INTEGRATION OF PRODUCTION, TRANSPORTATION AND INVENTORY DECISIONS IN SUPPLY CHAINS

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DOCTOR OF PHILOSOPHY

By
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January, 2012

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of doctor of philosophy.

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ABSTRACT

INTEGRATION OF PRODUCTION, TRANSPORTATION AND INVENTORY DECISIONS IN SUPPLY CHAINS

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This dissertation studies the integration of production, transportation and inventory decisions in supply chains, while utilizing the same vehicles in the inbound and outbound. The details of integration is studied in two levels: operational and tactical. In the first part of the thesis, we provide an operational level model for coordination of production and shipment schedules in a single stage supply chain. The production scheduling problem at the facility is modelled as belonging to a single process. Jobs that are located at a distant origin are carried to this facility making use of a finite number of capacitated vehicles. These vehicles, which are initially stationed close to the origin, are also used for the return of the jobs upon completion of their processing. In the first part, a model is developed to find the schedules of the facility and the vehicles jointly, allowing effective utilization of the vehicles for both in the inbound and outbound transportation.

In the second part of the dissertation, we provide a tactical level model and study a manufacturer's production planning and outbound transportation problem with production capacities to minimize transportation and inventory holding costs. The manufacturer in this setting can use two vehicle types for outbound shipments. The first type of vehicle is available in unlimited number. The availability of the second type, which is less expensive, changes over time. For each possible combination of operating policies affecting the problem structure, we either provide a pseudo-polynomial algorithm for general cost structure or prove that no such algorithm exists even for linear cost structure. We develop general optimality properties, propose a generic model formulation that is valid

for all problems and evaluate the effects of the operating policies on the system performance.

The third part of the dissertation considers one of the problems defined in the second part in detail. Motivated by some industry practices, we present formulations for three different solution approaches, which we refer to as the *uncoordinated solution*, the *hierarchically-coordinated solution* and the *centrally-coordinated solution*. These approaches vary in how the underlying production and transportation subproblems are solved, i.e., sequentially versus jointly, or, heuristically versus optimally. We provide intractability proofs or polynomial-time exact solution procedures for the subproblems and their special cases. We also compare the three solution approaches to quantify the savings due to integration and explicit consideration of transportation availabilities.

Keywords: supply chain scheduling, coordinated schedules, outbound transportation, hierarchical solution, integrated solution, tabu search, beam search.

ÖZET

TEDARİK ZİNCİRLERİNDE ÜRETİM, TAŞIMA VE ENVANTER KARARLARININ ENTEGRASYONU

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Bu tezde tedarik zincirlerinde üretim, taşıma ve envanter kararlarının entegrasyonu üzerine çalışılmıştır. Entegrasyon detayları iki düzeyde ele alınmaktadır. Tezin ilk aşamasında, tek aşamalı bir tedarik zincirinde üretim ve sevkiyat programlarının koordinasyonunu sağlayan operasyonel seviyede bir model kullanılmıştır. Tesisin üretim planlaması problemi tek bir süreç olarak modellenmiştir. Tesisten uzakta bulunan işler sonlu sayıda kapasiteli araçlar kullanılarak tesise getirilmektedir. İşlerin kaynağına yakın olarak konuşlandırılmış olan bu araçlar, işlenmesi bitmiş işlerin teslimatında (dağıtım) da kullanılmaktadır. Tezin ilk aşamasında hem üretim tesisinin hem de araçların çizelgelerini oluşturan bir model geliştirilmiştir. Bu model aynı araçların hem tedarik hem de dağıtımda etkin olarak kullanılmalarına olanak sağlamaktadır.

Tezin ikinci aşamasında, üretim kapasitelerini göz önüne alan, üretim planlama ve dağıtım problemi için taktik seviyede bir model geliştirilmiştir. Modelin amacı toplam taşıma ve envanter maliyetlerini en azlamaktır. Bu sistemdeki üretici, dağıtımı iki tip araç kullanarak yapabilmektedir. İlk tip araç sınırsız sayıda kullanılabilirken, maliyeti daha düşük olan ikinci tip araçların sayısı zamana bağlı olarak değişmektedir. Problem yapısını etkileyen operasyonel faktörlerin her bir kombinasyonu için ya en genel maliyet yapısı için sözde polinom bir algoritma geliştirilmiş ya da doğrusal maliyet fonksiyonları için bile böyle bir algoritmanın var olamayacağı ispatlanmıştır. Tüm kombinasyonlar için geçerli en iyilik koşulları incelenmiş, tüm problemler için geçerli kapsamlı bir formülasyon geliştirilmiş ve operasyonel faktörlerin sistem maliyetleri üzerine etkileri incelenmiştir.

Tezin üçüncü aşamasında, önceki aşamada önerilen problemlerden biri daha detaylı olarak incelenmiştir. Sanayi uygulamalarından esinlenerek üç çözüm yaklaşımı önerilmiş (*koordine-edilmemiş*, *aşamalı-koordineli* ve *merkezi-koordineli*) ve bunların formülasyonu yapılmıştır. Bu yaklaşımlar arasındaki temel fark alt problemlerin çözüm şeklidir (bütünleşik veya sırayla, sezgisel veya en iyi). Alt problemler ve bunların özel durumları için tam çözüm yöntemleri geliştirilmiş ve bunların zorlukları ispatlanmıştır. Bu üç yaklaşım sayısal analizler kullanılarak karşılaştırılmış, bu sayede entegrasyonun kıymeti farklı taşıma koşulları için değerlendirilmiştir.

Anahtar sözcükler: tedarik zinciri çizelgelemesi, koordine çizelgeler, dağıtım, hiyerarşik çözüm, bütünleşik çözüm, tabu taraması, ışın taraması.

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hayatıma en büyük anlamı, neşeyi, huzuru ve mutluluğu katan biricik aşkıma...

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Chapter 1

Introduction

Supply, production and delivery are among the key functions for manufacturing companies. Although these functions are managed independently in many traditional systems, recent studies in supply chain management show that there is significant opportunity for savings if the related decisions are coordinated (Thomas and Griffin [23], Dawande et al. [8]). Coordination of decisions among the various stages and functions of the supply chain is an issue that prevails at different phases of planning. Some examples are: innovation, pricing at the strategic level; inventory control, lot sizing at the tactical level; and scheduling at the operational level.

Transportation of finished goods to the customers is an important logistical activity that has to be planned by companies along with production and inventory management. Efficient utilization of transportation alternatives provides a great opportunity in reducing costs, energy consumption and pollution. In traditional supply-chain research and in many industries, planning activities revolve around production, and transportation decisions typically follow the production and inventory decisions. A growing body of research, on the other hand, emphasize the importance of making these decisions in an integrated manner, and in particular accounting for transportation issues (vehicle routing, cost, delivery time, etc.) at earlier stages of production planning, to reduce overall costs and to increase service levels (Hall and Potts [11], Chen [7]). Such integration can take place at various circumstances: Joint decision making for production and vehicle schedules, coordination of scheduling, batching and delivery decisions, integration of inventory and inbound/outbound transportation decisions, etc.

In keeping with this trend, we consider the production scheduling problem of a company with transportation considerations in a single stage supply chain. In particular, we solve the joint transportation and production planning problem of a company for different transportation circumstances. Specifically, in the first part of this dissertation, we focus on coordination of production and transportation schedules of a company, where a finite number of capacitated vehicles are used for both inbound and outbound transportation activities. In the later parts, we consider production and outbound transportation problem of a company that faces varying vehicle availabilities. For the problems studied in this dissertation, we consider the length of the planning horizon to be in the order of a month.

The problems studied in this dissertation are motivated by production, supply and delivery activities of a worldwide home appliance manufacturer in Turkey, which imports a significant amount of its raw materials and exports a major portion of its end products. The company uses maritime transportation for import and export. The manufacturing facility is located inland whereas the two warehouses–one for holding the imported raw materials and one for holding the end products to be exported, are located at the harbor. Transportation of materials between the manufacturing facility and the harbor is done via containers. Traditionally, the company arranges for transportation after the production schedule is made. This hierarchical decision making results in many containers being used only one way and travelling empty the other way. The company thinks that transportation costs can be reduced significantly if the inbound and outbound shipment schedules are coordinated so that the containers are utilized both ways.

In practice, using the same vehicle for both inbound and outbound transportation is reasonable since some suppliers and customers are close to each other. Especially when import and export is done by sea, both suppliers and customers are reached at the ports. Hence, inbound vehicles can be used for outbound transportation to reduce supply chain costs. Coordination of inbound and outbound transportation schedules with the production schedule by utilizing the same vehicles in both ways is a great opportunity to decrease costs. Moreover, economical utilization of commercial vehicles naturally leads to a decrease in energy consumption and pollution as well. Coordinating inbound and outbound transportation decisions with the production schedule is especially suitable when part of the production process is outsourced or the supplier and customer locations are close.

Despite the broad literature on supply chain scheduling with transportation consideration, there are only few studies that consider using the same vehicle for both inbound and outbound transportation. This research aims at solving the production planning and transportation problem while utilizing inbound vehicles for outbound transportation. Specifically, inspired by the recent developments in the literature and the above real practice, we seek answers for the following questions throughout the dissertation:

- How the production and transportation activities can be coordinated if the same vehicles are utilized for both inbound and outbound transportation?
- How the production and outbound transportation problem can be integrated with the inbound transportation schedule? What are the possible generalizations?
- What are the factors that affect the structure of production and outbound transportation problem? How do these factors affect the system performance?
- What are the alternative solution approaches, and how do these approaches vary? What are the benefits of solving production and transportation problems jointly? How do the problem parameters affect the value of integration?

These are the basic motivations behind our study that we formulate the optimization problems and develop exact and heuristic procedures, and test their performances under various experimental conditions. Considering different problem structures and solution procedures, the dissertation is divided into three consecutive parts, each corresponding to a problem domain.

1.1 Scheduling-Transportation Problem

Shipment schedules of incoming materials and outbound delivery schedules in any system are linked to the production schedule through the inventories of unprocessed and processed jobs, respectively. In this research, our focus is on coordination of scheduling decisions involving production as well as inbound and outbound transportation. We consider a setting consisting of two close warehouses–one for unprocessed jobs and the other for processed jobs, and a production facility far away from the warehouses. The unprocessed jobs are transferred to the production facility using a finite number of capacitated vehicles. Each unprocessed job requires processing in the production facility which is represented by a single process. Upon completion of the process, the end products are delivered to the warehouse using the same set of vehicles allowing effective utilization of the same vehicles both in the inbound and outbound transportation. This kind of planning offers an opportunity but at the same time it turns out to be a challenge, because there is a limit on the time that a vehicle can be held at the facility. In this particular setting, the inventory holding costs for both types of jobs at the production facility, transportation costs and times between the facility and the warehouses are significant. Therefore, planning for effective interaction of the schedules for the production facility and the vehicles, serves as an important tool for lowering total inventory holding and transportation costs. The objective of the proposed model is to minimize the sum of transportation costs and inventory holding costs. Transportation characteristics such as travel times, vehicle capacities, waiting limits are explicitly accounted for.

The first part of the dissertation contributes to the literature on supply chain scheduling under transportation considerations by modeling a practically motivated problem, proving that it is strongly $\mathcal{NP}$-Hard, and conducting an analytical and a numerical investigation of its solution. In particular, properties of the solution space are explored, lower bounds on the optimal costs of the general and the one-vehicle cases are developed, polynomially solvable cases are explored, and a computationally-efficient heuristic is proposed for solving large-size instances. The performances of the heuristic and the lower bounds are examined with an extensive numerical analysis.

1.2 Production-Delivery Problem

In the second part of this dissertation, we study a specific problem in which production planning and outbound transportation decisions are coordinated. The system considered here can be viewed as a manufacturer that schedules a certain number of orders on a single machine. Jobs have to be completed and delivered to customers before their deadlines. Holding costs are incurred for items that stay in the inventory. Deliveries can be made using a combination of heterogeneous vehicles. Mainly, there are two vehicle types that are different in their availability and costs over time. We study the manufacturer's scheduling problem to minimize total inventory holding and outbound transportation costs.

This coordination problem is motivated by a practice of home appliance manufacturer in Turkey. This company produces hundreds of different types of products in their facilities, however, many of the raw materials needed for production are the same in their product spectrum. Thus, the company plans procurement of raw materials in advance, without regarding the exact product mix. Hence, from the production planning perspective, it can be assumed that production facility has a predetermined inbound transportation schedule which is almost known at the beginning of the planning horizon. The vehicles arriving at the facility according to the predetermined inbound transportation schedule can also be utilized for outbound shipments.

Note that, this common input characteristic can also be observed in automotive and furniture industries. Although the final products are different, raw materials are common for all end products. Plastic, lumber and steel are examples for common raw materials. For home appliance industry, certain plastic materials are used in most of the products. Similarly, the same type of lumber can be used to produce a variety of furniture. In all these industries, supply decisions for common raw materials can be made in advance, allowing effective utilization of inbound vehicles in the outbound transportation.

The manufacturing company in our setting, delivers the finished goods to the customers by utilizing newly hired vehicles and/or by arranging for extended use of incoming vehicles that have been already hired for inbound shipments. When the manufacturer resorts to the latter option, an additional fee is paid in proportion to the extended usage time of a vehicle. Using an already hired vehicle may be less costly than hiring a new vehicle depending on this extra time. There is no limit on the number of vehicles that can be hired, however, the number of incoming vehicles is limited and changes over time. The manufacturer decides the composition of vehicles to be used for each delivery after a production plan is made and given the arrival times of incoming vehicles.

