(t,no\text{-}marshmallow\text{-}bullets) \wedge \ldots \wedge \Diamond$other mundane conditions $\supset \Box(t+1,noise), \forall t$.
>
> Roughly these lines state that if at time t the gun is loaded, fired, surrounded by air, provided with proper firing pin, and loaded with bullets not made of marshmallow, and in addition other mundane conditions are also satisfied, then at time t+1 a noise will be heard. That this is a nonsolution, however, is made visible most prominently by the appearance of the 'other mundane conditions' clause. This clause clearly will have to take care of the rest of the world. But the world cannot be captured in terms of predicates."

Are all the things in the rest of the world relevant to the process of inferring noise? No matter how large this set of qualifications, the context of reasoning limits this set, leaving the rest as assumptions. One cannot talk about a context "capturing the whole world." McCarthy states [16, p. 1034]:

> "Whenever we write an axiom, a critic can say it is true only in a certain context. With a little ingenuity, the critic can usually devise a more general context in which the precise form of the axiom does not hold. [...] We encounter Socratic puzzles over what the concepts mean in complete generality and encounter examples that never arise in life. There simply is not a most general context."

Appendix B

A CRITIQUE OF SHOHAM'S ACCOUNT OF CAUSATION

It is noted that the terminology introduced in Chapter 1 is followed. The reader can refer to [35, pp. 142-172] for details of Shoham's account of causation.

B.1 Shoham's account of causation

According to Shoham, causal statements in any domain involve the concept of knowledge with temporal dimension. Furthermore, he asserts that "causal statements are based on the logic of chronological ignorance" [30, p. 158]. Hence, he finds *causal rules* an appropriate formalization of causal statements. As stated in Section 2.3, a causal rule has the form $\Phi \wedge \Theta \supset \Box\varphi$ where Φ must be a non-empty conjunction of $\Box$-conditions. Shoham interprets Φ as the cause of the effect $\Box\varphi$ relative to the causal theory that contains this causal rule. He calls the object in the real world which is denoted by a temporal proposition (be it a fact, event, process, or whatever) a *happening*. Identification of causes with respect to a causal theory is a fine approach since the causal theory forms a *causal context* in which only a finite number of happenings can cause or be caused. For example, firing a gun at some time is a happening. Saying that firing a loaded gun at time 1 causes a loud noise at time 2 is equivalent to having the following causal rule:

(*) $\Box(1,\text{fire}) \wedge \Box(1,\text{loaded}) \wedge \Diamond(1,\text{air}) \wedge \Diamond(1,\text{firing-pin}) \wedge \ldots \supset \Box(2,\text{noise})$.

But saying that firing a loaded gun causes a noise to be heard at the next instant of time is a more general causal statement, and it is equivalent to having the following causal rule:

$$(**)\ \Box(t,\text{fire}) \wedge \Box(t,\text{loaded}) \wedge \Diamond(t,\text{air}) \wedge \Diamond(t,\text{firing-pin}) \wedge \ldots$$

$$\supset \Box(t+1,\text{noise}),\ \forall t.$$

Statements such as "TRUE(t_1,t_2,p) is an *actual cause* of TRUE(t_3,t_4,q)" can be judged as follows. Let M be the cmi model of the causal theory Ψ. TRUE(t_1,t_2,p) is an *actual cause* of TRUE(t_3,t_4,q) iff the following conditions hold:

1. Ψ contains a causal rule $\Box(t_1,t_2,p) \wedge \Theta \supset \Box(t_3,t_4,q)$,

   where $\Theta = \bigwedge^{n}_{i=1} \Diamond(t_{i1},t_{i2},r_i)$.
2. $M \models \Box(t_1,t_2,p)$.
3. $M \models \Diamond(t_{i1},t_{i2},r_i)$, $i=1,\ldots,n$.

One can say that TRUE(t_1,t_2,p) *actually caused* TRUE(t_3,t_4,q) iff TRUE(t_1,t_2,p) is the *only* actual cause of TRUE(t_3,t_4,q). This view of actual causation provides a *uniqueness* criterion in the identification of causes [2, p. 8 and p. 39]. Uniqueness of causation enables finding out what caused what in an unambiguous way.

Statements such as "TRUE(t_1,t_2,p) actually caused TRUE(t_3,t_4,q)" are what Mackie calls *singular causal statements* [38, pp. 15-38]. This type of causation, having only one cause and a single effect, is called *simple causation* [2, p. 119]. Then, one can ask about the possibility of *multiple causation*, that is, having a set of causes and a set of effects. Shoham answers this question by stating that causal rules permit multiple causation since there exist a set of causes and only one effect. Additionally, he suggests two categories: *conjunctive* and *disjunctive causation* (also cf. [2, pp. 119-147]). He does not give formal definitions of these categories; this we shall now do.

Conjunctive causation requires a set of happenings be identified as the cause of a single effect. Turning the ignition key of a car and pressing the gas pedal cause the car to run. Firing the gun and its being loaded cause a loud noise. Causal rules having more than one $\Box$-condition reflect conjunctive causation.

The statement "The happenings $TRUE(t_{i1},t_{i2},p_i)$ (for i=1,..,n) *(actually) conjunctively caused* $TRUE(t_3,t_4,q)$" is true with respect to Ψ if the following hold:

1. Ψ contains $\Phi \wedge \Theta \supset \Box(t_3,t_4,q)$, where $\Phi=\bigwedge^{n}_{i=1}\Box(t_{i1},t_{i2},p_i)$ and $\Theta=\bigwedge^{m}_{j=1}\Diamond(t_{j1},t_{j2},r_j)$.
2. $M \models \Box(t_{i1},t_{i2},p_i)$, i=1,...,n.
3. $M \models \Diamond(t_{j1},t_{j2},r_j)$, j=1,...,m.

The other category, *disjunctive causation*, transpires when there exists a plurality of causes producing the same effect (the effect is produced by each cause alone). Furthermore, each cause may actually be a conjunction of single causes. Striking a match or exposition of the match to heat cause the match to light. Firing a loaded gun or explosion of a bomb cause a loud noise. Disjunctive causation is then reflected by causal rules with distinct $\Box$-conditions on their l.h.s. and the same r.h.s. For example, the following causal rules represent disjunctive causation:

$$\Box(t,fire) \wedge \Diamond(t,loaded) \wedge \Diamond(t,air) \wedge \ldots \supset \Box(t+1,noise), \quad \forall t.$$
$$\Box(t,explosion) \wedge \Diamond(t,air) \wedge \ldots \supset \Box(t+1,noise), \quad \forall t.$$

In this case, the statement "The happenings $TRUE(t_{i1},t_{i2},p_i)$ (for i=1,..,n) *(actually) disjunctively caused* $TRUE(t_3,t_4,q)$" is true relative to Ψ if the following hold:

For each $TRUE(t_{i1},t_{i2},p_i)$,

1. Ψ contains $\Box(t_{i1},t_{i2},p_i) \wedge \Theta \supset \Box(t_3,t_4,q)$, where $\Theta=\bigwedge^{m}_{j=1}\Diamond(t_{j1},t_{j2},r_j)$.
2. $M \models \Box(t_{i1},t_{i2},p_i)$.
3. $M \models \Diamond(t_{j1},t_{j2},r_j)$, j=1,...,m.

Multiple causation is not restricted to disjunctive and conjunctive causation in which one single effect is produced. A set of causes can produce a set of effects. Causal statements such as "hitting the table with a hammer or with an axe cause both noise to be heard and the table to be damaged" exemplify this. One can still judge causal statements of the form "TRUE(t_1,t_2,p) was the *common cause* of the happenings TRUE(t_{i3},t_{i4},q_i) (for i=1,..,n)" relative to Ψ. This is true if the following hold:

For each $\Box(t_{i1},t_{i2},q_i)$,

1. Ψ contains $\Box(t_3,t_4,p) \wedge \Theta_i \supset \Box(t_{i1},t_{i2},q_i)$, where

   $\Theta_i=\bigwedge^{m}_{j=1}\Diamond(t_{j1},t_{j2},r_j)$.
2. $M \models \Box(t_3,t_4,p)$.
3. $M \models \Diamond(t_{j1},t_{j2},r_j)$, j=1,...,m.

This type of causation is weak in the sense that the conjunction of $\Diamond$-conditions in each causal rule can be different. Occurrence of one effect does not necessitate the occurrence of another. In the example above, if one hits a steel table with a hammer, a noise will be heard although the table remains intact. However, there might exist dependencies among effects of a cause so that whenever one of the effects occurs, the others unconditionally occur. Such a view suggests an economical way to represent the causal relations between happenings; a causal rule can have more than one $\Box$-condition on its r.h.s. These rules then can have the form $\Phi \wedge \Theta \supset \omega$:

1. $\Phi=\bigwedge^{l}_{i=1}\Box(t_{i1},t_{i2},p_i)$, $\Theta=\bigwedge^{m}_{j=1}\Diamond(t_{j1},t_{j2},q_j)$, and

   $\omega=\bigwedge^{n}_{k=1}\Box(t_{k1},t_{k2},r_k)$.
2. $t_{i2} < t_{k1}$, i=1,...,l and k=1,...,n.
3. $t_{j2} < t_{k1}$, j=1,...,m and k=1,...,n.

Then, whenever the cmi models of some causal theory Ψ satisfy the l.h.s. of such a causal rule, all the conjuncts on its r.h.s. immediately follow. Permitting this type of causal rule does not spoil the properties of causal theories and provides efficiency in representation and computation.

Shoham treats notions that are indirectly involved in causation. These are "prevention" and "enabling." It is said that "a prevented b from happening" or "a enabled b to happen." These notions can also be defined relative to some causal theory Ψ. For example, TRUE(t_1,t_2,p) is an *(actual) direct preventing condition* of TRUE(t_3,t_4,q) with respect to a causal theory Ψ if the following hold:

1. Ψ contains a causal rule $\Phi \wedge \Theta \supset \Box(t_3,t_4,q)$, where $\Phi = \bigwedge^{n}_{i=1} \Box(t_{i1},t_{i2},r_i)$.
2. $\Diamond(t_1,t_2,\neg p)$ is a conjunct of Θ.
3. $M \models \Box(t_{i1},t_{i2},r_i)$, i=1,...,n.
4. $M \models \Box(t_1,t_2,p)$.
5. $M \not\models \Box(t_3,t_4,q)$.

Then, it is said that TRUE(t_1,t_2,p) *(actually) directly prevented* TRUE(t_3,t_4,q) if TRUE(t_1,t_2,p) is the only (actual) direct preventing condition of TRUE(t_3,t_4,q). Consider axiom (*). If only TRUE(t_1,loaded), TRUE(t_1,fire), and TRUE(t_1,$\neg$air) are known, then it is said that absence of air (TRUE(t_1,$\neg$air)) prevented a loud noise to be heard (TRUE(t_2,noise)).

Similarly, one would say that TRUE(t_1,t_2,$\neg$p) is an *(actual) direct enabling condition* of TRUE(t_3,t_4,q) just in case TRUE(t_1,t_2,p) is an (actual) direct preventing condition of TRUE(t_3,t_4,q). The formal definitions for "enabling" are omitted. Clearly, notions such as "prevention" and "enabling" are not directly related to causes (that appear as $\Box$-conditions in the causal rules), but belong to the class of causal factors (that appear as $\Diamond$-conditions in the causal rules).

