UNIT DEMANDS INVENTORY SYSTEM WITH ACCEPTANCE SAMPLING

A THESIS
SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING
AND THE INSTITUTE OF ENGINEERING AND SCIENCES
OF BILKENT UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
MASTER OF SCIENCE

By
Zeki Akbaş
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I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Prof. Izzet Sahin (Principal Advisor)

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Nesim Erkip

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Asst. Prof. Cemal Dincer

I certify that I have read this thesis and in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Asst. Prof. Levent Onur

Approved for the Institute of Engineering and Sciences:

Prof. Mehmet Baray

Director of Institute of Engineering and Sciences

ABSTRACT

UNIT DEMANDS INVENTORY SYSTEM WITH ACCEPTANCE SAMPLING

Zeki Akbaş

M.S. in Industrial Engineering

Supervisor : Prof. Dr.İzzet Şahin

June, 1989

In this study, an extension to the unit demands inventory model with exponentially distributed interdemand times is considered. In this extension orders may arrive in two shipments due to an acceptance sampling scheme. The defective items that are detected by the plan will arrive through a second shipment. A reorder-point, order-quantity (s,Q) type control plan is adopted. The corresponding cost-rate function is constructed and numerically optimized for the best operating policy.

ÖZET

Kabul Örneklemeli Birim İstem

Envanter Sistemi

Zeki Akbaş

Endüstri Mühendisliği Bölümü Yüksek Lisans

Tez Yöneticisi: Prof. İzzet Şahin

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Bu çalışmada istem ara zamanlarının üssel dağıldığı birim istem envanter sisteminin bir uzantısı ele alınmıştır. Gelen siparişler kabul örneklemesinden geçirilmekte ve örnekleme planının saptadığı bozuk malların ikinci bir sevkıyat ile gönderildiği varsayılmaktadır. Tekrar ısmarlama noktası, ısmarlama miktarı (s,Q) tipi envanter kontrol planı kullanılmıştır. Bu sisteme karşılık gelen maliyet fonksiyonu oluşturularak sayısal olarak eniyilenmiştir.

CONTENTS

CHAPTER 1

INTRODUCTION

In this study, we consider a continuous review inventory system in which units are demanded one at a time and interarrival times between successive demands are exponentially distributed. We allow for complete backlogging of unfilled demand. We assume that a reorder-point, order-up-to-level type control policy is active. That is, when the inventory position (inventory on hand plus on order less backorders) falls below the reorder point s (i.e. when it becomes $s-1$), a constant order of size $\Delta+1$ ($\Delta=S-s$) is placed to bring it back up to S. We also assume that orders may arrive in two shipments due to an acceptance sampling scheme.

There is a considerable amount of literature on the analysis of continuous review (s,S) inventory systems with various extensions (Beckman (1965), Sivazlian (1974), Sahin (1979), etc.). In most of the studies it is implicitly assumed that quantity received is the same as quantity ordered. However, there might be cases where quantity received may not be the same as ordered due to various reasons (see, for example, Silver (1976)).

The situation where the quantity received is uncertain is investigated by various authors. Silver (1976) considers the case where the amount received is random under the assumptions of classical Economic Order Quantity (EOQ) model with no lead time. Later, Karlo and Gohil (1982) extend his work to include partial backlogging of unfilled demand with constant lead time. In both of these works, it is assumed that amount received is a proportion of the amount ordered and received quantity may exceed quantity ordered. Lee and Rosenblatt (1985) consider a similar system under two inspection policies assuming constant and continuous demand rate with no procurement lead time. However, in these studies, there is no second shipment for the cases where the amount received is less than the amount ordered.

Peters, Schneider and Tang (1988) work on the problem of determining the best inventory and quality control policy simultaneously. They use a cost model which is a combination of fixed order quantity (s,Q) inventory control system and a lot-by-lot attribute acceptance sampling system. They also give an algorithm that finds the best operating policy.

In these studies the basic idea is to extend the classical EOQ model to consider various situations. In most of the cases, one or more of the assumptions of the EOQ model are relaxed and some solution procedures for the new problems are presented. The size of the coming order is random and this causes the source of uncertainty.

Moinzadeh and Lee (1989) consider the case where a random part of the order is first received and the remaining items arrive through a second shipment. They give the time dependent and stationary distributions of the inventory on hand and use it to construct the appropriate cost-rate function assuming the demands are generated by a Poisson process. The cost-rate function that they developed turns out to be computationally cumbersome and they make an assumption to obtain an approximation. Based on that assumption, they derive an approximate inventory level and use it in the cost-rate function. Sahin (1989) generalizes their model to the case where demands are generated by a general renewal process.

In this study, we consider a situation where the arriving lot may contain some defective items. We assume that the supplier produces the items by a process that yields a proportion of defectives that is known or estimated. The coming units are inspected by using an acceptance sampling plan and this causes the two shipments. That is, the arriving shipment is sampled and it is either accepted or rejected. If the lot is rejected, a rectifying inspection scheme is applied. We assume that the rejected lots are 100% inspected and defective items found both in sampling and in full inspection are replaced by good ones through a second shipment. We also assume that second shipment contains no defective items.

Our aim is to incorporate a particular acceptance sampling scheme with the unit demands inventory system. By doing this, we are specializing on the source of uncertainty in the size of the first shipment. We follow Moinzadeh and Lee (1989) to give the time dependent and stationary distributions of inventory position. However they use an approximate inventory level in the cost-rate function. We derive the distribution of the size of the first shipment by using a single sampling acceptance plan. This distribution is used to construct the stationary distribution of on-hand inventory. Then, the exact cost-rate function is established and numerically optimized to find the best inventory policy under various inventory parameter configurations.

While Moinzadeh and Lee (1989) is based on an approximation for the cost-rate function, we construct and optimize the exact function. Their approximation is based on an assumption. They assume that the probability of more than one outstanding order at any time is negligible. This is a strong assumption and it particularly holds when the lead times are small. However, we find that when only the second lead time is small, the classical one-shipment model also gives a good approximation. Our findings indicate that this second approximation is quite accurate in a number of situations. Since the approximation of Moinzadeh and Lee is particularly valid for small lead times and in these cases one-shipment model provides good results, this approximation becomes somewhat redundant.

