PORTFOLIO SELECTION METHODS :

AN APPLICATION TO ISTANBUL SECURITIES EXCHANGE

A THESIS
SUBMITTED TO THE DEPARTMENT OF MANAGEMENT
AND THE GRADUATE SCHOOL OF BUSINESS ADMINISTRATION
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF BUSINESS ADMINISTRATION

By
İ. Tunç Seler
February 1989

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Business Administration.

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I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Business Administration.

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ABSTRACT

PORTFOLIO SELECTION METHODS :

AN APPLICATION TO İSTANBUL SECURITIES EXCHANGE

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Supervisor: Assistant Prof. Dr. Kürşat Aydoğan
February 1989

In this study, Modern Portfolio Theory tools are used for constructing efficient portfolios. The Markowitz mean-variance model and Sharpe single index model are presented and calculated, for the construction of efficient portfolios from the İstanbul Securities Exchanges' first market stocks for the 1986 - 1987 period. Constructed efficient portfolios are compared on the risk and return scales.

Keywords : Portfolio, Efficient Frontier, Diversification, Return, Risk, Capital Markets, Mathematical Programming Structure.

ÖZET :

Portföy Seçim Yöntemleri :

İstanbul Menkul Kıymetler Borsası İçin Bir Uygulama

İ. Tunç Seler
İşletme Yönetimi Yüksek Lisans
Tez Yöneticisi : Kürşat Aydoğan
Şubat 1989

Bu çalışmada Modern Portföy Teorisi araçları , Etkinlik Sınırı oluşturulması için kullanılmıştır. Markowitz ortalama varyans modeli ve Sharpe tekli index modeli açıklanmış ve 1986 - 1987 dönemi için, İstanbul Menkul Kıymetler Borsası birinci pazar hisse senetleri için hesaplanmıştır. Elde edilen etkinlik öncüleri risk ve getiri boyutlarında karşılaştırılmıştır.

Anahtar Kelimeler : Portföy, Etkinlik Sınırı , Çeşitlendirme, Getiri , Risk , Sermaye Piyasaları , Matematiksel Programlama Yapısı.

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NOMENCLATURE :

The notation used in this study is presented below.

A_i	: Alpha coefficient of the i^{th} asset.
A_p	: Portfolio Alpha .
B_i	: Beta coefficient for the i^{th} asset.
B_p	: Portfolio Beta .
C_i	: The error term.
$d_{i,t}$	: Dividend and other payments for the i^{th} asset on the t^{th} period.
E_i	: Expected return of the i^{th} asset.
E_p	: Expected rate of return on the portfolio.
I_i	: The market index.
x_i	: Proportion to invest form the i^{th} asset.
$P_{i,n}$	: Price of i^{th} asset on the n^{th} period .
P_i	: Price of old quotation on the i^{th} period.
P_{ssr}	: Price of the stock split right owning quotation.
$\bar{R}_i$	: Average rate of return on the i^{th} asset.
$r_{i,t}$	: Rate of return on the i^{th} asset on the t^{th} period.
R_f	: Riskfree rate of return.
$\bar{R}_p$	: Average Portfolio Return.
R_p	: Portfolio rate of return.
R_i	: Rate of return on the i^{th} asset.
m	: Number of stocks to be received as the result of the stock split.
n	: Number of periods.
λ	: Risk preferance coefficient.
x_i	: Proportion to invest on the i^{th} asset.
$\sigma^2_{e,i}$	: Standart error of the estimator.
σ^2_m	: Variance of the market index.
σ^2_p	: Portfolio Variance.
$\sigma_{i,j}$	: Covariance between i^{th} and j^{th} assets' returns.
σ_p	: Portfolio Standart Deviation.
β_i	: Beta value of the i^{th} security.

1. INTRODUCTION :

The purpose of this study is the construction of efficient portfolios by using stocks listed in İstanbul Securities Exchange . In this study, Markowitz full covariance model and Sharpe's single index model are utilized for the construction of efficient portfolios. The constructed efficient portfolios are compared on the risk and return spaces and policy recomendations for investors are put forward as a result of this study.

1.1 Financial Markets :

Financial markets have significant impacts on economic systems, because of their vital economic functions. The function of financial markets is to provide a convenient medium for savings to be done and investments to be realized. Another major function of financial markets is the creation of new wealth by providing a connection between savings and investments. Wealth can be defined as the summation of real and financial assets minus liabilities or net worth plus liabilities.

Although financial markets can create new wealth, a great portion of the transactions that takes place in these markets have no effect on the creation of new wealth. This can better be seen in the markets for corporate debt and markets for corporate equity. The transactions in these markets, only affect the ownership of liabilities. However, wealth can be created in these markets by issuing new stocks or debt.

Financial markets can be analyzed under two major submarkets. These submarkets are money markets and capital markets.

Financial markets classification :

1. Money markets .
2. Capital Markets .
   2.1 markets for goverment securities
   2.2 markets for local goverment securities
   2.3 mortgage markets
   2.4 markets for corporate debt
   2.5 markets for corporate equity

Capital markets have certain distinguishing properties. These are:

1. having a maturity of one year or more
2. being both a wholesale and retail market

There are primary and secondary markets which carry out the operations for transactions in financial markets. Securities that are available for the first time are offered through the primary markets. This is important because, primary markets create new funds for the issuers. After their purchase in primary markets, securities are traded in the Secondary markets. Secondary markets have two major segments. Those are the organized exchanges and over-the-counter markets. Organized exchanges are physical market places where auction can take place between the representatives of the buyers and sellers. Over-the-counter market is not a physical market. The securities that are traded in the over-the-counter markets are the unlisted securities. This market can be characterized by a network of buyers and sellers over a country, connected by communication links for negotiating the buying or selling prices.

The market for corporate debt can be further divided into two submarkets according to their organizational structures. These are :

1. organized security exchanges
2. over-the-counter markets or over-the-telephone markets.

1.2 Turkish Financial System :

The Turkish financial system is dominated by commercial banks. The extensive power and the domination of commercial banks in the Turkish financial system is due to the legislators concern for the protection of the investors [29] .

Another feature of the Turkish financial system is the goverment's entering to the financial markets to finance the budget deficits.

1.2.1 Turkish Securities Markets :

Supply Side :

The public sector securities dominates the supply side of the market. This can be seen in Table 1.1 [8] as percent increase and relative market share during 1983 to 1987. Public sector market share in the primary market has been over 85 % for the last five years, which indicates a domination in the new issues market.

Years	1983	1984	1985	1986	1987
Public Sector	-	158.1	177.94	61.4	103.2
Market Share of Public Sector	0.853	0.93	0.94	0.933	0.905
Market Share of Private Sector	0.147	0.07	0.06	0.067	0.095

Table 1.1: Public and Private sectors' relative percentages in primary markets.

In the corporate sector, there is a reluctance for changing its financing behavior. Corporate sector depends heavily on bank financing. This reluctance of the corporate sector is closely related with the small size of financial markets in Turkey. However, there is a recent growth in the corporate sector's bond issues. The recent tendencies of the corporate sector to finance its activities through security markets, is a direct result of the change in the behavior of the banks. Banks do advice corporations to issue securities and particularly bonds, to finance their operations.

The yearly percent changes in the primary market transactions according to the previous year is presented in Table 1.2 [8] . These figures include both the public and private primary market issues that are allowed by the Capital Markets Board.

Year	1984	1985	1986	1987
Primary market transactions	136.78	174.28	62.56	109.46

Table 1.2: Yearly percent changes in primary market transactions during 1984 to 1987

Demand Side :

To flourish the level of demand in securities markets, certain tax incentives are granted to individual investors. Those tax incentives are presented in Table 1.3, for the individual and collective portfolios [29] . These incentives remained the same up to January 1989 together with one new rule. The interest income for the corporations obtained from the securities will be taxed according to this new application and the taxation will be % 10. The concept of Collective portfolio in the Table 1.3, in general terms refers to mutual funds.

To flourish the level of demand, the double taxation of dividends is also prevented. Corporate tax becomes the final tax on these earnings. Capital gains becomes taxfree, if and only if, the security in question is listed on an exchange or sold through licensed intermediary agencies.

The changes in the demand side of the market have been slow. Individual investors react rather slowly in investing their savings on a group of totally new, hence unknown instruments. Institutional investors in Turkey are practically non-existent in

Tax Incentive	Individual portfolio	Collective portfolio
Bank deposits	10.4 % withholding tax	can not deposit
Shares	Tax exempt	Tax exempt
Corporate bonds	10.4 % withholding tax	Tax exempt
Profit Loss Sharing Cert.	10.4 % withholding tax	Tax exempt
Bank Bills	10.4 % withholding tax	can not deposit
Finance bills	10.4 % withholding tax	Tax exempt
F shares	Tax exempt	can not deposit
T-Bills	Tax exempt	Tax exempt
Goverment Bonds	Tax exempt	Tax exempt
Revenue Sharing C.	Tax exempt	Tax exempt

Table 1.3: Incentives granted to investors

the capital markets. Institutional investors are corporations which collect funds for investing. In Turkey, Social Security Board and Insurance corporations can be given as examples for institutional investors. However their rules in the capital markets have been limited so far [29].

1.3 İstanbul Securities Exchange :

İstanbul Securities Exchange (*İstanbul Menkul Kıymetler Borsası*) started operation in January 1986. Stocks are traded in these markets according to their transaction volumes in the exchange. The organization of the İstanbul securities exchange can be analyzed under two headings :

1. First Market , (*Birinci Piyasa*)
2. Second Market , (*İkinci Piyasa*)

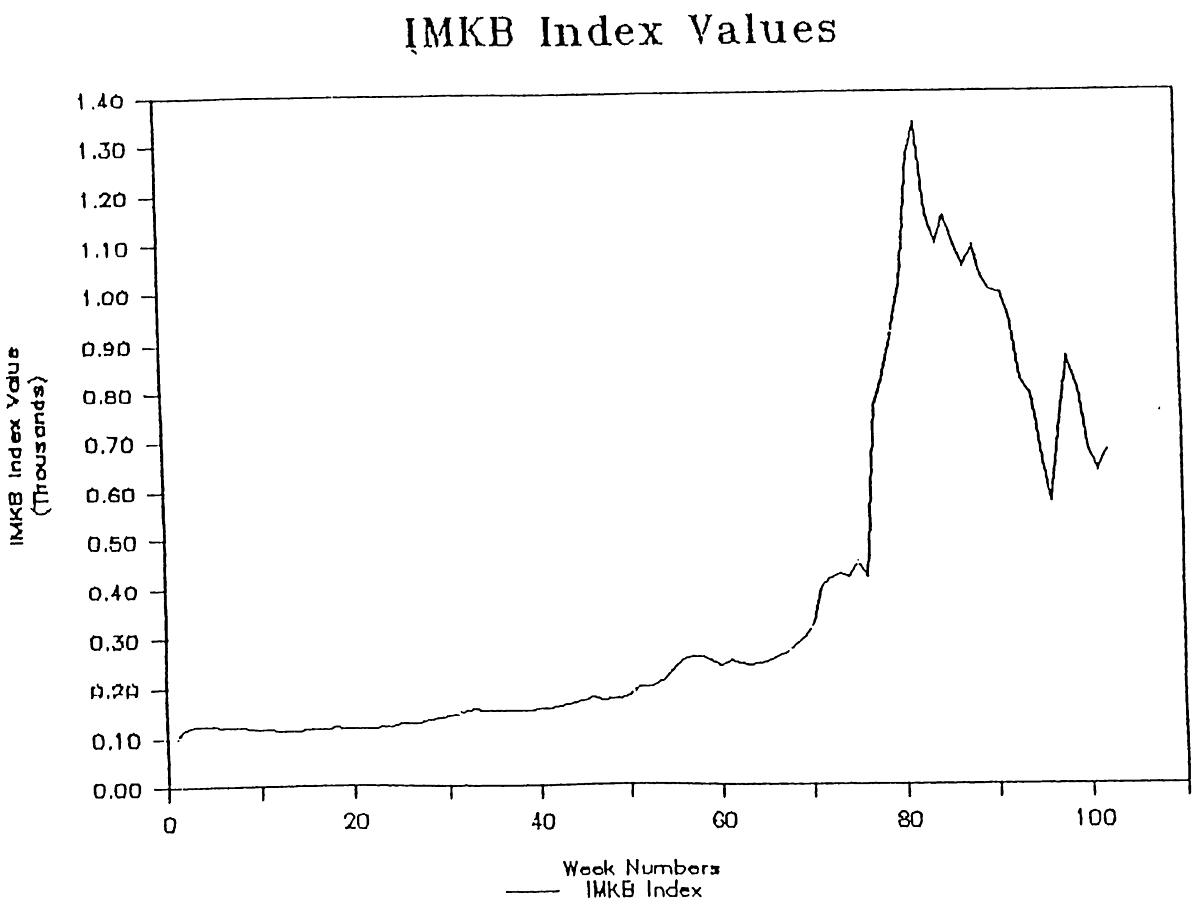

Figure 1.1: İMKB index values during Jan. 1986 - Dec. 1987

1.3.1 İMKB Index :

The İMKB index is an average of the stock prices, weighted with the corporate capital amounts and dividend payments. The İMKB index is calculated for the first 43 stocks that were traded in the first market in January 1986. January 1986 is taken as the base period (January 1986 = 100) for this index.