Von Wright [38, pp. 95-113] introduced one type of preventing condition which he calls *counteracting cause* or *intervening cause*. Causal verbs defining causal factors other than the cause involved in causation do not normally contain the word "cause." What von Wright calls counteracting cause cannot generally be identified as a cause of some effect, but only a preventing condition of it. He defines *counteracting cause* as "a cause which operates against some operating cause of the prevented change" [39, p. 101] and adds "the *effect* of the occurrence of a counteracting cause is that another change, which also happens, fails to produce the effect which, we think, it would have produced had the preventive cause not intervened" [39, p. 82]. Consider our previous example: a block resting at the center of a table. There are two actions; *push-left* and *push-right*. It is assumed that the forces applied on the block are of equal magnitude. If one pushes the block left, but at the same time someone else pushes it right, the block remains at the center of the table. It is said that pushing the block right prevented it from moving (pushing it right was a counteracting cause).

Then, the statement "TRUE(t_3,t_4,p) is a *counteracting cause for* TRUE(t_1,t_2,q)" relative to Ψ is true just in case the following hold (p and q denote actions):

1. Ψ contains $\Box(t_1,t_2,p) \wedge \Theta \supset \Box(t_5,t_6,s)$.
2. $\Diamond(t_3,t_4,\neg q)$ is a conjunct of Θ.
3. $M \models \Box(t_1,t_2,p)$.
4. $M \models \Box(t_3,t_4,q)$.
5. $M \not\models \Box(t_5,t_6,s)$.

But this definition is incomplete for the definition of counteracting cause requires persistence of some happening. Then, it is said that TRUE(t_3,t_4,p) is a *counteracting cause for* TRUE(t_1,t_2,q) and TRUE(t_1,t_2,p) *caused* TRUE(t_5,t_6,r) to *persist* relative to some causal theory Ψ just in case the following hold (p and q denote actions):

1. Ψ contains $\Box(t_1,t_2,p) \wedge \Box(t_5,t_6,r) \wedge \Theta \supset \Box(t_7,t_8,s)$.
2. $\Diamond(t_3,t_4,\neg q)$ is a conjunct of Θ'.
3. $M \models \Box(t_1,t_2,q)$.
4. $M \models \Box(t_3,t_4,q)$.
5. $M \models \Box(t_5,t_6,r)$.
6. $M \not\models \Box(t_7,t_8,s)$.
7. Ψ contains $\Box(t_5,t_6,r) \wedge \Pi(\Diamond(t_1,t_2,\neg p),\Diamond(t_3,t_4,\neg q)) \wedge \Theta' \supset$ $\Box(t_9,t_{10},r)$ (Definition 3.3).
8. $M \models \Theta'$.
9. $M \models \Box(t_9,t_{10},r)$.

B.2 What else is needed?

B.2.1 Causal determinism

Causal determinism asserts that "everything has a cause," and that "nothing can exist or cease to exist without a cause." If causal determinism is taken as a doctrine, Shoham's account of causation fails in that some facts can be introduced into causal theories for no reason, and one fails to determine their causes since, as Shoham [35, p. 110] says, "those items of knowledge that come into being *for no reason*, simply because one posits their truth." They are the boundary conditions of the causal theory at hand. This contradicts causal determinism.

In fact, this does no big harm since causal determinism is not commonly agreed upon. Any causal theory can be looked upon as a representation of some part of the world isolated from the rest. The boundary conditions can be viewed as external factors that trigger causal processes inside the causal theories. If it is assumed that there exist causal theories which subsume other causal theories or which interact with each other, then when the cause of a happening cannot be found in a causal theory it can be looked for in other related causal theories. Consequently, this suggests joining various causal theories to obtain a representation of a broader part of the world.

B.2.2 Self-causation

Consider axiom (*). If interpreted in causal terms, the gun's being loaded at t_1 caused the gun's being loaded at t_2. Obviously, this does not mean that the antireflexivity property of causation is lost here since the temporal gap between two happenings directly provides this property. Then, what is wrong? Actually, people do not talk about the causes of persistences. For example, assume that a rod is supported by hand so that it stands in a vertical position, and whenever the support is removed, it falls. Then, assuming everything remains the same one would say that supporting of the rod by hand causes it to remain in a vertical position. But, what can be said to be the cause of the gun remaining loaded is unclear. If there exist counteracting causes as explained before and if they interfere with each other, then they might be identified as causes of a constancy. This is not, however, a satisfactory way of determining causes of constancies.

Another point is that people generally talk about the cause of changes. They look for the cause of something coming into existence which had not been there before or disappearance of something which had been there before. Von Wright remarks [39, p. 73]:

> "Let us assume that the presence of oxygen in the environment is a causally necessary condition for a human body staying alive. Thus in the absence of oxygen no human body can stay alive (for more than a short time) [... This] statement amounts to saying that the disappearance (removal) of oxygen from the environment will cause [...] the extinction of life in a (living) human body which happens to be in this environment."

Then, one would have the following sentence in his causal theory:

$$\Box(t_1,\text{alive}) \wedge \Diamond(t_1,\text{oxygen}) \wedge \ldots \wedge \Diamond\text{other mundane conditions} \supset \Box(t_2,\text{alive}).$$

B.2.3 Causality: A theory of change

Shoham's characterization of causation admits other counterintuitive statements. As Delgrande notes [3, p. 58]:

> "It is thus possible to assert that some condition causes some other condition, where the second condition already happens to be true. Hence, in this account if I painted a fire hydrant red, which was already red, I could nonetheless claim that I caused the hydrant to be red."

Delgrande also points out that some strange results may arise in Shoham's account [3, p. 58]:

> "I have a friend Art who is an avid drummer. If we assume that he catches cold, it seems that we can equally well assert catching the cold on Saturday caused Art's blocked sinuses and drumming on Saturday caused Art's blocked sinuses."

But this is not really true. The cause of Art's blocked sinuses must be identified with respect to some causal theory. This means that there must be causal rules with r.h.s. indicating Art's blocked sinuses as their effects. Then, if an actual cause is found (this may well be catching cold on Saturday), then an indirect actual cause can be found. This is possible if there is a causal rule whose r.h.s. is the actual cause found, and whose l.h.s. is satisfied (this may be a statement asserting that drumming on Saturday causes catching cold on Saturday). In this case, the indirect actual cause (drumming on Saturday) is an actual cause of another actual cause (catching cold on Saturday) which is also an actual cause of an effect (Art's blocked sinuses).

B.2.4 Limitations of temporal propositions

It has been mentioned in Section B.1 that happenings are propositions associated with time intervals. In Shoham's account, only certain happenings (primitive propositions) can cause others. However, this has been subject of a debate [2, pp. 31-53]. Causal statements are primitively in the form *if C then E* where *C* and *E* are regarded as designators of "singulars belonging to any classes of concrete objects - events, processes, conditions, and so on" [2, p. 36]. This view of causation suggests the idea of talking about properties of singulars belonging to a certain class, instead of relying on only singulars.

To exemplify the situation, consider (**). Assume that the gun was loaded when it was fired. Indeed, absence of air in the environment prevents hearing a loud noise. Moreover, a loud noise would not be inferred if the gun had no firing pin, or if the gun was immersed into the water, or if the bullets were made from marshmallow, or if the gun powder was moist, and so on. But how does one deal with the bulk of these conditions?

First of all, what makes something a real bullet is its having a part that has the property of being explosive. Then, a "marshmallow bullet" or a "bullet lacking explosives" would not be called a real bullet. For the gun powder, being immersed in water or orange juice would not be the crux of the problem. What matters is that it becomes moist and that being moist directly affects the explosiveness property. One can list many things (the gun powder being in touch with all liquids in this case) that actually have the same result. Therefore, the condition $\diamond$(t,¬gun-powder-moist), when added to the l.h.s. of (**), should suffice to handle the effects of all liquids that make the gun powder moist.

Also firing the gun is not necessary for the explosion of the bullet. If the bullet is in direct contact with something hot, it may again explode and produce noise. Then, all happenings having the same property (being hot) can cause the bullet to explode. However, propositions are weak for representation of relations in this respect. Another source of difficulty is to find out the factors relevant to the phenomenon under consideration. A bear jumping in the north-pole cannot have causal connections with hearing a loud noise when a gun is fired in the south-pole. The approach taken here is that of causal determinism. This requires there be some restriction on the range of possibly relevant factors to the phenomenon. In particular, the relevant factors must be limited at least to the spatio-temporally neighboring ones. These assumptions allow one to deal with only a finite number of factors.

B.2.5 $\Box$- versus $\diamond$-conditions

The division between $\Box$- and $\diamond$-conditions in causal rules is made by statistical observations about the domain of reasoning. If a happening rarely occurs in the domain, then it is supposed to be a □-condition. If it occurs more often, then it is supposed to be a ◇-condition. The term "more often" is somewhat unclear.

Consider causal rule (**). One can start with the hypothesis that firing a gun causes a loud noise. If the gun is continuously fired by a patient agent in our world, then firing of the gun would always occur and would be a ◇-condition. However, firing the gun in the real world is considered as an action occurring less frequently, and hence as a □-condition in (**). Then, one can go on adding other conditions to obtain a more general causal rule in this way. Shoham [35, p. 167] claims that "after a while □-conditions of the causal rule become relatively stable." But assume that fifty percent of the guns are defective, e.g., they lack firing pins. Further assume that the guns are used one after the other. In this case, one may again start with (**). If he fails to predict noise with one gun, then he will obviously want to know whether the next gun has a firing pin. This means that "firing-pin" in (**) must now be a □-condition. It will remain as a □-condition until he is satisfied with the frequency of the working guns. Then, it will again become a ◇-condition.

This means that the □- and ◇-conditions of a causal rule may not become stable after some time. The distinction between these two classes of conditions is somewhat dependent on the particular context. More important is that in the same context, something can be identified either as a cause or just as an "enabling condition" as Shoham calls it [35, p. 163]. For example, in causal rule (**), firing the gun and its being loaded are identified as the causes of hearing a loud noise. However, if the gun's having a firing pin temporarily passes from the set of ◇-conditions to the set of □-conditions as illustrated above, the firing of the gun will be identified as the only cause of hearing the noise.

B.2.6 Simultaneous cause and effect

Shoham does not permit simultaneous causation. Since the ltp's of base sentences on the l.h.s. of a causal rule are required to be earlier than the etp of the base sentence on the r.h.s., the time intervals associated with cause and effect are not allowed to overlap. The principle *causes cannot succeed their effects in time* is taken for granted. However, this does not mean that causes strictly precede their effects in time since they may coincide in time. Then, Shoham's account lacks this property of causation.

Appendix C

PROOFS

C.1 Theorem 3.1

Part I. To prove that ζ has a cmi model, a construction procedure is devised for a model M for ζ. The construction is built upon the time-bounded Kripke interpretation M/t for a time point t (see Definition 2.16). It starts with a time-bounded Kripke interpretation at ltp t_1 of the initial boundary condition $\Box\varphi_s$ and it proceeds by augmenting this interpretation. (Φ' and Θ' denote the conjunction of base sentences obtained by replacing the variable t in each conjunct by a constant.)