Moinzadeh and Lee basically study the two-shipment model.

Although they indicate that the reason for the second shipment may be due to the defective items in the first shipment, they do not give any structure or result that are specific for the problem we consider in this study. However, one needs the particular distribution of on-hand inventory in order to use the model and this distribution may be computationally complex. In our case, the distribution of the size of the first shipment (that is the lot size after the quality control) that we derive turns out to be complicated. Furthermore, we need the convolutions of this distribution with itself for the distribution of on-hand inventory. In Moinzadeh and Lee this distribution is simply expressed as a convolution and no specific expression is provided. Another complication is related to the shape of the cost-rate function. Our numerical results indicate that this function is convex. However, we also provide a methodology that locates the global without using the convexity.

After the optimization, we add the quality control costs to the model and investigate the changes in the cost-rate function when one of the acceptance sampling parameters is altered. We change the acceptance number for some of the cases keeping the sample sizes as before. In the last part of the study, the numerical results are presented and conclusions are derived.

CHAPTER 2

MODELS

2.1. Basic Unit Demands Model

In this section, we present the basic unit demands inventory model with a constant lead time. This model is well-studied in the literature; the outline below follows Sahin (1979).

We assume that unit demands are generated by a renewal process. We analyze a continuous review (s,S) inventory system with a constant procurement lead time, L. We also assume complete backlogging of unfilled demand.

Let $I_p(t)$ be the inventory position and $I(t)$ be the on-hand inventory at time t and let $L \geq 0$ denote the constant lead time. The control policy is an (s,S) (reorder-point, order-up-to level) policy; that is, whenever the inventory position $I_p(t) < s$ (i.e. $I_p(t) = s-1$), we place an order to raise it back to S. The order comes after the lead time, L. Since we assume unit demands, the size of each order is $\Delta+1$ where $\Delta=S-s$. This makes the (s,S) policy operate like a reorder-point $(s-1)$, order-quantity $(\Delta+1)$ policy.

Figure 1 represents a possible realization.

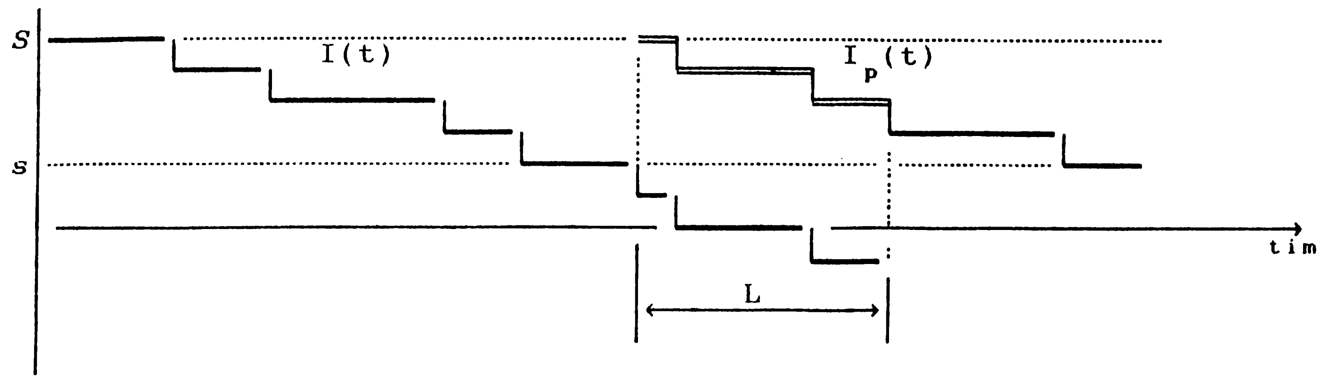

Figure 1. Basic unit demands inventory system with constant lead tim

2.1.1. Distributions

In this section, we give the time-dependent and stationary distributions of inventory position and inventory on-hand. The following notation is used throughout:

$I_p(t)$, $I(t)$	: Inventory position and inventory on-hand at time t.
I_p, I	: Inventory position and inventory on-hand in a stationary process.
$f_p(t,.)$, $f(t,.)$	: p.d.f.s of $I_p(t)$ and $I(t)$.
$f_p(.)$, $f(.)$	: p.d.f.s of I_p and I.
$a(.)$, $A(.)$	: p.d.f. and c.d.f. of interarrival times.
$a_k(.)$, $A_k(.)$	: k-fold convolutions of a(.) and A(.).
μ_a	: Mean of the interarrival distribution.
$D(t,t+u)$, $D(u)$	: Total demand from time t to t+u, demand during u starting right after a withdrawal point (that is after a unit is taken).
$\bar{D}(u)$	: Total demand during u in a stationary process.

By the assumptions that interarrival times are i.i.d. and demands are of size one, we may conclude that demand during $(u,u+t]$ is a delayed renewal process. If u is an arrival point, we have from renewal theory that:

$$P[\ D(t) = n\ ] = A_n(t) - A_{n+1}(t).$$

Time points at which the inventory position becomes S define inventory cycles. Assume $I_p(0)=S$ and consider the length of the first cycle, T_1:

$$\begin{aligned} P[\ T_1 \le x\ ] &= P[\ D(x) \ge \Delta + 1\ ] \\ &= \sum_{n=\Delta+1}^{\infty} [\ A_n(x) - A_{n+1}(x)\ ] \\ &= A_{\Delta+1}(x). \end{aligned}$$

Since the sequence $\{T_n,\ n=1,2,...\}$ is that of i.i.d. random variables,

$$P[\ T_n \le x\ ] = A_{\Delta+1}(x) \quad , x \ge 0, \quad n = 1,2,...$$

Therefore, the p.d.f. of cycle length is $a_{\Delta+1}(x)$. The expected cycle length is given by:

$$\begin{aligned} E[\ T_n\ ] &= \int_0^{\infty} P[\ T_n > x\ ]\, dx \\ &= \int_0^{\infty} [\ 1 - A_{\Delta+1}(x)\ ]\, dx = \mu_a(\Delta + 1). \end{aligned}$$

Let m(.) be the renewal density of the renewal process $\{T_n, n=1,2,...\}$ formed by cycle lengths, then, from renewal theory:

$$m(t) = a_{\Delta+1}(t) + \int_0^t m(t-u)\, a_{\Delta+1}(u)\, du.$$

By conditioning on the renewal process of cycles, we can write the distribution of inventory position as:

$$f_p(t,n) = \begin{cases} P[D(t)=S-n] + \int_0^t m(t-u)\, P[D(u)=S-n]\, du, & s\le n<S \\ 1 - A(t) + \int_0^t m(t-u)\,[1 - A(u)]\, du, & n=S. \end{cases} \quad (1)$$

The first branch can be verified as follows. The first term is the probability that total demand during (0,t] is S-n , $s\le n<S$. The second term accounts for cases where there was a cycle end at time t-u, followed by a total demand during u of S-n. Similarly, first term in the second branch represents the probability that no demand occurred in the time interval (0,t], and second term stands for the probability that a cycle ended at time t-u and no demand occurred from that point to time t.

