OPTIMAL DETECTOR RANDOMIZATION IN COGNITIVE RADIO RECEIVERS IN THE PRESENCE OF IMPERFECT SENSING DECISIONS

A THESIS

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By
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August, 2013

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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In cognitive radio systems, spectrum sensing is one of the crucial tasks to be performed by secondary users in order to limit the interference to primary users. Therefore various spectrum sensing methods have been proposed in the literature. Once secondary users make a sensing decision, they adapt their communication parameters accordingly, which means that they perform communications when the channel is sensed as idle whereas they either do not transmit at all or transmit at a reduced power when the channel is sensed as busy. However, in practical systems, sensing decisions of secondary users are never perfect; hence, there can be cases in which the sensing decision is idle (busy) but primary user activity actually exists (does not exist). Therefore, the optimal design of secondary systems requires the consideration of imperfect sensing decisions.

In this thesis, optimal detector randomization is developed for secondary users in a cognitive radio system in the presence of imperfect spectrum sensing decisions. Also, suboptimal detector randomization is proposed under the assumption of perfect sensing decisions. It is shown that the minimum average probability of error can be achieved by employing no more than four maximum a-posteriori probability (MAP) detectors at the secondary receiver. Optimal and suboptimal MAP detectors and generic expressions for their average probability of error are derived in the presence of possible sensing errors. Numerical results are presented and the importance of taking possible sensing errors into account is illustrated in terms of average probability of error optimization.

Keywords: Cognitive radio, spectrum sensing, detector randomization, probability of error.

ÖZET

HATALI SEZİM KARARLARININ VARLIĞINDA BİLİŞSEL RADYO ALICILARDA OPTİMAL SEZİCİ RASTGELELEŞTİRME

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Bilişsel radyo sistemlerinde spektrum sezimi, birincil kullanıcılara olan girişimi limitlemek için ikincil kullanıcılar tarafından gerçekleştirilen önemli görevlerden biridir. Bu nedenle literatürde çeşitli spektrum sezim yöntemleri önerilmiştir. İkincil kullanıcılar bir sezim kararına vardıktan sonra, karara göre iletişim parametrelerini uyarlarlar. Yani kanal meşgul olarak algılandığında ya hiç yayın yapmaz ya da düşük güçte yayın yaparken, kanal boş olarak algılandığında iletişimi gerçekleştirirler. Fakat uygulanabilir sistemlerde ikincil kullanıcıların sezim kararı hiç bir zaman mükemmel olmaz. Bu yüzden sezim kararının boş (meşgul) olduğu ama aslında birincil kullanıcı faaliyetinin olduğu (olmadığı) durumlar olabilir. Bu nedenle ikincil sistemlerin optimal tasarımı, hatalı sezim kararlarının göz önünde bulundurulmasını gerektirir.

Bu tezde optimal sezici rastgeleleştirme, bilişsel radyo sistemindeki ikincil kullanıcılar için hatalı spektrum sezim kararlarının varlığında geliştirilmektedir. Ayrıca, optimal olmayan sezici rastgeleleştirme mükemmel sezim kararları altında tasarlanmaktadır. İkincil alıcıdaki dörtten fazla olmayan maksimum sonsal olasılık (MAP) sezicilerinin çalıştırılmasıyla en düşük ortalama hata olasılığına ulaşılacağı gösterilmektedir. Optimal ve optimal olmayan MAP seziciler ve bunların ortalama hata olasılıklarının genel ifadeleri muhtemel sezim hatalarının varlığında elde edilmektedir. Sayısal sonuçlar sunulmakta ve muhtemel sezim hatalarını hesaba katmanın önemi ortalama hata olasılığı eniyilemesi açısından gösterilmektedir.

Anahtar sözcükler: Bilişsel radyo, spektrum sezimi, sezici rastgeleleştirme, hata olasılığı.

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Chapter 1

INTRODUCTION

As the electromagnetic radio spectrum is a limited natural resource, it is important to improve spectrum utilization. Based on the report prepared by the Spectrum Policy Task Force and published by the Federal Communications Commission, radio spectrum is not being fully utilized due to spectrum holes unoccupied by the licensed users. To increase spectrum efficiency, this report proposes a solution that users other than the licensed users can also access spectrum holes on a time, frequency, bandwidth, or space basis [1]. This solution can be applicable by means of cognitive radios [2].

Cognitive radio was first proposed by Mitola in his article [2] and it was defined in his PhD thesis [3] in 2000 as: "The point in which wireless personal digital assistants (PDAs) and the related networks are sufficiently computationally intelligent about radio sources and related computer-to-computer communications to detect user communications needs as a function of use context, and to provide radio resources and wireless services most appropriate to those needs."

According to another definition mentioned in the P1900.1 Standard, a cognitive radio is "a type of radio in which communication systems are aware of their environment and internal state and can make decisions about their radio operating behavior based on that information and predefined objectives [4]."

In cognitive radio systems, primary users, i.e., licensed users, have the right to use the allocated spectrum. Besides primary users, secondary users also have the right to access this licensed spectrum when it is not occupied by the primary users to provide highly reliable communications and to increase efficiency of utilization of the radio spectrum when needed [5].

One of the main problems in cognitive radio system is spectrum sensing. Since secondary users first sense the channel to decide whether the licensed spectrum is available or not, they need to sense the channel before starting communications in order to prevent interference caused to primary users. In the literature, various spectrum sensing methods such as energy detection, waveform detection, cyclostationary detection and matched filtering have been proposed [6]. As a common sensing method, energy detection is a simple way of deciding whether primary user's signal is present or not since it does not require any prior knowledge about the signal [7, 8, 9]. In a generic energy detection scheme, the signal filtered at the center frequency is squared and integrated over an interval. Then the output is compared with a threshold level in order to identify the absence and presence of primary user activity. Cyclostationary detection is the second method which relies on the cyclostationary feature of received signal, such as periodicity, autocorrelation, and spectral correlation [10]. As a third method, waveform detection (coherent detection) can be applied when the primary users' signal includes known patterns such as preambles, midambles, and pilot tone [11]. Based on waveform detection, matched filtering is the optimum method for perfectly known signals, which requires frequency, bandwidth and modulation scheme of the received signal [12]. In addition to these methods, cooperative sensing methods implemented via centralized and distributed collaboration among cognitive radios are studied in [13, 14].

In this thesis, the aim is to design the optimal secondary communications system in the presence of detector randomization by taking imperfect channel sensing decisions into account. In most of the studies in the literature, communications systems of secondary users are designed independently of the sensing decision, or, the sensing decisions are considered as perfect. However, the spectrum sensing methods discussed above do not provide perfect sensing in general; hence, the optimal secondary systems need to be designed in the presence of sensing errors. In this thesis, possible spectrum sensing errors are taken into consideration in the optimal design of secondary systems.

Detector randomization is a technique to employ multiple detectors at the receiver with certain probabilities (certain fractions of time) [15, 16, 17]. By adapting the transmitted power level according to the employed detector at the receiver, performance improvements can be achieved via detector randomization (i.e., via switching between multiple transmit power-detector pairs). In [18], it is shown that an average power-limited transmitter cannot improve its error performance via detector randomization when the channel noise has a unimodal probability density function. However, as investigated in [16, 17], benefits of detector randomization are observed commonly in non-Gaussian channels. By noting that secondary users in cognitive radio systems experience non-Gaussian channels in practice due to imperfect sensing decisions, the use of detector randomization for the design of secondary communications systems is proposed in this thesis.

The main contributions of this thesis are as follows:

1. Detector randomization is studied for cognitive radio systems for the first time.
2. Optimal detector randomization is developed both in the presence of imperfect sensing decisions and under the assumption of perfect sensing decisions, and it is shown that the minimum average probability of error can be achieved by employing no more than four maximum a-posteriori probability (MAP) detectors at the secondary receiver.
3. Optimal MAP detectors are derived and generic probability of error expressions are obtained in the presence of possible sensing errors.
4. Effects of ignoring possible sensing errors are illustrated in terms of degraded error performance.

Chapter 2

DETECTOR RANDOMIZATION IN COGNITIVE RADIO RECEIVERS

2.1 Motivation and System Model

In a cognitive radio system including two groups of users, primary and secondary users, secondary users first sense the channel to decide whether the channel is being occupied by primary users. Assume that $\mathcal{H}_0$ and $\mathcal{H}_1$ represent the hypotheses that correspond to the absence and presence of primary user activity, respectively. In addition to $\mathcal{H}_0$ and $\mathcal{H}_1$, $\hat{\mathcal{H}}_0$ and $\hat{\mathcal{H}}_1$ denote the events in which the secondary user declares $\mathcal{H}_0$ and $\mathcal{H}_1$ as the true hypothesis, respectively. Since there are two channel sensing decision states, $\{\hat{\mathcal{H}}_0, \hat{\mathcal{H}}_1\}$, and two states of the channel (i.e., the presence and absence of primary user activity), $\{\mathcal{H}_0, \mathcal{H}_1\}$, four scenarios exist:

$$(\mathcal{H}_1, \hat{\mathcal{H}}_1) \ : \ \text{Detection of active primary user} \tag{2.1}$$
$$(\mathcal{H}_1, \hat{\mathcal{H}}_0) \ : \ \text{Miss-detection of active primary user} \tag{2.2}$$
$$(\mathcal{H}_0, \hat{\mathcal{H}}_1) \ : \ \text{False alarm} \tag{2.3}$$
$$(\mathcal{H}_0, \hat{\mathcal{H}}_0) \ : \ \text{Detection of inactive primary user} \tag{2.4}$$

After the channel sensing phase, cognitive secondary users start digital communications. Specifically, the secondary transmitter sends information carrying signals to the secondary receiver in a certain manner depending on the channel sensing decision. When the channel sensing decision is $\hat{\mathcal{H}}_0$ (i.e., no primary user activity is detected), the information symbol power is set to P_0. On the other hand, the symbol power is set to P_1 when the channel sensing decision is $\hat{\mathcal{H}}_1$. The selection of two different power levels is employed for the protection of primary users. In practice, lower power levels are employed in the presence of primary user activity; hence, $P_1 < P_0$. In this way, the interference caused to primary users is limited. It is noted that when $P_1 = 0$ is employed, the considered generic scenario reduces to the special case in which no secondary user communications are allowed when primary users are active. For the theoretical investigations in this thesis, generic values for P_0 and P_1 are considered.

In this study, the secondary radio channel is assumed to be subject to slow frequency-flat fading. Then, depending on the channel sensing decision and the true state of the channel (i.e., the presence and absence of primary user activity), the following four scenarios exist:

$$(\mathcal{H}_1, \hat{\mathcal{H}}_1) \ : \ x = h\sqrt{P_1}\, d + n + s \tag{2.5}$$
$$(\mathcal{H}_1, \hat{\mathcal{H}}_0) \ : \ x = h\sqrt{P_0}\, d + n + s \tag{2.6}$$
$$(\mathcal{H}_0, \hat{\mathcal{H}}_1) \ : \ x = h\sqrt{P_1}\, d + n \tag{2.7}$$
$$(\mathcal{H}_0, \hat{\mathcal{H}}_0) \ : \ x = h\sqrt{P_0}\, d + n \tag{2.8}$$

where $(\mathcal{H}_i, \hat{\mathcal{H}}_j)$ denotes the scenario in which the sensing decision is $\hat{\mathcal{H}}_j$ while the true hypothesis is $\mathcal{H}_i$. Also, x is the observation at the receiver of the secondary user, h denotes the fading coefficient of the channel between the secondary transmitter and receiver, n denotes the zero-mean complex Gaussian noise with variance σ_n^2, s is the sum of the faded primary users' signals arriving at the secondary receiver, and d denotes the complex information symbol. In addition, as discussed in the previous paragraph, P_i denotes the power level of the information symbol when the sensing decision is $\hat{\mathcal{H}}_i$. Without loss of generality, it is assumed that $\mathbb{E}\{|d|^2\} = 1$. Considering M-ary modulation, the complex information symbol d takes values from set $\{d_0, d_1, \ldots, d_{M-1}\}$. Furthermore, it is assumed that the channel coefficient h is known; i.e., channel estimation is performed perfectly before the communications start.

It is noted that in the presence of primary user activity, the additive disturbance is noise plus the primary users' received sum signal, i.e., $n + s$, as in (2.5) and (2.6), while only additive noise is present when the channel is not occupied by the primary users. Since errors are possible in channel sensing, the true state of the channel (busy or idle) and consequently the statistics of the additive disturbance are not perfectly known by the secondary receiver. Hence, optimal communications system needs to be designed in the presence of such sensing errors and ambiguities.