1. Let $M/t_1 \models \Box\varphi_s$. For any other φ appearing on the l.h.s. of the axiom schemata, let $M/t_1 \not\models \Box\varphi'$ where φ' is obtained by replacing the variable t in φ by t_1.

2. In order to augment M/t_1 to M/t_1+1 such that $t_1+1 < t_2$, let

$$\begin{aligned}\mathrm{Cons}_{t_1+1} = \\ \{\Box(t_1+1,p):\ & \Box\varphi_p \wedge \Theta_p \supset \Box(t+1,p) \in \zeta \text{ and } M/t_1 \models \Box\varphi_p' \wedge \Theta_p' \text{ for } t=t_1, \\ & \text{or } \Phi_c \wedge \Theta_c \supset \Box(t+1,p) \in \zeta \text{ and } M/t_1 \models \Phi_c' \wedge \Theta_c' \text{ for } t=t_1\}.\end{aligned}$$

Augmentation M/t_1 is the result of making the wff in Cons_{t_1+1} true, and for any other φ whose ltp is t_1+1, making $\Box\varphi$ false. Note that M/t_1 can satisfy either $\Box\varphi_p' \wedge \Theta_p'$ or $\Phi_c' \wedge \Theta_c'$ for $t=t_1$ since the only base sentence that the time-bounded Kripke interpretation M/t_1 satisfies is $\Box\varphi_s$. And if $M/t_1 \models \Box\varphi_p' \wedge \Theta_p'$ for $t=t_1$, then $M/t_1 \models \Box\varphi_p'$ where $\Box\varphi_p'$ is obtained by replacing t in $\Box\varphi_p$ on the r.h.s. of the persistence axiom schema by t_1. This implies that $\Box\varphi_s = \Box\varphi_p'$ for $\Box\varphi_p'$ in $\Box\varphi_p' \wedge \Theta_p'$. In this case, $M/t_1 \not\models \Phi'_c \wedge \Theta'_c$ since for $M/t_1 \models \Phi'_c$ to hold it must be the case that $\Box\varphi_s \in \Phi'_c$, implying that $\Box\varphi'_p \in \Phi'_c$ for $t=t_1$. Then, there must exist at least one conjunct of Φ'_c distinct from $\Box\varphi_p'$, say $\Box\varphi_q'$. Consequently, $M/t_1 \not\models \Box\varphi_q'$ and hence $M/t_1 \not\models \Phi_c' \wedge \Theta_c'$. Similarly, if $M/t_1 \models \Phi_c' \wedge \Theta_c'$, then $M/t_1 \not\models \Box\varphi_p' \wedge \Theta_p'$. An immediate conclusion is that $Cons_{t_1+1}$ can have at most one wff; either $\Box\varphi_p'$ or $\Box\varphi_c'$ with ltp=t_1+1.

3. Augmentation of M/t_1+1 to M/t_2, where t_2 is the ltp of the final boundary condition $\Box\varphi_f$:

$$Cons_{t_2} = \{\Box(t_2,p){:}\ \Box\varphi_f = \Box(t_2,p),\ \text{or}\ \Box\varphi_p \wedge \Theta_p \supset \Box(t+1,p) \in \zeta \text{ and } M/t_1+1 \models \Box\varphi_p' \text{ such that } \Box\varphi_p' \in Cons_{t_1+1} \text{ for } t=t_1+1\}.$$

M/t_2 is obtained by making all wff in $Cons_{t_2}$ true and by letting $M/t_2 \models \Box\varphi_p'$, $t_1 < t \leq t_2$, if the latter holds in the specification of $Cons_{t_2}$. Finally, for any other φ' appearing in axiom schemata, let $M/t_2 \not\models \Box\varphi'$, $t_1 < t < t_2$.

4. Construction ends with the augmentation of M/t_2 for the next time point, i.e., t_2+1.

$$Cons_{t_2+1} = \{\Box(t_2+1,p){:}\ \Box\varphi_p \wedge \Theta_p \supset \Box(t+1,p) \in \zeta \text{ such that } \Box\varphi_p' \in Cons_{t_2} \text{ and } \Diamond\neg\varphi_f \notin \Theta_p' \text{ for } t=t_2, \text{ or } \Phi_c \wedge \Theta_c \supset \Box(t+1,p) \in \zeta \text{ such that } \Box\varphi_f \in \Phi_c' \text{ and } M/t_2 \models \Phi_c' \wedge \Theta_c' \text{ for } t=t_2\}.$$

By making all wff in $Cons_{t_2+1}$ true and for any other φ' appearing in the axiom schemata letting $M/t_2+1 \not\models \Box\varphi'$ for $t=t_2+1$, M/t_2+1 is obtained. Thus the construction of the cmi model M is completed.

Part II. In order to complete the proof, it must be shown that the cmi model thus constructed is unique. Assume that a model M' exists and it differs from M on the truth value of $\Box\varphi$ for some φ. There are two possibilities:

1. $M \not\models \Box\varphi$ while $M' \models \Box\varphi$ for a φ with ltp $\leq t_1$. By Definition 2.7, this means that $M' \subset_{ci} M$.

2. M and M' differ on the truth value of $\Box\varphi$ with ltp=t_3+1, $t_1 \leq t_3 \leq t_2$. There are two possibilities:

(i) M $\models\Box\varphi$ while M' $\not\models\Box\varphi$. Let $\Box\varphi$ be of the form $\Box(t_3+1,p)$:

First, if $t_3=t_1$, since M $\models\Box\varphi$, there exists either an axiom schema $\Box\varphi_p \wedge \Theta_p \supset \Box(t+1,p) \in \zeta$ such that for $t=t_3$ M $\models\Box\varphi_p' \wedge \Theta_p'$ or an axiom schema $\Phi_c \wedge \Theta_c \supset \Box(t+1,p) \in \zeta$ such that M $\models\Phi_c' \wedge \Theta_c'$ for $t=t_3$. Since M and M' agree on the knowledge of all base sentences with ltp's $\leq t_1$, by the second construction step M' $\models\Box\varphi$. This is a contradiction.

If $t_1+1 < t_3 < t_2$, then by the third step of the construction procedure there exists a persistence axiom schema $\Box\varphi_p \wedge \Theta_p \supset \Box(t+1,p) \in \zeta$ such that M/t_1+1 $\models\Box(t+1,p)$ for $t=t_1$. Since there exists no known atomic base sentence with ltp < t_3 other than φ, it will always be the case that M/t $\models\Box\varphi_p' \wedge \Theta_p'$, $t_1+1 < t < t_3$. Then, M/t_3 $\models\Box(t_3,p)$ and hence M/t_3 $\models\Box\varphi$. Since M and M' agree on the knowledge of all base sentences with ltp's $\leq t_1$, M'/t_1+1 $\models\Box\varphi$. But by the discussion above, this implies that M'/t_3 $\models\Box\varphi$. This contradicts the assumption that M'/t_3 $\not\models\Box\varphi$.

For $t_3=t_2$, M/t_2+1 $\models\Box\varphi$. This is true iff one of the following conditions hold: M/t_2 $\models\Box\varphi_p' \wedge \Theta_p'$ for $t=t_2$ or M/t_2 $\models\Phi_c' \wedge \Theta_c'$ for $t=t_2$. But it is known that M and M' have the same atomic base sentences whose ltp's $\leq t_2$. Then, M'/t_2+1 $\models\Box\varphi$, contradicting the assumption that M'/t_2+1 $\not\models\Box\varphi$.

(ii) M $\not\models\Box\varphi$ while M' $\models\Box\varphi$. Again by Definition 2.7, it follows that M' $\subset_{ci}$ M. ■

C.2 Proposition 3.1

The construction procedure of Shoham in the proof of Theorem 2.1 will yield M/t_1 $\models\Box\varphi_s$ at its first step since $\Box\varphi_s$ is the only "boundary condition" with ltp $t_1 > t_0$.

Then, the augmentation of M/t_1 into M/t_1+1 (as specified by Shoham in the proof of Theorem 2.1) will yield $\Box\varphi_p'$ or $\Box\varphi_c'$ iff M/t_1 $\models\Box\varphi_p' \wedge \Theta_p'$

or $M/t_1 \models \Phi_c' \wedge \Theta_c'$ for $t=t_1$. But for any of them to hold $\Box\varphi_s$ must appear as a single $\Box$-condition in only one of $\Box\varphi_p' \wedge \Theta_p'$ and $\Phi_c' \wedge \Theta_c'$. If $\Box\varphi_s=\Box\varphi_p'$ and $\Box\varphi_p' \in \Phi_c'$, there must be another $\Box$-condition in Φ_c'. But since only $\Box\varphi_p'$ is known, it will be the case that $M/t_1 \models \Box\varphi_p' \wedge \Theta_p'$. If $\Box\varphi_s \neq \Box\varphi_p'$ and $\Box\varphi_s \in \Phi_c'$ as its unique $\Box$-condition, then it will only be the case that $M/t_1 \models \Phi_c' \wedge \Theta_c'$. Thus, $Cons_{t_1+1}$ will contain either $\Box\varphi_p'$ or $\Box\varphi_c'$ with $ltp=t_1+1$.

The augmentation of M/t into M/t+1 for $t_1+1 < t < t_2$ will yield the following. If $M/t_1+1 \models \Box\varphi_c'$ for $t=t_1$ and $\Box\varphi_c \neq \Box\varphi_p$, then $M/t_1+2 \not\models \Box\varphi_c'$ for $t=t_1+1$, and no other base sentences will be known until time point t_2. Otherwise, if $M/t_1+1 \models \Box\varphi_p'$ where φ_p' has the ltp t_1+1, then it will be the case that $M/t_1+2 \models \Box\varphi_p'$ such that φ_p' has the ltp t_1+1. Since this is the only base sentence known, the augmentation into the next time point will yield $M/t_1+3 \models \Box\varphi_p'$ where φ_p' has the ltp t_1+3. The iteration, thus obtained, will result in $M/t_2 \models \Box\varphi_p'$ such that φ_p' has the ltp t_2. Moreover, by construction $M/t_2 \models \Box\varphi_f$.

The base sentences with ltp's $\leq t_2$ are then exactly the same as the ones following the construction in Theorem 3.1. The last step of this construction specifies the same augmentation procedure as the one introduced in Theorem 3.1. However, the augmentation in Theorem 1 is more detailed than in Theorem 2.1 since this approach will simplify the procedure that will be introduced later for computing the atomic base sentences in the unique cmi models of ζ. The augmentation of M/t_2 into M/t_2+1 will end the construction. The set of atomic base sentences known in the cmi models of Ψ and ζ will be the same. ■

C.3 Theorem 3.2

The algorithm below follows the construction in the proof of Theorem 3.1.

1. Let KNOWN and CONS be two lists. KNOWN contains $\Box\varphi_s$ and CONS is empty.

2. Find the axiom schema containing $\Box\varphi_s$ as its unique $\Box$-condition. If there exists such an axiom schema, then let CONS contain the r.h.s. of this schema such that the ltp of the r.h.s. becomes t_1+1 when t is replaced by t_1. Add this atomic base sentence into KNOWN.