The stationary distribution of inventory position follows by taking the limit of $f_p(t,n)$ as t tends to infinity and by using the result from renewal theory that $\lim_{t\to\infty} m(t)=\frac{1}{\mu_a(\Delta+1)}$. We find the well-known result that the limiting distribution of inventory position is uniform:

$$\lim_{t\to\infty} f_p(t,n) = f_p(n) = \frac{1}{\Delta+1}, \quad n = s, s+1, \ldots, S \tag{2}$$

Richards (1965) proved that the stationary distribution of inventory position is uniform over (Δ+1) under an (s,S) type control policy, if and only if the demand sizes are unity.

For the distribution of inventory on-hand, we can use the following well-known relationship between inventory on-hand, inventory position and demand during the lead time:

$$I(t+L) = I_p(t) - D(t,t+L).$$

$I_p(t)$ and $D(t,t+L)$ are independent if demands are generated by a Poisson process. Also, it is shown in Sahin (1979) that I_p and $\bar{D}(L)$ are asymptotically independent. This result, and the above relationship between I and I_p (that is $I=I_p-\bar{D}(L)$) enable us to obtain the stationary distribution of on-hand inventory as:

$$f(n) = \frac{1}{\Delta+1} \sum_{k=\max(n,s)}^{S} P[\ \bar{D}(L) = k-n\ ] \quad , \quad -\infty < n \le S$$

2.1.2. Cost Rate Function

A common objective in designing an inventory system is to minimize the total cost involved in operations. There are three basic costs in an inventory system: procurement, holding and shortage costs. Here, the procurement cost can be represented by ordering cost K ($/order), since we assume that the unit price of the item is constant. We also take the unit shortage and holding costs to be P and H ($/unit/unit time) respectively.

The expected cost-rate function representing the total expected cost per unit time can be given as:

$$E(s,\Delta) = \frac{K}{\mu_a(1+\Delta)} + H\sum_{n=0}^{s} n\ f(n) - P\sum_{n=-\infty}^{0} n\ f(n)$$

Note that, $\mu_a(1+\Delta)$ is the expected cycle length. We can optimize this function to get the best operating policy (s^*, Δ^*). It turns out that the cost-rate function is pointwise convex and therefore the optimization is straightforward (Sahin (1989)).

2.2. Two-Shipment Model

In this section, we present the case where the orders may arrive in two shipments, following Moinzadeh and Lee (1989), and Sahin (1989) .

In some cases, the incoming orders are not directly accepted. Instead, a quality control plan is used to ensure a specific input quality level. According to the result of this plan, either the lot is accepted and placed in inventory or rejected with a corrective action. In the rejection case, a second shipment is used to recover the defectives found in the first shipment. We assume that the second shipment contains no defective items.

The two-shipment model under the assumption of unit demands generated by a Poisson process is worked out by Moinzadeh and Lee (1989). Unit demands case results in the constant order size and the assumptions of the basic continuous review model still applies.

In the version of this model that is of interest to us, the orders come after a constant lead time of L, L>0. Then, according to the sampling plan, either the lot is accepted or it is rejected. In the rejection case, the whole lot is inspected and a second shipment is used to compensate for the defectives found. This shipment arrives after another constant lead time, ℓ, $\ell > 0$. There is no relation between ℓ and L. Possible realizations of $I_p(t)$ and $I(t)$ are given in Figure 2.

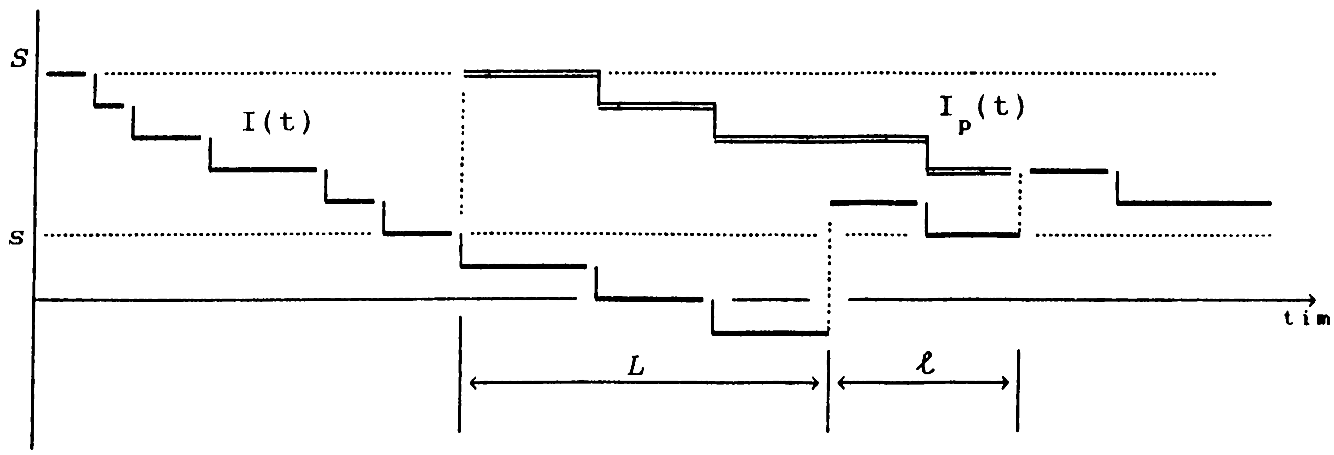

Figure 2. Unit demands inventory model with two shipments

2.2.1. Distributions

Under the assumption of a Poisson demand process, we give in this section the time-dependent and stationary distributions of inventory position and inventory on-hand, following Moinzadeh and Lee (1989).