İMKB index values are presented in Figure 1.1 during the January 1986 - December 1987 period [18]. This makes a times series consisting of 102 weekly observations.

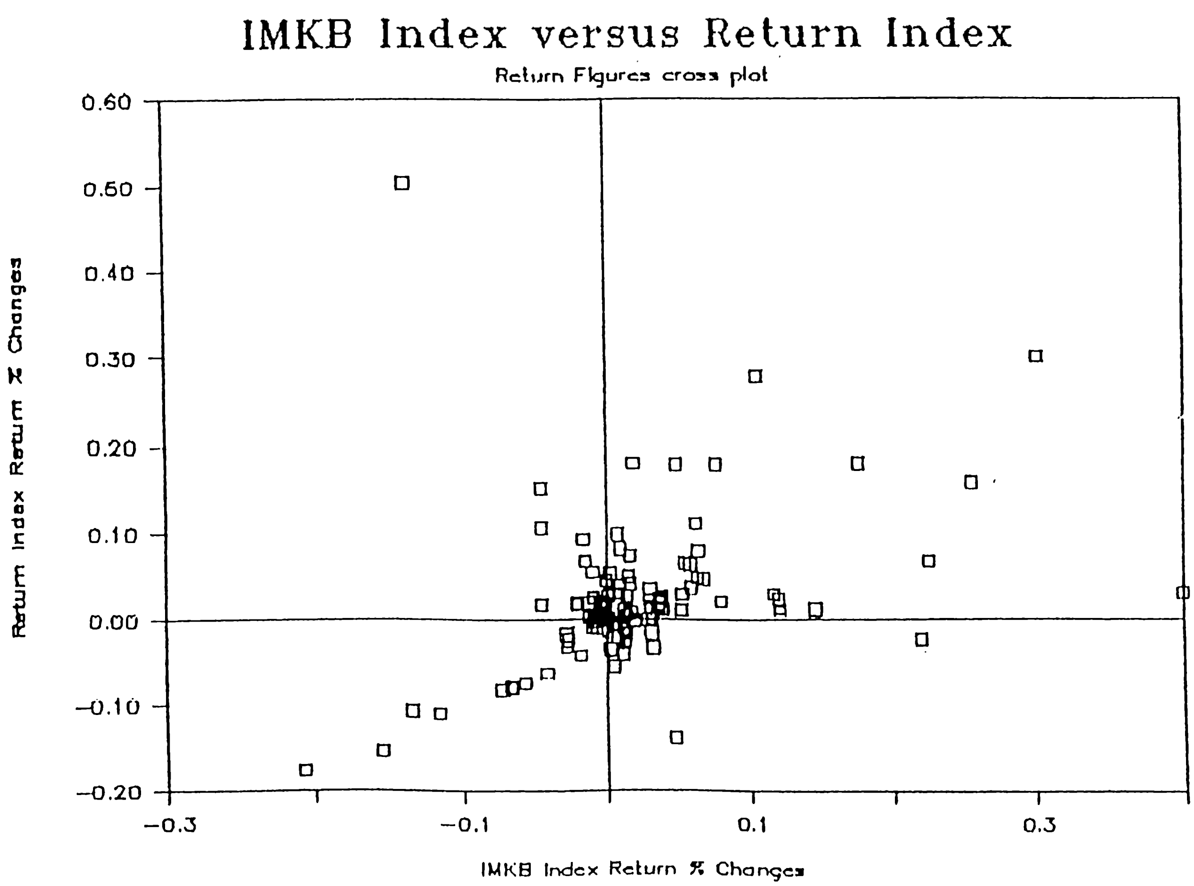

Figure 1.2: İMKB index returns versus Return index values during January 1986 to December 1987

1.3.2 Return Index :

To measure the return trends in the İstanbul securities exchange, another index is developed and used in this study. This index is named as the Return index, which is constructed from the average returns for a given period.

The İMKB index returns and Return index are plotted against each other to present the different natures of the indexes and their potentials to show the actual return level in the market in Figure 1.2 . Since the Return index is an average of the returns, and if the İMKB index is argued to be showing the overall return level, then, a distribution around a 45° line would be expected. It can be concluded that İMKB index is not an appropriate measure for the applications that utilize the market index concept, which measures the market's price level.

1.4 Investing in Equity Markets :

Investment is a commitment of funds made in the expectation of some positive rate of return. Investment and speculation can be distinguished by the time horizon and risk return characteristics.

Investments in equity markets can be realized by investing in common stocks of corporations. A common stock is a represantation of ownership for the firms. Common stocks have no maturity date at which a fixed value will be realized like bonds and IOUs (I owe you). Investing in common stocks gives the voting right for the corporation's decisions, and receiving earnings of the firm, as dividends .

Return from investing in common stocks is realized in two forms ;

1. Dividends,
2. Capital gains.

Dividends can be paid out by the decision of the board of directors of the corporations, if there are sufficient funds available for such an action. Capital gains is the difference between the buying price of stocks and price at a specific time. It can be stated as ,

$$CG = \frac{P_2 - P_1}{P_1} \tag{1}$$

where,

P_2 :selling price
P_1 :buying price.

The possibility of high volatility in capital gains and dividends requires the careful analysis of the stocks to be invested and traded. There are two approaches;

1. Traditional Investment Analysis,
2. Modern Security Analysis approaches.

The *Traditional approach* is interested in the projection of prices and dividend amounts for forecasting the future dividends and market value of the stocks. The projected amounts for prices and dividend amounts are discounted back to present and named as the *intrinsic value*. The comparison of the *intrinsic value* and common stock's market value determines the buying or selling decision for stocks.

Modern security analysis approach, on the other hand, is interested with the risk return estimates of the common stocks. Generally two approaches can be applied.

These two approaches are ;

1. Fundamental analysis ,

2. Technical analysis .

Fundamental analysis emphasizes the analysts should consider major factors effecting the economy, the industry, and the company in order to determine the investment decisions. Fundamentalists make a judgement of the stock's value within a risk-return framework based upon earning power and economic environment. At any time, the price of a security is equal to the discounted value of the stream of income for that security. Therefore the price of a security can be stated as a function of the anticipated returns plus the anticipated capitalization rates corresponding to future time periods.

Technical analysis approach to investment is essentially a reflection of the idea that the stock market moves in trends which are determined by the changing attitudes of investors to a variety of economic, monetary, political and psychological forces. The objective of technical analysis is to identify changes in potential trends at an early stage and to maintain an investment posture [33] .

2. LITERATURE REVIEW :

The era of modern portfolio theory started with two papers published in 1952. These papers were from Roy and Markowitz.

Roy [34] defined the best portfolio with the probability of producing a rate of return below some desired level. If portfolio return is represented by R_p and the desired level of return with R_d, then Roy's formulation can be stated as, $\min Pr(R_p < R_d)$.

The second major contribution to the modern portfolio theory was from Markowitz [23]. Markowitz proposed an objective portfolio selection criterion in his article. This criterion is known as the mean-variance portfolio selection method. Also, graphical mean-variance portfolio selection was presented by Markowitz in this article. In 1957, Markowitz presented the generalized solution methodology for the mean-variance portfolio selection problem. [27]

2.1 Mean - Variance portfolio selection method :

The mean-variance portfolio selection method uses mean and variance of returns for measures to capture and evaluate the relevant information about the opportunity set. The logic of the mean-variance portfolio selection method can be explained by the following postulates :

Let R_i denote the return on i^{th} asset and σ_i denote the standart deviation of the return for the i^{th} asset. Then for certain cases, investors' decisions can be predetermined according to the mean-variance criterion. Here the investors are assumed to be rational, who prefer more to less.

1. If the returns of two assets are equal, then investors will prefer the asset with lower variance on return.

$R_1 = R_2 \rightarrow$ prefer minimum variance.

2. If the variances of two assets are equal, then investors will prefer the asset with the higher rate of return.

$\sigma_1 = \sigma_2 \rightarrow$ prefer maximum return.

The Markowitz model's data requirement for an n security set is as follows ;

1. n return terms,
2. n variance terms,
3. $\frac{(n^2-n)}{2}$ covariance terms,

There is another measure which is required in this analysis, and that is the investors risk preferance measure. This measure determines the relative importance of unit return compared with unit risk, from the investors point of view. This measure is named as the lambda coefficient and used as λ in this study.
As the risk preference coefficient, lambda (λ) , will change for $\lambda \geq 0$, modicient portfolios, which are superior to all other combinations under mean-variance criterion will be traced for that opportunity and constraint set.

Markowitz model when used for *ex ante* analysis has certain inconveniences. The user of this model should be able to predict the n return terms and should also be able to predict the n number of return variation coefficients for an n asset case. In addition to these predicitions, the analysts should predict $\frac{(n^2-n)}{2}$ covariance terms or serial correlation coefficients between assets or securities , which is very difficult and time consuming method and infact practically is an impossible task. This property is the major drawback of the Markowitz mean-variance portfolio selection method.

2.2 Sharpe - Single Index Method :

In 1963, another approach to the portfolio selection problem was developed by Sharpe [30]. This approach was named as index models or Sharpe index models.

As the name implies, this models depend on indexes that measure the volatility in security markets. The logic of the index method can be explained as follows. The securities are affected by the overall fluctuations in markets. Generally some stocks are more sensitive to overall changes than others, and if so, then investors can forecast the fluctuations in stocks returns as a function of the overall market fluctuations. The measure used for this is named as the Beta coefficient (β_i or B_i). The stock price observations indicates that, the stock prices intend to move together. The price fluctuations in markets are measured with an index which is an average of the stock returns, weighted with some other information content on the gain that was provided to the stock holders.
The Single index model utilizes this relationship between the market fluctuations and stock returns. Sharpe [30] defined the functional relationship between the market and stock returns in a linear functional form as it is presented in section 3.1.3 in equation (21) .

Index model uses return and variation of returns as a function of the securities responsiveness to the overall market fluctuations. As index models have certain computational efficiencies when compared with Markowitz model , index models became well known and applied.

The volatility measures are also different in these two models. The Markowitz model uses the standart deviation of return which is presented in section 3.1.1 and in equation (5) as the volatility measure where the index model uses the beta coefficient for this purpose which is presented in section 3.1.3 and in equation (21).

A brief emprical comparison of the Markowitz model and index models can be found in Cohen and Pogue [10], and a comparison of the index models with each other (Single index versus Multiple index models) can be found in Kwan [20] .

2.3 Other Methods :

There are various other approaches and studies which utilize different techniques for the portfolio selection problem. Those studies utilize different types of assumptions and many of them are not as widely known and applied as the Index and Markowitz models. Some of the different assumption based studies are presented below for presenting a wider picture of the techniques used for the portfolio problem.

Jacobs [19] argued that the mean-variance and index models are only suitable for the institutional investors' portfolio selection process, with holding many number of securities with some in very small proportions. However, the situation that the small or non-institutional investors are facing is that, they are forced to select between the mutual fund shares and direct investments in a relatively small proportion from the securities. Jacobs proposed a mean-variance portfolio selection procedure with turn over constraints that would minimize the commisions to be paid and bring out a managable size for the small investors' portfolio.

Faaland [15] formulated the portfolio selection problem with a quadratic integer programming structure. Faaland, in his article, presented a generalized version of the approach that Jacobs used for the small investors. In this article computational summaries about CPU time and iterations can be found.

Another interesting portfolio selection method is presented by Canto [7]. The Fat CATS (Capital Tax Sensitivity) approach is an encouraging approach for the portfolio selection problem for beating the market [1]. This approach considers macroeconomic shocks for the selection process.

Portfolio selection based on the price earnings ratio is another widely used method. British academicians Keown, Pin and Chen [9] used this approach for the British stock market. In that study the price earnings ratio based applications' history and performance records can also be found.

A review of the related literature on portfolio theory can determine the classes of problems that have been analyzed by the modern portfolio theory. These classes of problems are :

1. Actual portfolio selection and money allocation based on mean-variance analysis. This application imposes complex constraints depending on the managerial attitudes and legal constraints.