3. If CONS is empty, then go to 5.

4. If the atomic base sentence in CONS is a $\Box$-condition in the persistence axiom schema, then add the atomic base sentences into KNOWN such that these atomic base sentences are obtained by replacing t in the $\Box$-condition by constants in the range t_1+2 to t_2. Then, first empty CONS, and let CONS contain only the atomic base sentence with ltp=t_2 obtained from the $\Box$-condition above.

5. Add $\Box\varphi_f$ into CONS and KNOWN.

6. If CONS contains $\Box\varphi_p'$ with ltp=t_2 and if $\Diamond\neg\varphi_f$ is not a $\Diamond$-condition in Θ_p, then add $\Box\varphi_p$ (by letting its ltp be t_2+1) into KNOWN.

7. Let $\Phi_c' \wedge \Theta_c' \supset \Box\varphi_c'$ be obtained by replacing t with t_2 in $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$. Check if each conjunct $\Box(t_i,p)$ of Φ_c' exists in CONS. If so, let the conjuncts of Θ_c' be of the form $\Diamond(t_j,p)$ (respectively $\Diamond(t_j,\neg p)$). If for each $\Diamond(t_j,p)$ (respectively $\Diamond(t_j,\neg p)$), $\Box(t_i,\neg p)$ (respectively $\Box(t_i,p)$) does not exist in CONS, then add $\Box\varphi_c'$ into KNOWN.

8. The set of atomic base sentences known in the unique cmi model of the YSP-like causal theory ζ are the ones in KNOWN.

Complexity:

Step 1: O(1) (initialization).

Step 2: O(n) (searching and matching).

Step 3: O(1) (testing).

Step 4: O(1) (matching two atomic base sentences).

Step 5: O(1) (add operation).

Step 6: O(n) (searching).

Step 7: O(n) (searching each condition of the sentence in a list of size at most 2).

Step 8: O(1) (reporting the set of atomic base sentences in KNOWN).

Consequently, the total time complexity of the algorithm is O(n). ■

C.4 Theorem 3.3

Part I. To prove that ζ' has a cmi model, a procedure to construct a model M for ζ is given. (As in the proof of Theorem 3.1, any primed base sentence φ' is obtained by replacing the variable t in φ by a constant. Φ',Θ', and Π_i' denote the conjunction of base sentences obtained by replacing the variable t in each conjunct by a constant.)

1. Let t_l be the ltp of the initial boundary conditions of ζ'. Let $M/t_l \models \Box\varphi_{si}$, i=1,...,n. For any other φ appearing on the l.h.s. of the axiom schemata, let $M/t_l \not\models \Box\varphi'$.

2. Augment M/t_l into M/t_l+1:

   $Cons_{t_l+1}$ =
   $\{\Box(t_l+1,p)$: $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t+1,p) \in \zeta'$ such that $M/t_l \models \Box\varphi_p'$, $M/t_l \models \Theta_p'$ and $M/t_l \models \Pi_i'$ $\forall \Pi_i'$ in $\varnothing_p'$ for $t=t_l$ (Definition 3.3), or $\Phi_c \wedge \Theta_c \supset \Box(t+1,p) \in \zeta'$ and $M/t_l \models \Phi_c' \wedge \Theta_c'$ for $t=t_l\}$.

   Make the wff in $Cons_{t_l+1}$ true and for any other φ whose ltp=t_l+1, make $\Box\varphi$ false.

3. $Cons_{t_l+1}$ contains the base wff appearing either on the r.h.s. of persistence axiom schemata or on the r.h.s. of the causal axiom schemata. The sentences in the latter can falsify the l.h.s. of the

persistence axiom schemata in which the former appear. Therefore, to find out what base sentences preserve their truth value for the next time point, one more iteration is needed. Then, augmentation of M/t_1+1 into M/t_1+2, $t_1+2 < t_2$, is done by letting

$$\text{Cons}_{t_1+2} = \{\Box(t_1+2,p):\ \Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t+1,p) \in \zeta' \text{ such that } M/t_1+1 \models \Box\varphi_p', M/t_1+1 \models \Theta_p', \text{ and } M/t_1+1 \models \Pi_i'\ \forall \Pi_i' \text{ in } \varnothing_p' \text{ for } t=t_1+1\},$$

making the wff in Cons_{t_1+2} true, and for any other φ whose ltp is t_1+2, making $\Box\varphi$ false.

4. Augmentation of M/t_1+2 into M/t_2 is specified first by letting $M/t_2 \models \Box\varphi_{f_j}$, $j=1,\ldots,m$ (note that all final boundary conditions have ltp t_2), and then letting $M/t_2 \models \Box\varphi''\ \forall \Box\varphi' \in \text{Cons}_{t_1+2}$ such that $\Box\varphi''$ is obtained by replacing the time constants in each $\Box\varphi'$ with the time constants in the range t_1+3 to t_2. For any other φ whose ltp is in the range t_1+3 to t_2, make $\Box\varphi$ false.

5. Finally, M/t_2 is augmented into M/t_2+1 by letting

$$\text{Cons}_{t_2+1} = \{\Box(t_2+1,p):\ \Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t+1,p) \in \zeta' \text{ such that } M/t_2 \models \Box\varphi_p', M/t_2 \models \Theta_p' \text{ and } M/t_2 \models \Pi_i'\ \forall \Pi_i' \text{ in } \varnothing_p' \text{ for } t=t_2, \text{ or } \Phi_c \wedge \Theta_c \supset \Box(t+1,p) \in \zeta' \text{ and } M/t_1 \models \Phi_c' \wedge \Theta_c' \text{ for } t=t_2\},$$

making the wff in Cons_{t_2+1} true, and for any other φ whose ltp is t_2+1, making $\Box\varphi$ false.

Part II. It must be shown that if there exists another model M' of ζ' which differs from M on the truth value of $\Box\varphi$ for some φ, then M' is not a cmi model for ζ. This would be very similar to the second part of the proof of Theorem 3.1. ■

C.5 Theorem 3.5

The steps of the construction procedure given in the proof of Theorem 3.3. are followed.

1. Let IC and FC denote the set of initial and final boundary conditions respectively. Let PS and CS be two lists containing the set of persistence and causal axiom schemata. Let CONS, TCONS, and KNOWN be three lists. CONS contains the set of initial boundary conditions. TCONS and KNOWN are empty.

2. Let TIMES and TIMEF be two variables, initially set to the respective ltp's of the initial and final boundary conditions.

3. Sort CONS in alphabetical order with respect to the propositions.

4. Call Sub-1, Sub-2, and Sub-3.

5. If TIMES = TIMEF, then go to step 9.

6. Sort CONS in alphabetical order with respect to the propositions.

7. Call Sub-1 and Sub-4.

8. Call Sub-3 and then set TIMES = TIMEF.

9. Add the final boundary conditions in FC to CONS. Sort CONS in alphabetical order with respect to the propositions.

10. Call Sub-1, Sub-2, and Sub-3.

11. Add all sentences in TCONS to KNOWN and halt. The atomic base sentences known in all cmi models of the given YSP'-like causal theory are in KNOWN.

Sub-1.

{This subroutine checks if the l.h.s. of all causal axiom schemata in CS are satisfied. If the l.h.s. of a schema is satisfied, its r.h.s. is added to TCONS.}

1. Let TM be an empty list.
2. Copy the contents of CS to TM.
3. If TM is empty, then return.
4. Remove the first causal axiom schema of TM, say $\Phi_c \wedge \Theta_c \supset \Box(t+1,r)$.
5. If any conjunct $\Box(t,pi)$ ($\Box(t,\neg p_i)$) of Φ_c is not in CONS, then go to 3.
6. Check if for any conjunct $\Diamond(t,q_j)$ (respectively $\Diamond(t,\neg q_j)$) of Θ_c, there exists a sentence $\Box(t,\neg q_j)$ (respectively $\Box(t,q_j)$) in CONS. If there is such sentence, then go to 3. Otherwise, add $\Box$(TIMES+1,r) to TCONS. Go to 3.

Sub-2.

{This subroutine checks if the l.h.s. of all persistence axiom schemata in PS are satisfied. If the l.h.s. of a schema is satisfied, its r.h.s. is added to TCONS.}

1. Let TM be an empty list.
2. Copy the contents of PS to TM.
3. If TM is empty, then return.
4. Remove the first persistence axiom schema of TM, say $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t+1,r)$.
5. If $\Box\varphi_p$ is not in CONS, then go to 3.
6. Let TP be an empty list.
7. Copy the contents of $\varnothing_p$ to TP.
8. If TP is empty, then go to 11.
9. Remove the first element of TP, say Π_i.
10. If for all $\Diamond(t,p_i)$ (respectively $\Diamond(t,\neg p_i)$) of Π_i, there exist sentences $\Box(t,\neg p_i)$ (respectively $\Box(t,p_i)$) in CONS, or there do not exist sentences $\Box(t,\neg p_i)$ (respectively $\Box(t,p_i)$) in CONS, then go to 8. Otherwise go to 3.
11. Check if for any conjunct $\Diamond(t,q_j)$ (respectively $\Diamond(t,\neg q_j)$) of Θ_p, there exists a sentence $\Box(t,\neg q_j)$ (respectively $\Box(t,q_j)$) in CONS. If there is such a sentence, then go to 3. Otherwise, add $\Box$(TIMES+1,r) to TCONS and go to 3.

Sub-3.

{This subroutine transfers base sentence between lists and sets time.}

Add all sentences in CONS to KNOWN. Empty CONS and copy the contents of TCONS to CONS. Then, empty TCONS. Set TIMES=TIMES+1. Return.

Sub-4.

{This subroutine performs base sentence transfer between lists and replaces time variables in sentences by constants.}

Add all sentences in CONS to KNOWN. For each sentence ⊔(TIMES,r) in CONS, add ⊔(TIMES+1,TIMEF-1,r) to KNOWN and □(TIMEF,r) to TCONS. Empty CONS and store all sentences in TCONS to CONS. Then, empty TCONS. Return.

Complexity:

Step 1: O(1) (initialization).

Step 2: O(1) (initialization).

Step 3: O($n$log$n$) (sorting a list of size at most n).

Step 4: O($n$log$n$) (subroutines Sub-1 and Sub-2 take O($n$log$n$) and Sub-3 takes O(1)).

Step 5: O(1) (testing).

Step 6: O($n$log$n$) (sorting a list of size at most n).

Step 7: O($n$log$n$) (Sub-1 takes O($n$log$n$) and Sub-4 takes O(n)).

Step 8: O(n) (Sub-3 takes O(n)).

Step 9: O($n$log$n$) (sorting a list of size at most n).

Step 10: O($n$log$n$) (Sub-1 and Sub-2 take O($n$log$n$) and Sub-3 takes O(n)).

Step 11: O(1) (reporting).

Sub-1: O($n$log$n$). For each causal axiom schema, all conjuncts on its l.h.s. are searched in a list of size at most n. For each conjunct, this takes O(log$n$).