The idea of utilizing inbound vehicles for outbound transportation can be generalized. In this extended setting, there are two types of vehicles with the same capacity. The first type represents the newly hired vehicles which is expensive and unlimited in number. Extended use of inbound vehicles are represented by a second type, which is cheaper but its availability changes over time. In other words, inbound vehicles that are used for outbound transportation are considered to be a different type with less cost and varying availability.

In the detailed analysis of the problem, we identify three operating policies that affect the structure of the problem. The combinations of the operating policies lead to six different problem settings. For each possible combination of operating policies affecting the problem structure, we either provide a pseudo-polynomial algorithm for general cost structure or prove that no such algorithm exists even for linear cost structure. We develop general optimality conditions and propose a generic model formulation that is valid for all possible combinations of operating policies. We also evaluate the effects of the operating policies on the system performance with an extensive computational analysis.

1.3 Hierarchical versus Central Coordination

The third part of the dissertation is dedicated to a detailed analysis of one of the problems defined in the second part. In this part, we assume that an order destined to a specific customer cannot be delivered in multiple batches and orders of different customers cannot be delivered in the same vehicle. We propose mathematical formulations representing different decision making approaches (i.e., sequential versus integrated, optimal versus heuristic) and compare their solutions in terms of overall costs.

As reported in many recent papers on supply chain scheduling (e.g., Chen [7], Chen and Vairaktarakis [6], Wang and Lee [27]) and evidenced in our relations with this manufacturer as well with others, we have come to the conclusion that it is a common practice in the industry that outbound transportation decisions (e.g., transport mode choice, schedules of vehicles, routing of vehicles) are made following a production plan. Furthermore, as objectives related to production and customer service are given more priority, transportation costs are either ignored, or it becomes too late to come up with a less costly delivery plan after the production is complete and orders are ready for delivery. In keeping with this observation, we have identified three solution approaches regarding the decision making process for planning the production and outbound transportation of orders. We refer to them as the *uncoordinated solution*, the *hierarchically-coordinated solution* and the *centrally-coordinated solution*. These approaches vary in how the underlying production and transportation subproblems are solved, i.e., sequentially versus jointly, or, heuristically versus optimally. We provide intractability proofs or polynomial-time exact solution procedures for the subproblems and their special cases. We also compare the three solution approaches over a numerical study to quantify the savings due to integration and explicit consideration of transportation availabilities.

The rest of the dissertation is organized as follows: next, we provide the review of the related literature in Chapter 2. In Chapter 3, we develop a model to find the schedules of the facility and the vehicles jointly, allowing effective utilization of the same vehicles for both in the inbound and outbound transportation. Chapter 4 is dedicated to the analysis of the integrated production and outbound transportation problem with varying vehicle availabilities. The explanations of different solution approaches within the specific context of our problems, and the value of centralization are discussed in Chapter 5. Our major findings and contributions are summarized and future research directions are discussed in Chapter 6.

Chapter 2

Literature Review

Supply chain scheduling with transportation considerations has received significant attention over the past decade (e.g., Chang and Lee [3], Chen and Vairaktarakis [18], Li and Ou [17], Hall and Potts [11]). A common property of the studies in this area is that they model the factory as performing a single process on one machine or parallel machines, and consider the scheduling of a group of jobs taking into account transportation times, capacities and/or costs in the inbound and/or the outbound. In these models, a job requires some processing at the shop floor (scheduling) and upon the completion of processing activities, each job needs to be delivered to a customer or next facility for further processing (transportation). The scheduling objectives are functions of delivery time rather than completion time. As far as transportation issues are concerned, most papers focus on the delivery side (e.g., Chang and Lee [3], Li et al. [18], Wang and Lee [27], Chen and Vairaktarakis [6], Chen and Pundoor [5], Wang and Cheng [28], Zhong et al. [30]) while a few take into account both the inbound and the outbound transportation (e.g., Li and Ou [17], Wang and Cheng [29]). Another feature that differentiates these studies from one another, is the objective function they consider. Many of the papers reviewed, optimize a scheduling related objective such as makespan or total tardiness (e.g., Chang and Lee [3], Li and Ou [17], Li et al. [18], Wang and Cheng [28], Zhong et al. [30], Wang and Cheng [29]) whereas others take account of a combined measure of transportation costs and scheduling objectives (e.g., Wang and Lee [27], Chen and Vairaktarakis [6], Chen and Pundoor [5], Hall and Potts [11]).

In terms of the above attributes, the first part of our study models transportation issues both in the inbound and the outbound as Li and Ou [17], Wang and Cheng [29] do. These two studies consider minimization of makespan whereas our study aims to minimize total inventory holding and transportation costs. Moreover, our study differs from Li and Ou [17] and Wang and Cheng [29] in the characteristics of the settings, concerning the number of vehicles used and the locations they operate in-between. Wang and Cheng [29] assume that there are two vehicles–one for carrying items in the inbound from the warehouse to the factory, and one for carrying items in the outbound from the factory to a single customer location. Another distinguishing feature of our study is that, the same vehicles are used for both inbound and outbound transportation. Li and Ou [17], on the other hand, model the availability of one vehicle travelling between a factory and a warehouse where both the unprocessed and processed jobs are held. In fact, within the context of supply chain scheduling with transportation considerations, Li and Ou [17] stands out as the only paper that models utilization of the same vehicle both in the inbound and outbound. Note that, in this kind of a setting, production and vehicle schedules affect one another, and hence, they should be made jointly.

In summary, the first part of our study is different from the existing literature in the following ways: (i) we consider detailed scheduling model with transportation and inventory costs rather than scheduling related costs, (ii) both inbound and outbound transportation decisions are coordinated with production schedule, (iii) a finite number of capacitated vehicles are used and (iv) the benefit of using the same vehicle for inbound and outbound transportation is explicitly modeled.

It is important to note that, a majority of the papers on supply chain scheduling with transportation considerations model the existence of a single type of transportation (e.g., Chang and Lee [3], Li et al. [18], Chen and Vairaktarakis [6], Wang and Cheng [28], Hall and Potts [11]). Chen and Lee [4], Stecke and Zhao [22], and Wang and Lee [27] are examples of the few studies that account

Table 2.1: Summary of the studies in the literature

Measure	Transportation		
	Outbound		Inbound & outbound
	Single type	Multiple types	
Scheduling	Chang & Lee [3] Li et al. [18] Wang & Cheng [28] Zhong et al. [30]		Li & Ou [17] Wang & Cheng [29]
Scheduling + Transportation	Chen & Vairaktarakis [6] Hall & Potts [11] Chen and Pundoor [5]	Wang & Lee [27] Chen & Lee [4]	
Transportation	Chen and Pundoor [5]	Stecke & Zhao [22]	

for different transportation choices. However, in all these studies the difference among the transportation choices stems from delivery time and cost. Mainly, it is assumed that the transportation alternative with a shorter delivery time is more costly. Transportation costs are part of the objective function, and delivery times of orders either contribute to the costs (see Chen and Lee [4], and the second problem in Wang and Lee [27]) or they are incorporated in a constraint allowing for no tardiness (see Stecke and Zhao [22], and the first problem in Wang and Lee [27]). In the second and third parts of our study, vehicle costs and capacities are explicitly modeled, and vehicles are considered as heterogeneous due to the differences in their costs and availabilities. Mainly, the less costly vehicle is less available. Furthermore, we take minimization of inventory holding and transportation costs as an objective and do not allow for any job to be tardy. A brief summary of the literature for supply chain scheduling with transportation considerations is provided in Table 2.1. The columns of the table correspond to different transportation considerations whereas the rows correspond to the objective measures each study consider. In the second row, the studies that consider a scheduling related objective such as makespan or tardiness are given. The studies in the third row consider a combined measure of scheduling related objectives and transportation costs.

Integrated production and transportation planning problems are extensively studied in the supply chain literature (e.g., Hwang and Jaruphongsa [12], Lee et al. [16], Cetinkaya and Lee [1], Cetinkaya et al. [2], Lee et al. [15]). A common characteristic for these studies is providing a lot sizing model to investigate the trade off between production and transportation or inventory holding costs (e.g., Hwang [13], Lee et al. [15], Cetinkaya and Lee [1]). Production cost, especially production setup cost, is an important part of the total cost for this line of research. In most of the studies in this literature, early deliveries are not allowed. There are studies that use demand time windows to allow early or tardy deliveries with a penalty cost (Hwang and Jaruphongsa [12], Lee et al. [16]). Hwang [13] and Lee et al. [15] are examples in which only late deliveries are allowed (backlogging) in order to save transportation costs.

In the second and third parts of our study, however, early deliveries are allowed without any cost. A variety of production and transportation cost functions are studied for the deterministic demand cases in the literature. Moreover, alternative stochastic demand structures are also studied (Cetinkaya and Lee [1], Cetinkaya et al. [2]). Although majority of the studies in the literature consider only outbound transportation decisions and ignore inbound activities, there are a few studies that consider inbound transportation (Toptal et al. [24], Jaruphongsa et al. [14], Lee et al. [15]). In the majority of the papers, production capacity is assumed to be infinite, however, there is a number of multilevel and multi facility models with finite production capacities (Hoesel et al. [25], Lee et al. [15], Eksioglu et al. [9]).

The second and third parts of this study are different from the literature in the following ways: (i) vehicles used for inbound transportation are utilized for outbound transportation, (ii) vehicles are considered as heterogenous due to the differences in their costs and availabilities, (iii) there is a finite production capacity with no production setup cost, (iv) multiple orders can be defined for the same period, and (v) early deliveries are allowed without penalty.

It is a common practice in the industry that outbound transportation decisions follow production decisions (e.g., Chen [7], Chen and Vairaktarakis [6], Wang and Lee [27]). This leads suboptimal transportation decisions. Although integration of production and transportation decisions reduces the total costs, the value of integration is not well studied in the literature except two papers (Chen and Vairaktarakis [6], Pundoor and Chen [19].

The third part of the dissertation contributes to the literature by quantifying the value of integration via comparing uncoordinated, hierarchically-coordinated and centrally-coordinated solutions over an extensive computational test bed.

We now continue with the analysis of the first problem.

Chapter 3

Scheduling-Transportation Problem

Coordination of Inbound and Outbound Transportation Schedules with the Production Schedule

In this chapter, we study the problem of jointly finding the production schedule of the facility and the schedules of a finite number of capacitated vehicles subject to a waiting limit constraint at the facility. The objective is to minimize the total inventory holding and transportation costs for a certain number of unprocessed jobs to travel from an origin to a distant facility, get processed and return back to the origin. All vehicles are assumed to be identical but their capacities, defined in terms of the number of jobs they can carry, are allowed to be different in the inbound and outbound.

The proposed model and its solution are also applicable in a setting where jobs travel to and from a subcontractor for some of their operations to be performed. The aforementioned appliance manufacturer outsources a portion of injection molding process from a number of small subcontractors. Due to economies of scale, the company imports and stores the raw materials in its facilities. When there is a need for injection process, the raw materials are sent to subcontractors and the molded parts are then shipped back to the factory using a finite number of vehicles. A similar situation is valid for the textile industry in the US. Some US textile manufacturers cut fabrics in the US and send cut fabrics to a low wage country for assembly. The assembled products are then returned to the US for finishing. This kind of manufacturing relations are so common that, there are even international agreements between the US and Mexico on reducing the duty for outsourcing textile production activities from a subcontractor (Sen [21]). In such cases, each production batch can be considered as a job, and our model may be of use if the objective is to minimize the sum of transportation costs and the inventory holding costs at the subcontractor.

The rest of the chapter is organized as follows: In the next section, we begin with a detailed description of the problem and present a mixed integer linear programming formulation. In Section 3.2, we establish the computational complexity of the problem and present lower bounds on the optimal value of the objective function. We also present some properties of a class of solutions for the general case and a special case of the problem. Polynomial algorithms for some special cases are provided in Section 3.3. This is followed by a description of the proposed heuristic in Section 3.4. In Section 3.5, we report the results of a computational study.

3.1 Problem Definition and Formulation

The system under consideration consists of two warehouses and a production facility. The warehouses, the first for unprocessed jobs and the second for end products, are close to each other. Therefore, they can be considered as in the same location, that is the origin. The production facility is far away from the warehouses. Unprocessed jobs are transferred from the first warehouse to the production facility and end products are transported from the facility to the second warehouse with m identical vehicles. The vehicle capacity of is k_1 for unprocessed jobs and k_2 for the processed jobs. Waiting time of a vehicle at the production facility is limited to l time units. A *tour* is referred to as the run made by a vehicle which starts and ends at the first warehouse, and visits the production facility and the second warehouse in that order. All vehicles are initially located in close proximity to the first warehouse. Total duration of a tour, excluding the waiting time, loading and unloading times, is called *tour time* and denoted by τ. The production facility is modeled as a single machine. An unprocessed job i requires p_i time units of processing at the facility. Loading and unloading times are negligible.