We consider a secondary communications system as in Figure 2.1, where the secondary transmitter can randomize the power levels, P_0 and P_1 in (2.5)-(2.8), and the secondary receiver can perform a corresponding randomization (time-sharing) among multiple MAP detectors.[1] The power levels P_0 and P_1 are generated according to PDFs f_{P_0} and f_{P_1}, respectively, depending on the sensing decision. Namely, if the secondary system decides that there are no primary users in the system $(\hat{\mathcal{H}}_0)$, the secondary transmitter generates the power levels according to f_{P_0}. Otherwise $(\hat{\mathcal{H}}_1)$, the power levels are generated based on f_{P_1}. It is assumed that for each possible power level used by the secondary transmitter, the secondary receiver can employ the corresponding optimal MAP detector for that power level. Hence, there exist as many MAP detectors at the secondary

[1]Please see [16, 17] for detailed discussions on detector randomization.

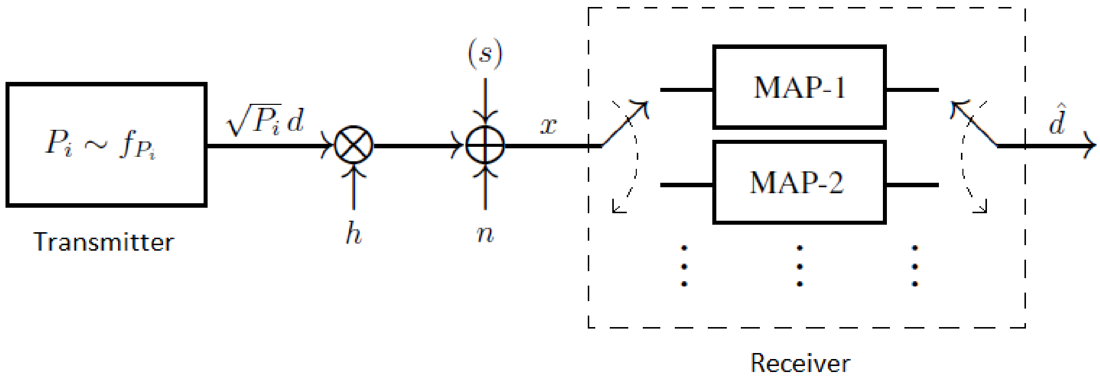

Figure 2.1: Basedband model of the communications system for the secondary users. The secondary transmitter generates a signal, the power P_i of which is determined according to the PDF f_{P_i} for $i \in \{0, 1\}$. The information signal, $\sqrt{P_i}\,d$ is multiplied with the complex channel coefficient h, and it is corrupted by additive noise n. Also, if primary users exist, their faded signals, denoted by s, interfere with the desired signal. The secondary receiver can perform randomization among multiple MAP detectors, each of which is optimized according to a possible power level of the transmit signal.

receiver as the number of different transmit power levels. Although we start with such a generic formulation in order to obtain the optimal error performance that can be achieved by the secondary system, we show in the following that no more than four MAP detectors are necessary for obtaining the overall optimal solution.

Remark 1: MAP detectors are employed in Figure 2.1 since they minimize the average probability of error among all possible detectors. It is also possible to start with generic detectors and then show that they must be MAP detectors in order to minimize the average probability of error of the system, by employing an approach similar to that in [17, 19]. ■

2.2 Problem Formulation

Based on the formulation in (2.5)-(2.8) and the system model in Figure 2.1, the aim is to find the optimal power distributions for P_0 and P_1 in order to minimize the average error probability of the secondary system under the following average and peak power constraints:

$$\mathbb{E}\{P_i\} \leq P_{\mathrm{av},i} \quad \text{and} \quad P_i \leq P_{\mathrm{pk},i} \quad \text{for} \quad i \in \{0,1\} \tag{2.9}$$

where $P_{\mathrm{av},i}$ and $P_{\mathrm{pk},i}$ are the limits on the average and peak powers, respectively. Note that the constraints in (2.9) also imply limits on the average transmit power at the secondary transmitter and on the average interference power to primary users [20]. Specifically, the average transmit power at the secondary user is expressed as

$$\Pr\{\hat{\mathcal{H}}_0\}\mathbb{E}\{P_0\} + \Pr\{\hat{\mathcal{H}}_1\}\mathbb{E}\{P_1\} \;, \tag{2.10}$$

and the average interference power to a primary user is given by

$$\left(\Pr\{\hat{\mathcal{H}}_0|\mathcal{H}_1\}\mathbb{E}\{P_0\} + \Pr\{\hat{\mathcal{H}}_1|\mathcal{H}_1\}\mathbb{E}\{P_1\}\right)\mathbb{E}\{|g|^2\} \;, \tag{2.11}$$

where g is the channel coefficient between the secondary transmitter and the primary receiver. (If there are multiple primary users in the system, the primary user with the maximum value of $\mathbb{E}\{|g|^2\}$ can be considered in (2.11) for determining the average interference power constraint.) It is noted from (2.10) and (2.11) that via the constraints in (2.9), the average transmit and interference powers can be constrained. In addition, it is observed that for practical cases with $\Pr\{\hat{\mathcal{H}}_0|\mathcal{H}_1\} < \Pr\{\hat{\mathcal{H}}_1|\mathcal{H}_1\}$, $A_1 < A_0$ is commonly employed in order to meet strict limits on the interference to primary users.

In obtaining the optimal power distributions for P_0 and P_1 under the average power constraints in (2.9), two scenarios are considered. In the first one, possible errors in the sensing decision are taken into consideration in designing the optimal secondary system (Section 2.3). In the second one, the secondary receiver assumes that the sensing decision is perfect (although it is not in general) and designs the MAP detectors accordingly (Section 2.4).

2.3 Optimal Detector Randomization in the Presence of Channel Sensing Errors

Consider the secondary system as shown in Figure 2.1. Let $\mathbb{P}_{e,i}$ denote the average probability of error for the secondary receiver when the sensing decision is $\hat{\mathcal{H}}_i$, where $i \in \{0, 1\}$. Then, the proposed optimal detector randomization problem can be formulated under the constraints in (2.9) as follows:

$$\begin{aligned} \min_{f_{P_0}, f_{P_1}} \quad & \Pr\{\hat{\mathcal{H}}_0\}\mathbb{P}_{e,0} + \Pr\{\hat{\mathcal{H}}_1\}\mathbb{P}_{e,1} \\ \text{subject to} \quad & \mathbb{E}\{P_i\} \leq P_{\text{av},i}\,,\ P_i \leq P_{\text{pk},i} \ \text{ for } \ i \in \{0, 1\}\,. \end{aligned} \tag{2.12}$$

where $\Pr\{\hat{\mathcal{H}}_i\}$ is the probability that the sensing decision is $\hat{\mathcal{H}}_i$, and f_{P_i} denotes the PDF of the power parameter P_i for $i \in \{0, 1\}$. In other words, the aim is to obtain the optimal power distributions that minimize the average error probability of the secondary system under the power constraints.

Due to the structure of the optimization problem in (2.12), the optimal power distributions can be obtained separately for P_0 and P_1 as follows:

$$\min_{f_{P_i}} \mathbb{P}_{e,i} \ \text{ subject to } \ \mathbb{E}\{P_i\} \leq P_{\text{av},i}\,,\ P_i \leq P_{\text{pk},i} \tag{2.13}$$

for $i \in \{0, 1\}$. In order to obtain a solution of the optimization problem in (2.13), $\mathbb{P}_{e,i}$ is evaluated for optimal MAP detectors in the following proposition (cf. Remark 1).

Proposition 1: *Consider a scenario in which the sensing decision is $\hat{\mathcal{H}}_i$. Suppose that the secondary transmitter employs a power randomization strategy according to PDF f_{P_i}, and the secondary receiver employs the corresponding randomization of MAP detectors. Then, $\mathbb{P}_{e,i}$ in (2.13) can be expressed as*

$$\mathbb{P}_{e,i} = 1 - \mathbb{E}\{\phi_i(P_i)\} \tag{2.14}$$

with[2]

$$\phi_i(P_i) \triangleq \int \max_{l\in\{0,1,\ldots,M-1\}} \big\{ \Pr\{d_l\} \big(\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_i\} f(x|d_l,\hat{\mathcal{H}}_i,\mathcal{H}_0) + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\} f(x|d_l,\hat{\mathcal{H}}_i,\mathcal{H}_1) \big) \big\}\, dx \tag{2.15}$$

where $\Pr\{d_l\}$ *is the prior probability of information symbol* d_l*,* $\Pr\{\mathcal{H}_j|\hat{\mathcal{H}}_i\}$ *is the conditional probability of* $\mathcal{H}_j$ *when the sensing decision is* $\hat{\mathcal{H}}_i$*, and* $f(x|d_l,\hat{\mathcal{H}}_i,\mathcal{H}_j)$ *denotes the conditional PDF of observation* x *when information symbol* d_l *is sent, the sensing decision is* $\hat{\mathcal{H}}_i$ *and the true hypothesis is* $\mathcal{H}_j$ *.*

Proof: When the sensing decision is $\hat{\mathcal{H}}_i$, the following MAP decision rule is employed in order to estimate the information symbol for a given value of P_i :

$$\hat{d} = d_k \quad \text{where} \quad k = \arg\max_{l\in\{0,1,\ldots,M-1\}} \Pr\{d_l|x,\hat{\mathcal{H}}_i\}\,. \tag{2.16}$$

Then, the following manipulations can be performed to derive alternative expressions:

$$k = \arg\max_{l\in\{0,1,\ldots,M-1\}} \Pr\{d_l,\hat{\mathcal{H}}_i\} f(x|d_l,\hat{\mathcal{H}}_i) \tag{2.17}$$

$$= \arg\max_{l\in\{0,1,\ldots,M-1\}} \Pr\{d_l\} f(x|d_l,\hat{\mathcal{H}}_i) \tag{2.18}$$

$$= \arg\max_{l\in\{0,1,\ldots,M-1\}} \Pr\{d_l\} \big(\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_i\} f(x|d_l,\hat{\mathcal{H}}_i,\mathcal{H}_0) + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\} f(x|d_l,\hat{\mathcal{H}}_i,\mathcal{H}_1) \big) \tag{2.19}$$

where (2.17) is obtained from (2.16) based on Bayes' rule, (2.18) follows from the independence of d_l and $\hat{\mathcal{H}}_i$, and (2.19) is obtained by conditioning on the true hypotheses.

When the sensing decision is $\hat{\mathcal{H}}_i$, the average probability of error for a given

[2]The expectation in (2.14) is taken with respect to the PDF of P_i; i.e., f_{P_i}.

value of P_i can be expressed as follows:

$$\mathbb{P}_{e,i}(P_i) = 1 - \sum_{l=0}^{M-1} \Pr\{d_l\} \Pr\{\hat{d} = d_l | d_l, \hat{\mathcal{H}}_i\} \tag{2.20}$$

$$= 1 - \sum_{l=0}^{M-1} \Pr\{d_l\} \int_{\Gamma_{l,i}} f(x|d_l, \hat{\mathcal{H}}_i)\, dx \tag{2.21}$$

$$= 1 - \sum_{l=0}^{M-1} \int_{\Gamma_{l,i}} \Pr\{d_l\}\big(\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_i\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_0) + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_1)\big)\, dx \tag{2.22}$$

where $\Gamma_{l,i}$ denotes the decision region for symbol l of the MAP decision rule corresponding to sensing decision $\hat{\mathcal{H}}_i$. Based on (2.19), $\Gamma_{l,i}$ is specified as the set of x for which $\Pr\{d_l\}(\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_i\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_0) + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_1)) \geq \Pr\{d_m\}(\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_i\}\ f(x|d_m, \hat{\mathcal{H}}_i, \mathcal{H}_0) + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\} f(x|d_m, \hat{\mathcal{H}}_i, \mathcal{H}_1)),\ \forall m \neq l$. Therefore, (2.22) can be stated as

$$\mathbb{P}_{e,i}(P_i) = 1 - \int \max_{l \in \{0,1,\ldots,M-1\}} \big\{ \Pr\{d_l\}\big(\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_i\} \times f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_0) + \Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_i\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_1)\big)\big\}\, dx. \tag{2.23}$$

Since the expression in (2.23) is conditioned on a given value of P_i, the average probability of error for a power randomization strategy corresponding to PDF f_{P_i} can be expressed as the expectation of (2.23), which results in

$$\mathbb{P}_{e,i} = \int f_{P_i}(t)\, \mathbb{P}_{e,i}(t)\, dt = 1 - \mathbb{E}\{\phi_i(P_i)\} \tag{2.24}$$

where $\phi_i(P_i)$ is as defined in (2.15).[3] ■

Proposition 1 provides an explicit expression for the average probabilities of error under both sensing decisions when a generic power randomization strategy (denoted by f_{P_0} or f_{P_1}) and the corresponding MAP detectors are employed as shown in Figure 2.1. Based on the proposition (specifically, based on the expression in (2.14)), the optimal detector randomization problems in (2.13) can

[3]The dependence of $\phi_i(P_i)$ in (2.15) on the value of P_i is through the conditional PDFs $f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_0)$ and $f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_1)$ (please see (2.5)-(2.8)).

be formulated as

$$\max_{f_{P_i}} \mathbb{E}\{\phi_i(P_i)\} \text{ subject to } \mathbb{E}\{P_i\} \le P_{\text{av},i}\,,\ P_i \le P_{\text{pk},i} \tag{2.25}$$

for $i \in \{0, 1\}$.