Sub-2: O($n$log$n$). For each persistence axiom schema, all conjuncts on its l.h.s. are searched in a list of size at most n. For each conjunct in the set of counteractions, two separate searches can be done. Any conjunct can be checked in O(log$n$).

Sub-3: O(n). After examination of all axioms and axiom schemata, at most n base sentences can be added to TCONS. Since all base sentences of TCONS are copied to CONS, CONS can have at most n base sentences. Add and copy operations take O(n).

Sub-4: O(n). CONS can contain at most n base sentences. Adding them to KNOWN takes O(n). Moreover, adding new base sentences to KNOWN and TCONS takes O(n). Copying all sentences in TCONS to CONS also takes O(n).

Therefore, the total time complexity of the algorithm is O($n$log$n$). Note that base sentences in KNOWN can take the form $\Box(t_0,t_1,p) = \{\Box(t_i,p) | t_0 \leq t_i \leq t_1\}$. The implementation of this algorithm given in Appendix D generates each $\Box(t_i,p)$ explicitly rather than producing a single base sentence $\Box(t_0,t_1,p)$. ■

C.6 Theorem 4.1

A construction procedure will be needed to build a model M for the extended causal theory Ω. This will be done by augmenting some time-bounded Kripke interpretation. This augmentation, however, cannot be used in the construction of M. There exist some technical defects that can destroy the unique-model property of extended causal theories. Starting with Shoham's augmentation and considering these technical problems, a new augmentation will be specified. Then, it will be used to show that in all cmi models of any extended causal theory the same atomic base sentences are known.

Shoham specifies an augmentation of a time-bounded Kripke interpretation M/t to M/t+1 as follows [35, p. 112]:

$Cons_{t+1} = \{\Box(t',t+1,x)$: $\Phi \wedge \Theta \supset \Box(t',t+1,x) \in \Omega$ and $M/t \models \Phi \wedge \Theta\}$, M/t+1 is obtained by making all wff in $Cons_{t+1}$ and all their tautological consequences true, and for any other φ' whose ltp is t+1, making $\Box\varphi'$ false.

Now, consider the following sentences in Ω. Note that they are all in TMS_{t_4}, but except the first one they are all in BS_{t_4} as well.

$\Box(t_1,t_2,s) \supset \Box(t_3,t_4,q)$.
$\Box(t_1,t_2,s) \wedge \Diamond(t_4,\neg r) \supset \Box(t_4,v)$.
$\Box(t_4,r) \supset \Box(t_4,u)$.
$\Box(t_1,t_2,p) \wedge \Diamond(t_3,t_4,q) \supset \Box(t_4,r)$.

Assume that $M/t_3 \models \Box(t_1,t_2,s)$ and $M/t_3 \models \Box(t_1,t_2,p)$. We would like to augment M/t_3 to M/t_4. Now, if all sentences are examined in the order they are written, $Cons_{t_4}$ will contain $\Box(t_3,t_4,q)$ since $M/t_3 \models \Box(t_1,t_2,s)$. However, none of the l.h.s. of the other sentences is satisfied since they contain base sentences with ltp t_4. Then, as a result of the augmentation only $\Box(t_3,t_4,q)$ is obtained. But the l.h.s. of other sentences can also be satisfied. For example, for $\Box(t_1,t_2,s) \wedge \Diamond(t_4,\neg r) \supset \Box(t_4,v)$, $M/t_4 \models \Box(t_4,v)$ iff $M/t_3 \models \Box(t_1,t_2,s)$ and $M/t_4 \not\models \Box(t_4,r)$. Therefore, in order to perform the augmentation successfully, one must also consider the sentences in $Cons_{t_4}$. That is, a possible augmentation might be:

$Cons_{t+1} =$
$\{\Box(t',t+1,x)$: $\Phi \wedge \Theta \supset \Box(t',t+1,x) \in \Omega$, but $\notin TMS_{t+1}$ and $M/t \models \Phi \wedge \Theta$, or
$\Phi \wedge \Theta \supset \Box(t',t+1,x) \in \Omega$ and $\in TMS_{t+1}$ such that $\forall \Box\varphi \in \Phi$, $\Box\varphi \in Cons_{t+1}$ if ltp of φ is t+1, $M/t \models \Box\varphi$ otherwise, and $\forall \Diamond\varphi \in \Theta$, $\Box\neg\varphi \notin Cons_{t+1}$ if ltp of φ is t+1, $M/t \not\models \neg\Box\varphi$ otherwise$\}$.

M/t+1 is obtained in the same way as in the first specification.

Returning to the example set of sentences above, the augmentation of M/t_3 to M/t_4 can be obtained:

1. For $\Box(t_1,t_2,s) \supset \Box(t_3,t_4,q)$, $Cons_{t_4}=\{\Box(t_3,t_4,q)\}$ since $M/t_3 \models \Box(t_1,t_2,s)$.

2. For $\Box(t_1,t_2,s) \wedge \Diamond(t_4,\neg r) \supset \Box(t_4,v)$, $Cons_{t_4}=\{\Box(t_3,t_4,q), \Box(t_4,v)\}$ since $M/t_3 \models \Box(t_1,t_2,s)$ and $\Box(t_4,r) \notin Cons_{t_4}$.

3. For $\Box(t_4,r) \supset \Box(t_4,u)$, $Cons_{t_4}=\{\Box(t_3,t_4,q), \Box(t_4,v)\}$ since $\Box(t_4,r) \notin Cons_{t_4}$.

4. For $\Box(t_1,t_2,p) \wedge \Diamond(t_3,t_4,q) \supset \Box(t_4,r)$,$Cons_{t_4}=\{\Box(t_3,t_4,q), \Box(t_4,v), \Box(t_4,r)\}$ since $M/t_3 \models \Box(t_1,t_2,p)$ and $\Box(t_3\ t_4,\neg q) \notin Cons_{t_4}$.

But $Cons_{t_4}$ does not have the right sentences. Since $Cons_{t_4}$ contains $\Box(t_4,r)$, it will cause the l.h.s. of the second sentence to fail and the l.h.s. of the third sentence to be satisfied. Hence, it must be the case that $Cons_{t_4}=\{\Box(t_3,t_4,q), \Box(t_4,u), \Box(t_4,r)\}$. Therefore, the augmentation specified above is incorrect.

Another technical problem has to do with the *pluri-extensionality* of the nonmonotonic systems. One of the properties of nonmonotonic systems is that they may produce several sets of possible conclusions. For example, consider the following set of premises where Unless(p) is true iff p cannot be inferred [18]:

$$S = \{p,\ p \wedge \text{Unless}(q) \supset r,\ p \wedge \text{Unless}(r) \supset q\}.$$

Depending on the order of in which inferences have been applied, one can obtain two conclusions: {p, r} as a result of the subset {p, p $\wedge$ Unless(q) $\supset$ r} and {p, q} as a result of the subset {p, p $\wedge$ Unless(r) $\supset$ q}. However, these two conclusions cannot be inferred conjointly; if r is inferred, then q cannot be inferred, and vice versa. If one is mainly interested in constructing only one of these possible sets, then the system is inconsistent in a sense that the intended model may not be obtained.

This is the case with extended causal theories. To illustrate the situation consider the following set of sentences that constitute TMS_{t_4} of an extended causal theory Ω.

TMS_{t4} = $\{\Box(t_4,p),$

$\Box(t_4,p) \wedge \Diamond(t_4,\neg q) \supset \Box(t_4,r),$

$\Box(t_4,p) \wedge \Diamond(t_4,\neg r) \supset \Box(t_4,q)\}.$

Assuming that $Cons_{t4}$ contains only $\Box(t_4,p)$ and assuming that the sentences in TMS_{t4} are examined in the order they are written, one finds out that $Cons_{t4} = \{\Box(t_4,p), \Box(t_4,r)\}$. If the order of the last two sentences in TMS_{t4} is changed, then $Cons_{t4} = \{\Box(t_4,p), \Box(t_4,q)\}$ is obtained. Thus, the order of these sentences is important.

In the following proof, an augmentation which will not cause such problems will be used.

Let there be two models M and M' such that $M' \subset_{ci} M$ and they differ on the truth value of some sentence $\Box\varphi$.

1. By definition, there exists a t_0 such that it precedes the ltp of any φ where $\Box\varphi$ appears as in the r.h.s. of a boundary condition in Ω. Then, $M/t_0 \models \Box\varphi$ for any φ with ltp $\leq t_0$, and $M/t_0 \not\models \Box\varphi'$ for any other φ' with ltp $\leq t_0$.

   M/t_0 *partially* satisfies all the boundary conditions of Ω since their ltp's are greater than t_0. Obviously, M/t_0 also *partially* satisfies all the causal rules since the truth values of sentences with ltp$>t_0$ depend on the sentences with ltp$\leq t_0$, and the l.h.s. of causal rules with ltp$\leq t_0$ are falsified.

2. The construction progresses iteratively over time.

   $Cons_{t+1}$ =
   $\{\Box(t',t+1,x)$: $\Phi \wedge \Theta \supset \Box(t',t+1,x) \in BS_{t+1}$, but $\notin TMS_{t+1}$, and $M/t \models \Phi \wedge \Theta\}$,

   $Cons'_{t+1}$ =
   $\{\Box(t',t+1,x)$: $\Phi \wedge \Theta \supset \Box(t',t+1,x) \in TDS_{t+1}$, such that $\forall \Box\varphi \in \Phi$, $\Box\varphi \in (Cons_{t+1} \cup Cons'_{t+1})$ if ltp of φ is t+1,$M/t \models \Box\varphi$

otherwise, and $\forall \Diamond\varphi \in \Theta$, $\Box\neg\varphi \notin (Cons_{t+1} \cup Cons'_{t+1})$ if ltp of φ is t+1, $M/t \not\models \neg\Box\varphi$ otherwise},

$Cons''_{t+1} =$

$\{\Box(t',t+1,x)$: $\Phi \wedge \Theta \supset \Box(t',t+1,x) \in TMS_{t+1}$, but $\notin TDS_{t+1}$ such that $\forall \Box\varphi \in \Phi$, $\Box\varphi \in (Cons_{t+1} \cup Cons'_{t+1} \cup Cons''_{t+1})$ if ltp of φ is t+1, $M/t \models \Box\varphi$ otherwise, and $\forall \Diamond\varphi \in \Theta$, $\Box\neg\varphi \notin (Cons_{t+1} \cup Cons'_{t+1} \cup Cons''_{t+1})$ if ltp of φ is t+1, $M/t \not\models \neg\Box\varphi$ otherwise}.

It is assumed that the sentences of Ω are examined in the order they are written. For this reason, although one can obtain more than one possible set $Cons'_{t+1}$, a unique set is constructed under this assumption. (It must be admitted that this is a very strong assumption.)

M/t+1 is obtained by first making all wff in $Cons_{t+1}$ true, then making all wff in $Cons'_{t+1}$ true, and finally making all wff in $Cons''_{t+1}$ true. For any other φ with ltp t+1, $\Box\varphi$ is made false.

The last step in the proof of the theorem is to show that this M is chronologically more ignorant than any M' which differs on the truth value of $\Box\varphi$ for some φ. There are two cases:

1. It may be that $M' \models \Box\varphi$ for some φ with ltp $\leq t_0$. But this, by Definition 2.7, implies that $M' \subset_{ci} M$.

