CARBON RESTRICTED NEWSVENDOR PROBLEM UNDER CVAR OBJECTIVE AND RESOURCE CONSTRAINTS

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By

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July, 2014

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ABSTRACT

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Newsboy problem has been studied in the literature extensively. The classical newsvendor, representing the risk neutral decision maker, determines the optimal order/production quantity by maximizing the expected profit or minimizing the expected total cost of a single period with stochastic demand. This approach is not suitable if one also aims to reduce the chances of facing unexpected losses due to demand uncertainty. In this thesis, two problems are investigated with a single product newsvendor under CVaR maximization objective. The first problem addresses the newsvendor model with two different carbon emission reduction policies, namely, mandatory emission allowance and carbon emission trading mechanism. In the second problem, as an extension of the first one, a newsvendor with multiple resource constraints is considered for the cases where the resources have quotas with trade options. Analytical expressions for optimal order/production quantities are determined together with the optimal trading policy and numerical examples are provided.

Keywords: Newsvendor, CVaR, Carbon Emissions, Cap and Trade.

ÖZET

KOŞULLU RİSKE MARUZ DEĞER AMACI VE KAYNAK KISITLARI ALTINDA KARBON SINIRLI GAZETECİ ÇOCUK PROBLEMİ

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Gazeteci çocuk problemi literatürde kapsamlı bir şekilde çalışılmıştır. Riske duyarsız karar alıcıyı temsil eden klasik gazeteci çocuk tek dönemli rassal talebe ait beklenen karı veya maliyeti en iyileyen üretim/sipariş miktarını belirler. Bu yöntem talep kesinsizliğinden kaynaklanan beklenmeyen kayıplarla karşılaşma ihtimalini azaltma amacına uygun değildir. Bu çalışmada, koşullu riske maruz değer en büyüklemesi amacı altında tek ürünlü gazeteci çocuk modeli iki problemde incelenmiştir. Birinci problem gazeteci çocuk modelini iki farklı karbon emisyonu azaltma politikası olan katı emisyon kotası ve emisyon üst sınırı ticareti ile ele almaktadır. Birinci problemin uzantısı olarak görülebilen ikinci problemde birden fazla kaynak kısıtı olan gazeteci çocuk problemi kaynakların alım-satımının yapıldığı durumlar için incelenmiştir. Problemi en iyileyen üretim/sipariş miktarları ve en uygun alım-satım politikaları belirlenmiş ve rakamsal örnekler verilmiştir.

Anahtar sözcükler: Gazeteci Çocuk Problemi, Koşullu Riske Maruz Değer, Karbon Emisyonu, Emisyon Üst Sınırı Ticareti .

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Chapter 1

Introduction

Greenhouse gas emissions have become one of the biggest threats of ecological system and humankind, among which carbon dioxide contributing the high percentage. The observations provided in the Intergovernmental Panel on Climate Change(IPCC) report declares that the increase in carbon dioxide emissions mainly originates from human activities, fossil fuel burning and activities related with land use [1]. Besides, European Commission's findings show that the road transportation is responsible for 20% of the carbon dioxide emissions of European Union [2]. As carbon dioxide emissions increase, the balance of nature changes and as a result of this, climate changes, extinction of animal species can be observed. When the increasing carbon dioxide emissions and their severe consequences are considered, it is understood that this problem is a big threat to the world which should not be underestimated [3].

There is a global consensus that the carbon dioxide emission problem must be intervened and some precautions must be taken at intergovernmental level in order to deal with this life threatening problem and mitigate carbon emissions. As a first step IPCC was founded by World Meteorological Organization (WMO) and United Nations Environment Programme (UNEP) in 1988 in order to have scientific information on climate change. As a result of the first report of the IPCC, the United Nations Framework Convention on Climate Change (UNFCCC) was established in May 1992. After several conventions of the parties (COP) to the UNFCCC a consensus was reached on Kyoto Protocol in 1997 providing the countries "flexibility mechanisms" to reduce the emissions efficiently. While helping to mitigate emissions, the mechanisms defined in the protocol also provide an economic perspective. The Kyoto Protocol has three mechanisms: emission trading, joint implementation and clean development mechanism.

Via an emissions trading mechanism; emission reductions are encouraged by market-based actions in which countries or corporations whose emissions are beyond the given carbon emission allowance can buy extra credits or the ones emitting less than given emission permission can sell their unused credits in domestic, regional or international markets.

Joint Implementation (JI) is a project-based mechanism which is carried out between two member countries of Kyoto Protocol as follows. One country makes an investment on an emission reduction project conducted in the other country in order to gain the emission reduction units (ERUs), which corresponds to one tonne of carbon dioxide, resulting from the project and use them towards increasing its emission capacity. JI can also be used within a country between two firms willing to curb their carbon emissions.

Clean Development Mechanism (CDM) is also a project-based mechanism enabling a member country of the Kyoto Protocol to conduct an emission reduction project in developing countries. The resulting emission reduction credits, called certified emission reduction credits (CERs), can be sold in the carbon market [4].

Commitments made at the intergovernmental level to mitigate carbon emissions have had impacts on companies' decision making processes. The firms investigating the ways to reduce their emissions came up with two alternatives: replacing the existing technology with more energy efficient and at the same time expensive one or adjusting their operations management decisions by means of production quantites, inventory levels and transportation mode selection. The general conclusion of the studies in the literature which have been conducted so far is that the reduction in carbon emission percentage is higher than the percentage increase in cost when companies modify their operations management activities.

The significance of operating policies mainly comes from inventory management mode of the companies, such as economic order quantity, just in time, lean production or single period production policies each of which has different contributions to carbon emissions. Hence; distinguishing the proper mode of inventory management is an important issue for those firms with the aim of curbing their carbon emissions. The awareness of the importance of inventory management choices of firms lead the researchers to consider joint implementation of inventory management and carbon emission reduction actions. Consequently, a growing research area is emerging with the basic interest of optimizing the inventory management activities together with the carbon emission restriction. As a part of this research area, we study the carbon emission restriction within a newsvendor model.

The recent studies show that the firms are in a quest to find new perspectives to the newsboy problem to take the possible losses into account which are due to the uncertainty of demand. Hence the risk measures are introduced to the newsvendor problem by means of objective function or constraints. The risk settings under which the newsboy problem has been examined in the literature so far are the satisfaction probability, expected utility maximization, mean-variance analysis, value-at-risk(VaR) and conditional value-at-risk(CVaR).

In this thesis we consider the CVaR criterion to study the newsboy problem with demand uncertainty. CVaR can be defined as the maximization of conditional expected profit falling below a certain level or minimization of conditional expected loss exceeding a threshold.

In our first problem, we combine both the risk averse behavior and carbon emissions reduction concerns of a newsvendor. The risk-aversion is introduced by altering the objective function as conditional value at risk (CVaR) maximization while the carbon emission reduction is analyzed under two carbon emissions control policies: strict carbon cap and cap and trade mechanism. Therefore, the first problem is analyzed by two sub-models investigating the single product newsvendor model for strict carbon cap policy and for cap and trade mechanism with the aim of CVaR maximization. In strict carbon cap policy the newsvendor cannot exceed the given carbon emission threshold and cannot turn the unused carbon emission credits into cash if there is any, whereas in the cap and trade mechanism one can sell its unused carbon capacity or buy any extra credits for fixed prices from the market. In both sub-problems, our aim is to find the optimal production quantity, threshold value for profit, VaR, and the trading amount maximizing CVaR. The analytical expressions of the optimal production quantity and VaR are determined. The trading behavior of the newsvendor is directly determined according to the optimal production quantity.

In the second problem, the newsvendor model with multiple resource constraints is investigated under CVaR maximization setting for the cases where resources have caps with trade options at pre-defined prices; which is a generalization of the first problem. A solution method is developed to find the optimal production quantity and the corresponding threshold for profit, VaR. The optimal solution of the problem under resources with strict caps setting is also determined as a special case.

In our numerical study, we basically investigate the impacts of changing problem parameters on the optimal decision variables and the corresponding CVaR, VaR, expected profit and service level values. We provide our findings from numerical experiment in three sections. First, we discuss the effect of the newsvendor problem parameters on the optimal order/production quantity of the unconstrained CVaR maximization problem in order to pick a reasonable parameter set for further analysis of the first and second problems. In this analysis, we see that the optimal order/production quantity and risk aversion level relation directly depends on the parameter setting. The optimal order/production quantity increases as the newsvendor becomes more risk averse at higher values of lost sales cost, which is a counter-intuitive outcome. Then, with a parameter set enabling high service levels, we study the first problem in order to determine the impact of increasing carbon cap tightness, which is the percentage reduction from the emission released at the unconstrained optimal solution, risk aversion level and carbon trading prices on CVaR, VaR, expected profit and service level values. The general outcome is that the CVaR increases with decreasing risk averse behavior while the behavior of expected profit for changing risk aversion level value depends on the relationship between optimal order/production quantity and risk aversion level. If the optimal order/production quantity decreases with increasing risk aversion level value then the expected profit decreases also. However, CVaR increases with increasing risk aversion level value since the higher the risk aversion value the less risk averse the newsvendor is. Another characteristic of the problems we observe is that the percentage decrease in CVaR is more sensitive to carbon cap tightness than percentage decrease in expected profit at a fixed risk aversion level. For example, under the strict cap policy decreasing the carbon emission by 6% and 10% causes 10.13% and 100.18% decreases in CVaR while the expected profit decreases by 0.81% and 6.14%, respectively. More drastical changes are observed as the tightness increases. The impact of increasing carbon prices is seen as high decreases in emission with respect to the emission of unconstrained order/production level while leading to low customer service levels. Changing carbon prices impacts the optimal policy that is determined at a fixed carbon cap tightness, also. The impact of risk aversion level is also analyzed for given carbon cap values at fixed carbon prices. Our observations are supported by graphs and detailed analyses are provided in tables in the numerical study section. Lastly, we provide a brief analysis for the second problem. We set the number of limited resources as two. The problem is examined under different risk aversion levels and the impacts of changing resource limits at a fixed risk aversion level are discussed.

The rest of this thesis is organized as follows: In Chapter 2, we briefly review the literature on the classical newsvendor problem, risk-averse newsvendor problem and inventory management problems with carbon emission consideration. In Chapter 3, we provide a preliminary study for the unconstrained and carbon constrained classical newsvendor problem and the mathematical background for VaR and CVaR are introduced together with their interpretations. In Chapter 4, a detailed analysis of the newsboy problem with the CVaR maximization objective under carbon emission reduction concerns is provided. Analytical expressions of the optimal policies for the unconstrained CVaR maximization, carbon constrained CVaR maximization problems are determined. In Chapter 5, the multiple resource constrained problem under the CVaR maximization setting is investigated and the optimal solution method is provided. In Chapter 6, the results of the numerical experiment regarding the two problems and unconstrained CVaR maximization problem are provided. Finally, in Chapter 7 we give the concluding remarks.

Chapter 2

Literature Review

In this section we provide our findings in the literature about the newsvendor problem under three categories named as classical newsvendor problem with extensions, risk-averse newsvendor models and carbon restricted inventory management.

In the first subsection we will briefly go through the single-item newsvendor model extensions that have been incorporated to literature ever since the introduction of the newsvendor problem.

In the second part, we will briefly review the studies in which the newsvendor problem is investigated under risk-aversion approach.

In the last section, the inventory management problems considering carbon emission policies will be presented.

2.1 Classical Newsvendor Problem with Extensions

The newsvendor problem is a single period inventory management problem with stochastic demands, in which the trade-off between the holding and lost sales cost is optimized. For each unsold item a unit holding cost is incurred and for each unsatisfied demand a unit lost sales cost is incurred. The main goal of the newsvendor is to find the optimal order quantity which maximizes the expected profit or minimizes the expected total cost.

The emergence of the newsvendor problem, one of the keystones of the inventory management framework, dates back to 1955 (see Whitin [5]). Since then many extensions of it are investigated in the literature. Khouja [6] provides a broad literature survey and categorizes the extensions to the single period stochastic demand problem. The extensions considered by means of different objective functions, pricing policies of supplier or newsvendor, discounting structures, multiple product with constraints or substitutions, multi-location models and different demand structures. Khouja [6] is a repository to gain insights about what had been done about the newsvendor model. We mention below some recent studies regarding the newsboy problem under different settings.

Chung et al. [7] analyze the newsvendor model under in-season price adjustment setting and stochastic demand environment with the aim of expected profit maximization for a fixed time horizon. Once the newsvendor orders an amount of product before the selling season starts, he cannot make orders during the season. The uncertain demand is observed for a pre-specified time interval starting from the beginning of the selling season in order to predict the unrealized demand better and modify the selling price accordingly. The demand is taken random during the observation period, after that it is approximated linearly by using demand-price curve. The decision variables are the optimal retailer price and the order quantity to be decided at the beginning of the season. The optimal price and a solution strategy to determine the optimal order quantity are provided. A heuristic method is also given as an alternative to obtain the optimal order quantity.

Grubbström [8] discusses the newsboy problem where the demand is considered as a compound renewal process. Under this setting, the optimal order quantity, maximizing the expected net present value (NPV) of the payments involved is determined explicitly. The surprising conclusion arrived at in this study is that the classical newsvendor problem and this special problem have the same solution under a specific demand distribution process.

Wu et al. [9] consider a framework which enables the newsvendor to set the price internally and assumes that product arrival has a quoted lead-time. This is the case for firms purchasing semi-processed products which will be customized according to the preferences of the customer. The stochasticity is introduced by the randomness of demand and the quoted lead-time with demand being linearly dependent upon the price of the good and as well as the quoted-lead time. A multiplicative model is set for the lead time distribution which is basically the product of the mean demand during the selling season and a random variable independent of demand. The decision variables in the problem are selected to be optimal order quantity, selling price and the quoted lead-time under the objective of maximizing expected profit. The analytical expressions of the decision variables are provided. Managerial insights about the effect of demand uncertainty and lead time are provided. The effect of lead time is analyzed for two cases where the demand is taken as certain and uncertain. They conclude that taking the loss arising from random demand into account causes an increase in optimal quoted lead time and mean demand while it leads to a decrease in optimal selling price and expected profit. Besides, random lead time results in an increase in optimal price while uncertain demand with an additive random error term brings about a decrease in optimal price.

Yu et al. [10] consider the stochastic price-dependent demand of the newsvendor model by enforcing a fuzzy price-dependent demand structure. With the aim of maximizing expected profit the optimal price and order quantity expressions are derived for certain cases where the problem is convex. The mathematical interpretation of the fuzzy price-dependent demand is discussed. Since the demand is assumed to be fuzzy, the profit is also a fuzzy number. Hence a solution algorithm is established by implementing the method of ranking fuzzy numbers with integral value. The model under consideration is compared with the one with price-dependent deterministic demand by analytically and numerically. In numerical experiment, the impacts of market potential, price sensitivity of the demand, wholesale price, salvage value and shortage cost are investigated for both models and the results are compared. An immediate result of the numerical analysis is that the fuzzy price-dependent demand model results in a higher expected profit than the price dependent deterministic demand model.

Jammernegg and Kischka [11] investigate a newsvendor model which is subject to financial and non-financial constraints under classical newsvendor and price-setter newsvendor settings. In particular, they consider service level constraints and probability bounds on negative profit. For the newsvendor model, the necessary circumstance that guarantees the existence of the optimal solution is provided. The optimal order quantity is determined and the effect of demand variability is discussed. In the price setting case, the price-dependent demand is assumed to have multiplicative structure. The optimal price and stocking amount are selected as the decision variables to be determined. The necessary condition for admissible solutions to exist is discussed and an algorithm to find optimal solution is developed. The impact of demand variability is investigated.

2.2 Risk-averse Newsvendor Models

Analyzing the newsvendor model under a risk averse approach is another diversification of the classical newsvendor model.

Satisfaction probability and expected utility maximization are the first adjustments to the classical newsvendor problem in order to take the risk-averse behavior into account. Recently, financial risk measures such as the value at risk (VaR) and conditional value at risk (CVaR) are commonly being considered as objective functions in inventory management problems.

Satisfaction probability is defined as the probability of going beyond a specified profit level which company aims to reach. The satisfaction probability maximization is discussed by Lau [12], Lau and Lau [13], Li et al.[14], [15] and Parlar and Weng [16]. Lau [12] and Lau and Lau [13] investigate the solution strategies for single and two-product newsvendor problems so as to maximize the satisfaction probability, respectively. Moreover, Li et al. [14], [15] provide analytical solutions of two-product newsvendor problem for both exponentially and uniformly distributed demand with the aim of satisfaction probability maximization. The study of Parlar and Weng [16] differs in this setting in terms of target profit since they replace the target profit with the expected profit.

Yang et al. [17] increase the complexity of the satisfaction probability maximization context by examining a price setter newsvendor who specifies the profit and revenue level probability of accomplishing which are maximized simultaneously. The decision variables to be determined at optimality are the order quantity and selling price. They indicate that the proportionate magnitudes of the profit margin and profit target to revenue target ratio have decisive role on probability of accomplishing both goals at the same time. Analytical expressions of the decision variables and the optimal target levels are also provided.

In the expected utility maximization framework for newsvendor problem, Bouakiz and Sobel [18] analyze the newsvendor problem under exponential utility setting and base-stock policy is shown to be optimal.

Dohi et al. [19] investigate the newsvendor model with the goal of utility maximization under two different contexts. First, they implement Taylor approximation to the logarithmic utility function which will be maximized under newsvendor setting. In their second model the objective is adjusted as maximizing the upper and lower bounds of the expected utility. The practical applicabilities of the two models are supported by the numerical examples.

Agrawal and Seshadri [20] determine the optimal price and order quantity of the risk-averse newsvendor aiming to maximize the expected utility. The price-dependent demand is assumed to be concave in selling price and two price-dependent demand models, namely additive and multiplicative demand models, are investigated. A solution method reducing the number of decision variables to one is developed. The optimal solutions are determined for both additive and multiplicative demand models and the comparisons of two models with the risk-neutral newsvendor are provided. The results conclude that under the multiplicative demand model assumption the newsvendor sets a higher price and orders less while under additive demand model assumption he sets a lower price as compared to the classical risk-neutral newsvendor. The associated impacts of the findings are investigated for products that differ in price dependency characteristics.

Mean-risk model is another concept of risk aversion in newsvendor setting. The choice of "risk "function is up to the newsvendor. Choi and Ruszczynski [21] suggest the mean-risk model as a unique formulation of risk-averse newsvendor problem where the risk measures are required to be law invariant and coherent. They conclude that the optimal order quantity is inversely proportinal to the risk aversion significance.

Under profit maximization setting, value at risk (VaR) is defined as the maximum value of the profit the probability of the expected profit falling below which is the fixed confidence level. In loss minimization VaR can be defined as the threshold for the loss such that the probability that loss will be lower than the threshold is the chosen confidence level. VaR can be utilized as an objective function or the constraint of the risk-averse newsvendor.

Özler et al. [22] construct a multi-product newsvendor model with a VaR constraint with the objective of maximizing the expected profit. The VaR constraint puts an upper bound to the probability of obtaining a profit that is less than a target value. Since the VaR constraint is represented as the probability distribution of the profit, deriving the profit distribution is one of the main works. To begin with, two-product newsvendor problem is examined then the problem is extended to multiple-product case. The distribution of the profit is derived under the assumption that two products have a joint demand distribution. In the multi-product case determining the profit distribution is the main challenge. The total profit distribution is approximated as a normal distribution by making use of central limit theorem with the assumption that the demand of each product is independently distributed. Feasibility of the both problems are discussed and a mathematical programming method is applied to solve each problem. A numerical experiment is conducted to observe the results of the two-product problem under independent exponentially distributed demand and bivariate exponentially distributed demand settings. For the multiple-product case the results for the exponentially distributed demand are illustrated with the corresponding graphs. In addition, a numerical experiment analyzing the effects of problem parameters and the precision of the method developed for multi-product case is provided.

Chiu and Choi [23] find the price and order quantity maximizing the profit of the newsvendor which lies in the pre-defined quantile. In other words, the VaR is utilized as the objective of the newsvendor. The value of the quantile of the profit represents the risk aversion level of the newsvendor. The optimal order quantity and selling price are the decision variables to be determined simultaneously. Demand is assumed to be price-dependent. The optimal solutions of risk-averse price-setter newsvendor are provided for both linear and multiplicative demand models. The results are compared with the risk-neutral newsvendor case. They also investigate the connection between the confidence level and the customer service level in an analytical manner.

Recently, CVaR, which is derived from VaR, is introduced as a risk measure in financial risk management field. CVaR is defined as the conditional expected loss that is beyond the threshold given by VaR or the conditional expected profit which stays below the threshold given by VaR.

Gotoh and Takano [24] analyze the newsvendor model under CVaR minimization setting where the aim is to minimize the losses going beyond the threshold value. For the loss minimization concept, they define two distinct loss functions: net loss (negative profit) and total cost. The single-product unconstrained newsvendor model is investigated in the context of net loss CVaR minimization, total cost CVaR minimization and mean-CVaR maximization. In the first two problems, the CVaR is the objective function of the newsvendor. The optimal order quantity and the corresponding threshold for loss are derived analytically for these two cases and they are compared by the help of a numerical study. In the mean-CVaR maximization problem mean is taken as the expected profit of the newsvendor and the risk is introduced as CVaR which takes the negative profit as the loss function. The optimal solution for this problem is searched in three different cases depending on the value of the threshold for loss. An algorithm to solve the problem is provided. They extend the mean-CVaR model to multi-product multiple constrained case where the constraints are represented as a set of linear inequalities. The demand distribution is taken as discrete and represented by a finite number of scenarios. The LP formulation of the problem and numerical analysis supporting the efficiency of the findings are provided.

Zhou et al. [25] discuss the multi-product stochastic demand newsvendor problem under risk averse approach. The newsvendor is also subjected to a budget constraint and the order quantity is bounded below and above. Risk aversion is controlled in the context of CVaR. A linear program minimizing the negative profit and subject to a CVaR constraint together with the budget and order quantity constraints is written for the multi-product newsvendor problem to determine the optimal order quantities of each product. A case study is conducted for the different values of the upper bound of the CVaR constraint in order to determine the impacts of the risk tolerance on the optimal order quantities, and expected profit values. The findings of the risk- averse multiple product newsvendor problem and classical multiple product newsvendor problem are compared.

Chahar and Taaffe [26] introduce CVaR as the objective function of the newsvendor who faces stochastic demands from multiple markets and chooses the demands to be satisfied. The selling price of the good is assumed to be unique for each market. Demand realization is assumed to behave as a Bernoulli experiment, where the demand is either realized at a pre-specified value with some probability or will not be realized at all, which is called all-or-nothing (AON) orders in the literature. There is a set of unconfirmed demands some of which will be met. The demands to be satisfied and the order quantity of the newsvendor are the decision variables which will maximize the CVaR. A mixed-integer linear programming model is written for both expected profit and CVaR maximization problems under the AON demand selection setting. A sensitivity analysis investigating the impacts of significance level, salvage value, material cost and shortage cost is provided to gain insights about the results. They also analyze the mean-CVaR model by investigating the problem under three different settings. The first problem is a convex combination of the expected profit and the CVaR formulated as a mean-CVaR maximization model. In the second problem, the newsvendor aims to maximize the expected profit under a CVaR constraint which puts a lower bound on the worst-case profit and this problem is called minimum acceptable CVaR level. In the third problem, a risk minimizer firm aiming to minimize the profits falling below a certain level is considered. The analysis of the three problems are supported by numerical results.

Chen et al. [27] determine the optimal price and the order quantity of the newsvendor aming to maximize CVaR. The price-dependent demand is represented by additive and multiplicative demand models for which the existence and uniqueness of the optimal policy are obtained under some conditions. They discuss the impacts of changes in the unit acquisition cost, salvage value and the risk aversion level on the decision variables for both demand models under CVaR and risk-neutral settings. The findings are also compared with the utility maximization approach. We use the same objective function with the one given in this study.

Ma et al. [28] investigate the risk-averse retailer's optimal order quantity in a multiple retailer and one supplier environment under a CVaR objective function.

Most of the papers presented so far, investigate the impact of incorporating one risk measure into newsvendor model on the decision variables and make comparison with the classical newsvendor approach. We next provide two studies that compare the results obtained under different risk measure settings.

Arcelus et al. [29] determine the optimal ordering and pricing policies of the newsvendor under four different objective function settings for varying risk aversion levels. The objectives are: maximization of risk adjusted expected profit, maximization of minimum guaranteed profit, maximization of the probability of exceeding expected profit and maximization of the expected profit under the constraint enforcing a lower bound for the probability of the profit to be higher than a target profit. The main properties of each problem are discussed analytically and numerically in order to make comparisons across the models.

Katariya et al. [30] determine the link between the traditional and risk-averse newsvendor in terms of optimal order quantities via utilizing three different risk measures, namely, expected utility maximization (EU), mean-variance (MV) analysis and

CVaR minimization. By investigating special cases, they show that the general understanding from risk-aversion that the risk-averse newsvendor orders less than risk neutral one is not valid all the time. Dependence of order quantity to problem parameters, demand distribution, decision criterion is revealed by analytical and numerical findings. Additionally, the consistency between the risk measures is observed under some circumstances.

2.3 Carbon Restricted Inventory Management

It has not been so long that the carbon emission reduction initiatives are investigated in the literature. Studies that will be reviewed in this section consider the carbon emission reduction issues under economic order quantity (EOQ) and newsvendor settings. We will present the economic order quantity models and then provide the newsvendor models concerning carbon emission reduction.

Hua et al. [3] merge the inventory management and carbon emission reduction concerns by analyzing the EOQ setting with cap and trade mechanism which is both economically and environmentally efficient way to constrain carbon emissions. The optimal order quantity is derived analytically. They draw analytical and numerical conclusions about how the order quantity, carbon emissions and total cost are influenced by carbon trade, carbon price and carbon emission allowance.

Hua et al. [31] increase the complexity of the carbon constrained EOQ model by incorporating the selling price as an additional decision variable. The price-dependent demand is considered by means of additive and multiplicative demand models. The outcomes of the carbon trading on the operational decisions and emission levels are discussed.

Chen et al. [32] determine the optimal order quantity that minimizes cost of economic order quantity model subject to a carbon emission constraint. The model is investigated under four different carbon emission policies: strict carbon cap, carbon tax, cap-and-offset and cap-and-price where the first two can be seen as special instances of the third policy. Impact of carbon cap on emission and the circumstances which enable one to reduce carbon emission without a significant increase in cost are investigated.

Benjaafar et al. [33] approach the cost minimization of conventional deterministic inventory management models with a carbon emission reduction concern under single firm and multiple firms within the same supply chain settings. Carbon emission initiatives are taken into account via different regulations such as strict carbon cap, carbon tax, carbon cap-and-trade and carbon offsets for single firm models while under multiple firms setting each firm is assumed to be subject to a strict carbon cap. The corresponding mathematical programming model of each problem is provided. A numerical experiment underlining the significance of the inventory management model and carbon emission reduction policy on cost and carbon emissions is provided.

Hoen et al. [34] balance the trade-off between transportation, inventory holding and carbon emission costs by considering a carbon emission sensitive transport mode selection problem for the newsvendor model with stochastic demand. The traditional transport mode selection problem is extended to a environmentally friendly case by adding carbon emission concern into the framework. The paper determines the unit carbon emission contribution of air, rail, road and water transport modes. Then, they develop a model accounting for the transportation, inventory holding and emission costs for each transport mode and decision criterion for the transport mode selection is discussed. Finally, the impacts of carbon emission policies such as strict carbon cap, cap-and-trade and carbon tax on the emission transport mode selection problem are provided. The insights are supported by a numerical study. It is concluded that, despite the fact that changing the transport mode yields significant carbon emission reductions, the major factor on carbon emission reduction decision is the regulatory policy applied for the emissions.

Song and Leng [35] extend the single product newsvendor problem by examining the optimal order quantity under three carbon emission reduction regulations: strict carbon cap, carbon tax, cap-and-trade mechanism. The analytical expression of optimal solution for each policy is provided. The analytical solutions are supported by a numerical study for each carbon emission regulation.

Rosic and Jammernegg [36] discuss carbon emission concern of a newsvendor who has offshore and onshore suppliers. The problem takes the carbon emissions resulting from the transportation into account. While constructing the problem setting it is assumed that the onshore supplier is used only when the demand exceeds the order quantity. The analytical expression of the optimal order quantity obtained from the offshore supplier is provided for two emission regulations: a transport emission tax and emission trading for transport. The analytical findings are used in a numerical experiment to visualize the impacts of different regulations. The conclusions of both analytical and numerical analyses are used to obtain some managerial and regulatory insights. For both perspectives the emission trading is found to be more beneficial.

Zhang and Xu [37] bring the carbon emission reduction issue to the multi-product newsvendor problem where each product has independent demand distribution. The optimal production amount and carbon trading policy is determined with the objective of expected profit maximization. The newsvendor has a resource and carbon constraint and is allowed to buy or sell carbon credits. After providing the optimal policy and a solution algorithm for this setting, the changes in total profit, carbon emissions, operational decisions, shadow price of the resource constraint for changing carbon price and carbon cap are investigated by analytical and numerical methods. The cap-and-trade mechanism is compared with the one where a tax is imposed on carbon emissions to give insights about each policy.

Toptal et al. [38] seek for the optimal order quantity and investment amount on green technology that jointly minimize the total average annual cost of a retailer operating in an EOQ environment. The classical EOQ model is enhanced by incorporating the carbon emission reduction investment opportunity and analyzed under three distinct carbon emission policies: carbon cap, tax and cap-and-trade mechanism. Determining the analytical expressions of the optimal solutions under each carbon policy, they compare different carbon emission reduction policies with regard to costs and carbon emissions. Effects of problem parameters are supported by a numerical experiment.

To our knowledge, in the literature there is not a study incorporating the CVaR setting to newsboy problem with carbon restrictions. In this respect, this research aims to make a contribution to the literature.

Chapter 3

Preliminaries

Before formally introducing the risk measures VaR and CVaR and carbon trading system to the newsboy problem, we shall briefly review the classical newsboy problem, the classical newsboy problem with carbon trading system which is also investigated by Song and Leng [35] .

3.1 Classical Newsboy Problem

The derivation of optimal production quantity for the classical newsboy problem will be reviewed briefly below. The parameters and the variables of the classical single-period newsboy problem are defined as follows: unit ordering cost c, unit selling price p, unit salvage value s, unit lost sales cost l, random demand X, order quantity Q, profit function $\pi(X, Q)$, probability distribution function of the demand $f(x)$ and the cumulative distribution function of the demand $F(x)$.

Throughout this thesis in the newsvendor model it is assumed that $p - s < l$. The profit function, $\pi(X,Q)$, and its expectation, $E[\pi(X,Q)]$ are determined as follows:

$$\pi(X,Q) = (p-c)Q - (p-s)(Q-X)^+ - l(X-Q)^+ \tag{3.1}$$

where, $(A)^+ = A$ if $A > 0$, zero otherwise for $A \in \mathbb{R}$.

$$\begin{aligned} E[\pi(X,Q)] &= \int_0^\infty [(p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+]dF(x) && (3.2)\\ &= (p-c)Q - (p-s)\int_0^Q (Q-x)dF(x) - l\int_Q^\infty (x-Q)dF(x) && (3.3)\\ &\equiv E_{NW}[\pi(X,Q)] && (3.4)\end{aligned}$$

As the objective is to maximize the expected profit.The unconstrained maximization problem of the newsvendor is given as follows:

$$\text{Max}_Q \;\; E[\pi(X,Q)] \tag{3.5}$$

Concavity of the expected profit function of the newsvendor leads the first order condition to imply optimality (see Khouja [6]). The corresponding analytical expression of optimal order quantity is given as follows:

$$Q^* = F^{-1}\left(\frac{p-c+l}{p-s+l}\right) \tag{3.6}$$

3.2 The Classical Newsvendor Problem with Cap and Trade Mechanism

In this section, the carbon emission restriction is taken into consideration within the classical newsboy problem. In literature, carbon emission restriction is combined with the newsboy problem in three cases: “Mandatory Carbon Emissions Capacity, ” “Carbon Emissions Tax, ” and “Cap-and-Trade ” (see Song and Leng [35]). In the first policy, the newsvendor has to set the production quantity so that the carbon emission is strictly less than or equal to the given emission allowance. In carbon emissions tax policy, the newsvendor pays a tax per unit produced which is responsible from carbon emission. In the cap-and-trade mechanism, the firms are provided by some free carbon allowances, denoted by K, which they can trade in a carbon market. The difference of this policy from mandatory carbon emissions policy is that the firms are allowed to sell the excess allowances if they emit less carbon than K or buy extra carbon units if they want to produce more. When the classical newsvendor problem is analyzed under cap-and-trade mechanism for a given carbon emission restriction, one shall find the optimal decision, buying or selling carbon credits, which maximizes the expected profit and determine the order quantity accordingly. In the literature, this problem is analyzed by Song and Leng [35] and here their findings will be revealed concisely.

For the newsvendor problem with cap and trade mechanism there are additional parameters coming into equation such as unit carbon buying cost, c_b, unit carbon selling price, c_s, and it is assumed that $c_b > c_s$, and the carbon cap, K.

It is assumed that each unit produced emits a carbon amount of α units. Therefore, the emission amount for an arbitrary order quantity Q will be calculated as $Q\alpha$.

As the given carbon emission allowance is K, if $Q\alpha > K$ then the amount of carbon bought is $(Q\alpha - K)$ else the amount of carbon sold is $(K - Q\alpha)$. Throughout all the calculations in this thesis, it is assumed that $p - c + l > c_b\alpha$, that is the cost of underage is greater than the cost of buying carbon credits to produce one unit product.

The profit of the newsvendor and its expectation, which is the objective function, is expressed as follows:

$$\pi(X, Q, K) = (p - c)Q - (p - s)(Q - X)^+ - l(X - Q)^+ - c_b(Q\alpha - K)^+ + c_s(K - Q\alpha)^+ \tag{3.7}$$

$$\begin{aligned} E[\pi(X, Q, K)] &= \int_0^\infty [(p - c)Q - (p - s)(Q - x)^+ - l(x - Q)^+ \\ &\quad - c_b(Q\alpha - K)^+ + c_s(K - Q\alpha)^+]dF(x) \\ &= \int_0^\infty (p - c)QdF(x) - \int_0^Q (p - s)(Q - x)dF(x) - \int_Q^\infty l(x - Q)dF(x) \\ &\quad - c_b(Q\alpha - K)^+ + c_s(K - Q\alpha)^+ \end{aligned} \tag{3.8}$$

Rearranging the objective function results in :

$$\begin{aligned} E[\pi(X, Q, K)] &= (p - c)Q - (p - s)\int_0^Q (Q - x)dF(x) - l\int_Q^\infty (x - Q)dF(x) \\ &\quad - c_b(Q\alpha - K)^+ + c_s(K - Q\alpha)^+ \\ &= E_{NW}[\pi(X, Q)] - c_b(Q\alpha - K)^+ + c_s(K - Q\alpha)^+ \\ &\equiv E_{CT}[\pi(X, Q)] \end{aligned} \tag{3.9}$$

The expected profit can be rewritten in three regions with respect to the quantity, Q.

$$E[\pi(X,Q,K)] = \begin{cases} E_{NW}[\pi(X,Q)] + c_s(K - Q\alpha) & \text{if } Q < K/\alpha \\ E_{NW}[\pi(X,Q)] & \text{if } Q = K/\alpha \\ E_{NW}[\pi(X,Q)] - c_b(Q\alpha - K) & \text{if } Q > K/\alpha \end{cases}$$

$$\equiv E_{NW}[\pi(X,Q)] + c_s(K - Q\alpha)I(K > Q\alpha) - c_b(Q\alpha - K)I(Q\alpha > K) \qquad (3.10)$$

The newsboy problem with carbon trading mechanism for a given carbon emission level K is given as follows:

$$\text{Max}_Q \;\; E[\pi(X,Q,K)] \qquad (3.11)$$

Proposition 3.2.1 For a given carbon emission allowance K, $E[\pi(X,Q,K)]$ is concave in Q.

Proof. First we shall prove that $E[\pi(X,Q,K)]$ is piecewise concave. The piecewise concavity is proven by analyzing the functions of three regions.

Region 1: $Q < K/\alpha$**.** The corresponding function is $E_{NW}[\pi(X,Q)]+c_s(K-Q\alpha)$. In order to show that the function is concave in Q the second derivative of the function with respect to Q must satisfy the relation $\frac{d^2E[\pi(X,Q,K)]}{dQ^2} \leq 0$. The first derivative is found by applying Leibniz Rule as follows:

$$\begin{aligned}\frac{dE[\pi(X,Q,K)]}{dQ} &= p-c-(p-s)\int_0^Q dF(x)+l\int_Q^\infty dF(x)-c_s\alpha \\ &= p-c-c_s\alpha-(p-s)F(Q)+l[1-F(Q)] \\ &= p-c+l-c_s\alpha-(p-s+l)F(Q) \qquad (3.12)\end{aligned}$$

Then the second derivative of the function w.r.t. Q is determined as:

$$\frac{d^2E[\pi(X,Q,K)]}{dQ^2} = -(p-s+l)f(Q) < 0 \qquad (3.13)$$

Hence, $E_{NW}[\pi(X,Q)] + c_s(K-Q\alpha)$ is concave in Q.

Region 2: $Q = K/\alpha$**.** The corresponding function is $E_{NW}[\pi(X,Q)]$ which is the expected profit function of the classical newsvendor. Since the expected profit of the classical newsvendor is concave in Q, we can directly say that the function is concave in Q in the second region.

Region 3: $Q > K/\alpha$**.** The corresponding function is $E_{NW}[\pi(X,Q)] - c_b(K-Q\alpha)$. The first derivative is again found by applying Leibniz Rule as:

$$\begin{aligned}\frac{dE[\pi(X,Q,K)]}{dQ} &= p-c-(p-s)\int_0^Q dF(x)+l\int_Q^\infty dF(x)-c_b\alpha \\ &= p-c-c_b\alpha-(p-s)F(Q)+l[1-F(Q)] \\ &= p-c+l-c_b\alpha-(p-s+l)F(Q) \qquad (3.14)\end{aligned}$$

and the second derivative is:

$$\frac{d^2E[\pi(X,Q,K)]}{dQ^2} = -(p-s+l)f(Q) < 0 \qquad (3.15)$$

Hence, $E_{NW}[\pi(X,Q)] - c_b(K - Q\alpha)$ is concave in Q.

Therefore, $E[\pi(X,Q,K)]$ is piecewise concave in Q. In order for $E[\pi(X,Q,K)]$ to be concave in Q the following relation must hold:

$$\frac{dE[\pi(X,Q,K)]}{dQ}\Big|_{Q=(K/\alpha)^+} < \frac{dE[\pi(X,Q,K)]}{dQ}\Big|_{Q=(K/\alpha)^-} \tag{3.16}$$

We shall determine the right and left derivatives of $E[\pi(X,Q,K)]$ w.r.t. Q at $Q = K/\alpha$ and ensure that the above inequality holds.

$$\frac{dE[\pi(X,Q,K)]}{dQ}\Big|_{Q=(K/\alpha)^+} = p - c + l - c_b\alpha - (p - s + l)F((K/\alpha)^+) \tag{3.17}$$

$$\frac{dE[\pi(X,Q,K)]}{dQ}\Big|_{Q=(K/\alpha)^-} = p - c + l - c_s\alpha - (p - s + l)F((K/\alpha)^-) \tag{3.18}$$

$F((K/\alpha)^+) > F((K/\alpha)^-)$ and $c_b > c_s$ imply equation(3.16). □

Proposition 3.2.2 For a given carbon emission allowance K, the optimal order quantity belongs to the following set:

$$Q^* \in \left(F^{-1}\left(\frac{p - c + l - c_s\alpha}{p - s + l}\right), K/\alpha, F^{-1}\left(\frac{p - c + l - c_b\alpha}{p - s + l}\right)\right) \tag{3.19}$$

Proof. Concavity of $E[\pi(X,Q,K)]$ in Q guarantees that there is a unique maximizer in one of the three regions. Since $E[\pi(X,Q,K)]$ is piecewise concave, the unique root of the corresponding function in each region is candidate for optimal solution. The roots of the functions in regions where $Q < K/\alpha$ and $Q > K/\alpha$ are found from first order conditions while in the second case $Q = K/\alpha$ is directly set.

If $Q < K/\alpha$, then the candidate for optimal solution is found from the first order condition of the corresponding objective function which is given in equation (3.12) and by equating the first order condition to 0 we have:

$$F(Q) = \left(\frac{p - c + l - c_s\alpha}{p - s + l}\right) \tag{3.20}$$

Hence the corresponding order quantity when $Q < K/\alpha$ is given by:

$$Q = F^{-1}\left(\frac{p - c + l - c_s\alpha}{p - s + l}\right) \tag{3.21}$$

The derivation of the root of the objective function when $Q > K/\alpha$ follows exactly the same steps as shown above and the expression of it is given as:

$$Q = F^{-1}\left(\frac{p - c + l - c_b\alpha}{p - s + l}\right) \tag{3.22}$$

□

Let Q_{down}, Q_{up} be the solution of the first order condition given in equations (3.12) and (3.14), respectively, and K_{down}, K_{up} be the corresponding carbon emission quantities given by:

$$Q_{down} = F^{-1}\left(\frac{p - c + l - c_s\alpha}{p - s + l}\right) \tag{3.23}$$

$$Q_{up} = F^{-1}\left(\frac{p - c + l - c_b\alpha}{p - s + l}\right) \tag{3.24}$$

$$K_{down} = Q_{down}\alpha \tag{3.25}$$

$$K_{up} = Q_{up}\alpha \tag{3.26}$$

From the non-decreasing property of cumulative distribution function, F, and the assumption that $c_b > c_s$ we immediately have the following result: $Q_{up} < Q_{down}$, and

$K_{up} < K_{down}$.

The objective of the newsvendor is to determine the optimal order quantity and trading amounts according to given carbon emission allowance, K. The following theorem gives the optimal policy of the newsvendor under carbon cap and trade mechanism.

Theorem 3.2.1 The optimal policy of the risk neutral newsvendor for a given carbon emission allowance, K, and carbon market prices c_b and c_s is given as:

$$Q^* = \begin{cases} Q_{up} & \text{if } K < K_{up} \\ K/\alpha & \text{if } K_{up} \leq K \leq K_{down} \\ Q_{down} & \text{if } K > K_{down} \end{cases} \tag{3.27}$$

Proof. As it is given in Proposition 3.2.1 there are three critical points: Q_{down}, Q_{up} and $Q = K/\alpha$. The optimal policy is determined by studying three cases for a given K.

Case 1: $K < K_{up} \equiv K/\alpha < Q_{up} < Q_{down}$

$E_{NW}[\pi(X,Q)]+c_s(K-Q\alpha)$ is concave and maximized at Q_{down}. Hence it is increasing in $(-\infty, Q_{down})$. $K/\alpha < Q_{up} < Q_{down}$ implies that $E_{NW}[\pi(X,Q)] + c_s(K - Q\alpha)$ is also increasing in $(-\infty, K/\alpha)$.

$E_{NW}[\pi(X,Q)] - c_b(Q\alpha - K)$ is concave and maximized at Q_{up}. Hence it is increasing in $(K/\alpha, Q_{up})$ and decreasing in (Q_{up}, ∞).

Thus, $E[\pi(X,Q,K)]$ is increasing in $(-\infty, Q_{up})$ and decreasing in (Q_{up}, ∞). This implies that $E[\pi(X,Q,K)]$ is maximized at $Q = Q_{up}$.

Case 2: $K_{up} \leq K \leq K_{down} \equiv Q_{up} \leq K/\alpha \leq Q_{down}$

$E_{NW}[\pi(X,Q)]+c_s(K-Q\alpha)$ is concave and maximized at Q_{down}. Hence it is increasing in $(-\infty, Q_{down})$. $K/\alpha < Q_{down}$ implies that $E_{NW}[\pi(X,Q)] + c_s(K - Q\alpha)$ is increasing in $(-\infty, K/\alpha)$.

$E_{NW}[\pi(X,Q)] - c_b(Q\alpha - K)$ is concave and maximized at Q_{up}. Hence it is decreasing in (Q_{up}, ∞). $Q_{up} \leq K/\alpha$ implies that it is also decreasing in $(K/\alpha, \infty)$.

Thus, $E[\pi(X,Q,K)]$ is increasing in $(-\infty, K/\alpha)$ and decreasing in $(K/\alpha, \infty)$. This implies that $E[\pi(X,Q,K)]$ is maximized at $Q = K/\alpha$.

Case 3: $K > K_{down} \equiv K/\alpha > Q_{down}$

$E_{NW}[\pi(X,Q)] + c_s(K - Q\alpha)$ is concave and maximized at Q_{down}. Hence it is increasing in $(-\infty, Q_{down})$ and decreasing in $(Q_{down}, K/\alpha)$ since $(K/\alpha > Q_{down})$.

$E_{NW}[\pi(X,Q)] - c_b(Q\alpha - K)$ is concave and maximized at Q_{up}. Hence it is decreasing in (Q_{up}, ∞). Since $Q_{up} < Q_{down}$ it is also decreasing in (Q_{down}, ∞).

Thus, $E[\pi(X,Q,K)]$ is increasing in $(-\infty, Q_{down})$ and decreasing in (Q_{down}, ∞). This implies that $E[\pi(X,Q,K)]$ is maximized at $Q = Q_{down}$. □

3.3 VaR and CVaR

We next introduce VaR and CVaR concepts formally. We shall start with the analysis of VaR and CVaR with loss minimization concept which is derived in Rockafellar and Uryasev [39], [40] in detail. Then we will provide the expressions for profit maximization case. The following mathematical representations are taken from Rockafellar and Uryasev [40].

Loss Minimization

Let X and Q represent random vector and the decision vector, respectively, where X is assumed to be independent of Q. The loss function for the random vector

X is defined as $l(X, Q)$ which is assumed to be continuous in Q, and has a finite expectation for each Q. Then, for a given Q the distribution function of the loss is written as:

$$\Psi(Q, \omega) = P\{l(X, Q) \leq \omega\} \tag{3.28}$$

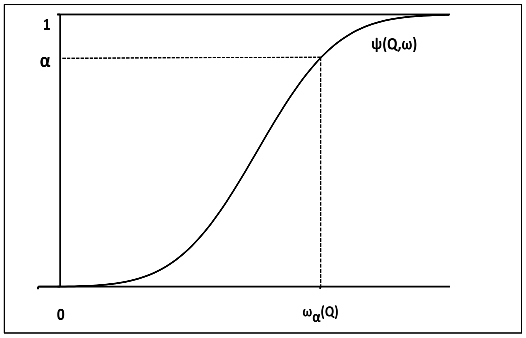
For a specified confidence level $\alpha \in (0, 1)$ the α-VaR is given by:

$$\alpha\text{-}VaR(Q) = \omega_\alpha(Q) = inf\{\omega | \Psi(Q, \omega) \geq \alpha\} \tag{3.29}$$

With the assumption of a continuous loss distribution equation (3.29) is written as:

$$\alpha\text{-}VaR(Q) = \omega_\alpha(Q) = min\{\omega | \Psi(Q, \omega) \geq \alpha\} \tag{3.30}$$

The assumption that $l(X, Q)$ is continuous in Q guarantees that the distribution function of loss $\Psi(Q, \omega)$ is also continuous and increasing. This implies that there is a unique minimum at which we observe $\Psi(Q, \omega) = \alpha$. For a risk aversion level α, continuous and discrete loss distributions and the corresponding α-VaR values are represented in the following figures.



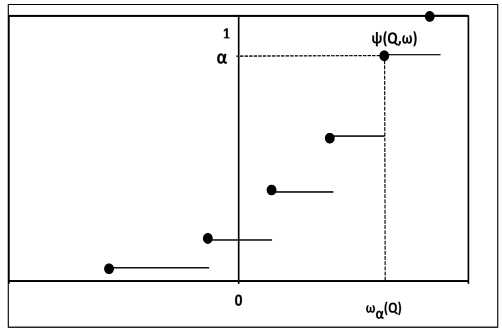


Figure 3.1: Distribution functions ,$\Psi(Q,\omega)$, VaR values at α, $\omega_\alpha(Q)$, for continuous and discrete loss functions $l(X,Q)$, respectively.

By definition, α-CVaR is the expectation of the α-tail distribution. Let $\Psi_\alpha(Q,\omega)$ denote the α-tail distribution of the loss $l(X,Q)$, consider the following analysis:

$$\begin{aligned}
\Psi_\alpha(Q,\omega) &= P(l(X,Q) \leq \omega | l(X,Q) \geq \omega_\alpha(Q)) \\
&= \frac{P(\omega_\alpha(Q) \leq l(X,Q) \leq \omega)}{P(l(X,Q) \geq \omega_\alpha(Q))} = \frac{P(l(X,Q) \geq \omega_\alpha(Q)) - P(l(X,Q) > \omega)}{1-\alpha} \\
&= \frac{1 - P(l(X,Q) > \omega) - [1 - P(l(X,Q) \geq \omega_\alpha(Q))]}{1-\alpha} \\
&= \frac{(P(l(X,Q) \leq \omega) - P(l(X,Q) < \omega_\alpha(Q))}{1-\alpha} = \frac{P(l(X,Q) \leq \omega) - \alpha}{1-\alpha} \\
&= \frac{\Psi(Q,\omega) - \alpha}{1-\alpha} \qquad (3.31)
\end{aligned}$$

which reduces the α-tail distribution to the following:

$$\Psi_\alpha(Q,\omega) = \begin{cases} 0 & \text{for } \omega < \omega_\alpha(Q) \\ [\Psi(Q,\omega) - \alpha] \,/\, [1-\alpha] & \text{for } \omega \geq \omega_\alpha(Q) \end{cases} \qquad (3.32)$$

Rockafellar and Uryasev [40] also define the following relation:

$$E\left[l(X,Q)|l(X,Q) \geq \omega_\alpha(Q)\right] \leq \phi_\alpha(Q) \leq E\left[l(X,Q)|l(X,Q) > \omega_\alpha(Q)\right] \qquad (3.33)$$

which basically implies that the α-$CVaR$ can be approximated by taking the expectation of the α-tail distribution of the loss.