2. Economic analysis usage, the analysis of economy under the assumption that investors are acting upon mean-variance efficiency. The usage of the mean-Variance model for the economic analysis typically assumes highly simplified constraint sets. Such can be seen in the Tobin-Sharpe-Litner model and Black's model.

[1] The term *Beating the Market* refers earning higher returns then the market index returns.

3. METHODOLOGY :

In this study *ex post* data is used for the construction of efficient portfolios. The methods that are used in this normative study are the tools of the Modern Portfolio theory. The Markowitz and Single index models are briefly explained below.

The objective of the Portfolio Analysis is to determine the set of efficient portfolios or efficient frontiers [23]. Mathematically, that is to maximize expected return and minimize risk.

Four classes of calculation approaches can be utilized for the construction of the efficient frontier. These are :

1. Short selling allowed with riskless lending and borrowing;
2. Short selling allowed with no riskless lending and borrowing;
3. No short selling allowed with riskless lending and borrowing;
4. No short selling allowed with no riskless lending or borrowing.

Short selling process is a strategy that investors use when it is believed that trading a security which is non existing in the investors portfolio, can provide positive returns. Short selling or going in a short position for a security requires the selling of a non owning stock and then buying that stock for physical or electronic delivery, the price difference between selling and buying; becomes the return for such a transaction. Selling of a non owning stock is generally supplying a security which is believed to decrease in price. Since one can provide the delivery of a stock sometime later, and during that period if the price of a security is believed to decrease, then an investor can go for a short position. Then the difference between the selling and buying prices becomes the positive return for the short seller, if the selling price is greater then the buying price.

In this study, riskless lending and borrowing with no short selling allowed approach is utilized for the selection methods.

This can generally be formulated as ;

$$\max \frac{\bar{R}_p - R_f}{\sigma_p} \tag{1}$$

Subject to ;

$$\sum_{i=1}^{k} x_i = 1 \ , \ x_i \geq 0, \tag{2}$$

$\bar{R}_p$: Portfolio return.
R_f : Riskfree rate.
x_i : Proportion to invest in the i^{th} asset.
σ_p : Portfolio standart deviation.

3.1 Formulation :

The formulation that is used in this study to delineate efficient frontiers in Markowitz and SIM is presented below.

3.1.1 General relationships and definitions :

1. **Return :**

$$r_{i,t} = \frac{P_{i,t} - P_{i,(t-1)} + d_{i,t}}{P_{i,(t-1)}} \tag{3}$$

$r_{i,t}$: Rate of return on the i^{th} stock on the t^{th} period.
$P_{i,n}$: Price of i^{th} asset on the n^{th} period .
$d_{i,t}$: Dividend and other payments for the i^{th} asset on the t^{th} period.

2. **Average return :**

$$\bar{R}_i = \frac{\sum_{j=1}^{n} r_{i,j}}{n} \tag{4}$$

$\bar{R}_i$: Average rate of return on the i^{th} asset.

n : Number of periods.

3. **Variance of Return :**

$$\sigma_i^2 = \frac{\sum_{i=1}^{n}(r_{i,j} - \bar{R}_i)^2}{n-1} \tag{5}$$

σ_i^2 : Variance of return for the i^{th} asset.

4. **Covariance :**

$$\sigma_{i,k} = \frac{\sum_{i=1}^{n}(r_{i,j} - \bar{R}_i)(r_{k,j} - \bar{R}_k)}{n} \tag{6}$$

$\sigma_{i,k}$: Covariance between i^{th} and k^{th} assets' returns.

5. **Correlation :**

$$\rho_{i,j} = \frac{\sigma_{i,j}}{\sigma_i \sigma_j} \tag{7}$$

$\rho_{i,j}$: Coefficient of correlation for the i^{th} and j^{th} assets.

σ_i : Standart deviation of the i^{th} asset's returns.

6. **Minimum Variance Portfolio (MVP) :**

   The minimum variance portfolio is a portfolio combination which has the lowest possible variance for a given set of objectives. In other words, minimum variance portfolio is the least risky portfolio for a given set of objectives.

7. **Feasible Portfolio :**

   A portfolio $x_1, x_2, ..., x_n$ which meets the requirements for the $\sum_{i=1}^{n} x_i = 1$ and $x_i \geq 0, \forall i$ is said to be a feasible portfolio for the standart model [25].

8. **Inefficiency of an risk-return combination :**

   An obtainable risk return combination is inefficient if another obtainable combination has either higher mean and no higher variance, or less variance and no less mean [25].

9. **Infeasibility :**

   A model is infeasible if, no portfolio can meet it's requirements.

10. **Mean variance (EV) space or Portfolio space :**

    Mean variance space is the n dimensional space whose points are mean-variance combinations. Portfolio space is the n dimensional space whose points are portfolios.

3.1.2 Mean - Variance model :

The objective of the Markowitz model is to trace the opportunity set for different risk preference levels for constructing efficient mean-variance combinations.

The Markowitz model or the mean-variance portfolio selection method uses mean and variance of returns for evaluating the opportunity set. The higher the rate of return and the less the variance of return, the more that asset is desired according to the mean-variance portfolio selection model [26].

This objective can be stated as :

max (Returns - Variance)

or

min (Variance - Returns)

The rate of return is calculated according to the equation (3) . Average rate of return and variance of return are calculated according to the equation (4) and (5) .

The shape of the efficient frontier is concave for the portion which is above the MVP, and convex for the portion which is below the MVP. The efficient frontier

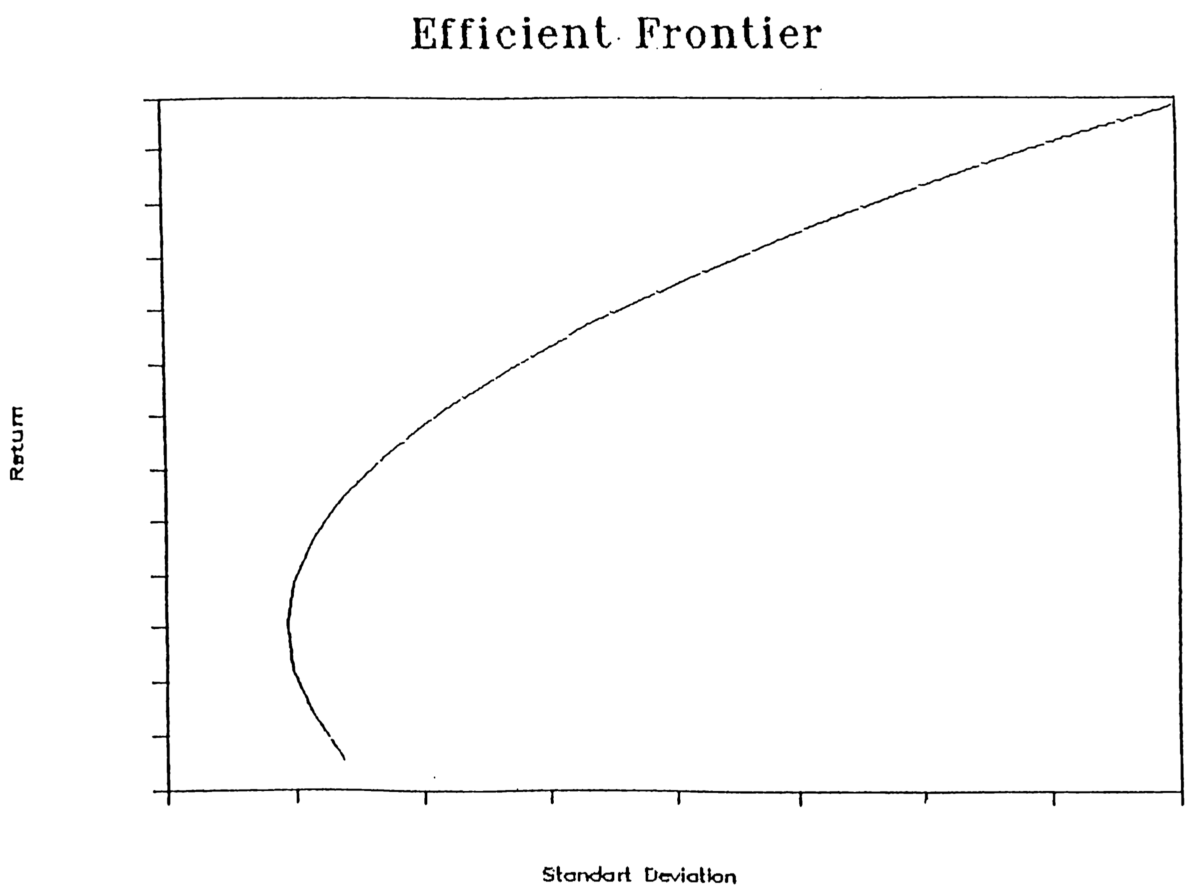

Figure 3.1: Efficient Frontier

can not be convex. This is because combinations of assets can not have more risk than the risk found on a straight line connecting those assets.
An efficient mean-variance combinations set is called as the efficient frontier. A typical efficient frontier is presented in Figure 3.1 .

The efficient frontier should be mean variance efficient when compared with other combinations. This can be explained in other words as, the efficient frontier has no higher mean of returns and no less variance for any combination of risk-return level for a given opportunity set.

The construction of an efficient frontier requires the development of average return and variance of return formulas adaptation to a more than one asset structure or to the portfolio structure. Below, return and variance formulations are presented for the n asset portfolio case.

1. **Portfolio Return :**

$$R_p = \sum_{i=1}^{n} x_i R_i \tag{8}$$

R_i : Rate of return on the i^{th} asset.
x_i : Proportion to invest from the i^{th} asset.

2. **Portfolio Variance :**

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{i,j} \tag{9}$$

σ_p^2 : Portfolio variance .
x_i : Proportion to invest on the i^{th} asset .
$\sigma_{i,j}$: Covariance between the i^{th} and j^{th} assets returns.

The mean-variance portfolio selection method can be formulated to obtain efficient frontiers, for a given set of investment objectives as follows.

$$\min -\lambda R_p + \sigma_p, \ \lambda \geq 0 \tag{10}$$

λ : Risk preference coefficient.
R_p : Portfolio rate of return.
σ_p : Portfolio variance.

This objective function can be written as ;

$$\min -\lambda(\sum_{i=1}^{n} x_i R_i) + (\sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{i,j}), \ \lambda \geq 0 \tag{11}$$

The constraint set for the Standart model includes :

Unity constraint, which forces all the resources to be invested :

$$\sum_{i=1}^{n} x_i = 1 \tag{12}$$

The required rate of return constraint, which ensures that a required rate of return which is denoted by R_D will be earned by the constructed portfolio.

$$\sum_{i=1}^{n} x_i R_i = R_D \tag{13}$$

Upper and Lower Bounds constraint, which allows the investor to determine the combination and proportion of the investment from a given set.

$$L_i \leq x_i \leq U_i, \ (i = 1, 2, ..., n) \tag{14}$$

This mathematical programming problem can be solved by using the quadratic programming techniques. The application of the quadratic programming structure implicitly brings the Kuhn-Tucker conditions of the classical optimization theory, with one or more inequality constraints.
The Kuhn-Tucker conditions for a two variable model can be written as,

$$\frac{min}{max} \ f(x_1, x_2) \tag{15}$$

Subject to ;

$$g(x_1, x_2) \geq 0 \tag{16}$$

$$\frac{\delta f}{\delta x_i} - \lambda \frac{\delta g}{\delta x_1} = 0, \forall i \tag{17}$$

$$\lambda g(x_1, x_2) = 0 \tag{18}$$

$$g(x_1, x_2) \geq 0 \tag{19}$$

$$\lambda \geq 0 \tag{20}$$

The usage of Kuhn-Tucker conditions for one or more inequality constraint, will ensure that, if the optimum can be found then all x_i's will be positive [14].

For the solution of quadratic programming problems, several software packages are available both for the mainframes and PCs. On mainframes Minos and on PCs Gams-Minos, Ginos, Hyper/Lindo can be given as well known examples.

In this study for the solution of the mean-variance portfolio selection problem Hyper/Lindo - 1987 version is used. Certain changes are made in the modelling in order to make the model acceptable by the software package [22].
The preparation of the model to be solved is done with a matrix generator. The matrix generator created models according to the changing lambda values in MPS (Mathematical Programming Structure) and these models are imported and solved for obtaining efficient portfolios and efficient frontiers.