2. It may be that there exists a time point t, $t_0 \leq t$, and that either $M' \not\models \Box\varphi$ and $M \models \Box\varphi$, or $M' \models \Box\varphi$ and $M \not\models \Box\varphi$ or for some φ with ltp=t+1. Now let $M \models \Box\varphi$. Then two cases must be examined:

(a) There exists a sentence $\Phi \wedge \Theta \supset \Box\varphi \in \Omega$ such that the ltp's of the base sentences in Φ and Θ are $\leq$ t, and $M/t \models \Phi \wedge \Theta$. It is known that M and M' agree on the knowledge of all base sentences having ltp $\leq$ t. Hence, it follows that $M' \models \Box\varphi$ since $M'/t \models \Phi \wedge \Theta$. But this contradicts $M' \not\models \Box\varphi$.

(b) There is a sentence of the form $\Phi \wedge \Theta \supset \Box\varphi \in \Omega$, such that the ltp's of the base sentences in Φ and Θ are $\leq$ t+1. Then, the etp of φ must be equal to its ltp (t+1). This implies that $M/t+1 \models \Phi \wedge \Theta$ and it is known from case (a) that for any φ' in $\Phi \wedge \Theta$ with ltp t+1, $M \models \Box\varphi'$ and $M' \models \Box\varphi'$. Since M and M' agree on the knowledge of all base sentences with ltp $\leq$ t and they agree on the knowledge of all base sentences in Φ and Θ with ltp $\leq$ t+1, it must be the case that $M' \models \Box\varphi$, contradicting $M' \not\models \Box\varphi$.

Similarly, if $M' \models \Box\varphi$ and $M \not\models \Box\varphi$, in light of the discussion above, for any φ with ltp=t+1, whenever $M' \models \Box\varphi$, it must be the case that $M \models \Box\varphi$.

Consequently, if there is a model M' differing from the model M constructed for Ω, then $M' \subset_{ci} M$. ■

C.7 Theorem 4.2

The algorithm below organizes the sentences of Ω in ascending order of their ltp's. Then, each set of sentences with the same ltp is reordered. This reordering is done by first dividing these sentences into classes and then rearranging these classes among themselves. Note that the classes are formed according to the definitions given in Chapter 4 (bounded sets, temporally meeting sets, and temporally dependent sets of sentences). As a final step, the sentences are examined to see if their l.h.s. are satisfied. If so, their r.h.s. are marked accordingly.

1. Let T be the list of all sentences in Ω. Let S be a list.

2. Gather all atomic base sentences appearing in T into a list S by dropping negation signs.

3. Sort T in ascending order by the ltp of the r.h.s. of the sentences in it. Also sort S in ascending order by the ltp of the base sentences.

4. Remove all duplicates of any atomic base sentence in S. Mark all members UNMARKED.

5. Gather all sentences in T into bounded sets of sentences, BS, such that if say $\Phi \wedge \Theta \supset \Box(t_5,t_6,[\neg]r)$ is a sentence in T, then it must be in the

bounded set of sentences at time t_6, BS_{t6}. At the end, BS contains the ordered sets BS_{ti} for i=1,..,n.

6. For each BS_{ti}, divide it into two groups; the temporally meeting set of sentences at t_i, TMS_{ti}, and the set of other sentences, NMS_{ti}. Replace BS_{ti} with these two sets such that NMS_{ti} appears before TMS_{ti}. That is, $BS_{ti} = NMS_{ti} \cup TMS_{ti}$. For example, if

$$\begin{aligned} BS_{t4} = \{&\Box(t_1,t_2,p) \supset \Box(t_3,t_4,q), \\ &\Box(t_4,u) \wedge \Diamond(t_3,t_4,\neg r) \supset \Box(t_4,v), \\ &\Box(t_4,p) \wedge \Diamond(t_3,t_4,q) \supset \Box(t_4,r)\}, \text{ then} \\ NMS_{t4} = \{&\Box(t_1,t_2,p) \supset \Box(t_3,t_4,q)\}, \text{ and} \\ TMS_{t4} = \{&\Box(t_4,u) \wedge \Diamond(t_3,t_4,\neg r) \supset \Box(t_4,v), \\ &\Box(t_4,p) \wedge \Diamond(t_3,t_4,q) \supset \Box(t_4,r)\}. \end{aligned}$$

7. For each TMS_{ti}, divide it into two groups: the temporally dependent set of sentences at t_i, TDS_{ti} and the set of other sentences, NDS_{ti}. Replace TMS_{ti} with these two sets such that TDS_{ti} appears before NDS_{ti}. That is, $TMS_{ti} = TDS_{ti} \cup NDS_{ti}$. For the above example,

$$\begin{aligned} TDS_{t4} &= \{\Box(t_4,p) \wedge \Diamond(t_3,t_4,q) \supset \Box(t_4,r)\}, \\ NDS_{t4} &= \{\Box(t_4,u) \wedge \Diamond(t_3,t_4,\neg r) \supset \Box(t_4,v)\}. \end{aligned}$$

(Now BS contains the sets BS_{ti}'s such that $BS_{ti} = NMS_{ti} \cup TDS_{ti} \cup NDS_{ti}$ for some t_i. All these sentences are still in increasing order of their ltp's. From now on, the names BS_{ti},NMS_{ti}, TDS_{ti}, and NDS_{ti} will not be used. Since the list BS has all the sentences, each sentence in BS will be examined.)

8. If BS is empty, then halt. The atomic sentences that are known in all cmi models of the extended causal theory are those sentences marked YES in S plus the negation of those marked NO in S.

9. Remove the first sentence of BS, and let this sentence be $\Phi \wedge \Theta \supset \Box(t_1,t_2,[\neg]p)$. For each conjunct $\Box(t_{i1},t_{i2},[\neg]p_i)$ of Φ and each conjunct $\Diamond(t_{i1},t_{i2},[\neg]p_i)$ of Θ, determine how $TRUE(t_{i1},t_{i2},p_i)$ is marked in S by performing binary search on S. If one of the following conditions is true:

(a) $\Box(t_{i1},t_{i2},p_i)$ is a conjunct of Φ and TRUE(t_{i1},t_{i2},p_i) is not marked YES in S,

(b) $\Box(t_{i1},t_{i2},\neg p_i)$ is a conjunct of Φ and TRUE(t_{i1},t_{i2},p_i) is not marked NO in S,

(c) $\Diamond(t_{i1},t_{i2},p_i)$ is a conjunct of Θ and TRUE(t_{i1},t_{i2},p_i) is marked NO in S,

(d) $\Diamond(t_{i1},t_{i2},\neg p_i)$ is a conjunct of Θ and TRUE(t_{i1},t_{i2},p_i) is marked YES in S,

then go to 8,

else mark TRUE(t_1,t_2,p) in S with YES if the r.h.s. is $\Box(t_1,t_2,p)$, and NO if it is $\Box(t_1,t_2,\neg p)$. Go to 8.

Complexity:

Step 1: O(1) (initialization).

Step 2: $O(n)$ (collection).

Step 3: $O(n\log n)$ (sorting).

Step 4: $O(n)$ (removing duplicates and marking).

Step 5: $O(n)$. Examining the ltp of the r.h.s. of each sentence in T suffices to classify the sentences. Note that the sentences in T are currently sorted with respect to the ltp of their r.h.s.

Step 6: $O(n)$. This step requires determining the sentences that have r.h.s. with the same etp and ltp such that at least one of the conjuncts on the l.h.s. has this etp (ltp) as its ltp. For each class BS_{t_i}, this can be done by examining the ltp of the l.h.s. of each sentence and comparing it to the etp and ltp of its r.h.s. Since this is done for all sentences in every class BS_{t_i}, this step requires all sentences to be examined at most once.

Step 7: $O(n^2)$. The r.h.s. of each sentence in every class TMS_{t_i} must be on the l.h.s. of all sentences in its corresponding class TMS_{t_i}.

Step 8: O(1) (testing and reporting).

Step 9: $O(n\log n)$. Label checking can be done at most n times. Determination of the label of each conjunct requires binary search. A new labeling can be done in time $O(n\log n)$ since it also requires binary search. There can be at most n new labeling operations during the execution of the algorithm.

Hence, the total time complexity of the algorithm is $O(n^2)$. ■

Appendix D

PROGRAM LISTINGS

This appendix contains the listings of the programs implemented in PC Scheme™ programming language.

D.1 Causal theories

The atomic base sentences known in all cmi models of a given causal theory are computed by the program below. The causal theory given below is input to the program. This causal theory represents a shooting scenario. The sentences TRUE(1,loaded), TRUE(2,loaded), TRUE(3,loaded), TRUE(4,loaded), TRUE(5,loaded), TRUE(5,fire), and TRUE(6,noise) are computed.

1. $\Box$(1,loaded).
2. $\Box$(1,loaded) $\wedge$ $\Diamond$(1,$\neg$fire) $\wedge$ $\Diamond$(1,$\neg$emptied-manually) $\supset$ $\Box$(2,loaded).
3. $\Box$(1,loaded) $\wedge$ $\Box$(1,fire) $\wedge$ $\Diamond$(1,air) $\wedge$ $\Diamond$(1,firing-pin) $\supset$ $\Box$(2,noise).
4. $\Box$(2,loaded) $\wedge$ $\Diamond$(2,$\neg$fire) $\wedge$ $\Diamond$(2,$\neg$emptied-manually) $\supset$ $\Box$(3,loaded).
5. $\Box$(2,loaded) $\wedge$ $\Box$(2,fire) $\wedge$ $\Diamond$(2,air) $\wedge$ $\Diamond$(1,firing-pin) $\supset$ $\Box$(3,noise).
6. $\Box$(3,loaded) $\wedge$ $\Diamond$(3,$\neg$fire) $\wedge$ $\Diamond$(3,$\neg$emptied-manually) $\supset$ $\Box$(4,loaded).
7. $\Box$(3,loaded) $\wedge$ $\Box$(3,fire) $\wedge$ $\Diamond$(3,air) $\wedge$ $\Diamond$(3,firing-pin) $\supset$ $\Box$(4,noise).
8. $\Box$(4,loaded) $\wedge$ $\Diamond$(4,$\neg$fire) $\wedge$ $\Diamond$(4,$\neg$emptied-manually) $\supset$ $\Box$(5,loaded).
9. $\Box$(4,loaded) $\wedge$ $\Box$(4,fire) $\wedge$ $\Diamond$(4,air) $\wedge$ $\Diamond$(4,firing-pin) $\supset$ $\Box$(5,noise).
10. $\Box$(5,fire).
11. $\Box$(5,loaded) $\wedge$ $\Diamond$(5,$\neg$fire) $\wedge$ $\Diamond$(5,$\neg$emptied-manually) $\supset$ $\Box$(6,loaded).
12. $\Box$(5,loaded) $\wedge$ $\Box$(5,fire) $\wedge$ $\Diamond$(5,air) $\wedge$ $\Diamond$(5,firing-pin) $\supset$ $\Box$(6,noise).