Although it is challenging to obtain a closed-form solution for the optimal f_{P_i} in (2.25), the form of an optimal solution can be obtained based on the arguments similar to those in [21, 22, 23]. Specifically, when ϕ_i's are continuous functions and P_i's take values from finite closed intervals (i.e., $[0, P_{\text{pk},i}]$), it can be shown that an optimal solution to (2.25) lies at the boundary of the convex hull of set U, which is defined as the set of all possible $(P_i, \phi_i(P_i))$ pairs [23]. Therefore, from Carathéodory's theorem [24, 25], an optimal solution can be obtained as the convex combination of at most two different pairs from set U. Hence, an optimal solution to (2.25) can be expressed in the form of

$$f_{P_i}^{\text{opt}}(P_i) = \lambda_i\,\delta(P_i - P_{i,1}) + (1-\lambda_i)\,\delta(P_i - P_{i,2})\,, \tag{2.26}$$

for $i \in \{0, 1\}$, where $\lambda_i \in [0, 1]$, and $\delta(\cdot)$ denotes the Dirac delta function.

The form of the optimal solution in (2.26) implies that, for each sensing decision, the secondary transmitter should perform randomization between at most two different power levels and the secondary receiver needs to perform corresponding detector randomization between at most two different MAP detectors. Therefore, the secondary receiver illustrated in Figure 2.1 should implement *at most four* different MAP detectors considering the two possible sensing decisions, which are the absence ($\hat{\mathcal{H}}_0$) and presence ($\hat{\mathcal{H}}_1$) of primary users.

Based on the expression in (2.26), the solutions of the optimization problems in (2.25) can be obtained from the following formulation:

$$\begin{aligned} \max_{\lambda_i, P_{i,1}, P_{i,2}}\ & \lambda_i\,\phi_i(P_{i,1}) + (1-\lambda_i)\,\phi_i(P_{i,2}) \\ \text{subject to}\ & \lambda_i\,P_{i,1} + (1-\lambda_i)\,P_{i,2} \le P_{\text{av},i}\,,\ \ \lambda_i \in [0,1] \\ & P_{i,1} \in [0, P_{\text{pk},i}]\,,\ \ P_{i,2} \in [0, P_{\text{pk},i}] \end{aligned} \tag{2.27}$$

for $i \in \{0, 1\}$. Compared to (2.25), the problems in (2.27) are significantly easier to solve since they require a search over three scalar parameters instead of a search over all possible PDFs.

Since generic probability distributions are considered in the derivations, the formulation in (2.27) may result in non-concave problems in some cases depending on the probability distributions of the noise and the interference from primary users. Therefore, global optimization algorithms such as particle swarm optimization (PSO) and differential evolution (DE) can be used to obtain the solution [26, 27].

2.4 Detector Randomization Assuming Perfect Sensing Decisions

Now consider a scenario in which the secondary receiver assumes that the sensing decision is perfect, and designs the optimal MAP detectors according to the signal models in (2.5) and (2.8). In other words, the secondary receiver considers the sensing decision as the true hypothesis corresponding to the absence or presence of primary users although the sensing decision may not always be correct. Although this approach is suboptimal compared to the one in Section 2.3, it is studied in this section for two main reasons. First, the performance of this suboptimal approach will be compared to that of the optimal one in Section 2.3 in order to quantify the performance improvements due to the optimal approach (i.e., due to considering possible channel sensing errors). Second, since most approaches in the literature do not take into account possible errors in sensing decisions when designing secondary receivers (except for some recent studies such as [28]), it is important to derive the optimal MAP detectors and analyze their error performance under the assumption of perfect sensing decisions.

Consider the secondary system model in Figure 2.1, where the secondary transmitter randomizes the power levels according to PDF f_{P_i} under sensing decision $\hat{\mathcal{H}}_i$, and the secondary receiver performs corresponding randomization of

MAP detectors. The main difference of this scenario from the one in Section 2.3 is that the receiver assumes that the sensing decisions are perfect and designs the MAP detectors according to that assumption. For this scenario, let $\tilde{\mathbb{P}}_{\text{e},0}$ and $\tilde{\mathbb{P}}_{\text{e},1}$ denote the average probabilities of error at the secondary receiver when the sensing decision is $\hat{\mathcal{H}}_0$ and $\hat{\mathcal{H}}_1$, respectively. The aim is to find the optimal power distributions, f_{P_0} and f_{P_1}, that minimize the average probability of error, $\Pr\{\hat{\mathcal{H}}_0\}\tilde{\mathbb{P}}_{\text{e},0} + \Pr\{\hat{\mathcal{H}}_1\}\tilde{\mathbb{P}}_{\text{e},1}$, under the average and peak power constraints as in (2.12). Due to the structure of the problem, the optimal probability distributions can be obtained separately for P_0 and P_1 as follows:

$$\min_{f_{P_i}} \tilde{\mathbb{P}}_{\text{e},i} \text{ subject to } \mathbb{E}\{P_i\} \leq P_{\text{av},i}\,,\; P_i \leq P_{\text{pk},i} \tag{2.28}$$

for $i \in \{0, 1\}$. Then, the following proposition can be employed to provide an explicit formulation of $\tilde{\mathbb{P}}_{\text{e},i}$ in (2.28).

Proposition 2: *Consider a scenario in which the sensing decision is $\hat{\mathcal{H}}_i$. Suppose that the secondary transmitter employs a power randomization strategy according to PDF f_{P_i}, and the secondary receiver employs the corresponding randomization of MAP detectors assuming that the sensing decision is perfect. Then,* $\tilde{\mathbb{P}}_{\text{e},i}$ *in* (2.28) *can be expressed as*

$$\tilde{\mathbb{P}}_{\text{e},i} = 1 - \mathbb{E}\{\varphi_i(P_i)\} \tag{2.29}$$

with

$$\begin{aligned}\varphi_i(P_i) \triangleq \Pr\{\mathcal{H}_i|\hat{\mathcal{H}}_i\} &\int \max_{l\in\{0,1,\ldots,M-1\}} \Big\{\Pr\{d_l\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i)\Big\}\, dx \\ &+ \Pr\{\mathcal{H}_{1-i}|\hat{\mathcal{H}}_i\} \sum_{l=0}^{M-1} \Pr\{d_l\} \int_{\tilde{\Gamma}_{l,i}} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_{1-i})\, dx \end{aligned}\tag{2.30}$$

where $\tilde{\Gamma}_{l,i} = \{x \mid \Pr\{d_l\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i) \geq \Pr\{d_m\} f(x|d_m, \hat{\mathcal{H}}_i, \mathcal{H}_i)\ , \forall m \neq l\}$.

Proof: When the sensing decision is $\hat{\mathcal{H}}_i$ and the receiver assumes that this decision is perfect (i.e., correct), the following MAP decision rule is employed in order to estimate the information symbol for a given value of P_i :

$$\hat{d} = d_k \quad \text{where} \quad k = \underset{l \in \{0,1,\ldots,M-1\}}{\arg\max} \ \Pr\{d_l | x, \hat{\mathcal{H}}_i, \mathcal{H}_i\} \ . \tag{2.31}$$

Then, after some manipulation, the following expression can be obtained:

$$k = \underset{l \in \{0,1,\ldots,M-1\}}{\arg\max} \ \Pr\{d_l\} f(x | d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i) \ . \tag{2.32}$$

When the sensing decision is $\hat{\mathcal{H}}_i$ and the decision rule in (2.32) is employed, the average probability of error for a given value of P_i can be calculated as follows:

$$\tilde{\mathbb{P}}_{\mathrm{e},i}(P_i) = 1 - \sum_{l=0}^{M-1} \Pr\{d_l\} \Pr\{\hat{d} = d_l | d_l, \hat{\mathcal{H}}_i\} \tag{2.33}$$

$$= 1 - \sum_{l=0}^{M-1} \Pr\{d_l\} \Big(\Pr\{\mathcal{H}_i | \hat{\mathcal{H}}_i\} \Pr\{\hat{d} = d_l | d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i\} + \Pr\{\mathcal{H}_{1-i} | \hat{\mathcal{H}}_i\} \Pr\{\hat{d} = d_l | d_l, \hat{\mathcal{H}}_i, \mathcal{H}_{1-i}\} \Big) \tag{2.34}$$

$$= 1 - \sum_{l=0}^{M-1} \Pr\{d_l\} \Big(\Pr\{\mathcal{H}_i | \hat{\mathcal{H}}_i\} \int_{\tilde{\Gamma}_{l,i}} f(x | d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i) \, dx + \Pr\{\mathcal{H}_{1-i} | \hat{\mathcal{H}}_i\} \int_{\tilde{\Gamma}_{l,i}} f(x | d_l, \hat{\mathcal{H}}_i, \mathcal{H}_{1-i}) \, dx \Big) \tag{2.35}$$

where $\tilde{\Gamma}_{l,i}$ is defined as $\tilde{\Gamma}_{l,i} = \{x \mid \Pr\{d_l\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i) \geq \Pr\{d_m\} f(x|d_m, \hat{\mathcal{H}}_i, \mathcal{H}_i) \ , \forall m \neq l\}$ due to the decision rule in (2.32).

After some manipulation, (2.35) becomes

$$\tilde{\mathbb{P}}_{\mathrm{e},i}(P_i) = 1 - \Pr\{\mathcal{H}_i | \hat{\mathcal{H}}_i\} \sum_{l=0}^{M-1} \int_{\tilde{\Gamma}_{l,i}} \Pr\{d_l\} f(x | d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i) \, dx - \Pr\{\mathcal{H}_{1-i} | \hat{\mathcal{H}}_i\} \sum_{l=0}^{M-1} \Pr\{d_l\} \int_{\tilde{\Gamma}_{l,i}} f(x | d_l, \hat{\mathcal{H}}_i, \mathcal{H}_{1-i}) \, dx \ . \tag{2.36}$$

Due to the definition of $\tilde{\Gamma}_{l,i}$, the term $\sum_{l=0}^{M-1} \int_{\tilde{\Gamma}_{l,i}} \Pr\{d_l\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i) \, dx$ in (2.36) can be expressed as $\int \max_{l\in\{0,1,\ldots,M-1\}} \{\Pr\{d_l\} f(x|d_l, \hat{\mathcal{H}}_i, \mathcal{H}_i)\} dx$. Hence, $\tilde{\mathbb{P}}_{\mathrm{e},i}(P_i)$ becomes equal to $1-\varphi_i(P_i)$, where $\varphi_i(P_i)$ is as defined in (2.30). Since the expression in (2.36) is conditioned on a given value of P_i, the average probability of error for a power randomization strategy corresponding to PDF f_{P_i} can be calculated as the expectation of (2.36), which results in $\tilde{\mathbb{P}}_{\mathrm{e},i} = 1 - \mathbb{E}\{\varphi_i(P_i)\}$, as claimed in the proposition. ■

From (2.29) in Proposition 2, the optimization problems in (2.28) can be expressed as

$$\max_{f_{P_i}} \mathbb{E}\{\varphi_i(P_i)\} \text{ subject to } \mathbb{E}\{P_i\} \leq P_{\mathrm{av},i}\,,\ P_i \leq P_{\mathrm{pk},i} \tag{2.37}$$

for $i \in \{0,1\}$. Since (2.37) is in the same form as (2.25), its solution can also be expressed as in (2.26) based on similar arguments to those in Section 2.3. Therefore, the optimal solutions of (2.37) can be obtained from the following formulation:

$$\begin{aligned} \max_{\lambda_i, P_{i,1}, P_{i,2}} \ & \lambda_i\, \varphi_i(P_{i,1}) + (1-\lambda_i)\varphi_i(P_{i,2}) \\ \text{subject to } & \lambda_i\, P_{i,1} + (1-\lambda_i)P_{i,2} \leq P_{\mathrm{av},i}\,,\ \lambda_i \in [0,1] \\ & P_{i,1} \in [0, P_{\mathrm{pk},i}]\,,\ P_{i,2} \in [0, P_{\mathrm{pk},i}] \end{aligned} \tag{2.38}$$

for $i \in \{0,1\}$.