A transportation cost c is incurred whenever a vehicle makes a tour, regardless of the number of jobs carried. An unprocessed job waiting at the facility incurs an inventory holding cost of $\$h_1$ per unit time until its processing starts. Similarly, the inventory holding cost per unit per time of an end product at the facility is denoted by h_2. No inventory holding cost is incurred for the jobs while they are being transported on the vehicles. The objective is to minimize the sum of inventory holding costs at the facility, and inbound and outbound transportation costs. A feasible solution to this problem should include the schedules of the vehicles and the production facility, and an assignment of the jobs to the vehicles for both inbound and outbound transportation.

The problem is first modeled as a nonlinear integer program. Then, an effective way for its linearization is proposed. Before presenting the model, we briefly summarize our main assumptions and introduce additional notation for decision variables.

Assumptions

- Each job occupies the same size on vehicle
- Tour cost and tour time are independent of the number of jobs carried
- All jobs have the same unit holding cost

N	:	Set of jobs	
σ_i	:	Starting time of the processing of job i.	$\forall i \in N$.
α_i	:	Arrival time of job i to the facility.	$\forall i \in N$.
δ_i	:	Departure time of job i from the facility.	$\forall i \in N$.
s_{ij}	:	$\begin{cases} 1, & \text{if job } i \text{ is to be processed before job } j \\ 0, & \text{otherwise} \end{cases}$	$\forall i, j \in N$
a_t	:	Arrival time of the vehicle in tour t to the facility.	$t = 1, \ldots, 2\|N\|$
d_t	:	Departure time of the vehicle in tour t from the facility.	$t = 1, \ldots, 2\|N\|$
ψ_t	:	$\begin{cases} 1, & \text{if } t^{th} \text{ tour is utilized} \\ 0, & \text{otherwise} \end{cases}$	$t = 1, \ldots, 2\|N\|$
x_{it}	:	$\begin{cases} 1, & \text{if job } i \text{ arrives at the facility with tour } t \\ 0, & \text{otherwise} \end{cases}$	$\forall i \in N, t = 1, \ldots, 2\|N\|$
y_{it}	:	$\begin{cases} 1, & \text{if job } i \text{ departs from the facility with tour } t \\ 0, & \text{otherwise} \end{cases}$	$\forall i \in N, t = 1, \ldots, 2\|N\|$
M	:	A very big number	

$$\min\ h_1 \sum_{i \in N} (\sigma_i - \alpha_i) + h_2 \sum_{i \in N} (\delta_i - (\sigma_i + p_i)) + c \sum_{t=1}^{2|N|} \psi_t$$

subject to

$$\sigma_j \geq \sigma_i + p_i s_{ij} - M(1 - s_{ij}) \qquad \forall i, j \in N \tag{3.1}$$

$$\sigma_i \geq \alpha_i \qquad \forall i \in N \tag{3.2}$$

$$\sigma_i + p_i \leq \delta_i \qquad \forall i \in N \tag{3.3}$$

$$s_{ij} + s_{ji} = 1 \qquad \forall i, j \in N \tag{3.4}$$

$$\sum_{t=1}^{2|N|} x_{it} = 1 \qquad \forall i \in N \tag{3.5}$$

$$\sum_{t=1}^{2|N|} y_{it} = 1 \qquad \forall i \in N \tag{3.6}$$

$$\sum_{i \in N} x_{it} \leq k_1 \psi_t \qquad t = 1, 2, .., 2\,|N| \tag{3.7}$$

$$\sum_{i \in N} y_{it} \leq k_2 \psi_t \qquad t = 1, 2, .., 2\,|N| \tag{3.8}$$

$$a_{t+m} \geq d_t + \tau \qquad t = 1, \ldots, 2\,|N| - m \tag{3.9}$$

$$d_t \geq a_t \qquad t = 1, \ldots, 2\,|N| \tag{3.10}$$

$$d_t \leq a_t + l \qquad t = 1, \ldots, 2\,|N| \tag{3.11}$$

$$\alpha_i = \sum_{t=1}^{2|N|} a_t x_{it} \qquad \forall i \in N \tag{3.12}$$

$$\delta_i = \sum_{t=1}^{2|N|} d_t y_{it} \qquad \forall i \in N \tag{3.13}$$

$$\sigma_i, \alpha_i, \delta_i, a_t, d_t \geq 0 \qquad \forall i \in N, t = 1, \ldots, 2\,|N| \tag{3.14}$$

$$s_{ij}, \psi_t, x_{it}, y_{it} \in \{0, 1\} \qquad \forall i, j \in N, t = 1, \ldots, 2\,|N| \tag{3.15}$$

The first and the second terms of the objective function are inventory holding costs for unprocessed and processed jobs, respectively. The third term corresponds to the transportation costs. Constraint set (3.1) assures that there is no overlap of the processing of different jobs. The set of constraints in (3.2) and (3.3) restrict the processing of a job to be between its arrival and departure times.

The sequence of jobs is maintained by Expression (3.4). Constraint sets (3.5) and (3.6) ensure that each job is assigned to a tour for its arrival to and departure from the production facility. Vehicle capacity constraints are modeled by (3.7) and (3.8). (3.9)–(3.11) establish the link between arrival and departure times of the tours. Finally, (3.12) and (3.13) make sure that arrival and departure times of the jobs are consistent with the arrival and departure times of the tours they are assigned to. Even though the constraint sets as defined by Expressions (3.12) and (3.13) are nonlinear, they can easily be linearized as follows:

$$\alpha_i \geq a_t - (1 - x_{it})M \quad \forall i \in N, t = 1, \ldots, 2\,|N|$$
$$\alpha_i \leq a_t + (1 - x_{it})M \quad \forall i \in N, t = 1, \ldots, 2\,|N|$$
$$\delta_i \geq d_t - (1 - y_{it})M \quad \forall i \in N, t = 1, \ldots, 2\,|N|$$
$$\delta_i \leq d_t + (1 - y_{it})M \quad \forall i \in N, t = 1, \ldots, 2\,|N|$$

Since the vehicles are identical, there is no need to provide a different schedule for each vehicle. Instead, we index the tours and decide on the arrival and departure times of each tour. The maximum number of tours is $2|N|$, in which case each job arrives and departs with a different tour. The indexed tours are assigned to vehicles in a uniform manner. If there are m vehicles, the first vehicle makes the $1^{st}, (m+1)^{st}, (2m+1)^{st}, \ldots$ tours, the second vehicle makes the $2^{nd}, (m+2)^{nd}, (2m+2)^{nd}, \ldots$ tours, etc. Without loss of generality, we assume that vehicle k makes the tours $k+mj$ where $j \in \mathcal{Z}^+ \cup \{0\}$. An optimal solution of the above integer program is post-processed and translated to an optimal solution of the original problem. The post-processing is briefly assigning arrival and departure times of the tours to the vehicles. If tour k is utilized (i.e., $\psi_k = 1$), its arrival and departure times, to and from the production facility, are taken as those of vehicle k at the first time it is used. Similarly, if tour $k+mj$ is utilized, then vehicle k is used at least j times, and the j^{th} arrival and departure times of this vehicle can be inferred from those of tour $k+mj$.

3.2 Analysis of the Problem

In this section, we first show that the problem described in Section 3.1 is $\mathcal{NP}$-Hard in the strong sense. Therefore, the rest of our analysis aims at identifying some properties of an optimal solution to reduce the set of feasible solutions. We also propose some lower bounds on the optimal objective function value.

Theorem 3.1 *The decision version of the problem (referred to as problem $\mathcal{P}$) is $\mathcal{NP}$-Complete in the strong sense.*

Proof: In the proof we consider the special case of one vehicle. Clearly the generalization is also $\mathcal{NP} - Complete$ and $\mathcal{P}$ is in $\mathcal{NP}$. Proof is done by a reduction from 3-Partition(3P) problem. 3P is defined as follows.

3P: Given a set $\mathcal{G}$ of $3t$ elements, a bound $B \in Z^+$, and a size $s(a) \in Z^+$ for each $a \in A$ such that $B/4 < s(a) < B/2$ and such that $\sum_{a\in\mathcal{G}} s(a) = tB$, can $\mathcal{G}$ be partitioned into t disjoint sets $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_t$ such that $\sum_{a\in\mathcal{G}_i} s(a) = B$ for $i = 1, 2, .., t$(note that each $\mathcal{G}_i$ must therefore contain exactly three elements from $\mathcal{G}$)?

REDUCTION: Given an instance of 3P, the instance of $\mathcal{P}$ is constructed as follows: for each element a in set $\mathcal{G}$, a job a is defined in set N with processing time equal to $s(a)$. Thus, $N = \mathcal{G}$, $|N| = 3t$, $p_a = s(a), \forall a \in \mathcal{G}$, $\tau = B$, $c = 4tB$, $h_1 = h_2 = 1$, $z^* = (t+1)c + \frac{c}{2}$, $k_1 = k_2 = 3$, $l = 0$. We prove that there is a solution to 3P if and only if there is a solution to $\mathcal{P}$ with objective less than or equal to z^*.

Suppose that there is a feasible solution to $\mathcal{P}$ such that the cost z is less than or equal to z^*. We show that there also exists a feasible solution to 3P. Since $l = 0$, the vehicle is not allowed to wait at the facility. Therefore, the first tour departs from the facility empty. As $k_1 = k_2 = 3$, the vehicle makes at least $t+1$ tours, with a transportation cost of $c(t+1)$. Since $z \le z^* < c(t+2)$, the vehicle makes exactly $t+1$ tours. Therefore, tour i $(i = 1, \ldots, t)$ carries exactly three jobs (whose total processing times is denoted by $\tilde{p}_i$) to the facility, which should be processed by the time of the next arrival of the vehicle. At tour i, whatever the processing sequence is, the inventory holding cost incurred is at least $2\tilde{p}_i$. This is because, each job waits for the other two either after or before being processed and $h_1 = h_2 = 1$. Then, the total inventory holding cost is at least $2\sum_{i=1}^{t}\tilde{p}_i = 2\sum_{a\in\mathcal{G}} p_a = 2tB = c/2$, that is $z = z^*$, which in turn implies that the total inventory holding cost is exactly $c/2$. Note that $\tilde{p}_i \geq \tau, \forall i$. Otherwise, there would be an extra inventory holding cost incurred by all three jobs waiting after or before being processed. However, $\sum_{i=1}^{t}\tilde{p}_i = tB$, thus, we should have $\tilde{p}_i = \tau, \forall i$. Then, one can obtain a feasible solution to 3P by taking $\mathcal{G}_i$ as the set which includes the processing times of the jobs arriving with tour i. Conversely, if there exists a feasible solution to 3P, a feasible solution to $\mathcal{P}$ can be obtained by assigning the jobs whose processing times are the numbers in $\mathcal{G}_i$ to arrive with tour i. Note that the parameter settings in the reduction are polynomial in the size of the problem. Consequently, decision version of $\mathcal{P}$ is $\mathcal{NP} - Complete$ in the strong sense. ■

The mathematical program in Section 3.1 formulates the problem of interest in its most general form. This leads to many alternative solutions. However, some of these solutions can be further eliminated by the following observation: Vehicles are allowed to wait l time units at the production facility. This may lead to alternative solutions in which some vehicles arrive early at the production facility or depart late without affecting the rest of the schedule and without exceeding the waiting time limit. In the rest of the section, we do not consider such alternative solutions that involve unnecessary waiting of the vehicles at the production facility. More specifically, we look into only the feasible solutions with the following characteristics:

- Every tour t departs from the production facility at $d_t = \max\left(a_t, \delta_{(t)}\right)$. Here, $\delta_{(t)}$ is the latest completion time of processing among those of all the jobs that depart from the production facility with tour t (if no such job exists, $\delta_{(t)}$ is taken as 0).
- Every tour t arrives at the production facility at $a_t = \min\left(d_t, \sigma_{(t)}\right)$ where $\sigma_{(t)}$ is the earliest start time of processing among those of all the jobs that

arrive to the production facility with tour t (if no such job exists, $\sigma_{(t)}$ is taken as ∞).

We note that a solution may be optimal even though $d_t > max\left(a_t, \delta_{(t)}\right)$ for some tour t as long as $d_t \leq a_t + l$. Similarly, a solution may be optimal even though $a_t < \min\left(d_t, \sigma_{(t)}\right)$ for some tour t as long as $a_t \geq d_t - l$. However, we eliminate these solutions for practical purposes. Furthermore, due to the identicalness of the vehicles, indexing the tours with $\psi_t = 1$ such that $a_1 \leq a_2 \leq \ldots$, an assignment of vehicles to the tours can be made for any solution to also have $d_1 \leq d_2 \leq \ldots$

The sequence of jobs in their nondecreasing order of arrival times to the facility is referred to as the *inbound transportation sequence*. As several items may arrive to the facility in the same vehicle, an inbound transportation sequence related to a production sequence may not be unique. The sequence of jobs in their nondecreasing order of departure times from the facility is referred to as the *outbound transportation sequence*. Similarly, an outbound transportation sequence related to a production sequence may not be unique. The following two theorems jointly imply that there is an optimal solution in which inbound and outbound transportation sequences are in compliance with the production sequence.