The expected value of the α-tail distribution is given as:

$$\phi_\alpha(Q) = \int_{\omega_\alpha(Q)}^{\infty} u d\Psi_\alpha(Q,u) = -\int_{\omega_\alpha(Q)}^{\infty} u d\bar{\Psi}_\alpha(Q,u) = -\int_{\omega_\alpha(Q)}^{\infty} u \frac{d\bar{\Psi}(Q,u)}{1-\alpha} \qquad (3.34)$$

Applying integration by parts leads to :

$$\begin{aligned} -\int_{\omega_\alpha(Q)}^{\infty} u \frac{d\bar{\Psi}(Q,u)}{1-\alpha} &= \frac{-1}{1-\alpha}\left(u\bar{\Psi}(Q,u) - \int_{\omega_\alpha(Q)}^{\infty} \bar{\Psi}(Q,u)du\right)\Bigg|_{\omega_\alpha(Q)}^{\infty} \\ &= \frac{-u\bar{\Psi}(Q,u)}{1-\alpha}\Bigg|_{\omega_\alpha(Q)}^{\infty} + \frac{1}{1-\alpha}\int_{\omega_\alpha(Q)}^{\infty} \bar{\Psi}(Q,u)du \\ &= \frac{\omega_\alpha(Q)\bar{\Psi}(Q,\omega_\alpha(Q))}{1-\alpha} + \frac{1}{1-\alpha}\int_{\omega_\alpha(Q)}^{\infty} \bar{\Psi}(Q,u)du \\ &= \omega_\alpha(Q) + \frac{1}{1-\alpha}\int_{\omega_\alpha(Q)}^{\infty} \bar{\Psi}(Q,u)du \qquad (3.35) \end{aligned}$$

For any distribution function F we will use $\bar{F} \equiv 1 - F$ which leads to $\bar{\Psi}(\omega_\alpha(Q)) = 1 - \Psi(\omega_\alpha(Q)) = 1 - \alpha$.

Next we consider the expected value of the positive part of a random variable. In particular, let

$$Y = (L - \alpha)^+ = \begin{cases} L - \alpha & \text{if } L > \alpha \\ 0 & \text{o.w.} \end{cases} \tag{3.36}$$

where L is a random variable with distribution function F. Then,

$$\begin{aligned} E[Y] &= -\int_{\alpha}^{\infty} (u - \alpha) d\bar{F}(u) \\ E[Y] &= E[(L - \alpha)^+] = \int_{\alpha}^{\infty} \bar{F}(u) du \end{aligned} \tag{3.37}$$

which follows by applying integration by parts.

Using Equations (3.35) and (3.37), $\phi_\alpha(Q)$ is given as follows:

$$\phi_\alpha(Q) = \omega_\alpha(Q) + \frac{1}{1-\alpha} E\left[[l(X, Q) - \omega_\alpha(Q)]^+\right] \tag{3.38}$$

Since the α-VaR is obtained as a by product of α-$CVaR$, the above expression is represented as an auxiliary function depending on two variables: ω and Q. Rockafellar and Uryasev [40] provide the following auxiliary function for the loss minimization.

$$F_\alpha(Q, \omega) = \omega + \frac{1}{1-\alpha} E\left[[l(X, Q) - \omega]^+\right] \tag{3.39}$$

Value at risk, VaR, and conditional value at risk, CVaR, are two commonly used risk measures, particularly used for portfolio investments in financial management. These tools are also encountered in operations management problems in which risk aversion is under consideration.

As a financial risk tool, VaR is defined as the maximum loss that can be observed on a portfolio over a fixed time horizon, at a specified confidence level (Luciano et al. [41]). With the loss minimization objective, the term α-VaR refers to the threshold below which the loss will fall with probability α (Rockafellar and Uryasev [40]). Therefore, the bigger the α the more risk averse we are.

VaR gives the information about the threshold value for loss at the specified confidence levels. Therefore, by using VaR one cannot predict what will happen beyond the threshold value. For instance, by using VaR as a risk tool a manager is not prepared for the losses above it.

CVaR is defined as the conditional average loss given that the loss is beyond the threshold. It gives information about the values that loss function can take beyond VaR. As the interpretation of it implies, the CVaR originates from VaR. Computation of CVaR gives VaR as a by-product. CVaR is interpreted as the conditional expected loss when the loss is greater than α-VaR. Therefore; if the loss is in the $1-\alpha$ quantile then the conditional expected loss gives α-$CVaR$. Moreover; CVaR minimization problem amounts to expected profit maximization when $\alpha = 0$.

Profit Maximization

Based on the results of Rockafellar and Uryasev [40] we next formulate the corresponding expressions for profit maximization. The rationale behind the above expression can be expressed as follows.

First note that if the loss function is replaced with the reward function then the threshold represents the maximum expected value of the reward that can be attained. Under this setting, the expected value of the reward function falling below the threshold is maximized. Let $\pi(X, Q)$ denote the reward (profit) function where

X represents the random demand and Q is the order quantity, the decision variable. In the profit maximization setting η is chosen to represent the risk aversion level in the literature. (see Chen et al [27]). Throughout our calculations η will be used as risk aversion parameter. The following observations are done by analogy of the ones provided in the loss minimization analysis.

For a given Q the distribution function of the profit is written as:

$$\Psi(Q, \omega) = P\{\pi(X, Q) \leq \omega\} \tag{3.40}$$

For $\eta \in (0, 1]$ the α-VaR is given by:

$$\eta\text{-}VaR(Q) = \omega_\eta(Q) = inf\{\omega | \Psi(Q, \omega) \geq \eta\} \tag{3.41}$$

With the assumption of a continuous profit distribution equation (3.41) is written as:

$$\eta\text{-}VaR(Q) = \omega_\eta(Q) = min\{\omega | \Psi(Q, \omega) \geq \eta\} \tag{3.42}$$

The assumption that $\pi(X, Q)$ is continuous in Q guarantees that the distribution function of profit $\Psi(Q, \omega)$ is also continuous and increasing. This implies that there is a unique minimum at which we observe $\Psi(Q, \omega) = \eta$. For a risk aversion level η, continuous and discrete profit distributions and the corresponding η-VaR values are represented in the following figures.

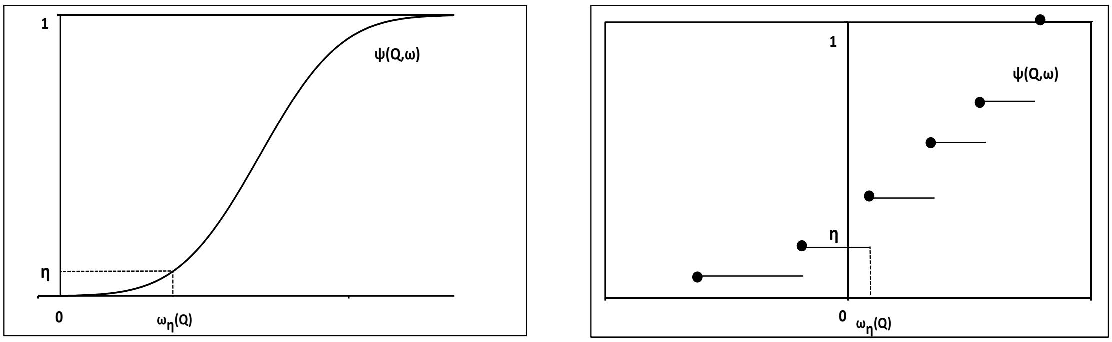


Figure 3.2: Distribution functions ,$\Psi(Q, \omega)$, VaR values at η, $\omega_\eta(Q)$, for continuous and discrete profit functions $\pi(X, Q)$, respectively.

In order to determine η-$CVaR$ we shall take the expectation of the η-tail distribution. Let $\Psi_\eta(Q, \omega)$ denote the η-tail distribution of the profit $\pi(X, Q)$, consider the following analysis:

$$
\begin{aligned}
\bar{\Psi}_\eta(Q, \omega) &= P(\pi(X, Q) > \omega | \pi(X, Q) \leq \omega_\eta(Q)) \\
&= \frac{P(\omega < \pi(X, Q) \leq \omega_\eta(Q))}{P(\pi(X, Q) \leq \omega_\eta(Q))} = \frac{P(\pi(X, Q) > \omega) - P(\pi(X, Q) > \omega_\eta(Q))}{\eta} \\
&= \frac{\bar{\Psi}(Q, \omega) - (1 - \eta)}{\eta} = \frac{1 - \Psi(Q, \omega) - (1 - \eta)}{\eta} \\
&= \frac{-\Psi(Q, \omega) + \eta}{\eta} \qquad (3.43)
\end{aligned}
$$

which reduces the η-tail distribution to the following:

$$
\Psi_\eta(Q, \omega) = \begin{cases} \frac{\Psi(Q, \omega)}{\eta} & \text{for } \omega < \omega_\eta(Q) \\ 0 & \text{for } \omega \geq \omega_\eta(Q) \end{cases} \qquad (3.44)
$$

The expected value of the η-tail distribution is given as:

$$\phi_\eta(Q) = \int_{-\infty}^{\omega_\eta(Q)} u d\Psi_\eta(Q,u) = -\int_{-\infty}^{\omega_\eta(Q)} u d\bar{\Psi}_\eta(Q,u) = \int_{-\infty}^{\omega_\eta(Q)} u \frac{d\Psi(Q,u)}{\eta} \tag{3.45}$$

Applying integration by parts leads to :

$$\begin{aligned}
\int_{-\infty}^{\omega_\eta(Q)} u \frac{d\Psi(Q,u)}{\eta} &= \frac{1}{\eta}\left(u\Psi(Q,u) - \int_{-\infty}^{\omega_\eta(Q)} \Psi(Q,u)du \right)\Bigg|_{-\infty}^{\omega_\eta(Q)} \\
&= \frac{u\Psi(Q,u)}{\eta}\Bigg|_{-\infty}^{\omega_\eta(Q)} - \frac{1}{\eta}\int_{-\infty}^{\omega_\eta(Q)} \Psi(Q,u)du \\
&= \frac{\omega_\eta(Q)\Psi(Q,\omega_\eta(Q))}{\eta} - \frac{1}{\eta}\int_{-\infty}^{\omega_\eta(Q)} \Psi(Q,u)du \\
&= \omega_\eta(Q) - \frac{1}{\eta}\int_{-\infty}^{\omega_\eta(Q)} \Psi(Q,u)du
\end{aligned} \tag{3.46}$$

Next we consider the expected value of the negative part of a random variable. In particular, let

$$Y = (L-\alpha)^- = \begin{cases} L-\alpha & \text{if } L < \alpha \\ 0 & \text{o.w.} \end{cases}$$

where L is a random variable with distribution function F. Then,

$$\begin{aligned} E[Y] &= -\int_{-\infty}^{\alpha} (u-\alpha)dF(u) \\ E[Y] &= E[(L-\alpha)^-] = -\int_{-\infty}^{\alpha} F(u)du \end{aligned} \quad (3.47)$$

which follows by applying integration by parts.

Using Equations (3.46) and (3.47), $\phi_\eta(Q)$ is given as follows:

$$\phi_\eta(Q) = \omega_\eta(Q) + \frac{1}{\eta} E\left[[\pi(X,Q) - \omega_\eta(Q)]^-\right] \quad (3.48)$$

The auxiliary function given in Equation (3.39) can be rewritten for the profit maximization setting as:

$$F_\eta(Q,\omega) = \omega + \frac{1}{\eta} E\left[[\pi(X,Q) - \omega]^-\right] \quad (3.49)$$

In operations management VaR represents a threshold for profit at a pre-specified confidence level. When the objective is to maximize the profit, VaR represents the η quantile of the profit distribution where η is the confidence level. Hence, η-VaR is the value, below which the profit falls with probability η. For this setting, the smaller the η the more risk averse we are.

CVaR is defined as the conditional average profit given that the profit is below the threshold. It gives information about the values that profit function can take below VaR. CVaR is the conditional expected value of the profit given that it falls below the η-VaR. Therefore; if the profit is in the η quantile, then the conditional expected profit gives the η-$CVaR$ value and the probability that the expected profit will be higher than the threshold is 1-η. CVaR maximization problem amounts to expected profit maximization when $\eta = 1$.

The CVaR is a coherent risk measure meeting the desirable convexity, monotonicty, subadditivity, translation equivariance and positive homogeneity properties (Artzner et al. [42]).

The papers of Rockafellar and Uryasev, Optimization of Conditional Value at Risk [39], [40] are the keystones in the literature for CVaR and its minimization, CVaR and VaR relation.

Proposition 3.3.1 The equation (3.49) is concave in Q and ω.

Proof. If the reward function, $\pi(X, Q)$, is concave with respect to Q, then $\phi_\eta(Q)$ is concave with respect to Q, as well. Certainly, in this case $F_\eta(Q, \omega)$ is jointly concave in Q and ω (See Rockafellar and Uryasev, [40]). Now, we shall prove that the profit function $\pi(X, Q)$ is concave in Q for a realized demand x.
The profit function $\pi(X, Q)$ can be written in three regions with respect to the order quantity, Q for a demand realization x.

$$\pi(x, Q) = \begin{cases} (p-c)Q - l(x-Q) & \text{if } Q < x \\ (p-c)Q & \text{if } Q = x \\ (p-c)Q - (p-s)(Q-x) & \text{if } Q > x \end{cases} \tag{3.50}$$

The piecewise profit function is linear in Q in each region which implies piecewise concavity. In order to guarantee the concavity of the overall profit function the following relation must be satisfied:

$$\frac{d\pi(x,Q)}{dQ}\Big|_{Q=x^+} < \frac{d\pi(x,Q)}{dQ}\Big|_{Q=x^-} \tag{3.51}$$

where; $\frac{d\pi(x,Q)}{dQ}\Big|_{Q=x^+} = -(c-s)$ and $\frac{d\pi(x,Q)}{dQ}\Big|_{Q=x^-} = (p-c+l)$.
$-(c-s) < (p-c+l)$ imply concavity of the profit function . □

Computing CVaR by the help of equation (3.49) eases the computation and helps to determine analytical expressions for order quantity and the η-VaR which will be represented as ω_η in our problem. Throughout this paper equation (3.49) will be used for optimization of CVaR.

Chapter 4

Problem 1: Newsvendor Problem in the CVaR Maximization Objective with Carbon Emission Concerns

Before moving on to carbon emission restrictions, it is useful to examine the newsvendor problem under unconstrained CVaR maximization setting. As discussed earlier, CVaR is the expected value of the function under consideration which falls below a threshold value for a fixed risk aversion level, $\eta \in (0, 1]$ in the maximization setting. The threshold value for profit is also a decision variable which is determined simultaneously with the order/production quantity.

4.1 Newsvendor Problem Under Unconstrained CVaR Maximization

The unconstrained CVaR maximization problem for the newsvendor at a fixed risk aversion level $\eta \in (0, 1]$ is given as follows:

$$\text{Max}_{Q,\omega\in\Re} \quad \{\omega + \frac{1}{\eta} E\left[[(\pi(X, Q) - \omega]^{-}\right]\} \tag{4.1}$$

where $\pi(X, Q)$ is the profit function of the classical newsvendor problem given in equation (3.1) in Chapter 3. The explicit form of the unconstrained problem is written as follows:

$$\text{Max}_{Q,\omega\in\Re} \quad \{\omega + \frac{1}{\eta} \int_0^\infty \left[(p - c)Q - (p - s)(Q - x)^{+} - l(x - Q)^{+} - \omega\right]^{-} dF(x)\} \tag{4.2}$$

where ω represents the threshold for profit in η quantile and the optimal ω value corresponds to the η-VaR.

The loss minimization version of the above problem is provided in Gotoh and Takano [24]. Here we obtain the analogous result for our optimization problem.

Proposition 4.1.1 At a fixed risk aversion level η the optimal solution of equation (4.2) is (Q_{unc},ω_{unc}) where;

$$Q_{unc} = (\frac{l}{p - s + l})F^{-1}(\frac{p - c + l + (c - s)(1 - \eta)}{p - s + l}) + (\frac{p - s}{p - s + l})F^{-1}(\frac{(p - c + l)(\eta)}{p - s + l}) \tag{4.3}$$

$$\omega_{unc} = (\frac{(p - s)(p - c + l)}{p - s + l})F^{-1}(\frac{(p - c + l)\eta}{p - s + l}) - (\frac{l(c - s)}{p - s + l})F^{-1}(\frac{p - c + l + (c - s)(1 - \eta)}{p - s + l}) \tag{4.4}$$

Proof. The concavity of CVaR in Q and ω is proven in Proposition 3.3.1 which implies existence of a unique pair (Q, ω) maximizing CVaR. In order to find the optimal Q and ω we make use of the first order conditions of the objective function w.r.t. Q and ω, which imply optimality.

First, we shall rewrite the equation (4.2) explicitly and investigate it.

$$CVaR_\eta(Q,\omega) = \omega + \frac{1}{\eta}\left(\int_0^\infty [(p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ - \omega]^- dF(x)\right)$$

$$= \omega + \frac{1}{\eta}\left(\int_0^Q [(p-c)Q - (p-s)(Q-x) - \omega]^- dF(x) + \int_Q^\infty [(p-c)Q - l(x-Q) - \omega]^- dF(x)\right)$$

Note that $\quad (p-c)Q - (p-s)(Q-x) - \omega \leq 0 \quad \text{if} \quad x \leq \dfrac{(c-s)Q+\omega}{p-s} = U$

and $\quad (p-c)Q - l(x-Q) - \omega \leq 0 \quad \text{if} \quad x \geq \dfrac{(p-c+l)Q-\omega}{l} = V$

Then, by rearranging the limits of integral we have:

$$CVaR_\eta(Q,\omega) = \omega + \frac{1}{\eta}\left(\int_0^U [-(c-s)Q + (p-s)x - \omega] dF(x) + \int_V^\infty [(p-c+l)Q - lx - \omega] dF(x)\right) \tag{4.5}$$

Taking the first derivative of the equation above with respect to Q and ω we have:

$$\frac{\partial CVaR_\eta(Q,\omega)}{\partial Q} = \frac{1}{\eta}\big((p-c+l)[1-F(V)] - (c-s)F(U)\big) \tag{4.6}$$

$$\frac{\partial CVaR_\eta(Q,\omega)}{\partial \omega} = 1 - \frac{1}{\eta}\big(1 - F(V) + F(U)\big) \tag{4.7}$$

Equating equations (4.6) and (4.7) to zero and solving them simultaneously gives the result. □

Corollary 4.1.1: As we discussed in Chapter 3, Section 3.3, the CVaR maximization problem amounts to expected profit maximization at η=1. When η=1 is inserted in equations (4.3) and (4.4) the equations reduces to the following expressions:

$$\begin{aligned} Q_{unc} &= F^{-1}(\frac{p-c+l}{p-s+l}) \\ \omega_{unc} &= (p-c)F^{-1}(\frac{p-c+l}{p-s+l}) \end{aligned} \tag{4.8}$$

4.2 Newsvendor Problem Under CVaR Maximization and Mandatory Carbon Cap Policy

Mandatory Cap policy is one of the common initiatives to curb carbon emissions of the companies. In this policy, firms are given a carbon emission quota and not allowed to exceed it. The carbon emission level of the company can at most be the given quota, which is also called the mandatory cap, and if there is any unused carbon credits the company cannot trade them. In this problem, the newsvendor's optimal order/production quantity is investigated under this setting with the aim of maximizing CVaR.

$\pi(X,Q)$ which is the well-known profit function of the newsvendor given as follows:

$$\pi(X,Q) = (p-c)Q - (p-s)(Q-X)^{+} - l(X-Q)^{+} \tag{4.9}$$

For a fixed risk aversion level $\eta \in (0, 1]$ and given strict carbon cap, K, the optimization problem of the newsvendor is given as follows:

$$\text{Max}_{Q,\omega\in\Re} \quad \{\omega + \frac{1}{\eta}E[[\pi(X,Q) - \omega]^-]\}$$
$$\text{s.t.}$$
$$Q\alpha \leq K \tag{4.10}$$

The carbon emission constraint can be moved to the objective function with a positive Lagrange multiplier, λ, which can be interpreted as the shadow price of producing one more unit that results in exceeding the carbon cap. The new problem which is equivalent to the one given in equation (4.10) is given as:

$$\text{Max}_{Q,\omega,\lambda\in\Re} \quad \{\omega + \frac{1}{\eta}E[[\pi(X,Q,\lambda,K) - \omega]^-]\} \tag{4.11}$$

where;

$$\pi(X,Q,\lambda,K) = (p-c)Q - (p-s)(Q-X)^+ - l(X-Q)^+ - \lambda(Q\alpha - K) \tag{4.12}$$

The objective function can explicitly be written as:

$$CVaR_\eta(Q,\omega,\lambda,K) = \omega + \frac{1}{\eta}\left(\int_0^\infty [(p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ - \lambda(Q\alpha - K) - \omega]^- dF(x)\right) \tag{4.13}$$

Proposition 4.2.1 For a given λ, the optimal order/production quantity and threshold for profit maximizing equation (4.11) are:

$$Q(\lambda) = \left(\frac{l}{p-s+l}\right)F^{-1}\left(\frac{(p-s+l)-(c-s+\lambda\alpha)\eta}{p-s+l}\right) + \left(\frac{p-s}{p-s+l}\right)F^{-1}\left(\frac{(p-c-\lambda\alpha+l)\eta}{p-s+l}\right) \quad (4.14)$$

$$\omega(\lambda) = \left(\frac{(p-s)(p-c-\lambda\alpha+l}{p-s+l}\right)F^{-1}\left(\frac{(p-c-\lambda\alpha+l)\eta}{p-s+l}\right) - \left(\frac{l(c-s+\lambda\alpha)}{p-s+l}\right)F^{-1}\left(\frac{(p-s+l)-(c-s+\lambda\alpha)\eta}{p-s+l}\right) + \lambda K \quad (4.15)$$

where at the optimal solution λ^* satisfies $K/\alpha = Q$.

Proof. The concavity of CVaR in Q and ω is proven in Proposition 3.3.1. In this case we add a term, $\lambda(Q\alpha - K)$, to CVaR which does not violate concavity since the term is linear in Q. Hence the optimal Q and ω are obtained from the FOC given as follows:

$$\frac{\partial CVaR_\eta(Q,\omega,K)}{\partial Q} = \frac{1}{\eta}\big((p-c-\lambda\alpha+l)[1-F(V)] - (c-s+\lambda\alpha)F(U)\big) \quad (4.16)$$

$$\frac{\partial CVaR_\eta(Q,\omega,K)}{\partial \omega} = 1 - \frac{1}{\eta}\big(1 - F(V) + F(U)\big) \quad (4.17)$$

where U and V are given as:

$$U = \frac{(c-s+\lambda\alpha)Q - \lambda K + \omega}{p-s} \quad \text{and} \quad V = \frac{(p-c-\lambda\alpha+l)Q + \lambda - \omega}{l}$$

Equating the equations (4.16) and (4.17) to zero and solving them simultaneously gives $Q(\lambda)$ and $\omega(\lambda)$. □

Remark 1: Note that $Q(\lambda)$ is optimal if and only if the given carbon cap, K, is less than the emission level of the unconstrained optimal order/production quantity, K_{unc}=$Q_{unc}\alpha$.

Remark 2: Let $Q_c = K/\alpha$, then it must hold that $Q_c = Q(\lambda)$ for the cases where $K < K_{unc}$.

4.3 Newsvendor Problem under CVaR Maximization and Cap and Trade Mechanism

In this problem a risk-averse newsvendor determines the optimal order/production quantity and threshold for the profit that maximizes the conditional expectation of the profit given that it is below the threshold with probability η with a carbon emission restriction. The newsvendor is given a carbon quota denoted by K. He is allowed to trade carbon if the given carbon quota is insufficient to produce the optimal amount or if excess carbon is left after the optimal amount is produced. The carbon trading setting is the same as the one given in the classical newsvendor problem in the preliminaries section.

The profit of the newsvendor is the same as the profit function given in equation (3.7) in classical newsvendor with cap and trade mechanism section which is:

$$\pi(X, Q, K) = (p-c)Q - (p-s)(Q-X)^+ - l(X-Q)^+ - c_b(Q\alpha - K)^+ + c_s(K - Q\alpha)^+ \tag{4.18}$$

The objective function of the carbon restricted risk-averse newsvendor is given as:

$$Max_{Q,\omega\in\Re} \quad \{\omega + \frac{1}{\eta}E\left[[\pi(X,Q,K)-\omega]^{-}\right]\} \tag{4.19}$$

Note that here in the profit function c_b can be interpreted as the penalty of emitting one unit more than the given carbon emission cap and c_s can be interpreted as the gain from emitting one unit less than the given carbon emission cap, K. In that sense the interpretation of c_b and c_s are the same as the interpretation of the Lagrange multipler λ defined in the previous section. Also, the assumption that $c_b > c_s$ given in Chapter 3, Section 3.2 holds in this setting.

Throughout this thesis, the problems are investigated for the case where $p-c+l \geq c_b\alpha > c_s\alpha$.

The explicit form of the objective function can be given as:

$$\begin{aligned} CVaR_\eta(Q,\omega,K) = \omega + \frac{1}{\eta}\Bigg(\int_0^\infty &[(p-c)Q-(p-s)(Q-x)^+ - l(x-Q)^+ \\ &- c_b(Q\alpha-K)^+ + c_s(K-Q\alpha)^+ - \omega]^- dF(x)\Bigg) \end{aligned} \tag{4.20}$$

Proposition 4.3.1 For a given carbon emission quota K and a demand realization x, $\pi(X,Q,K)$ given in equation (4.18) is concave in Q.

Proof. The profit function $\pi(x,Q,K)$ can be written in three regions with respect to the order/production quantity, Q for a given carbon emission quota, K.

$$\pi(x,Q,K) = \begin{cases} (p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ + c_s(K-Q\alpha) & \text{if } Q < K/\alpha \\ (p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ & \text{if } Q = K/\alpha \\ (p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ - c_b(Q\alpha - K) & \text{if } Q > K/\alpha \end{cases} \tag{4.21}$$

In order to prove concavity of equation (4.21) in Q, we shall examine the three regions specified with respect to a demand realization x as : $Q < x$, $Q = x$, $Q > x$.

Region 1: $\mathbf{Q < x}$

In this region equation (4.21) reduces to:

$$\pi(x,Q,K) = \begin{cases} (p-c)Q - l(x-Q) + c_s(K-Q\alpha) & \text{if } Q < K/\alpha \\ (p-c)Q - l(x-Q) & \text{if } Q = K/\alpha \\ (p-c)Q - l(x-Q) - c_b(Q\alpha - K) & \text{if } Q > K/\alpha \end{cases} \tag{4.22}$$

equation (4.22) is piecewise concave in Q since the function in each region is linear in Q. In order to show overall concavity it must be satisfied that:

$$\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^+} < \frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^-} \tag{4.23}$$

where; $\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^+} = (p-c+l-c_b\alpha)$ and $\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^-} = (p-c+l-c_s\alpha)$. The assumption that $c_b\alpha > c_s\alpha$ implies the concavity of $\pi(x,Q,K)$ for the case $Q < x$ respect to Q.

Region 2: $\mathbf{Q = x}$

Rewriting the equation (4.21) gives:

$$\pi(x,Q,K) = \begin{cases} (p-c)Q + c_s(K - Q\alpha) & \text{if } Q < K/\alpha \\ (p-c)Q & \text{if } Q = K/\alpha \\ (p-c)Q - c_b(Q\alpha - K) & \text{if } Q > K/\alpha \end{cases} \tag{4.24}$$

Piecewise concavity of equation (4.24) is guaranteed with the same reasoning as in Case 1. The concavity is satisfied by:

$$\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^+} < \frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^-} \tag{4.25}$$

where; $\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^+} = (p - c - c_b\alpha)$ and $\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^-} = (p - c - c_s\alpha)$. The assumption that $c_b\alpha > c_s\alpha$ implies the concavity of $\pi(x,Q,K)$ for the case $Q = x$ respect to Q.

Region 3: $\mathbf{Q > x}$

The equation (4.21) is rearranged as:

$$\pi(x,Q,K) = \begin{cases} (p-c)Q - (p-s)(Q-x) + c_s(K - Q\alpha) & \text{if } Q < K/\alpha \\ (p-c)Q - (p-s)(Q-x) & \text{if } Q = K/\alpha \\ (p-c)Q - (p-s)(Q-x) - c_b(Q\alpha - K) & \text{if } Q > K/\alpha \end{cases} \tag{4.26}$$

Piecewise concavity of equation (4.26) is guaranteed with the same reasoning as in Case 1 and 2. The concavity is satisfied by:

$$\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^+} < \frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K/\alpha)^-} \tag{4.27}$$

where; $\frac{d\pi(x,Q,K)}{dQ}\bigg|_{Q=(K/\alpha)^+} = -(c-s+c_b\alpha)$ and $\frac{d\pi(x,Q,K)}{dQ}\bigg|_{Q=(K/\alpha)^-} = -(c-s+c_s\alpha)$. The assumption that $c_b\alpha > c_s\alpha$ implies the concavity of $\pi(x,Q,K)$ for the case $Q > x$ respect to Q.

Concavity in each three region implies concavity of equation $\pi(x,Q,K)$ in Q. □

Proposition 4.3.2 For a given carbon emission quota, K, $CVaR_\eta(Q,\omega,K)$ is jointly concave in Q and ω.

Proof. In Proposition 4.3.1 it is shown that $\pi(x,Q,K)$ is concave in Q. Hence, Proposition 3.3.1 implies that $CVaR_\eta(Q,\omega,K)$ is concave in Q and ω. □

The objective function can be rewritten in three regions given below with respect to the order/production quantity Q as follows:

$$CVaR_\eta(Q,\omega,K) = \begin{cases} \omega + \frac{1}{\eta}\Big(\int_0^\infty [(p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ \\ \qquad + c_s(K-Q\alpha) - \omega]^- dF(x)\Big) & \text{if } Q < K/\alpha \\ \omega + \frac{1}{\eta}\Big(\int_0^\infty [(p-c)Q - (p-s)(Q-x)^+ \\ \qquad - l(x-Q)^+ - \omega]^- dF(x)\Big) & \text{if } Q = K/\alpha \\ \omega + \frac{1}{\eta}\Big(\int_0^\infty [(p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ \\ \qquad - c_b(Q\alpha - K) - \omega]^- dF(x)\Big) & \text{if } Q > K/\alpha \end{cases} \tag{4.28}$$

Proposition 4.3.3 For a given carbon emission cap, K, and at a fixed risk aversion level, η, the optimal order/production quantity and threshold for profit belong to the following set:

$$(Q^*, \omega^*) \in \Big((Q_{up}, \omega_{up}), (Q = K/\alpha, \omega(K/\alpha), (Q_{down}, \omega_{down})\Big) \tag{4.29}$$

where;

$$\begin{aligned} Q_{up} &= \left(\frac{l}{p-s+l}\right) F^{-1}\left(\frac{(p-s+l)-(c-s+c_b\alpha)\eta}{p-s+l}\right) \\ &+ \left(\frac{p-s}{p-s+l}\right) F^{-1}\left(\frac{(p-c-c_b\alpha+l)\eta}{p-s+l}\right) \end{aligned} \tag{4.30}$$

$$\begin{aligned} \omega_{up} &= \left(\frac{(p-s)(p-c-c_b\alpha+l}{p-s+l}\right) F^{-1}\left(\frac{(p-c-c_b\alpha+l)\eta}{p-s+l}\right) \\ &- \left(\frac{l(c-s+c_b\alpha)}{p-s+l}\right) F^{-1}\left(\frac{(p-s+l)-(c-s+c_b\alpha)\eta}{p-s+l}\right) + c_b K \end{aligned} \tag{4.31}$$

$Q = K/\alpha$ and $\omega(K/\alpha)$ is the solution of

$$Max_{\omega\in\Re} \quad \{\omega + \frac{1}{\eta} E\Big[\big[(p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ - \omega\big]^-\Big]\} \tag{4.32}$$

satisfying the relation:

$$F\Big(\frac{(p-c+l)Q-\omega}{l}\Big) - F\Big(\frac{(c-s)Q+\omega}{p-s}\Big) = 1-\eta \tag{4.33}$$

$$Q_{down} = \left(\frac{l}{p-s+l}\right) F^{-1}\left(\frac{(p-s+l)-(c-s+c_s\alpha)\eta}{p-s+l}\right) + \left(\frac{p-s}{p-s+l}\right) F^{-1}\left(\frac{(p-c-c_s\alpha+l)\eta}{p-s+l}\right) \tag{4.34}$$

$$\omega_{down} = \left(\frac{(p-s)(p-c-c_s\alpha+l}{p-s+l}\right)F^{-1}\left(\frac{(p-c-c_s\alpha+l)\eta}{p-s+l}\right) - \left(\frac{l(c-s+c_s\alpha)}{p-s+l}\right)F^{-1}\left(\frac{(p-s+l)-(c-s+c_s\alpha)\eta}{p-s+l}\right) + c_sK \tag{4.35}$$

Proof. Concavity of $CVaR_\eta(Q,\omega,K)$ in Q guarantees the piecewise concavity in Q in each region which implies that each region has a unique maximizer. The unique root of the corresponding function in each region is candidate for the optimal solution. The roots of the functions in regions where $Q < K/\alpha$ and $Q > K/\alpha$ are found from the first order conditions while in the second region $Q = K/\alpha$ is directly set.
For the region where $Q < K/\alpha$ the corresponding first order conditions w.r.t. Q and ω are determined by using the Leibniz' rule. The first derivative of the objective function w.r.t. Q and ω when $Q < K/\alpha$ are determined as:

$$\frac{\partial CVaR_\eta(Q,\omega,K)}{\partial Q} = \frac{1}{\eta}\big((p-c+l-c_s\alpha)[1-F(V)]-(c-s+c_s\alpha)F(U)\big) \tag{4.36}$$

$$\frac{\partial CVaR_\eta(Q,\omega,K)}{\partial \omega} = 1+\frac{1}{\eta}\big(F(V)-F(U)-1\big) \tag{4.37}$$

where; $V = \frac{(c-s+c_s\alpha)Q-c_sK+\omega}{p-s}$ and $U = \frac{(p-c+l-c_s\alpha)Q+c_sK-\omega}{l}$.

Equating the equations (4.36) and (4.37) to zero and solving simultaneously gives (Q_{down},ω_{down}).
Solution of the region in which $Q > K/\alpha$ can be found by changing the objective

function with the one given in the equation (4.28) and following the same steps given above.

When the order/production quantity is set $Q = K/\alpha$ then the corresponding optimal threshold for profit, $\omega(K/\alpha)$ which maximizes the equation (4.32) is found from the first order condition.

$$\frac{\partial CVaR_\eta(Q,\omega,K)}{\partial\omega} = 1 - \frac{1}{\eta}\big(1 - F\Big(\frac{(p-c+l)Q-\omega}{l}\Big) + F\Big(\frac{(c-s)Q+\omega}{p-s}\Big)\big)$$

Hence the optimal $\omega(K/\alpha)$ when $Q = K/\alpha$ satisfies:

$$F\Big(\frac{(p-c+l)Q-\omega}{l}\Big) - F\Big(\frac{(c-s)Q+\omega}{p-s}\Big) = 1-\eta \tag{4.38}$$

□

Let K_{up} and K_{down} represent the corresponding emission levels of the cases $Q > K/\alpha$ and $Q < K/\alpha$, respectively, given by:

$$K_{up} = Q_{up}\alpha \qquad \text{and} \qquad K_{down} = Q_{down}\alpha \tag{4.39}$$

(Q_{up}, ω_{up}), and $(Q_{down}, \omega_{down})$ are inversely proportional to c_b and c_s, respectively and the distribution function of demand is a non-decreasing function. These properties help us conclude that $Q_{up} < Q_{down}$, $\omega_{up} < \omega_{down}$ and $K_{up} < K_{down}$ for non-negative values of c_b and c_s.

The optimal policy of the risk-averse newsvendor for a given carbon emission cap, K, is provided in the following Theorem.

Theorem 4.3.1 The optimal policy of the risk averse newsvendor for a given carbon emission allowance, K, at the specified risk aversion level, η, and carbon market prices c_b , c_s is given as:

$$(Q^*, \omega^*) = \begin{cases} (Q_{up}, \omega_{up}) & \text{if } K < K_{up} \\ (Q = K/\alpha, \omega(K/\alpha)) & \text{if } K_{up} \leq K \leq K_{down} \\ (Q_{down}, \omega_{down}) & \text{if } K > K_{down} \end{cases} \tag{4.40}$$

where $\omega(K/\alpha)$ satisfies the relation given in equation (4.33) provided in Proposition 4.3.2.

Proof. As it is given in Proposition 4.3.3 there are three critical points: $(Q_{down}, \omega_{down})$, (Q_{up}, ω_{up}) and $(Q = K/\alpha, \omega(K/\alpha))$. The optimal policy is determined by studying three cases for a given K.

Let us rewrite the objective function given in equation (4.28) in a shorter representation:

$$CVaR_\eta(Q, \omega, K) = \begin{cases} (CVaR_{sell})_\eta(Q, \omega, K) & \text{if } Q < K/\alpha \\ (CVaR_{notrade})_\eta(Q, \omega, K) & \text{if } Q = K/\alpha \\ (CVaR_{buy})_\eta(Q, \omega, K) & \text{if } Q > K/\alpha \end{cases} \tag{4.41}$$

Case 1: $K < K_{up} \equiv K/\alpha < Q_{up}$

$(CVaR_{sell})_\eta(Q, \omega, K)$ is concave and maximized at Q_{down}. Hence it is increasing in $(-\infty, Q_{down})$. $K/\alpha < Q_{up} < Q_{down}$ implies that $(CVaR_{sell})_\eta(Q, \omega, K)$ is also increasing in $(-\infty, K/\alpha)$.

$(CVaR_{buy})_\eta(Q, \omega, K)$ is concave and maximized at Q_{up}. Hence it is increasing in

$(K/\alpha, Q_{up})$ and decreasing in (Q_{up}, ∞).

Thus, $CVaR_\eta(Q, \omega, K)$ is increasing in $(-\infty, Q_{up})$ and decreasing in (Q_{up}, ∞). This implies that $CVaR_\eta(Q, \omega, K)$ is maximized at $Q = Q_{up}$.

Case 2: $K_{up} \leq K \leq K_{down} \equiv Q_{up} \leq K/\alpha \leq Q_{down}$

$(CVaR_{sell})_\eta(Q, \omega, K)$ is concave and maximized at Q_{down}. Hence it is increasing in $(-\infty, Q_{down})$. $K/\alpha < Q_{down}$ implies that $(CVaR_{sell})_\eta(Q, \omega, K)$ is increasing in $(-\infty, K/\alpha)$.

$(CVaR_{buy})_\eta(Q, \omega, K)$ is concave and maximized at Q_{up}. Hence it is decreasing in (Q_{up}, ∞). $Q_{up} \leq K/\alpha$ implies that it is also decreasing in $(K/\alpha, \infty)$.

Thus, $CVaR_\eta(Q, \omega, K)$ is increasing in $(-\infty, K/\alpha)$ and decreasing in $(K/\alpha, \infty)$. This implies that $CVaR_\eta(Q, \omega, K)$ is maximized at $Q = K/\alpha$.

Case 3: $K > K_{down} \equiv K/\alpha > Q_{down}$

$(CVaR_{sell})_\eta(Q, \omega, K)$ is concave and maximized at Q_{down}. Hence it is increasing in $(-\infty, Q_{down})$ and decreasing in $(Q_{down}, K/\alpha)$ since $(K/\alpha > Q_{down})$.

$(CVaR_{buy})_\eta(Q, \omega, K)$ is concave and maximized at Q_{up}. Hence it is decreasing in (Q_{up}, ∞). Since $Q_{up} < Q_{down}$ it is also decreasing in (Q_{down}, ∞).

Thus, $CVaR_\eta(Q, \omega, K)$ is increasing in $(-\infty, Q_{down})$ and decreasing in (Q_{down}, ∞). This implies that $CVaR_\eta(Q, \omega, K)$ is maximized at $Q = Q_{down}$. $\square$

Remark: Note that we previously defined $K_{up}{=}Q_{up}\alpha$, $K_{down}{=}Q_{down}\alpha$ and $K{=}Q(\lambda)\alpha$. Analytical expressions imply that for a continuous demand distribution Q_{up}, Q_{down}, and $Q(\lambda)$ are decreasing functions of c_b, c_s and λ, respectively, since the cumulative distribution function is a one to one and non-decreasing function. Therefore; $Q(\lambda) < Q_{up}$ implies $\lambda > c_b$, $Q(\lambda) > Q_{down}$ implies $c_s > \lambda$ and $Q_{up} < Q(\lambda) < Q_{down}$ implies $c_b > \lambda > c_s$.

According to the relations above, K_{up}, K_{down} and K are also decreasing functions of c_b, c_s and λ, respectively. Therefore, we can say that the rationale behind the policy given in Theorem 4.3.1 comes from the comparison of the relation between carbon buying price, c_b, carbon selling price, c_s and the Lagrange multiplier value, λ, of the order/production quantity when the strict cap policy is implemented.

Corollary 4.3.1: Comparison of the given carbon cap, K with the thresholds K_{up} and K_{down} is similar to the comparison of the carbon prices with the Lagrange multiplier value of using strictly the given cap. Then the following relation holds:
i) if $K < K_{up}$ then $Q(\lambda) < Q_{up}$ which implies $\lambda > c_b$
ii) if $K_{up} \leq K \leq K_{down}$ then $Q_{up} \leq Q(\lambda) \leq Q_{down}$ which implies$c_s \leq \lambda \leq c_b$
iii) if $K > K_{down}$ then $Q(\lambda) > Q_{down}$ which implies $c_s > \lambda$

A special case occurs when $c_b = c_s$. Since the pairs (Q_{up}, ω_{up}) and $(Q_{down}, \omega_{down})$ given in Proposition 4.3.3 are the functions of c_b and c_s, respectively, in the special case of $c_b = c_s$ it is observed that $Q_{up} = Q_{down}$ and $\omega_{up} = \omega_{down}$. Accordingly $K_{up} = K_{down}$. Let $Q_T = Q_{up} = Q_{down}$, $\omega_T = \omega_{up} = \omega_{down}$ and $K_T = K_{up} = K_{down}$.

Corollary 4.3.2: The optimal policy for the special case of $c_b = c_s$ is given as follows:

$$(Q^*, \omega^*) = (Q_T, \omega_T) \;\; \forall \quad \text{K} \tag{4.42}$$

Proof. Since $K_{up} = K_{down}$, the region for given carbon emission quota, K, between K_{up} and K_{down} is reduced to a point which is $K_T = K_{up} = K_{down}$. Hence we analyze the optimal policy for a given carbon cap, K for the cases $K < K_T$, $K > K_T$ and

$K = K_T$ as follows:

$$(Q^*, \omega^*) = \begin{cases} (Q = K/\alpha = Q_T, \omega(K/\alpha) = \omega_T) & \text{if } K = K_T \\ (Q_T, \omega_T) & \text{o.w.} \end{cases} \tag{4.43}$$

The proof follows from the proof of Theorem 4.3.1. □

Remark: Recall that at the beginning of this section we make the assumption that $p - c + l \geq c_b\alpha > c_s\alpha$. All the analytical expressions of Q and ω we determine in this section can be calculated for a parameter set satisfying this assumption. Otherwise, when $c_b\alpha > c_s\alpha > p - c + l$ or $c_b\alpha > p - c + l > c_s\alpha$, we cannot calculate the analytical expressions of Q and ω since in these cases the expression written in the inner side of the inverse cumulative distribution function obtains negative values, (see expressions of Q_{up}, Q_{down}, ω_{up}, ω_{down}), and these are ill-defined cases.

Chapter 5

Problem 2: Newsvendor Problem under CVaR Maximization with Multiple Constraints

In the previous problem the carbon emission restriction can be thought as a resource restriction and the newsvendor is allowed to buy or sell the resource according to his optimal policy. In this section, the newsvendor problem is analyzed under multiple resource constraints where all the resources can be traded at pre-specified trading prices with the aim of CVaR maximization.

Suppose that the newsvendor has N limited resources which he can trade at fixed prices. At a fixed risk-aversion level $\eta \in (0, 1]$, our aim is to find the optimal production quantity, Q, and the threshold for profit, ω, that maximize the conditional value at risk and accordingly the optimal trading behavior is determined.

Let i, $i = 1, 2, ..N$ correspond to the index of N resources. The maximum amount of resource i that can be used in production is represented by K_i. The set of limited resources is represented by $\bar{K}$, where $\bar{K} = \{K_1, K_2, ..K_N\}$. The unit acquisition and selling prices of resource i are defined as c_b^i and c_s^i, respectively, with the assumption of $c_b^i > c_s^i$. It is assumed that α_i is the amount of resource i required to produce a unit product. The profit of the newsvendor, $\pi(X, Q, \bar{K})$, is given as:

$$\pi(X, Q, \bar{K}) = (p-c)Q-(p-s)(Q-X)^+-l(X-Q)^+-\sum_{i=1}^{N} c_b^i(Q\alpha_i-K_i)^+ + \sum_{i=1}^{N} c_s^i(K_i-Q\alpha_i)^+ \tag{5.1}$$

The objective function of the newsvendor is written as:

$$CVaR_\eta(Q, \omega, \bar{K}) = \omega + \frac{1}{\eta}\Bigg(\int_0^\infty [(p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ - \sum_{i=1}^{N} c_b^i(Q\alpha_i - K_i)^+ + \sum_{i=1}^{N} c_s^i(K_i - Q\alpha_i)^+ - \omega]^- dF(x)\Bigg) \tag{5.2}$$

Under this setting the optimization problem of the newsvendor is given as:

$$Max_{Q,\omega\in\Re} \quad \{\omega + \frac{1}{\eta}E\left[\left[\pi(X, Q, \bar{K}) - \omega\right]^-\right]\} \tag{5.3}$$

In order to find the optimal production quantity and threshold for profit maximizing the Equation (5.2), the newsvendor needs to find the best trading option among the possible choices.

Solution Method for Binding Constraints

This problem is an extension of the one with mandatory cap policy which is discussed earlier. Let $Q_{(i)} = K_i/\alpha_i$ be the binding production quantity of resource i and $\omega_{(i)}$ be η-VaR value for the corresponding profit distribution. Since the resources are binding, the optimal production quantity will be either the $min_i\{Q_{(i)}\}$ or the unconstrained solution, Q_{unc} whichever is smaller.

Solution Method for Trading Option

Consider the ordered binding production quantities of resources as $Q_{(1)} < Q_{(2)} < .. < Q_{(N)}$ where the resource giving the minimum binding production quantity is labelled as (1) and the one giving the maximum binding production quantity is labelled as (N). The resource caps and production coefficients, $K_{(i)}$, and $\alpha_{(i)}$, are also labelled accordingly as $K_{(1)}$, $K_{(2)}$, .., $K_{(N)}$ and $\alpha_{(1)}$, $\alpha_{(2)}$, .., $\alpha_{(N)}$. It is assumed that $Q_{(0)}$ is the point $Q = 0$. The set of binding production quantities is represented by $B = \{Q_{(1)}, Q_{(2)}, .., Q_{(N)}\}$. Consider the following action set:

$$A_i = \{\text{buy resources } (j) \text{ for } j \leq i, \text{ sell of resources } (j) \text{ for } j > i; \text{ i=0,1,N}\} \tag{5.4}$$

where $i = 0$ corresponds to the action of selling all resources and $i = N$ corresponds to the action of buying from all of the resources, while $i = k$ is the case of buying from the resources (1) to (k) and selling the resources $(k+1)$ to (N).

The objective function under action i is written as:

$$CVaR^i_\eta(Q,\omega,\bar{K}) = \omega + \frac{1}{\eta}\Bigg(\int_0^\infty [(p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ \\ - \sum_{j=(1)}^{(i)} c_b^j(Q\alpha_j - K_j) + \sum_{j=(i+1)}^{(N)} c_s^j(K_j - Q\alpha_j) - \omega]^- dF(x)\Bigg) \tag{5.5}$$

where a sum is set to zero if lower index is bigger than the upper index.

Proposition 5.1.1 For given ordered resource quotas $K_{(i)}, \quad i = 1, .., N$ and a demand realization of x, Equation (5.1) is concave in Q.

Proof. The profit function $\pi(x, Q, \bar{K})$ can be written in $N + 1$ regions with respect to the ordered resource quotas $K_{(i)}, \quad i = 1, .., N$ as:

$$\pi(x,Q,\bar{K}) = \begin{cases} (p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ + \sum_{i=(1)}^{(N)} c_s^i(K_i - Q\alpha_i) & \text{if } Q \leq K_{(1)}/\alpha_{(1)} \\ (p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ - c_b^{(1)}(Q\alpha_{(1)} - K_{(1)}) + \sum_{i=(2)}^{(N)} c_s^i(K_i - Q\alpha_i) & \text{if } K_{(1)}/\alpha_{(1)} < Q \leq K_{(2)}/\alpha_{(2)} \\ (p-c)Q - (p-s)(Q-x)^+ - l(x-Q)^+ - \sum_{i=(1)}^{(N)} c_b^i(Q\alpha_i - K_i) & \text{if } Q > K_{(N)}/\alpha_{(N)} \end{cases} \tag{5.6}$$

In order to prove concavity of Equation (5.1) in Q, we shall examine three regions specified with respect to a demand realization of x as : $Q < x$, $Q = x$, $Q > x$.