2.5 Performance Evaluation

In order to investigate the error performance of the optimal and suboptimal detector randomization approaches in the previous sections, consider a scenario in which noise n in (2.7) and (2.8) is modeled as zero-mean, circularly symmetric, complex Gaussian noise, and the sum of primary signal and noise, $s+n$, in (2.5) and (2.6) is modeled as a mixture of complex Gaussian components each with independent real and imaginary parts having equal variances. That is, the PDFs of n and $s+n \triangleq \varepsilon$ are expressed, respectively, as

$$p_n(x) = \frac{1}{\pi\sigma_n^2}\exp\left(-\frac{|x|^2}{\sigma_n^2}\right), \tag{2.39}$$

$$p_\varepsilon(x) = \sum_{j=1}^{N_m}\frac{\nu_j}{\pi\sigma_j^2}\exp\left(-\frac{|x-\mu_j|^2}{\sigma_j^2}\right). \tag{2.40}$$

where σ_n^2 is the variance of noise n, N_m is the number of Gaussian components in the mixture ε, μ_j and σ_j^2 are, respectively, the mean and the variance of the jth component in the mixture, and $\sum_{j=1}^{N_m}\nu_j = 1$ with $\nu_j \geq 0$, $\forall j$.

The main motivation for employing the Gaussian mixture model in (2.40) is that the sum of noise and interference from primary users can accurately be modeled by a non-Gaussian random variable as discussed in [29]-[33]. In addition, the Gaussian mixture model in (2.40) is quite generic since it can model various probability density functions by a suitable selection of its parameters. Specifically, as the number of components, N_m, increases, it can approximate any probability density function as accurately as desired [34].

Based on (2.39) and (2.40), the conditional PDFs in Proposition 1 and Proposition 2 (please see (2.15) and (2.30)) can be expressed as follows:

$$f(x|d_l,\hat{\mathcal{H}}_i,\mathcal{H}_0) = \frac{1}{\pi\sigma_n^2}\exp\left(-\frac{\left|x-h\sqrt{P_i}\,d_l\right|^2}{\sigma_n^2}\right) \tag{2.41}$$

$$f(x|d_l,\hat{\mathcal{H}}_i,\mathcal{H}_1) = \sum_{j=1}^{N_m}\frac{\nu_j}{\pi\sigma_j^2}\exp\left(-\frac{|x-h\sqrt{P_i}\,d_l-\mu_j|^2}{\sigma_j^2}\right) \tag{2.42}$$

for $i \in \{0,1\}$.

For the simulations, the receiver is assumed to have perfect channel state information (CSI), and h in (2.5)-(2.8) is set to 1 without loss of generality. In addition, $\Pr\{\mathcal{H}_0\} = 0.75$, $\Pr\{\mathcal{H}_1\} = 0.25$, $\Pr\{\hat{\mathcal{H}}_1|\mathcal{H}_1\} = 0.6$, and $\Pr\{\hat{\mathcal{H}}_0|\mathcal{H}_0\} = 0.8$ are employed. From these parameters, $\Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_1\}$ and $\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_0\}$ can be obtained via Bayes' rule as $\Pr\{\mathcal{H}_1|\hat{\mathcal{H}}_1\} = 0.5$ and $\Pr\{\mathcal{H}_0|\hat{\mathcal{H}}_0\} = 0.8571$.

In order to quantify the improvements obtained via detector randomization, systems that do not employ any detector randomization are considered as well. Similar to the cases in Section 2.3 and Section 2.4, the following two scenarios are investigated in the simulations:

Optimal Single Detector in the Presence of Channel Sensing Errors: In this case, no detector randomization is employed, and the optimal MAP detector is obtained by taking the channel sensing errors into account. Since this scenario is a special case of the one in Section 2.3 when there is only a single detector, the optimal power values can be obtained as (cf. (2.25))

$$\max_{P_i} \phi_i(P_i) \text{ subject to } P_i \leq \min\{P_{\mathrm{av},i}, P_{\mathrm{pk},i}\} \tag{2.43}$$

for $i \in \{0, 1\}$, and the resulting conditional probabilities of error can be calculated from $1 - \phi_i(P_i^*)$ (cf. (2.14)), where P_i^* denotes the maximizer of (2.43).[4]

Single Detector Assuming Perfect Sensing Decisions: In this case, no detector randomization is employed, and the MAP detector is obtained by assuming that the channel sensing decision is correct. Since this scenario is a special case of the one in Section 2.4 when there is only a single detector, the optimal power values can be obtained as (cf. (2.28)-(2.29))

$$\max_{P_i} \varphi_i(P_i) \text{ subject to } P_i \leq \min\{P_{\mathrm{av},i}, P_{\mathrm{pk},i}\} \tag{2.44}$$

for $i \in \{0, 1\}$, and the resulting conditional probabilities of error can be calculated from $1 - \varphi_i(P_i^\star)$ (cf. (2.29)), where $P_i^\star$ denotes the maximizer of (2.44).[5] (This approached is called *suboptimal single detector* in the following.)

First, consider binary phase-shift keying (BPSK), where $d \in \{-1, 1\}$ with equal priors, and assume that the power levels are limited by the peak power constraint which is set as $P_{\mathrm{pk},i} = 3$ for $i \in \{0, 1\}$. In Figure 2.2, the average probabilities of error are plotted versus $1/\sigma^2$ for the four approaches described above, where $\sigma^2 = \sigma_n^2 = \sigma_j^2 \ \forall j$ in (2.39) and (2.40), and the parameters of the complex Gaussian mixture in (2.40) are given by $N_m = 3$, $\boldsymbol{\mu} = [\mu_1 \ \mu_2 \ \mu_3] =$

[4]For practical cases, $\min\{P_{\mathrm{av},i}, P_{\mathrm{pk},i}\} = P_{\mathrm{av},i}$ in (2.43).
[5]For practical cases, $\min\{P_{\mathrm{av},i}, P_{\mathrm{pk},i}\} = P_{\mathrm{av},i}$ in (2.44).

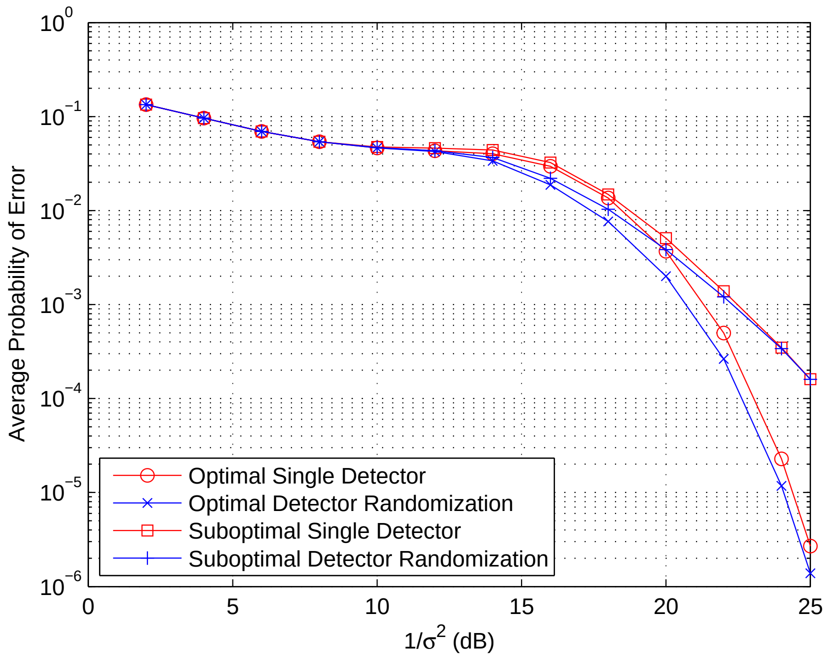

Figure 2.2: Average probability of error versus $1/\sigma^2$ for different approaches when $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

$[-1\ 0\ 1]$, and $\boldsymbol{\nu} = [\nu_1\ \nu_2\ \nu_3] = [0.25\ 0.5\ 0.25]$. Also, the average power limits $P_{\text{av},0}$ and $P_{\text{av},1}$ in (2.9) are set to $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$. From Figure 2.2, it is observed that the proposed optimal detector randomization approach achieves the lowest average probabilities of error among all the approaches for reasonably low values of σ^2 (namely, when $1/\sigma^2$ is larger than 10 dB), which correspond to practical error rates. Also, it is concluded that it can be crucial to take possible sensing errors into account when designing the detector. Specifically, the average probabilities of error are significantly larger for the suboptimal approaches, which assume that the sensing decision is perfect.

In Table 2.1, the solutions of the optimal single detector and optimal detector randomization approaches are presented for the scenario in Figure 2.2. The solution of the optimal single detector approach, which is obtained from (2.43), is denoted by P_0^* and P_1^*, which correspond to the optimal power levels employed when the sensing decision is $\hat{\mathcal{H}}_0$ and $\hat{\mathcal{H}}_1$, respectively. On the other hand, the solution of the optimal detector randomization approach, calculated from (2.27),

Table 2.1: Solutions of optimal single detector and optimal detector randomization approaches for the scenario in Figure 2.2.

$1/\sigma^2$	Single Detector		Detector Randomization					
(dB)	P_0^*	P_1^*	λ_0^*	$P_{0,1}^*$	$P_{0,2}^*$	λ_1^*	$P_{1,1}^*$	$P_{1,2}^*$
2	1.300	0.400	1	1.300	N/A	1	0.400	N/A
4	1.300	0.400	1	1.300	N/A	1	0.400	N/A
6	1.300	0.400	1	1.300	N/A	1	0.400	N/A
8	1.300	0.400	1	1.300	N/A	1	0.400	N/A
10	1.300	0.400	1	1.300	N/A	1	0.400	N/A
12	1.300	0.400	1	1.300	N/A	0.691	0.296	0.633
14	1.300	0.400	0.223	0.765	1.453	0.397	0.096	0.600
16	1.300	0.082	0.320	0.704	1.581	0.399	0.078	0.614
18	0.667	0.073	0.337	0.659	1.627	0.388	0.071	0.608
20	0.629	0.068	0.333	0.626	1.637	0.373	0.068	0.598
22	0.605	0.066	0.323	0.604	1.632	0.359	0.066	0.587
24	0.590	0.065	0.312	0.589	1.622	0.348	0.065	0.579
25	0.584	0.064	0.306	0.584	1.616	0.344	0.064	0.576

is expressed by λ_i^*, $P_{i,1}^*$, and $P_{i,2}^*$ for $i \in \{0, 1\}$ (please see (2.26)). That is, when the sensing decision is $\hat{\mathcal{H}}_i$, the optimal detector randomization approach employs power levels $P_{i,1}^*$ and $P_{i,2}^*$ for λ_i^* and $(1 - \lambda_i^*)$ fractions of time, respectively, with the corresponding MAP detectors. From the table, it is observed that the two approaches result in the same solution for large σ values whereas randomization between two different power levels and two MAP detectors becomes the optimal solution for small values of σ.

Similarly, in Table 2.2, the solutions of the suboptimal single detector and suboptimal detector randomization approaches are presented for the scenario in Figure 2.2. $P_0^\star$ and $P_1^\star$ are the solution of the suboptimal single detector approach obtained from (2.44). Also, $\lambda_i^\star$, $P_{i,1}^\star$, and $P_{i,2}^\star$ for $i \in \{0, 1\}$ represent the solution of the suboptimal detector randomization approach obtained from (2.38) (please see (2.26)). It is observed from the table that the solution of both approaches are the same as the case in which no primary activity is detected by the secondary user (i.e., the channel sensing decision is $\hat{\mathcal{H}}_0$). However, for the channel sensing decision that primary user activity exists, the solution differs for small values of σ.

Table 2.2: Solutions of suboptimal single detector and suboptimal detector randomization approaches for the scenario in Figure 2.2.