Proposition 3.1 *Every feasible solution can be converted to an alternative one in which for all job pairs (i, j), if job i precedes job j in the production sequence, job i arrives at the facility no later than job j.*

Proof: Let S be a feasible solution such that job i precedes job j in the production sequence but arrives at the facility later (i.e., $\sigma_i < \sigma_j$ and $\alpha_j < \alpha_i$). We have $\alpha_j < \alpha_i \leq \sigma_i < \sigma_j$. Consider a new solution S' in which job i and job j are swapped for their assignment to vehicles in inbound transportation. That is, we now have $\alpha'_i = \alpha_j$ and $\alpha'_j = \alpha_i$, where α'_i and α'_j are the arrival times of jobs i and j in solution S', respectively. Note that S and S' have the same outbound transportation and production schedules. Let $TC(S)$ denote the cost of solution S. $TC(S)$ and $TC(S')$ differ only in terms of inventory holding costs of jobs i and j while they are waiting as unprocessed at the production facility. It follows that $TC(S) - TC(S') = (\sigma_i - \alpha_i + \sigma_j - \alpha_j)h_1 - (\sigma_i - \alpha'_i + \sigma_j - \alpha'_j)h_1 = 0$. Thus, S' is equivalent to S in its objective function value. Continuing in this fashion and swapping the inbound vehicle assignments all such (i, j) in S, results in another feasible solution in which production sequence is in compliance with the inbound transportation sequence. ■

Proposition 3.2 *Every feasible solution can be converted to an alternative one in which for all job pairs (i, j), if job i precedes job j in the production sequence, job i departs from the facility no later than job j.*

Proof: Similar to that of Proposition 3.1. ■

Proposition 3.1, Proposition 3.2 and their proofs imply that there exists an optimal solution in which if job i precedes job j in the production sequence, then job i arrives at the facility and departs from the facility no later than job j does. This can be accomplished by a pairwise interchange of job assignments to the vehicles for their inbound and outbound transportation. The following two propositions present additional properties involving the jobs that arrive at and depart from the production facility together.

Proposition 3.3 *If $h_1 < h_2$, there exists an optimal solution in which jobs that arrive at and depart from the production facility together, are processed in LPT (Longest Processing Time first) order.*

Proof: We know from Proposition 3.1, Proposition 3.2 and their proofs that there exists an optimal solution in which if job i precedes job j in the production sequence, then job i arrives at the facility and departs from the facility no later than job j. The proof of the current theorem will follow by showing that, if $h_1 < h_2$, in such an optimal solution, jobs that arrive to and depart from the facility together are processed in LPT order. Hence, in case of $h_1 < h_2$, there exists an optimal solution with the property stated in the theorem.

Take an optimal solution S in which inbound, outbound and production sequences are in compliance. Note that, in this solution, jobs that arrive to and depart from the production facility together are processed consecutively. Assume, by contradiction, that S does not comply with the theorem. Therefore, there exists at least a pair of adjacent jobs i and j in the production schedule that arrive to and depart from the facility together ($\alpha_i = \alpha_j, \delta_i = \delta_j$), however, job i precedes job j in the production schedule ($\sigma_i < \sigma_j = \sigma_i + p_i$) despite $p_i < p_j$.

Construct another feasible solution S' from S by interchanging jobs i and j in the production sequence. We now have $\sigma'_j = \sigma_i, \sigma'_i = \sigma'_j + p_j$, where σ'_i and σ'_j are the starting times of processing of jobs i and j in S', respectively. Note that, S and S' are only different in their production schedules of these two jobs. Let $TC(S)$ denote the total cost of solution S. We have

$$\begin{aligned} TC(S) \quad & -TC(S') = \\ & [(\sigma_i - \alpha_i + \sigma_j - \alpha_j)h_1 + (\delta_i - (\sigma_i + p_i) + \delta_j - (\sigma_j + p_j))\, h_2] \\ & - \left[(\sigma'_i - \alpha_i + \sigma'_j - \alpha_j)h_1 + \left(\delta_i - (\sigma'_i + p_i) + \delta_j - (\sigma'_j + p_j)\right) h_2\right], \end{aligned}$$

which leads to

$$\begin{aligned} TC(S) - TC(S') &= (\sigma_i + \sigma_j - \sigma'_i - \sigma'_j)(h_1 - h_2) \\ &= (p_j - p_i)(h_2 - h_1). \end{aligned}$$

Since $p_j > p_i$ and $h_2 > h_1$, the above expression is greater than zero. This implies $TC(S') < TC(S)$, which contradicts with the optimality of S. Therefore, if $h_1 < h_2$, jobs that arrive to and depart from the production facility together, should be processed in LPT order. ■

Proposition 3.4 *If $h_1 > h_2$, there exists an optimal solution in which jobs that arrive at and depart from the production facility together are processed in SPT (Smallest Processing Time first) order.*

Proof: Similar to that of Proposition 3.3. ■

3.2.1 Lower Bound Scheme

In this section, we propose two lower bounds on the optimal value of the objective function. The first lower bound, which is presented in Corollary 3.1, concerns the general case where there may be more than one vehicle. The second lower bound, which is presented in Corollary 3.2, applies to the case of one vehicle. Recall that, the objective function is composed of inventory holding and transportation costs. Given the number of tours, which will be denoted by ω, transportation cost is fixed and is equal to $c \times \omega$. Note that, ω may range from $\left\lceil \frac{|N|}{\min(k_1,k_2)} \right\rceil$ to $2|N|$. For a specified value of ω, Theorem 3.2 and Theorem 3.3 introduce lower bounds on inventory holding costs considering the general case and the one-vehicle case, respectively. A lower bound on the objective function value of an optimal solution in each case is then given by the minimum, over all possible ω values, of the sum of lower bound on inventory holding costs and the value $c \times \omega$. The lower bounds in Corollary 3.1 and Corollary 3.2 rely on this fact.

We start with presenting a lower bound on inventory holding costs for the general case.

Theorem 3.2 *Given the number of tours, i.e. ω, the following is a lower bound on the total inventory holding costs:*

$$LB'_{\mathcal{I}}(\omega) = \left\{ \sum_{i=1}^{|N|} \left\lfloor \frac{i-1}{\omega} \right\rfloor p_{(i)} \right\} (h_1 + h_2).$$

Here, $\lfloor x \rfloor$ refers to the largest integer that is smaller than or equal to x, and, (i) refers to the index of the job with the i^{th} longest processing time.

Proof: Total inventory holding costs are composed of inventory holding costs for unprocessed jobs and processed jobs. For the proof of the theorem, we will first find lower bounds individually for each component, and later, we will sum them up. In reaching a lower bound for unprocessed jobs, we will ignore the effect of any scheduling decision on the inventory holding costs of the processed jobs. This is equivalent to momentarily assuming that $h_2 = 0$. Likewise, in deriving a lower bound for processed jobs, we will assume that $h_1 = 0$.

Let us start with the inventory holding costs of the unprocessed jobs. The production facility will never be idle as long as there is some job waiting to be processed. Therefore, the inventory holding costs of unprocessed jobs are given by $\sum_{i=1}^{|N|} \mu_i^1 p_i h_1$, where μ_i^1 is the number of jobs that wait for job i as unprocessed. Since there are ω tours, we have at most ω jobs with $\mu_i^1 = 0$, at most ω jobs with $\mu_i^1 = 1$ and so on. The expression $\sum_{i=1}^{|N|} \mu_i^1 p_i h_1$ is minimized when jobs with longer processing times have smaller μ_i^1 values as multipliers. That is, when the longest ω number of jobs are chosen to have $\mu_i^1 = 0$, the next longest ω number of jobs are chosen to have $\mu_i^1 = 1$ and so on. This is achieved by assigning each of the first ω jobs with longer processing times to a different tour and processing it the last among all the jobs in that tour. Similarly, each of the next longest ω number of jobs is assigned to one of ω different tours, and placed as second from the end in the processing sequence of all the jobs in that tour, and so on. This leads to

$$\sum_{i=1}^{|N|} \mu_i^1 p_i h_1 \geq \sum_{i=1}^{|N|} \mu_{(i)}^1 p_{(i)} h_1, \tag{3.16}$$

where $\mu_{(i)}^1 = \left\lfloor \frac{i-1}{\omega} \right\rfloor$ and (i) is the index of the job with the i^{th} largest processing time. Hence, the right side of the above inequality is a lower bound on the inventory holding costs of unprocessed jobs.

A lower bound on the inventory holding costs of the processed jobs can be derived in a similar way. Let μ_i^2 be the number of jobs that wait for job i as processed. Then, the inventory holding costs of the processed jobs are given by $\sum_{i=1}^{|N|} \mu_i^2 p_i h_2$. With a similar argument as in the case of unprocessed jobs, we have

$$\sum_{i=1}^{|N|} \mu_i^2 p_i h_2 \geq \sum_{i=1}^{|N|} \mu_{(i)}^2 p_{(i)} h_2,$$

where $\mu_{(i)}^2 = \left\lfloor \frac{i-1}{\omega} \right\rfloor$ and (i) is the index of the job with the i^{th} largest processing time. The right side of the above inequality is a lower bound on the inventory holding costs of the processed jobs. Therefore, its summation with the right side of inequality (3.16) gives a lower bound on the total inventory holding costs for a given value of number of tours (i.e., w). ∎

Next, based on the above theorem, we present a lower bound on the objective function value of an optimal solution.

Corollary 3.1 *A lower bound on the total cost of an optimal solution is given* *by*

$$LB^1 = \min_{\left\lceil \frac{|N|}{\min(k_1,k_2)} \right\rceil \leq \omega \leq 2|N|} \{LB'_{\mathcal{I}}(\omega) + c\omega\}.$$

The following theorem provides a lower bound on inventory holding costs for the one-vehicle case.

Theorem 3.3 *Given the number of tours, i.e. ω, the following is a lower bound on the total inventory holding costs when there is a single vehicle:*

$$LB''_I(\omega) = \sum_{i=1}^{|N|} \left\{ I_{(i)} \left(\tau - p_{(i)}\right) min(h_1, h_2) + \left\lfloor \frac{i-1}{\omega} \right\rfloor p_{(i)} |h_1 - h_2| \right\}.$$

Here, (i) refers to the index of the job with the i^{th} longest processing time and $I_{(i)}$ is an indicator variable with the following value:

$$I_{(i)} = \begin{cases} 1, & \text{if } \tau > p_{(i)} > l \\ 0, & \text{otherwise.} \end{cases}$$

Proof: The proof of Theorem 3.3 follows based on a similar idea which underlies the proof of Theorem 3.2. In general, a job may contribute to the total inventory holding costs in two ways; one is due to the waiting of the job for its delivery until the departure of next available vehicle (it may wait processed or unprocessed), and the other is the inventory holding cost of a job while it waits for the processing of the other jobs. Note that some of these waiting times may overlap. Theorem 3.2 and its proof build on a consideration of the second cause for waiting of any job. Herein, we will also take into account the waiting of jobs for their pickup until a vehicle becomes available. Notice that, this is easier to do in case of one vehicle, because in this case, we know that the time between the drop-off and pick-up of a job, if $\tau > p_j > l$, is at least τ. The remaining part of the proof relies on this observation and accounts for the two reasons of waiting.

If $\tau > p_j > l$ for some job j, the job has to wait for the return of the vehicle as long as at least $\tau - p_j$ time units. Ignoring other jobs at the facility momentarily, if $h_1 < h_2$, the inventory holding cost due to the waiting of this job for the return of the vehicle can be minimized if the job is held unprocessed during its waiting time. That is, the machine is kept idle for $\tau - p_j$ time units, during which the job contributes to the total inventory holding costs in an amount of at least $h_1(\tau - p_j)$. If $h_2 < h_1$, the job's contribution to the total inventory holding costs is decreased if it is held processed. This, in turn, leads to an inventory holding cost of at least $h_2(\tau - p_j)$. Thus, the inventory holding cost incurred by this job due to the first reason is at least $(\tau - p_j)min(h_1, h_2)$, and this is valid for all jobs for which $\tau > p_j > l$.

Note that summing up $(\tau - p_j)min(h_1, h_2)$ for all jobs, we already include the waiting time of a job either in its unprocessed or processed state. Recall that Theorem 3.2 proposes $\sum_{i=1}^{|N|} \left\lfloor \frac{i-1}{\omega} \right\rfloor p_{(i)}(h_1 + h_2)$ as a lower bound on inventory holding costs due to the waiting of the jobs for one another. The cost of waiting due to the vehicle unavailability is incorporated in the above calculations by considering a job's state at which the inventory holding cost rate is minimum. Therefore, the waiting of jobs in their minimum cost state is already penalized. To that, we add the term $\left\lfloor \frac{i-1}{\omega} \right\rfloor p_{(i)}|h_1 - h_2|$ for each job to account for the incremental cost of waiting of jobs for one another, which has not been incorporated in the $(\tau - p_j)min(h_1, h_2)$ term. ■

Based on Theorem 3.3, the following corollary provides a lower bound on the objective function value of an optimal solution when $m = 1$.

Corollary 3.2 *In case of a single vehicle, a lower bound on the total cost of an optimal solution is given by*

$$LB^2 = \min_{\left\lceil \frac{|N|}{\min(k_1,k_2)} \right\rceil \leq \omega \leq 2|N|} \{max(LB'_{\mathcal{I}}(\omega), LB''_{\mathcal{I}}(\omega)) + c\omega\}.$$

3.2.2 A Special Case: Restricted Outbound Transportation Policy

For the problem of interest, a mathematical model is presented in Section 3.1. Even in small-sized instances, this model has very long solution times (e.g., in the order of a week for 10 jobs). Due to the proposed lower bounds and some characteristics of the optimal solutions, computational times decrease significantly. However, they are still too long to be considered as practical. Upon the analysis of the optimal solutions for some small sized instances (i.e., up to 10 jobs), we have detected a property which reveals itself commonly. It involves a certain relation between inbound and outbound transportation sequences. In the rest of this section, we restrict our analysis to the set of solutions which exhibit this property. The heuristic approach that will be presented in Section 3.4 also utilizes this property. We next present it as an assumption.