Region 1: $\mathbf{Q < x}$

In this region Equation (5.6) reduces to:

$$\pi(x,Q,\bar{K}) = \begin{cases} (p-c)Q - l(x-Q) + \sum_{i=(1)}^{(N)} c_s^i(K_i - Q\alpha_i) & \text{if } Q \leq K_{(1)}/\alpha_{(1)} \\ (p-c)Q - l(x-Q) - c_b^{(1)}(Q\alpha_{(1)} - K_{(1)}) + \sum_{i=(2)}^{(N)} c_s^i(K_i - Q\alpha_i) & \text{if } K_{(1)}/\alpha_{(1)} < Q \leq K_{(2)}/\alpha_{(2)} \\ (p-c)Q - l(x-Q) - \sum_{i=(1)}^{(N)} c_b^i(Q\alpha_i - K_i) & \text{if } Q > K_{(N)}/\alpha_{(N)} \end{cases} \tag{5.7}$$

Equation (5.7) is piecewise concave in Q since the function in each region is linear in Q. In order to show overall concavity it must be satisfied that:

$$\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K_i/\alpha_i)^+} < \frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K_i/\alpha_i)^-} \tag{5.8}$$

for all $i = (1), .., (N)$.
For i=1;
$\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K_{(1)}/\alpha_{(1)})^+} = (p-c+l-c_b^{(1)}\alpha^{(1)} - \sum_{i=(2)}^{(N)} c_s^i\alpha_i)$ and $\frac{d\pi(x,Q,K)}{dQ}\Big|_{Q=(K_{(1)}/\alpha_{(1)})^-} = (p-c+l-\sum_{i=(1)}^{(N)} c_s^i\alpha_i)$.
The assumption that $c_b^{(1)}\alpha_{(1)} > c_s^{(1)}\alpha_{(1)}$ implies the concavity.
This logic holds for all boundary points: $(K_{(1)}/\alpha_{(1)}), .., (K_{(N)}/\alpha_{(N)})$.
For i=N;

$\left.\frac{d\pi(x,Q,K)}{dQ}\right|_{Q=(K_{(N)}/\alpha_{(N)})^+} = (p - c + l - \sum_{i=(1)}^{(N)} c_b^i \alpha_i)$ and $\left.\frac{d\pi(x,Q,K)}{dQ}\right|_{Q=(K_{(N)}/\alpha_{(N)})^-} = (p - c + l - \sum_{i=(2)}^{(N)} c_b^i \alpha_i - c_s^{(N)} \alpha_{(N)}$.
The assumption that $c_b^{(N)} \alpha_{(N)} > c_s^{(N)} \alpha_{(N)}$ implies the concavity. Hence; $\pi(x, Q, \bar{K})$ is concave in Q for the case $Q < x$.
Region 2: $\mathbf{Q = x}$
Rewriting the Equation (5.6) gives:

$$\pi(x,Q,\bar{K}) = \begin{cases} (p-c)Q + \sum_{i=(1)}^{(N)} c_s^i (K_i - Q\alpha_i) & \text{if } Q \leq K_{(1)}/\alpha_{(1)} \\ (p-c)Q - c_b^{(1)}(Q\alpha_{(1)} - K_{(1)}) + \sum_{i=(2)}^{(N)} c_s^i (K_i - Q\alpha_i) & \text{if } K_{(1)}/\alpha_{(1)} < Q \leq K_{(2)}/\alpha_{(2)} \\ (p-c)Q - \sum_{i=(1)}^{(N)} c_b^i (Q\alpha_i - K_i) & \text{if } Q > K_{(N)}/\alpha_{(N)} \end{cases} \tag{5.9}$$

Piecewise concavity of Equation (5.9) is guaranteed with the same reasoning in Case 1. The concavity requirements are checked by the same methodology given in Case 1. Hence; $\pi(x, Q, \bar{K})$ is concave in Q for the case $Q = x$
Region 3: $\mathbf{Q > x}$ The Equation (5.6) is rearranged as:

$$\pi(x,Q,\bar{K}) = \begin{cases} (p-c)Q - (p-s)(Q-x) + \sum_{i=(1)}^{(N)} c_s^i (K_i - Q\alpha_i) & \text{if } Q \leq K_{(1)}/\alpha_{(1)} \\ (p-c)Q - (p-s)(Q-x) - c_b^{(1)}(Q\alpha_{(1)} - K_{(1)}) + \sum_{i=(2)}^{(N)} c_s^i (K_i - Q\alpha_i) & \text{if } K_{(1)}/\alpha_{(1)} < Q \leq K_{(2)}/\alpha_{(2)} \\ (p-c)Q - (p-s)(Q-x) - \sum_{i=(1)}^{(N)} c_b^i (Q\alpha_i - K_i) & \text{if } Q > K_{(N)}/\alpha_{(N)} \end{cases} \tag{5.10}$$

Piecewise concavity of Equation (5.10) is guaranteed with the same reasoning in Case 1 and 2. The concavity is satisfied by again following the same path given in Case 1. Hence; $\pi(x, Q, \bar{K})$ is concave in Q for the case $Q > x$. Concavity of $\pi(x, Q, K)$ in each three region implies concavity of Equation (5.1) in Q. □

Proposition 5.1.2 For given resource limits $K_{(i)}, i = 1, 2, ..N$, the optimal solution of action i is (Q_i, ω_i) where;

$$\begin{aligned} Q_i &= \left(\frac{l}{p-s+l}\right) F^{-1}\left(\frac{(p-s+l)-(c'-s)\eta}{p-s+l}\right) + \left(\frac{p-s}{p-s+l}\right) F^{-1}\left(\frac{(p-c'+l)\eta}{p-s+l}\right) \\ \omega_i &= \left(\frac{(p-s)(p-c'+l)}{p-s+l}\right) F^{-1}\left(\frac{(p-c'+l)\eta}{p-s+l}\right) - \left(\frac{l(c'-s)}{p-s+l}\right) F^{-1}\left(\frac{(p-s+l)-(c'-s)\eta}{p-s+l}\right) \\ &\quad + \sum_{j=1}^{i} c_b^j K_j + \sum_{j=i+1}^{N} c_s^j K_j \end{aligned} \tag{5.11}$$

where $c' = c + \sum_{j=(1)}^{(i)} c_b^j \alpha_j + \sum_{j=(i+1)}^{(N)} c_s^j \alpha_j$.

Proof. Concavity of Equation (5.1) implies the concavity of Equation 5.2) and accordingly the concavity of Equation (5.5) which follows from Proposition 4.3.2. Hence, the objective function under action i is concave in Q and ω ensuring that there is a unique maximizer of action i which will be obtained from the first order conditions. The first derivatives of the objective function w.r.t. Q and ω are given as:

$$\frac{\partial CVaR_\eta^i(Q,\omega,K_i)}{\partial Q} = \frac{1}{\eta}\big((p-c')[1-F(V)] - (c'-s)F(U)\big) \tag{5.12}$$

$$\frac{\partial CVaR_\eta^i(Q,\omega,K_i)}{\partial \omega} = 1 + \frac{1}{\eta}\big(F(V) - F(U) - 1\big) \tag{5.13}$$

where;

$$\begin{aligned} U &= \frac{(c - s + \sum_{j=(1)}^{(i)} c_b^j \alpha_j + \sum_{j=(i+1)}^{(N)} c_s^j \alpha_j)Q - \sum_{j=(1)}^{(i)} c_b^j K_j - \sum_{j=(i+1)}^{(N)} c_s^j K_j + \omega}{p-s} \\ V &= \frac{(p - c - \sum_{j=(1)}^{(i)} c_b^j \alpha_j - \sum_{j=(i+1)}^{(N)} c_s^j \alpha_j + l)Q + \sum_{j=(1)}^{(i)} c_b^j K_j + \sum_{j=(i+1)}^{(N)} c_s^j K_j - \omega}{l} \\ c' &= c + \sum_{j=(1)}^{(i)} c_b^j \alpha_j + \sum_{j=(i+1)}^{(N)} c_s^j \alpha_j \end{aligned}$$

Equating equations (5.12) and (5.13) and solving simultaneously gives Q_i and ω_i. $\square$

Remark: The expressions given in equation 5.11 are determined with the assumption that the relation $p-c+l \geq max_i\{\sum_{j=(1)}^{(i)} c_b^j\alpha_j + \sum_{j=(i+1)}^{(N)} c_s^j\alpha_j\}$ where i=0, 1, .., N holds.

The optimal production quantity Q_i is determined for the action i which refers to the action of buying from the resources $(j) \leq i$ and selling the resources $(j) > i$ according to actions set A_i. This implies that the amounts of the resources $(j) \leq i$ are insufficient and there is more than enough of the resources $(j) > i$ to produce the optimal amount Q_i. As the resources $(j) \leq i$ are insufficient and the resources $(j) > i$ have excess to produce Q_i, one intuitively expects that the binding production quantities of the resources $(j) \leq i$ will be less than Q_i since these resources are inadequate, and the binding production quantities of the resources $(j) > i$ will be greater than Q_i since they have excess credits. Hence, for each Q_i the requirement that $Q_i \in (Q_{(i)}, Q_{(i+1)})$ must be checked in order not to face a contradiction. Suppose that $Q_i \notin [Q_{(i)}, Q_{(i+1)}]$ and $Q_i > Q_{(i+2)}$. This means that the given credits of resource $(i+2)$ is inadequate to produce Q_i and the newsvendor buys extra credits from the resource $(i+2)$ to produce Q_i. However, Q_i is determined according to the actions set A_i implying that the newsvendor sells unused credits of the resource $(i+2)$. This creates a contradiction. Hence, Q_i is an infeasible production quantity under this setting.

Let $(Q_{if}, \omega_{if}) = (Q_i, \omega_i)$ if $Q_i \in [Q_{(i)}, Q_{(i+1)}]$. This implies that the action i and the corresponding optimal production quantity are feasible under the given setting. Let T be the set of optimal production quantities of the actions set A_i, $i = 1, .., N$

that are feasible which is represented as follows:

$$T = \{(Q_{if}, \omega_{if}) | (Q_{if}, \omega_{if}) = (Q_i, \omega_i) \quad \text{if} \quad Q_i \in [Q_{(i)}, Q_{(i+1)}], i = 0, 1, .., N\} \tag{5.14}$$

The sets B and T are the two feasible sets of production quantities that can be searched to find the optimal solution. Let S represent the set of feasible production quantities and the corresponding optimal thresholds for profit which is formally given as:

$$\begin{aligned} S &= B \cup T \\ S &= \{(Q_{(i)}, \omega_{(i)}); i = 1, .., N, (Q_{if}, \omega_{if}); i = 0, .., N\} \end{aligned} \tag{5.15}$$

Optimal Policy

In order to find the optimal policy the objective function value is determined at each element of the feasible set S. Then, the optimal production quantity, Q^*, and the corresponding threshold for profit, ω^*, of the risk-averse newsvendor is found by picking the (Q, ω) pair with the maximum objective function value. The optimal policy can be summarized as:

$$(Q^*, \omega^*) = \{(Q', \omega') | CVaR_\eta \Big|_{(Q',\omega') \in S} > CVaR_\eta \Big|_{(Q,\omega) \in S - (Q',\omega')}\} \tag{5.16}$$

Let us consider the following numerical example in order to gain some insight about the solution method.

Numerical Example: Let the risk-averse problem parameters be: p=2, c=1, s=0.8, l=3, η=0.01 and demand to be normally distributed with $mean$=500 and *variance*=10000. Suppose that the newsvendor has two limited resources: carbon and cash. Let the resource related parameters are given as *carbon buying price*=1.4, *carbon selling price*=1.33, *production coefficient* (α) *of carbon*=1, *carbon cap*=600, *cash buying price*=1.125, *cash selling price*=1.1, *production coefficient* (α) *of cash*=1, *cash limit*=650. We shall find the optimal production quantity under this setting.

According to the solution method discussed earlier, we shall first find the binding production quantities of carbon and cash. Then, the resources and resource related parameters, decision variables are labelled as 1 and 2 according to the ordered binding production quantities.

- The binding production quantity of carbon resource is $600/1 = 600$
- The binding production quantity of cash resource is $650/1 = 650$

Since $600 < 650$, we label carbon as resource 1, cash as resource 2 and the set of binding production quantities is $B = \{600, 650\}$.

Next, we shall consider the trading actions by analyzing the actions set A_i given in Equation (5.4). According to A_i, the possible actions of this problem and their corresponding production quantities are listed as:

- Q_0: The optimal production quantity when the newsvendor sells both carbon and cash.
- Q_1: The optimal production quantity when the newsvendor buys from carbon and sells cash.
- Q_2: The optimal production quantity when the newsvendor buys both from carbon and cash.

By using Equation (5.11) we calculate the optimal production quantities of trading actions as $Q_0 = 601.9$, $Q_1 = 600.8$, $Q_2 = 600.4$. In order to determine the set T given in Equation (5.14) which includes the optimal production quantities of the corresponding trading actions which are feasible the following relations must hold:

- Q_0 is feasible $\leftrightarrow Q_0 \in [Q_{(0)}, Q_{(1)}] \equiv Q_0 \in [0, 600)$
- Q_1 is feasible $\leftrightarrow Q_1 \in [Q_{(1)}, Q_{(2)}] \equiv Q_1 \in (600, 650)$
- Q_2 is feasible $\leftrightarrow Q_2 \in [Q_{(2)}, Q_{(3)}] \equiv Q_2 \in (650, \infty)$

Recall that we set $Q_{(0)} = 0$ in the solution method part. The following figure clarifies the requirements defined above as follows:

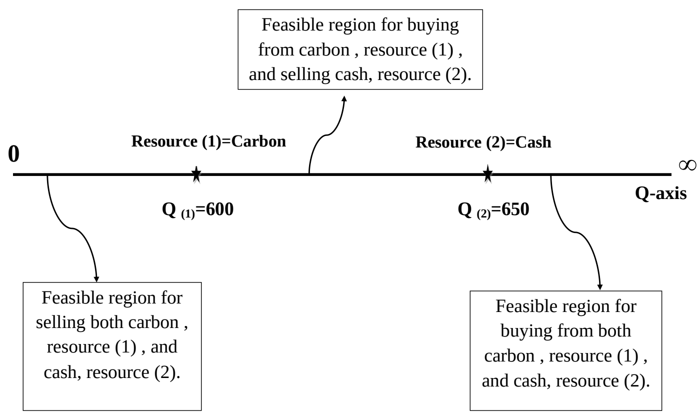


Figure 5.1: Feasible regions for optimal production quantities of actions set A_i.

The feasibility results are summarized by the table given below.

Table 5.1: Trading Actions, Optimal Production Quantities of Actions and Their Feasiblity

Action	Optimal Production Quantity	Feasibility Requirement	Feasibility Check
0	601.9	$601.9 \in [0, 600)$	Infeasible
1	600.8	$600.8 \in (600, 650)$	Feasiblee
2	600.4	$600.4 \in (650, \infty)$	Infeasible

According to the Table 5.1, $T = \{600.8\}$. Then, the set of feasible production quantities S is determined as $S = B \cup T = \{600, 600.8, 650\}$. The corresponding objective function values of the production quantities given in the set S are tabulated as follows:

Table 5.2: Feasible Production Quantities and Corresponding Objective Function Values

Feasible Production Quantity	Objective Function Value
600	134.3308
600.8	134.3529
650	70.223

According to Table 5.2, the optimal production quantity is $Q_1 = 600.8$ giving the maximum objective function value. The corresponding VaR value which is denoted by ω_i can be calculated from Equation (5.11).

Chapter 6

Numerical Study

In this chapter, we provide the results of the numerical experiments conducted to analyze the impacts of problem parameters on the optimal policies of the problems we discussed in previous chapters. The numerical study findings and discussions are provided under three sections.

In section 6.1, the effects of the problem parameters on the optimal order/production quantity, Q^*, and the corresponding service level, SL^*, of the unconstrained newsvendor model under CVaR maximization objective are investigated. Then, for specified sets of problem parameters the impact of changing risk aversion level on Q^* and the optimal value of the objective function, $CVaR^*$ are determined.

In Section 6.2, we study the sensitivity of the newsvendor model with CVaR maximization objective and carbon emission concerns to changing parameters. In Sections 6.2.1 and 6.2.2 we provide and discuss the results of the mandatory carbon cap policy and cap and trade policy, respectively. The changes in Q^*, optimal carbon policy and the optimal values of objective function, $CVaR^*$, value-at-risk, ω^*, and the expected profit, EP^* for changing risk aversion level, η, carbon buying and selling prices, c_b and c_s respectively, given carbon cap, K and carbon cap tightness, τ are provided.

In Section 6.3, we examine the newsvendor problem with CVaR maximization objective and multiple resource constraints for a case where the number of limited resources is 2. We conduct a numerical experiment in order to observe the changes in the optimal production strategy with respect to risk aversion level for the binding resource constraints and resources with trading option. We also analyze the optimal strategy for tradeable resources for changing resource limits.

We obtained the closed form expressions for the optimal values of the decision variables in the previous chapters. In our numerical experiment we implement those expressions in MATLAB and calculate the values for changing problem parameters.

6.1 Unconstrained CVaR Maximization

Before conducting a numerical experiment for our problems we first need to select reasonable parameter sets. For this reason we determine the customer service level at different parameter settings. First, for a normally distributed demand with μ=500 and σ^2=100^2 we specify the selling price to acquisition cost ratio, p/c, as 1.25, 1.5, 1.75, and 2 where c=1. Then, at a fixed η we determine Q^* and SL^* values of the unconstrained CVaR maximization problem for changing salvage, s, and lost sales cost, l, values. We consider s as the percentages of c. The set of salvage values we consider are s=0.7c, 0.75c, 0.8c, 0.85c and 0.85c. Since c = 1 throughout all calculations, the set of salvage values are s=0.7, 0.75, 0.8, 0.85 and 0.85. The values of l are determined as the multiples of profit mark-up. For each p/c value the set of l values is fixed as l=0, $(p-c)$, $2(p-c)$,and $3(p-c)$. We calculate Q^* and SL^* for η=0.01, 0.1, and 0.25. Hence we examine $4 \times 5 \times 4 \times 3$=240 Q^* and SL^* values which are provided in Tables 6.1, 6.2 and 6.3. For a fixed η, we observe that Q^* and SL^* increases with increasing l and s. The same outcome is valid for increasing p/c value. For example, in Table 6.1 we see that at p=1.25, c=1, s=0.7 and l=0 setting Q^*=239.14, SL^*=0.0045 while increasing s value to 0.9 under the same setting gives Q^*=255, SL^*=0.0071. Also, at p=2, c=1, s=0.7 and l=0 we see that Q^*=257.68, SL^*=0.0077. As can be seen in Tables 6.2 and 6.3 this result is valid for each η.

When the impact of risk aversion is investigated, intuitively one expects Q^* to increase with decreasing risk aversion, meaning increasing η value. However, Q^* and η relation comes out to be parameter sensitive which is parallel with findings of Katariya et al. [30]. As can be observed in Tables 6.1, 6.2, 6.3 this relation depends on l value. Q^* increases with increasing η under the settings with small values of l while it increases with decreasing η with high values of l. In order to focus on this behavior we analyze the following settings:

1) p=1.5, c=1, s=0.8 for l=0.5, 1, and 1.5

2) p=1.75, c=1, s=0.8 for l=0.75, 1.5, and 2.25

3) p=2, c=1, s=0.8 for l=1, 2, and 3

for a normally distributed demand with μ=500 and $\sigma^2=100^2$.

Figures 6.1, 6.2, and 6.3 summarize the impact of risk aversion level on Q^* and $CVaR^*$ values for each set defined above, respectively. The corresponding data of the figures are tabulated in Tables 6.4, 6.5 and 6.6. In Figures 6.1, 6.2, and 6.3 we see that Q^* and η relation affirms the intuition of obtaining higher order quantities at higher η values when l=$p-c$, where $p-c$ is the profit mark-up value. The counter-intuitive result is observed for the cases where l=2($p-c$) and 3($p-c$). According to Tables 6.4, 6.5 and 6.6 Q^* increases when η decreases from 0.1 to 0.01 and when η increases from 0.1 to 1 for the settings with l=2($p-c$) while it increases when η decreases from 0.4 to 0.01 and when η increases from 0.4 to 1 for the settings with l=3($p-c$). Also, we see that when l=3($p-c$) Q^* at η=0.01 is greater than the Q^* value at the risk neutral case , at $\eta = 1$ which is another counter-intuitive result.

Since CVaR maximization problem amounts to expected profit maximization at $\eta = 1$, one expects CVaR to increase with increasing η value for any parameter setting. This expectation is verified by Figures 6.1, 6.2 and 6.3.

6.2 CVaR Maximization with Carbon Emission Concerns

The optimal order quantity and customer service level analysis we provide in the previous section helps us to pick a justifiable parameter setting for a detailed investigation. By the help of data provided in Tables 6.1, 6.2, 6.3, we set the newsvendor problem parameters as p=2, c=1, s=0.8, l=3 giving SL^*=0.9545, 0.9303 and 0.9209 at η=0.01, 0.1, 0.25, respectively, for a normally distributed demand with μ=500 and σ^2=100^2. We set the carbon emission coefficient α=1. Therefore for the strict cap and cap and trade analyses we have three parameter sets:

1) p=2, c=1, s=0.8, l= 3, α=1, η=0.01

2) p=2, c=1, s=0.8, l= 3, α=1, η=0.1

3) p=2, c=1, s=0.8, l= 3, α=1, η=0.25

All of the calculations in the numerical experiment of this thesis are conducted for a normally distributed demand with μ=500 and σ^2=100^2.
In order to investigate the impact of carbon cap on the optimal solution we consider a given carbon cap tightness which can be defined as the percentage reduction of the carbon cap that is emitted at the unconstrained optimal solution. Let τ, K_{unc} and K represent the carbon cap tightness, carbon emission at the unconstrained optimal solution and the given carbon cap, respectively, then K=$(1-\tau)K_{unc}$.

6.2.1 CVaR Maximization with Strict Cap Policy

As we discussed earlier, under the mandatory cap policy the newsvendor cannot emit more than the given cap. Hence, for a given carbon cap, K, Q^*=$min(Q_{unc}, K/\alpha)$ where Q_{unc} is the optimal solution of the unconstrained problem. For all values of $K > K_{unc}$, where K_{unc}=$Q_{unc}\alpha$, Q^*=Q_{unc}. In order to investigate the impact of K, we study the cases where $K < K_{unc}$ by introducing carbon cap tightness, τ, explained above.

In this section we study the impacts of increasing τ on $CVaR^*$, SL^* and EP^* at η=0.01, 0.1, 0.25. For the same risk aversion levels we also study %Decrease in $CVaR^*$, %Decrease in EP^*, %Decrease in $CVaR^*$ / %Decrease in Q^*, and % Decrease in EP^* / %Decrease in Q^* and τ relation.

SL^*, $CVaR^*$, and EP^* versus τ relations are illustrated in Figures 6.4, 6.5 and 6.6 for η=0.01, 0.1, 0.25, respectively. The general conclusion is that the tighter the given cap, the less value SL^*, $CVaR^*$, and EP^* obtain.

Tables 6.7, 6.8 and 6.9 provides the data of Figures 6.4, 6.5 and 6.6 and % Decrease in Q^*, $CVaR^*$, EP^*, %Decrease in $CVaR^*$ / %Decrease in Q^*, and % Decrease in EP^* / %Decrease in Q^* for further analysis.

In Tables 6.7, 6.8 and 6.9 the range of τ values investigated are different. Recall that the τ is defined as the percentage reduction from the emission level of the unconstrained CVaR maximization problem which is a function of η. At each η we first determine the emission level of unconstrained CVaR maximization problem and then determine the carbon cap values corresponding to each τ value. Since the carbon emission coefficient per unit production,α, is set to be 1, those carbon cap values are the optimal production quantities also. As given in Chapter 4, Section 4.2, producing according to a binding constraint incurs a Lagrange multiplier value, λ. In order to determine the VaR^* and accordingly $CVaR^*$ we determine the λ values of each τ for a fixed η by the help of FMINCON function in MATLAB to solve the problem:

$$Min \quad \lambda$$

$$\text{s.t.} \quad Q(\lambda) = K\alpha$$

The optimal λ values can take the values up to the cost of underage value, $p - c + l$ since when $\lambda = p - c + l$ the $Q(\lambda)$ given in Equation (4.14) in Chapter 4, converges to $-\infty$ for a demand distribution with a domain of $(-\infty, \infty)$. At each η , the τ value at which $Q(\lambda)$ approaches to $-\infty$ and $\lambda = p - c + l$ is different. Hence we observe τ up to 22 at $\eta = 0.01$, τ up to 29 at $\eta = 0.1$ and τ up to 34 at $\eta = 0.25$.

In Table 6.4 we see that % Decrease in $CVaR^*$ > %Decrease in EP^* for all τ at η=0.01. However; Tables 6.5 and 6.6 show that at η=0.1 % Decrease in $CVaR^*$ < %Decrease in EP^* for τ=1 and 2 and at η=0.25 % Decrease in $CVaR^*$ < %Decrease in EP^* for $\tau=1, 2, .., 5$. For higher values of τ we see % Decrease in $CVaR^*$ > %Decrease in EP^*.

From a managerial perspective we shall examine the trade off between the % Decrease in carbon emission and % Decrease in $CVaR^*$ or % Decrease in carbon emission and % Decrease in EP^*. Tables 6.7, 6.8 and 6.9 signify that if a manager tolerates only a small % Decrease in $CVaR^*$, then the % Decrease in carbon emission will be small which is supported by the following instances taken from them:

For η=0.01:

a) % Decrease in Emission = 4, % Decrease in $CVaR^*$= 3.67, % Decrease in EP^*=0.53

b) % Decrease in Emission = 5, % Decrease in $CVaR^*$= 6.35, % Decrease in EP^*=0.31

c) % Decrease in Emission = 22, % Decrease in $CVaR^*$= 247.58, % Decrease in EP^*=19.13

For η=0.1:

a) % Decrease in Emission = 5, % Decrease in $CVaR^*$= 2.35, % Decrease in EP^*=1.41

b) % Decrease in Emission = 7, % Decrease in $CVaR^*$= 5.32, % Decrease in EP^*=2.49

c) % Decrease in Emission = 22, % Decrease in $CVaR^*$= 102.27, % Decrease in EP^*=25.19

For η=0.25:

a) % Decrease in Emission = 5, % Decrease in $CVaR^*$= 1.57, % Decrease in EP^*=1.74

b) % Decrease in Emission = 10, % Decrease in $CVaR^*$= 8.34, % Decrease in EP^*=5.56

c) % Decrease in Emission = 22, % Decrease in $CVaR^*$= 64.30, % Decrease in EP^*=27.19

It is obvious that CVaR is more sensitive to increasing τ than the expected profit is. If a manager allows reduction in emission by considering % Decrease in $CVaR^*$ then the reduction will be small. However if one takes % Decrease in EP^* as a basis then higher reductions in emission will be possible as the given instances imply. Note that % Decrease in $CVaR^*$ decreases as η increases. This allows us to conclude that a less risk-averse newsvendor can make more reduction in emissions with a relatively less % Decrease in $CVaR^*$ under strict cap policy.

In addition, %Decrease in EP^* for increasing τ increases when η increases from 0.01 to 0.25 while %Decrease in $CVaR^*$for increasing τ decreases in the same range of η. The reason for this consequence is our parameter set. As we discussed in Section 6.1, under this setting Q^* increases as η decreases from 0.4 to 0.01 in order to maximize CVaR. Hence, intuitively we expect to see the CVaR to increase with increasing η. However, we have $Q^*|_{\eta=0.01} > Q^*|_{\eta=0.25}$ causing the % Decrease in EP^* to increase when η increases from 0.01 to 0.25. The reason for this is the fact that EP is concave in Q and increases with increasing Q for $Q < Q_{opt}$ where Q_{opt} is the maximizer of the expected profit. With a parameter set where l is equal to profit mark-up we can observe the results that are parallel to our intuition. In order to support this claim we make the same analysis for the set p=2, c=1, s=0.8, l= 1 for the same demand distribution and η range. The results of the analysis are illustrated in Figures 6.7, 6.8 and 6.9 and the corresponding data are available in Tables 6.10, 6.11 and 6.12. It is seen in the tables that Q^* increases with increasing η and accordingly % Decrease in EP^* decreases with increasing η. % Decrease in $CVaR^*$ again decreases with increasing η is it is in the previous parameter set.

%Decrease in $CVaR^*$ / %Decrease in Q^*, and % Decrease in EP^* / %Decrease in Q^* and τ relations for the parameter sets p=2, c=1, s=0.8, l= 3 and p=2, c=1, s=0.8, l= 1 at η=0.01, 0.1 and 0.25 are represented in Figures 6.10 and. 6.11, respectively.

6.2.2 CVaR Maximization with Cap and Trade Policy

In the cap and trade policy the newsvendor searches the optimal production policy according to relation between the given cap and the thresholds of emission under carbon trading. As it is claimed in Chapter 4 under Section 4.3, the rationale behind this analysis comes from comparison of the Lagrange multiplier, λ, c_b and c_s values. The parameter settings that will be used throughout the calculations in this section in order to investigate the optimal policy for specified c_b and c_s values are given as:

1) p=2, c=1, s=0.8, l= 3, α=1, η=0.01

2) p=2, c=1, s=0.8, l= 3, α=1, η=0.1

3) p=2, c=1, s=0.8, l= 3, α=1, η=0.25

for a normally distributed demand with μ=500 and σ^2=100^2.

We determine the c_b and c_s values as the percentages of the cost of underage value, $p-c+l$, by setting c_b=$(p-c+l)\Delta$ and c_s=$0.95c_b$ for Δ=0.02, 0.10, 0.35, 0.50 and 0.9. Hence the c_b and c_s values we analyzed under each parameter setting given above are (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42)$ for Δ=0.02, 0.10, 0.35, 0.50 and 0.9, respectively.

As shown in Section 6.1, Q^* vs. η relation is parameter sensitive. In order to see the impact of carbon trading prices on this relation we first analyze Q_{up}, the optimal production quantity when the newsvendor buys carbon, and Q_{down}, the optimal production quantity when the newsvendor sells carbon, vs. η relation for changing carbon prices. We make this analysis for the following parameter sets:

1) p=2, c=1, s=0.8, l= 3, α=1, for (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38)$, $(0.8, 0.76), (1.4, 1.33), (3, 2.85), (3.6, 3.42)$

2) p=2, c=1, s=0.85, l= 1, α=1, for (c_b, c_s)=$(0.08, 0.076), (0.2, 0.19)$, $(0.4, 0.38), (0.8, 0.76), (1.4, 1.33)$

The data of the first set is tabulated in Table 6.13 and the Q_{up}, Q_{down} vs. η relation is illustrated in Figures 6.12 and 6.13 for the carbon price set given above. Recall that, the newsvendor problem parameters, p, c, s and l are the same as the ones give the counter-intuitive Q^* vs. η behavior in Section 6.1. Hence we expect Q_{up} and Q_{down} to decrease with increasing η up to a certain η and then start to increase up to $\eta = 1$. In Figures, we clearly see that the Q_{up}, Q_{down} vs. η relation is also parameter sensitive. For carbon prices (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38), (0.8, 0.76)$, the Q_{up} and Q_{down} first decrease with increasing η up to a certain η and then they increase with increasing η which is parallel with our expectation. However, when carbon prices are further increased we see that Q_{up} and Q_{down} decrease with increasing η as illustrated in Figure 6.13.

The similar analysis is conducted for the second parameter set at which Q^* increases with increasing η for the unconstrained CVaR Maximization problem. In this setting we expect the Q_{up}, Q_{down} to increase with increasing η at a fixed carbon price, however we again see the counter intuitive impact of carbon prices. The Q_{up}, Q_{down} data for each η at a fixed carbon price are available in Table 6.14. In Figures 6.14 and 6.15 we observe that Q_{up} and Q_{down} increase with increasing η at carbon prices (c_b, c_s)=$(0.08, 0.076), (0.2, 0.19), (0.4, 0.38), (0.8, 0.76)$ while they decrease with increasing η at (c_b, c_s)=$(1.4, 1.33)$.

Analyses of both parameter sets show that the impact of carbon trading prices is similar to the one of lost sales cost observed in Section 6.1. Hence, we again see that the optimal production quantity and risk aversion level relation directly depends on our choice of problem parameters.

The Lagrange multiplier values which are calculated in the strict cap policy section are compared with the specified carbon prices in order to find the optimal policy. We have 3 parameter sets and 5 different carbon price values, hence we make analysis of $3 \times 5 = 15$ cases. We provide the detailed analysis of the cap and trade policy in Tables 6.15, 6.16,..,6.23. We present the results of the cases with (c_b, c_s)=$(0.08, 0.076)$, $(1.4, 1.33)$, $(3.6, 3.42)$. We pick minimum, moderate and maximum values of the carbon prices in order to gain a general understanding. The data provided in the tables imply that for a fixed η the Q^* value at a fixed τ decreases with increasing c_b and c_s. For example from Tables 6.15, 6.16 and 6.17 we see that at $\eta = 0.01$ and $\tau = 1$; Q^*=662.3, 620.5, 584.6 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$, respectively. However, the $CVaR^*$, ω^* and EP^* depends on the trading action. Hence in order to draw a conclusion we must take the optimal action into consideration and compare the optimal values on the same basis with respect to optimal action. For example at $\eta = 0.01$ and $\tau = 5$ the newsvendor buys carbon at (c_b, c_s)=$(0.08, 0.076)$ while he sells carbon at (c_b, c_s)=$(1.4, 1.33), (3.6, 3.42)$. Here we can compare the cases where (c_b, c_s)=$(1.4, 1.33), (3.6, 3.42)$ and it is obvious that $CVaR^*$, ω^* and EP^* increases with increasing c_s. The opposite is valid for the case where the newsvendor buys carbon. $CVaR^*$, ω^* and EP^* decreases with increasing c_b.

$CVaR^*$, ω^* and EP^* versus τ relation for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ is represented in Figures 6.16, 6.17, and 6.18. In Figures, the left y-axis represents $CVaR^*$, ω^* and the right y-axis gives the EP^* values. From the Figures 6.16, 6.17, and 6.18 and Tables 6.15, 6.16 to 6.23 it is clear that the relation $EP^* > CVaR^* > \omega^*$ holds at each parameter setting and (c_b, c_s) value.

More detailed investigation of $CVaR^*$ and τ relation for the parameter settings defined in the begining of the section under the set of carbon prices (c_b, c_s)= =$(0.08, 0.076), (0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42)$ are represented in Figure 6.19. The corresponding data of the Figures are available in Tables 6.24, 6.25 and 6.26. In this analysis we see that the range of $CVaR^*$ values realized within the sprecified τ range increases with increasing c_b and c_s. For instance the difference $CVaR|_{\tau=1}$-$CVaR|_{\tau=22}$ at (c_b, c_s)=$(0.08, 0.076)$ is less than $CVaR|_{\tau=1}$-$CVaR|_{\tau=22}$ at (c_b, c_s)=$(3.6, 3.42)$ as Tables 6.24, 6.25 and 6.26 imply.

The impact of changing c_b and c_s on $CVaR^*$ is further analyzed by studying % Decrease in $CVaR^*$ for increasing τ at the set of carbon prices (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42)$ which is summarized by Figure 6.20. The data related to figure is provided in Tables 6.27, 6.28 and 6.29. While calculating the % Decrease in $CVaR^*$ we take the τ=0 as a reference point. For a specified (c_b, c_s) value, we first determine the optimal action and corresponding $CVaR^*$ at each τ. Then, at τ=t the % Decrease in $CVaR^*$ is calculated as:

% Decrease in $CVaR^*$=$\left(\frac{CVaR^*|_{\tau=0} - CVaR^*|_{\tau=t}}{CVaR^*|_{\tau=0}}\right) \times 100$

At higher carbon prices we observe higher % Decrease in $CVaR^*$ at each τ value. This consequence is parallel with our conclusion in $CVaR^*$ carbon price observation in the previous paragraph. Since increasing the carbon tightness forces the newsvendor to buy carbon up to the corresponding threshold value, the negative gain increases with increasing c_b and $CVaR^*$ decreases, accordingly the % Decrease in $CVaR^*$ increases.

All the observations we provide up to this point are based on the carbon cap tightness. Since the tightness is considered as the percentage reduction from the emission released at the unconstrained optimal solution, $\tau = t$ corresponds to different K values at each risk aversion level. In order to examine the risk aversion level impact on the $CVaR^*$ under cap and trade policy exactly, we consider the problem at different K values. For the set of (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ we calculate $CVaR^*$ by determining the optimal action and corresponding Q^* values for K=300,325,..,1000 at η=0.01, 0.1 and 0.25. The $CVaR^*$ values at each K for specified carbon prices are available in Table 6.30 and illustrated in Figure 6.21. As the data and the Figure indicate that higher values of $CVaR^*$ are attained at higher η values for each K and (c_b, c_s) couple which is an intuitive result since $CVaR$ increases with decreasing risk aversion, increasing η.

Next, we consider the % Decrease in Emission with respect to the emission released at unconstrained optimal solution at the given K values with the carbon prices of (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ at each η we specified. At a fixed η, for each (c_b, c_s) the newsvendor has different threshold values of carbon to decide whether to buy, sell or use all carbon without trading. For each of K=300,325,..,1000 the newsvendor finds the optimal policy implying to buy carbon up to K_{up} if $K < K_{up}$, sell carbon down to K_{down} if $K > K_{down}$ and otherwise use all carbon without trading. Hence the newsvendor emits: K_{up} for $K < K_{up}$, K for $K_{up} < K < K_{down}$, and K_{down} for $K > K_{down}$. Note that K_{up} and K_{down} values depend on the carbon price as it is discussed earlier. First, we determine the carbon buying and selling thresholds for each carbon price couple at the specified η values. Then the % Decrease in Emission is calculated as follows:

1)$\mathbf{K < K_{up}}$: $\%DecreaseinEmission$=$\left(\frac{K_{unc}-K_{up}}{K_{unc}}\right) \times 100$

2)$\mathbf{K_{up} \leq K \leq K_{down}}$: $\%DecreaseinEmission$=$\left(\frac{K_{unc}-K}{K_{unc}}\right) \times 100$

3) $\mathbf{K > K_{down}}$: $\%DecreaseinEmission$=$\left(\frac{K_{unc}-K_{down}}{K_{unc}}\right) \times 100$

The % Decrease in Emission and $CVaR^*$ value realized at each given carbon cap, K, for different carbon prices and risk aversion level analysis for the parameter set given at the beginning of this section is illustrated by Figures 6.22, 6.23 and 6.24. According to the figures higher values of % Decrease in Emission are attained at higher η values for a fixed (c_b, c_s) couple. In addition at a fixed η value % Decrease in Emission increases with increasing carbon prices since the optimal order quantity is inversely proportional to carbon prices. The range of the values $CVaR^*$ attains within the K=$[300, 1000]$ interval increases as the carbon prices increases for a fixed η. Tables 6.31, 6.32, 6.33 providing the data of Figures 6.22, 6.23 and 6.24 can be analyzed for detailed information.

The same experiment is conducted for the parameter set p=2, c=1, s=0.85, l= 1, α=1, for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ and the same conclusions are reached. The % Decrease in Emission and $CVaR^*$ vs. K relations are provided in Figures 6.25, 6.26 and 6.27 and the corresponding data is available in Tables 6.34, 6.35 and 6.36.

In Figures 6.28, 6.29 and 6.30 we examine the trade-off between the customer satisfaction and environmental welfare by determining the $\%SL^*$ and % Decrease in Emission with respect to the emission level of unconstrained optimal order/production quantity that correspond to each given carbon cap, K=300, 325, .., 1000. This study is conducted for three carbon trading prices (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ at each η=0.01, 0.1 and 0.25 with p=2, c=1, s=0.8, l= 3, α=1. The calculation method is as follows. At a specified carbon trading price and risk aversion level we calculate the thresholds K_{up} and K_{down}. Then, for each given carbon cap, K, the optimal policy is determined according to relation between K, K_{up} and K_{down} as given in Theorem 4.3.1. Determining the optimal policy provides the optimal order/production quantity and the corresponding carbon emission level values directly. Therefore, we calculate the $\%SL^*$ at the optimal order/ production quantity and % Decrease in Emission with respect to the emission level of unconstrained optimal order/production quantity as it is explained in the previous analysis.

According to Figures 6.28, 6.29 and 6.30, we see that at a fixed η, increasing the carbon trading prices provides the opportunity of decreasing carbon emissions, however; it decreases the customer satisfaction at the same time. For example, according to Figure 6.28 we see that at (c_b, c_s)=$(0.08, 0.076)$, $\%SL^*$ changes in $[94.74, 94.77]$ and % Decrease in Emission takes values in $[1.01, 1.05]$ while at (c_b, c_s)=$(3.6, 3.42)$, $\%SL^*$

changes in $[78.85, 80.12]$ and % Decrease in Emission takes values in $[12.6, 13.3]$.

Since the optimal order/production quantity decreases from η=0.01 to η=0.4 under our parameter setting, as discussed in Section 6.1, we see that at a fixed carbon price the $\%SL^*$ decreases and % Decrease in Emission increases when η is increased from 0.01 to 0.1 and 0.25. For instance, at (c_b, c_s)=$(0.08, 0.076)$, $\%SL^*$ is in $[94.74, 94.77]$ and % Decrease in Emission is in $[1.01, 1.05]$ at η=0.01, while $\%SL^*$ is in $[90.52, 90.6]$ and % Decrease in Emission is in $[1.47, 1.54]$ at η=0.25. Therefore, we conclude that there is a trade-off between being environmentally friendly and satisfying the customers. If one wants to emit less then he must takes the risk of losing some of the customers. For further information Tables 6.37, 6.38 and 6.39 can be analyzed which tabulates the data related to Figures 6.28, 6.29 and 6.30.

This analysis is made also for the parameter set p=2, c=1, s=0.85, l= 1, α=1, for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ which is summarized by the Figures 6.31, 6.32 and 6.33. The corresponding data is provided in Tables 6.40, 6.41 and 6.42. Contrary to the outcomes of previous parameter set, in this analysis we see that a less risk averse newsvendor can increase the customer service level and decrease the carbon emission at the same time at the carbon trading prices (c_b, c_s)=$(0.08, 0.076)$ since the optimal production quantity increases with increasing η. However for (c_b, c_s)=$(1.4, 1.33)$ the customer service level first increases when η increases from 0.01 to 0.1 and then it decreases when η increases from 0.1 to 0.25. However, % Decrease in Emission increases with increasing η for both (c_b, c_s)=$(0.08, 0.076)$ and $(1.4, 1.33)$.

Lastly, for the cap and trade problem we discuss the effect of c_b on the service level and % Decrease in Emission. As discussed above each c_b gives a K_{up} value at a fixed η which is always less than K_{unc}. Hence, imposing a cap and trade policy decreases the carbon emission level even when the newsvendor buys carbon. The logic of the % Decrease in Emission calculation is the same as case given above for $K < K_{up}$. In order to gain some managerial insights on the cap and trade policy we also consider the customer service level values corresponding to each c_b value. The summary of this analysis is provided by Figure 6.34 and Table 6.43. In this analysis there is a trade-off between high % Decrease in emission and service level since increasing c_b decreases service level while decreasing the emission level with respect to the case where carbon emissions are not taken into consideration.

6.3 CVaR Maximization with Multiple Resource Constraints

In this section we provide a basic analysis of a multiple resource constrained problem by analyzing the response of the optimal policy and the objective value to changing η and resource limits. We first make the analysis of the impact of η on Q^* and $CVaR^*$ for the binding resources model and tradeable resources model. For the sake of simplicity we consider 2 limited resources: carbon allowance and cash.

The impact of η on Q^* and $CVaR^*$ under the binding resources model is examined by using two parameter sets given below:

1) p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon}=600, K_{cash}=650.

2) p=2, c=1, s=0.8, l= 1, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon}=450, K_{cash}=510 for a normally distributed demand with μ=500 and σ^2=100^2.

The Q^* and $CVaR^*$ versus η relation at the binding resource cap model for two parameter sets given above are illustrated in Figures 6.35 and 6.36,respectively. The corresponding data is available in Tables 6.44 and 6.45. For both of the data sets $'$use all carbon$'$ action is optimal and it is clear that $CVaR^*$ increases with increasing η.

Also, the Q^* and $CVaR^*$ versus η relation under the tradeable resources model is demonstrated in Figure 6.37. Again, we see that the $CVaR^*$ increases with increasing η. The detailed analysis of the problem is provided in Tables 6.46. We also see that Q^* decreases with increasing η in both binding resource cap and tradeable resources models which is the outcome of our parameter setting.

We also consider the CVaR maximization problem with tradeable resources under the same parameter setting given above for changing resource limits at η=0.01 and 0.1. The values of carbon and cash caps at which we investigate the optimal policy are given as K_{carbon}=550, 580, 600, 620 and K_{cash}=550, 580, 610, 625. When we consider the impact of changing carbon cap we fix K_{cash}=650 and K_{carbon}=600 is fixed while analyzing the changing cash cap values. The optimal policy analyses of the changing resource cap under trading policy are provided in Tables 6.47, 6.48 and 6.49, 6.50 for changing K_{carbon} and K_{cash}, at η=0.01 and 0.1 respectively. The Q^* and $CVaR^*$ values obtained for changing K_{carbon} and K_{cash} is illustrated in Figures 6.38, 6.39 and 6.40, 6.41. In both cases we observe that at $Q^*|_{\eta=0.1} < Q^*|_{\eta=0.01}$ however $CVaR^*|_{\eta=0.1} > CVaR^*|_{\eta=0.01}$. The reason for the Q^* to decrease with increasing η is the parameter setting as discussed in Section 6.1. For each case it is valid that $CVaR^*$ increases with increasing η. Also it is understood that changing the resource caps changes the optimal trading policy as the changing carbon cap analysis implies. In Table 6.47 we see that for K_{carbon}=550, 580, 600 the optimal policy is buy carbon sell cash while increasing the cap to K_{carbon}= 620 changes the optimal policy as sell carbon sell cash. Besides, if we consider a carbon cap $K > 650$ in this case we will see that feasibility of buying carbon and selling cash option is not considered as it is done for the cap values $K < 650$. Hence the optimal policy may change. This case is observed in cash cap change analysis(see Table 6.49). For the cash cap values K_{cash}=550, 580 feasibility of the action buy cash sell carbon is considered while for K_{cash}=610, 625 feasibility of buy carbon sell cash action is considered. The feasibility of buying and selling from two of the resources is always checked. However, the feasibility of other actions in the action set depends on the ordering of resources.

FIGURES OF THE NUMERICAL STUDY

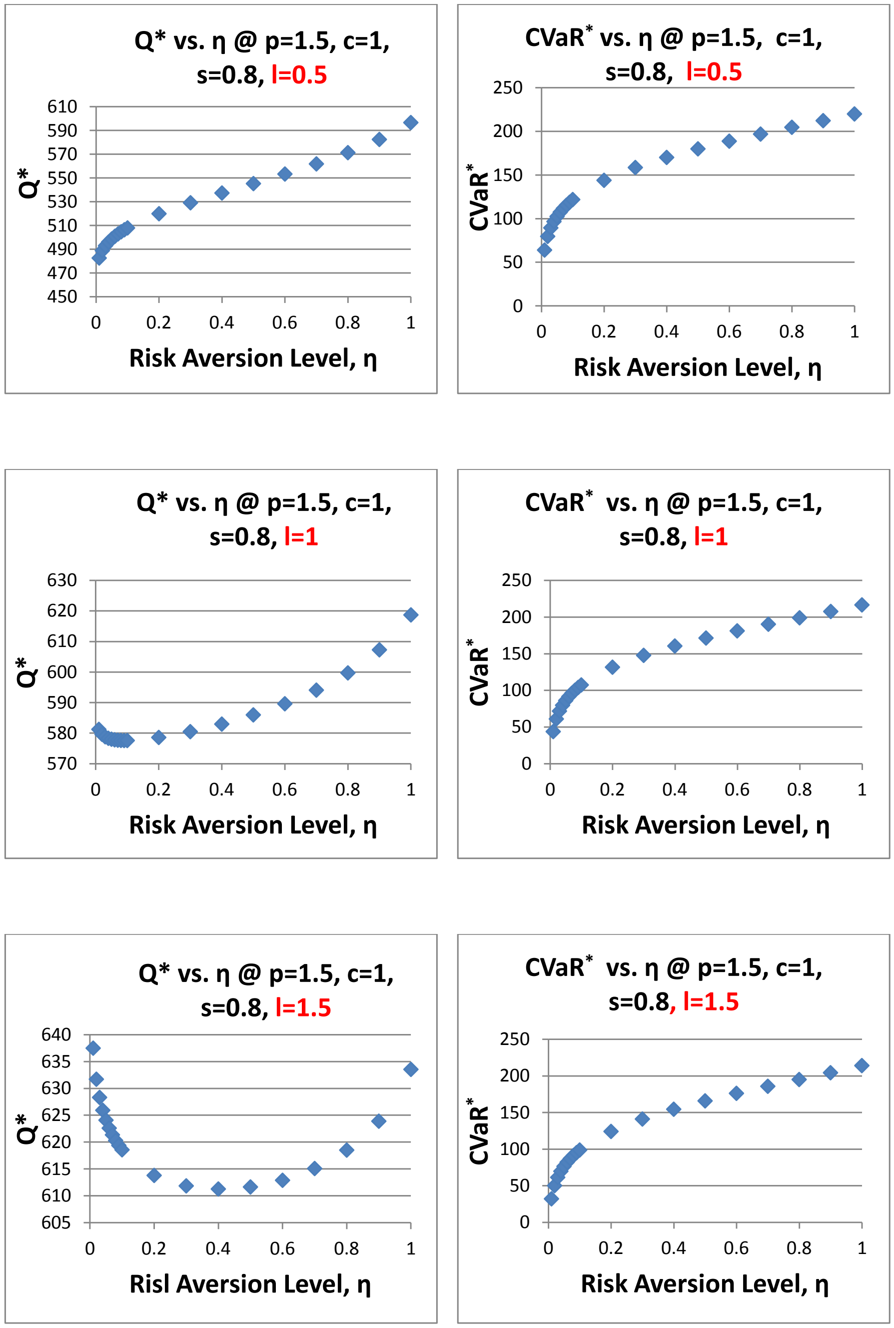


Figure 6.1: Q^* and $CVaR^*$ vs. η at p=1.5, c=1, s=0.8 with l=0.5, 1, and 1.5 under Unconstrained CVaR Maximization.