3.1.3 Single Index Model :

The Single index model is a simplified model for the portfolio selection problem. This model relays on a market index which measures the fluctuations in the market and its effects on the individual security returns. Market fluctuations' effects are measured by the Beta (B_i) coefficent.
The Single index model or the diagonal model is defined by Sharpe as follows [30] :

$$R_i = A_i + B_i I + c \tag{21}$$

$$I = A_{n+1} + C_{n+1} \tag{22}$$

$$E_i = A_i + B_i(A_{n+1}) \tag{23}$$

$$V_i = (B_i^2)(Q_{n+1} + Q_i) \tag{24}$$

$$C = (B_i)(B_j)(Q_{n+1}) \tag{25}$$

R_i : Rate of return on the i^{th} asset.
A_i : A constant return term which is independent of the market fluctuations.
B_i : The beta coefficient.
I_i : The market index.
C_i : The error term.
E_i : Expected return of the i^{th} asset.

1. Portfolio Return :

$$E_p = \sum_{i=1}^{n} x_i E_i = \sum_{i=1}^{n} x_i (A_i + B_i I + C_i) \tag{26}$$

E_p : Portfolio rate of return.

2. **Portfolio Variance :**

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j B_i B_j \sigma_m^2 + \sum_{i=1}^{n} x_i^2 \sigma_{e,i}^2 \tag{27}$$

or ,

$$\sigma_p^2 = \sum_{i=1}^{n} x_i^2 B_i^2 \sigma_m^2 + \sum_{i=1}^{n} \sum_{J=1\ i\neq j}^{n} x_i x_j B_i B_j \sigma_m^2 + \sum_{i=1}^{n} x_i^2 \sigma_{e,i}^2 \tag{28}$$

σ_p^2 : Portfolio Variance.

σ_m : Variance of the market index.

$\sigma_{e,i}^2$: Standart error of the estimator.

3. **Portfolio Beta :**

As individual securities have beta coefficients, constructed portfolios have beta coefficients as well. Portfolio beta will measure the percent change in the portfolio's return when there is a one percent change in the market index.

$$B_p = \sum_{i=1}^{n} x_i B_i \tag{29}$$

B_p : Portfolio beta .

4. **Portfolio Alpha :**

Portfolio alpha measures the rate of portfolio return which is independent of the market fluctuations that a portfolio can gain.

$$A_p = \sum_{i=1}^{n} x_i A_i \tag{30}$$

A_p : Portfolio alpha .

In this study the Single index model is applied together with the Excess Return to Beta algorithm presented by Elton-Gruber and Padberg [12].

The Excess Return to Beta algorithm :

The excess return to beta algorithm provides considerable computational conveniences for the portfolio selection problem under the index model [12] .
The objective of the excess return to beta algorithm can be summarized as finding a set that would maximize the objective of ;

$$\phi = \frac{(R_p - R_f)}{\sigma_p} \tag{31}$$

$$\phi = \frac{\sum_{i=1}^{n} x_i(\bar{R}_i - R_f)}{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j B_i B_j \sigma_m^2 + \sum_{i=1}^{n} x_i^2 \sigma_{e,i}^2} \tag{32}$$

σ_p^2 : Portfolio Variance.
σ_m : Variance of the market index.
$\sigma_{e,i}^2$: Standart error of the estimator.

To find the optimal set that would maximize ϕ, the derivative of ϕ is taken with respect to each x_i and set equal to zero.

The optimal allocation will be performed according to excess return to beta formula :

$$z_i = (\frac{\bar{R}_i - R_f}{\sigma_{e,i}^2}) - \left(\sigma_m^2 \frac{\sum_{j=1}^{n} [\frac{\bar{R}_j - R_f}{\sigma_{e,j}^2} \beta_j]}{1 + \sigma_m^2 \sum_{j=1}^{n} \frac{\beta_j^2}{\sigma_{e,j}^2}} \right) \frac{\beta_j}{\sigma_{e,i}^2} \tag{33}$$

The proportion to invest from the i^{th} security is calculated according to equation (34) which is presented below.

$$x_i^o = \frac{z_i}{\sum_{i=1}^{n} |z_i|} \tag{34}$$

The x_i values that will be obtained from the algorithm will ensure the constraint portfolio to be an optimal portfolio for a standart model.

This model presented by the equations (32) , (33) ,(34) allows short selling of any security that is taken into analysis. However, if equation (33) is modified with equations (35) and (36) , then the Excess Return to Beta algorithm can handle models with no short sales. Basically, this modification is the inclusion of the Kuhn-Tucker conditions to the optimization problem.

$$\phi_k = \sigma_m^2 \frac{\sum_j^k \frac{\bar{R}_i - R_f}{\sigma_{e,i}^2} \beta_j}{1 + \sigma_m^2 \sum_j^k \frac{\beta_j^2}{\sigma_{e,j}^2}} \tag{35}$$

Equation (35) uses set k as the set of stocks to be selected. And the inclusion rule for the algorithm is determined by the μ_i , in equation (36) .

The inclusion rule for the algorithm is; select i as long as $\mu_i \geq 0$.

$$z_i = \frac{\beta_i}{\sigma_{e,i}^2} \left[\frac{\bar{R}_i - R_f}{\beta_i} - \phi_k \right] + \mu_i \tag{36}$$

The proportions to be invested according to the excess return to beta algorithm can be calculated according to the equation (34) again. However, the absolute value operator in the denominator becomes redundant, since short selling is not allowed.

3.2 Assumptions of the Study :

The assumptions behind the mean-variance model and single index model are important for the evaluation of this study's findings and potential real life applications.

In both of the models applied, the investors are assumed to be rational, who prefer more to less and assumed to be risk averse. Also all of the relevant information should be quantifiable by the investors. Also, in both models all of the funds were forced to be invested and no short sales are allowed.

3.2.1 Mean - Variance Model :

In this study, for the Markowitz model, a single period, utility maximizing strategy is used together with the usage of *ex post* data.

The single period utility maximization restricts the portfolio selection process as a one period act, which should, infact be a continous process of reviewing and reallocating. Also the usage of *ex post* data is another important point to be considered when interpreting the findings of this study.

3.2.2 Single Index Model :

Single index model totally depends on the index model selected and used. Therefore the calculations with different indexes will possibly present different results. There are certain assumptions of the single index model, these are:

1. $E(e_i) = 0 \ , \quad \forall i$
   The mean of error terms should be normally distributed.

2. $E(e_i(R_m - \bar{R_M})) = 0 \ , \quad \forall i$
   Index should be unrelated to unique return for the securities analyzed.

3. $E(e_i, e_j) = 0 \ , \quad \forall i, j$
   Securities should only be related through a common response to market. Error terms should not be correlated.

4. $E(e_i, e_i)^2 = \sigma^2_{e,i}$
   By definition variance of e_i is σ^2_{ei} which is a constant.

5. $E(e_i(R_m - \bar{R_M}))^2 = \sigma^2_m$
   By definition variance of R_m is σ^2_m .

4. DATA :

The data used for this study is obtained from the İstanbul Securities Exchange publications. Weekly closing prices are used for the calculation of capital gains on stocks. The stock split and capital increase data is obtained from the the Capital Markets Board publications , and used for the modification of the closing prices for the correct calculation of capital gains. This modification is presented in this section , in equation (1) .

The time series used for this study covers 102 weekly observations from January 1986 to December 1987.

The stocks analyzed are selected from the first market of the İstanbul Securities exchange. For consistency, a set of 43 stocks are taken throughout this period. Company names are presented in Table 4 .

The classification that was used in the İstanbul Securities Exchange required certain modifications to be made for obtaining correct capital gains figures on the *price data.* As the corporations announce capital increases and issue stock splits, four types of quotations are made in the market till the end of the capital increase period according to their right contents.

These quotation types are;

1. Old,
2. Preemptive rights on ,
3. Stock split right on ,
4. New .

The rate of return at the end of the capital increase period should be corrected according to the following equation .

$$r_i = \frac{(P_{i+1} - (P_{ssr}/m))}{(P_{ssr}/m)} \quad (1)$$

r_i : Rate of return on the i^{th} period.

P_i : Price of old quotation on the i^{th} period.

P_{ssr} : Price of the stock split right on quotation.

m : number of shares to be received as the result of the stock split.

If such a modification is not performed, then because of the enourmous price changes at the end of the capital increase periods, superflorous negative rate of returns can be observed.

For the single index model, the beta coefficients are estimated by using linear regression technique, with the functional structure of the equation (21) in section 3.1.3.

Stock No.	Stock's Name:
1	Akçimento
2	Anadolu Cam
3	Arçelik
4	Aymar
5	Bağfas
6	Bolu Çimento
7	Çelik Halat
8	Çimsa
9	Çukurova Holding
10	Döktaş
11	Eczacıbası Yatırım
12	Ege Biracılık
13	Ege Gübre
14	Enka
15	Ereğli Demir Çelik
16	Goodyear
17	Gübre Fabrikaları
18	Güney Biracılık
19	Hektaş
20	İzmir Demir Çelik
21	İzocam
22	Kartonsan
23	Kav
24	Koç Holding
25	Koç Yatırım
26	Kordsa
27	Koruma Tarım
28	Lassa
29	Makina Takım
30	Metaş
31	Nasaş
32	Olmuksa
33	Otosan
34	Pimaş
35	Polylen
36	Rabak
37	Sarkuysan
38	Sifaş
39	Türk Demir Döküm
40	İş Bankası-A
41	İş Bankası-B
42	Siemens
43	Şişe Cam

Table 4.1: List of Stocks' analyzed.

The coefficients calculated for the İMKB index is presented below in Table 4.2.

Stock Id.No.	R Square	F	B (1)	B (0)	t(B(1))	t(B(0))	D.W.
1	0.2559	27.858	0.7127	0.0103	5.278	0.892	1.7092
2	0.0000	0.002	-0.0157	0.0567	-0.044	1.840	2.6079
3	0.1579	16.316	0.6389	0.0184	4.039	1.361	2.0312
4	0.0018	0.087	-0.0846	0.0387	-0.296	1.586	2.1746
5	0.1293	14.116	0.7604	0.0159	3.757	0.920	2.5350
6	0.0395	1.894	0.3589	0.0222	1.376	0.998	1.2588
7	0.1461	16.594	0.5925	0.0008	4.074	0.070	2.1633
8	0.1857	18.703	0.6447	0.0205	4.325	1.604	1.7568
9	0.1213	13.389	0.6393	0.0219	3.659	1.463	2.3625
10	0.1961	20.007	0.7751	0.0104	4.473	0.701	1.6269
11	0.0032	0.271	0.0904	0.0197	0.521	1.323	1.3339
12	0.0211	1.622	0.2994	0.0204	1.274	1.010	1.5381
13	0.0777	5.979	0.5362	0.0210	2.445	1.119	1.8025
14	0.0009	0.031	0.1122	0.0454	0.179	0.850	1.7729
15	0.0004	0.033	-0.0487	0.0237	-0.183	1.043	1.8880
16	0.1431	6.346	0.9160	0.0296	2.519	0.957	2.2285
17	0.0458	3.749	0.3540	0.0135	1.936	0.865	1.8562
18	0.0009	0.055	0.0816	0.0330	0.234	1.106	1.5741
19	0.0735	5.001	0.5649	-0.0013	2.236	-0.059	1.8558
20	0.0131	0.875	-0.4057	0.0430	-0.935	1.158	1.8445
21	0.0797	7.279	0.5072	0.0189	2.698	1.177	1.6911
22	0.0738	7.512	0.3860	0.0133	2.752	1.108	2.3203
23	0.1444	12.154	0.5387	0.0156	3.486	1.182	1.8355
24	0.1461	13.685	0.5261	0.0130	3.699	1.069	1.9916
25	0.0955	9.512	0.5004	0.0167	3.084	1.202	1.8758
26	0.3060	41.446	0.8231	0.0074	6.438	0.670	1.9603
27	0.0218	2.165	0.2787	0.0232	1.471	1.429	2.4065
28	0.3089	40.678	1.0448	0.0157	6.378	1.121	1.9289
29	0.0263	1.455	-0.4686	0.0473	-1.207	1.425	1.2841
30	0.0078	0.355	0.3238	0.0488	0.595	1.051	1.8879
31	0.0048	0.411	0.1834	0.0249	0.641	1.018	1.4747
32	0.1985	19.320	0.9429	0.1664	4.395	0.905	1.8832
33	0.0380	3.401	0.4306	0.0228	1.844	1.139	1.7576
34	0.0042	0.059	-0.8786	0.2553	-0.244	0.846	2.1734
35	0.0031	0.068	-0.1758	-0.0039	-0.261	-0.069	1.5548
36	0.0525	5.323	0.3994	0.0175	2.307	1.175	1.7848
37	0.0922	9.753	0.4768	0.0168	3.123	1.284	1.9492
38	0.2276	6.484	1.4597	-0.0175	2.546	-0.360	1.3861
39	0.1734	20.150	0.6726	0.0163	4.489	1.269	2.2049
40	0.0033	0.171	0.0922	0.0328	0.413	1.716	1.5682
41	0.0000	0.000	0.0010	0.0113	0.005	0.627	1.8359
42	0.0210	1.655	0.1952	0.0188	1.287	1.450	2.0066
43	0.0581	5.485	0.4887	0.0123	2.342	0.686	1.7129

Table 4.2: Regression results for İMKB index Returns

Beta statistics calculated presents poor F and t test values for the İMKB index. Therefore, another index is developed as the weekly returns' average. The coefficients calculated for the Return index is presented below in Table 4.3.