We note that the distribution of inventory position is uniform over $(\Delta+1)$ and it is not affected by order splitting. Thus, the time dependent and stationary distributions of inventory position remain to be (1) and (2). But the relation between inventory on-hand and inventory position is more complicated and is given as:

$$I(t+L+\ell) = \begin{cases} I_p(t) - D(t, t+\ell+L), & \text{if} \quad I_p(t)-D(t,t+\ell) \geq s \\ I_p(t) - D(t,t+\ell+L)+V(t,t+\ell), & \text{if} \quad I_p(t)-D(t,t+\ell) < s \end{cases}$$

where $V(t,t+u)$ is the units received through the first shipment of the orders placed during $(t,t+u]$.

The above relation is verified by noting that in the first case, no orders are placed during $(t,t+\ell]$ and the equation holds. In the second case, $I_p(t) - D(t,t+\ell) < s$ results ordering during $(t,t+\ell]$. Since the first shipment comes after the lead time, L, we receive them during $(t+\ell,t+\ell+L]$. But there may be more than one order placed during $(t,t+\ell]$, so that $V(t,t+\ell)$ represents the sum of such units received by $t+\ell+L$. It can also be justified by noting that :

$$I(t+\ell+L) = I(t) + M(t,t+\ell+L) - D(t,t+\ell+L)$$

where $M(t,t+\ell+L)$ is the total units received during $(t,t+\ell+L]$.

Also:

$$I_p(t) = I(t) + O(t),$$

where O(t) is the outstanding orders by time t, and

$$M(t,t+\ell+L) = V(t,t+\ell) + O(t).$$

$V(t,t+\ell)$ can be expressed as

$$V(t,t+\ell) = \sum_{i=1}^{M} O_i$$

where O_is are the i.i.d. random variables representing the first shipment sizes of the orders placed during $(t,t+\ell]$ and M is the number of such orders. Since each order is of size $\Delta+1$, M can be given as:

$$M = \text{integer}\left[\frac{S - I_p(t) + D(t,t+\ell)}{\Delta + 1}\right]$$

The numerator is the total deficit (relative to S) in inventory position during $(t,t+\ell]$.

To obtain the distribution of on-hand inventory, we first note that $I_p(t)$, $D(t,t+\ell)$ and $D(t+\ell,t+\ell+L)$ are independent if the demands are generated by a Poisson process. Now defining the joint distributions of time-dependent and stationary processes, - $q(.,.,.;.,.,.)$ and $q(.,.,.)$ -, we get:

$$q(t,\ell,L\ ;\ n,k,i) = P[I_p(t)=n,\ D(t,t+\ell)=k,\ D(t+\ell,t+\ell+L)=i]$$

$$= P[I_p(t)=n]\ \ e^{-\mu_a \ell}\ \frac{(\mu_a \ell)^k}{k\,!} \cdot e^{-\mu_a L}\ \frac{(\mu_a L)^i}{i\,!}$$

and

$$q(n,k,i) = \lim_{t\to\infty} q(t,\ell,L\ ;\ n,k,i) = P[\ I_p=n,\ D(\ell)=k,\ D(\ell,\ell+L)=i]$$

$$= \frac{1}{1+\Delta} \cdot e^{-\mu_a \ell}\ \frac{(\mu_a \ell)^k}{k\,!} \cdot e^{-\mu_a L}\ \frac{(\mu_a L)^i}{i\,!}$$

Using the limiting distribution and the relationships noted above, we can obtain the stationary distribution of on-hand inventory - f(.) - for the two-shipment model as follows (cf Moinzadeh and Lee (1989), Sahin (1989)):

$$f(j) = \frac{1}{1+\Delta} \cdot \Bigg[e^{-\mu_a (L+\ell)} \cdot \sum_{n=\max(j,s)}^{S} \frac{(\mu_a L)^n}{n\,!} \cdot \sum_{k=0}^{\min(n-j,n-s)} \frac{(\mu_a \ell)^k}{k\,!} +$$

$$e^{-\mu_a \ell} \cdot \sum_{n=s}^{S} \sum_{k=n-s+1}^{\infty} \frac{(\mu_a \ell)^k}{k\,!} \cdot \sum_{i=i_1}^{i_2} \phi_{m^*}(j-n+k+i) \Bigg] \quad ,\ j \le S\ .$$

where $m^* =$ integer $[(k+s-n)/(\Delta+1)]$, $\phi(\cdot)$ is the distribution of the first shipment size and $\phi_{m^*}(\cdot)$ is the m^*th convolution of $\phi(\cdot)$. $i_1 = [n-j-k]^+$ and $i_2 = S-j-\mathrm{mod}_{\Delta+1}(k+s-n)$. Note that m^* represents the number of orders placed during $(t,t+\ell]$.

The lower limit for i is i_1 because the argument of $\phi_{m^*}(\cdot)$ should be nonnegative. The upper limit for i is i_2 since there are m^* orders and the maximum size of each order is $\Delta+1$, that is $m^*(\Delta+1) \ge j-n+k+i$.

2.3 Acceptance Quality Control

In the previous sections, the basic model and the two-shipment model are presented and the general form of the stationary distribution of on-hand inventory is given due to Moinzadeh and Lee (1989) and Sahin (1989). Note that, in order to use this expression, we need the convolutions of the distribution of first shipment sizes with itself ($\phi_m*(.)$). In this section, we derive this distribution based on an acceptance sampling scheme. By this way, we are specializing on the cause for two shipments and we are providing the explicit form of the distribution of the first shipment sizes. In addition, we construct the convolutions of this distribution with itself which is needed in the stationary distribution of on-hand inventory for the two-shipment model.

In the first part, we give the details of acceptance quality control of the incoming orders. Then, single sampling by attributes is described and the necessary distributions are derived.

Individual sampling plans are used to decide the acceptance or rejection of the lot without inspecting the whole lot (see, for example, Burr (1976) and Duncan (1974)).There are three measures in which the quality is expressed:

Attributes. A two class classification of units into defective and non-defective.