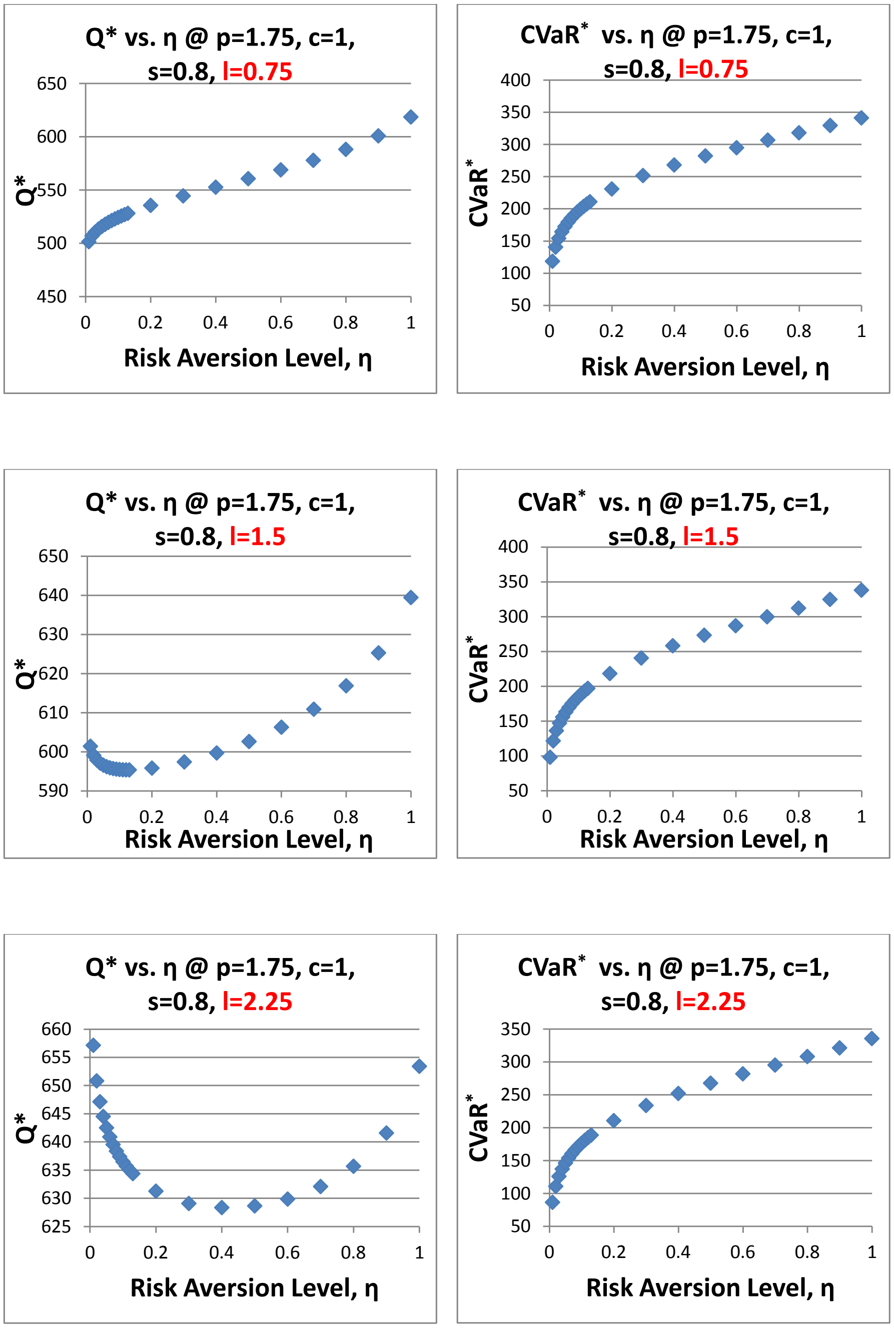


Figure 6.2: Q^* and $CVaR^*$ vs. η at p=1.75, c=1, s=0.8 with l=0.75, 1.5, and 2.25 under Unconstrained CVaR Maximization.

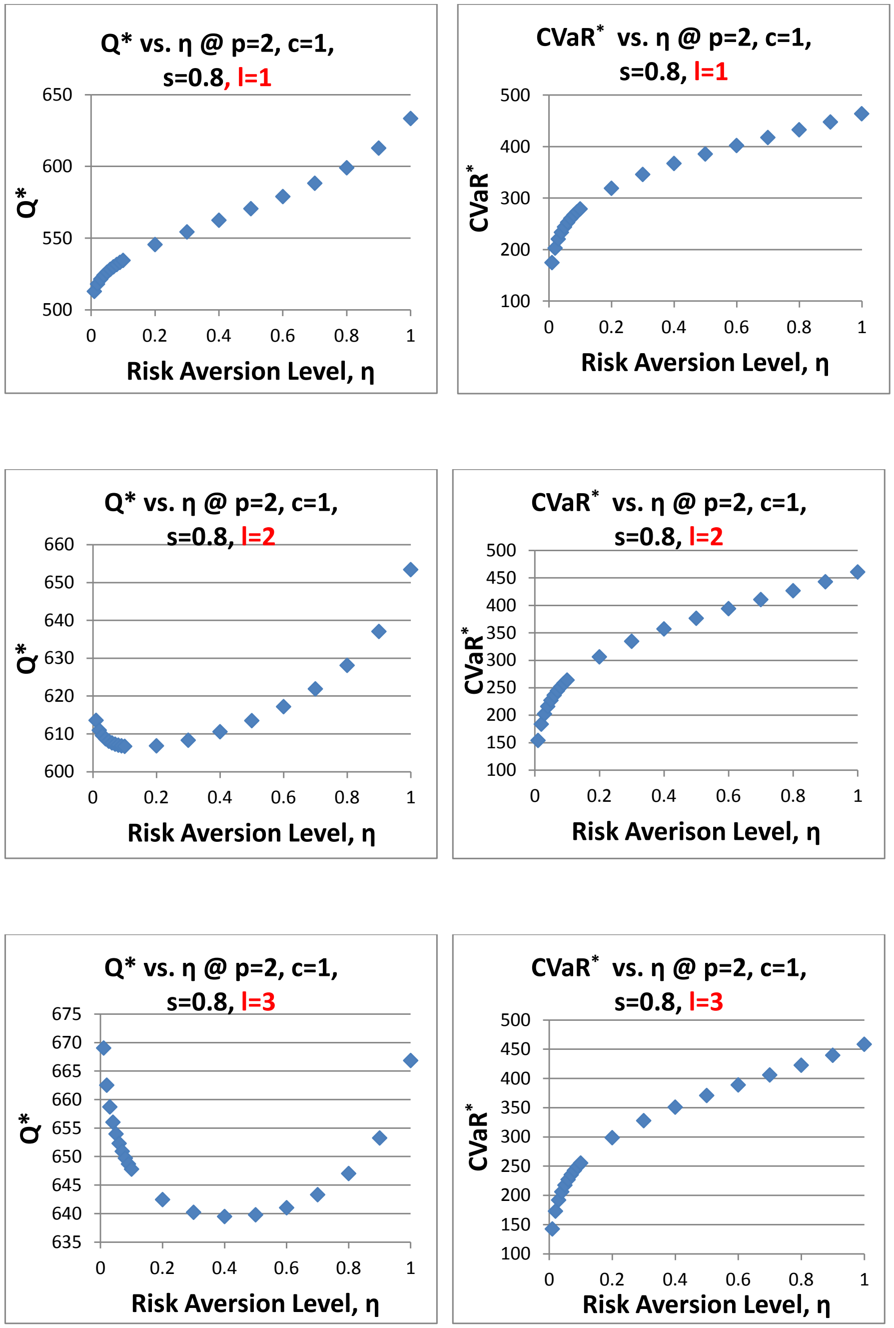


Figure 6.3: Q^* and $CVaR^*$ vs. η at p=2, c=1, s=0.8 with l=1, 2, and 3 under Unconstrained CVaR Maximization.

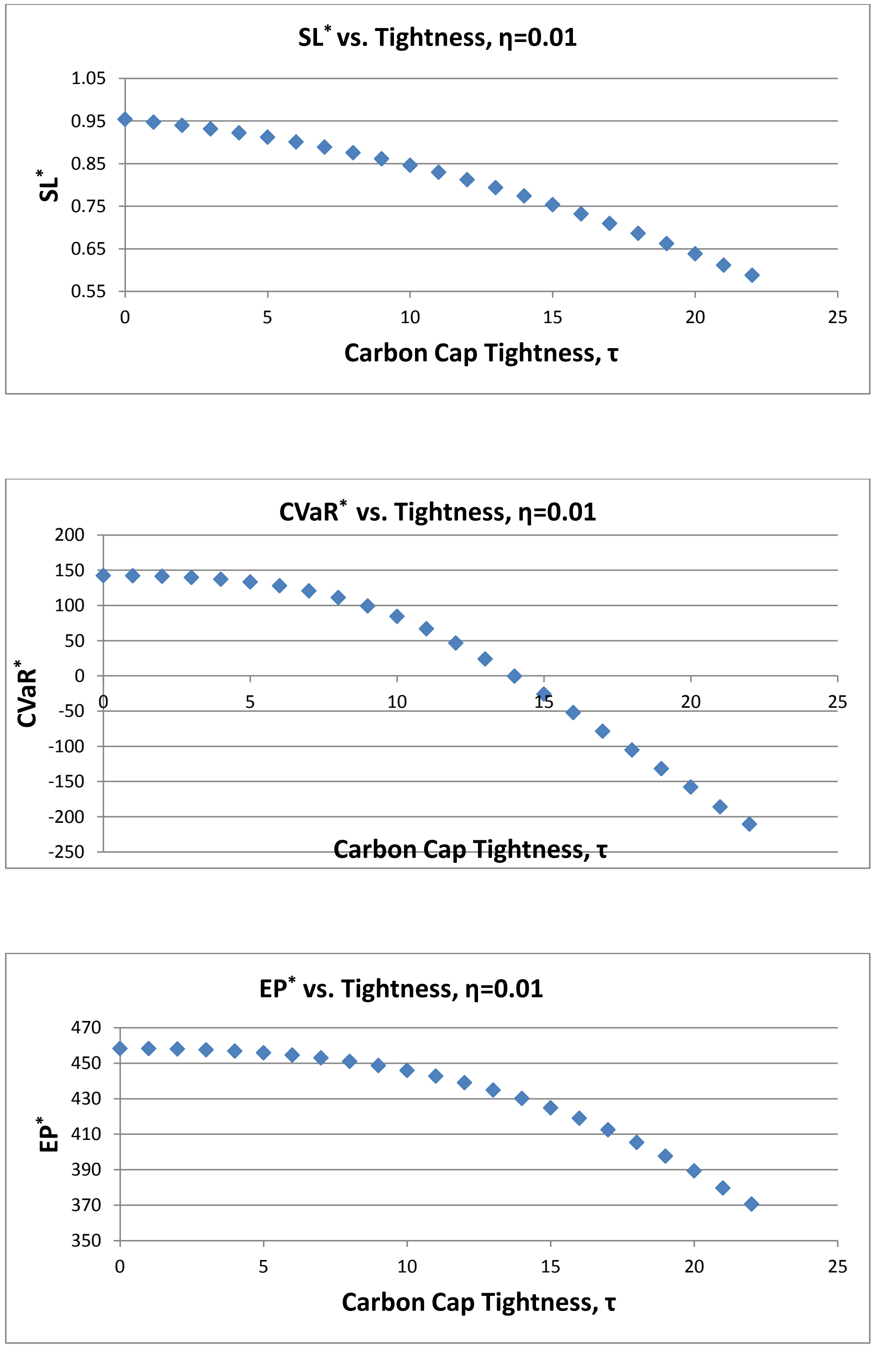


Figure 6.4: SL^*,$CVaR^*$ and EP^* vs. τ at p=2, c=1, s=0.8, l=3, η=0.01 under Strict Cap Policy.

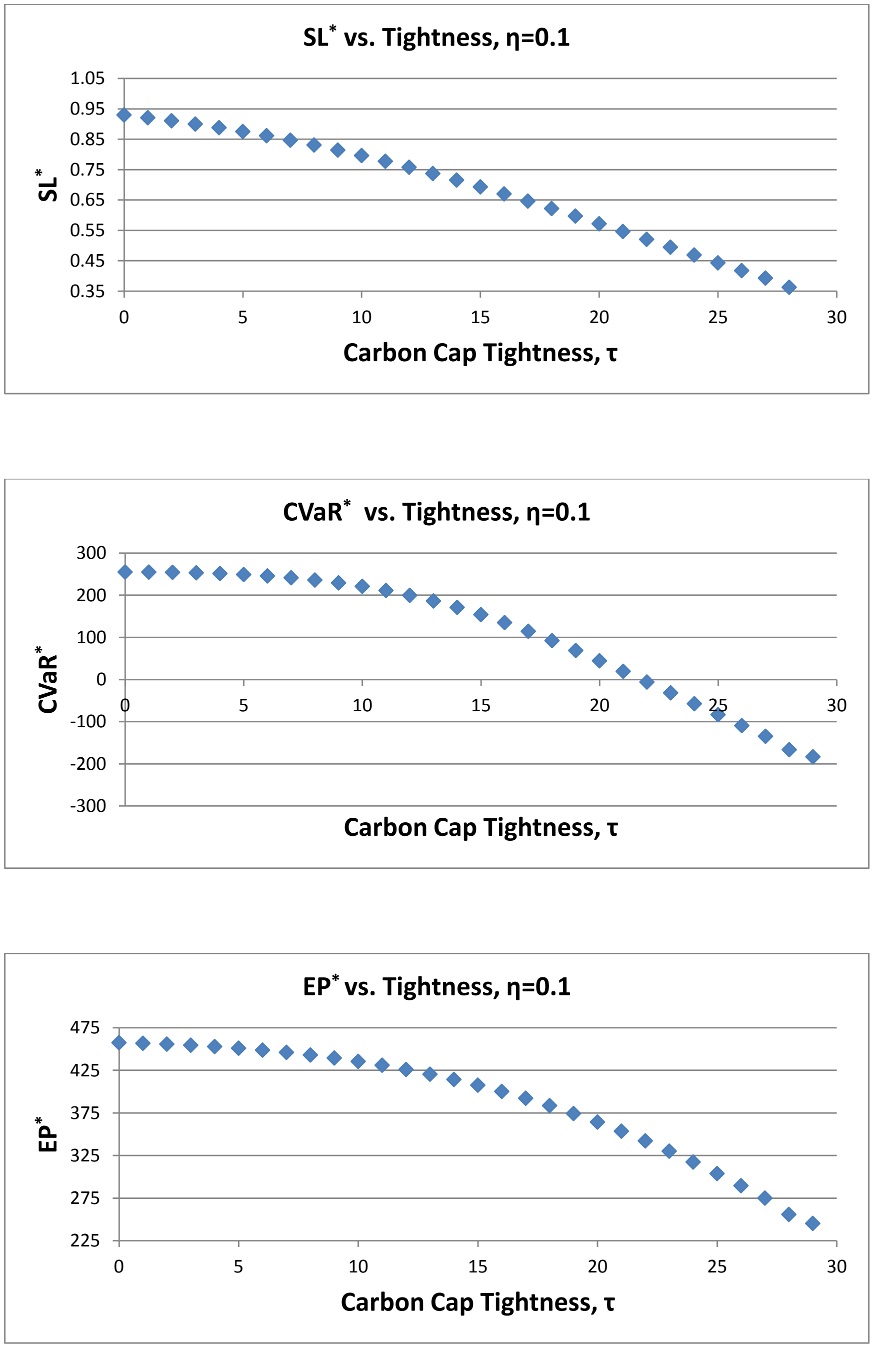


Figure 6.5: SL^*,$CVaR^*$ and EP^* vs. τ at p=2, c=1, s=0.8, l=3, η=0.1 under Strict Cap Policy.

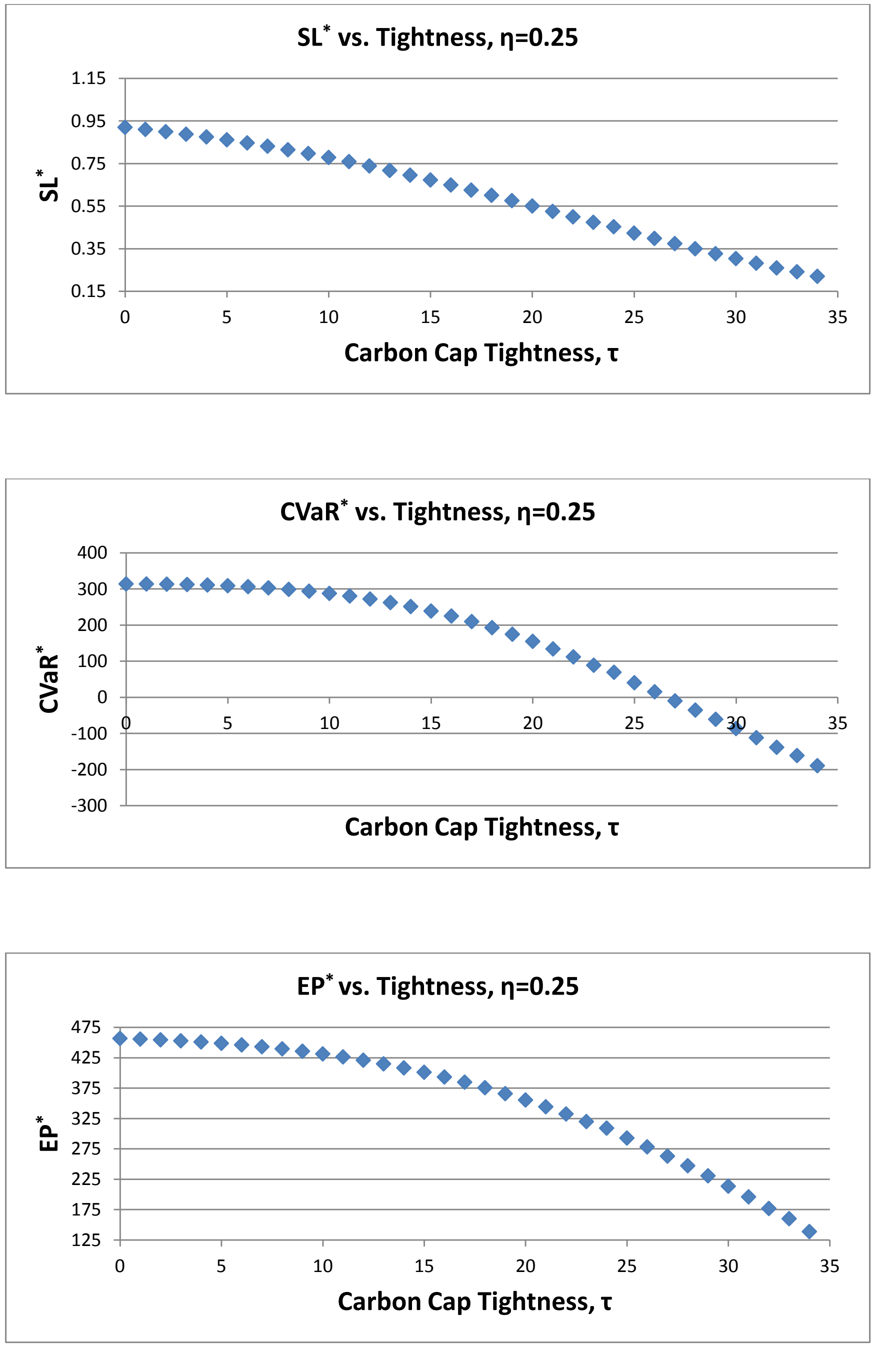
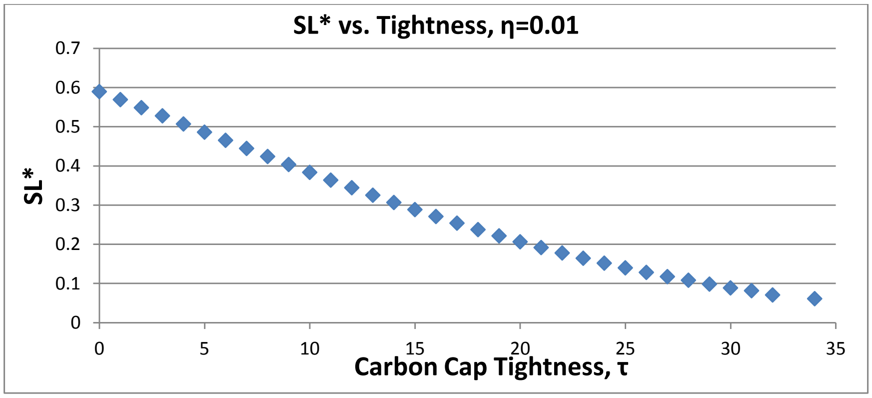
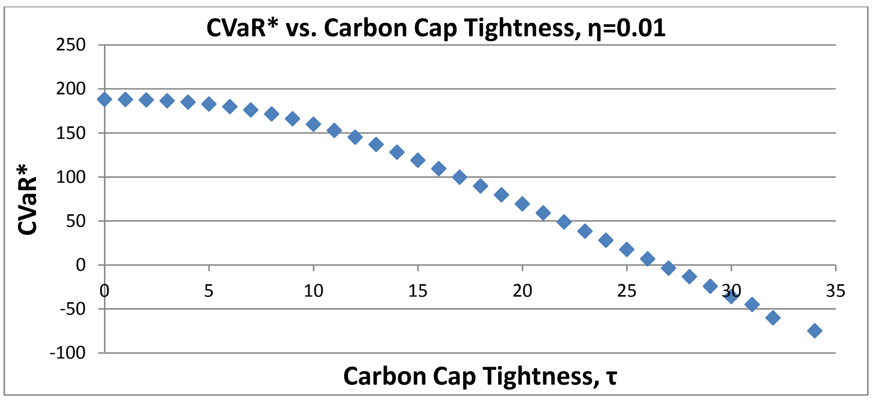


Figure 6.6: SL^*,$CVaR^*$ and EP^* vs. τ at p=2, c=1, s=0.8, l=3, η=0.25 under Strict Cap Policy.





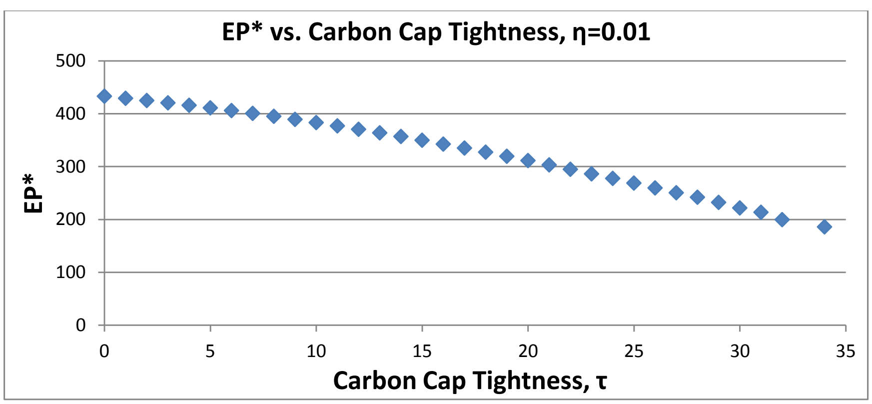


Figure 6.7: SL^*,$CVaR^*$ and EP^* vs. τ at p=2, c=1, s=0.8, l=1, η=0.01 under Strict Cap Policy.

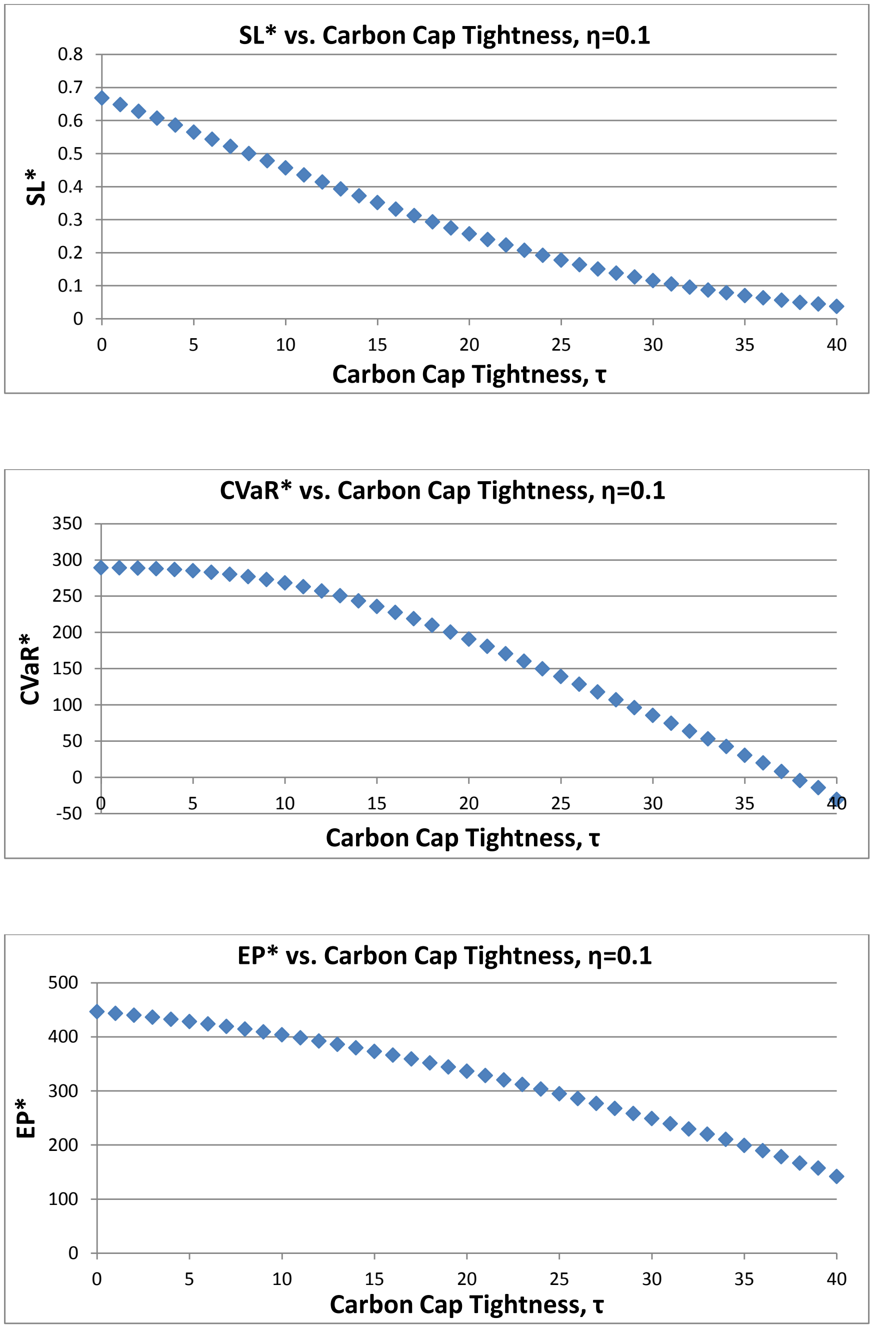


Figure 6.8: SL^*,$CVaR^*$ and EP^* vs. τ at p=2, c=1, s=0.8, l=1, η=0.1 under Strict Cap Policy.

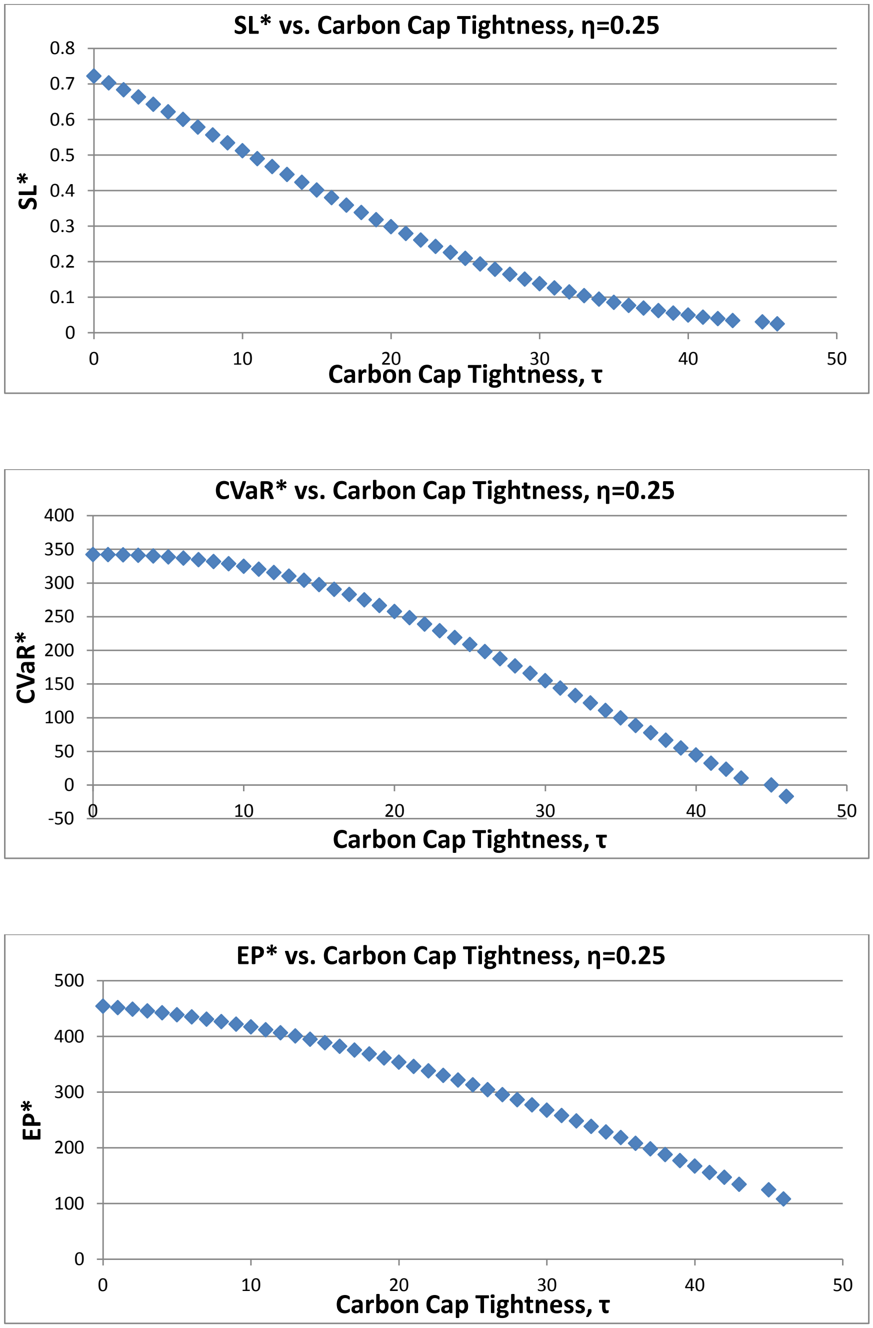
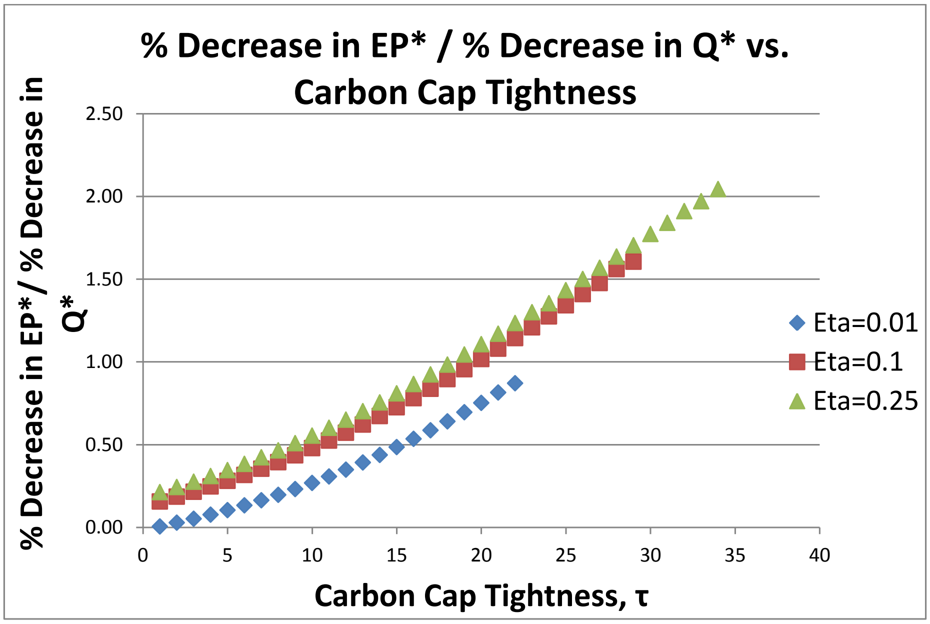


Figure 6.9: SL^*,$CVaR^*$ and EP^* vs. τ at p=2, c=1, s=0.8, l=1, η=0.25 under Strict Cap Policy.



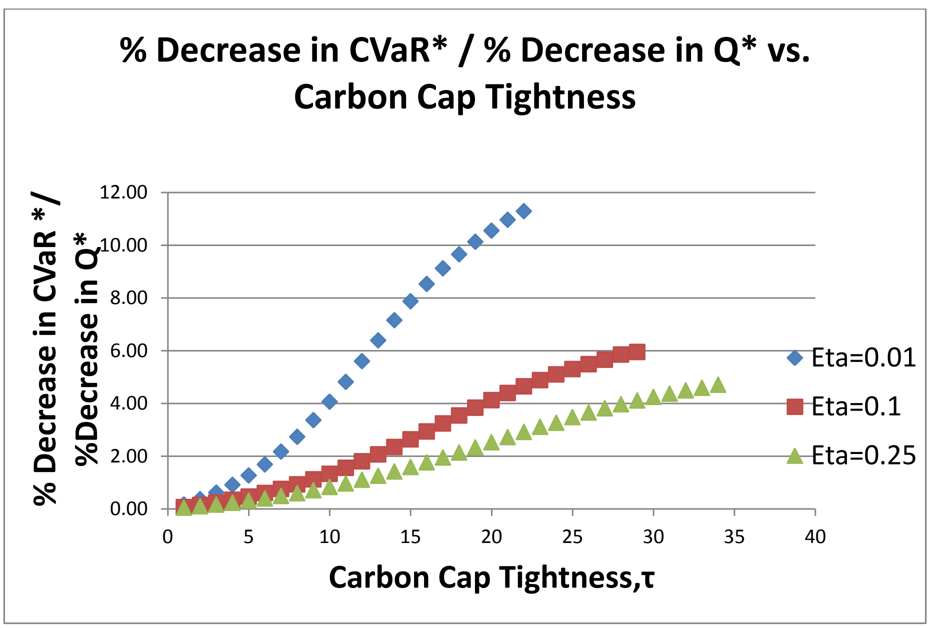
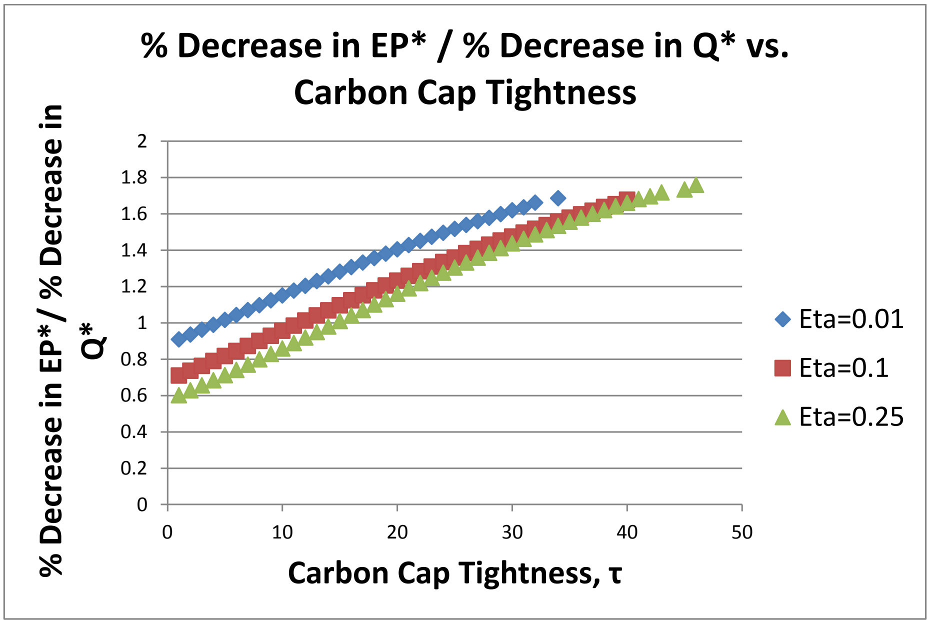


Figure 6.10: %Decrease in EP^*/ %Decrease in Q^* , %Decrease in $CVaR^*$/ %Decrease in Q^* vs. τ at p=2, c=1, s=0.8, l=3, η=0.01 ,0.1 and 0.25 under Strict Cap Policy.



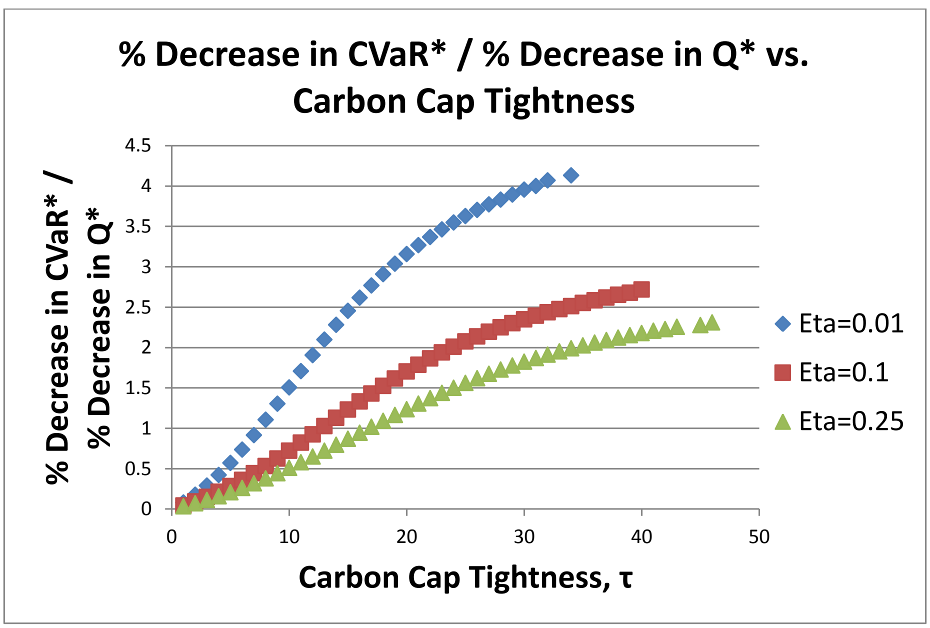


Figure 6.11: %Decrease in EP^*/ %Decrease in Q^* , %Decrease in $CVaR^*$/ %Decrease in Q^* vs. τ at p=2, c=1, s=0.8, l=1, η=0.01 ,0.1 and 0.25 under Strict Cap Policy.

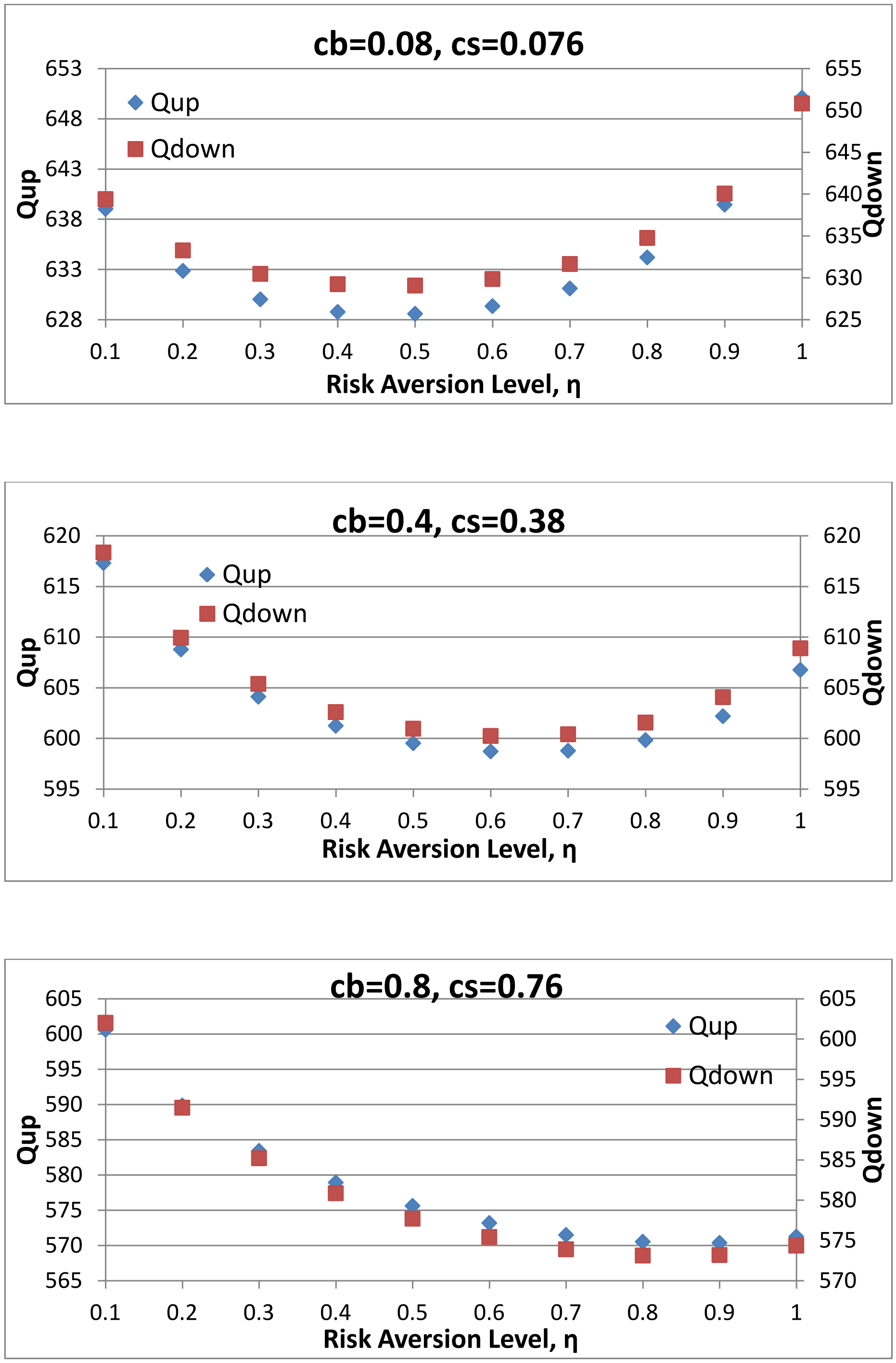
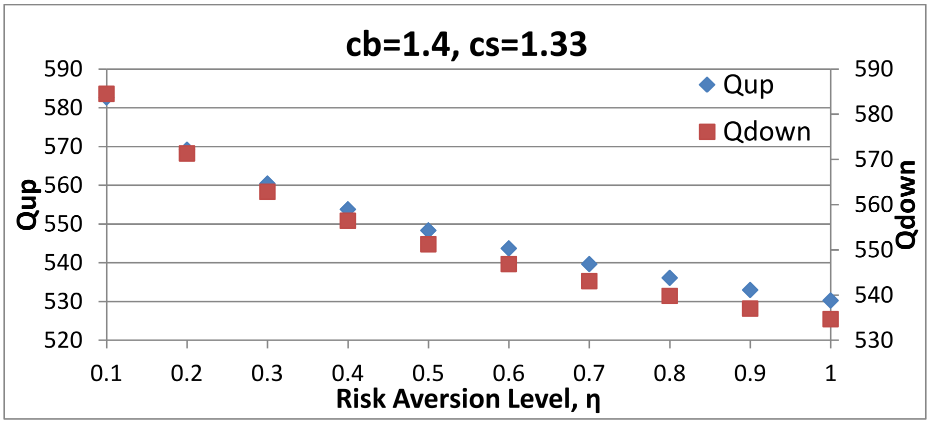
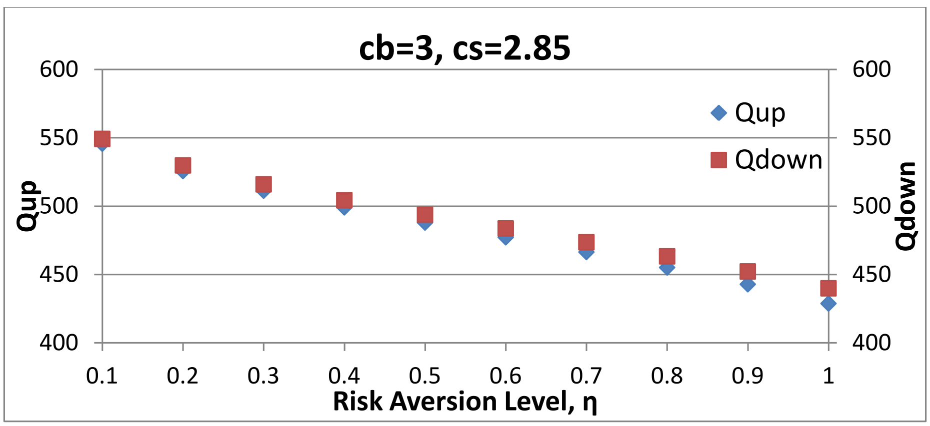


Figure 6.12: Q_{up} and Q_{down} vs. η at p=2, c=1, s=0.8, l=3 for (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38), (0.8, 0.76)$ under Cap and Trade Policy.





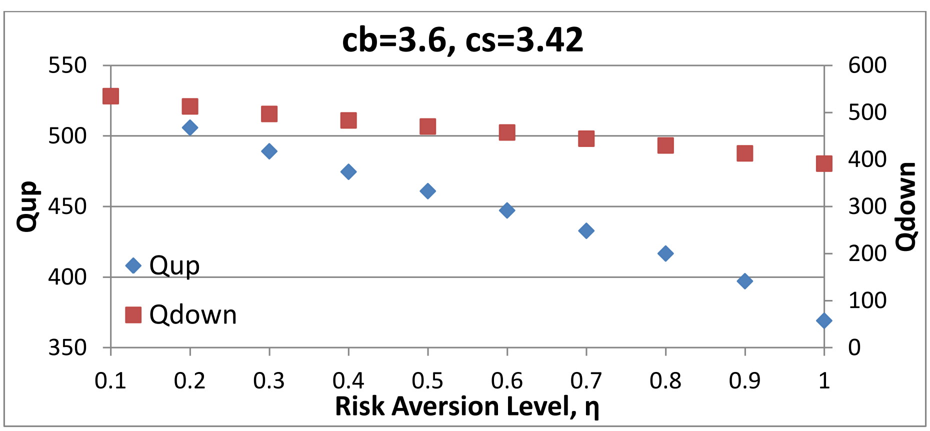


Figure 6.13: Q_{up} and Q_{down} vs. η at p=2, c=1, s=0.8, l=3 for (c_b, c_s)=(1.4, 1.33), (3, 2.85), (3.6, 3.42) under Cap and Trade Policy.

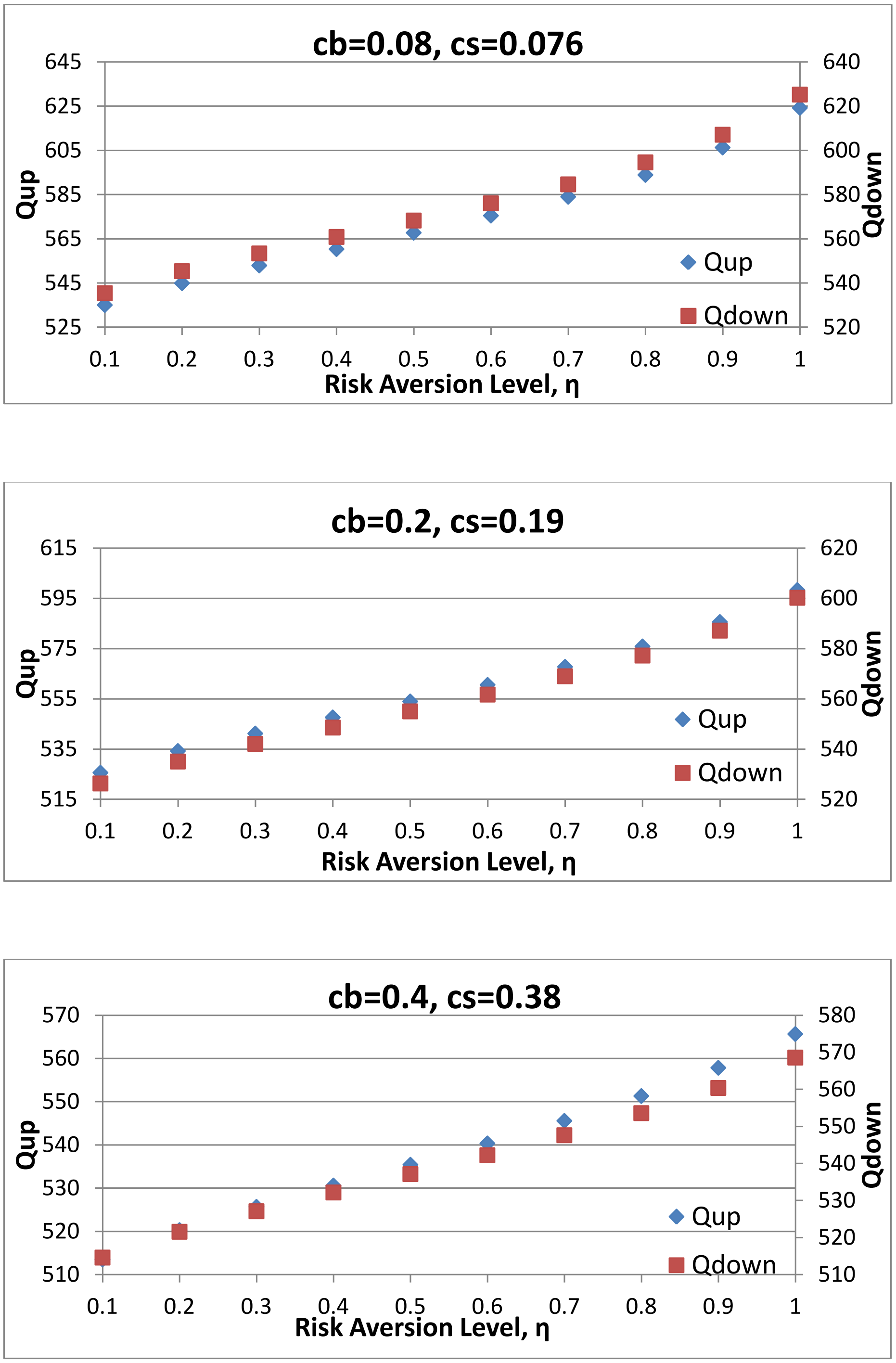
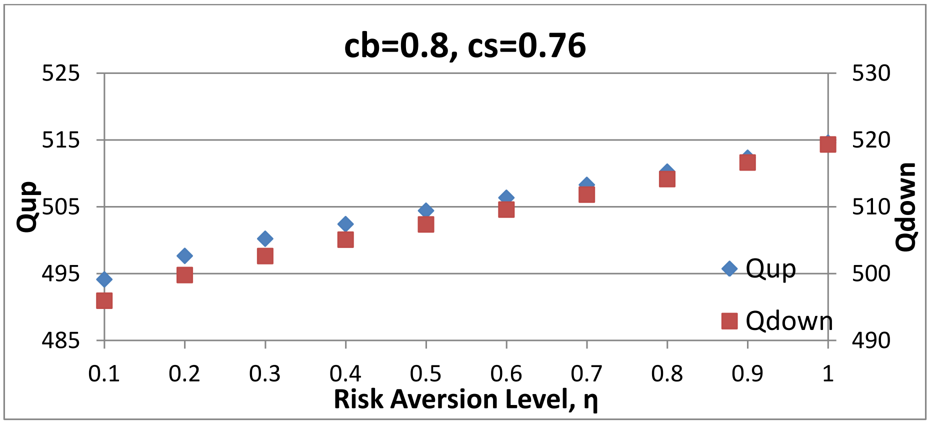


Figure 6.14: Q_{up} and Q_{down} vs. η at p=2, c=1, s=0.85, l=1 for (c_b, c_s)=(0.08, 0.076), (0.2, 0.19), (0.4, 0.38) under Cap and Trade Policy.