Stock Id.No.	R Square	F	B (1)	B (0)	t(B(1))	t(B(0))	D.W.
1	0.2923	33.4618	1.5135	0.0081	5.785	0.718	1.2341
2	0.0732	6.0059	1.6864	0.0361	2.451	1.212	2.5903
3	0.4164	62.0812	2.0613	0.0080	7.879	0.711	2.1192
4	0.1555	8.8416	1.5513	0.0182	2.973	0.807	2.5718
5	0.3369	48.2842	2.4384	0.0037	6.949	0.248	2.6200
6	0.1085	5.5995	1.1812	0.0161	2.366	0.747	1.4730
7	0.4079	66.8334	1.9662	-0.0094	8.175	-0.904	1.9981
8	0.1733	16.4785	1.2158	0.0203	4.059	1.569	1.6649
9	0.2947	40.5292	1.9798	0.0125	6.366	0.927	2.5475
10	0.4478	66.5025	2.3270	-0.0001	8.155	-0.010	1.7642
11	0.4436	66.9745	2.1098	-0.0035	8.184	-0.321	1.5785
12	0.3483	40.0911	2.4132	-0.0018	6.332	-0.115	1.9475
13	0.3445	37.3150	2.2439	0.0061	6.109	0.385	1.8607
14	0.0491	1.7045	1.5888	0.0288	1.306	0.552	2.0247
15	0.5019	88.6802	3.5093	-0.0194	9.417	-1.202	2.0488
16	0.2239	10.9677	2.2769	0.0228	3.312	0.772	1.5834
17	0.6692	157.8199	2.6867	-0.0107	12.563	-1.159	1.9992
18	0.5648	79.1911	4.0709	-0.0139	8.899	-0.706	1.8940
19	0.6059	96.8646	3.2217	-0.0272	9.842	-1.924	2.0053
20	0.4662	57.6611	4.8133	-0.0238	7.593	-0.868	2.1346
21	0.2627	29.9293	1.8292	0.0084	5.471	0.580	1.7619
22	0.2517	31.9645	1.4163	0.0050	5.654	0.460	2.7243
23	0.2872	29.0155	1.5095	0.0096	5.387	0.793	1.5801
24	0.2882	32.3925	1.4683	0.0072	5.691	0.646	2.4403
25	0.5822	125.4318	2.4539	-0.0014	11.200	-0.156	2.6268
26	0.3962	61.6957	1.8610	0.0034	7.855	0.339	1.9456
27	0.3748	58.1562	2.2948	0.0019	7.626	0.149	2.7524
28	0.5080	93.9738	2.6621	0.0072	9.694	0.608	1.8004
29	0.4402	42.4708	3.8136	-0.0089	6.517	-0.353	1.9332
30	0.5326	51.2836	5.3108	-0.0076	7.161	-0.239	2.3172
31	0.2979	36.0813	2.8664	-0.0053	6.007	-0.256	1.9966
32	0.3225	37.1395	2.3881	0.0091	6.094	0.538	2.1593
33	0.3943	55.9964	2.7546	-0.0006	7.483	-0.037	2.1146
34	0.0791	1.2035	7.5604	0.1449	1.097	0.497	1.9881
35	0.1485	3.8394	2.4228	-0.0369	1.959	-0.699	1.8205
36	0.5701	127.3408	2.6139	-0.0049	11.285	-0.492	2.5751
37	0.5635	123.9712	2.3422	-0.0006	11.134	-0.064	2.0456
38	0.0935	2.2693	1.8587	-0.0070	1.506	-0.133	2.0447
39	0.4149	68.0860	2.0668	0.0066	8.251	0.608	2.1731
40	0.0929	5.3292	0.9774	0.0231	2.309	1.263	1.5614
41	0.1852	20.6945	1.7096	-0.0092	4.549	-0.566	1.9862
42	0.1705	15.8359	1.1044	0.0099	3.979	0.830	1.9848
43	0.5839	124.9176	3.0799	-0.0137	11.177	-1.148	2.0292

Table 4.3: Regression results for Return index Returns

5. FINDINGS OF THE STUDY AND CONCLUSIONS :

5.1 Mean - Variance Model :

The calculated efficient frontier is presented for the Markowitz model, below in Figure 5.1 for $x_i \geq 0$ and $x_i \leq 1$ and $\sum x_i = 1$ together with no short sales constraint.

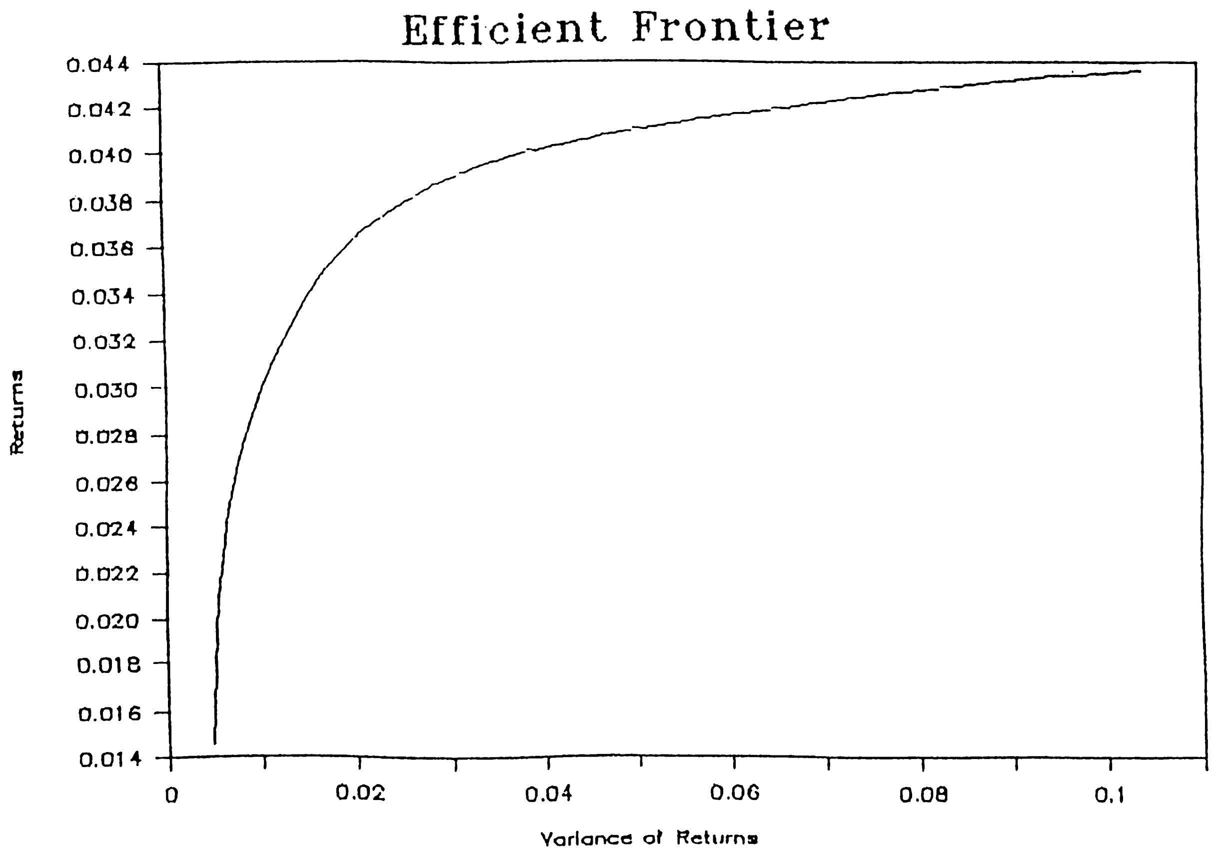

Figure 5.1: Efficient Frontier under the Markowitz model

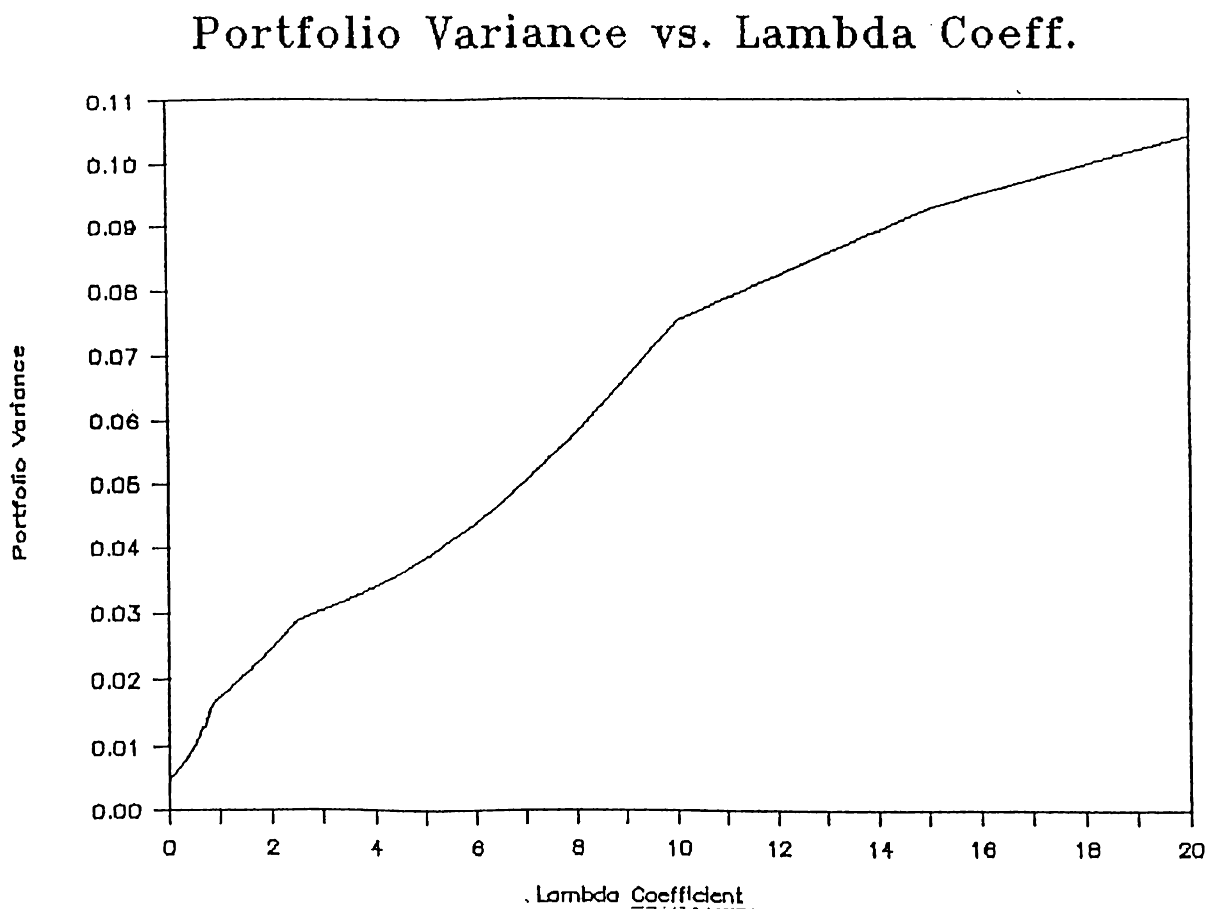

Figure 5.2: Efficient Frontier's Risk preferance graph

The risk and return characteristics of the constructed efficient frontier together with the objective function value , iterations performed and number of variables in basic are presented in Table 5.1 . The characteristic properties of the constructed efficient frontier in Figure 5.1 for the λ value , iteration number, basic variable set and the minimum variance portfolio allocation under the Markowitz model are presented in Figures 5.2, 5.3 , 5.4 and 5.5.