Counting. An enumeration of occurrences of a given characteristic per given number number of units counted.

Variables. The measurement of some characteristic along a continuous scale.

In this study, sampling by attributes is adopted and we assume that the items are produced by a process with proportion defective, p.

There are three basic sampling plans used in practice. These are (*i*) single, (*ii*) double-multiple and (*iii*) sequential sampling plans. We selected the single sampling plan for simplicity and since it is the most widely used one.

2.3.1 Single Sampling by Attributes

The simplest form of sampling plan is the single-sampling plan. It is employed in inspection by evaluating the proportion defective from the process from which the lots are coming. Single sampling plan operates by taking a random sample of size n from a lot of size N (In our case $N=\Delta+1$). The sample is intended to represent the process used to produce the lot. The sample is inspected and the number of defective items, d, is compared to an acceptance number, c. If the number d is less than or equal to c, the lot is accepted; otherwise, it is rejected. The defectives found both in sampling and in full inspection are replaced by good ones through the second shipment.

The random sample of size n taken from the lot and the remainder of the lot of size $\Delta+1-n$ can be considered as two independent samples from the same population. Consequently, the number of defective items in the sample and in the remainder are two independent random variables. The distribution of the number of defectives in the sample is given by the binomial distribution as:

$$P[\text{ \# of defectives } = j\ ] = \binom{n}{j} p^j q^{n-j}, \quad j = 0,1,\ldots,n,$$

where p is the proportion defective in the process and $q = 1-p$. We need the joint density of the number of defectives in the sample of size n and in the remainder of the population of $\Delta+1-n$ elements to obtain the distribution of the first-shipment size.

Let D_n denote the number of defectives in a sample of size n, $\bar{D}_{\Delta+1-n}$ the number of defectives in the rest of the population, and S_1 the number of non-defectives in the (first) shipment. We have:

$$P[\ S_1 = \Delta+1 - x] = \sum_j P[\ D_n = j,\ \bar{D}_{\Delta+1-n} = x-j\ ]$$

Since D_n and $\bar{D}_{\Delta+1-n}$ are independent,

$$P[\ D_n = j,\ \bar{D}_{\Delta+1-n} = x-j\ ] = \binom{n}{j} p^n\ q^{n-j} \binom{\Delta+1-n}{x-j} p^{x-j}\ \ q^{\Delta+1-n-x+j}$$

Thus,

$P[\ S_1 = \Delta+1-x\ ] =$

$$\begin{cases} \binom{n}{x} p^x q^{n-x} & , \quad x=0,\ldots,c \\ \displaystyle\sum_{j=\max(c+1,\,x-(\Delta+1)+n)}^{x} \binom{n}{j} p^j q^{n-j} \binom{\Delta+1-n}{x-j} p^{x-j} q^{\Delta+1-n-x+j} & , \quad x=c+1,\ldots,n \\ \displaystyle\sum_{j=\max(c+1,\,x-(\Delta+1)+n)}^{n} \binom{n}{j} p^j q^{n-j} \binom{\Delta+1-n}{x-j} p^{x-j} q^{\Delta+1-n-x+j} & , \quad x=n+1,\ldots,\Delta+1 \end{cases}$$

Making the change of variable, $z=\Delta+1-x$,

$$P[\ S_1 = z\ ] = \begin{cases} \displaystyle\sum_{j=\max(c+1,\,n-z)}^{n} \binom{n}{j} \binom{\Delta+1-n}{\Delta+1-z-j} p^{\Delta+1-z} q^{z} & , \quad z=0,\ldots,\Delta-n \\ \displaystyle\sum_{j=\max(c+1,\,n-z)}^{\Delta+1-z} \binom{n}{j} \binom{\Delta+1-n}{\Delta+1-z-j} p^{\Delta+1-z} q^{z} & , \quad z=\Delta+1-n,\ldots, \\ \binom{n}{\Delta+1-z} p^{\Delta+1-z} q^{n-\Delta-1+z} & , \quad z=\Delta+1-c,\ldots, \end{cases}$$

The lower branch is for the case where the number of defectives found in the sample, n, is less than or equal to the acceptance number, c, when the shipment is accepted. The middle and top branches are for the rejection case. The bounds on the index j are adjusted so that the binomial expressions are meaningful.

We need the convolutions of this distribution with itself for the distribution of on-hand inventory. They can recursively be given by the following relationship:

$$\Phi_k(i) = \sum_j \Phi(j) \cdot \Phi_{k-1}(i-j) \tag{3}$$

where $\Phi(j) = P[S_1 = j]$. Writing (3) in a more open form:

$$\Phi_k(i) = \begin{cases} \sum_{j=0}^{i} \Phi(j) \cdot \Phi_{k-1}(i-j) & , \; i=0,\ldots,\Delta+1 \\ \sum_{j=0}^{\Delta+1} \Phi(j) \cdot \Phi_{k-1}(i-j) & , \; i=\Delta+2,\ldots,(k-1)(\Delta+1) \\ \sum_{j=1-(k-1)(\Delta+1)}^{\Delta+1} \Phi(j) \cdot \Phi_{k-1}(i-j) & , \; i=(k-1)(\Delta+1)+1,\ldots,k(\Delta+1) \end{cases}$$

Again, the bounds on j are adjusted so that the arguments of $\Phi(\cdot)$ and $\Phi_{k-1}(\cdot)$ make sense. Unfortunately, since the distribution of $\Phi(x) = P[S_1 = x]$ is complicated, there is no closed form for $\Phi_k(i)$.

2.4 Quality Costs:

Up to this point, we only consider the inventory control costs. However, there are also quality related costs present in the system due to the application of the single sampling acceptance quality control. We assume that the quality cost is composed of three basic parts : cost of sampling, cost of inspection and warranty cost. Costs of sampling and inspection are directly proportional to the number of units involved. The last cost is incurred for the cases where there are defective units in the accepted lot. Based on these, we may derive the expected quality cost function as :

$$QC = n\, c_s + c_i \left(n + (\Delta+1 - n)\, P_{rejection} \right)$$

$$+ c_w \left(\sum_{x=0}^{\Delta+1-n} x\, P\{ D_{\Delta+1-n} = x \} \right) . P_{acceptance}$$

where

c_s : unit cost of sampling,

c_i : unit cost of inspection,

c_w : unit warranty cost (cost of defective units in the accepted lot),

n : sample size,

c : acceptance number,

$P_{rejection}$: probability of rejection ($= \sum_{x=c+1}^{n} P\{ D_n = x \}$).