Assumption 3.1 *A job arriving with the t^{th} tour either departs with the same tour (i.e., tour t) or the next tour (i.e., tour $t+1$).*

The set of solutions restricted to the above assumption does not always include an optimal one. However, numerical evidence shows that the cost of an optimal solution under this policy is close to that of a global optimum in practical cases. Moreover, if the number of vehicles is one or the waiting limit is zero, the set of solutions that have the above property would include an optimal solution. Furthermore, combining Proposition 3.1 and Proposition 3.2, one can conclude that there exists an optimal solution under this assumption with the following characteristics: The sequence of jobs in the production schedule can be grouped into blocks such that the first block consists of the jobs that both arrive and depart with the first tour, the second block consists of the jobs that arrive with the first tour and depart with the second tour, and so on. We refer to this characteristic of a sequence as a *block structure*. In Figure 3.1, an illustration of a sequence displaying this structure is presented. The arrows pointing inwards the figure coincide with inbound transportation times and the arrows pointing outwards coincide with the outbound transportation times.

In the next two propositions, we present some characteristics of an optimal solution exhibiting the block structure under Assumption 1.

Proposition 3.5 *For a setting where $h_1 \geq h_2$, consider an optimal solution under Assumption 1 which exhibits the block structure. In this solution, if two*

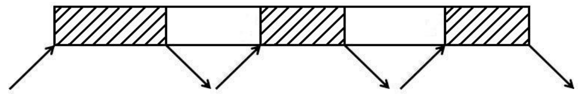

Figure 3.1: Block structure of a solution.

jobs arrive at the facility together but depart from the facility with different tours, then the processing time of the job which departs later must be greater than that of the other.

Proof: Consider an optimal solution S under Assumption 1 which exhibits the block structure. Assume, in contradiction to the proposition, that there exist two jobs u and v that arrive at the facility together, u departs earlier than v, and $p_u > p_v$. Figure 3.2 is an illustration of such a solution. A, B, C and D in the figure refer to sets of jobs with certain common characteristics. More specifically, A and B are groups of jobs that arrive at the facility with job u at time t_0 and leave the facility with job u at time t_1. Jobs in A are processed before job u and jobs in B are processed after job u. Jobs in C and D also arrive at the facility with job u, however, they leave the facility with job v and at time t_2. In mathematical terms,

$$\begin{aligned} \alpha_i &= t_0 \quad \forall i \in A \cup B \cup C \cup D \cup \{u, v\}, \\ \delta_i &= t_1 \quad \forall i \in A \cup B \cup \{u\}, \\ \delta_i &= t_2 \quad \forall i \in C \cup D \cup \{v\}. \end{aligned}$$

Note that, any of the sets A, B, C and D may be empty.

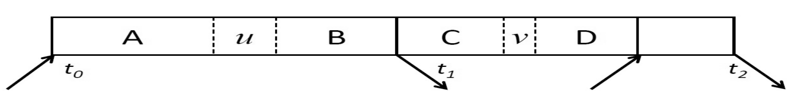

Figure 3.2: An illustration of a solution in contradiction to Proposition 3.5.

Now, consider a new solution S' that is formed by interchanging the positions of jobs u and v in the production sequence and their assignments to vehicles in the outbound transportation. Figure 3.3 is an illustration of such a solution.

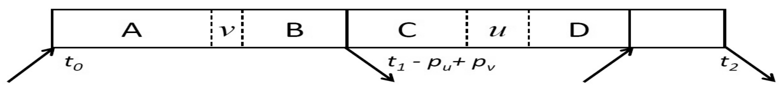

Figure 3.3: An illustration of the updated solution S'.

Denoting σ_i as the starting time of processing of job i in solution S, in the new schedule S' we have

$$\begin{aligned}
\alpha_i' &= t_0 && \forall i \in A \cup B \cup C \cup D \cup \{u, v\}, \\
\sigma_i' &= \sigma_i && \forall i \in A \cup D, \\
\sigma_i' &= \sigma_i - p_u + p_v && \forall i \in B \cup C, \\
\delta_i' &= t_1 - p_u + p_v && \forall i \in A \cup B \cup \{v\}, \\
\delta_i' &= t_2 && \forall i \in C \cup D, \\
\delta_u' &= t_2, \quad \sigma_u' = \sigma_v - p_u + p_v, && \sigma_v' = \sigma_u.
\end{aligned}$$

As the number of tours in S' remains the same as the one in S, the total costs of the two solutions differ only in their inventory holding cost component, and the difference is

$$TC(S) - TC(S') = \sum_{i \in A \cup B \cup C \cup D \cup \{u,v\}} (\sigma_i - t_0)h_1 + (\delta_i - \sigma_i - p_i)h_2 - (\sigma_i' - t_0)h_1 - (\delta_i' - \sigma_i' - p_i)h_2,$$

which reduces to

$$TC(S) - TC(S') = \sum_{i \in \in A \cup B \cup C \cup D \cup \{u,v\}} (\sigma_i - \sigma_i')h_1 + \{(\delta_i - \delta_i') + (\sigma_i' - \sigma_i)\}h_2.$$

When the values of δ_i, σ_i', δ_i' are plugged in the above expression for each group of jobs, it can be rewritten as

$$
\begin{aligned}
TC(S)- \quad & TC(S') = \\
& \sum_{i\in A}(\sigma_i-\sigma_i)h_1+\{t_1-(t_1-p_u+p_v)+(\sigma_i-\sigma_i)\}h_2 \\
& +\sum_{i\in B}(\sigma_i-(\sigma_i-p_u+p_v))h_1 \\
& +\sum_{i\in B}\{(t_1-(t_1-p_u+p_v))+((\sigma_i-p_u+p_v)-\sigma_i)\}h_2 \\
& +\sum_{i\in C}(\sigma_i-(\sigma_i-p_u+p_v))h_1+\{(t_2-t_2)+((\sigma_i-p_u+p_v)-\sigma_i)\}h_2 \\
& +\sum_{i\in D}(\sigma_i-\sigma_i)h_1+\{(t_2-t_2)+(\sigma_i-\sigma_i)\}h_2 \\
& +(\sigma_u-(\sigma_v-p_u+p_v))h_1+\{(t_1-t_2)+((\sigma_v-p_u+p_v)-\sigma_u)\}h_2 \\
& +(\sigma_v-\sigma_u)h_1+\{(t_2-(t_1-p_u+p_v))+(\sigma_u-\sigma_v)\}h_2.
\end{aligned}
$$

After some cancelations and rearrangement of terms, the above expression reduces to

$$
\begin{aligned}
& TC(S)-TC(S') = \\
& \quad \sum_{i\in A}(p_u-p_v)h_2+\sum_{i\in B}(p_u-p_v)h_1+\sum_{i\in C}(p_u-p_v)(h_1-h_2)+(p_u-p_v)h_1,
\end{aligned}
$$

which is equivalent to

$$
TC(S)-TC(S') \quad = \quad (p_u-p_v)\left(|A|\,h_2+|B|\,h_1+|C|\,(h_1-h_2)+h_1\right).
$$

Note that under the $h_1 \geq h_2$ condition of this proposition, we assume $h_1 > 0$ because, otherwise we would have $h_1 = h_2 = 0$, which would be trivial. Combining with $p_u > p_v$, we conclude that $TC(S) - TC(S') > 0$. This contradicts with the optimality of S. ■

Proposition 3.6 *For a setting where $h_2 \geq h_1$, consider an optimal solution under Assumption 1 which exhibits the block structure. In this solution, if two jobs depart from the facility together but arrive at the facility with different tours, then the processing time of the job which arrives earlier must be greater than that of the other.*

Proof: Similar to that of Proposition 3.5. ■

3.3 Polynomial Algorithms for Special Cases

In this section we identify two polynomially solvable versions of the problem. In the first one, we assume that the production schedule of the facility is given and we provide an exact algorithm to find inbound and outbound transportation schedules for a single vehicle. If the number of vehicles (m) is greater than 1, the algorithm can be modified to find a good feasible solution by dividing τ by m. In the second version, we have the number of tours and the production sequence known, an exact algorithm is developed under Assumption 1 for the cases where the number of vehicles is one or each vehicle makes at most one tour.

3.3.1 Exact Solution when Production Schedule is Known

Production plan of a facility does not solely depend on transportation or inventory decisions. In order to optimize some other performance measures, the production schedule may be predetermined. Moreover, the integrated problem is $\mathcal{NP} - Hard$ in the strong sense as proven in Theorem 3.1. For this type of complex problems, practitioners usually use a hierarchical approach and solve the subproblems sequentially. In such a setting, production scheduling decision is made first in the hierarchy. Then, the transportation scheduling decision is made according to the production schedule. In this section we develop a dynamic programming formulation that can be used to solve the transportation problem in polynomial time for a single vehicle. For the sake of clarity of the exposition, first a pseudo-polynomial version is introduced, then it is proven than the algorithm may in fact run in polynomial time after some modifications.

3.3.1.1 Pseudo-polynomial Algorithm

By Propositions 3.1 and 3.2 we know that there is an optimal solution in which the processing and transportation sequences are the same. We relabel jobs according to the processing sequence such that the first job in the sequence is labeled as $job1$, and so on. We assume that all temporal data is integer. The time spent by a vehicle for the transportation from the first warehouse to production facility is denoted by τ_1, and let σ_i and ϕ_i be the start and completion times of job i, respectively, at the given schedule.

Algorithm 3.1 *Define:*

$C(t,i,j,1)$: *Minimum cost accumulated by time t if the first i jobs have arrived to the facility and the first j jobs ($j \le i$) have departed from the facility and the vehicle is at the facility.*

$C(t,i,j,0)$: *Minimum cost accumulated by time t if the first i jobs have arrived to the facility and the first j jobs ($j \le i$) have departed from the facility and the vehicle is at the warehouse.*

$h(t_1,t_2,i,j)$ = *Inventory holding cost incurred between times t_1 and t_2 if the first i jobs have arrived to the facility and first j jobs ($j \le i$) have departed from the facility.*

Specifically, we have

$$h(t_1,t_2,i,j) = \sum_{w=j+1}^{i} [h_2(t_2 - \max(\phi_w,t_1))^+ + h_1(\min(\sigma_w,t_2) - t_1)^+]$$

where $X^+ := \max(0,X)$.

The recursion is as follows:

$$C(t,i,j,1) = \min_{0\le\lambda\le l}\{\min_{0\le k\le k_1} \left\{ \begin{array}{l} C(t-\tau_1-\lambda,i-k,j,0)+ \\ h(t-\tau_1-\lambda,t-\lambda,i-k,j)+ \\ h(t-\lambda,t,i,j) \end{array} \right\}\}$$

$$C(t,i,j,0) = \min \left\{ \begin{array}{l} \min_{0\le\lambda\le t}\{C(t-\lambda,i,j,0)+h(t-\lambda,t,i,j)\} \\ \min_{0\le k\le k_2} \left\{ \begin{array}{l} C(t-(\tau-\tau_1),i,j-k,1)+ \\ C+h(t-(\tau-\tau_1),t,i,j) \end{array} \right\} \end{array} \right.$$

Note that the calculation of function h takes $O(|N|)$ time. The optimal objective function value is $C(T,|N|,|N|,0)$ where T is the makespan of the schedule. This value can be found by the above recursions and initial conditions. An optimal solution to the problem can be found by standard backtracking techniques.

If the vehicle is at the facility, assume that the vehicle is waiting at the facility for λ time units ($0 \leq \lambda \leq l$). The vehicle must have departed from the facility at time $t - \tau_1 - \lambda$ and arrived at the facility at time $t - \lambda$. If the vehicle is carrying any jobs, then the last k ($0 \leq k \leq k_1$) jobs must have been arrived at the facility with this tour. The vehicle was at the facility at time $t - \tau_1 - \lambda$ with jobs $(1, \ldots, i - k)$ arrived at the facility and j jobs departed from the facility. At that time the cost was $C(t - \tau_1 - \lambda, i - k, j, 0)$. Two kinds of inventory costs accumulates during $(t - \tau_1 - \lambda, t)$. The first one is while the vehicle is on the way to the facility. This is between $t - \tau_1 - \lambda$ and $t - \lambda$ and in this time interval, jobs $j+1, j+2, \ldots, i-k$ incur inventory holding cost of $h(t-\tau_1-\lambda, t-\lambda, i-k, j)$. The second inventory cost is incurred between the times $t - \lambda$ and t. Jobs $j + 1, \ldots, i$ incur inventory holding cost of $(t - \lambda, t, i, j)$ units.