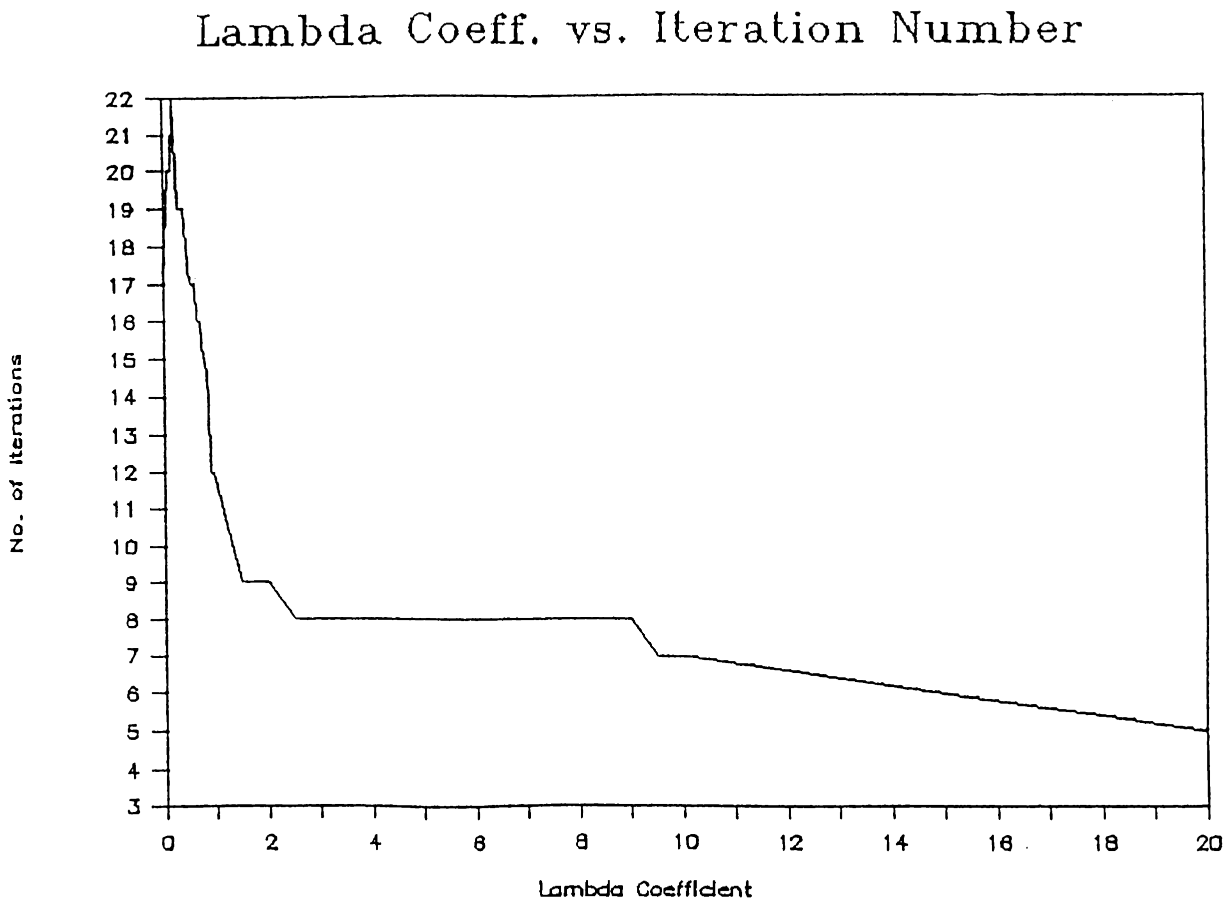

Figure 5.3: Efficient Frontier's Iteration Summary graph

5.2 Single Index Model :

The calculated efficient frontier is presented in Figure 5.2 for the Single index model under the Excess return to beta algorithm.

Portfolio Return	Portfolio Variance	Lambda	Objective Function Value	Iterations	Basic Variable Number
0.014614	0.004793	0.00	0.002395	17	11
0.020921	0.005430	0.10	0.000621	20	14
0.023381	0.006037	0.15	-0.000490	20	12
0.024741	0.006500	0.20	-0.001700	22	12
0.025747	0.006952	0.25	-0.002960	20	12
0.026752	0.007505	0.30	-0.004270	19	11
0.027526	0.008008	0.35	-0.005630	19	11
0.028300	0.008588	0.40	-0.007020	19	11
0.029097	0.009265	0.45	-0.008460	18	12
0.029917	0.010044	0.50	-0.009930	17	13
0.030673	0.010837	0.55	-0.011450	17	13
0.031534	0.011828	0.60	-0.013000	17	13
0.032361	0.012862	0.65	-0.014600	16	12
0.032361	0.012862	0.70	-0.014600	16	12
0.033578	0.014565	0.75	-0.017890	15	11
0.034144	0.015442	0.80	-0.019570	15	11
0.034661	0.016288	0.85	-0.021280	14	10
0.034923	0.016744	0.90	-0.023010	12	8
0.035119	0.017105	0.95	-0.024790	12	8
0.036725	0.020920	1.50	-0.044620	9	5
0.037817	0.024741	2.00	-0.063260	9	5
0.038778	0.029011	2.50	-0.082430	8	4
0.039030	0.030395	3.00	-0.101890	8	4
0.039281	0.032030	3.50	-0.121470	8	4
0.039533	0.033916	4.00	-0.141170	8	4
0.039784	0.036053	4.50	-0.161000	8	4
0.040035	0.038442	5.00	-0.180950	8	4
0.040287	0.041083	5.50	-0.201030	8	4
0.040538	0.043974	6.00	-0.221240	8	4
0.040790	0.047118	6.50	-0.241570	8	4
0.041041	0.050512	7.00	-0.262030	8	4
0.041292	0.054158	7.50	-0.282610	8	4
0.041544	0.058055	8.00	-0.303320	8	4
0.041795	0.062204	8.50	-0.324160	8	4
0.042047	0.066604	9.00	-0.345120	8	4
0.042301	0.071314	9.50	-0.366200	7	3
0.042519	0.075571	10.00	-0.387410	7	3
0.043273	0.092907	15.00	-0.602630	6	2
0.043600	0.104146	20.00	-0.819920	5	1
0.043600	0.104146	30.00	-1.255920	3	1
0.043600	0.104146	40.00	-1.691920	3	1
0.043600	0.104146	50.00	-2.127920	3	1