$P_{acceptance}$: probability of acceptance ($= \sum_{x=0}^{c} P\{ D_n = x \}$).

Note that the quality costs are directly related to the sampling plan used. One may explicitly try to find the minimum cost sampling plans for product attributes (see, for example, Moskowitz and Berry (1976)). Although the joint determination of inventory and quality control policy is out of the scope of this work, we still make some experiments to see the effect of the quality control parameters on the overall cost value. We change one of the sampling plan parameters and evaluate the combined costs for a number of problem instances. For this purpose we use acceptance number, c. In most of the cases, the increase in the acceptance number causes an increase in the combined cost function. The detailed results of this experimentation are presented in the last chapter.

CHAPTER 3

OPTIMIZATION AND RESULTS

In chapter 2, we established the stationary distribution of on-hand inventory which is needed in the evaluation of the cost-rate function. In this chapter, we explain the optimization process and its results.

Since we do not know the shape of the cost-rate function for the two-shipment case, we need to establish some bounds on s^* and Δ^* in the search routine. They can be found as follows: Suppose that the proportions defective, p, is 1. In this case we always reject the lot and everything will arrive in the second shipment. Then this case coincides with the one-shipment case with lead time equal to $L+\ell$. The optimal (s,Δ) for lead time $L+\ell$ then gives the upper bounds on (s^*,Δ^*) for the two-shipment case. Similarly, if the proportions defective, p, is 0, then everything will be accepted in the first shipment and this case becomes equivalent to the one-shipment case with lead time equal to L. Then lower bound on (s^*,Δ^*) will be the optimal (s,Δ) for the one-shipment case with lead time L.

Since demands are of size one, (s,Δ) pairs are integers. This results in evaluating the cost-rate function at discrete points. To avoid possible errors, we take loose bounds for both s^* and Δ^*. For example, if the original bounds on s^* is s1 to s2, we take s1-1 and s2+1. After the optimization, the values of s and Δ are compared with the bounds. If they are on the boundaries, then bounds are relaxed by one unit and the search is repeated. This way, we make sure that the pair s^* and Δ^* found is within the bounds. The cost-rate function is evaluated for each (s,Δ) pair within the bounds. The pair which gives the minimum cost-rate value is selected as the operating policy for the two-shipment case.

We developed a code in Pascal for the optimization of the exact cost-rate function. The routine first evaluates the distribution of the first shipment size. Its convolutions are then computed and stored in a two dimensional array. Then, the distribution of on-hand inventory is evaluated and used in the cost-rate function to get the cost value for that particular (s,Δ) pair. This procedure is repeated for each (s,Δ) pair within the bounds and the pair which gives the minimum cost value is selected as the two-shipment inventory policy. A relative error measure (cf. equation (4) below) is then calculated and the results are given through an output file. A typical problem takes about 10 minutes of CPU time on the mainframe.

The routine is used for a range of problem parameters. The customer arrival rate is taken as 1 throughout. The unit shortage cost P, and ordering cost K, are expressed in terms of H. The range of the parameters are :

$P/H = 1 , 5 , 20$ and $K/H = 1 , 5 , 10.$

$L/\ell = 5 , 2.5 , 1$ and $L = 5 , 10.$

$p = 0.05 , 0.10 , 0.25.$

We assume that if the two-shipment model is not used, then the inventory policy will be determined by the single lead time model corresponding to the same parameter values. We can utilize this assumption to define the relative error measure :

$$R = \frac{E(s^{o},\Delta^{o}) - E(s^{*}, \Delta^{*})}{E(s^{*}, \Delta^{*})} .100 \qquad (4)$$

where (s^{*},Δ^{*}) is the two-shipment optimum policy, (s^{o}, Δ^{o}) is the one-shipment optimum policy, and E(.,.) is the cost rate function for the two-shipment model. R is a measure of the relative increase in inventory cost induced by ignoring defectives in the first shipment.

Computational results are presented in Tables 1 to 18.

For L=5, relative errors lie between 0% and 13.5%, typically below 5%. The maximum errors occur in the case where $L=5,\ell=5$ and proportion defective p=0.25 in which they vary between 8 and 13 percent except for one case. In all the other cases for L=5, relative errors are between 0 and 4 percent. The errors generally increase as L/ℓ decreases.

In some cases, policy changes occur both in s^* and Δ^*, but the order up to level S^*, stays the same (See, for example, p=0.05, L=5, ℓ=1,ℓ=2.5; and P=20, K=1 in Tables 1 & 2). In these situations, the relative errors are very small and we may conclude that the policy is not significantly affected by the second shipment.

For L=10, relative errors become larger as compared with L=5. Larger errors occur for L=10, ℓ=10, p=0.25, which is an extreme case. In this situation, both s^* and Δ^* values increase and the change in the order-up-to level S^* reach 3 to 4 units (a 30 percent increase) whereas for lower ℓ values, this change drops to 1 to 3 units (a 5 to 20 percent increase).

Relative errors do not seem to be affected by changes in P/H or K/H. They are more sensitive to the changes in proportion defective, L and L/ℓ. For example as L/ℓ varies , relative errors (R) behave as follows (p=0.25 and L=10):

for L/ℓ = 5, R=0 to 2.2percent,

for L/ℓ = 2, R=2.4 to 7.0 percent,

for L/ℓ = 1, R=12.5 to 34.2 percent.

Note the increase in error as L/ℓ decreases. A similar behavior may be observed in other cases as well. As expected, relative errors also increase with p. As an example, for L=ℓ=5, we find:

for p=0.05, R= 0 to 1.3 percent

for p=0.10, R= 0.8 to 4.4 percent

for p=0.25, R= 12.5 to 34.2 percent.

Finally, we see that relative errors also increase with L. For example, for p=0.25 and $L=\ell$, we have:

for L = 5, R= 8.1 to 13.5 percent,

for L = 10, R= 12.5 to 34.2 percent.

These patterns of change for relative errors are not observable for small proportions defective and high L/ℓ values, since in those cases the policies do not change from the one-shipment model.