If the vehicle is at the origin, there are two possibilities. Either the vehicle is at the origin waiting for some time (λ), or has just arrived. If the vehicle is waiting for the last λ time units, the cost is $C(t - \lambda, i, j, 0)$ plus the accumulated inventory cost during this interval which is $h(t - \lambda, t, i, j)$. If the vehicle has just arrived at the warehouse, it must have been departed from the facility at time $t - (\tau - \tau_1)$ and it carries the last k ($0 \leq k \leq k_2$) jobs from the facility to the warehouse. The cost corresponding to this case is $C(t - (\tau - \tau_1), i, j - k, 1)$. The jobs $j - k + 1, j - k + 2, \ldots, j$ are carried with this last tour. As these jobs depart from the facility at time $t - (\tau - \tau_1)$ the inventory cost incurred during this time is $h(t - (\tau - \tau_1), t, i, j)$. As the tour has just completed, single tour cost C is added to the total cost.

Without loss of generality we assume that the schedule starts at time 0, and the first tour departs from the first warehouse at time $-\tau_1$.

$$
\begin{aligned}
&C(-\tau_1, 0, 0, 0) &&= 0 && && \\
&C(t, i-1, j, 0) &&= C(t, i-1, j, 1) &&= \infty && \forall t, i, j : t > \sigma_i \\
&C(t, i, j, 0) &&= C(t, i, j, 1) &&= \infty && \forall t, i, j : t < \phi_j \\
&C(t, i, j, 0) &&= C(t, i, j, 1) &&= \infty && \forall t, i, j : i < j \\
&C(t, i, j, 0) &&= C(t, i, j, 1) &&= \infty && \forall t, i, j : -\tau_1 \neq t < 0
\end{aligned}
$$

Note that calculation of $C(t, i, j, 0/1)$ requires $O(T|N|^2)$ operations, and is calculated for $O(T|N|^2)$ times, resulting in a time complexity of $O(T^2|N|^4)$. Thus, this algorithm is unary-polynomial.

3.3.1.2 Polynomial Algorithm

This algorithm is similar to the previous one except, $C(t, i, j, 0/1)$ values are calculated only at a polynomial number of time points.

Recall that, we only consider solutions in which, every tour t departs from the production facility either as soon as it arrives or at the latest completion time of processing among those of all the jobs that depart from the production facility with tour t. Similarly, we assume that in feasible solutions, every tour t arrives at the production facility either at the earliest start time of processing among those of all the jobs that arrive to the production facility with tour t or it coincides with the departure time.

Hence, we only need to calculate $C(t, i, j, 0/1)$ values at possible arrival and departure times, one of which should correspond to the starting or completion time of some job. One can, therefore, form the set of possible arrival and departure times of the tours by offsetting the starting and/or completion times of the jobs in the production schedule by integer multiples (including negative ones) of tour time τ. Note that there are $O(|N|)$ starting or completion times in the production schedule which forms $O(|N|)$ intervals on the real line. At each interval, there can be $O(|N|)$ possible arrivals or departures. For example, consider the interval during which job j receives its processing. In that interval, there can be at most $j-1$ departures (jobs $1, 2, ..., j-1$) and at most $|N|-j$ arrivals (jobs $j+1, ..., |N|$). Thus, there are $O(|N|)$ intervals and $O(|N|)$ possible arrival and departure times within each interval leading to a total of $O(|N|^2)$ possible arrival and departures. Consequently, we have $O(|N|)$ offsets within each interval, resulting in $O(|N|^3)$ offsets (i.e. possible arrival or departure times). With this modification, evaluating $C(t, i, j, 0/1)$ at $O(|N|^3)$ instead of $O(T)$ points is enough. Therefore, replacing $O(T)$ with $O(|N|^3)$, the time complexity of this algorithm is $O(|N|^{10})$. Note that for this algorithm there is no need for temporal data to be integer.

Thus, if the production schedule is known in advance and there is a single vehicle, an optimal transportation schedule can be found in $O(|N|^{10})$ time which is polynomial in the size of the problem.

3.3.2 Exact Solution when Production Sequence and Number of Tours is Known under Assumption 1

In this section, we develop a polynomial time algorithm when the production sequence (rather than schedule) is known and the number of tours made is predetermined. The algorithm works either for the case with a single vehicle or the number of vehicles is not less then the number of tours (i.e, each vehicle makes at most one tour). The algorithms are first derived for the single vehicle cases, then, a modification of the algorithms is proposed for the case where each vehicle makes at most one tour. Throughout this section, without loss of generality, we assume that the sequence of jobs is $(1, 2, \ldots, n)$ where $n = |N|$.

3.3.2.1 Number of tours is 2

The sequence can be divided into three blocks as shown in Figure 3.4. The first block arrives at and departs from the facility with the first tour. The second block arrives with the first tour but departs with the second tour. The jobs in the last block arrives at and departs from the facility with the second tour. Solving the problem is equivalent to deciding jobs i and j (i.e. the last jobs of the first and second blocks). We define a partial cost function for each block of jobs in Figure 3.4 (C_1 for the first block, C_2 for the second block and C_3 for the last block). Define partial cost functions and the feasibility set as follows.

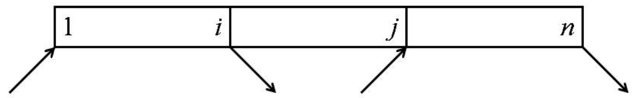

Figure 3.4: Solution with 2 tours

Algorithm 3.2

$$\begin{aligned}
C_1(i) &= \sum_{u=1}^{i}\left\{\sum_{v=1}^{u-1} h_1 p_v + \sum_{v=u+1}^{i} h_2 p_v\right\} \\
C_2(i,j) &= \sum_{u=i+1}^{j}\left\{\sum_{v=1}^{u-1} h_1 p_v + \sum_{v=u+1}^{n} h_2 p_v + (\tau - \sum_{v=i+1}^{j} p_v)^+ \min(h_1, h_2)\right\} \\
C_3(j) &= \sum_{u=j+1}^{n}\left\{\sum_{v=j+1}^{u-1} h_1 p_v + \sum_{v=u+1}^{n} h_2 p_v\right\} \\
\mathcal{X} &:= \left\{ (i,j) : \begin{array}{l} 1 \le i \le j \le n, \\ \sum_{v=1}^{i} p_v \le l, \sum_{v=j+1}^{n} p_v \le l, \\ j \le k_1, n-j \le k_1, \\ i \le k_2, n-i \le k_2 \end{array} \right\}
\end{aligned}$$

The optimal objective function value with 2 tours is

$$z^* = \min_{(i,j)\in\mathcal{X}} \{C_1(i) + C_2(i,j) + C_3(j) + 2c\}$$

Note that the complexity of the algorithm is $O(n^4)$ as calculation of C_1, C_2 and C_3 takes $O(n^2)$ operations and the minimization is taken over $O(n^2)$ values. If the number of vehicles is greater than 1, the problem can still be solved optimally with the modification of $C_2(i,j)$ as follows:

$$C_2(i,j) = \sum_{u=i+1}^{j}\left\{\sum_{v=1}^{u-1} h_1 p_v + \sum_{v=u+1}^{n} h_2 p_v\right\}$$

3.3.2.2 Number of tours is 3

The sequence can be divided into five parts as shown in Figure 3.5. The first part arrives at and departs from the facility with the first tour. The second part arrives with the first tour but departs with the second tour, and so on. In other words, (odd, even) consecutive pairs arrive at the facility and (even, odd) consecutive pairs depart from the facility together. Similar to the previous case define partial cost functions and feasibility set as follows.

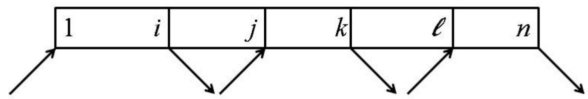

Figure 3.5: Solution with 3 tours

Algorithm 3.3

$$
\begin{aligned}
C_1(i) &= \sum_{u=1}^{i}\left\{\sum_{v=1}^{u-1} h_1 p_v + \sum_{v=u+1}^{i} h_2 p_v\right\}\\
C_2(i,j,k) &= \sum_{u=i+1}^{j}\left\{\sum_{v=1}^{u-1} h_1 p_v + \sum_{v=u+1}^{k} h_2 p_v + (\tau - \sum_{v=i+1}^{j} p_v)^+ \min(h_1,h_2)\right\}\\
C_3(j,k) &= \sum_{u=j+1}^{k}\left\{\sum_{v=j+1}^{u-1} h_1 p_v + \sum_{v=u+1}^{k} h_2 p_v\right\}\\
C_4(j,k,\ell) &= \sum_{u=k+1}^{\ell}\left\{\sum_{v=j+1}^{u-1} h_1 p_v + \sum_{v=u+1}^{n} h_2 p_v + (\tau - \sum_{v=k+1}^{\ell} p_v)^+ \min(h_1,h_2)\right\}\\
C_5(\ell) &= \sum_{u=\ell+1}^{n}\left\{\sum_{v=\ell+1}^{u-1} h_1 p_v + \sum_{v=u+1}^{n} h_2 p_v\right\}\\
\mathcal{X} &:= \left\{(i,j,k,\ell) : \begin{array}{l} 1 \le i \le j \le k \le \ell \le n, \\ \sum_{v=1}^{i} p_v \le l, \sum_{v=j+1}^{k} p_v \le l, \sum_{v=\ell+1}^{n} p_v \le l, \\ j \le k_1, \ell - j \le k_1, n-\ell \le k_1, \\ i \le k_2, k-i \le k_2, n-k \le k_2 \end{array}\right\}
\end{aligned}
$$

The optimal objective function value when number of tours is 3 is

$$z^* = \min_{(i,j,k,\ell)\in\mathcal{X}} \{C_1(i) + C_2(i,j,k) + C_3(j,k) + C_4(j,k,\ell) + C_5(\ell) + 3c\}$$

Note that the calculation of z^* can be done in $O(n^6)$ operations. This is the case for single vehicle. If the number of vehicles is greater than 3, the problem can still be solved optimally with a proper modification of $C_2(i, j, k)$ and $C_4(j, k, \ell)$ similar to the case where the number of tours is 2.

In general, if the number of tours is ω, one can formulate the partial cost functions and feasibility set by dividing the production sequence into $2\omega-1$ parts. The first and the last partial costs are functions of a single variable. The odd and even numbered partial costs are functions of two and three variables, respectively. Each partial cost can be evaluated in $O(n^2)$ time. The minimization is done on $O(n^{2\omega-2})$ total cost values, each having ω summations, leading to a $O(\omega n^{2\omega})$ time complexity. Given the number of tours, the problem can be solved in polynomial time.

3.4 Heuristic Procedure

In this section, we present a heuristic based on Assumption 3.1, Proposition 3.3 and Proposition 3.4. Recall that, due to Proposition 3.1 and Proposition 3.2, there exists a solution with the block structure which is optimal under Assumption 3.1. The underlying idea behind the proposed heuristic is to find this solution, which obviously is restricted to the set of policies satisfying Assumption 3.1–and hence not necessarily optimal for the original problem. Furthermore, the procedure for finding an optimal solution that exhibits the block structure is based on beam search. Therefore, the output of the proposed procedure constitutes a heuristic solution for this problem as well.

The heuristic evolves over a search tree with the following characteristics: At level 0 of the search tree, there is a single node with no information, that is the root node. We first branch on the number of tours ω. Note that w may range from $\left\lceil \frac{|N|}{\min(k_1,k_2)} \right\rceil$ to $2|N|$. Figure 3.6 illustrates part of the search tree for a sample problem with $\left\lceil \frac{|N|}{\min(k_1,k_2)} \right\rceil = 1$. Conditioning on the value of w, Theorem 3.2 implies that $LB'_{\mathcal{I}}(\omega) + c\omega$ is a lower bound on total costs when $m > 1$. Similarly, Theorem 3.3 suggests that $\max\{LB'_{\mathcal{I}}(\omega), LB''_{\mathcal{I}}(\omega)\} + c\omega$ is a lower bound on the total costs when $m = 1$. In subsequent parts of the search tree, we branch on different blocks for a given w value, and at each level, we consider the assignment of a job to one of the blocks. Figure 3.6 shows how further branching is performed at the second level conditioning on $w = 4$. Note that, in this case, there are seven blocks, each block referring to a different pair of assignments of a job to a tour for its inbound and outbound transportation. For example, when a job is assigned to block 2-3, it is implied that the job arrives at the facility with the second tour and leaves the facility with the third tour. In general, if there are w tours, then there are $2w - 1$ number of different blocks that a job can be assigned to.

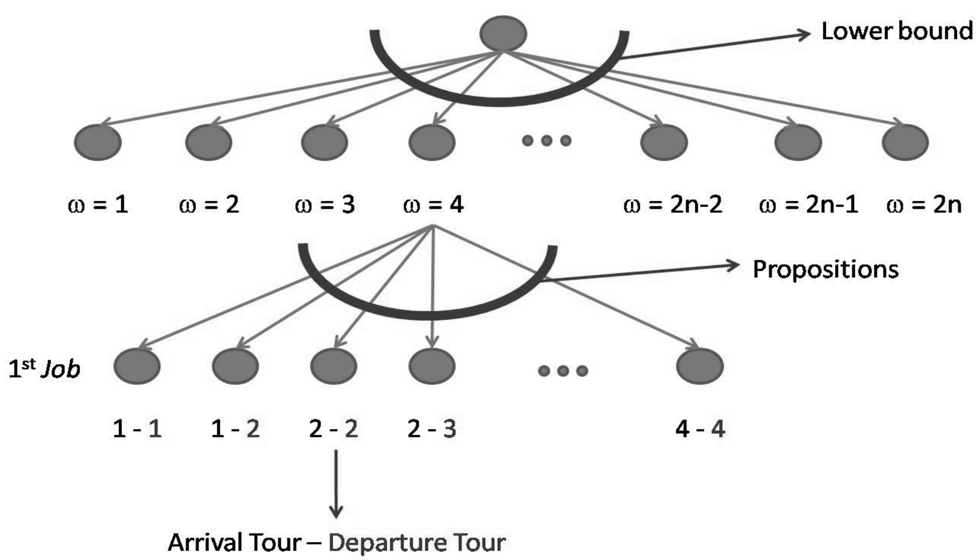

Figure 3.6: An illustration of the search tree.