Table 5.1: Markowitz model Efficient Frontier properties

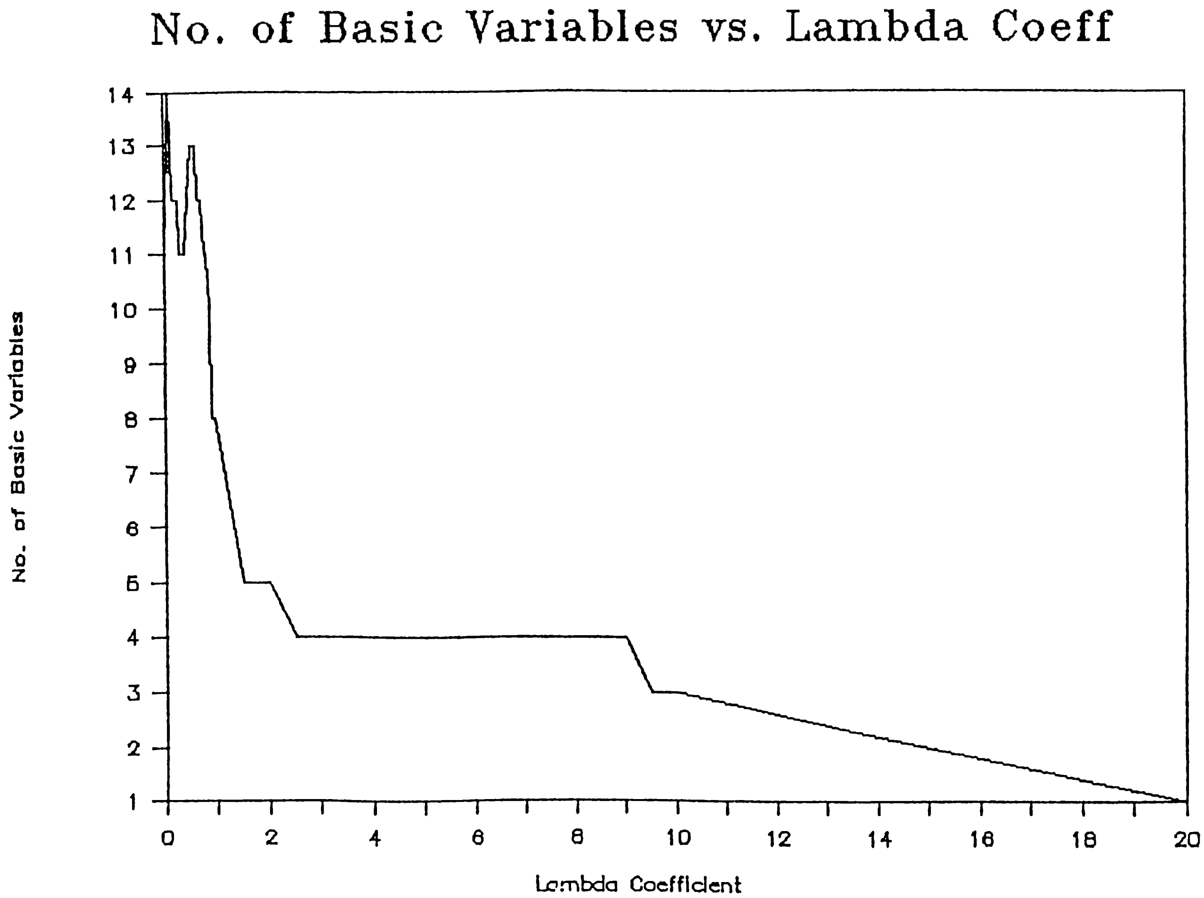

Figure 5.4: Efficient Frontier's Basic variable summary graph

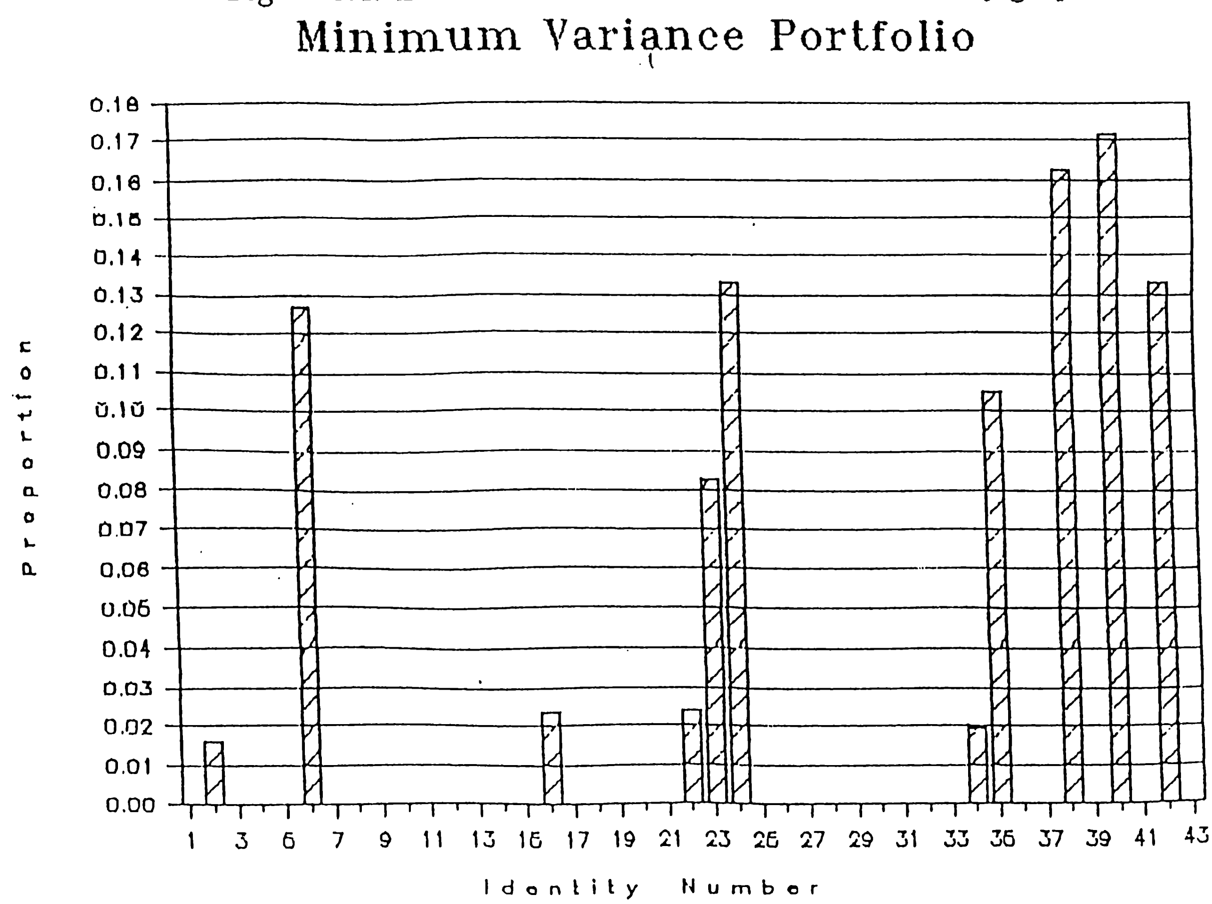

Figure 5.5: MVP combination under the Markowitz model

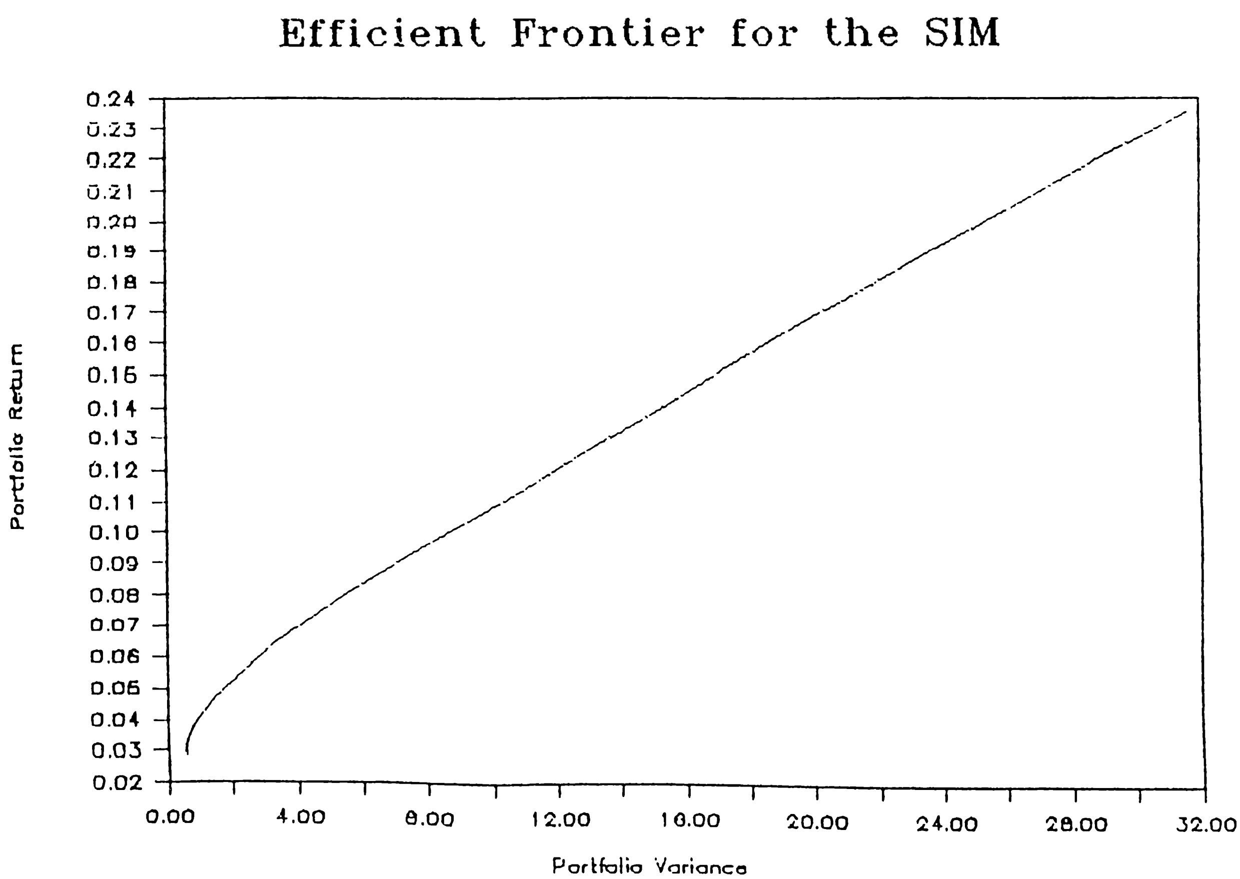

Figure 5.6: Efficient Frontier under the Single Index model

Portfolio beta and alpha are calculated for the single index model and presented below in Table 5.2 and in Figures 5.8, 5.7.

λ value	Portfolio Beta	Portfolio Alpha
0.000	1.719	0.0087
0.001	1.726	0.0088
0.002	1.733	0.0089
0.004	1.760	0.0092
0.006	1.816	0.0093
0.008	1.856	0.0097
0.010	1.909	0.0101
0.015	2.017	0.0113
0.020	2.093	0.0128
0.025	2.244	0.0160
0.030	2.478	0.0218
0.035	3.083	0.0326
0.040	3.531	0.0441
0.045	4.455	0.0551
0.050	6.854	0.0901
0.055	7.283	0.1096
0.060	7.648	0.1343
0.065	7.799	0.1447

Table 5.2: Portfolio Alpha and Beta under the Single Index model

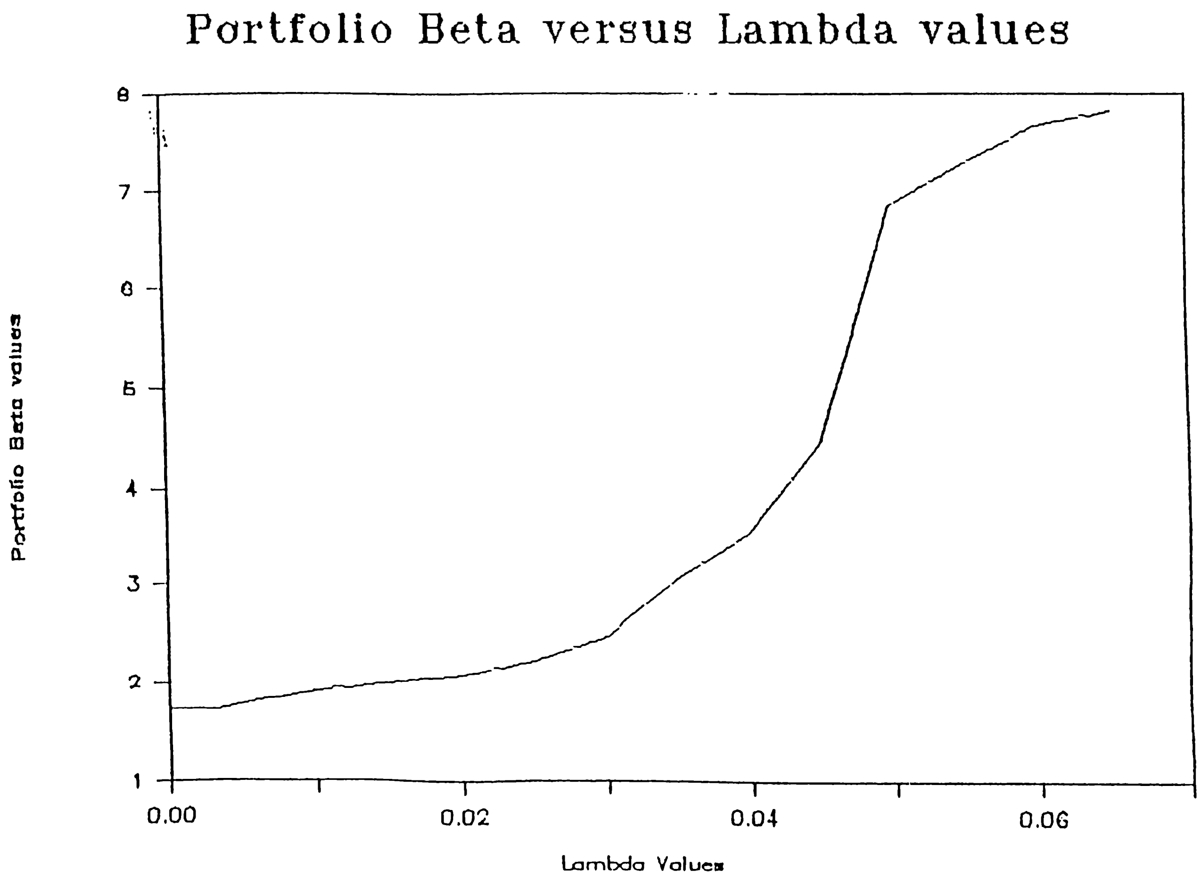

Figure 5.7: SIM portfolio beta versus Lambda coefficient.

The minimum variance allocation (MVP) under the single index model is presented in Figure 5.9 .

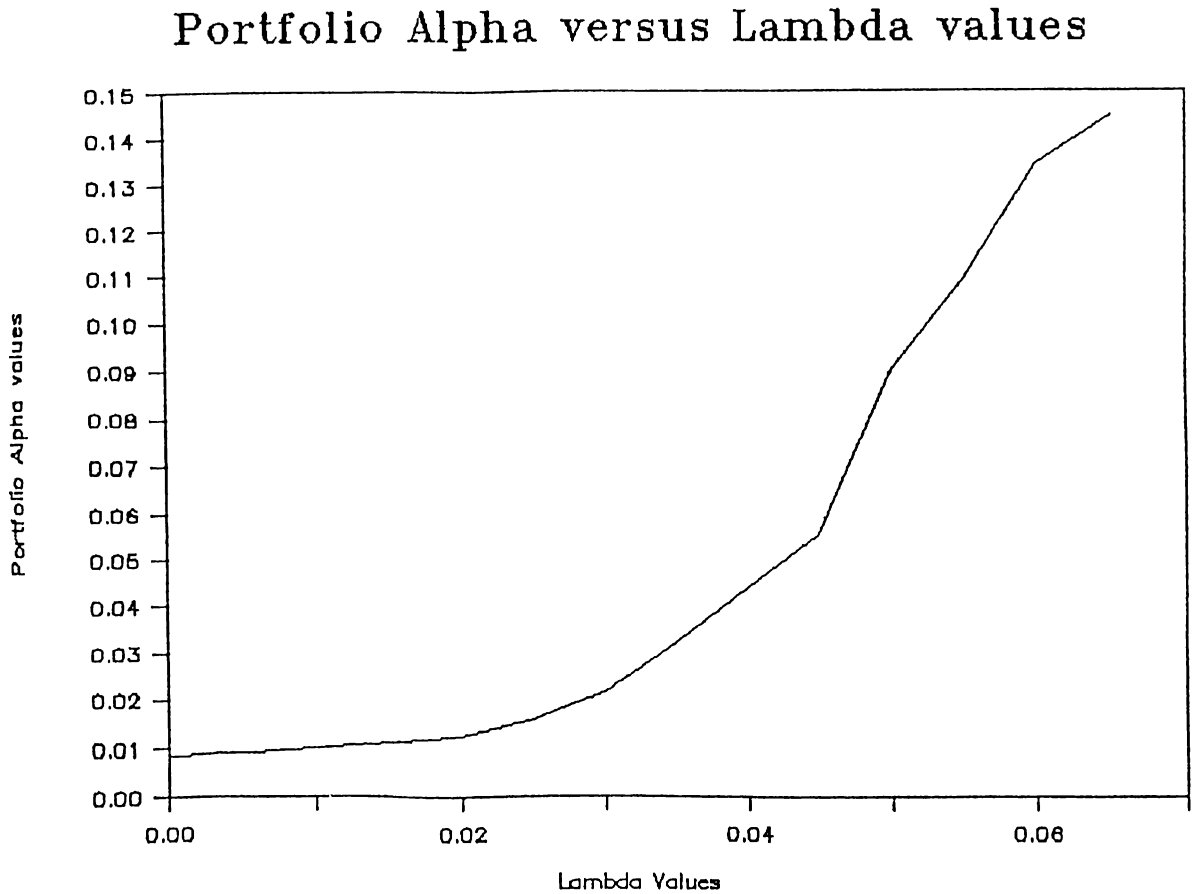

Figure 5.8: SIM alpha and beta values compared.