By observing the results, we may conclude that for small proportions defective (i.e. $p \le 0.05$), the two-shipment policies are similar to one-shipment policies. For proportions defective that are not so small (i.e., $p > 0.05$), the two-shipment model should be used when the second lead time is relatively longer (i.e., $L/\ell < 2$). It should also be used when the proportion defective is very large (i.e., $p \ge 0.25$) even though the second lead time is relatively short.

Based on these results, we may state that, when the second lead time is small, there is no need to use the two-shipment model and classical single shipment model gives good approximations. Moinzadeh and Lee (1989) suggest an approximation for the on-hand inventory and use this simplified expression in the cost-rate function. They make a somewhat strong assumption that probability of more than one order outstanding at any time is negligible. This assumption particularly holds when the lead times are small. However, when only the second lead time is small, single lead time model can be successfully applied and their approximation becomes somewhat redundant in these cases.

After the optimization, one of the acceptance sampling plan parameters (acceptance number) is altered to see its effect on the cost-rate function. For this purpose, the following problem instances are used :

$p=0.25$, $L=10$, $\ell=5$,

$p=0.25$, $L=10$, $\ell=10$,

$p=0.25$, $L=5$, $\ell=2.5$.

Quality and inventory costs are then evaluated for the optimal (s^*, Δ^*) pairs of these parameter settings. Three acceptance numbers, c, are used : 0, 1 and 2. The quality cost is expressed in terms of unit warranty cost, c_w which is taken as a multiple of unit holding cost, H. In all cases $c_w = 0.20$ H is used. The other cost parameters for the quality cost function are also given in terms of unit warranty cost. The parameter values are :

c_i / c_w = 0.2, 0.01, 0.002

c_s / c_w = 0.02, 0.001, 0.0002,

where c_i and c_s are the unit inspection and sampling costs, respectively. These parameter values are similar to the ones used in Tagaras and Lee (1987).

The results of this experimentation show that there is no obvious relation between the acceptance number and combined cost values. In some rare instances, as the acceptance number is increased from 0 to 2, the quality cost increases but overall cost decreases. In all cases, the increase in c causes an increase in the quality cost. In general, the overall cost increases as acceptance number increases but inventory cost decreases. However, these patterns are not observable in all cases. Results for one case is presented in table 19.

FUTURE RESEARCH

The following extensions could be considered as follow-ups.

1. Quality control costs may be included in the model and the model may jointly be optimized for the best inventory and quality control policies. In this study, we change the acceptance number for various problem instances to see its affect on the cost-rate function. All of the quality control parameters may be considered jointly.

2. The shape of the cost-rate function is not known but the computational results suggest that it may be unimodal. One may try to establish the unimodality of the cost-rate function.

3. We only utilized the single sampling acceptance plan for the inspection of the incoming orders. Other sampling plans may be incorporated within the model, such as double-multiple and sequential sampling plans.

p = 0.05   L = 5

P/H	K/H	Base Model s*	Base Model Δ*	Two-shipment s*	Two-shipment Δ*	R
1	1	4	2	4	2	0.0
	5	3	4	3	4	0.0
	10	2	6	2	6	0.0
5	1	7	1	6	2	1.2
	5	6	4	6	4	0.0
	10	5	5	5	6	0.01
20	1	9	1	8	2	1.1
	5	8	3	8	3	0.0
	10	8	4	7	6	1.1

Table 1. $L/\ell = 5$ ($\ell=1$)

P/H	K/H	Base Model s*	Base Model Δ*	Two-shipment s*	Two-shipment Δ*	R
1	1	4	2	4	2	0.0
	5	3	4	2	6	0.2
	10	2	6	2	6	0.0
5	1	7	1	6	2	0.5
	5	6	4	6	4	0.0
	10	5	5	5	6	0.4
20	1	9	1	8	2	0.9
	5	8	3	8	3	0.0
	10	8	4	7	6	1.4

Table 2. $L/\ell = 2$ ($\ell=2.5$)

P/H	K/H	Base Model s*	Base Model Δ*	Two-shipment s*	Two-shipment Δ*	R
1	1	4	2	4	2	0.0
	5	3	4	3	5	0.7
	10	2	6	2	7	0.4
5	1	7	1	7	2	0.1
	5	6	4	6	4	0.0
	10	5	5	5	6	1.0
20	1	9	1	9	1	0.0
	5	8	3	8	4	0.3
	10	8	4	7	6	0.5

Table 3. $L/\ell = 1$ ($\ell=5$)

p = 0.10   L = 5

P/H	K/H	Base Model s^*	Base Model Δ^*	Two-shipment s^*	Two-shipment Δ^*	R
1	1	4	2	4	2	0.0
	5	3	4	2	6	0.04
	10	2	6	2	6	0.0
5	1	7	1	6	2	1.0
	5	6	4	6	4	0.0
	10	5	5	5	6	0.1
20	1	9	1	8	2	0.9
	5	8	3	8	3	0.0
	10	8	4	7	6	1.2

Table 4. $L/\ell = 5$ ($\ell=1$)

P/H	K/H	Base Model s^*	Base Model Δ^*	Two-shipment s^*	Two-shipment Δ^*	R
1	1	4	2	4	2	0.0
	5	3	4	3	5	1.2
	10	2	6	2	7	0.5
5	1	7	1	6	3	0.6
	5	6	4	6	4	0.0
	10	5	5	5	6	1.1
20	1	9	1	8	2	0.09
	5	8	3	8	4	0.8
	10	8	4	7	6	1.7

Table 5. $L/\ell = 2$ ($\ell=2.5$)

P/H	K/H	Base Model s^*	Base Model Δ^*	Two-shipment s^*	Two-shipment Δ^*	R
1	1	4	2	4	3	0.9
	5	3	4	3	5	2.3
	10	2	6	2	7	1.7
5	1	7	1	7	2	1.9
	5	6	4	6	4	0.0
	10	5	5	5	6	2.3
20	1	9	1	9	2	1.0
	5	8	3	8	4	1.8
	10	8	4	8	5	1.4