$1/\sigma^2$	Single Detector		Detector Randomization					
(dB)	$P_0^\star$	$P_1^\star$	$\lambda_0^\star$	$P_{0,1}^\star$	$P_{0,2}^\star$	$\lambda_1^\star$	$P_{1,1}^\star$	$P_{1,2}^\star$
2	1.300	0.400	1	1.300	N/A	1	0.400	N/A
4	1.300	0.400	1	1.300	N/A	1	0.400	N/A
6	1.300	0.400	1	1.300	N/A	1	0.400	N/A
8	1.300	0.400	1	1.300	N/A	1	0.400	N/A
10	1.300	0.380	1	1.300	N/A	0.967	0.381	0.969
12	1.300	0.312	1	1.300	N/A	0.800	0.312	0.751
14	1.300	0.283	1	1.300	N/A	0.557	0.212	0.637
16	1.300	0.074	1	1.300	N/A	0.408	0.073	0.625
18	1.300	0.069	1	1.300	N/A	0.397	0.068	0.619
20	1.300	0.066	1	1.300	N/A	0.378	0.066	0.604
22	1.300	0.065	1	1.300	N/A	0.363	0.066	0.591
24	1.300	0.064	1	1.300	N/A	0.350	0.064	0.581
25	1.300	0.064	1	1.300	N/A	0.346	0.064	0.578

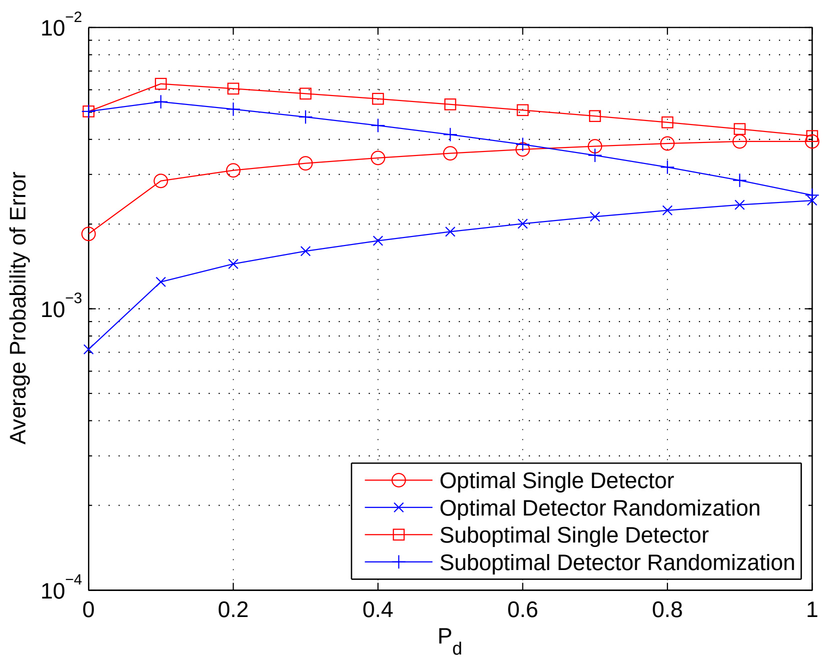

Figure 2.3: Average probability of error versus P_d for different approaches when $\sigma = 0.1$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

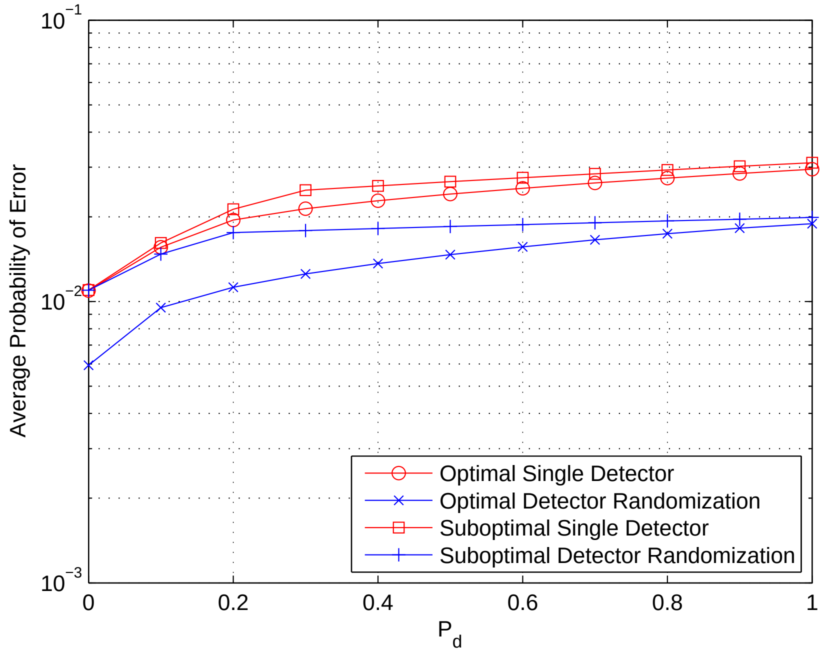

Figure 2.4: Average probability of error versus P_d for different approaches when $\sigma = 0.15$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

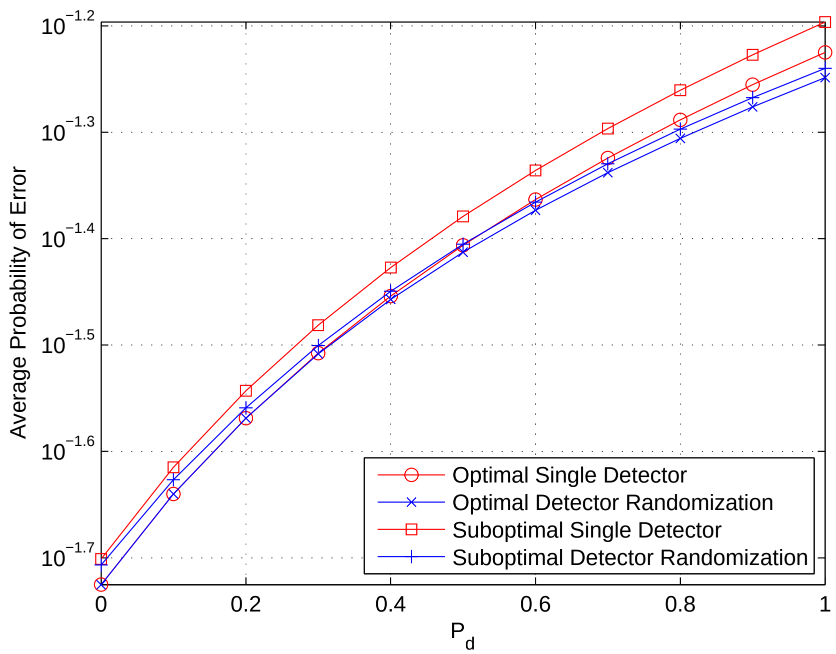

Figure 2.5: Average probability of error versus P_d for different approaches when $\sigma = 0.25$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

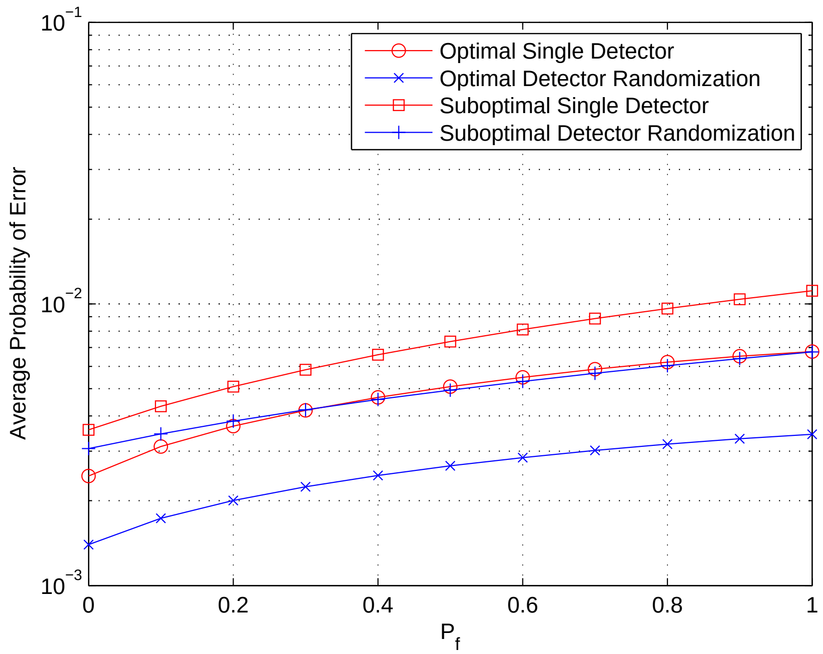

Figure 2.6: Average probability of error versus P_f for different approaches when $\sigma = 0.1$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

In Figures 2.3, 2.4, and 2.5, the average probabilities of error are plotted versus P_d for $\sigma = 0.1, 0.15$ and 0.25, respectively, where P_d is the probability of detection of active primary user (i.e., $P_d = \Pr\{\hat{\mathcal{H}}_1|\mathcal{H}_1\}$). Also, the probability of false alarm, P_f is set to 0.2 (i.e., $P_f = \Pr\{\hat{\mathcal{H}}_1|\mathcal{H}_0\}$).

Figures 2.6, 2.7, and 2.8 illustrate the average probabilities of error versus P_f for $\sigma = 0.1, 0.15$ and 0.25 where $P_d = 0.6$.

In order to investigate the effects of the mean values of the Gaussian components in (2.40) on the average probability of error performance of optimal and suboptimal detector randomization approaches, Figure 2.9, 2.10, and 2.11 are presented for $\sigma = 0.1, 0.15$ and 0.25 where $\mu = [\mu_1\ \mu_2\ \mu_3] = [-\mu'\ 0\ \mu']$.

It is evident from Figures 2.3-2.11 that the optimal detector randomization approach that takes possible channel sensing errors into account achieves the best probability of error performance among all the approaches. Also, the suboptimal approaches has worse error performance than the optimal approaches as expected.

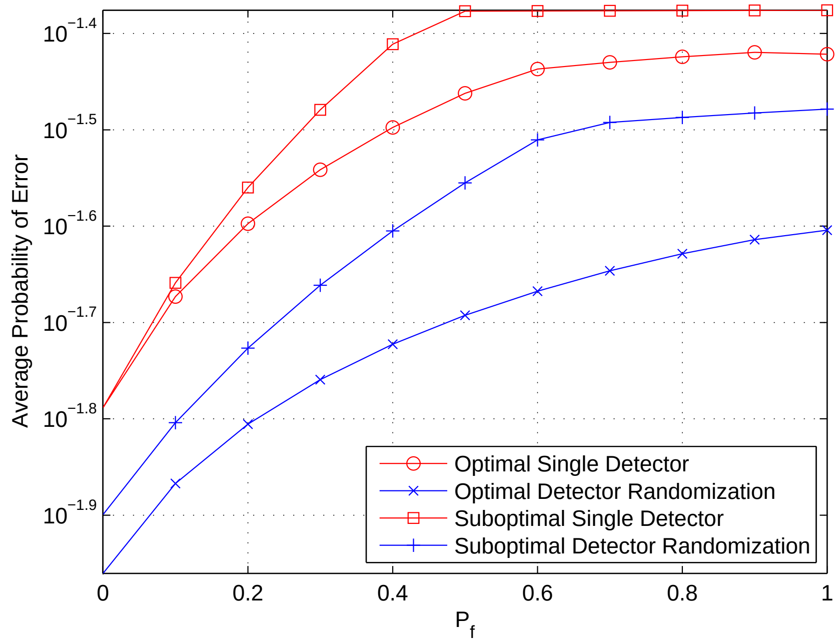

Figure 2.7: Average probability of error versus P_f for different approaches when $\sigma = 0.15$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