The atomic base sentences generated by the program are given at the end of the listing.

```
; Sample causal theory.
(define Causal_Theory
'((() () ((loaded + 1 1)))
  (() () ((fire + 5 5)))
  (((loaded + 1 1)) ((fire - 1 1) (emptiedmanually - 1 1)) ((loaded + 2 2)))
  (((loaded + 1 1) (fire + 1 1)) ((firingpin + 1 1) (air + 1 1)) ((noise + 2 2)))
  (((loaded + 2 2)) ((fire - 2 2) (emptiedmanually - 2 2)) ((loaded + 3 3)))
  (((loaded + 2 2) (fire + 2 2)) ((firingpin + 2 2) (air + 2 2)) ((noise + 3 3)))
  (((loaded + 3 3)) ((fire - 3 3) (emptiedmanually - 3 3)) ((loaded + 4 4)))
  (((loaded + 3 3) (fire + 3 3)) ((firingpin + 3 3) (air + 3 3)) ((noise + 4 4)))
  (((loaded + 4 4)) ((fire - 4 4) (emptiedmanually - 4 4)) ((loaded + 5 5)))
  (((loaded + 4 4) (fire + 4 4)) ((firingpin + 4 4) (air + 4 4)) ((noise + 5 5)))
  (((loaded + 5 5)) ((fire - 5 5) (emptiedmanually - 5 5)) ((loaded + 6 6)))
  (((loaded + 5 5) (fire + 5 5)) ((firingpin + 5 5) (air + 5 5)) ((noise + 6 6)))))

; "Tset" will hold all sentences of the causal theory sorted by their ltp's.
(define Tset '())

; "Sset" will contain all the base sentences of the causal
; theory by their ltp's.
(define Sset '())

; Base sentences in a given list are collected.
define (Construct_Sset_Level0 Baselist)
  (cond ((null? Baselist) ())
        (T (append (list (append '(()
                                  (append (list (caar Baselist)) '(+)
                                          (cddr (car Baselist)))))
                   (Construct_Sset_Level0 (cdr Baselist))))))

; The l.h.s. (box-conditions and diamond-conditions) and
; r.h.s. of a sentence are examined one by one.
(define (Construct_Sset_Level1 Axiom)
  (cond ((null? Axiom) ())
        (T (append (Construct_Sset_Level0 (car Axiom))
                   (Construct_Sset_Level1 (cdr Axiom))))))

; Each sentence in the causal theory is examined.
(define (Construct_Sset_Level2 Theory)
  (cond ((null? Theory) ())
        (T (append (Construct_Sset_Level1 (car Theory))
                   (Construct_Sset_level2 (cdr Theory))))))

; Duplicates in a given list are eliminated.
(define (Eliminate_Duplicates Atomiclist)
  (cond ((null? Atomiclist) ())
        ((member (car Atomiclist) (cdr Atomiclist))
          (Eliminate_Duplicates (cdr Atomiclist)))
        (T (append (list (car Atomiclist))
                   (Eliminate_Duplicates (cdr Atomiclist))))))

; The sort criterion is defined for the base sentences in "Sset".
(define (Sort_Condition_S Atomic1 Atomic2)
```

```
  (<= (cadddr (cdr Atomic1)) (cadddr (cdr Atomic2))))

; "Sset" is constructed by first extracting base sentences from the causal
; theory, and then eliminating duplicates. Finally, the set is sorted with
; respect to ltp's of the base sentences.
(define (Construct_Sset)
  (set! Sset (Eliminate_Duplicates
                    (Construct_Sset_Level2 Causal_Theory)))
  (sort! Sset Sort_Condition_S)
  (set! Sset (car Sset)))

; The sorting criterion is defined for the base sentences in "Tset".
(define (Sort_Condition_T Axiom1 Axiom2)
  (<= (cadddr (car (caddr Axiom1))) (cadddr (car (caddr Axiom2)))))

; "Tset" is constructed by first making a copy of the causal theory and
; then sorting its sentences with respect to their ltp's.
(define (Construct_Tset)
  (set! Tset (copy Causal_Theory)) (sort! Tset Sort_Condition_T)
  (set! Tset (car Tset)))

; Tests whether a given item is in a given list.
(define (Member_Test Atomic Atomlist)
  (cond ((null? Atomlist) ())
        ((equal? (append (list (car Atomic)) '(+) (cddr Atomic))
           (cdr (car Atomlist))) (car Atomlist))
        (T (Member_Test Atomic (cdr Atomlist)))))

; "Sset" is searched to determine how a given base sentence is marked
; The given sentence is assumed to be a box-condition.
(define (Search_Box Atomic)
  (let ((Temp (car (Member_Test Atomic Sset))))
    (cond ((and (equal? (cadr Atomic) '+)
            (equal? Temp 'YES)) T)
           ((and (equal? (cadr Atomic) '-) (equal? Temp 'NO)) T)
          (T ()))))

; Check if all the box-conditions of a sentence are satisfied.
(define (Test_Boxes Baselist)
  (cond ((null? Baselist) T)
        (T (and (Search_Box (car Baselist)) (Test_Boxes (cdr Baselist))))))

; "Sset" is searched to determine how a given base sentence is marked
; The given sentence is assumed to be a diamond-condition.
(define (Search_Diamond Atomic)
  (let ((Temp (car (Member_Test Atomic Sset))))
    (cond ((and (equal? (cadr Atomic) '+)
            (equal? Temp 'NO)) ())
           ((and (equal? (cadr Atomic) '-) (equal? Temp 'YES)) ())
          (T T))))

; Check if all the diamond-conditions of a sentence are satisfied.
(define (Test_Diamonds Baselist)
```

```
  (cond ((null? Baselist) T)
        (T (and (Search_Diamond (car Baselist))
                (Test_Diamonds (cdr Baselist))))))

; If the l.h.s. of a sentence is satisfied, the base sentence
; on its r.h.s. in "Sset" is marked according to its sign.
(define (Set_Truth_Value Basesentence)
  (let ((Sign 'NO))
    (if (equal? (cadr Basesentence) '+) (set! Sign 'YES))
    (set-car! (Member_Test Basesentence Sset) Sign)))

; Tests if the l.h.s. of a sentence is satisfied. If so, its r.h.s.
; is sent for marking.
(define (Test_Antecedents Axiom)
  (if (and (Test_Boxes (car Axiom)) (Test_Diamonds (cadr Axiom)))
     (Set_Truth_Value (caadr (cdr Axiom)))))

; Each sentence in "Tset" are examined to see if their l.h.s.
; are satisfied by the base sentences in "Sset".
(define (Reason Axiomlist)
  (cond ((null? Axiomlist) ())
        (T (let ()
               (Test_Antecedents (car Axiomlist))
               (Reason (cdr Axiomlist))))))

; The atomic base sentences known in all cmi models of the given
; causal theory reside in "Sset". These known sentences are extracted
; from "Sset" and reported.
(define (Collect_Known_Atomics Atomlist)
  (cond ((null? Atomlist) T)
        ((not (null? (caar Atomlist)))
           (let ((Sign '(+)) (Temp (car Atomlist)))
             (if (equal? (car Temp) 'NO) (set! Sign '(-)))
             (write (append (list (cadr Temp)) Sign (cdddr Temp)))
              (Collect_Known_Atomics (cdr Atomlist))))
         (T (Collect_Known_Atomics (cdr Atomlist)))))

; Main procedure to compute the atomic base sentences
; known in all cmi models of the given causal theory.
(define (Compute_CMI_Model)
  (Construct_Tset)
  (Construct_Sset)
  (Reason Tset)
  (writeln "The Atomic Base Sentences Known in the CMI Models")
  (writeln "of the Given Causal Theory are as Follows:")
  (Collect_Known_Atomics Sset))
```

The Atomic Base Sentences Known in the CMI Models
of the Given YSP-Like Causal Theory are as Follows:

(loaded + 1) (loaded + 2) (loaded + 3) (loaded + 4) (loaded + 5) (fire + 5)
(noise + 6)

D.2 YSP-like causal theories

The following is an implementation of the algorithm proposed in Appendix C for computing the atomic base sentences known in all cmi models of YSP-like causal theories. It contains the example shooting scenario:

1. $\Box(1,\text{loaded})$.
2. $\Box(10,\text{fire})$.
3. $\Box(t,\text{loaded}) \wedge \Diamond(t,\neg\text{fire}) \wedge \Diamond(t,\neg\text{emptied-manually}) \supset \Box(t+1,\text{loaded}), \quad \forall t.$
4. $\Box(t,\text{loaded}) \wedge \Box(t,\text{fire}) \wedge \Diamond(t,\text{air}) \wedge \Diamond(t,\text{firing-pin}) \supset \Box(t+1,\text{noise}), \quad \forall t.$

The intended model will contain the sentences TRUE(1,loaded), TRUE(2,loaded), ..., TRUE(10,loaded), TRUE(10,fire), and TRUE(11,noise). The output of the program is given at the end of the listing.

```
; Sample YSP-like causal theory.
; Initial boundary condition.
(define Initial '(loaded + 1))
; Final boundary condition.
(define Final '(fire + 10))
; Persistence axiom schema.
(define Pschema '((loaded + time)
                  ((fire - time) (emptiedmanually - time))
                 (loaded + (+ 1 time))))
; Causal axiom schema.
(define Cschema '(((loaded + time) (fire + time))
                 ((firingpin + time) (air + time))
                (noise + (+ 1 time))))

; "time" denotes the time variable.
(define time '())

; The list that will contain atomic base sentences
; known in all cmi models of the theory.
(define Known '())

; The list that will keep consequences of augmentations.
(define Conseq '())

; Replaces time variable in a sentence with current time point symbol.
(define (Replace Atomic)
 (append (list (car Atomic))
```

```
        (list (cadr Atomic)) (list (eval (eval (caddr Atomic))))))

; Check persistence schema for existence of a box condition.
(define (Check_Pschema Box)
  (if (equal? (cdr (reverse Box))
              (cdr (reverse (car Pschema)))) (caddr Pschema) ()))

; Check causal axiom schema for existence of a unique box condition.
(define (Check_Cschema Box)
  (if (and (= (length (car Cschema)) 1)
           (equal? (cdr (reverse Box)) (cdr (reverse (caar Cschema)))))
    (caddr Cschema) ()))

; Augmentation Step I.
(define (Check_Schemas Box)
  (let ((Atomic '()))
    (set! Atomic (Check_Pschema Box))
    (if (null? Atomic) (set! Atomic (Check_Cschema)))
    (cond ((null? Atomic) ())
          (T (let () (set! Conseq (Replace Atomic))
                 (set! Known (append Known (list Conseq)))))))))

; Augmentation Step II.
(define (Expand)
  (cond ((= time (caddr Final)) (set! Conseq (car (reverse Known))))
        (T (let () (set! time (add1 time))
             (if (not (equal? time (caddr Final)))
              (set! Known (append Known (list (Replace (caddr Pschema))))))
                 (Expand)))))

; Member test on a given list.
(define (Member_Test M L)
  (cond ((null? L) ())
        ((equal? (cdr (reverse M)) (cdr (reverse (car L)))) T)
        (T (Member_Test M (cdr L)))))

; Augmentation Step III for persistence axiom schema.
(define (Augment_P)
  (let ((Temp '()) (Sign '-))
    (if (equal? (cadr Final) '-) (set! Sign '+))
      (set! Temp (append (list (car Final)) (list Sign) (cddr Final)))
    (if (and (Member_Test (car Pschema) Conseq)
             (not (Member_Test Temp (cadr Pschema))))
      (set! Known (append Known (Replace (caddr Pschema)))))))

; Check Boxes of the causal axiom schema.
  (define (Check_Boxes Blist)
    (cond ((null? Blist) T)
      (T (and (Member_Test (car Blist) Conseq) (Check_boxes (cdr Blist))))))

; Check Diamonds of the causal axiom schema.
(define (Check_Diamonds Blist)
  (let ((Temp (car Blist)) (Sign '-))
```

```
    (if (not (null? Temp))
       (let () (if (equal? (cadr Temp) '-) (set! Sign '+))
                (set! Temp (append (list (car Temp)) (list Sign) (cddr Temp)))))
             (cond ((null? Blist) T)
                (T (and (not (Member_Test Temp Conseq))
                        (Check_Diamonds (cdr Blist)))))))

; Augmentation Step III for causal axiom schema.
(define (Augment_C)
   (if (and (Check_Boxes (car Cschema)) (Check_Diamonds (cadr Cschema)))
    (set! Known (append Known (list (Replace (caddr Cschema)))))))

; The main procedure that computes the atomic base sentences known
; in all cmi models of a given YSP-like causal theory.
(define (Compute_CMI_Model)
  (set! Known (append Known (list Initial)))
  (set! time (caddr Initial))
  (if (and (Check_Schemas Initial) (Check_Pschema Conseq)) (Expand))
  (set! Known (append Known (list Final)))
  (set! Conseq (append (list Conseq) (list Final)))
  (set! time (caddr Final))
  (Augment_P)
  (Augment_C)
  (writeln "The Atomic Base Sentences Known in the CMI Models")
  (writeln "of the Given YSP-Like Causal Theory are as Follows:")
  (writeln Known))
```

```
The Atomic Base Sentences Known in the CMI Models
of the Given YSP-Like Causal Theory are as Follows:

(loaded + 1) (loaded + 2) (loaded + 3) (loaded + 4) (loaded + 5) (loaded + 6)
(loaded + 7) (loaded + 8) (loaded + 9) (loaded + 10) (fire + 10) (noise + 11)
```