Table 6. $L/\ell = 1$ ($\ell=5$)

p = 0.25   L = 5

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	4	2	4	2	0.0
	5	3	4	3	5	1.1
	10	2	6	2	7	0.3
5	1	7	1	6	3	1.4
	5	6	4	6	4	0.0
	10	5	5	5	6	0.8
20	1	9	1	8	3	0.7
	5	8	3	8	4	0.7
	10	8	4	7	6	1.6

Table 7. $L/\ell = 5$ ($\ell=1$)

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	4	2	4	3	2.8
	5	3	4	3	5	4.1
	10	2	6	2	7	2.7
5	1	7	1	7	2	3.5
	5	6	4	6	5	0.4
	10	5	5	5	7	3.8
20	1	9	1	9	2	2.1
	5	8	3	8	5	3.3
	10	8	4	7	7	2.6

Table 8. $L/\ell = 2$ ($\ell=2.5$)

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	4	2	5	3	8.1
	5	3	4	3	6	9.8
	10	2	6	2	9	8.7
5	1	7	1	8	2	11.2
	5	6	4	7	4	3.8
	10	5	5	5	8	12.7
20	1	9	1	10	2	11.5
	5	8	3	9	4	13.5
	10	8	4	10	6	8.2

Table 9. $L/\ell = 1$ ($\ell=5$)

p = 0.05 L = 10

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	9	2	0.0
	5	7	6	7	6	0.0
	10	6	8	6	8	0.0
5	1	13	1	12	2	1.8
	5	11	5	11	5	0.0
	10	11	6	11	6	0.0
20	1	15	2	15	2	0.0
	5	14	4	14	4	0.0
	10	14	5	14	5	0.0

Table 10. $L/\ell = 5$ $(\ell=2)$

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	9	2	0.0
	5	7	6	7	6	0.0
	10	6	8	6	8	0.0
5	1	13	1	12	3	1.2
	5	11	5	11	5	0.0
	10	11	6	11	6	0.0
20	1	15	2	15	2	0.0
	5	14	4	14	4	0.0
	10	14	5	14	5	0.0

Table 11. $L/\ell = 2$ $(\ell=5)$

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	9	3	1.0
	5	7	6	8	5	0.1
	10	6	8	7	7	0.4
5	1	13	1	13	2	1.3
	5	11	5	12	4	0.5
	10	11	6	11	6	0.0
20	1	15	2	15	2	0.0
	5	14	4	14	5	0.4
	10	14	5	14	6	0.2

Table 12. $L/\ell = 1$ $(\ell=10)$

p = 0.10 L = 10

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	9	3	0.4
	5	7	6	7	6	0.0
	10	6	8	6	8	0.0
5	1	13	1	12	3	1.7
	5	11	5	11	5	0.0
	10	11	6	11	6	0.0
20	1	15	2	15	2	0.0
	5	14	4	14	4	0.0
	10	14	5	14	5	0.0

Table 13. $L/\ell = 5$ ($\ell=2$)

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	9	3	1.2
	5	7	6	7	6	0.0
	10	6	8	6	9	0.05
5	1	13	1	13	2	1.4
	5	11	5	11	5	0.0
	10	11	6	11	6	0.0
20	1	15	2	15	2	0.0
	5	14	4	14	5	0.6
	10	14	5	14	6	0.4

Table 14. $L/\ell = 2$ ($\ell=5$)

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	9	3	3.2
	5	7	6	8	6	2.1
	10	6	8	7	8	1.8
5	1	13	1	13	2	3.4
	5	11	5	12	5	3.3
	10	11	6	11	7	6.8
20	1	15	2	16	2	3.1
	5	14	4	15	5	4.4
	10	14	5	15	5	1.7

Table 15. $L/\ell = 1$ ($\ell=10$)

p = 0.25   L = 10

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	9	3	1.8
	5	7	6	7	7	0.08
	10	6	8	6	8	0.0
5	1	13	1	12	3	2.2
	5	11	5	11	6	0.1
	10	11	6	11	6	0.0
20	1	15	2	15	3	0.1
	5	14	4	11	5	0.8
	10	14	5	14	5	0.0

Table 16. $L/\ell = 5$ ($\ell=2$)

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	9	4	5.8
	5	7	6	7	8	4.2
	10	6	8	6	10	3.9
5	1	13	1	13	3	6.0
	5	11	5	12	6	4.9
	10	11	6	11	7	2.4
20	1	15	2	16	2	5.4
	5	14	4	15	5	7.0
	10	14	5	14	7	1.4

Table 17. $L/\ell = 2$ ($\ell=5$)

P/H	K/H	Base Model s^*	Δ^*	Two-shipment s^*	Δ^*	R
1	1	9	2	11	3	20.4
	5	7	6	9	7	15.1
	10	6	8	8	9	12.5
5	1	13	1	14	3	24.5
	5	11	5	13	6	23.4
	10	11	6	13	7	15.0
20	1	15	2	18	3	33.3
	5	14	4	16	6	34.2
	10	14	5	16	7	23.2

Table 18. $L/\ell = 1$ ($\ell=10$)

P/H	K/H	c	Inventory Cost	Quality Cost	Total Cost
1	1	0	2.24	0.15	2.40
		1	2.23	0.17	2.41
		2	2.22	0.18	2.40
	5	0	2.99	0.21	3.20
		1	3.00	0.25	3.25
		2	2.99	0.27	3.26
	10	0	3.62	0.28	3.90
		1	3.64	0.32	3.96
		2	3.64	0.34	3.99
5	1	0	4.18	0.11	4.30
		1	4.15	0.13	4.28
		2	4.13	0.13	4.27
	5	0	5.05	0.21	5.26
		1	5.07	0.25	5.32
		2	5.06	0.27	5.34
	10	0	5.77	0.28	6.05
		1	5.80	0.32	6.12
		2	5.81	0.34	6.16
20	1	0	5.89	0.11	6.01
		1	5.85	0.13	5.98
		2	5.83	0.13	5.96
	5	0	6.82	0.21	7.03
		1	6.84	0.25	7.09
		2	6.83	0.27	7.10
	10	0	7.60	0.28	7.89
		1	7.64	0.32	7.96
		2	7.65	0.34	8.00

Table 19. Inventory, quality and total costs for L=5, ℓ=2.5, p=0.25 case. (c_w=0.2 H, c_s=0.02 c_w, c_i=0.2 c_w)

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