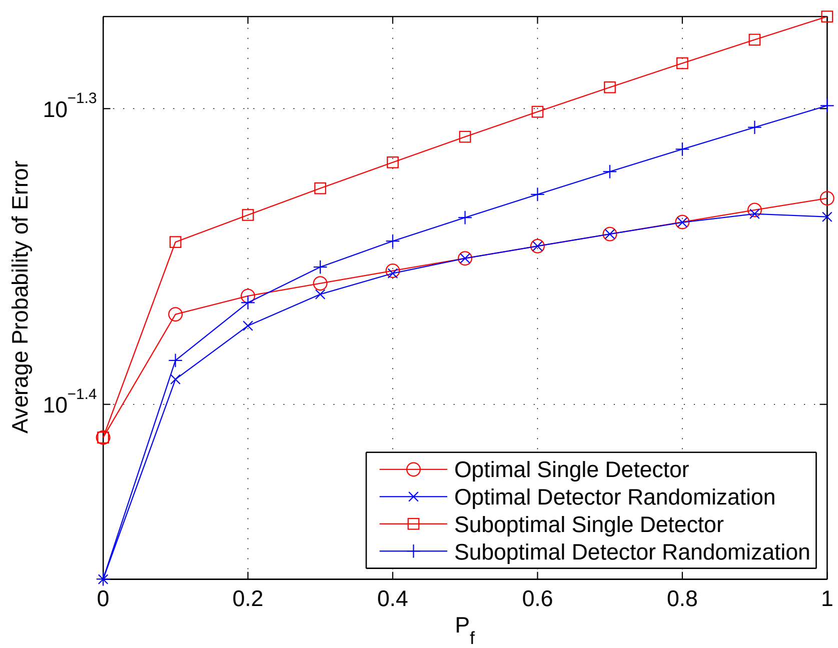

Figure 2.8: Average probability of error versus P_f for different approaches when $\sigma = 0.25$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

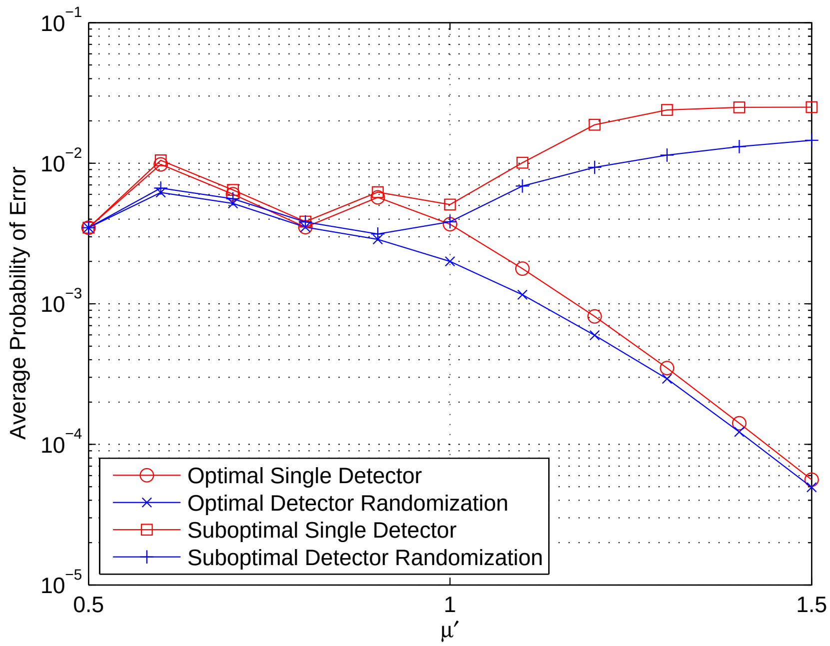

Figure 2.9: Average probability of error versus $\mu = [-\mu' \ 0 \ \mu']$ for different approaches when $\sigma = 0.1$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

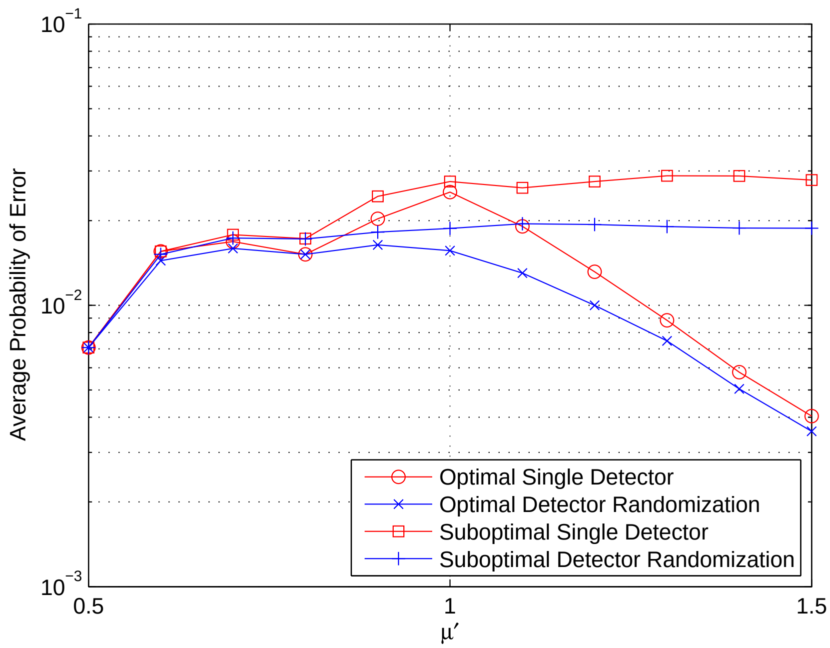

Figure 2.10: Average probability of error versus $\mu = [-\mu' \ 0 \ \mu']$ for different approaches when $\sigma = 0.15$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

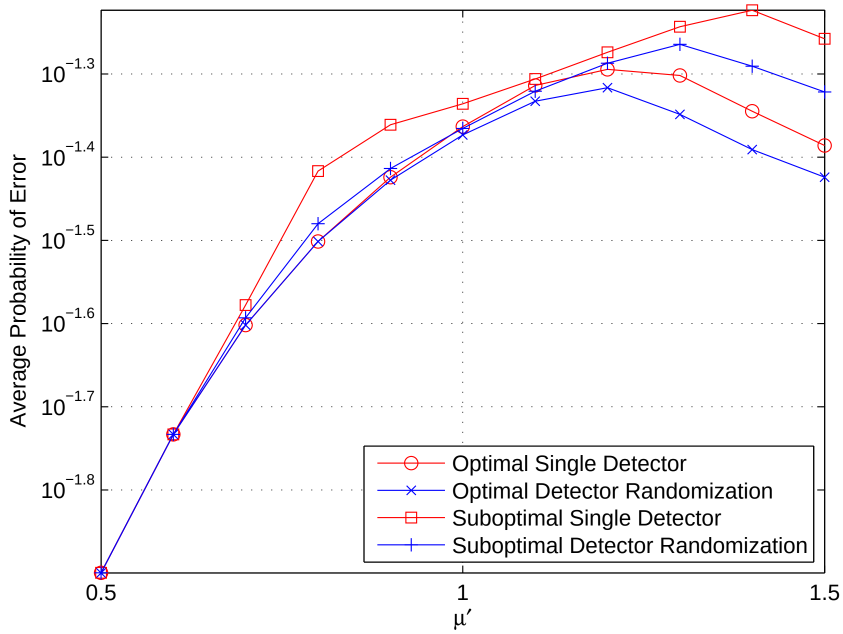

Figure 2.11: Average probability of error versus $\mu = [-\mu' \ 0 \ \mu']$ for different approaches when $\sigma = 0.25$, $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

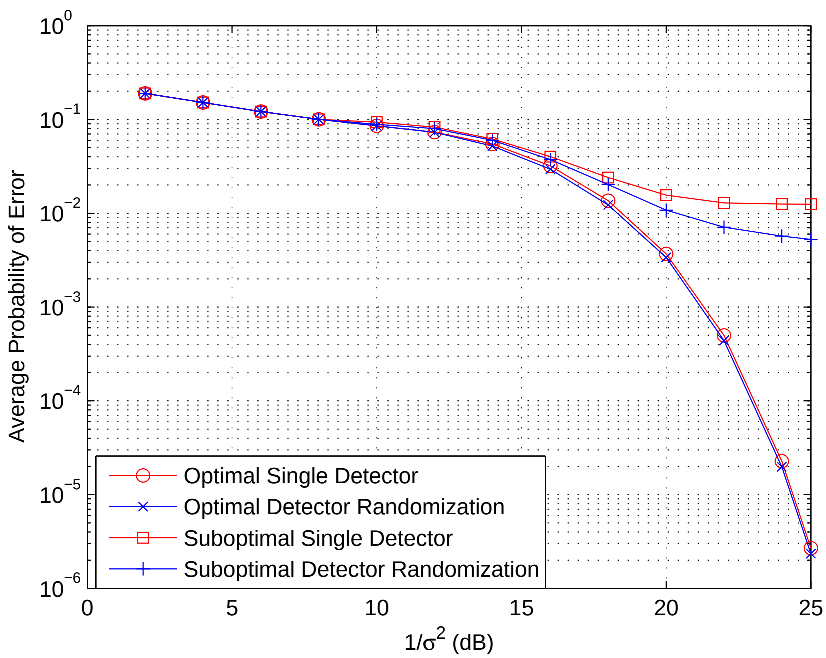

Figure 2.12: Average probability of error versus $1/\sigma^2$ for different approaches when $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

Table 2.3: Solutions of optimal single detector and optimal detector randomization approaches for the scenario in Figure 2.12.

$1/\sigma^2$	Single Detector		Detector Randomization					
(dB)	P_0^*	P_1^*	λ_0^*	$P_{0,1}^*$	$P_{0,2}^*$	λ_1^*	$P_{1,1}^*$	$P_{1,2}^*$
2	1.000	0.100	1	1.000	N/A	1	0.100	N/A
4	1.000	0.100	1	1.000	N/A	1	0.100	N/A
6	1.000	0.100	1	1.000	N/A	1	0.100	N/A
8	1.000	0.100	1	1.000	N/A	1	0.100	N/A
10	1.000	0.100	1	1.000	N/A	1	0.100	N/A
12	1.000	0.100	0.457	0.854	1.122	0.559	0.092	0.110
14	0.843	0.100	0.658	0.765	1.453	0.993	0.096	0.600
16	0.728	0.082	0.663	0.704	1.581	0.959	0.078	0.614
18	0.667	0.073	0.647	0.659	1.627	0.947	0.071	0.608
20	0.629	0.068	0.630	0.626	1.637	0.028	0.068	0.068
22	0.605	0.066	0.615	0.604	1.632	0.934	0.066	0.587
24	0.590	0.065	0.602	0.589	1.622	0.931	0.065	0.579
25	0.584	0.064	0.597	0.584	1.616	0.930	0.064	0.576

Table 2.4: Solutions of suboptimal single detector and suboptimal detector randomization approaches for the scenario in Figure 2.12.

$1/\sigma^2$	Single Detector		Detector Randomization					
(dB)	$P_0^\star$	$P_1^\star$	$\lambda_0^\star$	$P_{0,1}^\star$	$P_{0,2}^\star$	$\lambda_1^\star$	$P_{1,1}^\star$	$P_{1,2}^\star$
2	1.000	0.100	1	1.000	N/A	1	0.100	N/A
4	1.000	0.100	1	1.000	N/A	1	0.100	N/A
6	1.000	0.100	1	1.000	N/A	1	0.100	N/A
8	1.000	0.100	1	1.000	N/A	1	0.100	N/A
10	1.000	0.100	1	1.000	N/A	0.173	0.033	0.114
12	1.000	0.100	1	1.000	N/A	0.705	0.066	0.181
14	1.000	0.090	0.222	0.339	1.189	0.828	0.077	0.212
16	1.000	0.074	0.266	0.225	1.280	0.952	0.073	0.631
18	1.000	0.069	0.256	0.155	1.291	0.942	0.068	0.619
20	1.000	0.066	0.233	0.106	1.271	0.936	0.066	0.604
22	1.000	0.065	0.207	0.072	1.242	0.935	0.066	0.591
24	1.000	0.064	0.181	0.049	1.210	0.930	0.064	0.581
25	1.000	0.064	0.168	0.040	1.195	0.929	0.064	0.578