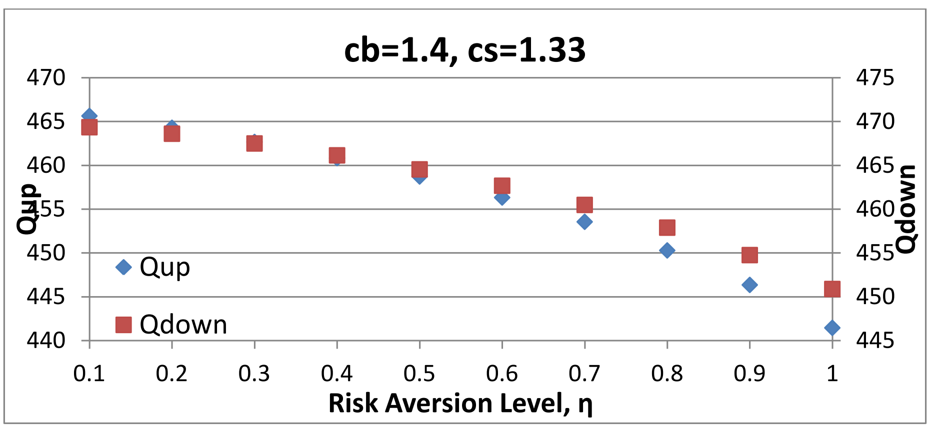


Figure 6.15: Q_{up} and Q_{down} vs. η at p=2, c=1, s=0.85, l=1 for (c_b, c_s)=$(0.8, 0.76), (1.4, 1.33)$ under Cap and Trade Policy.

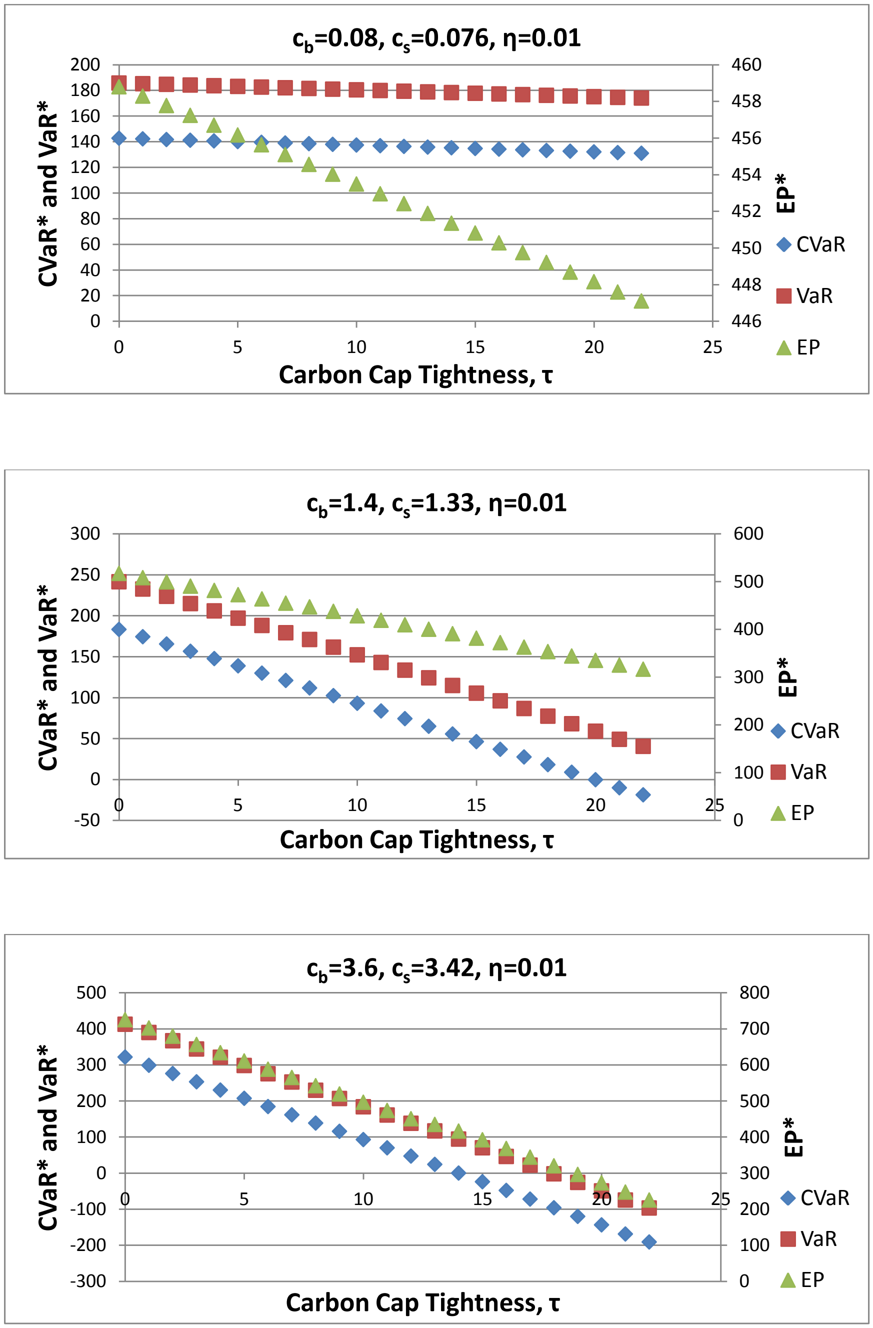


Figure 6.16: $CVaR^*$, ω^* and EP^* vs. τ at p=2, c=1, s=0.8, l=3, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

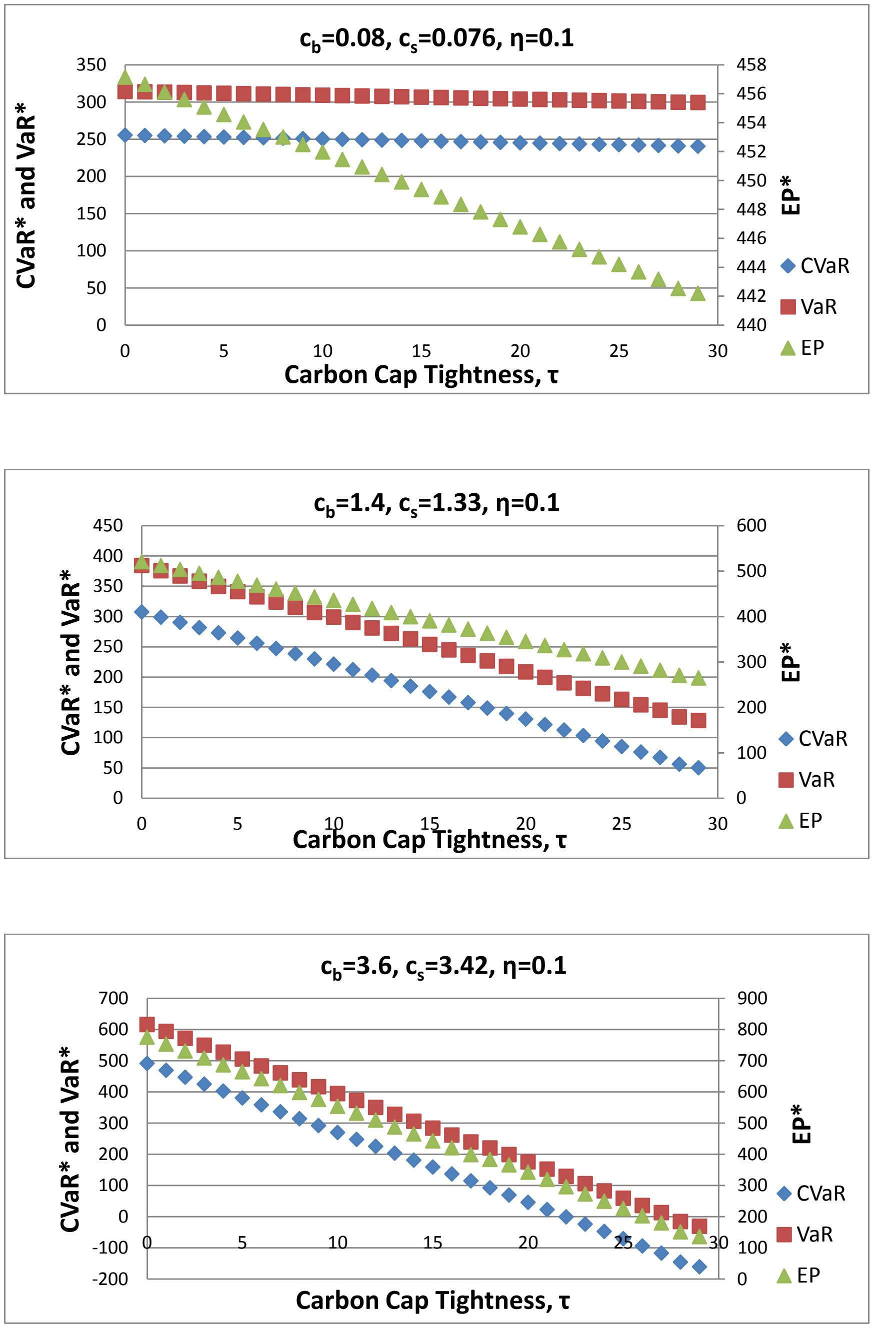


Figure 6.17: $CVaR^*$, ω^* and EP^* vs. τ at p=2, c=1, s=0.8, l=3, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

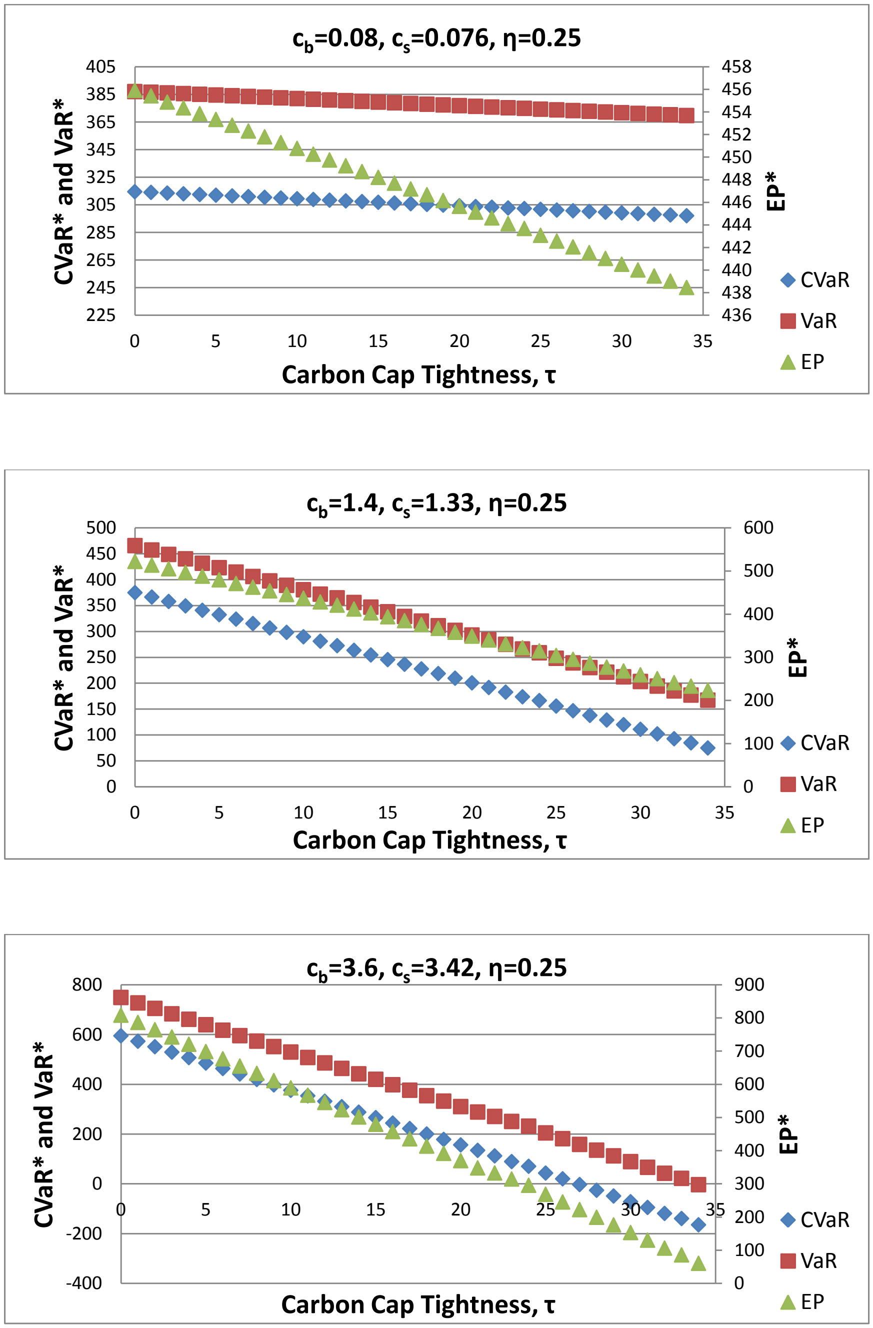


Figure 6.18: $CVaR^{*}$, ω^{*} and EP^{*} vs. τ at p=2, c=1, s=0.8, l=3, η=0.25 for (c_b, c_s)=(0.08, 0.076), (1.4, 1.33), (3.6, 3.42) under Cap and Trade Policy.

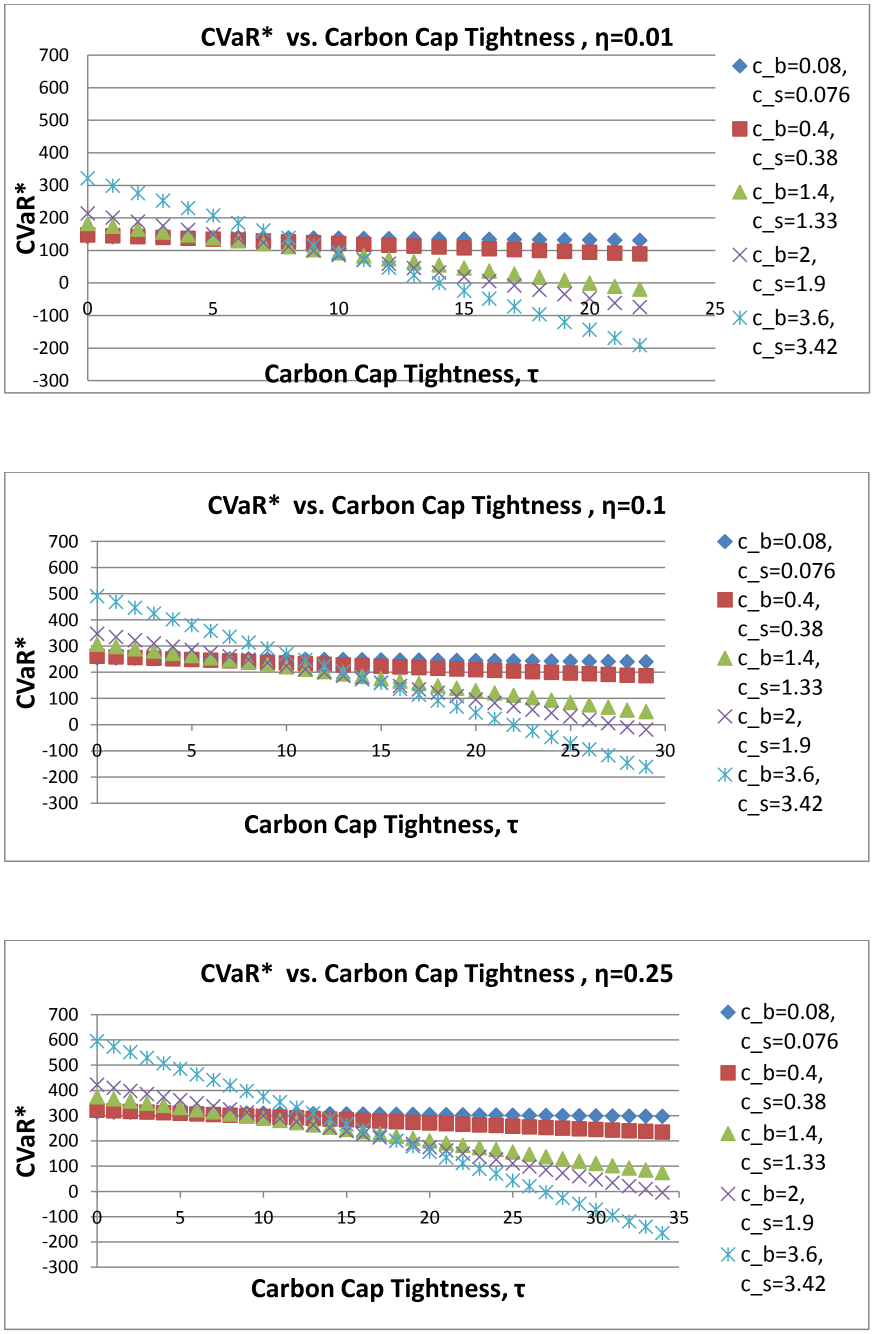


Figure 6.19: $CVaR^*$ vs. τ at p=2, c=1, s=0.8, l=3, η=0.01, 0.1, 0.25 for (c_b, c_s)=(0.08, 0.076), (0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42) under Cap and Trade Policy.

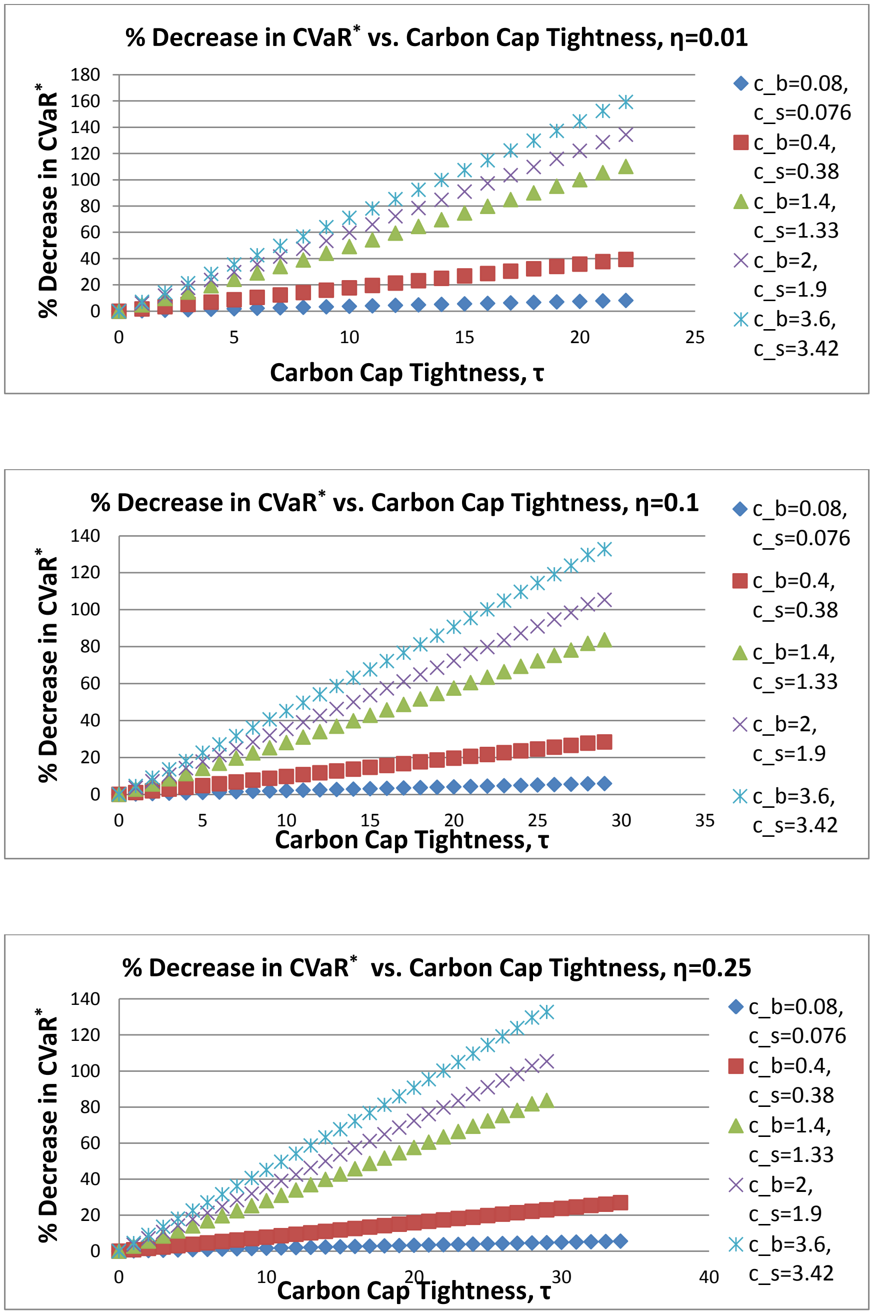
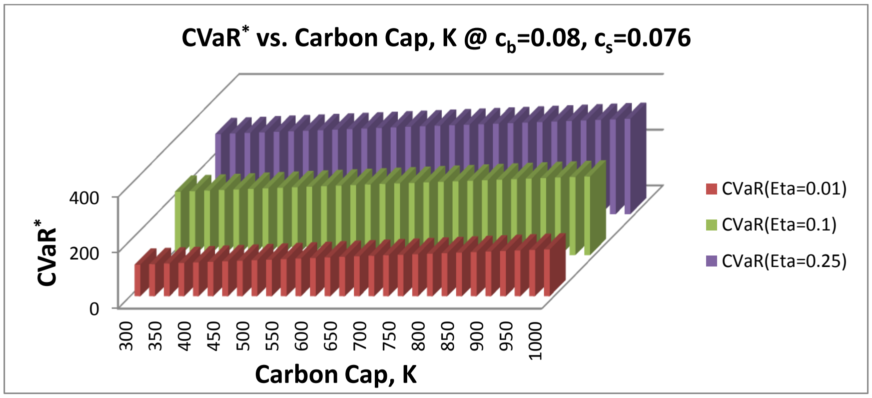
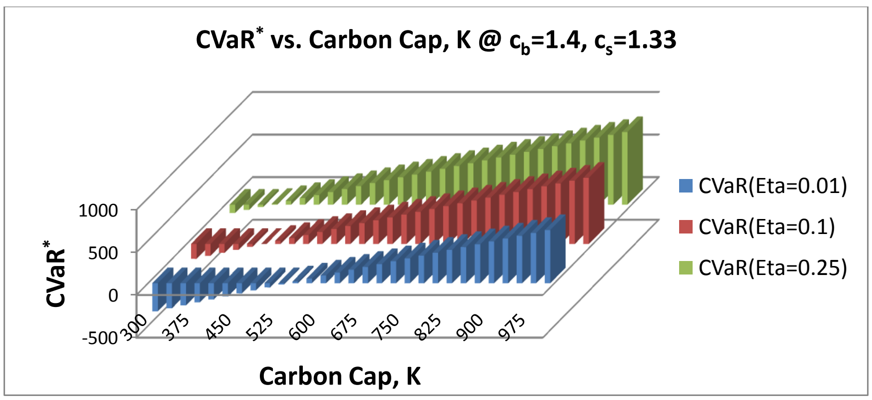


Figure 6.20: % Decrese in $CVaR^*$ vs. τ at p=2, c=1, s=0.8, l=3, η=0.01, 0.1, 0.25 for (c_b, c_s)=(0.08, 0.076), (0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42) under Cap and Trade Policy.





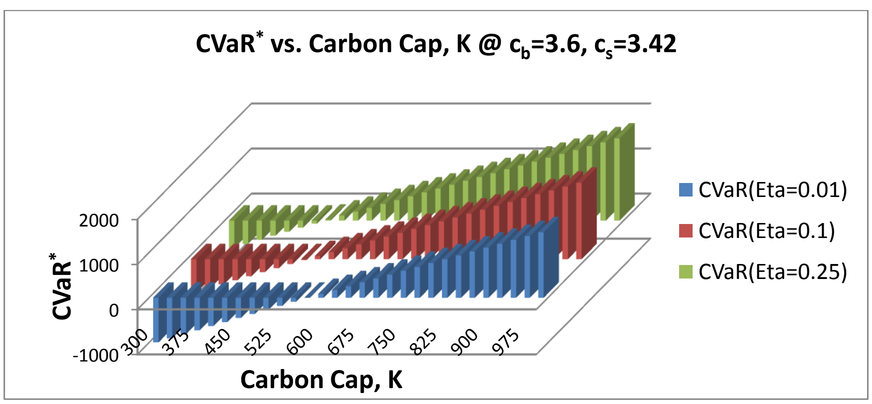


Figure 6.21: $CVaR^*$ vs. K at p=2, c=1, s=0.8, l=3, η=0.01, 0.1, 0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

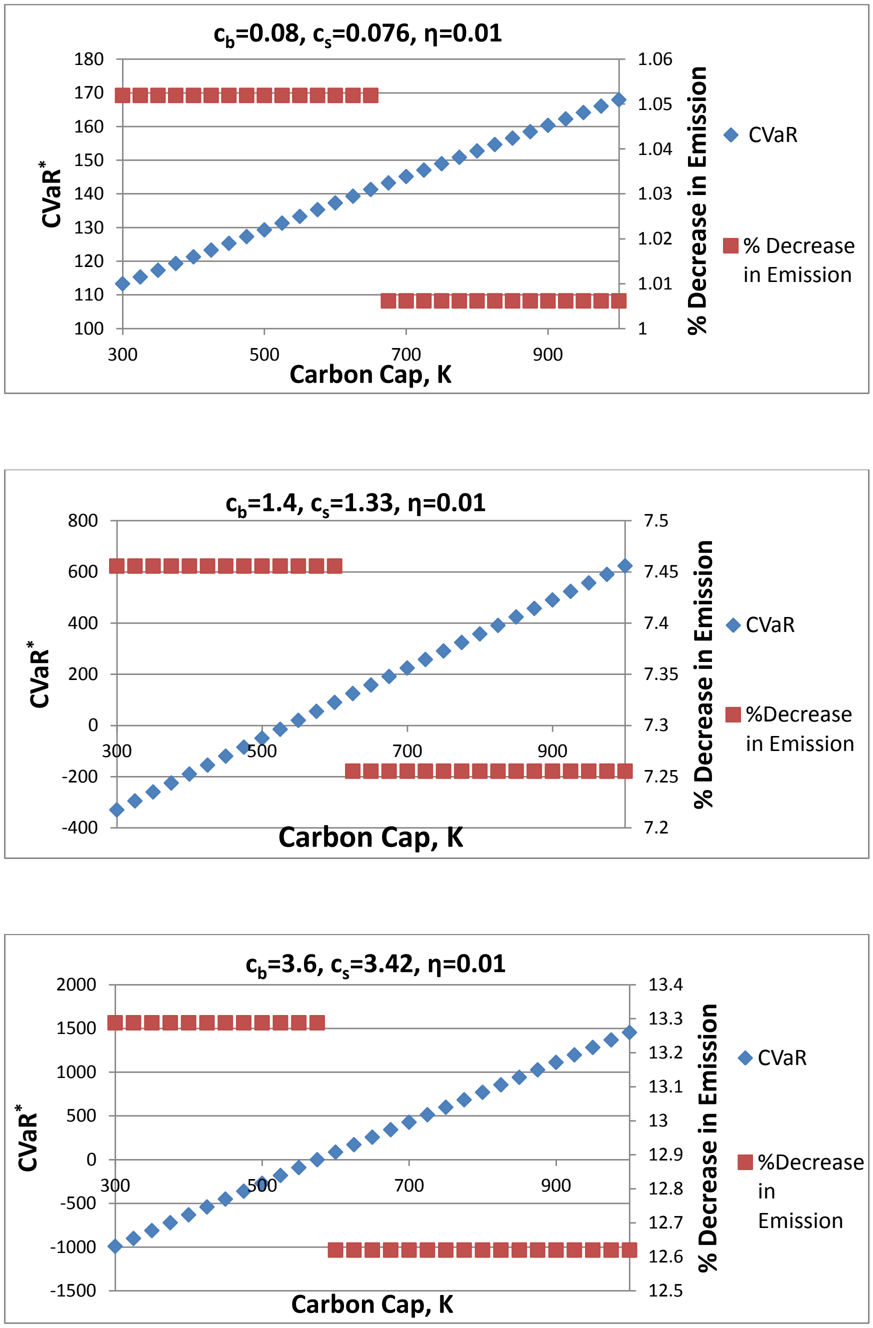


Figure 6.22: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.01 for (c_b, c_s)=(0.08, 0.076), (1.4, 1.33), (3.6, 3.42) under Cap and Trade Policy.

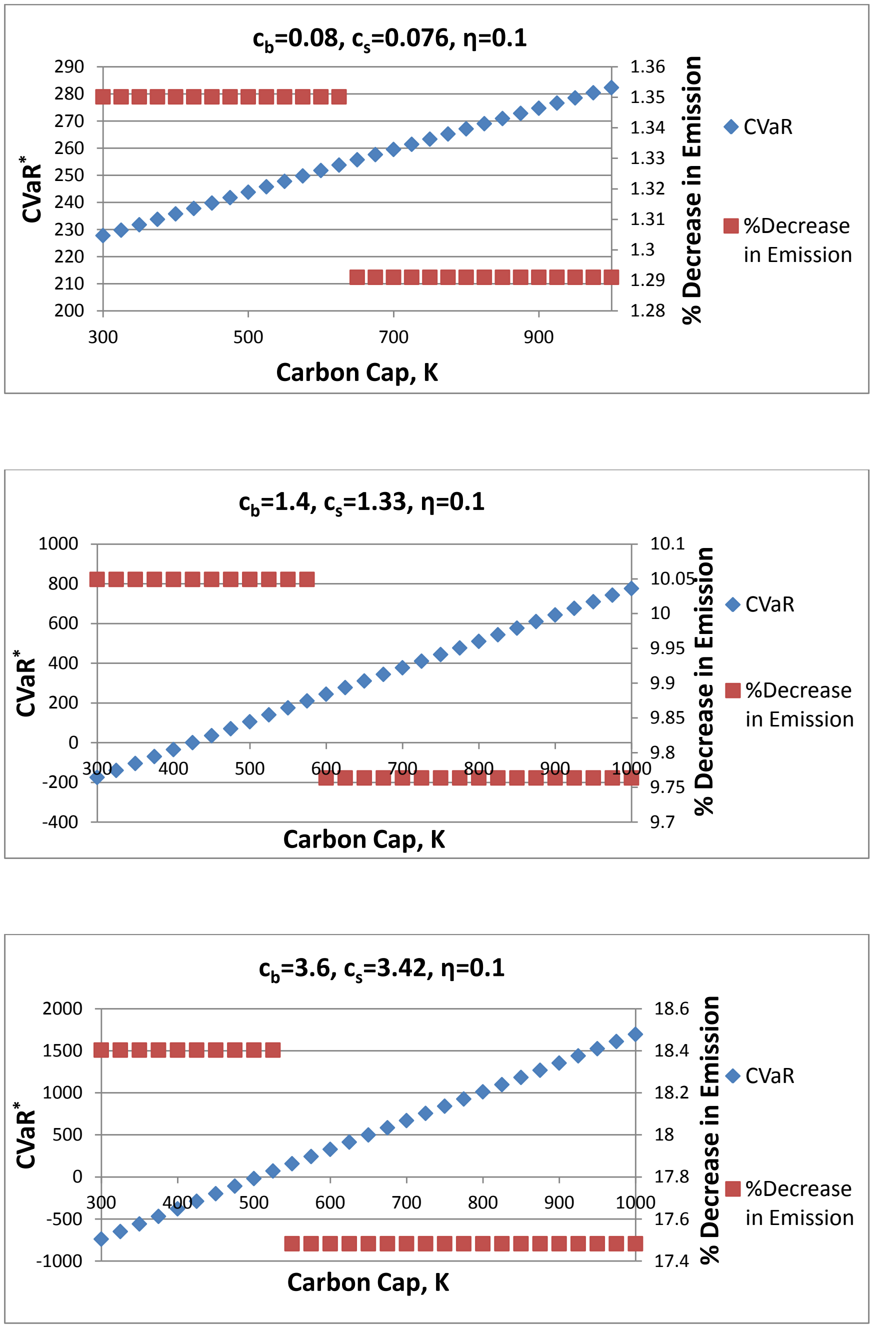


Figure 6.23: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

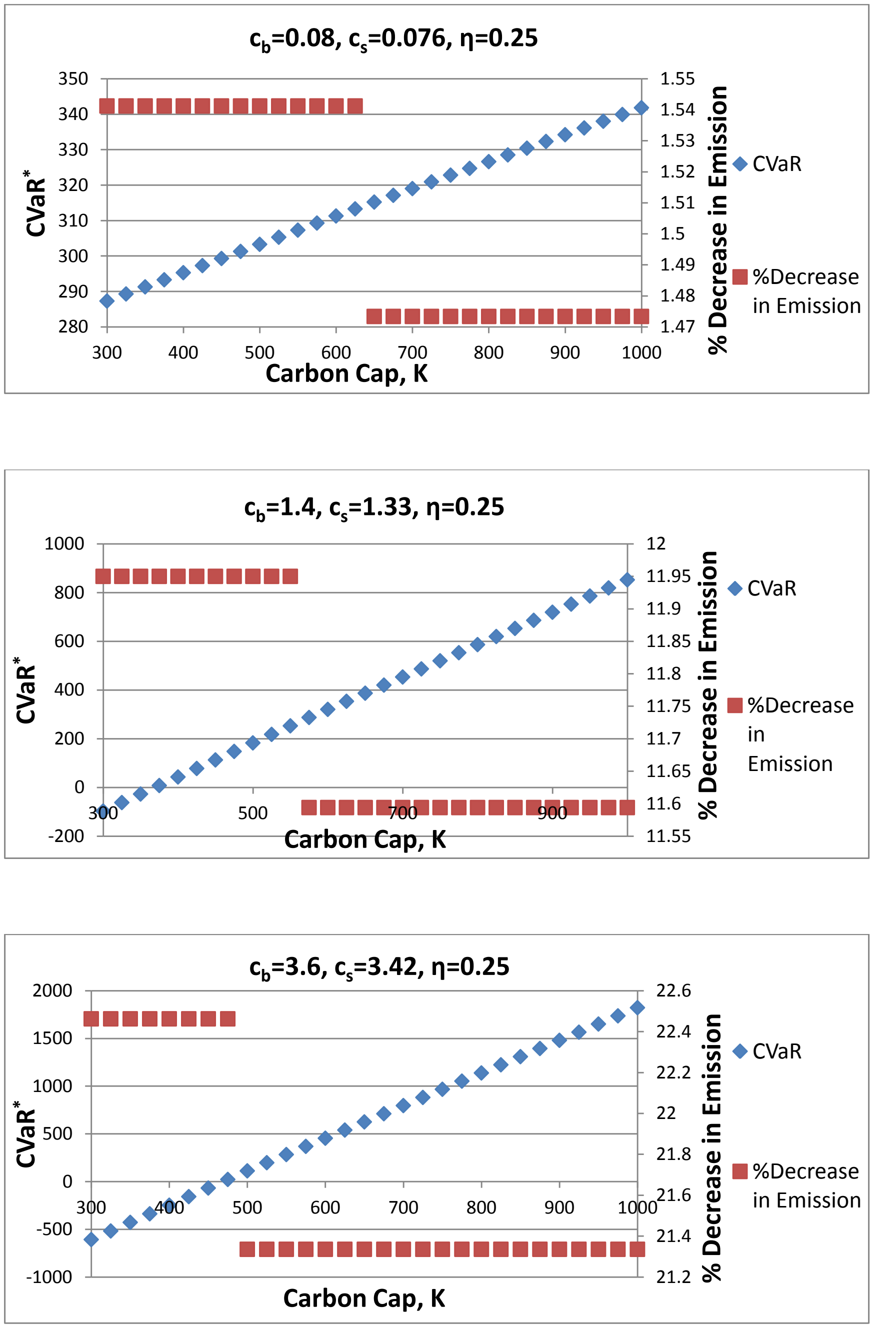
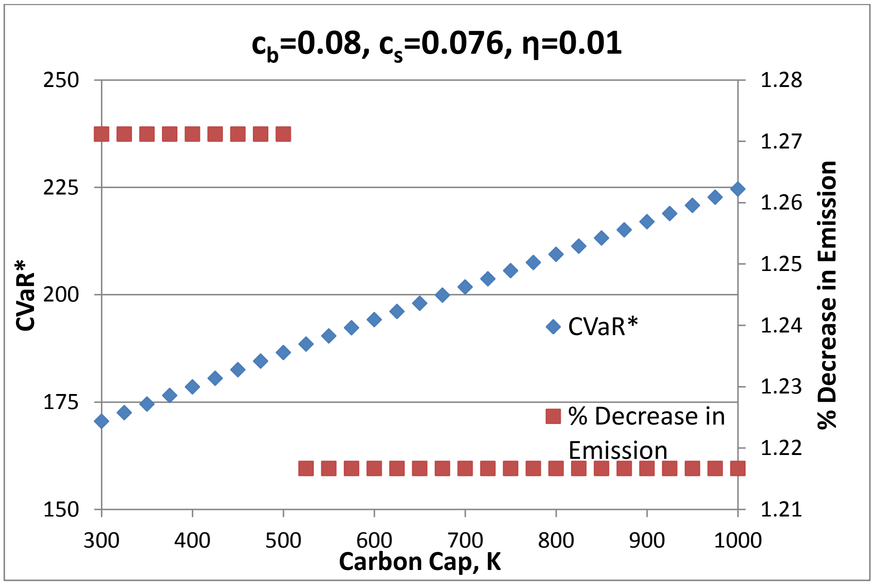


Figure 6.24: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.



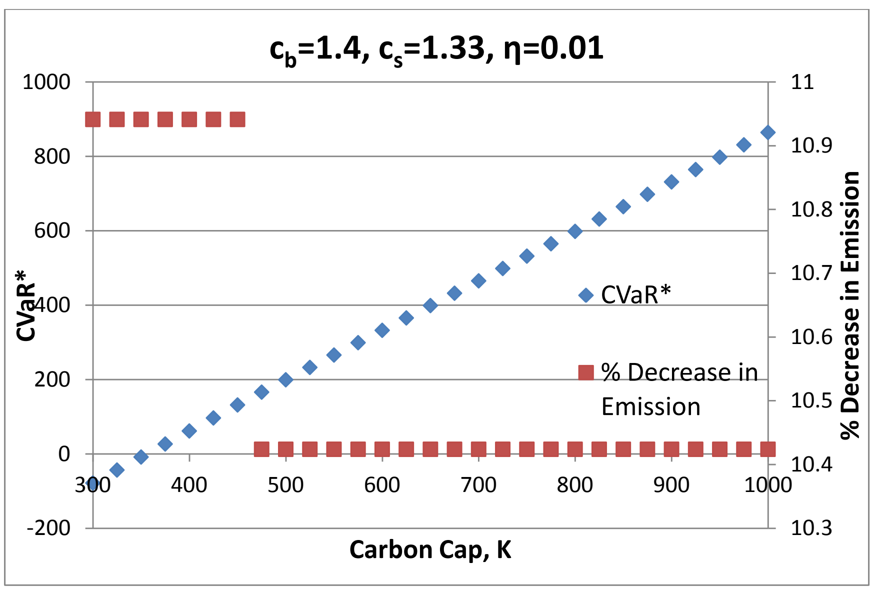
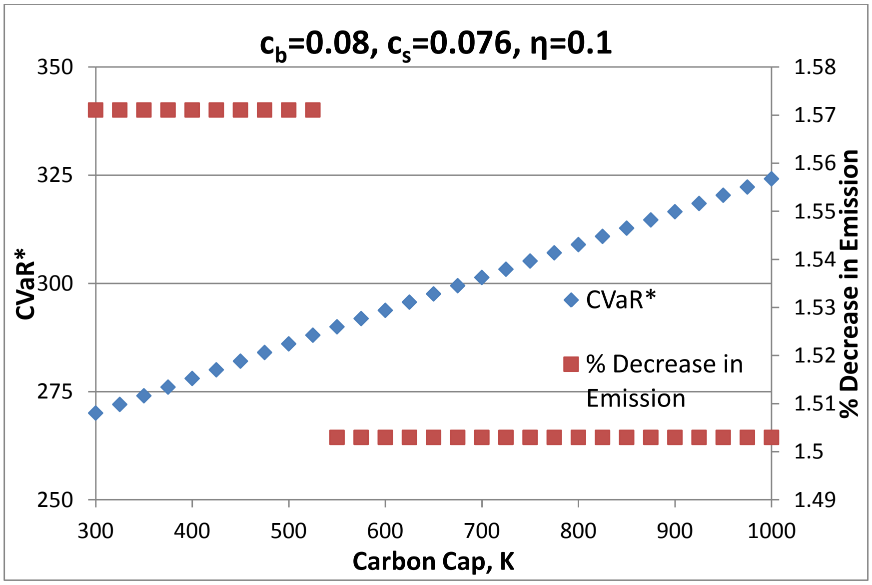


Figure 6.25: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.



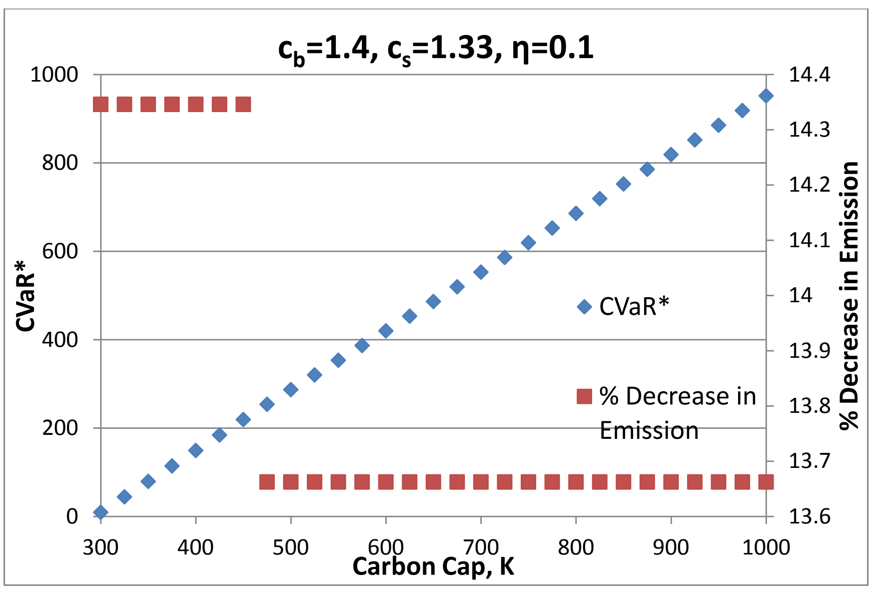


Figure 6.26: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

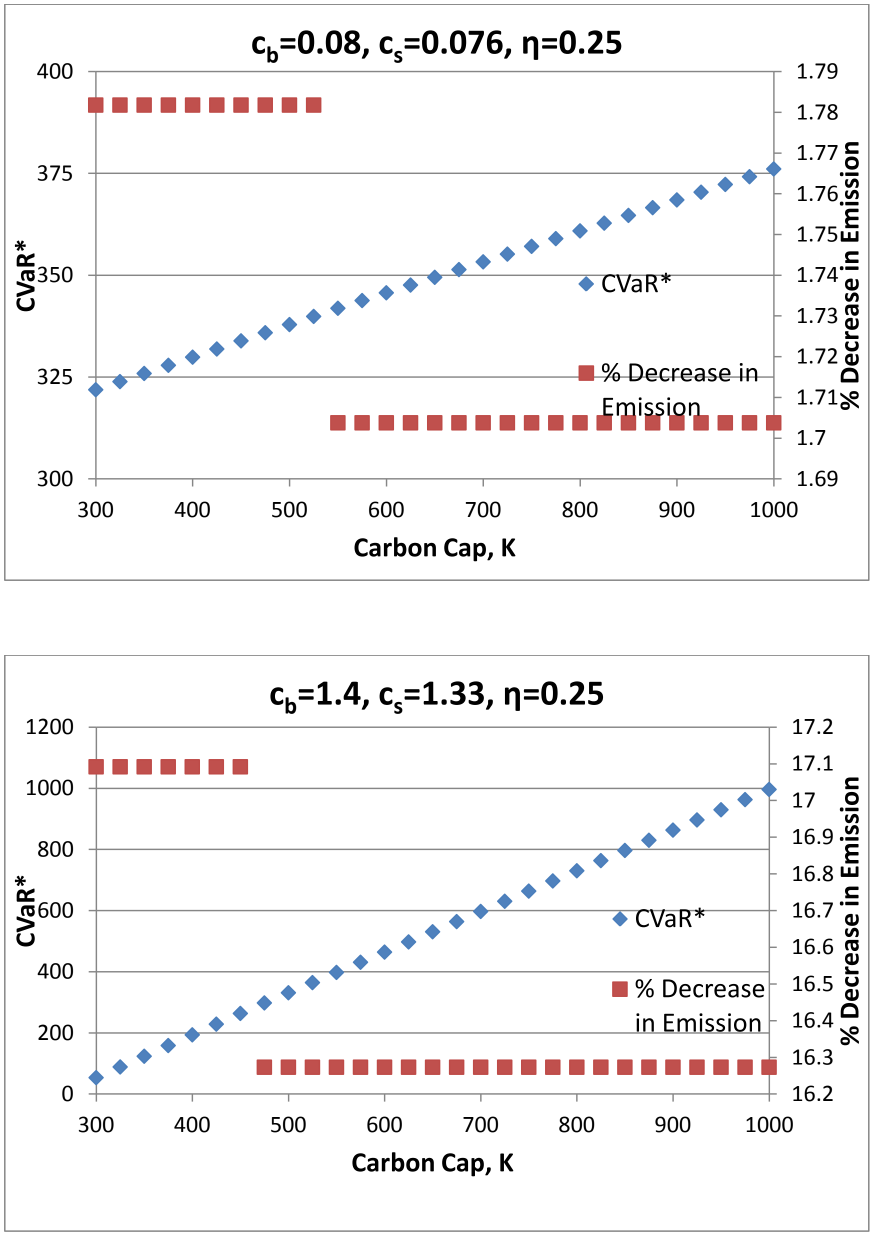


Figure 6.27: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

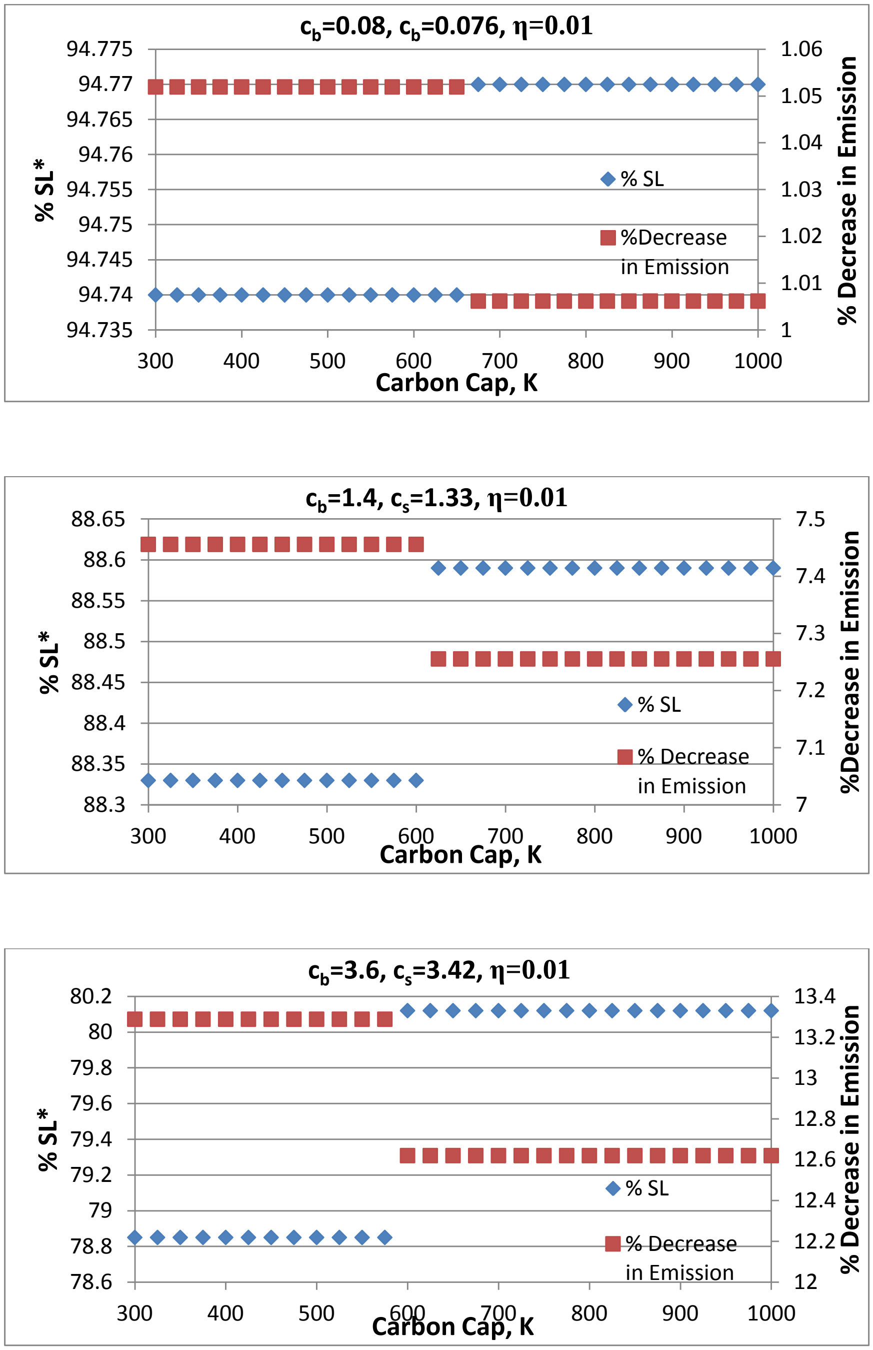


Figure 6.28: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

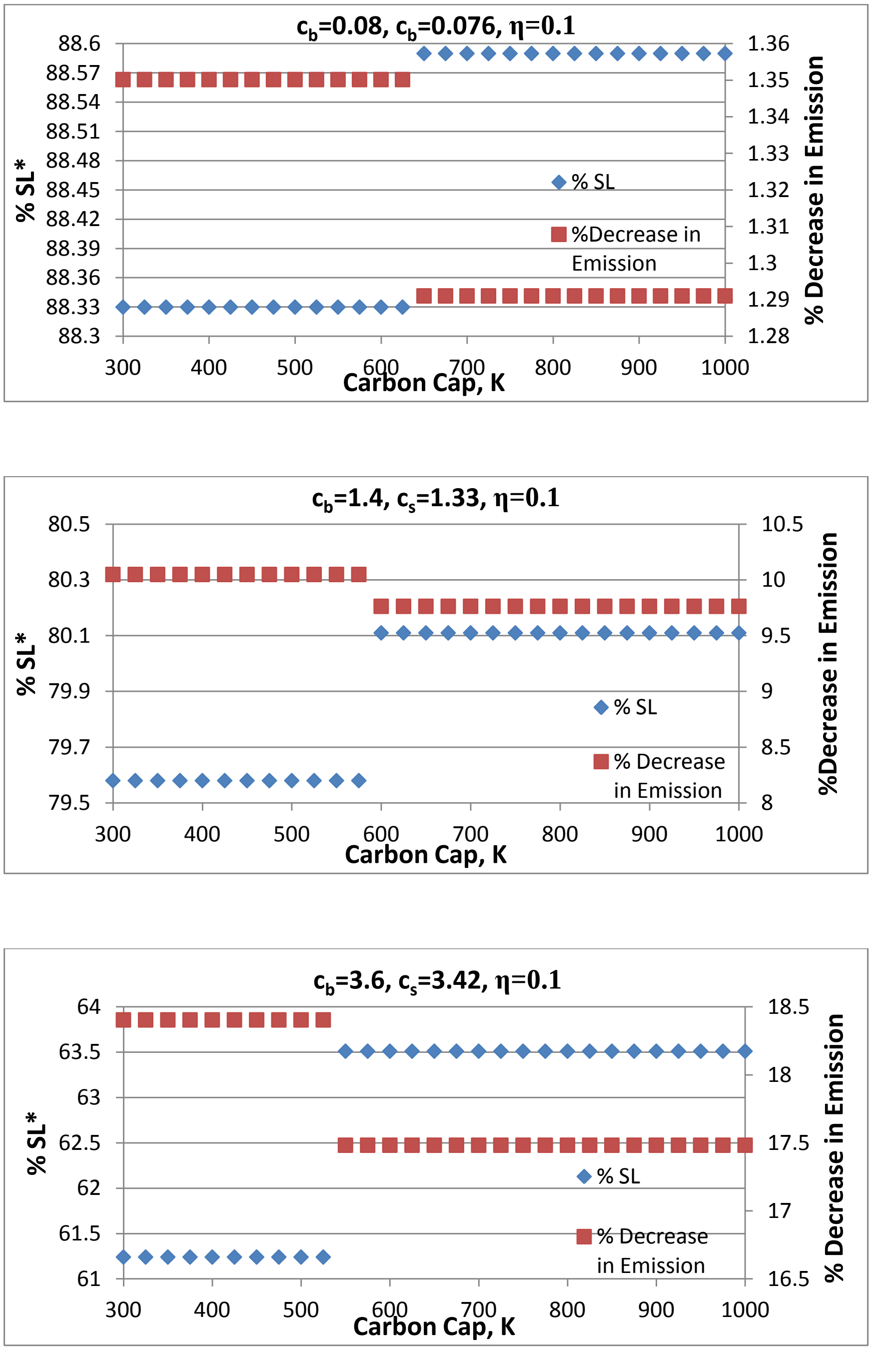


Figure 6.29: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.1 for (c_b, c_s)=(0.08, 0.076), (1.4, 1.33), (3.6, 3.42) under Cap and Trade Policy.