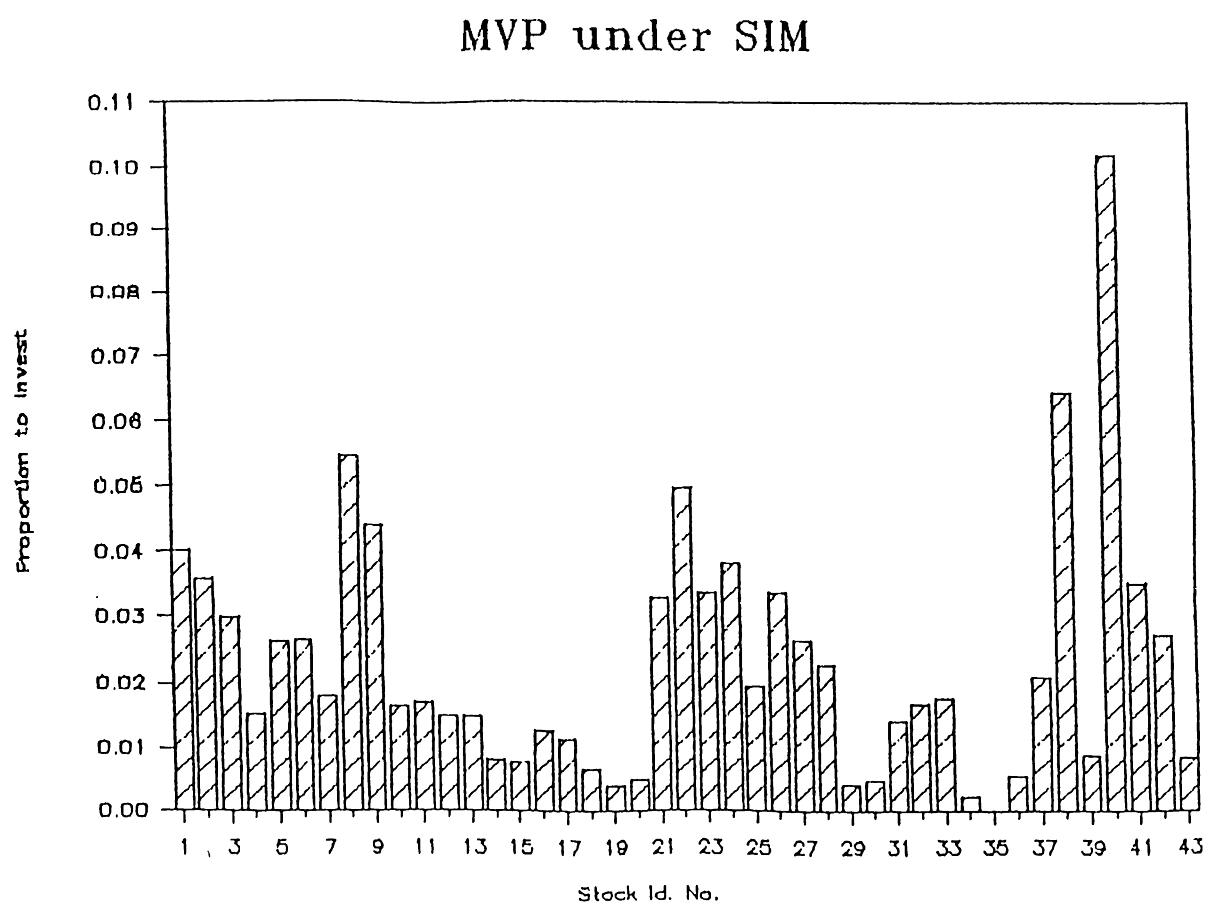

Figure 5.9: MVP combination under the SIM model

The constructed efficient frontiers are plotted below to present the basic differences in their risk return characteristics.

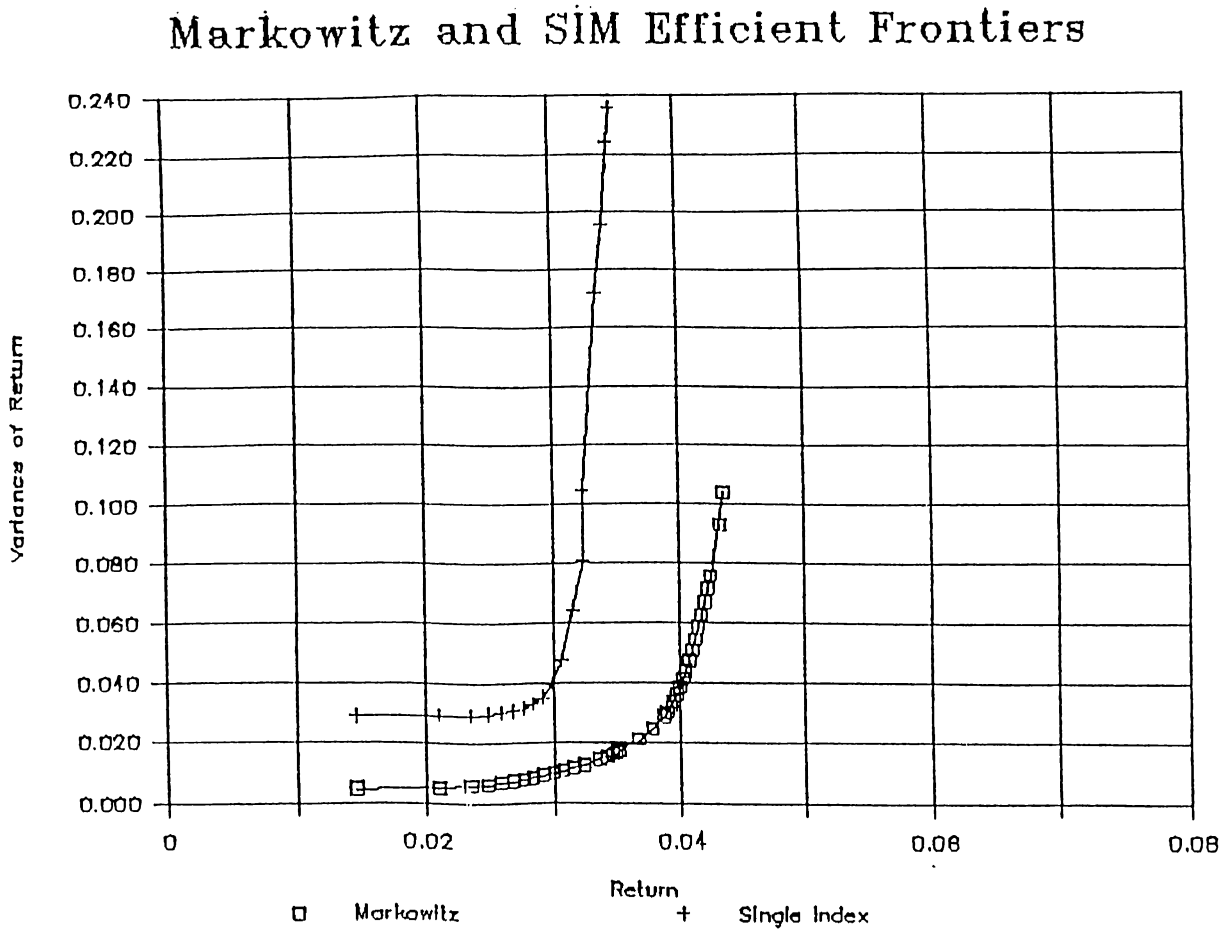

Figure 5.10: SIM and Markowitz Efficient Frontiers comparison

The comparison of these two models according to the mean-variance criterion implies that the investors should prefer the Markowitz efficient frontier to the single index model efficient frontier, because for a given level of return Markowitz model has a lower level of risk and for a given level of risk, has a higher level of return. This result is because of the index that is utilized in the single index model. However, with an appropriate index that will satisty the assumptions of the single index model the findings for the index model will surely change, and approach to the markowitz model efficient set.

5.3 Conclusions :

In this study Markowitz and Single index portfolio selection models are applied to the İstanbul Securities Exchange market securities.
The Markowitz model and Single index model portfolios, because of their different structures and assumptions, constructed different portfolios and efficient frontiers.

5.3.1 Findings :

The findings of the both models are as expected from the theory. As the level of the return from the portfolio increased, the level of the portfolio risk increased as well. Also as the portfolios become more risk taking ($\lambda \uparrow$) the number of securities decreased and the corner portfolio one is reached for the both models.

Markowitz Model Findings :

The Markowitz model accepted 14 securities as basic for the maximum and one security to basic for the minimum. Therefore a set out of the 43 securities are continiously preffered to another set, or to the inefficient set, In Markowitz model some of the securities are never accepted to the basic set. Those securities are presented below ;

1. Bağfaş
2. Çelik Halat
3. Döktaş
4. Eczacıbaşı Yatırım
5. Ege Biracılık
6. Ege Gübre
7. Gübre Fabrikaları
8. Güney Biracılık
9. Hektaş
10. İzmir Demir Çelik

11. İzocam
12. Koç Yatırım
13. Kordsa
14. Makina Takım
15. Metaş
16. Nasaş
17. Olmuksa
18. Rabak
19. Sarkuysan
20. İş Bankası - B
21. Şişe Cam

The MVP of the Markowitz model ($\lambda = 0$) , the following securities are taken into the basic ;

1. Anadolu Cam
2. Bolu Çimento
3. Goodyear
4. Kartonsan
5. Kav
6. Koç Holding
7. Pimaş
8. Polylen
9. Sifaş
10. İş Bankası - A
11. Siemens

Through out the delineation process three securities are continiously taken into the basic. Those are ;

1. Anadolu Cam
2. Koruma Tarım
3. Pimaş

The upper right end of the efficient frontier is only made of one security. That is the security of the Anadolu Cam corporation. These results should be interpreted carefully because, Pimaş stocks are traded on a very short period of time, out of the 102 periods.

Single Index Model Findings :

The SIM findings are different then the Markowitz model findings because of the model's nature. In SIM the MVP consists of 42 securities, however the MVP consisted of 11 securities in the Markowitz model. The MVP combination for the SIM is presented in Figure 5.9 of this section. MVP of the SIM only eliminated the 35^{th} security, which belongs to the Polylen Corporation.
As the R_f increased, the basic variables changed and for most cases the following securities are left in the basic of the Excess Return to Beta model;

1. Anadolu Cam
2. Goodyear
3. Metaş
4. Pimaş
5. Lassa

The upper right corner of the SIM efficient frontier includes only one security. That is the Pimaş cooperation's security. As it was mentioned in the previous part, Pimaş stocks being in the basic is because of it's transaction periods being very short and this should be interpreted carefully.
The single index model coefficients β_i and α_i that are calculated and used for the excess return to beta algorithm calculations, are not statistically significant for some of the securities. The β_i and α_i coefficients are not statistically significant because of the returns' not having a linear functional format which is used in equation (21) . In the scope of this study, β_i and α_i coefficients are used for the demonstration of the excess return to beta algorithm and no best functional format fiting study is performed.
Some of the coefficients being statistically insignificant can be a drawback for real

life users. Non linear functional formats should be tried for better functional form fitings. One other point to be mentioned for the single index model is that, the β_i and α_i coefficients totally depend on the market index that is used.
Here, for the correct estimation of the beta coefficients some of the following relationships should also be checked:

1. Dividend payout ,
2. Asset growth ,
3. Leverage ,
4. Liquidity ,
5. Asset size ,
6. Earning variability ,
7. Accounting beta ,

These variables can be used to develop a Fundamental beta coefficient for a stock [3] .

The single index model excess return to beta algorithm is a simple but powerfull tool for the portfolio analysis. Once the value of the equation (34) or (35) is calculated according to the assumptions used, in section 3.1.3 , then the calculation of proportions can be done very simply and quickly.
This application can either be done with an application specific software or with a spreadsheet software. As it is mentioned, once the program or spreadsheet is prepared then the calculation of the optimal proportions will become very fast. The single index problem can also be solved as a Mathematical Programming problem by replacing the variance with beta coefficient and calculating the rate of return according to the single index formulations as presented in section 3.1.3 in equations (21) to (25) . Such a formulation can be found in Jacob's and Faaland's study [19] [15] .

The potential investors or users of such tools should always keep in mind that each method has its limiting assumptions. Also each method will be looking at the problem from certain point of view. Therefore the best approach to the portfolio selection process should be a combined and revised methodology that would also include the fundamental analysis and technical analysis as well. Since in the real applications one other problem will be estimation of an *ex ante* data set, a combined methodology will more possibily come up with more accurate solutions, since one method will take into account the facts that the other method didn't.

Potential investors should also keep in mind that, investing in stocks is probabily the riskiest investment approach in financial markets. Although the presented techniques take care of the covariance structures of stocks, portfolios constructed portions with public and /or private sector bonds will surely decrease the possibility of wide return fluctuations that can be expected from portfolios.

Altough the number of securities analyzed in this study is relatively small, the models used for the construction of the efficient frontiers can be used for large scale portfolio problems that the institutional investors or individuals can face. However, the models and data requirements for the real life application must satisfy the assumptions that are stated, or appropriate modifications should be performed on the criterion used. Forecasting several parameters that the models require becomes the most challenging part for a real life application.

The computer application of this study can be revised to provide a fully automated and intelligent portfolio selection and construction program. The modifications for such an objective should start with the preperation of appropriate data structure.

The relaxation of the single period utility maximization assumption and applying continious utility maximization will be the first future study topic for the student.

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