We refer to the tree structure that emanates from a node at the first level a *subtree*. Notice that, there are at most $2|N|$ number of subtrees in a search tree. Our search for the best solution over the search tree gives full consideration to all the subtrees in order of increasing w. However, only a certain number of nodes are kept for further consideration at each level of a subtree. Therefore, our search for the best solution conditioning of a value of w, unfolds in accordance with the beam search approach. The number of nodes that are explored further at each level of the subtree is a parameter of this approach, and is referred to as the *beam width*.

Since the subtrees corresponding to different values of w are explored sequentially, a feasible solution may be obtained from the search of each subtree. The total costs associated with such feasible solutions set upper bounds on the minimum cost. Therefore, if a lower bound at any node in upcoming steps of the search exceeds the smallest upper bound, then this node is pruned. The nodes at the first level of each subtree (i.e., the second level of the main search tree) store partial solutions incorporating the possible assignments of the job with the longest processing time to a block. In general, at level i ($i = 1, \cdots, |N|$) of a subtree, an assignment of the i^{th} longest job to a block is made. We would like to note that in assigning jobs to blocks, two issues are taken into account. First, the vehicle capacity constraints should not be exceeded. Secondly, the waiting time of a vehicle at the facility should be less than or equal to the limit l. When a new assignment is made to a block, the sequence of the jobs in that block are updated using Proposition 3.3 and Proposition 3.4, and if the new sequence improves the lower bound, it is revised based on the underlying approach of Theorem 3.2 and its proof.

The search for a solution conditioning on a w value, evolves using the following approach recursively at each level of the corresponding subtree: All the children nodes are created and their corresponding lower bounds are updated based on the partial solutions they carry. The children nodes with lower bounds greater than or equal to the objective value of the best known solution are eliminated. Remaining partial solutions in the promising nodes are then rapidly completed to a full solution. The completion algorithm is simply scheduling the next job to the position where the lower bound is minimum. The value of the global evaluation function for each child is the objective function value of the completed solution. The children nodes are then sorted according to the global evaluation function values. If a completed solution has a better objective value than the best known solution, the smallest upper bound is updated. When all the nodes at the current level are examined, the most promising beam-width number of them are chosen for further exploration. At this point, since more than one child node originating from the same parent node can be kept for further consideration, the proposed method constitutes a dependent beam search.

After all jobs are assigned to blocks, the assignments are converted to a schedule in terms of the arrival and departure times of vehicles, and start and completion times of processing. As an example of such an assignment and how it is converted to a schedule, consider the illustrative representation in Figure 3.7. There are 5 jobs with the following processing times: $p_1 = 1$, $p_2 = 2$, $p_3 = 3$, $p_4 = 4$ and $p_5 = 5$. The jobs are assigned to 3 blocks, which implies that the number of tours is 2. Jobs 4 and 1 arrive at and depart from the facility with the same tour. Job 5 reaches to the facility with the same tour as of jobs 4 and 1, but it leaves the facility with the second tour. Jobs 3 and 2 arrive at and depart from the facility with the second tour. Figure 3.7 also shows the sequence of processing among the jobs that are in the same block. That is, job 4 is processed before job 1, and job 3 is processed before job 2. Proposition 3.3 hints that in this example $h_1 < h_2$.

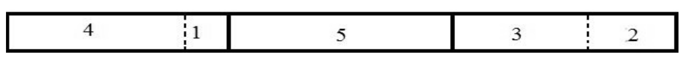

Figure 3.7: An illustration of block assignments to jobs.

Let us first assume that there are 2 vehicles (i.e., $m = 2$), tour time is 5 units (i.e., $\tau = 5$), and waiting time limit is 5 (i.e., $l = 5$). First, the tours are assigned to vehicles. Vehicle 1 makes the odd numbered tours (1, 3, 5, ...) and vehicle 2 makes the even numbered tours (2, 4, 6, ...). Since there are only two tours, each vehicle makes a single tour. Jobs 4, 1 and 5 arrive at time 0 with vehicle 1, and the vehicle waits at the facility until the processing of jobs 4 and 1 finishes. At time 5, vehicle 1 departs from the facility with jobs 4 and 1. Job 5 is then processed until time 10. Vehicle 2 arrives at the facility with jobs 3 and 2 at time 10, and the processing of job 3 starts immediately. Job 2 follows job 3 starting at time 13 and jobs 5, 3 and 2 depart from the facility with vehicle 2 at time 15.

For the same assignment illustrated in Figure 3.7, now assume that only the number of vehicles and the tour time attain different values, those are $m = 1$ and $\tau = 10$. In this case, job 5 waits an extra 5 time units for the return of the vehicle and there is an inserted idleness in the production schedule in front of job 5. Since $h_1 < h_2$, idleness is inserted before job 5, otherwise the job has to wait for 5 time units after its processing is completed.

We close this section by noting that beam search is an approach that has been successfully used to solve various complex scheduling problems. We cite Erenay et al. [10], Sabuncuglu and Karabuk [20] as examples of beam-search applications in the scheduling area.

3.5 Computational Experiments

In this section, we discuss the design and the results of our numerical analysis. The objectives of this analysis are: i) to test how the lower bounds, Proposition 3.1 and Proposition 3.2 affect the running time of the optimization model presented in Section 3.1, ii) to assess the tightness of the lower bounds, iii) to evaluate the quality of the proposed heuristic.

All the computational experiments have been carried out on a 2.6 GHz 2xAMD Opteron 252 Server running Centos version 2.6.9 with 2 GBs of physical memory. GAMS version 22.6 has been used to solve the mixed integer programming formulation of the problem.

3.5.1 The Effects of the Lower Bounds and the Propositions on the Computational Time

The integer programming models provided in Section 3.1 can only be used to solve small size problems. This is due to the large number of alternative feasible solutions and the slow progress of the LP relaxations through the branch and bound tree. The number of alternative solutions can be decreased utilizing Proposition 3.1 and Proposition 3.2. Similarly the progress through the branch and bound tree can be improved based on the lower bounds provided in Corollary

3.1 and Corollary 3.2. Our objective in this section is to test the effects of the results provided in these propositions and corollaries on the computational time of the integer programming formulation, under different problem parameters.

With the above objective in mind, 720 instances are generated based on 72 experimental settings and 10 instances for each setting. These experimental settings are given by the different combinations of the parameter values summarized in Table 3.1. The number of jobs in all the instances is taken as 5. The processing times of the jobs are sampled from a discrete uniform distribution $U[5, 25]$. As seen in Table 3.1, we have $h_1 \leq h_2$ in all the (h_1, h_2) pairs under consideration. It is important to note that an optimal solution to a problem where $h_1 > h_2$ can be obtained by first exchanging the values of h_1 and h_2, and the values of k_1 and k_2; and secondly, reversing the schedule of an optimal solution for this new problem. Therefore, $(1, 0)$ and $(2, 1)$ are not considered among the different levels of (h_1, h_2).

Table 3.1: Parameter Settings

Parameter	Levels
l	0, 30, 250
(k_1, k_2)	(3,3), (3,6), (6,3), (6,6)
(h_1, h_2)	(0,1), (1,2), (1,1)
m	1, 3
τ	15
c	25

In order to see the effects of the lower bounds and the properties stated in the propositions, all instances are solved using the following four models:

Model I: Linearized version of the integer programming formulation presented in Section 3.1.

Model II: Linearized version of the integer programming formulation with the incorporation of Proposition 3.1 and Proposition 3.2.

Model III: Linearized version of the integer programming formulation with the incorporation of the lower bounds.

Model IV: Linearized version of the integer programming formulation with the incorporation of Proposition 3.1, Proposition 3.2 and the lower bounds.

In Model II and Model IV, the following constraints are added to the formulation to incorporate Proposition 3.1 and Proposition 3.2:

$$\alpha_i \leq \alpha_j + (1 - s_{ij})M \qquad \forall i, j \in N,$$
$$\delta_i \leq \delta_j + (1 - s_{ij})M \qquad \forall i, j \in N.$$

In Model III and Model IV, to employ the lower bounding scheme, the following set of constraints are included in the formulation for all $t = 1, \ldots, 2|N|$;

$$h_1 \sum_{i \in N} (\sigma_i - \alpha_i) + h_2 \sum_{i \in N} (\delta_i - (\sigma_i + p_i)) \geq (\psi_t - \psi_{t+1}) * LB(t)$$

where $LB(t) = max(LB'(t), LB''(t))$. The left hand side of the inequality is the total inventory holding cost. If the number of tours is t, then $(\psi_t - \psi_{t+1}) = 1$ and the total inventory holding cost is bounded from below by $LB(t)$.

Table 3.2 presents the average solution times over 10 instances for each experimental setting. The rows and the columns of the table correspond to different settings of m, k_1, k_2 and h_1, h_2, l, respectively. There are four values in each cell. The first value is the average time spent in CPU seconds to solve Model I, the second value is the average time to solve Model II, and so on. As can be seen in the table, in general, the second and third values are smaller than the first one. This indicates that both the properties stated in the propositions and the lower bounds save from the computational time when they are considered one at a time. Also, the decrease in the computational time is much more significant due to the usage of lower bounds, and hence, lower bounds are more effective than the properties stated in the propositions.

Another observation is that, in settings where $h_1 = 0$, solving the problem is easier (see columns 3, 6 and 9). When rows 1-4 are compared to rows 5-8, it can further be concluded that, the lower bounds and the properties become more effective in reducing the computational time as the number of vehicles increase. If vehicles are not allowed to wait at the facility, and inventory holding costs for both

Table 3.2: Comparison of the computational times of the four models (CPU seconds)

		1	2	3	4	5	6	7	8	9
		h_1 1	h_1 1	h_1 0	h_1 1	h_1 1	h_1 0	h_1 1	h_1 1	h_1 0
		h_2 1	h_2 2	h_2 2	h_2 1	h_2 2	h_2 2	h_2 1	h_2 2	h_2 2
		l 250	l 250	l 250	l 30	l 30	l 30	l 0	l 0	l 0
1		24.06	23.18	4.50	16.08	11.94	3.21	18.05	12.10	3.38
	m 1	22.89	24.49	6.26	11.88	12.36	3.26	15.49	10.67	5.45
	k_1 3	5.82	8.03	0.27	3.93	5.77	0.24	12.85	5.77	0.34
	k_2 3	5.95	10.19	0.30	3.25	5.22	0.25	12.42	6.07	0.30
2		24.86	23.15	3.61	11.65	10.55	3.06	15.47	10.26	10.33
	m 1	22.83	20.87	5.38	11.90	13.43	2.98	10.99	10.18	7.39
	k_1 3	4.65	7.19	0.29	3.68	4.24	0.30	13.90	6.05	0.33
	k_2 6	6.28	7.18	0.38	3.97	4.97	0.38	12.25	5.53	0.25
3		24.58	22.33	2.60	11.45	10.69	1.87	17.14	12.14	2.37
	m 1	22.47	20.01	3.35	10.30	9.20	2.22	14.14	9.68	2.76
	k_1 6	6.56	6.96	0.21	3.77	4.51	0.20	11.71	6.81	0.19
	k_2 3	6.78	8.13	0.32	3.67	4.23	0.24	11.40	5.97	0.16
4		27.61	29.02	2.92	13.52	15.82	1.89	16.74	9.51	2.71
	m 1	29.12	38.66	3.21	10.41	11.09	2.19	11.82	8.69	3.70
	k_1 6	5.80	6.20	0.22	3.94	4.39	0.22	11.91	5.88	0.20
	k_2 6	5.53	8.54	0.27	3.50	4.36	0.21	12.31	6.56	0.21
5		75.71	101.63	7.66	19.28	35.82	4.44	32.96	17.53	7.20
	m 3	75.22	96.37	8.65	20.88	31.46	4.98	25.66	17.20	8.28
	k_1 3	1.47	0.45	0.20	2.27	0.54	0.27	22.61	9.61	0.35
	k_2 3	1.12	0.82	0.32	2.79	1.16	0.44	24.42	8.30	0.50
6		71.61	114.92	7.23	22.65	35.69	4.78	32.37	21.08	12.34
	m 3	65.04	103.45	8.05	18.68	29.33	4.63	27.75	21.95	8.16
	k_1 3	1.36	0.40	0.19	1.85	0.70	0.25	23.07	7.27	0.30
	k_2 6	0.98	0.68	0.28	2.04	0.88	0.33	19.44	9.08	0.35
7		74.99	100.57	9.62	20.28	35.34	4.15	30.25	19.36	4.71
	m 3	83.11	95.77	8.18	19.47	36.27	4.07	26.40	15.74	4.62
	k_1 6	1.74	0.34	0.20	1.54	0.71	0.21	20.88	8.86	0.19
	k_2 3	1.35	0.77	0.28	2.16	0.88	0.31	20.34	7.44	0.28
8		96.78	141.43	5.58	20.33	23.71	3.46	27.30	19.24	3.62
	m 3	95.01	159.14	8.37	20.64	32.95	3.75	25.30	18.01	3.94
	k_1 6	1.08	0.64	0.23	2.10	0.54	0.23	20.89	8.54	0.23
	k_2 6	0.96	0.50	0.30	2.41	0.70	0.23	21.63	8.86	0.20

raw materials and finished goods are the same, the effects of the lower bounds and the properties diminish (see column 7). In some cases, the computational time that Model IV requires is more than that of Model III. This may be due to the possibility that, in small problems, the additional computational burden of processing both types of constraints simultaneously does not justify their benefits.