D.3 YSP'-like causal theories

The following is an implementation of the algorithm proposed in Appendix C for computing the atomic base sentences known in all cmi models of YSP'-like causal theories. The following blocks world example is executed by the program:

1. $\Box(1,\text{at-center})$.
2. $\Box(10,\text{push-left})$.
3. $\Box(10,\text{push-right})$.
4. $\Box(t,\text{at-center}) \wedge \Pi(\Diamond(t,\neg\text{push-left}), \Diamond(t,\neg\text{push-right})) \supset \Box(t+1,\text{at-center}), \quad \forall t.$

5. $\Box(t,\text{at-center}) \wedge \Box(t,\text{push-left}) \wedge \Diamond(t,\neg\text{push-right}) \supset \Box(t+1,\text{at-left}), \quad \forall t.$

6. $\Box(t,\text{at-center}) \wedge \Box(t,\text{push-right}) \wedge \Diamond(t,\neg\text{push-left}) \supset \Box(t+1,\text{at-right}), \quad \forall t.$

The sentences TRUE(1,at-center), TRUE(2,at-center),..., TRUE(10,at-center), TRUE(10,push-left), TRUE(10,push-right), and TRUE(11,at-center) are known in all cmi models of this YSP'-like causal theory. The output of the program is given at the end of the listing.

```
; Sample YSP'-like causal theory.
; The set of initial boundary conditions.
(define Initials '((block-at-center + 1)))
; The set of final boundary conditions.
(define Finals '((pushleft + 10) (pushright + 10)))
; The set of persistence axiom schemata.
(define Pschemas '((((block-at-center + time))
                    (((pushleft - time) (pushright - time)))
                   ()
                    (block-at-center + (+ 1 time)))))
; The set of causal axiom schemata.
(define Cschemas '((((block-at-center + time) (pushleft + time))
                    ((pushright - time)) (block-at-left + (+ 1 time)))
                   (((block-at-center + time) (pushright + time))
                    ((pushleft - time)) (block-at-right + (+ 1 time)))))

; "time" denotes the time variable.
(define time '())

; The time points mentioned in the initial and final boundary conditions.
(define time1 (caddr (car Initials)))
(define time2 (caddr (car Finals)))

; The list that will contain atomic base sentences
; known in all cmi models of the theory.
(define Known '())

; The lists that will keep consequences of augmentations.
(define Conseq (copy Initials))
(define Tconseq '())

; Test for the existence of a sentence in a list of sentences.
(define (Member_Test Base Baselist)
  (cond ((null? Baselist) ())
    ((equal? Base (append (list (caar Baselist)) (list (cadr (car Baselist))))) T)
    (T (Member_Test Base (cdr Baselist)))))

; Check to see whether a box-condition is satisfied.
(define (Check_Box Base)
```

```
   (Member_Test (append (list (car Base)) (list (cadr Base))) Conseq))

; Check to see whether all box-conditions in a given set are satisfied.
(define (Check_Boxes Baselist)
  (cond ((null? Baselist) T)
    (T (and (Check_Box (car Baselist)) (Check_Boxes (cdr Baselist))))))

; Check to see whether a diamond-condition is satisfied.
(define (Check_Diamond Base)
  (let ((Sign '-))
    (if (equal? (cadr Base) '-) (set! Sign '+))
      (not (Member_Test (append (list (car Base)) (list Sign)) Conseq))))

; Check to see if all diamond-conditions in a given set are satisfied.
(define (Check_Diamonds Baselist)
  (cond ((null? Baselist) T)
          (T (and (Check_Diamond (car Baselist))
                  (Check_Diamonds (cdr Baselist))))))

; Test if a diamond-condition in a given set of counteractions is satisfied.
(define (Check_CAct Base)
  (let ((Sign '-))
    (if (equal? (cadr Base) '-) (set! Sign '+))
      (Member_Test (append (list (car Base)) (list Sign)) Conseq)))

; Test if all diamond-conditions in a given set of
; counteractions are satisfied.
(define (Check_CActions Baselist)
  (cond ((null? Baselist) T)
          (T (and (Check_CAct (car Baselist))
                  (Check_CActions (cdr Baselist))))))

; Check if one of the two conditions holds for a given set
; of counteractions to be satisfied.
(define (Check_CActionSet Baselist)
  (or (Check_Diamonds Baselist) (Check_CActions Baselist)))

; Test if all counteraction sets succeed.
(define (Check_All_CActionSets Cset)
 (cond ((null? Cset) T)
         (T (and (Check_CActionSet (car Cset))
                 (Check_All_CActionSets (cdr Cset))))))

; If the l.h.s. of a causal axiom schema is satisfied, make its r.h.s. true.
(define (Check_Cschema Schema)
  (let ((Temp (caddr Schema)))
  (if (and (Check_Boxes (car Schema)) (Check_Diamonds (cadr Schema)))
     (set! Tconseq (append Tconseq
                           (list (append (list (car Temp))
                                         (list (cadr Temp))
                                         (list (eval (eval (caddr Temp))))))))))

; Examine all causal axiom schemata.
```

```
(define (Check_All_Cschemas Schemalist)
  (cond ((null? Schemalist) T)
        (T (let () (Check_Cschema (car Schemalist))
                   (Check_All_Cschemas (cdr Schemalist))))))

; If the l.h.s. of a persistence axiom schema is satisfied,
;  make its r.h.s. true.
(define (Check_Pschema Schema)
  (let ((Temp (cadddr Schema)))
    (if (and (Check_Boxes (car Schema))
             (Check_All_CActionSets (cadr Schema))
             (Check_Diamonds (caddr Schema)))
    (set! Tconseq (append Tconseq
                          (list (append (list (car Temp))
                          (list (cadr Temp))
                          (list (eval (eval (caddr Temp))))))))))

; Examine all persistence axiom schemata.
(define (Check_All_Pschemas Schemalist)
  (cond ((null? Schemalist) T)
        (T (let () (Check_Pschema (car Schemalist))
                   (Check_All_Pschemas (cdr Schemalist))))))

; Given a sentence, generate new sentences with
; time points in the range timex to timey (inclusive).
(define (Replace timex timey Base)
  (cond ((> timex timey) ())
        (T (let () (set! Tconseq (append Tconseq
                                   (list (append (list (car Base))
                                   (list (cadr Base))
                                   (list timex)))))
  (Replace (add1 timex) timey Base)))))

; Given a set of sentences, generate new sentences with
; time points in the range timex to timey (inclusive).
(define (Replace_All timex timey Baselist)
  (cond ((null? Baselist) ())
        (T (let () (Replace timex timey (car Baselist))
                   (Replace_All timex timey (cdr Baselist))))))

; Eliminates duplicates in a given list.
(define (Eliminate Elist)
  (cond ((null? Elist) ())
        (T (let () (if (not (member (car Elist) Tconseq))
                       (set! Tconseq (append Tconseq (list (car Elist)))))
                   (Eliminate (cdr Elist))))))

; Sort condition for sorting sentences in ascending order
; of their time points.
(define (Sort_Condition Base1 Base2) (<= (caddr Base1) (caddr Base2)))

; Transfers the base sentences in each level to one level up.
(define (Transfer)
```

```
    (set! Known (append Known Conseq))
    (set! Conseq Tconseq) (set! Tconseq '()))

 ; The main procedure that computes the atomic base sentences
 ; known in all cmi models of a given YSP'-like causal theory.
(define (Compute_CMI_Model)
   (set! time time1)
    (Check_All_Pschemas Pschemas)
    (Check_All_Cschemas Cschemas)
   (Transfer)
   (set! time (add1 time))
   (if (< time time2)
      (let ((Temp '()))
         (Check_All_Pschemas Pschemas)
        (Transfer)
        (set! time (+ 2 time))
        (Replace_All time time2 Conseq)
        (set! Temp (copy Conseq))
        (Transfer)
        (Replace_All time2 time2 Temp))
      (set! Tconseq Conseq))
   (set! Tconseq (append Tconseq Finals))
   (Transfer)
   (set! time time2)
    (Check_All_Pschemas Pschemas)
    (Check_All_Cschemas Cschemas)
   (Transfer)
   (set! Known (append Known Conseq))
   (Eliminate Known)
   (sort! Tconseq Sort_Condition)
   (writeln "The Atomic Base Sentences Known in the CMI Models")
   (writeln "of the Given YSP'-Like Causal Theory are as Follows:")
   (writeln Tconseq))
```

The Atomic Base Sentences Known in the CMI Models
of the Given YSP'-Like Causal Theory are as Follows:

(atcenter + 1) (atcenter + 2) (atcenter + 3) (atcenter + 4) (atcenter + 5)
(atcenter + 6) (atcenter + 7) (atcenter + 8) (atcenter + 9) (atcenter + 10)
(pushleft + 10) (pushright + 10) (atcenter + 11)

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