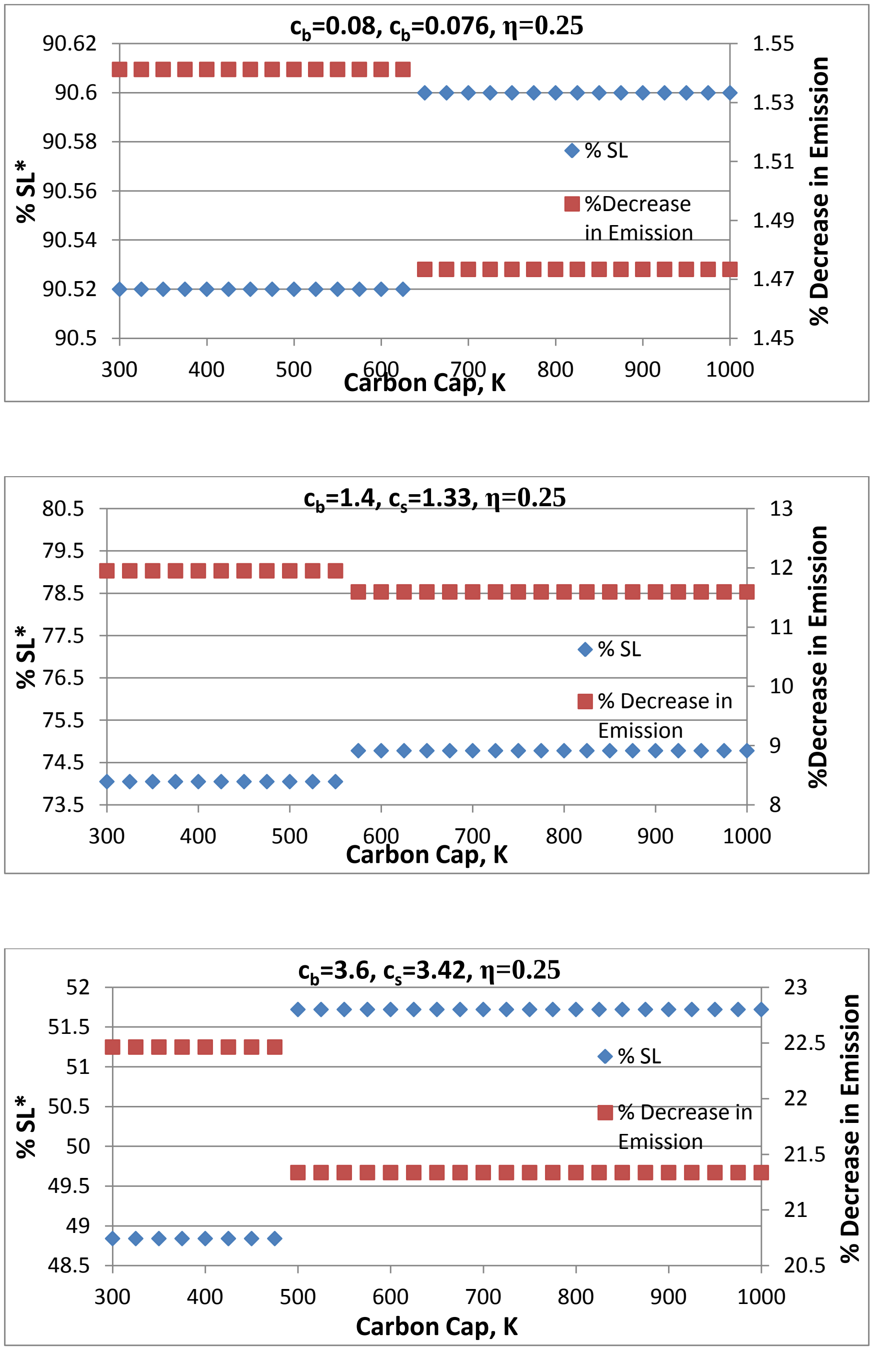


Figure 6.30: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.25 for (c_b, c_s)=(0.08, 0.076), (1.4, 1.33), (3.6, 3.42) under Cap and Trade Policy.

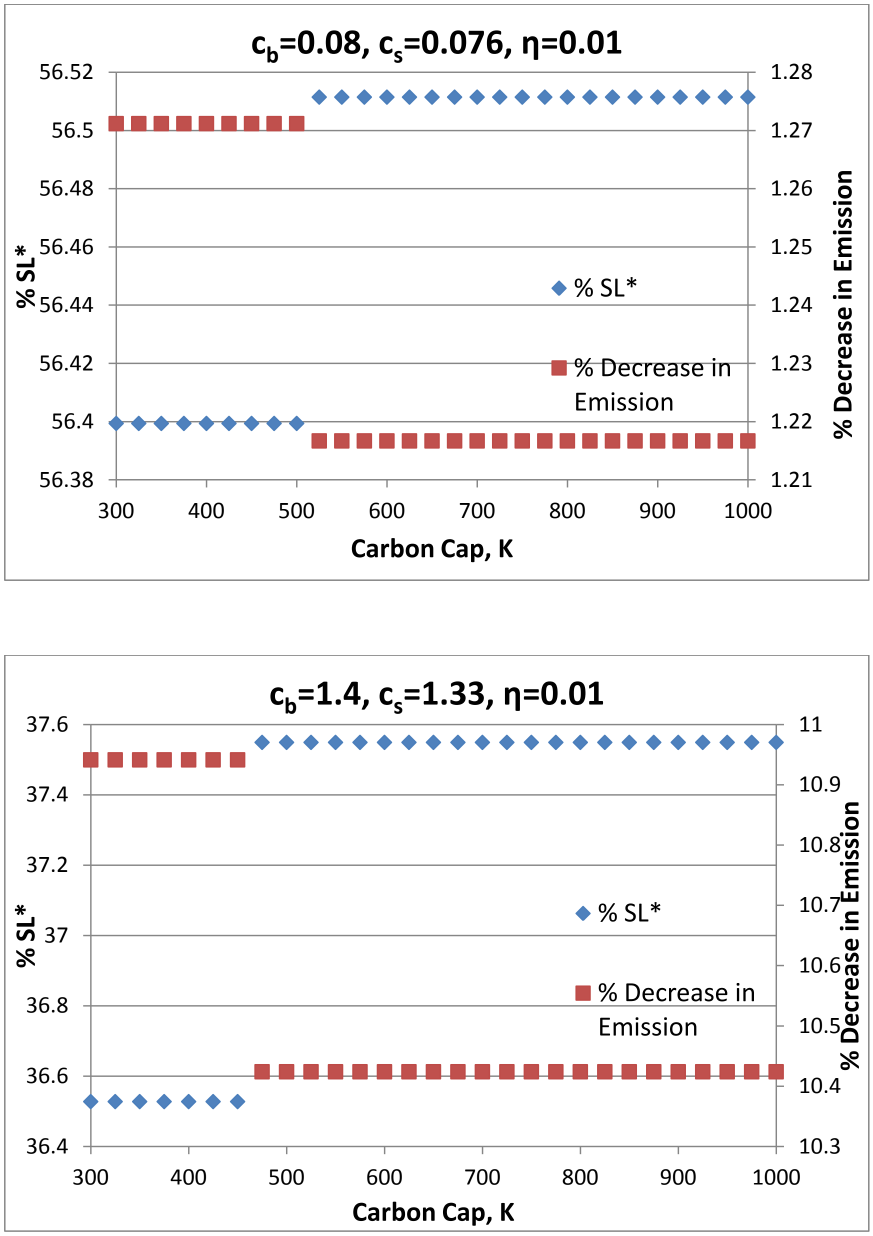
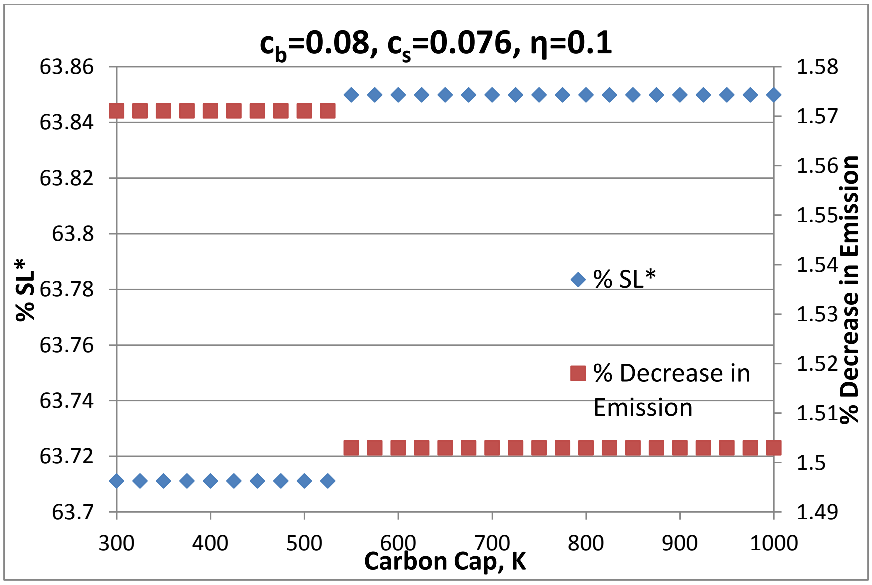


Figure 6.31: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.



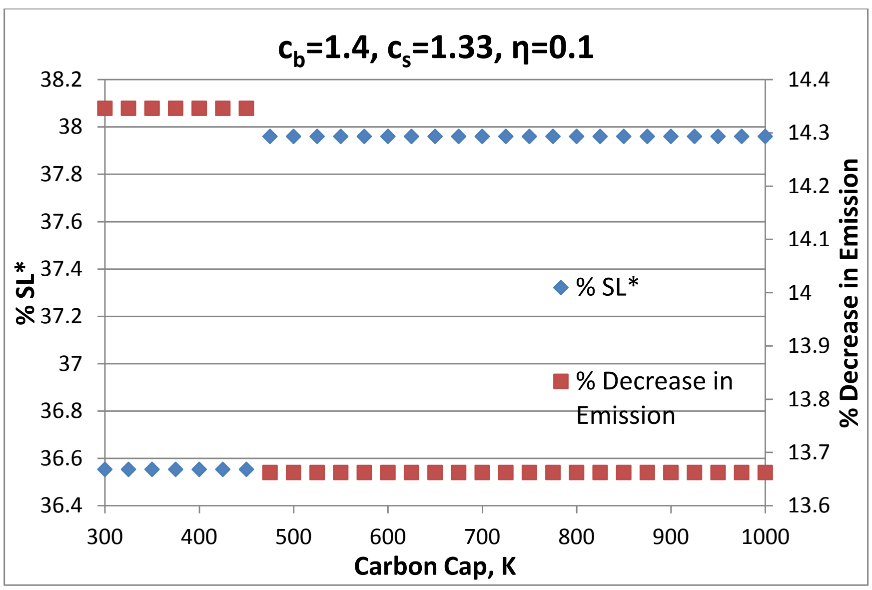
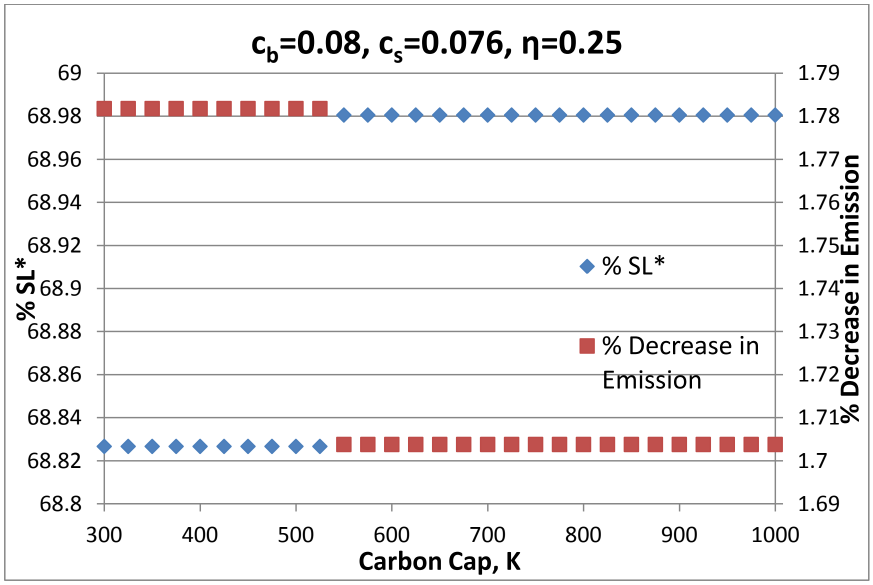


Figure 6.32: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.



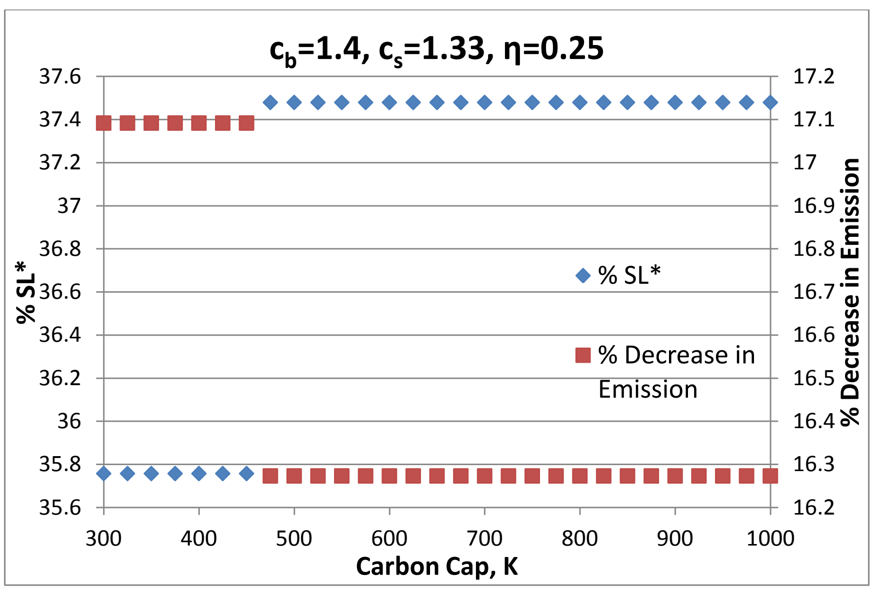


Figure 6.33: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

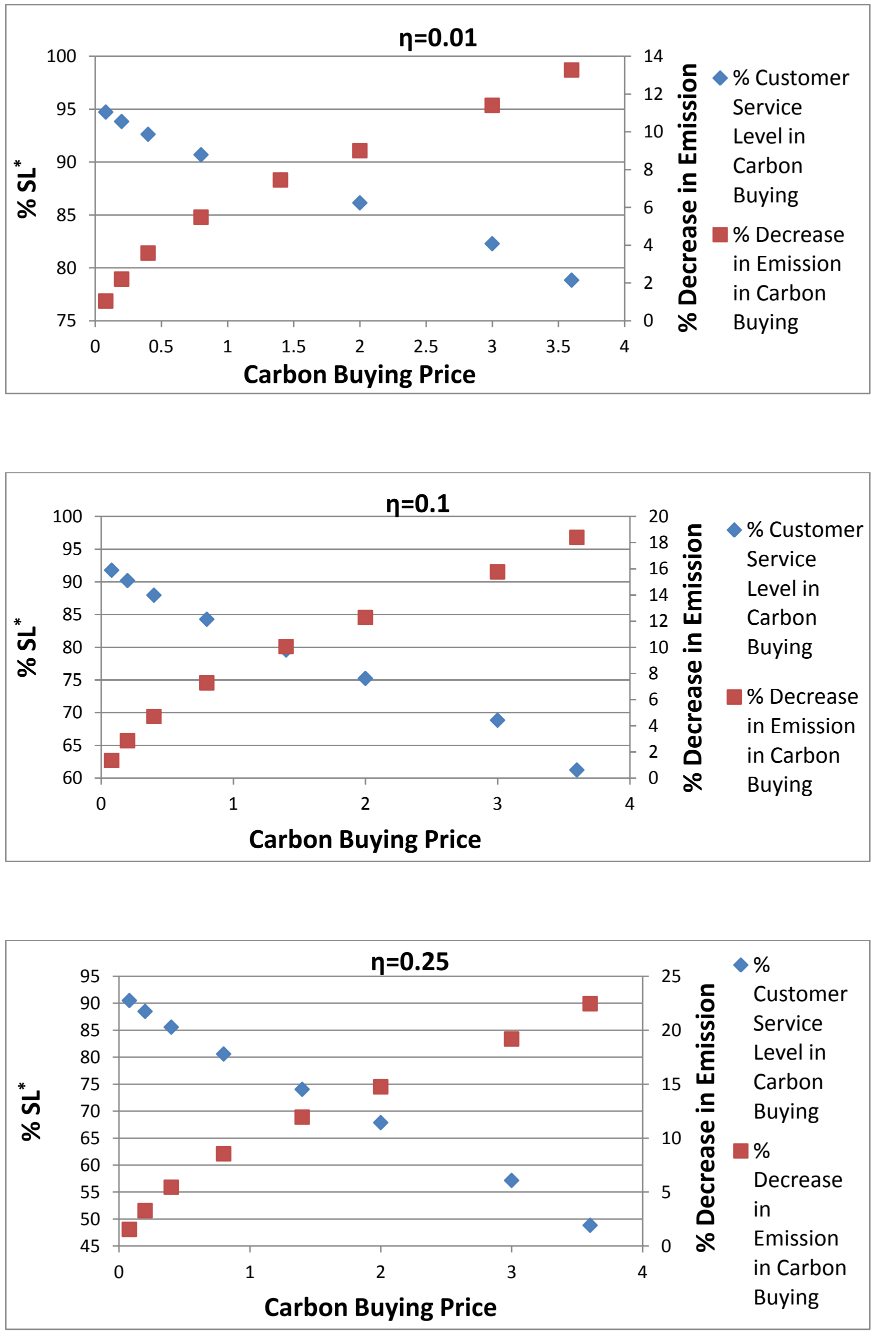


Figure 6.34: SL^* and % Decrease in Emission vs. c_b at p=2, c=1, s=0.8, l=3, η=0.01, 0.1, 0.25 and c_b=(0.08, 0.2, 0.4, 0.8, 1.4, 2, 3, 3.6) under Cap and Trade Policy.

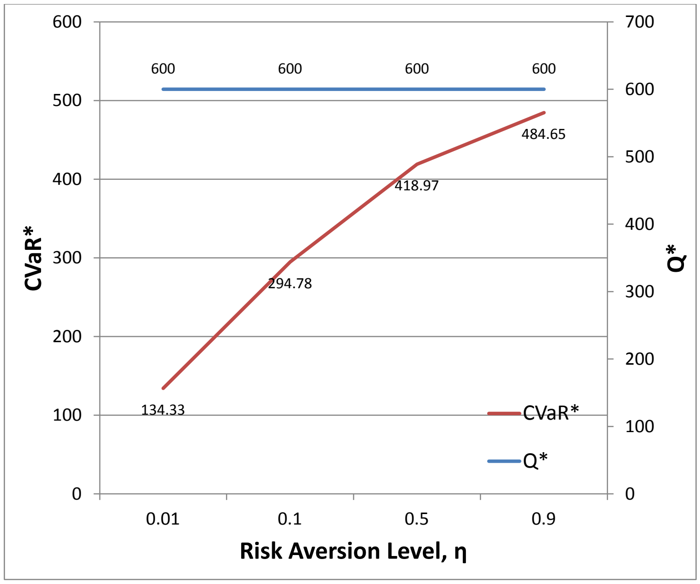


Figure 6.35: Q^{*} and $CVaR^{*}$ vs. η at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 600, K_{cash} = 650 under Binding Resources Policy.

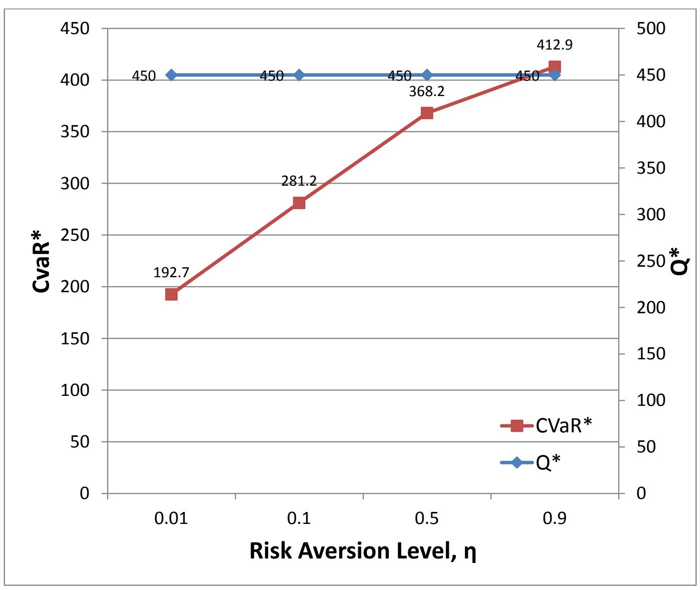


Figure 6.36: Q^{*} and $CVaR^{*}$ vs. η at p=2, c=1, s=0.8, l= 1, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 450, K_{cash} = 510 under Binding Resources Policy.

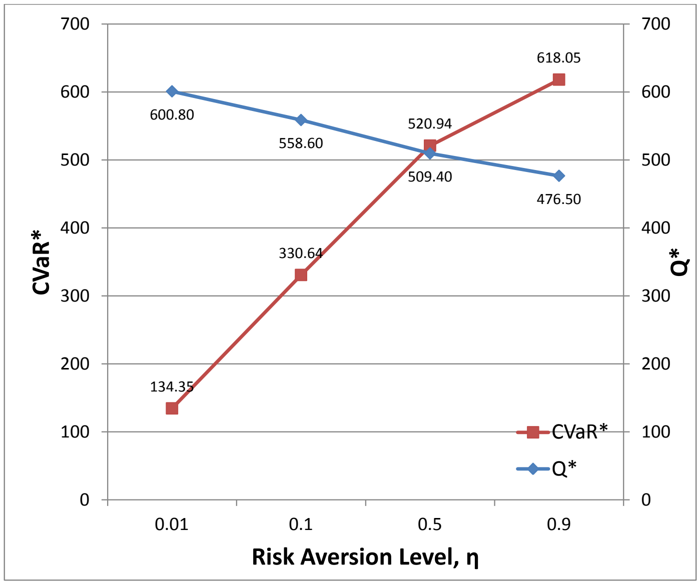


Figure 6.37: Q^{*} and $CVaR^{*}$ vs. η at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 600, K_{cash} = 650 under Resource Trading Policy.

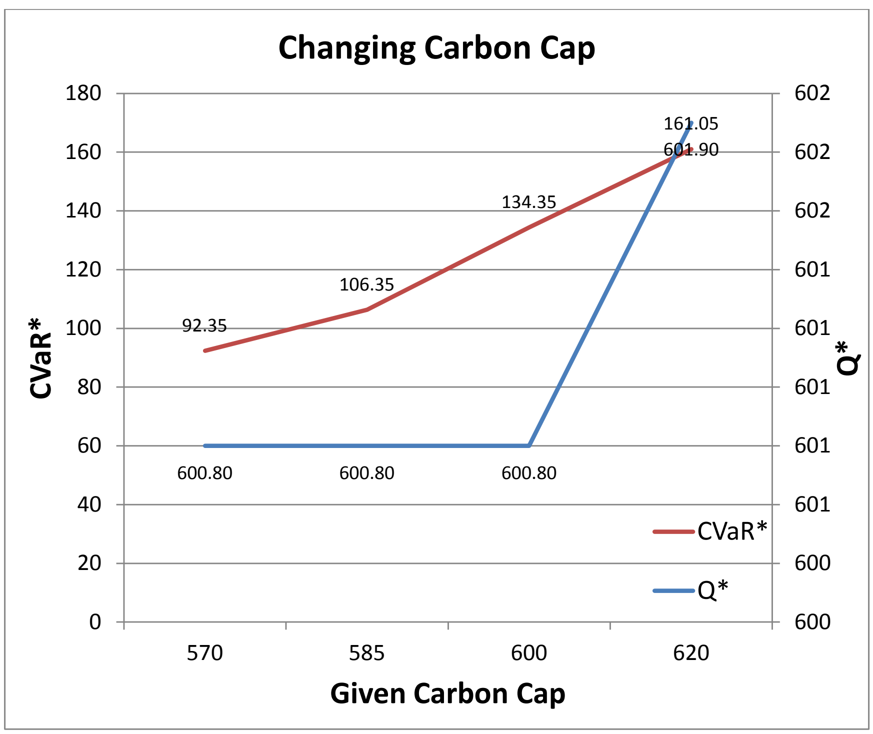


Figure 6.38: Optimal Policy analysis at K_{carbon}=550, 580, 600, 620 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{cash} = 650, η=0.01 under Resource Trading Policy.

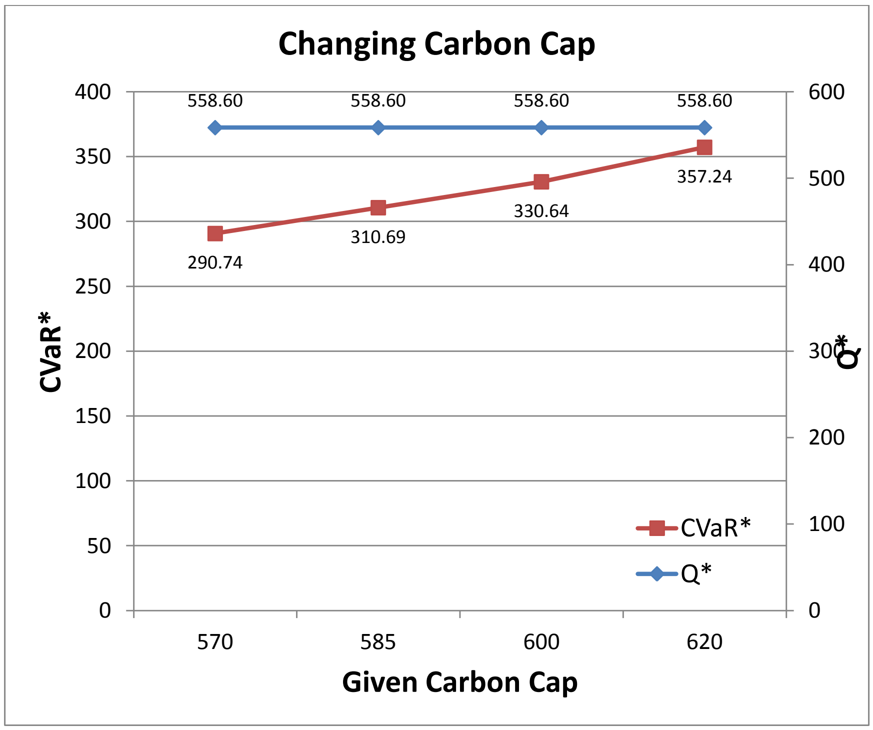


Figure 6.39: Optimal Policy analysis at K_{carbon}=550, 580, 600, 620 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{cash} = 650, η=0.1 under Resource Trading Policy.

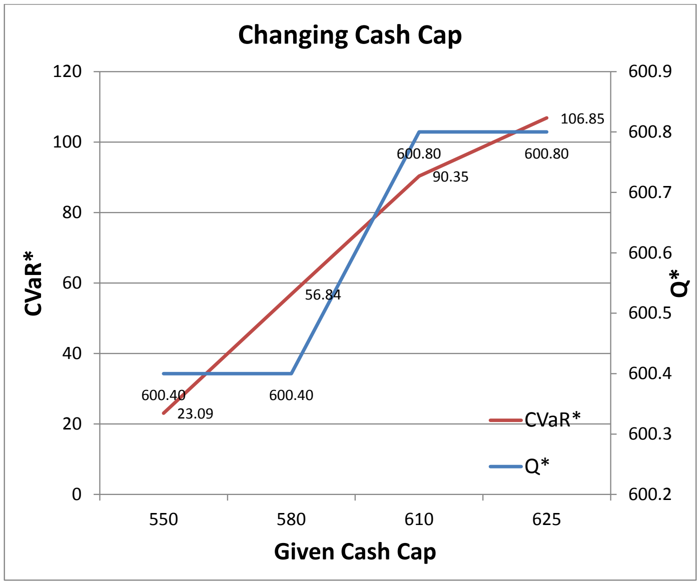


Figure 6.40: Optimal Policy analysis at K_{cash}=550, 580, 610, 625 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 600, η=0.01 under Resource Trading Policy.

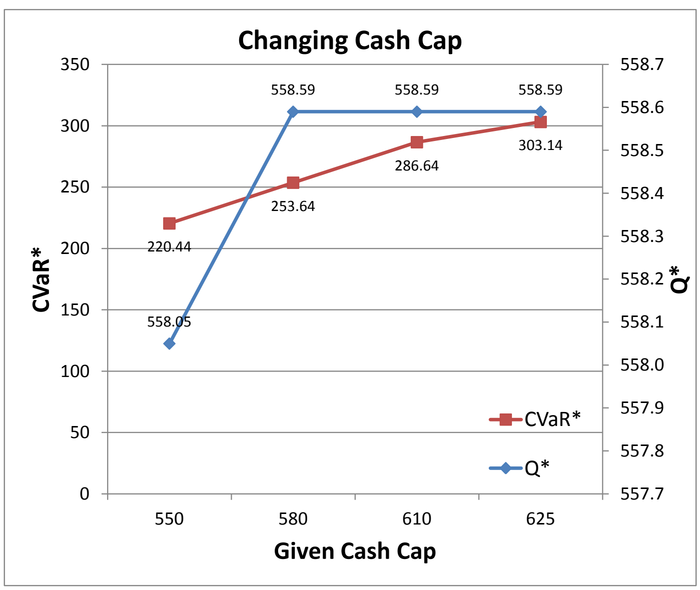


Figure 6.41: Optimal Policy analysis at K_{cash}=550, 580, 610, 625 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 600, η=0.1 under Resource Trading Policy.

p=1.25, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*
0.7	239.14	0.0045	411.84	0.189	503.27	0.5131	560.91	0.7288
0.75	242.42	0.005	425.45	0.228	518.73	0.5743	576.33	0.7774
0.8	246.08	0.0056	441.21	0.2783	536.29	0.6417	593.68	0.8256
0.85	250.23	0.0062	459.83	0.344	556.67	0.7145	613.61	0.872
0.9	255.00	0.0071	482.65	0.4311	581.21	0.7916	637.47	0.9154

p=1.5, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*
0.7	250.23	0.0062	459.83	0.344	556.67	0.7145	613.61	0.872
0.75	252.53	0.0067	470.59	0.3843	568.28	0.7526	624.91	0.8942
0.8	255.00	0.0071	482.65	0.4311	581.21	0.7916	637.47	0.9154
0.85	257.68	0.0077	496.46	0.4859	595.95	0.8313	651.80	0.9355
0.9	260.60	0.0083	512.92	0.5514	613.56	0.8719	669.03	0.9545

p=1.75, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*
0.7	255.00	0.0071	482.65	0.4311	581.21	0.7916	637.47	0.9154
0.75	256.76	0.0075	491.62	0.4666	590.79	0.818	646.78	0.9289
0.8	258.63	0.0079	501.58	0.5063	601.42	0.8448	657.13	0.9419
0.85	260.60	0.0083	512.92	0.5514	613.56	0.8719	669.03	0.9545
0.9	262.71	0.0088	526.41	0.6041	628.18	0.9	683.51	0.9668

p=2, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*
0.7	257.68	0.0077	496.46	0.4859	595.95	0.8313	651.80	0.9355
0.75	259.11	0.008	504.27	0.517	604.29	0.8515	659.94	0.9451
0.8	260.60	0.0083	512.92	0.5514	613.56	0.8719	669.03	0.9545
0.85	262.17	0.0087	522.76	0.59	624.20	0.8929	679.54	0.9637
0.9	263.81	0.0091	534.53	0.6351	637.14	0.9149	692.51	0.9729

Table 6.1: Q^* and SL^* values obtained at η=0.01 under Unconstrained CVaR Maximization.

p=1.25, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q*	SL*	Q*	SL*	Q*	SL*	Q*	SL*
0.7	330.94	0.0455	450.17	0.3091	513.83	0.555	554.70	0.7078
0.75	335.51	0.05	461.06	0.3485	526.02	0.6027	567.02	0.7486
0.8	340.68	0.0556	473.74	0.3964	540.08	0.6557	581.14	0.7914
0.85	346.59	0.0625	488.93	0.4559	556.76	0.7148	597.81	0.836
0.9	353.48	0.0714	507.99	0.5319	577.56	0.781	618.56	0.8821

p=1.5, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q*	SL*	Q*	SL*	Q*	SL*	Q*	SL*
0.7	346.59	0.0625	488.93	0.4559	556.76	0.7148	597.81	0.836
0.75	349.89	0.0667	497.85	0.4914	566.49	0.747	607.52	0.8588
0.8	353.48	0.0714	507.99	0.5319	577.56	0.781	618.56	0.8821
0.85	357.39	0.0769	519.88	0.5788	590.55	0.8174	631.54	0.9058
0.9	361.70	0.0833	534.53	0.6351	606.68	0.857	647.78	0.9303

p=1.75, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q*	SL*	Q*	SL*	Q*	SL*	Q*	SL*
0.7	353.48	0.0714	507.99	0.5319	577.56	0.781	618.56	0.8821
0.75	356.05	0.075	515.68	0.5623	585.95	0.805	626.94	0.8978
0.8	358.78	0.0789	524.38	0.5963	595.48	0.8302	636.49	0.9139
0.85	361.70	0.0833	534.53	0.6351	606.68	0.857	647.78	0.9303
0.9	364.83	0.0882	547.06	0.681	620.70	0.8863	662.05	0.9474

p=2, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q*	SL*	Q*	SL*	Q*	SL*	Q*	SL*
0.7	357.39	0.0769	519.88	0.5788	590.55	0.8174	631.54	0.9058
0.75	359.49	0.08	526.76	0.6055	598.09	0.8367	639.12	0.9179
0.8	361.70	0.0833	534.53	0.6351	606.68	0.857	647.78	0.9303
0.85	364.03	0.087	543.62	0.6686	616.82	0.8786	658.09	0.943
0.9	366.48	0.0909	554.88	0.7084	629.60	0.9025	671.21	0.9566

Table 6.2: Q^* and SL^* values obtained at η=0.1 under Unconstrained CVaR Maximization.

p=1.25, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*
0.7	379.26	0.1136	471.75	0.3888	521.54	0.5853	554.05	0.7056
0.75	384.97	0.125	481.61	0.427	532.35	0.6268	565.03	0.7422
0.8	391.47	0.1389	493.13	0.4726	544.94	0.6734	577.77	0.7816
0.85	399.00	0.1563	507.05	0.5281	560.11	0.7261	593.08	0.824
0.9	407.92	0.1786	524.77	0.5978	579.43	0.7865	612.60	0.8699

p=1.5, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*
0.7	399.00	0.1563	507.05	0.5281	560.11	0.7261	593.08	0.824
0.75	403.26	0.1667	515.30	0.5608	569.09	0.7552	602.14	0.8465
0.8	407.92	0.1786	524.77	0.5978	579.43	0.7865	612.60	0.8699
0.85	413.06	0.1923	536.02	0.6406	591.78	0.8206	625.11	0.8945
0.9	418.78	0.2083	550.14	0.6919	607.45	0.8587	641.09	0.9209

p=1.75, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*
0.7	407.92	0.1786	524.77	0.5978	579.43	0.7865	612.60	0.8699
0.75	411.29	0.1875	532.02	0.6256	587.38	0.8089	620.65	0.8862
0.8	414.89	0.1974	540.32	0.6566	596.53	0.8328	629.94	0.9031
0.85	418.78	0.2083	550.14	0.6919	607.45	0.8587	641.09	0.9209
0.9	422.98	0.2206	562.47	0.7339	621.38	0.8876	655.43	0.9399

p=2, c=1	l=0		l=p-c		l=2(p-c)		l=3(p-c)	
s	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*	Q^*	SL^*
0.7	413.06	0.1923	536.02	0.6406	591.78	0.8206	625.11	0.8945
0.75	415.84	0.2	542.61	0.665	599.06	0.8391	632.51	0.9074
0.8	418.78	0.2083	550.14	0.6919	607.45	0.8587	641.09	0.9209
0.85	421.90	0.2174	559.06	0.7226	617.49	0.88	651.41	0.935
0.9	425.21	0.2273	570.31	0.759	630.38	0.9039	664.78	0.9503

Table 6.3: Q^* and SL^* values obtained at η=0.25 under Unconstrained CVaR Maximization.

p=1.5, c=1, s=0.8	l=0.5		l=1		l=1.5	
η	Q*	CVaR*	Q*	CVaR *	Q*	CVaR*
0.01	482.65	64.11	581.21	43.71	637.47	32.08
0.02	488.91	79.77	579.59	60.93	631.69	50.13
0.03	493.00	89.53	578.78	71.65	628.30	61.36
0.04	496.14	96.76	578.30	79.60	625.91	69.69
0.05	498.74	102.58	577.99	85.99	624.07	76.38
0.06	500.98	107.47	577.79	91.36	622.58	82.01
0.07	502.98	111.71	577.66	96.03	621.34	86.90
0.08	504.79	115.47	577.59	100.16	620.28	91.23
0.09	506.45	118.86	577.56	103.89	619.36	95.13
0.1	507.99	121.96	577.56	107.29	618.56	98.69
0.2	519.98	143.99	578.56	131.53	613.79	124.09
0.3	529.19	158.66	580.46	147.68	611.84	141.03
0.4	537.42	170.21	582.93	160.42	611.26	154.41
0.5	545.35	180.04	585.95	171.30	611.64	165.84
0.6	553.40	188.83	589.59	181.06	612.88	176.13
0.7	561.93	196.97	594.05	190.15	615.07	185.73
0.8	571.41	204.73	599.69	198.87	618.49	195.00
0.9	582.53	212.33	607.25	207.52	623.88	204.26
1	596.74	220.02	618.68	216.47	633.52	214.01

Table 6.4: Q^* and $CVaR^*$ values at p=1.5, c=1, s=0.8 for l=0.5, 1, and 1.5 for changing η under Unconstrained CVaR Maximization.

p=1.75, c=1, s=0.8	l=0.75		l=1.5		l=2.25	
η	Q*	CVaR*	Q*	CVaR *	Q*	CVaR*
0.01	501.58	118.76	601.42	98.06	657.13	86.55
0.02	507.07	140.70	599.11	121.54	650.82	110.82
0.03	510.70	154.38	597.89	136.18	647.12	125.94
0.04	513.52	164.54	597.12	147.04	644.51	137.17
0.05	515.87	172.70	596.59	155.77	642.51	146.19
0.06	517.92	179.57	596.20	163.12	640.89	153.79
0.07	519.74	185.54	595.93	169.50	639.53	160.38
0.08	521.41	190.83	595.72	175.17	638.38	166.24
0.09	522.94	195.60	595.58	180.27	637.37	171.51
0.1	524.38	199.96	595.48	184.93	636.49	176.33
0.2	535.70	231.08	595.84	218.23	631.26	210.75
0.3	544.63	251.89	597.42	240.52	629.08	233.80
0.4	552.77	268.34	599.71	258.17	628.35	252.07
0.5	560.76	282.43	602.64	273.32	628.65	267.76
0.6	569.04	295.11	606.29	286.99	629.86	281.95
0.7	578.02	306.95	610.88	299.80	632.09	295.28
0.8	588.31	318.35	616.87	312.22	635.68	308.26
0.9	600.96	329.69	625.27	324.72	641.57	321.41
1	618.68	341.47	639.42	338.02	653.41	335.65

Table 6.5: Q^* and $CVaR^*$ values at p=1.75, c=1, s=0.8 for l=0.75, 1.5, and 2.25 for changing η under Unconstrained CVaR Maximization.

p=2, c=1, s=0.8	l=1		l=2		l=3	
η	Q*	CVaR*	Q*	CVaR *	Q*	CVaR*
0.01	512.92	174.90	613.56	154.03	669.03	142.57
0.02	518.03	203.04	610.95	183.69	662.51	173.00
0.03	521.45	220.59	609.56	202.19	658.70	191.97
0.04	524.12	233.62	608.67	215.92	656.02	206.06
0.05	526.36	244.10	608.05	226.96	653.95	217.39
0.06	528.31	252.93	607.60	236.27	652.29	226.93
0.07	530.06	260.60	607.26	244.35	650.90	235.22
0.08	531.66	267.41	607.01	251.52	649.72	242.58
0.09	533.14	273.55	606.82	257.99	648.69	249.21
0.1	534.53	279.15	606.68	263.89	647.78	255.27
0.2	545.58	319.24	606.82	306.15	642.44	298.62
0.3	554.41	346.13	608.30	334.50	640.21	327.72
0.4	562.55	367.45	610.55	357.02	639.47	350.85
0.5	570.62	385.77	613.48	376.40	639.78	370.77
0.6	579.06	402.31	617.18	393.94	641.02	388.82
0.7	588.34	417.83	621.88	410.45	643.30	405.86
0.8	599.15	432.87	628.10	426.55	647.01	422.52
0.9	612.87	447.99	637.07	442.90	653.25	439.54
1	633.52	464.01	653.41	460.65	666.84	458.34

Table 6.6: Q^* and $CVaR^*$ values at p=2, c=1, s=0.8 with l=1, 2, and 3 for changing η under Unconstrained CVaR Maximization.

η= 0.01										
τ	Q*	CVaR*	VaR*, ω*	EP*	SL*	% Decrease in Q*	% Decrease in EP*	% Decrease in CVaR*	(% Decrease in CVaR*)/ (% Decrease in Q*)	(% Decrease in EP*)/ (% Decrease in Q*)
0	669.02	142.57	184.84	458.33	0.95	0.00	0.00	0	-	-
1	662.34	142.33	185.33	458.30	0.95	1.00	0.01	0.17	0.17	0.01
2	655.65	141.51	185.52	458.06	0.94	2.00	0.06	0.74	0.37	0.03
3	648.96	139.92	185.32	457.60	0.93	3.00	0.16	1.86	0.62	0.05
4	642.26	137.34	184.63	456.90	0.92	4.00	0.31	3.67	0.92	0.08
5	635.57	133.51	183.29	455.92	0.91	5.00	0.53	6.35	1.27	0.11
6	628.88	128.13	181.13	454.64	0.90	6.00	0.81	10.13	1.69	0.13
7	622.19	120.87	177.92	453.04	0.89	7.00	1.15	15.22	2.17	0.16
8	615.51	111.38	173.39	451.08	0.88	8.00	1.58	21.88	2.74	0.20
9	608.82	99.37	167.19	448.74	0.86	9.00	2.09	30.30	3.37	0.23
10	602.13	84.61	158.90	445.98	0.85	10.00	2.69	40.65	4.07	0.27
11	595.44	67.03	148.05	442.77	0.83	11.00	3.39	52.98	4.82	0.31
12	588.75	46.74	134.18	439.09	0.81	12.00	4.20	67.22	5.60	0.35
13	582.05	24.13	116.98	434.90	0.79	13.00	5.11	83.07	6.39	0.39
14	575.36	-0.25	96.56	430.18	0.77	14.00	6.14	100.18	7.16	0.44
15	568.67	-25.80	73.48	424.90	0.75	15.00	7.29	118.10	7.87	0.49
16	561.98	-52.00	48.60	419.02	0.73	16.00	8.58	136.47	8.53	0.54
17	555.31	-78.47	22.75	412.54	0.71	17.00	9.99	155.04	9.12	0.59
18	548.62	-105.12	-3.63	405.40	0.69	18.00	11.55	173.73	9.65	0.64
19	541.99	-131.62	-30.02	397.67	0.66	18.99	13.24	192.32	10.13	0.70
20	535.46	-157.73	-56.09	389.39	0.64	19.96	15.04	210.63	10.55	0.75
21	528.43	-185.85	-84.20	379.73	0.61	21.01	17.15	230.36	10.96	0.82
22	522.29	-210.4	-108.7	370.65	0.59	21.93	19.13	247.58	11.29	0.87

Table 6.7: Q^*, $CVaR^*$, ω^*, EP^*, SL^*, % Decrease in Q^*, % Decrease in EP^*, % Decrease in $CVaR^*$, % Decrease in $CVaR^*$/% Decrease in Q^*, % Decrease in EP^*/% Decrease in Q^* for changing τ at p=2, c=1, s=0.8, l=3, η=0.01 under Strict Cap Policy.