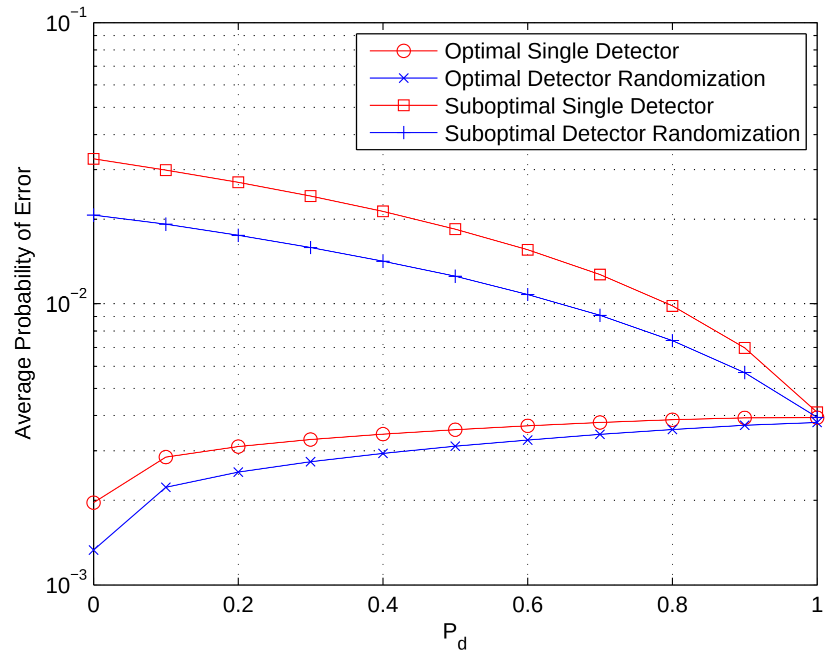

Figure 2.13: Average probability of error versus P_d for different approaches when $\sigma = 0.1$, $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

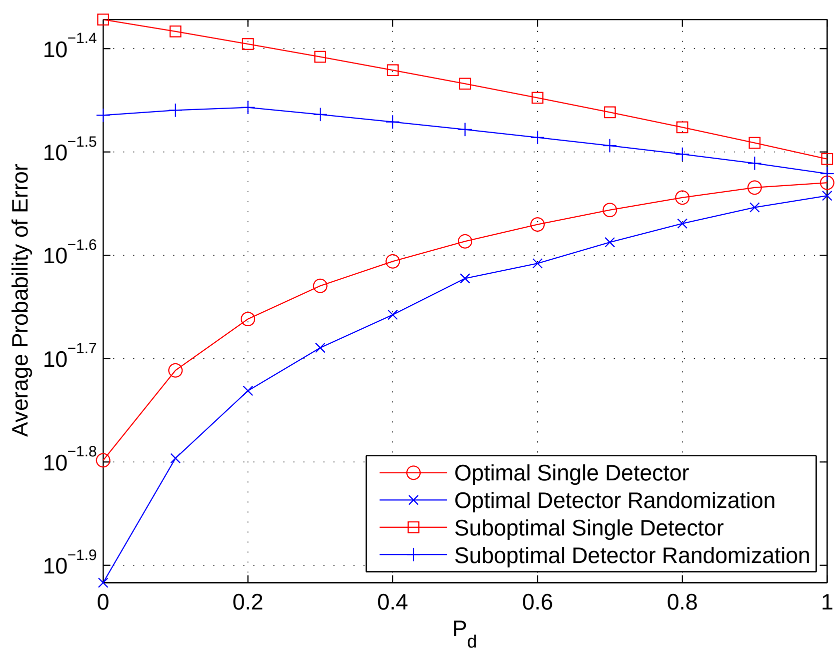

Figure 2.14: Average probability of error versus P_d for different approaches when $\sigma = 0.15$, $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

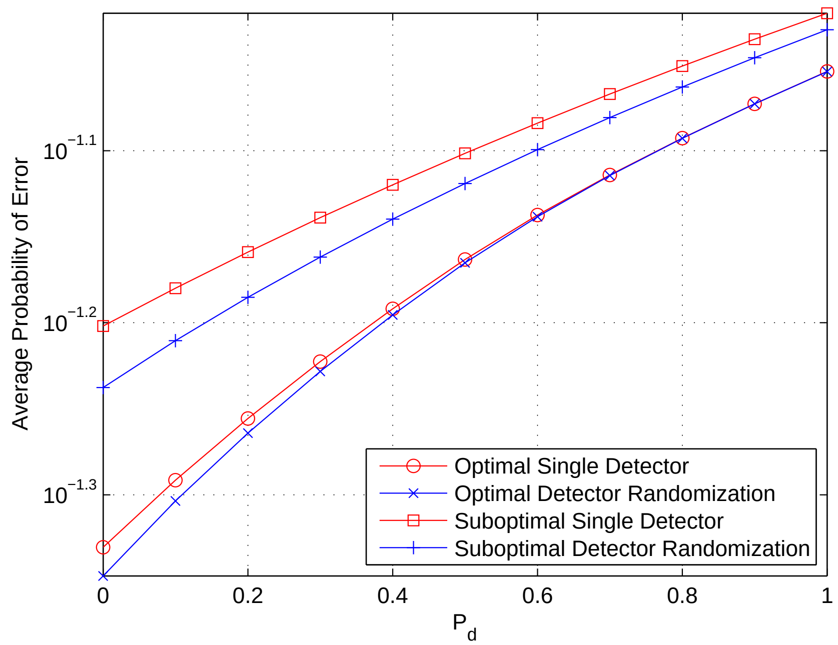

Figure 2.15: Average probability of error versus P_d for different approaches when $\sigma = 0.25$, $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

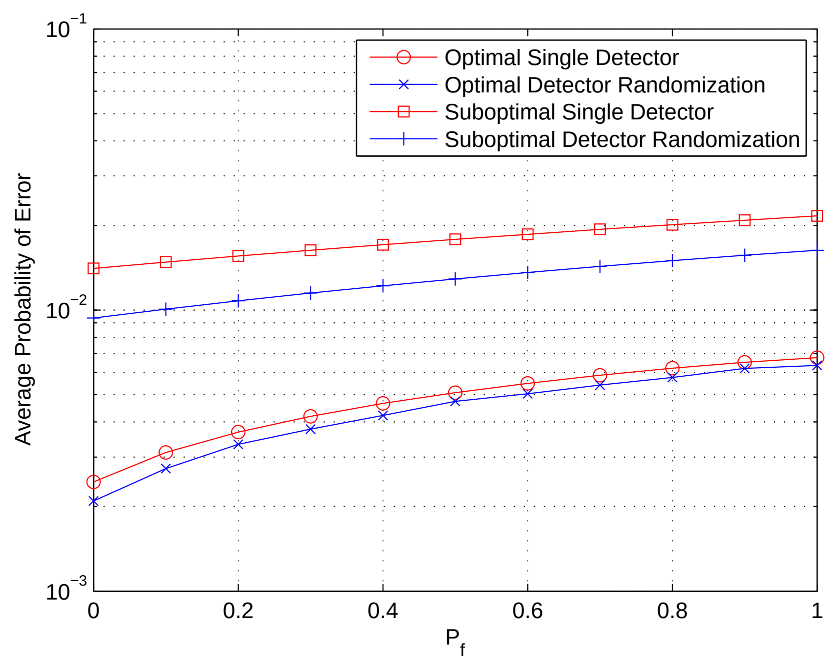

Figure 2.16: Average probability of error versus P_f for different approaches when $\sigma = 0.1$, $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

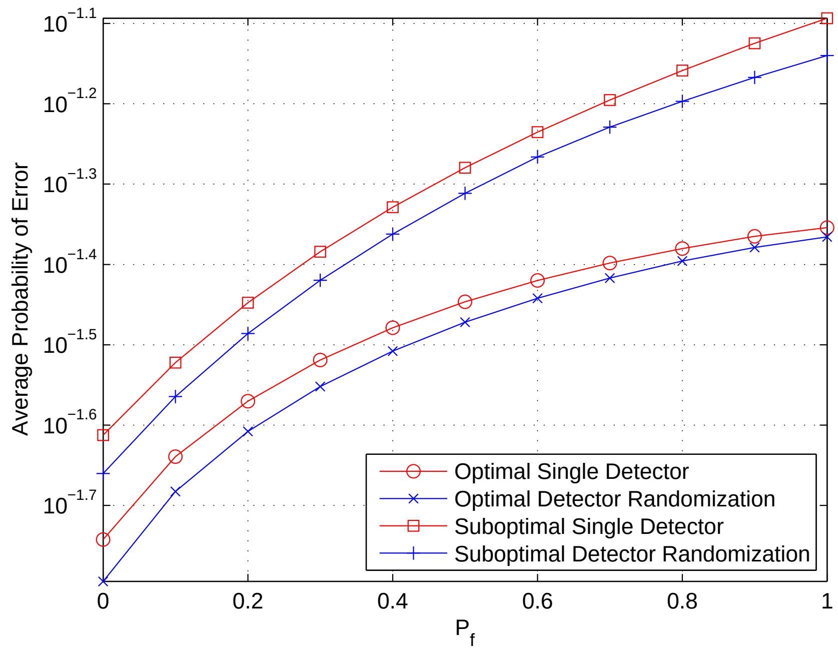

Figure 2.17: Average probability of error versus P_f for different approaches when $\sigma = 0.15$, $P_{\mathrm{av},0} = 1.0$ and $P_{\mathrm{av},1} = 0.1$.

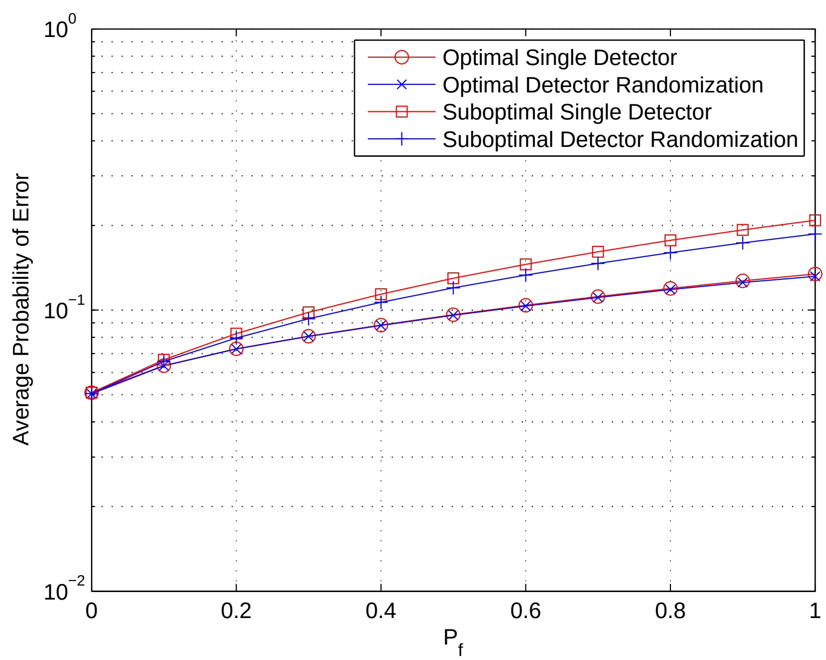

Figure 2.18: Average probability of error versus P_f for different approaches when $\sigma = 0.25$, $P_{\mathrm{av},0} = 1.0$ and $P_{\mathrm{av},1} = 0.1$.

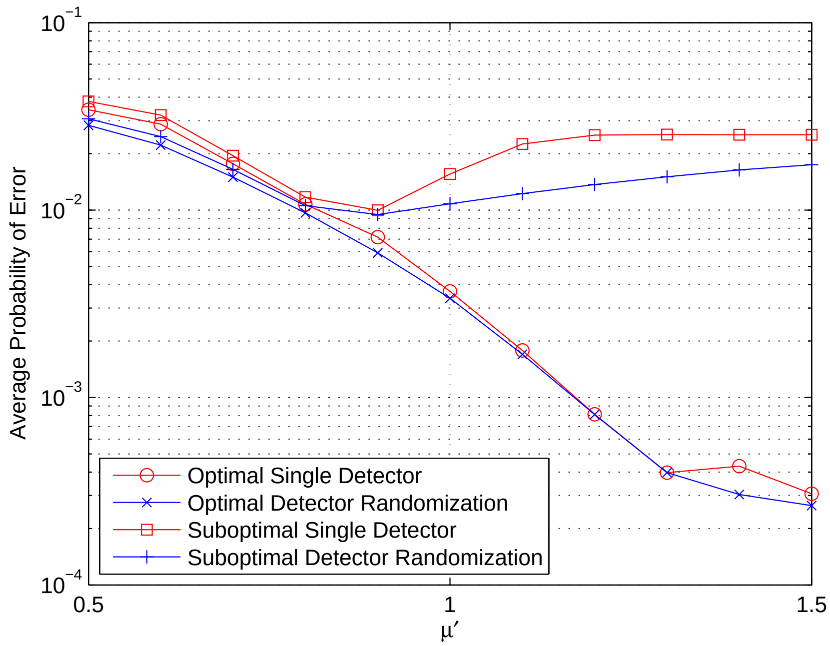

Figure 2.19: Average probability of error versus $\mu = [-\mu' \ 0 \ \mu']$ for different approaches when $\sigma = 0.1$, $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