η= 0.1										
τ	**Q***	**CVaR***	**VaR*, ω***	**EP***	**SL***	**% Decrease in Q***	**% Decrease in EP***	**% Decrease in CVaR***	**(% Decrease in CVaR*)/ (% Decrease in Q*)**	**(% Decrease in EP*)/ (% Decrease in Q*)**
0	647.78	255.27	313.34	457.50	0.93	0.00	0.00	0	-	-
1	641.31	255.09	313.67	456.78	0.92	1.00	0.16	0.07	0.07	0.16
2	634.83	254.50	313.76	455.79	0.91	2.00	0.37	0.30	0.15	0.19
3	628.35	253.41	313.57	454.53	0.90	3.00	0.65	0.73	0.24	0.22
4	621.87	251.71	313.03	452.95	0.89	4.00	0.99	1.39	0.35	0.25
5	615.39	249.28	312.10	451.04	0.88	5.00	1.41	2.35	0.47	0.28
6	608.92	245.99	310.72	448.77	0.86	6.00	1.91	3.64	0.61	0.32
7	602.44	241.69	308.80	446.11	0.85	7.00	2.49	5.32	0.76	0.36
8	595.96	236.24	306.27	443.04	0.83	8.00	3.16	7.45	0.93	0.40
9	589.48	229.48	303.04	439.52	0.81	9.00	3.93	10.10	1.12	0.44
10	583.01	221.27	299.00	435.53	0.80	10.00	4.80	13.32	1.33	0.48
11	576.53	211.46	294.06	431.04	0.78	11.00	5.78	17.16	1.56	0.53
12	570.05	199.92	288.07	426.03	0.76	12.00	6.88	21.68	1.81	0.57
13	563.57	186.57	280.89	420.47	0.74	13.00	8.09	26.91	2.07	0.62
14	557.09	171.33	272.33	414.33	0.72	14.00	9.44	32.88	2.35	0.67
15	550.62	154.21	262.19	407.60	0.69	15.00	10.91	39.59	2.64	0.73
16	544.14	135.26	250.25	400.25	0.67	16.00	12.51	47.01	2.94	0.78
17	537.66	114.60	236.29	392.26	0.65	17.00	14.26	55.11	3.24	0.84
18	531.18	92.43	220.11	383.61	0.62	18.00	16.15	63.79	3.54	0.90
19	524.71	69.03	201.65	374.30	0.60	19.00	18.19	72.96	3.84	0.96
20	518.23	44.64	180.98	364.30	0.57	20.00	20.37	82.51	4.13	1.02
21	511.75	19.61	158.46	353.61	0.55	21.00	22.71	92.32	4.40	1.08
22	505.27	-5.80	134.59	342.24	0.52	22.00	25.19	102.27	4.65	1.15
23	498.80	-31.47	109.76	330.15	0.50	23.00	27.84	112.33	4.88	1.21
24	492.34	-57.22	84.45	317.39	0.47	24.00	30.63	122.42	5.10	1.28
25	485.82	-83.23	58.65	303.83	0.44	25.00	33.59	132.60	5.30	1.34
26	479.33	-109.17	32.79	289.61	0.42	26.00	36.70	142.77	5.49	1.41
27	472.99	-134.53	7.47	275.05	0.39	26.98	39.88	152.70	5.66	1.48
28	465.04	-166.35	-24.33	255.87	0.36	28.21	44.07	165.17	5.85	1.56
29	460.83	-183.19	-41.17	245.32	0.35	28.86	46.38	171.76	5.95	1.61

Table 6.8: Q^*, $CVaR^*$, ω^*, EP^*, SL^*, % Decrease in Q^*, % Decrease in EP^*, % Decrease in $CVaR^*$, % Decrease in $CVaR^*$/% Decrease in Q^*, % Decrease in EP^*/% Decrease in Q^* for changing τ at p=2, c=1, s=0.8, l=3, η=0.1 under Strict Cap Policy.

η= 0.25										
τ	Q*	CVaR*	VaR*, ω*	EP*	SL*	% Decrease in Q*	% Decrease in EP*	% Decrease in CVaR*	(% Decrease in CVaR*)/ (% Decrease in Q*)	(% Decrease in EP*)/ (% Decrease in Q*)
0	641.09	314.21	386.29	456.75	0.92	0	0	0	-	-
1	634.67	314.05	386.43	455.77	0.91	1.00	0.21	0.05	0.05	0.21
2	628.26	313.55	386.34	454.51	0.90	2.00	0.49	0.21	0.10	0.25
3	621.85	312.64	385.98	452.95	0.89	3.00	0.83	0.50	0.17	0.28
4	615.44	311.25	385.32	451.06	0.88	4.00	1.25	0.94	0.24	0.31
5	609.03	309.29	384.34	448.82	0.86	5.00	1.74	1.57	0.31	0.35
6	602.63	306.69	382.98	446.20	0.85	6.00	2.31	2.39	0.40	0.39
7	596.21	303.35	381.21	443.17	0.83	7.00	2.97	3.46	0.49	0.42
8	589.80	299.19	379.00	439.70	0.82	8.00	3.73	4.78	0.60	0.47
9	583.39	294.10	376.31	435.78	0.80	9.00	4.59	6.40	0.71	0.51
10	576.98	287.99	373.10	431.37	0.78	10.00	5.56	8.34	0.83	0.56
11	570.57	280.78	369.32	426.45	0.76	11.00	6.63	10.64	0.97	0.60
12	564.16	272.36	364.92	421.00	0.74	12.00	7.83	13.32	1.11	0.65
13	557.75	262.67	359.87	414.98	0.72	13.00	9.15	16.40	1.26	0.70
14	551.34	251.63	354.09	408.38	0.70	14.00	10.59	19.92	1.42	0.76
15	544.93	239.19	347.53	401.17	0.67	15.00	12.17	23.88	1.59	0.81
16	538.51	225.30	340.10	393.34	0.65	16.00	13.88	28.30	1.77	0.87
17	532.10	209.95	331.71	384.88	0.63	17.00	15.74	33.18	1.95	0.93
18	525.69	193.12	322.23	375.76	0.60	18.00	17.73	38.54	2.14	0.99
19	519.28	174.85	311.55	365.97	0.58	19.00	19.88	44.35	2.33	1.05
20	512.87	155.21	299.49	355.51	0.55	20.00	22.17	50.60	2.53	1.11
21	506.46	134.28	285.90	344.37	0.53	21.00	24.60	57.26	2.73	1.17
22	500.05	112.18	270.60	332.54	0.50	22.00	27.19	64.30	2.92	1.24
23	493.64	89.08	253.45	320.02	0.47	23.00	29.94	71.65	3.12	1.30
24	488.38	69.49	237.95	309.23	0.45	23.82	32.30	77.88	3.27	1.36
25	480.82	40.58	213.51	292.92	0.42	25.00	35.87	87.09	3.48	1.43
26	474.41	15.59	191.07	278.36	0.40	26.00	39.06	95.04	3.66	1.50
27	467.99	-9.71	167.39	263.12	0.37	27.00	42.39	103.09	3.82	1.57
28	461.60	-35.10	142.93	247.28	0.35	28.00	45.86	111.17	3.97	1.64
29	455.20	-60.62	117.91	230.78	0.33	29.00	49.47	119.29	4.11	1.71
30	448.78	-86.23	92.55	213.63	0.30	30.00	53.23	127.44	4.25	1.77
31	442.44	-111.57	67.33	196.07	0.28	30.99	57.07	135.51	4.37	1.84
32	435.77	-138.26	40.69	176.99	0.26	32.03	61.25	144.00	4.50	1.91
33	430.08	-161.02	17.95	160.23	0.24	32.91	64.92	151.25	4.60	1.97
34	423.06	-189.11	-10.13	138.96	0.22	34.01	69.58	160.19	4.71	2.05

Table 6.9: Q^*, $CVaR^*$, ω^*, EP^*, SL^*, % Decrease in Q^*, % Decrease in EP^*, % Decrease in $CVaR^*$, % Decrease in $CVaR^*$/% Decrease in Q^*, % Decrease in EP^*/% Decrease in Q^* for changing τ at p=2, c=1, s=0.8, l=3, η=0.25 under Strict Cap Policy.

η= 0.01								
τ	Q*	CVaR*	VaR*, ω*	EP*	SL*	% Decrease in Q*	% Decrease in EP*	% Decrease in CVaR*
0	522.76	188.10	225.95	433.07	0.59	0.00	0.00	0.00
1	517.53	187.95	225.42	429.13	0.57	1.00	0.91	0.08
2	512.30	187.44	224.46	424.96	0.55	2.00	1.87	0.35
3	507.07	186.47	222.97	420.56	0.53	3.00	2.89	0.87
4	501.84	184.93	220.87	415.92	0.51	4.00	3.96	1.69
5	496.62	182.74	218.11	411.05	0.49	5.00	5.08	2.85
6	491.39	179.80	214.62	405.95	0.47	6.00	6.26	4.41
7	486.16	176.06	210.37	400.61	0.45	7.00	7.49	6.40
8	480.93	171.47	205.36	395.04	0.42	8.00	8.78	8.84
9	475.71	166.04	199.61	389.24	0.40	9.00	10.12	11.73
10	470.48	159.79	193.15	383.21	0.38	10.00	11.51	15.05
11	465.25	152.78	186.02	376.96	0.36	11.00	12.96	18.78
12	460.02	145.10	178.29	370.49	0.34	12.00	14.45	22.86
13	454.80	136.82	170.03	363.80	0.33	13.00	16.00	27.26
14	449.57	128.05	161.31	356.89	0.31	14.00	17.59	31.93
15	444.34	118.88	152.22	349.79	0.29	15.00	19.23	36.80
16	439.11	109.36	142.79	342.48	0.27	16.00	20.92	41.86
17	433.89	99.61	133.12	334.98	0.25	17.00	22.65	47.05
18	428.65	89.64	123.23	327.28	0.24	18.00	24.43	52.35
19	423.44	79.57	113.23	319.43	0.22	19.00	26.24	57.70
20	418.19	69.33	103.05	311.36	0.21	20.00	28.10	63.14
21	412.95	59.03	92.79	303.12	0.19	21.00	30.01	68.62
22	407.77	48.78	82.57	294.81	0.18	22.00	31.92	74.07

Table 6.10: Q^*, $CVaR^*$, ω^*, EP^*, SL^*, % Decrease in Q^*, % Decrease in EP^*, % Decrease in $CVaR^*$, % Decrease in $CVaR^*$/% Decrease in Q^*, % Decrease in EP^*/% Decrease in Q^* for changing τ at p=2, c=1, s=0.8, l=1, η=0.01 under Strict Cap Policy.

η= 0.1								
τ	**Q***	**CVaR***	**VaR*, ω***	**EP***	**SL***	**% Decrease in Q***	**% Decrease in EP***	**% Decrease in CVaR***
0	543.62	289.19	341.39	446.54	0.67	0.00	0.00	0.00
1	538.18	289.07	340.64	443.37	0.65	1.00	0.71	0.04
2	532.74	288.65	339.53	439.96	0.63	2.00	1.47	0.19
3	527.30	287.89	337.99	436.31	0.61	3.00	2.29	0.45
4	521.87	286.72	336.00	432.42	0.59	4.00	3.16	0.85
5	516.43	285.09	333.52	428.27	0.57	5.00	4.09	1.42
6	511.00	282.94	330.53	423.88	0.54	6.00	5.07	2.16
7	505.56	280.22	327.01	419.24	0.52	7.00	6.11	3.10
8	500.13	276.89	322.97	414.35	0.50	8.00	7.21	4.25
9	494.69	272.93	318.39	409.20	0.48	9.00	8.36	5.62
10	489.25	268.31	313.29	403.79	0.46	10.00	9.57	7.22
11	483.82	263.05	307.66	398.14	0.44	11.00	10.84	9.04
12	478.38	257.13	301.53	392.23	0.41	12.00	12.16	11.09
13	472.94	250.59	294.89	386.08	0.39	13.00	13.54	13.35
14	467.51	243.46	287.79	379.69	0.37	14.00	14.97	15.81
15	462.07	235.77	280.22	373.05	0.35	15.00	16.46	18.47
16	456.64	227.57	272.22	366.18	0.33	16.00	18.00	21.31
17	451.20	218.91	263.82	359.08	0.31	17.00	19.59	24.30
18	445.76	209.83	255.03	351.74	0.29	18.00	21.23	27.44
19	440.33	200.42	245.91	344.20	0.28	19.00	22.92	30.70
20	434.89	190.69	236.47	336.43	0.26	20.00	24.66	34.06
21	429.45	180.72	226.77	328.47	0.24	21.00	26.44	37.51
22	424.02	170.55	216.85	320.31	0.22	22.00	28.27	41.02

Table 6.11: Q^*, $CVaR^*$, ω^*, EP^*, SL^*, % Decrease in Q^*, % Decrease in EP^*, % Decrease in $CVaR^*$, % Decrease in $CVaR^*$/% Decrease in Q^*, % Decrease in EP^*/% Decrease in Q^* for changing τ at p=2, c=1, s=0.8, l=1, η=0.1 under Strict Cap Policy.

η= 0.25								
τ	Q^*	$CVaR^*$	VaR^*, ω^*	EP^*	SL^*	% Decrease in Q^*	% Decrease in EP^*	% Decrease in $CVaR^*$
0	559.06	342.22	407.14	454.32	0.72	0.00	0.00	0.00
1	553.46	342.11	406.13	451.71	0.70	1.00	0.60	0.03
2	547.88	341.74	404.76	448.87	0.68	2.00	1.26	0.14
3	542.28	341.07	403.01	445.78	0.66	3.00	1.97	0.34
4	536.69	340.06	400.86	442.46	0.64	4.00	2.74	0.63
5	531.10	338.67	398.30	438.88	0.62	5.00	3.56	1.04
6	525.52	336.86	395.33	435.05	0.60	6.00	4.45	1.57
7	519.92	334.59	391.94	430.96	0.58	7.00	5.39	2.23
8	514.33	331.84	388.14	426.61	0.56	8.00	6.40	3.03
9	508.74	328.57	383.92	421.98	0.53	9.00	7.47	3.99
10	503.15	324.77	379.31	417.10	0.51	10.00	8.59	5.10
11	497.56	320.43	374.31	411.94	0.49	11.00	9.78	6.37
12	491.97	315.54	368.91	406.52	0.47	12.00	11.04	7.80
13	486.38	310.09	363.12	400.83	0.45	13.00	12.35	9.39
14	480.79	304.09	356.96	394.88	0.42	14.00	13.73	11.14
15	475.20	297.56	350.43	388.66	0.40	15.00	15.16	13.05
16	469.61	290.52	343.52	382.19	0.38	16.00	16.66	15.11
17	464.02	282.97	336.25	375.45	0.36	17.00	18.21	17.31
18	458.42	274.95	328.61	368.46	0.34	18.00	19.83	19.66
19	452.84	266.51	320.63	361.24	0.32	19.00	21.49	22.12
20	447.25	257.65	312.29	353.76	0.30	20.00	23.22	24.71
21	441.65	248.43	303.62	346.06	0.28	21.00	25.00	27.41
22	436.07	238.89	294.64	338.13	0.26	22.00	26.83	30.19

Table 6.12: Q^*, $CVaR^*$, ω^*, EP^*, SL^*, % Decrease in Q^*, % Decrease in EP^*, % Decrease in $CVaR^*$, % Decrease in $CVaR^*$/% Decrease in Q^*, % Decrease in EP^*/% Decrease in Q^* for changing τ at p=2, c=1, s=0.8, l=1, η=0.25 under Strict Cap Policy.

	(cb,cs) (0.08,0.076)		(cb,cs) (0.4,0.38)		(cb,cs) (0.8,0.76)		(cb,cs) (1.4,1.33)		(cb,cs) (3,2.85)		(cb,cs) (3.6,3.42)	
η	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**
0.1	639.04	639.42	617.31	618.34	600.59	602.01	582.69	584.54	545.63	549.16	528.57	534.54
0.2	632.88	633.30	608.77	609.93	589.85	591.47	569.19	571.35	525.64	529.78	505.87	512.71
0.3	630.04	630.49	604.11	605.37	583.42	585.20	560.47	562.87	511.25	515.94	489.12	496.72
0.4	628.79	629.26	601.24	602.59	578.92	580.86	553.82	556.47	499.08	504.31	474.58	482.93
0.5	628.60	629.10	599.52	600.95	575.61	577.70	548.38	551.27	487.93	493.75	460.88	470.06
0.6	629.36	629.88	598.71	600.24	573.18	575.43	543.74	546.88	477.19	483.65	447.19	457.33
0.7	631.13	631.67	598.78	600.40	571.49	573.90	539.70	543.11	466.38	473.60	432.78	444.12
0.8	634.21	634.78	599.82	601.56	570.53	573.12	536.14	539.84	455.10	463.25	416.67	429.71
0.9	639.47	640.09	602.19	604.07	570.37	573.19	533.01	537.04	442.83	452.19	397.08	412.91
1	650.11	650.85	606.76	608.89	571.24	574.36	530.30	534.70	428.76	439.87	369.08	391.11

Table 6.13: Q_{up} and Q_{down} vs. η at p=2, c=1, s=0.8, l=3, (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38), (0.8, 0.76), (1.4, 1.33), (3, 2.85), (3.6, 3.42)$ under Strict Cap Policy.

	(cb,cs) (0.08,0.076)		(cb,cs) (0.2,0.19)		(cb,cs) (0.4,0.38)		(cb,cs) (0.8,0.76)		(cb,cs) (1.4,1.33)	
η	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**	**Q_{up}**	**Q_{down}**
0.1	535.07	535.45	525.58	526.28	513.48	514.57	494.16	495.98	465.63	469.35
0.2	544.97	545.39	534.21	535.01	520.28	521.54	497.69	499.82	464.29	468.60
0.3	552.97	553.42	541.15	542.03	525.67	527.08	500.25	502.66	462.66	467.49
0.4	560.39	560.88	547.57	548.53	530.59	532.14	502.44	505.11	460.82	466.13
0.5	567.78	568.31	553.93	554.97	535.40	537.10	504.45	507.39	458.72	464.53
0.6	575.53	576.10	560.54	561.68	540.32	542.19	506.38	509.61	456.32	462.67
0.7	584.05	584.67	567.71	568.95	545.55	547.59	508.31	511.85	453.55	460.48
0.8	593.94	594.62	575.84	577.21	551.29	553.55	510.29	514.18	450.28	457.88
0.9	606.34	607.12	585.57	587.14	557.83	560.36	512.38	516.65	446.34	454.75
1	624.28	625.29	598.31	600.21	565.63	568.55	514.63	519.36	441.44	450.88

Table 6.14: Q_{up} and Q_{down} vs. η at p=2, c=1, s=0.85, l=1, (c_b, c_s)=$(0.08, 0.076), (0.2, 0.19), (0.4, 0.38), (0.8, 0.76), (1.4, 1.33)$ under Strict Cap Policy.

c_b=0.08, c_s=0.076, η=0.01							
τ	λ	λ, c_b, c_s comparison	Optimal Action	Q^*	$CVaR^*$	VaR^*, ω^*	EP^*
0	0.000	c_b>c_s >λ	Sell	662.3	142.8	185.8	458.8
1	0.076	c_b>c_s = λ	Sell	662.3	142.3	185.3	458.3
1.02	0.077	c_b> λ >c_s	No Trade	662.2	142.3	185.3	458.3
1.03	0.078	c_b> λ >c_s	No Trade	662.1	142.3	185.3	458.3
1.04	0.079	c_b> λ >c_s	No Trade	662.1	142.3	185.3	458.3
2	0.175	λ >c_b>c_s	Buy	661.9	141.8	184.8	457.8
3	0.305	λ >c_b>c_s	Buy	661.9	141.3	184.3	457.2
4	0.471	λ >c_b>c_s	Buy	661.9	140.7	183.8	456.7
5	0.680	λ >c_b>c_s	Buy	661.9	140.2	183.2	456.2
6	0.937	λ >c_b>c_s	Buy	661.9	139.7	182.7	455.6
7	1.244	λ >c_b>c_s	Buy	661.9	139.1	182.2	455.1
8	1.600	λ >c_b>c_s	Buy	661.9	138.6	181.6	454.6
9	1.997	λ >c_b>c_s	Buy	661.9	138.0	181.1	454.0
10	2.418	λ >c_b>c_s	Buy	661.9	137.5	180.6	453.5
11	2.837	λ >c_b>c_s	Buy	661.9	137.0	180.0	453.0
12	3.218	λ >c_b>c_s	Buy	661.9	136.4	179.5	452.4
13	3.528	λ >c_b>c_s	Buy	661.9	135.9	179.0	451.9
14	3.746	λ >c_b>c_s	Buy	661.9	135.4	178.4	451.4
15	3.878	λ >c_b>c_s	Buy	661.9	134.8	177.9	450.8
16	3.947	λ >c_b>c_s	Buy	661.9	134.3	177.3	450.3
17	3.979	λ >c_b>c_s	Buy	661.9	133.8	176.8	449.8
18	3.992	λ >c_b>c_s	Buy	661.9	133.2	176.3	449.2
19	3.997	λ >c_b>c_s	Buy	661.9	132.7	175.7	448.7
20	3.999	λ >c_b>c_s	Buy	661.9	132.2	175.2	448.2
21	3.9997	λ >c_b>c_s	Buy	661.9	131.6	174.7	447.6
22	3.9999	λ >c_b>c_s	Buy	661.9	131.1	174.2	447.1

Table 6.15: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, c_b = 0.08, c_s = 0.76, η=0.01 under Cap and Trade Policy.

c_b=1.4, c_s=1.33, η=0.01							
τ	λ	λ, c_b, c_s comparison	Optimal Action	Q^*	$CVaR^*$	VaR^*, ω^*	EP^*
0	0.000	c_b>c_s >λ	Sell	620.5	183.2	241.5	517.1
1	0.076	c_b>c_s >λ	Sell	620.5	174.3	232.6	508.2
2	0.175	c_b>c_s >λ	Sell	620.5	165.4	223.7	499.3
3	0.305	c_b>c_s >λ	Sell	620.5	156.5	214.8	490.4
4	0.471	c_b>c_s >λ	Sell	620.5	147.6	205.9	481.5
5	0.680	c_b>c_s >λ	Sell	620.5	138.7	197.0	472.6
6	0.937	c_b>c_s >λ	Sell	620.5	129.8	188.1	463.7
7	1.244	c_b>c_s >λ	Sell	620.5	120.9	179.2	454.8
7.31	1.350	c_b> λ >c_s	No Trade	620.1	118.2	176.7	452.5
7.37	1.370	c_b> λ >c_s	No Trade	619.7	117.6	176.4	452.4
7.43	1.390	c_b> λ >c_s	No Trade	619.3	117.1	176.2	452.2
8	1.600	λ >c_b>c_s	Buy	619.2	111.7	170.9	447.1
9	1.997	λ >c_b>c_s	Buy	619.2	102.4	161.6	437.7
10	2.418	λ >c_b>c_s	Buy	619.2	93.0	152.2	428.4
11	2.837	λ >c_b>c_s	Buy	619.2	83.6	142.8	419.0
12	3.218	λ >c_b>c_s	Buy	619.2	74.3	133.5	409.6
13	3.528	λ >c_b>c_s	Buy	619.2	64.9	124.1	400.3
14	3.746	λ >c_b>c_s	Buy	619.2	55.5	114.7	390.9
15	3.878	λ >c_b>c_s	Buy	619.2	46.2	105.4	381.5
16	3.947	λ >c_b>c_s	Buy	619.2	36.8	96.0	372.2
17	3.979	λ >c_b>c_s	Buy	619.2	27.5	86.7	362.8
18	3.992	λ >c_b>c_s	Buy	619.2	18.1	77.3	353.5
19	3.997	λ >c_b>c_s	Buy	619.2	8.8	68.0	344.2
20	3.999	λ >c_b>c_s	Buy	619.2	-0.3	58.9	335.0
21	3.9997	λ >c_b>c_s	Buy	619.2	-10.2	49.0	325.2
22	3.9999	λ >c_b>c_s	Buy	619.2	-18.8	40.4	316.6

Table 6.16: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, c_b = 1.4, c_s = 1.33, η=0.01 under Cap and Trade Policy.

c_b=3.6, c_s=3.42, η=0.01							
τ	λ	λ, c_b, c_s comparison	Optimal Action	Q^*	$CVaR^*$	VaR^*, ω^*	EP^*
0	0.000	c_b>c_s >λ	Sell	584.6	321.7	412.7	725.3
1	0.076	c_b>c_s >λ	Sell	584.6	298.8	389.8	702.4
2	0.175	c_b>c_s >λ	Sell	584.6	276.0	366.9	679.6
3	0.305	c_b>c_s >λ	Sell	584.6	253.1	344.0	656.7
4	0.471	c_b>c_s >λ	Sell	584.6	230.2	321.1	633.8
5	0.680	c_b>c_s >λ	Sell	584.6	207.3	298.3	610.9
6	0.937	c_b>c_s >λ	Sell	584.6	184.4	275.4	588.0
7	1.244	c_b>c_s >λ	Sell	584.6	161.6	252.5	565.1
8	1.600	c_b>c_s >λ	Sell	584.6	138.7	229.6	542.3
9	1.997	c_b>c_s >λ	Sell	584.6	115.8	206.7	519.4
10	2.418	c_b>c_s >λ	Sell	584.6	92.9	183.9	496.5
11	2.837	c_b>c_s >λ	Sell	584.6	70.0	161.0	473.6
12	3.218	c_b>c_s >λ	Sell	584.6	47.1	138.1	450.7
12.83	3.480	c_b> λ >c_s	No Trade	583.2	28.2	120.2	435.7
13	3.528	c_b> λ >c_s	No Trade	582.1	24.1	117.0	434.9
13.21	3.580	c_b> λ >c_s	No Trade	580.7	19.2	113.0	434.0
14	3.746	λ >c_b>c_s	Buy	580.1	0.1	94.3	416.5
15	3.878	λ >c_b>c_s	Buy	580.1	-24.0	70.2	392.4
16	3.947	λ >c_b>c_s	Buy	580.1	-48.1	46.1	368.3
17	3.979	λ >c_b>c_s	Buy	580.1	-72.1	22.1	344.3
18	3.992	λ >c_b>c_s	Buy	580.1	-96.2	-2.0	320.2
19	3.997	λ >c_b>c_s	Buy	580.1	-120.0	-25.9	296.3
20	3.999	λ >c_b>c_s	Buy	580.1	-143.5	-49.4	272.8
21	3.9997	λ >c_b>c_s	Buy	580.1	-168.9	-74.7	247.5
22	3.9999	λ >c_b>c_s	Buy	580.1	-190.9	-96.8	225.4

Table 6.17: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, c_b = 3.6, c_s = 3.42, η=0.01 under Cap and Trade Policy.

c_b=0.08, c_s=0.076, η=0.1							
τ	λ	λ, c_b, c_s comparison	Optimal Action	Q^*	$CVaR^*$	VaR^*, ω^*	EP^*
0	0.000	c_b>c_s >λ	Sell	639.4	255.6	314.4	457.2
1	0.057	c_b>c_s >λ	sell	639.4	255.1	313.9	456.7
1.31	0.077	c_b> λ >c_s	No Trade	639.3	255.0	313.7	456.5
1.32	0.078	c_b> λ >c_s	No Trade	639.2	255.0	313.7	456.5
1.34	0.079	c_b> λ >c_s	No Trade	639.1	254.9	313.7	456.5
2	0.127	λ >c_b>c_s	Buy	639.0	254.6	313.4	456.1
3	0.213	λ >c_b>c_s	Buy	639.0	254.1	312.9	455.6
4	0.316	λ >c_b>c_s	Buy	639.0	253.6	312.4	455.1
5	0.438	λ >c_b>c_s	Buy	639.0	253.0	311.8	454.6
6	0.582	λ >c_b>c_s	Buy	639.0	252.5	311.3	454.1
7	0.748	λ >c_b>c_s	Buy	639.0	252.0	310.8	453.5
8	0.938	λ >c_b>c_s	Buy	639.0	251.5	310.3	453.0
9	1.152	λ >c_b>c_s	Buy	639.0	251.0	309.8	452.5
10	1.388	λ >c_b>c_s	Buy	639.0	250.5	309.2	452.0
11	1.645	λ >c_b>c_s	Buy	639.0	249.9	308.7	451.5
12	1.919	λ >c_b>c_s	Buy	639.0	249.4	308.2	450.9
13	2.206	λ >c_b>c_s	Buy	639.0	248.9	307.7	450.4
14	2.498	λ >c_b>c_s	Buy	639.0	248.4	307.2	449.9
15	2.787	λ >c_b>c_s	Buy	639.0	247.9	306.7	449.4
16	3.062	λ >c_b>c_s	Buy	639.0	247.3	306.1	448.9
17	3.312	λ >c_b>c_s	Buy	639.0	246.8	305.6	448.4
18	3.526	λ >c_b>c_s	Buy	639.0	246.3	305.1	447.8
19	3.696	λ >c_b>c_s	Buy	639.0	245.8	304.6	447.3
20	3.820	λ >c_b>c_s	Buy	639.0	245.3	304.1	446.8
21	3.901	λ >c_b>c_s	Buy	639.0	244.8	303.5	446.3
22	3.950	λ >c_b>c_s	Buy	639.0	244.2	303.0	445.8
23	3.976	λ >c_b>c_s	Buy	639.0	243.7	302.5	445.2
24	3.989	λ >c_b>c_s	Buy	639.0	243.2	302.0	444.7
25	3.996	λ >c_b>c_s	Buy	639.0	242.7	301.5	444.2
26	3.998	λ >c_b>c_s	Buy	639.0	242.2	301.0	443.7
27	3.999	λ >c_b>c_s	Buy	639.0	241.7	300.4	443.2
28	3.9998	λ >c_b>c_s	Buy	639.0	241.0	299.8	442.5
29	3.9999	λ >c_b>c_s	Buy	639.0	240.7	299.5	442.2

Table 6.18: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, c_b = 0.08, c_s = 0.76, η=0.1 under Cap and Trade Policy.

c_b=1.4, c_s=1.33, η=0.1							
τ	**λ**	**λ, c_b, c_s comparison**	**Optimal Action**	**Q^***	**$CVaR^*$**	**VaR^*, ω^***	**EP^***
0	0.000	c_b>c_s >λ	Sell	584.5	307.5	384.2	520.6
1	0.057	c_b>c_s >λ	Sell	584.5	298.9	375.5	512.0
2	0.127	c_b>c_s >λ	Sell	584.5	290.2	366.9	503.4
3	0.213	c_b>c_s >λ	Sell	584.5	281.6	358.3	494.8
4	0.316	c_b>c_s >λ	Sell	584.5	273.0	349.7	486.2
5	0.438	c_b>c_s >λ	Sell	584.5	264.4	341.1	477.6
6	0.582	c_b>c_s >λ	Sell	584.5	255.8	332.5	468.9
7	0.748	c_b>c_s >λ	Sell	584.5	247.2	323.8	460.3
8	0.938	c_b>c_s >λ	Sell	584.5	238.5	315.2	451.7
9	1.152	c_b>c_s >λ	Sell	584.5	229.9	306.6	443.1
9.85	1.350	c_b> λ >c_s	No Trade	584.0	222.6	299.7	436.2
9.93	1.370	c_b> λ >c_s	No Trade	583.5	221.9	299.3	435.8
10.01	1.390	c_b> λ >c_s	No Trade	583.0	221.2	299.0	435.5
11	1.645	λ >c_b>c_s	Buy	582.7	212.2	290.2	426.7
12	1.919	λ >c_b>c_s	Buy	582.7	203.1	281.1	417.6
13	2.206	λ >c_b>c_s	Buy	582.7	194.1	272.0	408.6
14	2.498	λ >c_b>c_s	Buy	582.7	185.0	263.0	399.5
15	2.787	λ >c_b>c_s	Buy	582.7	175.9	253.9	390.4
16	3.062	λ >c_b>c_s	Buy	582.7	166.9	244.8	381.4
17	3.312	λ >c_b>c_s	Buy	582.7	157.8	235.7	372.3
18	3.526	λ >c_b>c_s	Buy	582.7	148.7	226.7	363.2
19	3.696	λ >c_b>c_s	Buy	582.7	139.7	217.6	354.2
20	3.820	λ >c_b>c_s	Buy	582.7	130.6	208.5	345.1
21	3.901	λ >c_b>c_s	Buy	582.7	121.5	199.5	336.0
22	3.950	λ >c_b>c_s	Buy	582.7	112.5	190.4	326.9
23	3.976	λ >c_b>c_s	Buy	582.7	103.4	181.3	317.9
24	3.989	λ >c_b>c_s	Buy	582.7	94.3	172.3	308.8
25	3.996	λ >c_b>c_s	Buy	582.7	85.2	163.2	299.7
26	3.998	λ >c_b>c_s	Buy	582.7	76.1	154.1	290.6
27	3.9993	λ >c_b>c_s	Buy	582.7	67.3	145.2	281.7
28	3.9998	λ >c_b>c_s	Buy	582.7	56.1	134.1	270.6
29	3.9999	λ >c_b>c_s	Buy	582.7	50.2	128.2	264.7

Table 6.19: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, $c_b = 1.4$, $c_s = 1.33$, η=0.1 under Cap and Trade Policy.

c_b=3.6, c_s=3.42, η=0.1							
τ	**λ**	**λ, c_b, c_s comparison**	**Optimal Action**	**Q^***	**$CVaR^*$**	**VaR^*, ω^***	**EP^***
0	0	c_b>c_s >λ	Sell	534.5	491.4	616.1	775.5
1	0.057	c_b>c_s >λ	Sell	534.5	469.2	593.9	753.3
2	0.127	c_b>c_s >λ	Sell	534.5	447.1	571.8	731.2
3	0.213	c_b>c_s >λ	Sell	534.5	424.9	549.6	709.0
4	0.316	c_b>c_s >λ	Sell	534.5	402.8	527.5	686.9
5	0.438	c_b>c_s >λ	Sell	534.5	380.6	505.3	664.7
6	0.582	c_b>c_s >λ	Sell	534.5	358.5	483.2	642.6
7	0.748	c_b>c_s >λ	Sell	534.5	336.3	461.0	620.4
8	0.938	c_b>c_s >λ	Sell	534.5	314.2	438.8	598.2
9	1.152	c_b>c_s >λ	Sell	534.5	292.0	416.7	576.1
10	1.388	c_b>c_s >λ	Sell	534.5	269.8	394.5	553.9
11	1.645	c_b>c_s >λ	Sell	534.5	247.7	372.4	531.8
12	1.919	c_b>c_s >λ	Sell	534.5	225.5	350.2	509.6
13	2.206	c_b>c_s >λ	Sell	534.5	203.4	328.1	487.5
14	2.498	c_b>c_s >λ	Sell	534.5	181.2	305.9	465.3
15	2.786	c_b>c_s >λ	Sell	534.5	159.1	283.8	443.2
16	3.062	c_b>c_s >λ	Sell	534.5	136.9	261.6	421.0
17	3.312	c_b>c_s >λ	Sell	534.5	114.8	239.5	398.9
17.77	3.480	c_b> λ >c_s	No Trade	532.7	97.7	224.1	385.7
18.01	3.526	c_b> λ >c_s	No Trade	531.1	92.2	220.0	383.5
18.29	3.580	c_b> λ >c_s	No Trade	529.3	85.7	215.0	381.0
19	3.696	λ >c_b>c_s	Buy	528.6	69.2	199.0	366.0
20	3.819	λ >c_b>c_s	Buy	528.6	45.9	175.7	342.7
21	3.901	λ >c_b>c_s	Buy	528.6	22.6	152.4	319.4
22	3.949	λ >c_b>c_s	Buy	528.6	-0.7	129.1	296.1
23	3.976	λ >c_b>c_s	Buy	528.6	-24.0	105.8	272.8
24	3.989	λ >c_b>c_s	Buy	528.6	-47.3	82.5	249.5
25	3.996	λ >c_b>c_s	Buy	528.6	-70.8	59.0	226.0
26	3.998	λ >c_b>c_s	Buy	528.6	-94.1	35.7	202.7
27	3.999	λ >c_b>c_s	Buy	528.6	-117.0	12.8	179.9
28	3.9998	λ >c_b>c_s	Buy	528.6	-145.6	-15.8	151.2
29	3.9999	λ >c_b>c_s	Buy	528.6	-160.8	-30.9	136.1

Table 6.20: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, $c_b = 3.6$, $c_s = 3.42$, η=0.1 under Cap and Trade Policy.

c_b=0.08, c_s=0.076, η=0.25							
τ	**λ**	**λ, c_b, c_s comparison**	**Optimal Action**	**Q^***	**$CVaR^*$**	**VaR^*, ω^***	**EP^***
0	0.000	c_b>c_s >λ	Sell	631.6	314.6	387.1	455.9
1	0.049	c_b>c_s >λ	Sell	631.6	314.1	386.6	455.4
1.49	0.077	c_b> λ >c_s	No Trade	631.53	313.9	386.4	455.2
1.51	0.078	c_b> λ >c_s	No Trade	631.42	313.8	386.4	455.2
1.52	0.079	c_b> λ >c_s	No Trade	631.31	313.8	386.4	455.1
2	0.108	λ >c_b>c_s	Buy	631.2	313.6	386.2	454.9
3	0.178	λ >c_b>c_s	Buy	631.2	313.1	385.7	454.4
4	0.259	λ >c_b>c_s	Buy	631.2	312.6	385.1	453.9
5	0.353	λ >c_b>c_s	Buy	631.2	312.1	384.6	453.3
6	0.461	λ >c_b>c_s	Buy	631.2	311.5	384.1	452.8
7	0.583	λ >c_b>c_s	Buy	631.2	311.0	383.6	452.3
8	0.719	λ >c_b>c_s	Buy	631.2	310.5	383.1	451.8
9	0.871	λ >c_b>c_s	Buy	631.2	310.0	382.6	451.3
10	1.037	λ >c_b>c_s	Buy	631.2	309.5	382.1	450.8
11	1.217	λ >c_b>c_s	Buy	631.2	309.0	381.6	450.3
12	1.410	λ >c_b>c_s	Buy	631.2	308.5	381.0	449.8
13	1.615	λ >c_b>c_s	Buy	631.2	308.0	380.5	449.2
14	1.830	λ >c_b>c_s	Buy	631.2	307.4	380.0	448.7
15	2.053	λ >c_b>c_s	Buy	631.2	306.9	379.5	448.2
16	2.281	λ >c_b>c_s	Buy	631.2	306.4	379.0	447.7
17	2.510	λ >c_b>c_s	Buy	631.2	305.9	378.5	447.2
18	2.738	λ >c_b>c_s	Buy	631.2	305.4	378.0	446.7
19	2.959	λ >c_b>c_s	Buy	631.2	304.9	377.5	446.2
20	3.168	λ >c_b>c_s	Buy	631.2	304.4	376.9	445.7
21	3.360	λ >c_b>c_s	Buy	631.2	303.9	376.4	445.1
22	3.530	λ >c_b>c_s	Buy	631.2	303.3	375.9	444.6
23	3.673	λ >c_b>c_s	Buy	631.2	302.8	375.4	444.1
24	3.768	λ >c_b>c_s	Buy	631.2	302.4	375.0	443.7
25	3.869	λ >c_b>c_s	Buy	631.2	301.8	374.4	443.1
26	3.926	λ >c_b>c_s	Buy	631.2	301.3	373.9	442.6
27	3.960	λ >c_b>c_s	Buy	631.2	300.8	373.4	442.1
28	3.980	λ >c_b>c_s	Buy	631.2	300.3	372.8	441.6
29	3.991	λ >c_b>c_s	Buy	631.2	299.7	372.3	441.0
30	3.996	λ >c_b>c_s	Buy	631.2	299.2	371.8	440.5
31	3.998	λ >c_b>c_s	Buy	631.2	298.7	371.3	440.0
32	3.999	λ >c_b>c_s	Buy	631.2	298.2	370.8	439.5
33	3.9997	λ >c_b>c_s	Buy	631.2	297.7	370.3	439.0
34	3.9999	λ >c_b>c_s	Buy	631.2	297.2	369.8	438.5

Table 6.21: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, c_b = 0.08, c_s = 0.76, η=0.25 under Cap and Trade Policy.

c_b=1.4, c_s=1.33, η=0.25							
τ	**λ**	**λ, c_b, c_s comparison**	**Optimal Action**	**Q^***	**$CVaR^*$**	**VaR^*, $ω^*$**	**EP^***
0	0.000	c_b>c_s >λ	Sell	566.7	374.8	465.6	522.1
1	0.049	c_b>c_s >λ	Sell	566.7	366.2	457.1	513.6
2	0.108	c_b>c_s >λ	Sell	566.7	357.7	448.6	505.1
3	0.178	c_b>c_s >λ	Sell	566.7	349.2	440.1	496.5
4	0.259	c_b>c_s >λ	Sell	566.7	340.7	431.5	488.0
5	0.353	c_b>c_s >λ	Sell	566.7	332.1	423.0	479.5
6	0.461	c_b>c_s >λ	Sell	566.7	323.6	414.5	471.0
7	0.583	c_b>c_s >λ	Sell	566.7	315.1	406.0	462.5
8	0.719	c_b>c_s >λ	Sell	566.7	306.6	397.4	453.9
9	0.871	c_b>c_s >λ	Sell	566.7	298.0	388.9	445.4
10	1.037	c_b>c_s >λ	Sell	566.7	289.5	380.4	436.9
11	1.217	c_b>c_s >λ	Sell	566.7	281.0	371.9	428.3
11.70	1.350	c_b> λ >c_s	No Trade	566.10	275.0	366.3	422.7
11.80	1.370	c_b> λ >c_s	No Trade	565.45	274.2	365.9	422.1
11.90	1.390	c_b> λ >c_s	No Trade	564.80	273.3	365.4	421.6
12	1.410	λ >c_b>c_s	Buy	564.5	272.4	364.7	420.8
13	1.615	λ >c_b>c_s	Buy	564.5	263.4	355.7	411.9
14	1.830	λ >c_b>c_s	Buy	564.5	254.4	346.8	402.9
15	2.053	λ >c_b>c_s	Buy	564.5	245.4	337.8	393.9
16	2.281	λ >c_b>c_s	Buy	564.5	236.5	328.8	384.9
17	2.510	λ >c_b>c_s	Buy	564.5	227.5	319.8	376.0
18	2.738	λ >c_b>c_s	Buy	564.5	218.5	310.9	367.0
19	2.959	λ >c_b>c_s	Buy	564.5	209.5	301.9	358.0
20	3.168	λ >c_b>c_s	Buy	564.5	200.6	292.9	349.0
21	3.360	λ >c_b>c_s	Buy	564.5	191.6	283.9	340.1
22	3.530	λ >c_b>c_s	Buy	564.5	182.6	275.0	331.1
23	3.673	λ >c_b>c_s	Buy	564.5	173.6	266.0	322.1
24	3.768	λ >c_b>c_s	Buy	564.5	166.3	258.6	314.7
25	3.869	λ >c_b>c_s	Buy	564.5	155.7	248.0	304.2
26	3.926	λ >c_b>c_s	Buy	564.5	146.7	239.1	295.2
27	3.960	λ >c_b>c_s	Buy	564.5	137.7	230.1	286.2
28	3.980	λ >c_b>c_s	Buy	564.5	128.8	221.1	277.3
29	3.991	λ >c_b>c_s	Buy	564.5	119.8	212.2	268.3
30	3.996	λ >c_b>c_s	Buy	564.5	110.8	203.2	259.3
31	3.998	λ >c_b>c_s	Buy	564.5	102.0	194.3	250.4
32	3.999	λ >c_b>c_s	Buy	564.5	92.6	185.0	241.1
33	3.9997	λ >c_b>c_s	Buy	564.5	84.7	177.0	233.1
34	3.9999	λ >c_b>c_s	Buy	564.5	74.8	167.2	223.3

Table 6.22: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, c_b = 1.4, c_s = 1.33, η=0.25 under Cap and Trade Policy.

c_b=3.6, c_s=3.42, η=0.25							
τ	λ	λ, c_b, c_s comparison	Optimal Action	Q^*	$CVaR^*$	VaR^*, ω^*	EP^*
0	0.000	c_b>c_s >λ	Sell	504.3	594.8	748.7	808.3
1	0.049	c_b>c_s >λ	Sell	504.3	572.8	726.8	786.3
2	0.108	c_b>c_s >λ	Sell	504.3	550.9	704.9	764.4
3	0.178	c_b>c_s >λ	Sell	504.3	529.0	683.0	742.5
4	0.259	c_b>c_s >λ	Sell	504.3	507.1	661.1	720.6
5	0.353	c_b>c_s >λ	Sell	504.3	485.1	639.1	698.6
6	0.461	c_b>c_s >λ	Sell	504.3	463.2	617.2	676.7
7	0.583	c_b>c_s >λ	Sell	504.3	441.3	595.3	654.8
8	0.719	c_b>c_s >λ	Sell	504.3	419.4	573.3	632.9
9	0.871	c_b>c_s >λ	Sell	504.3	397.4	551.4	610.9
10	1.037	c_b>c_s >λ	Sell	504.3	375.5	529.5	589.0
11	1.217	c_b>c_s >λ	Sell	504.3	353.6	507.6	567.1
12	1.410	c_b>c_s >λ	Sell	504.3	331.7	485.7	545.2
13	1.615	c_b>c_s >λ	Sell	504.3	309.7	463.7	523.2
14	1.830	c_b>c_s >λ	Sell	504.3	287.8	441.8	501.3
15	2.053	c_b>c_s >λ	Sell	504.3	265.9	419.9	479.4
16	2.281	c_b>c_s >λ	Sell	504.3	244.0	397.9	457.5
17	2.510	c_b>c_s >λ	Sell	504.3	222.0	376.0	435.5
18	2.738	c_b>c_s >λ	Sell	504.3	200.1	354.1	413.6
19	2.959	c_b>c_s >λ	Sell	504.3	178.2	332.2	391.7
20	3.168	c_b>c_s >λ	Sell	504.3	156.3	310.2	369.8
21	3.360	c_b>c_s >λ	Sell	504.3	134.3	288.3	347.8
21.69	3.480	c_b> λ >c_s	No Trade	502.05	336.3	275.6	119.2
21.98	3.528	c_b> λ >c_s	No Trade	500.16	332.7	270.9	112.6
22.33	3.580	c_b> λ >c_s	No Trade	497.96	328.5	265.2	104.7
23	3.673	λ >c_b>c_s	Buy	497.1	89.2	250.5	314.4
24	3.768	λ >c_b>c_s	Buy	497.1	70.3	231.6	295.5
25	3.869	λ >c_b>c_s	Buy	497.1	43.0	204.3	268.3
26	3.926	λ >c_b>c_s	Buy	497.1	20.0	181.3	245.2
27	3.960	λ >c_b>c_s	Buy	497.1	-3.1	158.2	222.1
28	3.980	λ >c_b>c_s	Buy	497.1	-26.1	135.2	199.1
29	3.991	λ >c_b>c_s	Buy	497.1	-49.2	112.1	176.0
30	3.996	λ >c_b>c_s	Buy	497.1	-72.3	89.0	153.0
31	3.998	λ >c_b>c_s	Buy	497.1	-95.1	66.2	130.1
32	3.999	λ >c_b>c_s	Buy	497.1	-119.1	42.2	106.1
33	3.9997	λ >c_b>c_s	Buy	497.1	-139.6	21.7	85.6
34	3.9999	λ >c_b>c_s	Buy	497.1	-164.9	-3.6	60.3

Table 6.23: λ, (λ, c_b, c_s) comparison, Optimal Action, Q^*, $CVaR^*$, ω^*, EP^*, for changing τ at p=2, c=1, s=0.8, l=3, c_b = 3.6, c_s = 3.42, η=0.25 under Cap and Trade Policy.

η=0.01	c_b=0.08 c_s=0.076	c_b =0.4 c_s=0.38	c_b =1.4 c_s=1.33	c_b =2 c_s=1.9	c_b =3.6 c_s=3.42
τ	**CVaR***	**CVaR***	**CVaR***	**CVaR***	**CVaR***
0	142.84	147.67	183.23	213.85	321.72
1	142.33	145.12	174.33	201.14	298.83
2	141.79	142.58	165.43	188.43	275.96
3	141.26	140.04	156.53	175.72	253.07
4	140.72	137.44	147.64	163.00	230.19
5	140.19	134.76	138.74	150.29	207.31
6	139.65	132.08	129.84	137.59	184.44
7	139.12	129.41	120.94	124.87	161.55
8	138.58	126.73	111.74	112.16	138.67
9	138.05	124.05	102.38	99.37	115.79
10	137.51	121.38	93.01	85.99	92.91
11	136.98	118.70	83.64	72.61	70.03
12	136.44	116.03	74.28	59.23	47.15
13	135.91	113.35	64.91	45.85	24.13
14	135.37	110.68	55.55	32.47	0.12
15	134.84	108.00	46.18	19.09	-23.97
16	134.30	105.32	36.81	5.71	-48.06
17	133.77	102.65	27.47	-7.64	-72.09
18	133.23	99.98	18.11	-21.01	-96.16
19	132.70	97.32	8.82	-34.28	-120.04
20	132.18	94.71	-0.32	-47.34	-143.55
21	131.62	91.90	-10.17	-61.40	-168.86
22	131.13	89.45	-18.76	-73.67	-190.95

Table 6.24: $CVaR^*$ vs. τ at p=2, $c = 1$, s=0.8, l=3, η=0.01 for (c_b, c_s)=$(0.08, 0.076)$, $(0.4, 0.38)$, $(1.4, 1.33)$, $(2, 1.9)$, $(3.6, 3.42)$ under Cap and Trade Policy

η =0.1	c_b=0.08 c_s=0.076	c_b =0.4 c_s=0.38	c_b =1.4 c_s=1.33	c_b =2 c_s=1.9	c_b =3.6 c_s=3.42
τ	CVaR*	CVaR*	CVaR*	CVaR*	CVaR*
0	255.60	261.67	307.47	347.62	491.39
1	255.11	259.21	298.86	335.32	469.24
2	254.60	256.75	290.24	323.00	447.07
3	254.08	254.29	281.63	310.70	424.94
4	253.57	251.82	273.01	298.39	402.78
5	253.05	249.31	264.39	286.08	380.61
6	252.53	246.72	255.78	273.78	358.47
7	252.01	244.13	247.16	261.46	336.30
8	251.49	241.54	238.55	249.16	314.16
9	250.97	238.95	229.93	236.85	292.00
10	250.46	236.36	221.27	224.54	269.84
11	249.94	233.77	212.20	212.23	247.69
12	249.42	231.18	203.14	199.92	225.54
13	248.90	228.59	194.07	187.04	203.39
14	248.38	225.99	185.00	174.08	181.23
15	247.86	223.40	175.93	161.13	159.08
16	247.35	220.81	166.86	148.17	136.92
17	246.83	218.22	157.79	135.22	114.77
18	246.31	215.63	148.72	122.26	92.43
19	245.79	213.04	139.66	109.31	69.22
20	245.27	210.45	130.58	96.35	45.88
21	244.76	207.85	121.51	83.39	22.55
22	244.24	205.27	112.45	70.45	-0.73
23	243.72	202.68	103.39	57.50	-24.05
24	243.20	200.09	94.34	44.57	-47.32
25	242.68	197.49	85.22	31.55	-70.76
26	242.16	194.89	76.13	18.56	-94.13
27	241.65	192.35	67.25	5.88	-116.96
28	241.02	189.17	56.11	-10.03	-145.61
29	240.68	187.49	50.22	-18.45	-160.76

Table 6.25: $CVaR^*$ vs. τ at p=2, $c = 1$, s=0.8, l=3, η=0.1 for (c_b, c_s)=$(0.08, 0.076)$, $(0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42)$ under Cap and Trade Policy

η =0.25	c_b=0.08 c_s=0.076	c_b =0.4 c_s=0.38	c_b =1.4 c_s=1.33	c_b =2 c_s=1.9	c_b =3.6 c_s=3.42
τ	CVaR*	CVaR*	CVaR*	CVaR*	CVaR*
0	314.58	321.50	374.78	422.23	594.76
1	314.09	319.06	366.25	410.04	572.82
2	313.60	316.62	357.73	397.87	550.91
3	313.08	314.19	349.20	385.69	528.98
4	312.57	311.75	340.68	373.51	507.07
5	312.06	309.31	332.15	361.33	485.14
6	311.54	306.80	323.63	349.15	463.23
7	311.03	304.23	315.10	336.97	441.30
8	310.52	301.67	306.57	324.79	419.37
9	310.01	299.10	298.05	312.61	397.45
10	309.49	296.54	289.52	300.43	375.52
11	308.98	293.98	280.99	288.25	353.60
12	308.47	291.41	272.36	276.07	331.67
13	307.95	288.85	263.39	263.89	309.74
14	307.44	286.28	254.41	251.70	287.81
15	306.93	283.72	245.44	239.23	265.89
16	306.42	281.15	236.46	226.41	243.96
17	305.90	278.59	227.49	213.59	222.05
18	305.39	276.02	218.51	200.77	200.12
19	304.88	273.46	209.54	187.94	178.19
20	304.36	270.89	200.56	175.12	156.26
21	303.85	268.33	191.59	162.30	134.34
22	303.34	265.77	182.61	149.48	112.18
23	302.83	263.20	173.64	136.66	89.21
24	302.40	261.10	166.27	126.14	70.26
25	301.80	258.07	155.68	111.01	43.04
26	301.29	255.51	146.71	98.19	19.96
27	300.77	252.94	137.73	85.37	-3.13
28	300.26	250.39	128.78	72.58	-26.14
29	299.75	247.83	119.82	59.77	-49.19
30	299.24	245.26	110.84	46.95	-72.28
31	298.73	242.72	101.96	34.27	-95.10
32	298.20	240.06	92.62	20.92	-119.13
33	297.74	237.78	84.65	9.54	-139.61
34	297.18	234.97	74.82	-4.51	-164.90

Table 6.26: $CVaR^*$ vs. τ at p=2, $c = 1$, s=0.8, l=3, η=0.25 for (c_b, c_s)=$(0.08, 0.076)$, $(0.4, 0.38)$, $(1.4, 1.33)$, $(2, 1.9)$, $(3.6, 3.42)$ under Cap and Trade Policy

η=0.01	c_b=0.08 c_s=0.076	c_b =0.4 c_s=0.38	c_b =1.4 c_s=1.33	c_b =2 c_s=1.9	c_b =3.6 c_s=3.42
τ	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*
0	0	0	0	0	0
1	0.36	1.72	4.86	5.95	7.12
2	0.73	3.44	9.71	11.89	14.22
3	1.10	5.17	14.57	17.83	21.34
4	1.48	6.93	19.43	23.78	28.45
5	1.85	8.74	24.28	29.72	35.56
6	2.23	10.55	29.14	35.66	42.67
7	2.60	12.37	33.99	41.61	49.78
8	2.98	14.18	39.01	47.55	56.90
9	3.35	15.99	44.13	53.53	64.01
10	3.73	17.80	49.24	59.79	71.12
11	4.10	19.61	54.35	66.05	78.23
12	4.48	21.43	59.46	72.30	85.34
13	4.85	23.24	64.57	78.56	92.50
14	5.23	25.05	69.68	84.81	99.96
15	5.60	26.86	74.80	91.07	107.45
16	5.98	28.68	79.91	97.33	114.94
17	6.35	30.48	85.01	103.57	122.41
18	6.72	32.29	90.12	109.83	129.89
19	7.10	34.09	95.19	116.03	137.31
20	7.46	35.86	100.18	122.14	144.62
21	7.86	37.77	105.55	128.71	152.49
22	8.20	39.43	110.24	134.45	159.35

Table 6.27: % Decrese in $CVaR^*$ vs. τ at p=2, c=1, s=0.8, l=3, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42)$ under Cap and Trade Policy.

η=0.1	c_b=0.08 c_s=0.076	c_b =0.4 c_s=0.38	c_b =1.4 c_s=1.33	c_b =2 c_s=1.9	c_b =3.6 c_s=3.42
τ	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*
0	0	0	0	0	0
1	0.19	0.94	2.80	3.54	4.51
2	0.39	1.88	5.61	7.08	9.02
3	0.59	2.82	8.41	10.62	13.52
4	0.80	3.76	11.21	14.16	18.03
5	1.00	4.72	14.01	17.70	22.54
6	1.20	5.71	16.81	21.24	27.05
7	1.41	6.70	19.62	24.79	31.56
8	1.61	7.69	22.42	28.32	36.07
9	1.81	8.68	25.22	31.87	40.58
10	2.01	9.67	28.04	35.41	45.09
11	2.22	10.66	30.98	38.95	49.59
12	2.42	11.65	33.93	42.49	54.10
13	2.62	12.64	36.88	46.19	58.61
14	2.83	13.63	39.83	49.92	63.12
15	3.03	14.62	42.78	53.65	67.63
16	3.23	15.61	45.73	57.38	72.14
17	3.43	16.60	48.68	61.10	76.64
18	3.64	17.60	51.63	64.83	81.19
19	3.84	18.59	54.58	68.55	85.91
20	4.04	19.58	57.53	72.28	90.66
21	4.24	20.57	60.48	76.01	95.41
22	4.45	21.56	63.43	79.73	100.15
23	4.65	22.55	66.38	83.46	104.89
24	4.85	23.53	69.32	87.18	109.63
25	5.06	24.53	72.28	90.93	114.40
26	5.26	25.52	75.24	94.66	119.16
27	5.46	26.49	78.13	98.31	123.80
28	5.71	27.71	81.75	102.89	129.63
29	5.84	28.35	83.67	105.31	132.72

Table 6.28: % Decrese in $CVaR^*$ vs. τ at p=2, c=1, s=0.8, l=3, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42)$ under Cap and Trade Policy.

η=0.25	c_b=0.08 c_s=0.076	c_b =0.4 c_s=0.38	c_b =1.4 c_s=1.33	c_b =2 c_s=1.9	c_b =3.6 c_s=3.42
τ	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*	%Decrease in CVaR*
0	0	0	0	0	0
1	0.15	0.76	2.28	2.89	3.69
2	0.31	1.52	4.55	5.77	7.37
3	0.48	2.27	6.83	8.66	11.06
4	0.64	3.03	9.10	11.54	14.74
5	0.80	3.79	11.38	14.42	18.43
6	0.97	4.57	13.65	17.31	22.12
7	1.13	5.37	15.92	20.19	25.80
8	1.29	6.17	18.20	23.08	29.49
9	1.45	6.96	20.47	25.96	33.18
10	1.62	7.76	22.75	28.85	36.86
11	1.78	8.56	25.02	31.73	40.55
12	1.94	9.36	27.33	34.62	44.24
13	2.11	10.16	29.72	37.50	47.92
14	2.27	10.95	32.12	40.39	51.61
15	2.43	11.75	34.51	43.34	55.29
16	2.60	12.55	36.91	46.38	58.98
17	2.76	13.35	39.30	49.41	62.67
18	2.92	14.14	41.70	52.45	66.35
19	3.09	14.94	44.09	55.49	70.04
20	3.25	15.74	46.49	58.53	73.73
21	3.41	16.54	48.88	61.56	77.41
22	3.57	17.33	51.27	64.60	81.14
23	3.74	18.13	53.67	67.63	85.00
24	3.87	18.79	55.64	70.13	88.19
25	4.06	19.73	58.46	73.71	92.76
26	4.23	20.52	60.85	76.74	96.64
27	4.39	21.32	63.25	79.78	100.53
28	4.55	22.12	65.64	82.81	104.40
29	4.72	22.91	68.03	85.84	108.27
30	4.88	23.71	70.43	88.88	112.15
31	5.04	24.50	72.79	91.88	115.99
32	5.21	25.33	75.29	95.05	120.03
33	5.35	26.04	77.41	97.74	123.47
34	5.53	26.91	80.04	101.07	127.73

Table 6.29: % Decrese in $CVaR^*$ vs. τ at p=2, c=1, s=0.8, l=3, η=0.25 for (c_b, c_s)=$(0.08, 0.076), (0.4, 0.38), (1.4, 1.33), (2, 1.9), (3.6, 3.42)$ under Cap and Trade Policy.

	c_b=0.08, c_s=0.076			c_b =1.4, c_s=1.33			c_b =3.6, c_s=3.42		
K	**CVaR* (η=0.01)**	**CVaR* (η =0.1)**	**CVaR* (η =0.25)**	**CVaR* (η=0.01)**	**CVaR* (η =0.1)**	**CVaR* (η =0.25)**	**CVaR* (η=0.01)**	**CVaR* (η =0.1)**	**CVaR* (η =0.25)**
300	113.3	227.8	287.3	-330.0	-174.9	-97.5	-991.2	-739.7	-607.9
325	115.3	229.8	289.3	-295.0	-139.9	-62.5	-901.2	-649.7	-517.9
350	117.3	231.8	291.3	-260.0	-104.9	-27.5	-811.2	-559.7	-427.9
375	119.3	233.8	293.3	-225.0	-69.9	7.5	-721.2	-469.7	-337.9
400	121.3	235.8	295.3	-190.0	-34.9	42.5	-631.2	-379.7	-247.9
425	123.3	237.8	297.3	-155.0	0.1	77.5	-541.2	-289.7	-157.9
450	125.3	239.8	299.3	-120.0	35.1	112.5	-451.2	-199.7	-67.9
475	127.3	241.8	301.3	-85.0	70.1	147.5	-361.2	-109.7	22.1
500	129.3	243.8	303.3	-50.0	105.1	182.5	-271.2	-19.7	112.2
525	131.3	245.8	305.3	-15.0	140.1	217.5	-181.2	70.3	197.7
550	133.3	247.8	307.3	20.0	175.1	252.5	-91.2	157.0	283.2
575	135.3	249.8	309.3	55.0	210.1	286.9	-1.2	242.5	368.7
600	137.3	251.8	311.3	90.0	243.9	320.1	85.6	328.0	454.2
625	139.3	253.8	313.3	124.7	277.2	353.4	171.1	413.5	539.7
650	141.3	255.8	315.3	157.9	310.4	386.6	256.6	499.0	625.2
675	143.3	257.7	317.2	191.2	343.7	419.9	342.1	584.5	710.7
700	145.2	259.6	319.1	224.4	376.9	453.1	427.6	670.0	796.2
725	147.1	261.5	321.0	257.7	410.2	486.4	513.1	755.5	881.7
750	149.0	263.4	322.9	290.9	443.4	519.6	598.6	841.0	967.2
775	150.9	265.3	324.8	324.2	476.7	552.9	684.1	926.5	1052.7
800	152.8	267.2	326.7	357.4	509.9	586.1	769.6	1012.0	1138.2
825	154.7	269.1	328.6	390.7	543.2	619.4	855.1	1097.5	1223.7
850	156.6	271.0	330.5	423.9	576.4	652.6	940.6	1183.0	1309.2
875	158.5	272.9	332.4	457.2	609.7	685.9	1026.1	1268.5	1394.7
900	160.4	274.8	334.3	490.4	642.9	719.1	1111.6	1354.0	1480.2
925	162.3	276.7	336.2	523.7	676.2	752.4	1197.1	1439.5	1565.7
950	164.2	278.6	338.1	556.9	709.4	785.6	1282.6	1525.0	1651.2
975	166.1	280.5	340.0	590.2	742.7	818.9	1368.1	1610.5	1736.7
1000	168.0	282.4	341.9	623.4	775.9	852.1	1453.6	1696.0	1822.2

Table 6.30: $CVaR^*$ vs. K at p=2, c=1, s=0.8, l=3, η=0.01, 0.1, 0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

Eta=0.01	Qup=661.99 Qdown=662.29 (cb,cs)=(0.08,0.076)		Qup=619.15 Qdown=620.48 (cb,cs)=(1.4, 1.33)		Qup=580.13 Qdown=584.59 (cb,cs)=(3.6, 3.42)	
Carbon Cap	**CVaR***	**%Decrease in Emission**	**CVaR***	**%Decrease in Emission**	**CVaR***	**%Decrease in Emission**
300	113.34	1.05	-329.97	7.46	-991.20	13.29
325	115.34	1.05	-294.97	7.46	-901.20	13.29
350	117.34	1.05	-259.97	7.46	-811.20	13.29
375	119.34	1.05	-224.97	7.46	-721.20	13.29
400	121.34	1.05	-189.97	7.46	-631.20	13.29
425	123.34	1.05	-154.97	7.46	-541.20	13.29
450	125.34	1.05	-119.97	7.46	-451.20	13.29
475	127.34	1.05	-84.97	7.46	-361.20	13.29
500	129.34	1.05	-49.97	7.46	-271.20	13.29
525	131.34	1.05	-14.97	7.46	-181.20	13.29
550	133.34	1.05	20.04	7.46	-91.20	13.29
575	135.34	1.05	55.04	7.46	-1.20	13.29
600	137.34	1.05	90.04	7.46	85.60	12.62
625	139.34	1.05	124.67	7.26	171.10	12.62
650	141.34	1.05	157.92	7.26	256.60	12.62
675	143.29	1.01	191.17	7.26	342.10	12.62
700	145.19	1.01	224.42	7.26	427.60	12.62
725	147.09	1.01	257.67	7.26	513.10	12.62
750	148.99	1.01	290.92	7.26	598.60	12.62
775	150.89	1.01	324.17	7.26	684.10	12.62
800	152.79	1.01	357.42	7.26	769.60	12.62
825	154.69	1.01	390.67	7.26	855.10	12.62
850	156.59	1.01	423.92	7.26	940.60	12.62
875	158.49	1.01	457.17	7.26	1026.10	12.62
900	160.39	1.01	490.42	7.26	1111.60	12.62
925	162.29	1.01	523.67	7.26	1197.10	12.62
950	164.19	1.01	556.92	7.26	1282.60	12.62
975	166.09	1.01	590.17	7.26	1368.10	12.62
1000	167.99	1.01	623.42	7.26	1453.60	12.62

Table 6.31: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

Eta=0.1	Qup=639.04 Qdown=639.42 (cb,cs)=(0.08,0.076)		Qup=582.68 Qdown=584.54 (cb,cs)=(1.4, 1.33)		Qup=528.57 Qdown=534.54 (cb,cs)=(3.6, 3.42)	
Carbon Cap	**CVaR***	**%Decrease in Emission**	**CVaR***	**%Decrease in Emission**	**CVaR***	**%Decrease in Emission**
300	227.82	1.35	-174.94	10.05	-739.70	18.40
325	229.82	1.35	-139.94	10.05	-649.70	18.40
350	231.82	1.35	-104.94	10.05	-559.70	18.40
375	233.82	1.35	-69.94	10.05	-469.70	18.40
400	235.82	1.35	-34.94	10.05	-379.70	18.40
425	237.82	1.35	0.06	10.05	-289.70	18.40
450	239.82	1.35	35.06	10.05	-199.70	18.40
475	241.82	1.35	70.06	10.05	-109.70	18.40
500	243.82	1.35	105.06	10.05	-19.70	18.40
525	245.82	1.35	140.06	10.05	70.30	18.40
550	247.82	1.35	175.06	10.05	157.00	17.48
575	249.82	1.35	210.06	10.05	242.50	17.48
600	251.82	1.35	243.92	9.76	328.00	17.48
625	253.82	1.35	277.17	9.76	413.50	17.48
650	255.77	1.29	310.42	9.76	499.00	17.48
675	257.67	1.29	343.67	9.76	584.50	17.48
700	259.57	1.29	376.92	9.76	670.00	17.48
725	261.47	1.29	410.17	9.76	755.50	17.48
750	263.37	1.29	443.42	9.76	841.00	17.48
775	265.27	1.29	476.67	9.76	926.50	17.48
800	267.17	1.29	509.92	9.76	1012.00	17.48
825	269.07	1.29	543.17	9.76	1097.50	17.48
850	270.97	1.29	576.42	9.76	1183.00	17.48
875	272.87	1.29	609.67	9.76	1268.50	17.48
900	274.77	1.29	642.92	9.76	1354.00	17.48
925	276.67	1.29	676.17	9.76	1439.50	17.48
950	278.57	1.29	709.42	9.76	1525.00	17.48
975	280.47	1.29	742.67	9.76	1610.50	17.48
1000	282.37	1.29	775.92	9.76	1696.00	17.48

Table 6.32: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

Eta=0.25	Qup=631.01 Qdown=631.64 (cb,cs)=(0.08,0.076)		Qup=564.47 Qdown=566.76 (cb,cs)=(1.4, 1.33)		Qup=497.08 Qdown=504.31 (cb,cs)=(3.6, 3.42)	
Carbon Cap	**CVaR***	**%Decrease in Emission**	**CVaR***	**%Decrease in Emission**	**CVaR***	**%Decrease in Emission**
300	287.33	1.54	-97.46	11.95	-607.90	22.46
325	289.33	1.54	-62.46	11.95	-517.90	22.46
350	291.33	1.54	-27.46	11.95	-427.90	22.46
375	293.33	1.54	7.54	11.95	-337.90	22.46
400	295.33	1.54	42.54	11.95	-247.90	22.46
425	297.33	1.54	77.54	11.95	-157.90	22.46
450	299.33	1.54	112.54	11.95	-67.90	22.46
475	301.33	1.54	147.54	11.95	22.10	22.46
500	303.33	1.54	182.54	11.95	112.20	21.34
525	305.33	1.54	217.54	11.95	197.70	21.34
550	307.33	1.54	252.54	11.95	283.20	21.34
575	309.33	1.54	286.89	11.59	368.70	21.34
600	311.33	1.54	320.14	11.59	454.20	21.34
625	313.33	1.54	353.39	11.59	539.70	21.34
650	315.26	1.47	386.64	11.59	625.20	21.34
675	317.16	1.47	419.89	11.59	710.70	21.34
700	319.06	1.47	453.14	11.59	796.20	21.34
725	320.96	1.47	486.39	11.59	881.70	21.34
750	322.86	1.47	519.64	11.59	967.20	21.34
775	324.76	1.47	552.89	11.59	1052.70	21.34
800	326.66	1.47	586.14	11.59	1138.20	21.34
825	328.56	1.47	619.39	11.59	1223.70	21.34
850	330.46	1.47	652.64	11.59	1309.20	21.34
875	332.36	1.47	685.89	11.59	1394.70	21.34
900	334.26	1.47	719.14	11.59	1480.20	21.34
925	336.16	1.47	752.39	11.59	1565.70	21.34
950	338.06	1.47	785.64	11.59	1651.20	21.34
975	339.96	1.47	818.89	11.59	1736.70	21.34
1000	341.86	1.47	852.14	11.59	1822.20	21.34

Table 6.33: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

Eta=0.01	Qup=516.11 Qdown=516.39 (cb,cs)=(0.08,0.076)		Qup=465.56 Qdown=468.27 (cb,cs)=(1.4,1.33)	
Carbon Cap	**CVaR***	**% Decrease in Emission**	**CVaR***	**% Decrease in Emission**
300	170.56	1.27	-78.57	10.94
325	172.56	1.27	-43.57	10.94
350	174.56	1.27	-8.57	10.94
375	176.56	1.27	26.43	10.94
400	178.56	1.27	61.43	10.94
425	180.56	1.27	96.43	10.94
450	182.56	1.27	131.43	10.94
475	184.56	1.27	165.87	10.42
500	186.56	1.27	199.12	10.42
525	188.53	1.22	232.37	10.42
550	190.43	1.22	265.62	10.42
575	192.33	1.22	298.87	10.42
600	194.23	1.22	332.12	10.42
625	196.13	1.22	365.37	10.42
650	198.03	1.22	398.62	10.42
675	199.93	1.22	431.87	10.42
700	201.83	1.22	465.12	10.42
725	203.73	1.22	498.37	10.42
750	205.63	1.22	531.62	10.42
775	207.53	1.22	564.87	10.42
800	209.43	1.22	598.12	10.42
825	211.33	1.22	631.37	10.42
850	213.23	1.22	664.62	10.42
875	215.13	1.22	697.87	10.42
900	217.03	1.22	731.12	10.42
925	218.93	1.22	764.37	10.42
950	220.83	1.22	797.62	10.42
975	222.73	1.22	830.87	10.42
1000	224.63	1.22	864.12	10.42

Table 6.34: $CVaR^{*}$ and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

Eta=0.1	Qup=535.07 Qdown=535.45 (cb,cs)=(0.08,0.076)		Qup=465.63 Qdown=469.35 (cb,cs)=(1.4,1.33)	
K	**CVaR***	**% Decrease in Emission**	**CVaR***	**% Decrease in Emission**
300	270.06	1.57	8.98	14.35
325	272.06	1.57	43.98	14.35
350	274.06	1.57	78.98	14.35
375	276.06	1.57	113.98	14.35
400	278.06	1.57	148.98	14.35
425	280.06	1.57	183.98	14.35
450	282.06	1.57	218.98	14.35
475	284.06	1.57	253.45	13.66
500	286.06	1.57	286.70	13.66
525	288.06	1.57	319.95	13.66
550	290.00	1.50	353.20	13.66
575	291.90	1.50	386.45	13.66
600	293.80	1.50	419.70	13.66
625	295.70	1.50	452.95	13.66
650	297.60	1.50	486.20	13.66
675	299.50	1.50	519.45	13.66
700	301.40	1.50	552.70	13.66
725	303.30	1.50	585.95	13.66
750	305.20	1.50	619.20	13.66
775	307.10	1.50	652.45	13.66
800	309.00	1.50	685.70	13.66
825	310.90	1.50	718.95	13.66
850	312.80	1.50	752.20	13.66
875	314.70	1.50	785.45	13.66
900	316.60	1.50	818.70	13.66
925	318.50	1.50	851.95	13.66
950	320.40	1.50	885.20	13.66
975	322.30	1.50	918.45	13.66
1000	324.20	1.50	951.70	13.66

Table 6.35: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

Eta=0.25	Qup=549.09 Qdown=549.53 (cb,cs)=(0.08,0.076)		Qup=463.50 Qdown=468.08 (cb,cs)=(1.4,1.33)	
K	CVaR*	% Decrease in Emission	CVaR*	% Decrease in Emission
300	321.91	1.78	53.35	17.09
325	323.91	1.78	88.35	17.09
350	325.91	1.78	123.35	17.09
375	327.91	1.78	158.35	17.09
400	329.91	1.78	193.35	17.09
425	331.91	1.78	228.35	17.09
450	333.91	1.78	263.35	17.09
475	335.91	1.78	297.71	16.27
500	337.91	1.78	330.96	16.27
525	339.91	1.78	364.21	16.27
550	341.91	1.70	397.46	16.27
575	343.81	1.70	430.71	16.27
600	345.71	1.70	463.96	16.27
625	347.61	1.70	497.21	16.27
650	349.51	1.70	530.46	16.27
675	351.41	1.70	563.71	16.27
700	353.31	1.70	596.96	16.27
725	355.21	1.70	630.21	16.27
750	357.11	1.70	663.46	16.27
775	359.01	1.70	696.71	16.27
800	360.91	1.70	729.96	16.27
825	362.81	1.70	763.21	16.27
850	364.71	1.70	796.46	16.27
875	366.61	1.70	829.71	16.27
900	368.51	1.70	862.96	16.27
925	370.41	1.70	896.21	16.27
950	372.31	1.70	929.46	16.27
975	374.21	1.70	962.71	16.27
1000	376.11	1.70	995.96	16.27

Table 6.36: $CVaR^*$ and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

Eta=0.01	Qup=661.99 Qdown=662.29 (cb,cs)=(0.08,0.076)		Qup=619.15 Qdown=620.48 (cb,cs)=(1.4, 1.33)		Qup=580.13 Qdown=584.59 (cb,cs)=(3.6, 3.42)	
Carbon Cap	**%Decrease in Emission**	**%SL***	**%Decrease in Emission**	**%SL***	**%Decrease in Emission**	**%SL***
300	1.05	94.74	7.46	88.33	13.29	78.85
325	1.05	94.74	7.46	88.33	13.29	78.85
350	1.05	94.74	7.46	88.33	13.29	78.85
375	1.05	94.74	7.46	88.33	13.29	78.85
400	1.05	94.74	7.46	88.33	13.29	78.85
425	1.05	94.74	7.46	88.33	13.29	78.85
450	1.05	94.74	7.46	88.33	13.29	78.85
475	1.05	94.74	7.46	88.33	13.29	78.85
500	1.05	94.74	7.46	88.33	13.29	78.85
525	1.05	94.74	7.46	88.33	13.29	78.85
550	1.05	94.74	7.46	88.33	13.29	78.85
575	1.05	94.74	7.46	88.33	13.29	78.85
600	1.05	94.74	7.46	88.33	12.62	80.12
625	1.05	94.74	7.26	88.59	12.62	80.12
650	1.05	94.74	7.26	88.59	12.62	80.12
675	1.01	94.77	7.26	88.59	12.62	80.12
700	1.01	94.77	7.26	88.59	12.62	80.12
725	1.01	94.77	7.26	88.59	12.62	80.12
750	1.01	94.77	7.26	88.59	12.62	80.12
775	1.01	94.77	7.26	88.59	12.62	80.12
800	1.01	94.77	7.26	88.59	12.62	80.12
825	1.01	94.77	7.26	88.59	12.62	80.12
850	1.01	94.77	7.26	88.59	12.62	80.12
875	1.01	94.77	7.26	88.59	12.62	80.12
900	1.01	94.77	7.26	88.59	12.62	80.12
925	1.01	94.77	7.26	88.59	12.62	80.12
950	1.01	94.77	7.26	88.59	12.62	80.12
975	1.01	94.77	7.26	88.59	12.62	80.12
1000	1.01	94.77	7.26	88.59	12.62	80.12

Table 6.37: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

Eta=0.1	Qup=639.04 Qdown=639.42 (cb,cs)=(0.08,0.076)		Qup=582.68 Qdown=584.54 (cb,cs)=(1.4, 1.33)		Qup=528.57 Qdown=534.54 (cb,cs)=(3.6, 3.42)	
Carbon Cap	**%Decrease in Emission**	**%SL***	**%Decrease in Emission**	**%SL***	**%Decrease in Emission**	**%SL***
300	1.35	88.33	10.05	79.58	18.40	61.24
325	1.35	88.33	10.05	79.58	18.40	61.24
350	1.35	88.33	10.05	79.58	18.40	61.24
375	1.35	88.33	10.05	79.58	18.40	61.24
400	1.35	88.33	10.05	79.58	18.40	61.24
425	1.35	88.33	10.05	79.58	18.40	61.24
450	1.35	88.33	10.05	79.58	18.40	61.24
475	1.35	88.33	10.05	79.58	18.40	61.24
500	1.35	88.33	10.05	79.58	18.40	61.24
525	1.35	88.33	10.05	79.58	18.40	61.24
550	1.35	88.33	10.05	79.58	17.48	63.51
575	1.35	88.33	10.05	79.58	17.48	63.51
600	1.35	88.33	9.76	80.11	17.48	63.51
625	1.35	88.33	9.76	80.11	17.48	63.51
650	1.29	88.59	9.76	80.11	17.48	63.51
675	1.29	88.59	9.76	80.11	17.48	63.51
700	1.29	88.59	9.76	80.11	17.48	63.51
725	1.29	88.59	9.76	80.11	17.48	63.51
750	1.29	88.59	9.76	80.11	17.48	63.51
775	1.29	88.59	9.76	80.11	17.48	63.51
800	1.29	88.59	9.76	80.11	17.48	63.51
825	1.29	88.59	9.76	80.11	17.48	63.51
850	1.29	88.59	9.76	80.11	17.48	63.51
875	1.29	88.59	9.76	80.11	17.48	63.51
900	1.29	88.59	9.76	80.11	17.48	63.51
925	1.29	88.59	9.76	80.11	17.48	63.51
950	1.29	88.59	9.76	80.11	17.48	63.51
975	1.29	88.59	9.76	80.11	17.48	63.51
1000	1.29	88.59	9.76	80.11	17.48	63.51

Table 6.38: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

Eta=0.25	Qup=631.01 Qdown=631.64 (cb,cs)=(0.08,0.076)		Qup=564.47 Qdown=566.76 (cb,cs)=(1.4, 1.33)		Qup=497.08 Qdown=504.31 (cb,cs)=(3.6, 3.42)	
Carbon Cap	**%Decrease in Emission**	**%SL***	**%Decrease in Emission**	**%SL***	**%Decrease in Emission**	**%SL***
300	1.54	90.52	11.95	74.05	22.46	48.84
325	1.54	90.52	11.95	74.05	22.46	48.84
350	1.54	90.52	11.95	74.05	22.46	48.84
375	1.54	90.52	11.95	74.05	22.46	48.84
400	1.54	90.52	11.95	74.05	22.46	48.84
425	1.54	90.52	11.95	74.05	22.46	48.84
450	1.54	90.52	11.95	74.05	22.46	48.84
475	1.54	90.52	11.95	74.05	22.46	48.84
500	1.54	90.52	11.95	74.05	21.34	51.72
525	1.54	90.52	11.95	74.05	21.34	51.72
550	1.54	90.52	11.95	74.05	21.34	51.72
575	1.54	90.52	11.59	74.78	21.34	51.72
600	1.54	90.52	11.59	74.78	21.34	51.72
625	1.54	90.52	11.59	74.78	21.34	51.72
650	1.47	90.60	11.59	74.78	21.34	51.72
675	1.47	90.60	11.59	74.78	21.34	51.72
700	1.47	90.60	11.59	74.78	21.34	51.72
725	1.47	90.60	11.59	74.78	21.34	51.72
750	1.47	90.60	11.59	74.78	21.34	51.72
775	1.47	90.60	11.59	74.78	21.34	51.72
800	1.47	90.60	11.59	74.78	21.34	51.72
825	1.47	90.60	11.59	74.78	21.34	51.72
850	1.47	90.60	11.59	74.78	21.34	51.72
875	1.47	90.60	11.59	74.78	21.34	51.72
900	1.47	90.60	11.59	74.78	21.34	51.72
925	1.47	90.60	11.59	74.78	21.34	51.72
950	1.47	90.60	11.59	74.78	21.34	51.72
975	1.47	90.60	11.59	74.78	21.34	51.72
1000	1.47	90.60	11.59	74.78	21.34	51.72

Table 6.39: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.8, l=3, η=0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33), (3.6, 3.42)$ under Cap and Trade Policy.

Eta=0.01	Qup=516.11 Qdown=516.39 (cb,cs)=(0.08,0.076)		Qup=465.56 Qdown=468.27 (cb,cs)=(1.4,1.33)	
K	% Decrease in Emiss	%SL*	% Decrease in Emiss	%SL*
300	1.27	56.40	10.94	36.53
325	1.27	56.40	10.94	36.53
350	1.27	56.40	10.94	36.53
375	1.27	56.40	10.94	36.53
400	1.27	56.40	10.94	36.53
425	1.27	56.40	10.94	36.53
450	1.27	56.40	10.94	36.53
475	1.27	56.40	10.42	37.55
500	1.27	56.40	10.42	37.55
525	1.22	56.51	10.42	37.55
550	1.22	56.51	10.42	37.55
575	1.22	56.51	10.42	37.55
600	1.22	56.51	10.42	37.55
625	1.22	56.51	10.42	37.55
650	1.22	56.51	10.42	37.55
675	1.22	56.51	10.42	37.55
700	1.22	56.51	10.42	37.55
725	1.22	56.51	10.42	37.55
750	1.22	56.51	10.42	37.55
775	1.22	56.51	10.42	37.55
800	1.22	56.51	10.42	37.55
825	1.22	56.51	10.42	37.55
850	1.22	56.51	10.42	37.55
875	1.22	56.51	10.42	37.55
900	1.22	56.51	10.42	37.55
925	1.22	56.51	10.42	37.55
950	1.22	56.51	10.42	37.55
975	1.22	56.51	10.42	37.55
1000	1.22	56.51	10.42	37.55

Table 6.40: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.01 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

Eta=0.1	Qup=535.07 Qdown=535.45 (cb,cs)=(0.08,0.076)		Qup=465.63 Qdown=469.35 (cb,cs)=(1.4,1.33)	
K	% Decrease in Emission	%SL*	% Decrease in Emission	%SL*
300	1.57	63.71	14.35	36.55
325	1.57	63.71	14.35	36.55
350	1.57	63.71	14.35	36.55
375	1.57	63.71	14.35	36.55
400	1.57	63.71	14.35	36.55
425	1.57	63.71	14.35	36.55
450	1.57	63.71	14.35	36.55
475	1.57	63.71	13.66	37.96
500	1.57	63.71	13.66	37.96
525	1.57	63.71	13.66	37.96
550	1.50	63.85	13.66	37.96
575	1.50	63.85	13.66	37.96
600	1.50	63.85	13.66	37.96
625	1.50	63.85	13.66	37.96
650	1.50	63.85	13.66	37.96
675	1.50	63.85	13.66	37.96
700	1.50	63.85	13.66	37.96
725	1.50	63.85	13.66	37.96
750	1.50	63.85	13.66	37.96
775	1.50	63.85	13.66	37.96
800	1.50	63.85	13.66	37.96
825	1.50	63.85	13.66	37.96
850	1.50	63.85	13.66	37.96
875	1.50	63.85	13.66	37.96
900	1.50	63.85	13.66	37.96
925	1.50	63.85	13.66	37.96
950	1.50	63.85	13.66	37.96
975	1.50	63.85	13.66	37.96
1000	1.50	63.85	13.66	37.96

Table 6.41: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.1 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

Eta=0.25	Qup=549.09 Qdown=549.53 (cb,cs)=(0.08,0.076)		Qup=463.50 Qdown=468.08 (cb,cs)=(1.4,1.33)	
K	**% Decrease in Emission**	**%SL***	**% Decrease in Emission**	**%SL***
300	1.78	68.83	17.09	35.76
325	1.78	68.83	17.09	35.76
350	1.78	68.83	17.09	35.76
375	1.78	68.83	17.09	35.76
400	1.78	68.83	17.09	35.76
425	1.78	68.83	17.09	35.76
450	1.78	68.83	17.09	35.76
475	1.78	68.83	16.27	37.48
500	1.78	68.83	16.27	37.48
525	1.78	68.83	16.27	37.48
550	1.70	68.98	16.27	37.48
575	1.70	68.98	16.27	37.48
600	1.70	68.98	16.27	37.48
625	1.70	68.98	16.27	37.48
650	1.70	68.98	16.27	37.48
675	1.70	68.98	16.27	37.48
700	1.70	68.98	16.27	37.48
725	1.70	68.98	16.27	37.48
750	1.70	68.98	16.27	37.48
775	1.70	68.98	16.27	37.48
800	1.70	68.98	16.27	37.48
825	1.70	68.98	16.27	37.48
850	1.70	68.98	16.27	37.48
875	1.70	68.98	16.27	37.48
900	1.70	68.98	16.27	37.48
925	1.70	68.98	16.27	37.48
950	1.70	68.98	16.27	37.48
975	1.70	68.98	16.27	37.48
1000	1.70	68.98	16.27	37.48

Table 6.42: SL^* and % Decrease in Emission vs. K at p=2, c=1, s=0.85, l=1, η=0.25 for (c_b, c_s)=$(0.08, 0.076), (1.4, 1.33)$ under Cap and Trade Policy.

	η=0.01		η =0.1		η =0.25	
Carbon Buying Price	% SL*	% Decrease in Emission	% SL*	% Decrease in Emission	% SL*	% Decrease in Emission
0.08	94.74	1.05	91.78	1.35	90.52	1.54
0.2	93.85	2.21	90.19	2.86	88.50	3.28
0.4	92.64	3.59	87.96	4.71	85.58	5.45
0.8	90.71	5.49	84.28	7.28	80.60	8.55
1.4	88.33	7.46	79.58	10.05	74.05	11.95
2	86.16	9.01	75.24	12.28	67.88	14.76
3	82.30	11.41	68.85	15.76	57.16	19.19
3.6	78.85	13.28	61.24	18.40	48.84	22.46

Table 6.43: % Decrease in Emission and SL^* values obtained at p=2, c=1, s=0.8, l=3, for η=0.01, 0.1, 0.25 and c_b=(0.08, 0.2, 0.4, 0.8, 1.4, 2, 3, 3.6) under Cap and Trade Policy.

Eta=0.01		
	Production Quantity	CVaR
Qunconstrained	669	142.57
Use all Carbon	600	134.33
Use all Cash	650	70.22

Eta=0.1		
	Production Quantity	CVaR
Qunconstrained	647.8	255.27
Use all Carbon	600	294.78
Use all Cash	650	185.25

Eta=0.5		
	Production Quantity	CVaR
Qunconstrained	639.7	370.77
Use all Carbon	600	418.97
Use all Cash	650	300.46

Eta=0.9		
	Production Quantity	CVaR
Qunconstrained	653.3	439.54
Use all Carbon	600	484.65
Use all Cash	650	369.51

Eta	Q*	CVaR*
0.01	600	134.33
0.1	600	294.78
0.5	600	418.97
0.9	600	484.65

Table 6.44: Q^* and $CVaR^*$ at η=0.01, 0.1, 0.5 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, $K_{carbon} = 600$, $K_{cash} = 650$ under Binding Resources Policy.

Eta=0.01		
	Production Quantity	CVaR
Qunconstrained	512.9	174.9
Use all Carbon	450	192.7
Use all Cash	510	90.9

Eta=0.1		
	Production Quantity	CVaR
Qunconstrained	534.5	279.2
Use all Carbon	450	281.2
Use all Cash	510	191.2

Eta=0.5		
	Production Quantity	CVaR
Qunconstrained	570.6	385.7
Use all Carbon	450	368.2
Use all Cash	510	283.4

Eta=0.9		
	Production Quantity	CVaR
Qunconstrained	612.8	447.9
Use all Carbon	450	412.9
Use all Cash	510	327.7

Eta	Q*	CVaR*
0.01	450	192.7
0.1	450	281.2
0.5	450	368.2
0.9	450	412.9

Table 6.45: Q^* and $CVaR^*$ at η=0.01, 0.1, 0.5 at p=2, c=1, s=0.8, l= 1, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 450, K_{cash} = 510 under Binding Resources Policy.

Eta=0.01			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	601.9	Infeasible	-
Buy Carbon Sell Cash	600.8	Feasible	134.35
Buy Carbon Buy Cash	600.4	Infeasible	-
Use All Carbon	600	Feasible	134.33
Use All Cash	650	Feasible	70.22

Eta=0.1			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	558.6	Feasible	330.64
Buy Carbon Sell Cash	557.1	Infeasible	-
Buy Carbon Buy Cash	556.5	Infeasible	-
Use All Carbon	600	Feasible	294.78
Use All Cash	650	Feasible	185.25

Eta=0.5			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	509.4	Feasible	520.94
Buy Carbon Sell Cash	506.8	Infeasible	-
Buy Carbon Buy Cash	505.9	Infeasible	-
Use All Carbon	600	Feasible	418.97
Use All Cash	650	Feasible	300.46

Eta=0.9			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	476.5	Feasible	618.05
Buy Carbon Sell Cash	472.6	Infeasible	-
Buy Carbon Buy Cash	471.2	Infeasible	-
Use All Carbon	600	Feasible	484.65
Use All Cash	650	Feasible	369.51

Eta	Q*	CVaR*
0.01	600.8	134.35
0.1	558.6	330.64
0.5	509.4	520.94
0.9	476.5	618.05

Table 6.46: Q^* and $CVaR^*$ at η=0.01, 0.1, 0.5, 0.9 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33,c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 600, K_{cash} = 650 under Resource Trading Policy.

K_{carbon}=570			
Action	Production Quantity	Feasibility	CVaR
Sell Carbon Sell Cash	601.9	Infeasible	-
Buy Carbon Sell Cash	600.8	Feasible	92.35
Buy Carbon Buy Cash	600.4	Infeasible	-
Use All Carbon	570	Feasible	67.33
Use All Cash	650	Feasible	28.22

K_{carbon} =585			
Action	Production Quantity	Feasibility	CVaR
Sell Carbon Sell Cash	601.9	Infeasible	-
Buy Carbon Sell Cash	600.8	Feasible	106.35
Buy Carbon Buy Cash	600.4	Infeasible	-
Use All Carbon	585	Feasible	105.84
Use All Cash	650	Feasible	49.22

K_{carbon} =600			
Action	Production Quantity	Feasibility	CVaR
Sell Carbon Sell Cash	601.9	Infeasible	-
Buy Carbon Sell Cash	600.8	Feasible	134.35
Buy Carbon Buy Cash	600.4	Infeasible	-
Use All Carbon	600	Feasible	134.33
Use All Cash	650	Feasible	70.22

K_{carbon} =620			
Action	Production Quantity	Feasibility	CVaR
Sell Carbon Sell Cash	601.9	Feasible	161.05
Buy Carbon Sell Cash	600.8	Infeasible	-
Buy Carbon Buy Cash	600.4	Infeasible	-
Use All Carbon	620	Feasible	151.02
Use All Cash	650	Feasible	98.22

K_{cash}	K_{carbon}	Q*	CVaR*
650	570	600.8	92.35
650	585	600.8	106.35
650	600	600.8	134.35
650	620	601.9	161.05

Table 6.47: Optimal Policy analysis at K_{carbon}=570, 585, 600, 620 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{cash} = 650, η=0.01 under Resource Trading Policy.

K_{carbon}=570			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	558.6	Feasible	290.74
Buy Carbon Sell Cash	557.1	Infeasible	-
Buy Carbon Buy Cash	556.5	Infeasible	-
Use All Carbon	570	Feasible	287.83
Use All Cash	650	Feasible	143.25

K_{carbon} =585			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	558.6	Feasible	310.69
Buy Carbon Sell Cash	557.1	Infeasible	-
Buy Carbon Buy Cash	556.5	Infeasible	-
Use All Carbon	585	Feasible	295.46
Use All Cash	650	Feasible	164.25

K_{carbon} =600			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	558.6	Feasible	330.64
Buy Carbon Sell Cash	557.1	Infeasible	-
Buy Carbon Buy Cash	556.5	Infeasible	-
Use All Carbon	600	Feasible	294.78
Use All Cash	650	Feasible	185.25

K_{carbon} =620			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	558.6	Feasible	357.24
Buy Carbon Sell Cash	557.1	Infeasible	-
Buy Carbon Buy Cash	556.5	Infeasible	-
Use All Carbon	620	Feasible	284.08
Use All Cash	650	Feasible	213.25

K_{cash}	K_{carbon}	**Q***	**CVaR***
650	570	558.6	290.74
650	585	558.6	310.69
650	600	558.6	330.64
650	620	558.6	357.24

Table 6.48: Optimal Policy analysis at K_{carbon}=570, 585, 600, 620 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{cash} = 650, η=0.1 under Resource Trading Policy.

K_{cash}=550			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	601.9	Infeasible	-
Buy Cash Sell Carbon	601.5	Infeasible	-
Buy Carbon Buy Cash	600.4	Feasible	23.09
Use All Carbon	600	Feasible	23.08
Use All Cash	550	Feasible	-33.12

K_{cash}=580			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	601.9	Infeasible	-
Buy Cash Sell Carbon	601.5	Infeasible	-
Buy Carbon Buy Cash	600.4	Feasible	56.84
Use All Carbon	600	Feasible	56.83
Use All Cash	580	Feasible	43.23

K_{cash}=610			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	601.9	Infeasible	-
Buy Carbon Sell Cash	600.8	Feasible	90.35
Buy Carbon Buy Cash	600.4	Infeasible	-
Use All Carbon	600	Feasible	90.33
Use All Cash	610	Feasible	87.69

K_{cash}=625			
Action	**Production Quantity**	**Feasibility**	**CVaR**
Sell Carbon Sell Cash	601.9	Infeasible	-
Buy Carbon Sell Cash	600.8	Feasible	106.85
Buy Carbon Buy Cash	600.4	Infeasible	-
Use All Carbon	600	Feasible	106.83
Use All Cash	625	Feasible	89.16

K_{carbon}	K_{cash}	Q*	CVaR*
600	550	600.4	23.09
600	580	600.4	56.84
600	610	600.8	90.35
600	625	600.8	106.85

Table 6.49: Optimal Policy analysis at K_{cash}=550, 580, 610, 625 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 600, η=0.01 under Resource Trading Policy.

K_{cash}=550			
Action	Production Quantity	Feasibility	CVaR
Sell Carbon Sell Cash	558.59	Infeasible	-
Buy Cash Sell Carbon	558.05	Feasible	220.44
Buy Carbon Buy Cash	556.49	Infeasible	-
Use All Carbon	600	Feasible	183.53
Use All Cash	550	Feasible	218.98

K_{cash}=580			
Action	Production Quantity	Feasibility	CVaR
Sell Carbon Sell Cash	558.59	Feasible	253.64
Buy Cash Sell Carbon	558.05	Infeasible	-
Buy Carbon Buy Cash	556.49	Infeasible	-
Use All Carbon	600	Feasible	217.28
Use All Cash	580	Feasible	243.53

K_{cash}=610			
Action	Production Quantity	Feasibility	CVaR
Sell Carbon Sell Cash	558.59	Feasible	286.64
Buy Carbon Sell Cash	557.05	Infeasible	-
Buy Carbon Buy Cash	556.49	Infeasible	-
Use All Carbon	600	Feasible	250.78
Use All Cash	610	Feasible	232.6

K_{cash}=625			
Action	Production Quantity	Feasibility	CVaR
Sell Carbon Sell Cash	558.59	Feasible	303.14
Buy Carbon Sell Cash	557.05	Infeasible	-
Buy Carbon Buy Cash	556.49	Infeasible	-
Use All Carbon	600	Feasible	267.28
Use All Cash	625	Feasible	217.61

K_{carbon}	K_{cash}	Q*	CVaR*
600	550	558.05	220.44
600	580	558.59	253.64
600	610	558.59	286.64
600	625	558.59	303.14

Table 6.50: Optimal Policy analysis at K_{cash}=550, 580, 610, 625 at p=2, c=1, s=0.8, l= 3, α_{carbon}=1, α_{cash}=1, c_b^{carbon}=1.4, c_s^{carbon}=1.33, c_b^{cash}=1.125, c_s^{cash}=1.1, K_{carbon} = 600, η=0.1 under Resource Trading Policy.

Chapter 7

Conclusion

In this thesis we investigate two problems with a single product newsvendor under CVaR maximization objective. In the first problem we take the carbon emission reduction concerns of the newsvendor into consideration. In the second problem, as an extension of the first one, multiple resource constraints are introduced under the binding and tradable resource constraints settings.

The first problem is introduced as " Problem under CVaR Maximization with Carbon Emission Concerns". The carbon emission concerns are taken into consideration via analyzing the strict cap and cap and trade policies. Under the strict cap policy the order/production quantity is fixed according to the given carbon cap. The analytical expressions of order /production quantity satisfying the emission constraint and maximizing CVaR and the corresponding threshold value for profit, VaR, are provided for this policy. For the cap and trade policy, we determine the optimal policy of the newsvendor for a given carbon emission cap. The closed form expressions of the order/production quantity and the corresponding VaR value at the optimal policy are derived.

As a generalization of the first problem, in the second problem we search for the optimal production policy of the newsvendor who is subject to multiple resource constraints. The optimal production policy and corresponding analytical expressions are derived for the cases with binding resource constraints and tradeable resource constraints.

In the numerical experiment we first analyze the effects of newsvendor problem parameters on the optimal order/production quantity versus risk aversion level relation. It is observed that the newsvendor tends to order more as he becomes more risk averse if the lost sales cost is equal to two or three times value of the profit mark-up which is a counter intuitive result. Then, for specified parameter settings the impact of risk aversion level, carbon trading prices, carbon cap tightness and given carbon cap values are analyzed for Problem 1. The strict cap policy analysis implies that CVaR is more sensitive to carbon cap tightness than expected profit which is supported by % Decrease in CVaR and expected profit analyses. We observe that CVaR tends to increase with increasing risk aversion level value and the range of the values CVaR attains increases for a specified tightness interval as the carbon prices increase. In addition, higher percentage reduction in emission with respect to the emission of unconstrained optimal solution is observed at the higher carbon prices under the cap and trade policy. For Problem 2 we conduct a small numerical experiment and similar observations are made.

To our knowledge, in the literature there is not a study incorporating the CVaR setting to newsboy problem with carbon restrictions or multiple tradable resource restrictions. In this respect, this research aims to make a contribution to the literature.

As a future study our problems can be investigated under the case where the carbon/resource prices are random. Another extension could be analyzing the problems with a multi- product setting. As we observe that the parameter setting affects the optimal production amount and risk aversion level relation, it would be interesting to search whether there is an interaction between multiple product paramaeters. Lastly, we can propose a setting for Problem 1 where the carbon cap is also a decision variable. Under this setting the problem can be considered as a two stage stochastic optimization problem.

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