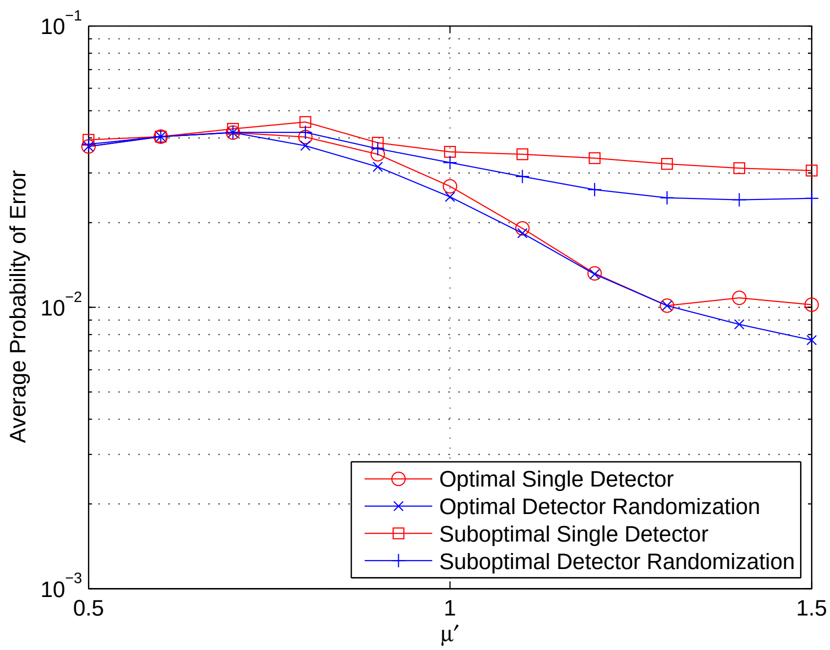

Figure 2.20: Average probability of error versus $\mu = [-\mu' \ 0 \ \mu']$ for different approaches when $\sigma = 0.15$, $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

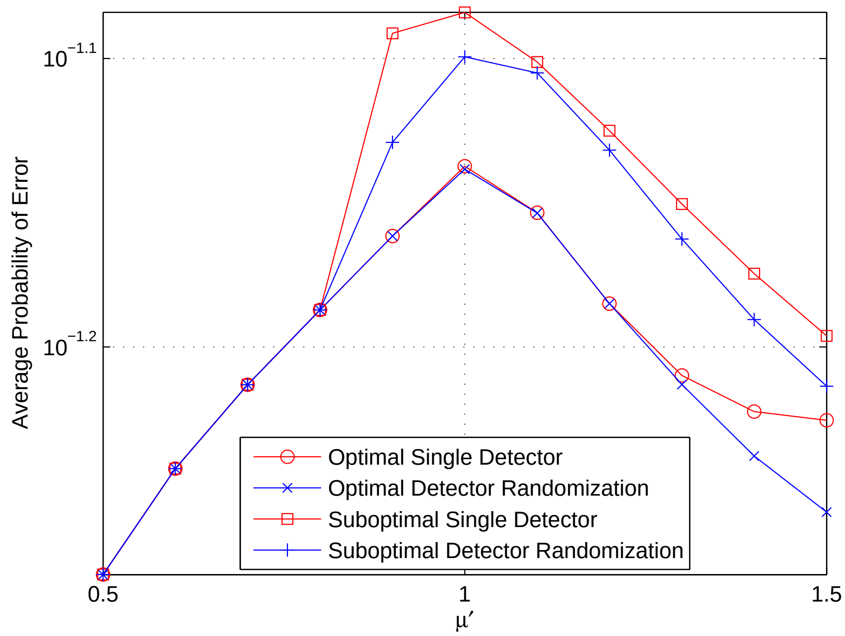

Figure 2.21: Average probability of error versus $\mu = [-\mu' \ 0 \ \mu']$ for different approaches when $\sigma = 0.25$, $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

Another example is obtained to indicate that the optimal detector randomization approach does not provide significant error performance improvements over the single detector approach in some scenarios. The same parameters as in Figure 2.2 are employed in Figure 2.12 except for $P_{\text{av},0}$ and $P_{\text{av},1}$. In this case, $P_{\text{av},0}$ and $P_{\text{av},1}$ are set to 1.0 and 0.1, respectively. It is observed from Figure 2.12 that optimal detector randomization results in slight performance improvement over the optimal single detector approach even though it achieves the lowest average probabilities of error among all the approaches. Similar to those in Figures 2.3-2.11, Figures 2.13-2.21 are presented for this example. Also, Table 2.3 and Table 2.4 present solutions of the optimal and suboptimal detector design approaches, respectively, for the scenario in Figure 2.12.

Next, consider quadrature phase-shift keying (QPSK), where $d \in \left\{\frac{-1-j}{\sqrt{2}}, \frac{-1+j}{\sqrt{2}}, \frac{1-j}{\sqrt{2}}, \frac{1+j}{\sqrt{2}}\right\}$ with equal prior probabilities. Figures 2.22, 2.23, and 2.24 show the average probability of error versus $1/\sigma^2$ where $P_{\text{av},0} = 1.3, 1.0, 1.0$ and $P_{\text{av},1} = 0.4, 0.1, 0.5$, respectively. Also, the parameters of the complex Gaussian

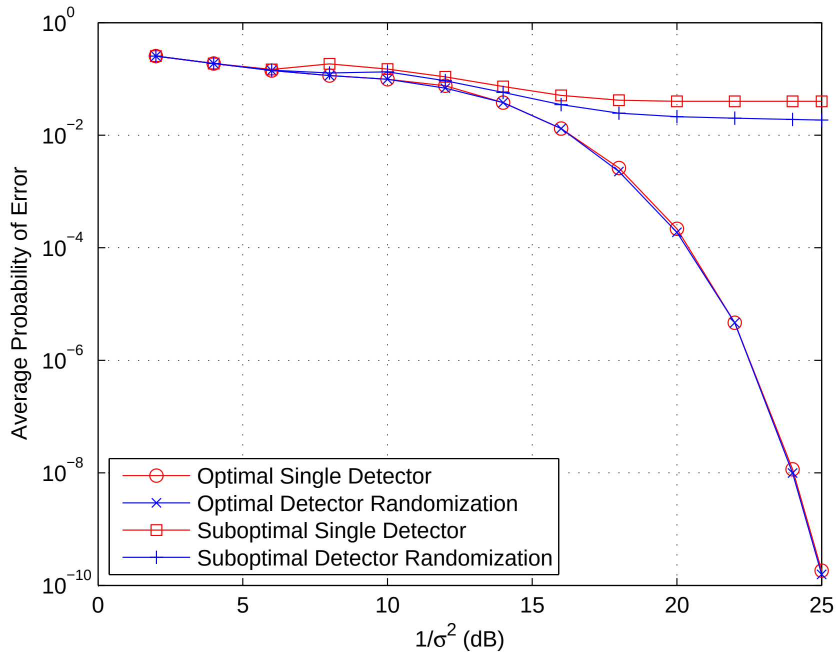

Figure 2.22: Average probability of error versus $1/\sigma^2$ for different approaches when $P_{\text{av},0} = 1.3$ and $P_{\text{av},1} = 0.4$.

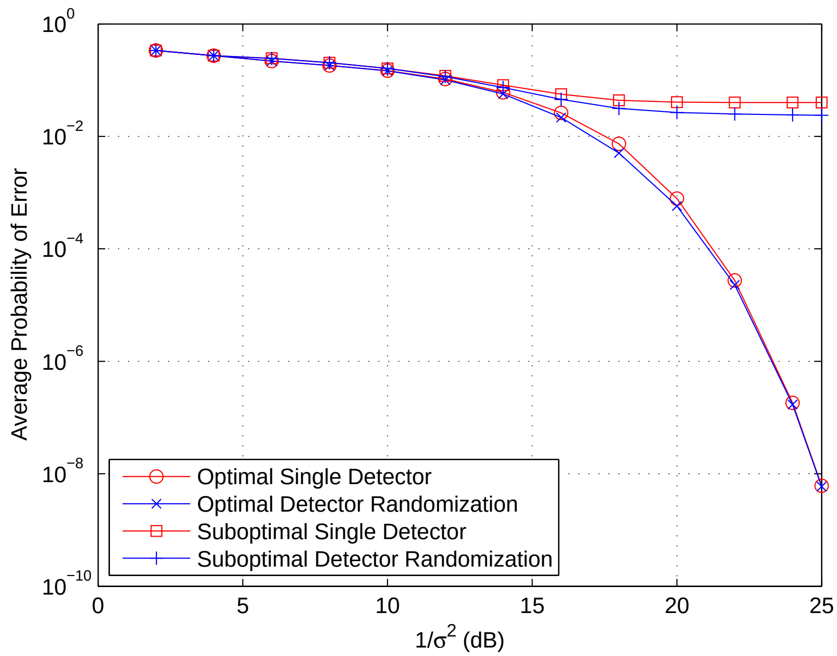

Figure 2.23: Average probability of error versus $1/\sigma^2$ for different approaches when $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.1$.

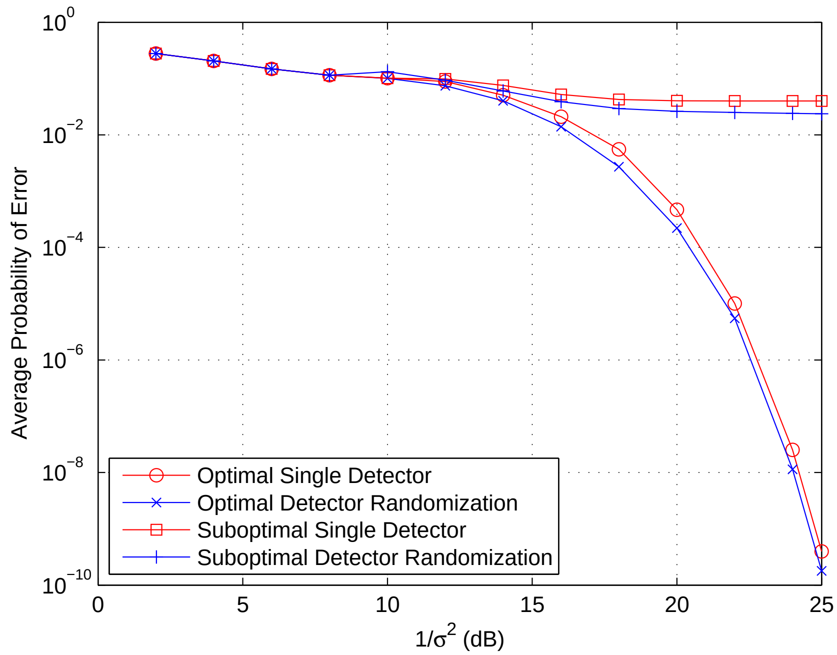

Figure 2.24: Average probability of error versus $1/\sigma^2$ for different approaches when $P_{\text{av},0} = 1.0$ and $P_{\text{av},1} = 0.5$.

mixture in (2.40) are given by $N_m = 5$, $\boldsymbol{\mu} = [\mu_1\ \mu_2\ \mu_3\ \mu_4\ \mu_5] = [-j\ \ -1\ 0\ 1\ j]$, and $\boldsymbol{\nu} = [\nu_1\ \nu_2\ \nu_3\ \nu_4\ \nu_5] = [0.2\ 0.2\ 0.2\ 0.2\ 0.2]$. The other parameters are kept the same as in Figure 2.2 and Figure 2.12. From the figures, it is observed that the optimal detector randomization approach achieves the best probability of error performance; however, the amount of error performance improvements obtained via detector randomization varies for different values of $P_{\text{av},0}$ and $P_{\text{av},1}$.

Chapter 3

CONCLUSION

In this thesis, detector randomization has been studied for secondary users in a cognitive radio system. Optimal and suboptimal detector randomization approaches both in the presence of possible sensing errors and under the assumption of perfect sensing have been analyzed in terms of average probability of error optimization. It has been concluded that the lowest average probability of error can be achieved via optimal detector randomization approach which takes possible sensing errors into account. Another result obtained via the solution of optimization problem is that at most four MAP detectors are needed at the secondary receiver to achieve the minimum average probability of error.

For future work, the detector randomization approach can be considered not only for secondary users but also for primary users. The optimization problem can be reformulated under the power constraints of both primary and secondary users and the optimal solution for both users can be investigated. Also, a new system can be modeled for secondary users by considering an undesired user such as a jammer, which tries to block communications among secondary users and prevents efficient spectrum utilization.

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