Our findings in this section imply that, the lower bounds and the properties stated in Proposition 3.1 and Proposition 3.2, are quite effective in decreasing the running time of the model presented in Section 3.1.

3.5.2 Quality of the Lower Bound

In this section, we discuss our findings on the assessment of the quality of the proposed lower bound. The experimental setting is based on extending the one described in the previous section to consider additional levels for the number of jobs. Namely, 3600 instances with number of jobs equal to 5, 10, 15, 20 or 25, are solved. The lower bound is compared to the objective function value of the best solution for each instance. The best solution is obtained by solving each instance using Model IV and an extended version of the proposed heuristic. The latter is simply the heuristic procedure applied with a large value of the beam width so that all nodes at each level are examined.

Both Model IV and the extended heuristic are limited to run for 10 minutes. Among all the feasible solutions obtained for an instance, the one with the minimum cost is chosen as the best solution. Table 3.3 presents a summary of the results. The values of m, k_1, k_2 are changed over the rows and the values of h_1, h_2, l are changed over the columns. In each cell, three statistics are reported based on the 50 instances, which include jobs of all sizes with varying processing times. The first statistic corresponds to the number of optimally solved problems. It can be observed that, the number of such instances is greater than or equal to 10 in each cell, because the 10 instances with 5 jobs are always optimally solved. The second value is the average gap between the objective function value and the lower bound over all problems for which an optimal solution is obtained. As a

Table 3.3: Summary of the Analysis for Measuring the Quality of the Lower Bound

	h_1 1 h_2 1 l 250	h_1 1 h_2 2 l 250	h_1 0 h_2 2 l 250	h_1 1 h_2 1 l 30	h_1 1 h_2 2 l 30	h_1 0 h_2 2 l 30	h_1 1 h_2 1 l 0	h_1 1 h_2 2 l 0	h_1 0 h_2 2 l 0
m 1	10	10	50	10	10	50	10	10	50
k_1 3	3.75%	3.25%	0.00%	3.75%	3.25%	0.00%	7.61%	5.07%	0.00%
k_2 3	7.85%	2.77%	0.00%	7.26%	2.77%	0.00%	9.23%	10.47%	0.00%
m 1	10	10	50	10	10	50	10	10	50
k_1 3	3.75%	3.25%	0.00%	3.75%	3.25%	0.00%	7.61%	5.07%	0.00%
k_2 6	7.94%	2.77%	0.00%	7.18%	2.77%	0.00%	9.20%	10.51%	0.00%
m 1	10	10	50	10	10	50	10	10	50
k_1 6	3.75%	3.25%	0.00%	3.75%	3.25%	0.00%	7.61%	5.07%	0.00%
k_2 3	7.88%	2.77%	0.00%	7.38%	2.77%	0.00%	9.25%	10.40%	0.00%
m 1	10	10	50	10	10	50	10	10	49
k_1 6	3.75%	3.25%	0.00%	3.75%	3.25%	0.00%	7.61%	5.07%	0.00%
k_2 6	7.93%	2.77%	0.00%	7.71%	2.77%	0.00%	9.25%	10.44%	0.14%
m 3	12	13	50	10	12	50	10	10	50
k_1 3	0.09%	0.00%	0.00%	0.57%	0.00%	0.00%	5.91%	2.68%	0.00%
k_2 3	2.08%	0.51%	0.00%	2.17%	0.57%	0.00%	7.71%	2.37%	0.00%
m 3	12	13	50	12	14	50	10	10	50
k_1 3	0.09%	0.00%	0.00%	0.47%	0.00%	0.00%	5.91%	2.68%	0.00%
k_2 6	2.08%	0.52%	0.00%	2.16%	0.56%	0.00%	7.71%	2.37%	0.00%
m 3	13	16	50	10	11	50	10	10	50
k_1 6	0.08%	0.00%	0.00%	0.57%	0.00%	0.00%	5.91%	2.68%	0.00%
k_2 3	2.02%	0.47%	0.00%	2.18%	0.52%	0.00%	7.71%	2.37%	0.00%
m 3	11	12	50	12	12	50	10	10	49
k_1 6	0.09%	0.00%	0.00%	0.47%	0.00%	0.00%	5.91%	2.68%	0.00%
k_2 6	2.08%	0.57%	0.00%	2.17%	0.53%	0.00%	7.71%	2.37%	0.06%

final statistic, we report the average gap between the lower bound and the best known solution over all the 50 instances.

Since the average gap between the lower bound and the best known solution over 50 instances in any cell is at most 10.51%, we conclude that the proposed lower bound is generally tight. When raw material inventory holding cost h_1 is zero, the lower bound is equal to the optimal objective function value for all the instances (see columns 3, 6 and 9). This is due to the way that the lower bound is constructed. Recall from Section 3.2.1 that, the lower bounds are computed based on minimization of costs assuming momentarily that either $h_1 = 0$ or

$h_2 = 0$. Therefore, the lower bounds are strictly tight in these cases. As another observation from Table 3.3, we note that the average gaps in columns 1, 4 and 7 are greater than those in columns 2, 5 and 8, respectively, which in turn, are greater than the ones in columns 3, 6 and 9. This implies the quality of the proposed lower bound decreases as h_1 approaches to h_2. Finally, observe that the average gaps in columns 1 and 2 are less than the ones in columns 4 and 5, in the same order. This implies that the lower bound gets tighter as the waiting limit l increases. When waiting is not allowed (i.e., $l = 0$), lower bounds are looser.

In the next section, we continue our numerical analysis with the objective of evaluating the quality of the heuristic. Since our analysis in the current section sets an evidence for the quality of the lower bound, the objective function value of the heuristic solution will be compared to the lower bound.

3.5.3 Quality of the Heuristic

In this section, the quality of the proposed heuristic is assessed with the help of lower bounds and over an extensive set of problems. Specifically, one more level is added for the number of jobs (i.e., 50), tour cost c (i.e., 150) and tour time τ (i.e., 100). Thus, a total of 1728 different experimental settings are considered. 10 random instances are generated for each experimental setting. As in Section 3.5.1 and Section 3.5.2, processing times of the jobs are sampled from a discrete uniform distribution $U[5, 25]$. The complete experimental design consists of 17280 problem instances.

In order to decide the beam width parameter, pilot runs are taken on sample instances of all sizes. The objective function values of the heuristic solutions and CPU times spent for several beam width values, are recorded. The solution time increases almost linearly as the beam width increases. However, the objective function value does not change for beam width values greater than eight. Furthermore, the marginal contribution of increasing the beam width beyond a value of five, does not justify the increase in the computational time. Thus, it is decided to fix the beam width at a value of five in the remaining part of the analysis.

We first start with analyzing the effect of inventory holding costs on the performance of the heuristic. Figure 3.8 shows the average percentage difference between the heuristic solution and the lower bound. The instances with 50 jobs are referred to as the large-size problems whereas the remaining instances, with 5, 10, 15, 20 or 25 jobs, are classified as small-size problems. The average solution times for the beam search algorithm for small-size and large-size problems are 0.69 and 139 CPU seconds, respectively. The maximum solution times are 25.5 and 1506 CPU seconds for small-size and large-size problems, respectively. It can be observed from Figure 3.8 that the heuristic performs slightly better for large-size problems. Furthermore, as the difference between the values of h_1 and h_2 increases, the quality of the heuristic improves. This is because the heuristic procedure makes use of the lower bound, and the quality of the lower bound itself is better at these values of h_1 and h_2.

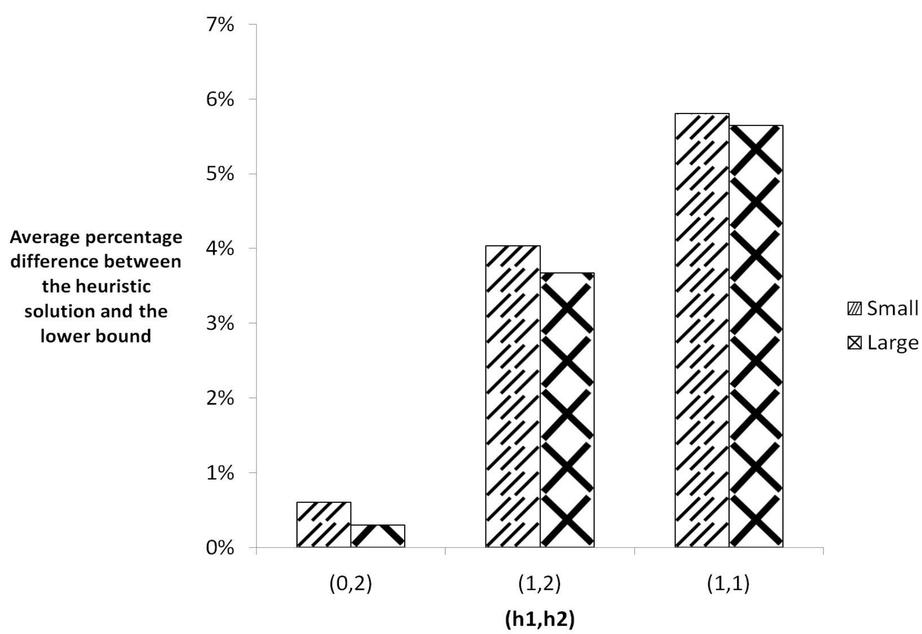

Figure 3.8: Effect of inventory holding costs on the heuristic performance for different problem sizes.

The effect of the number of vehicles depends on tour time τ. Figure 3.9 shows how the quality of the heuristic changes with respect to inventory holding costs at different combinations of m and τ. The general behavior observed in Figure 3.8 does not change. However, if τ is large and there is a single vehicle, the effect of close values of h_1 and h_2 on the gap between the heuristic and the lower bound is amplified. This is due to the fact that the computation of lower bounds does not take the tour time into account. If m is small and τ is large, the availability of a vehicle for timely pickup and delivery decreases, which increases the waiting times of the jobs at the facility. Since the lower bounds are not constructed to address the waiting time due to the unavailability of vehicles, their quality is not as good as it is in the other cases. This also results in a decrease in the performance of the heuristic, which explains a higher gap at $(h_1, h_2) = (1, 1)$ in Figure 3.9.

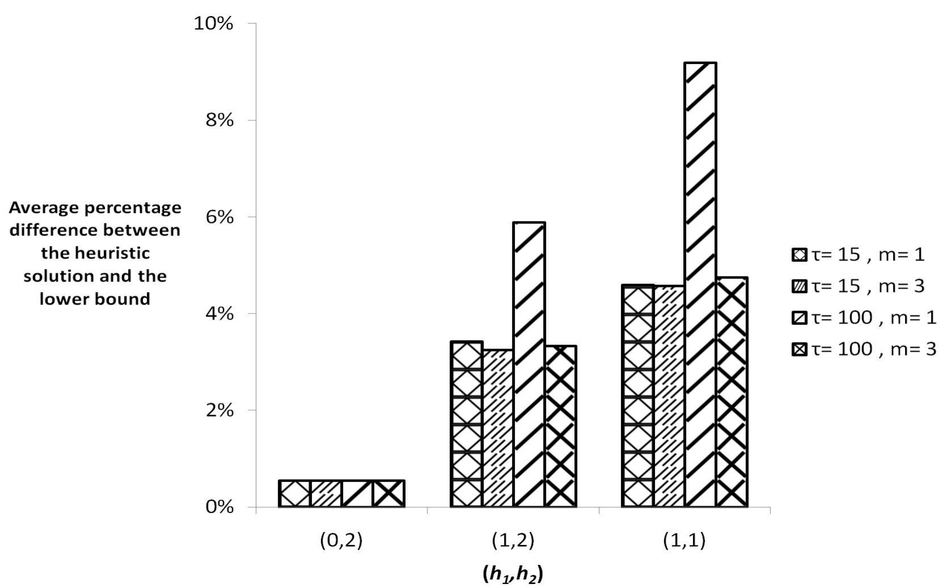

Figure 3.9: Effect of inventory holding costs on the heuristic performance for varying τ and m values.

The effect of the vehicle capacities on the performance of the heuristic is demonstrated in Figure 3.10. When tour cost c is low, the capacities have no effect because the vehicles are not fully utilized. With high tour costs, the vehicles are fully utilized to decrease the total number of tours. In this case, vehicle capacities become more constraining and they have an elevated effect on the heuristic performance. We observe that for the cases where $k_1 \neq k_2$, the heuristic performs better.

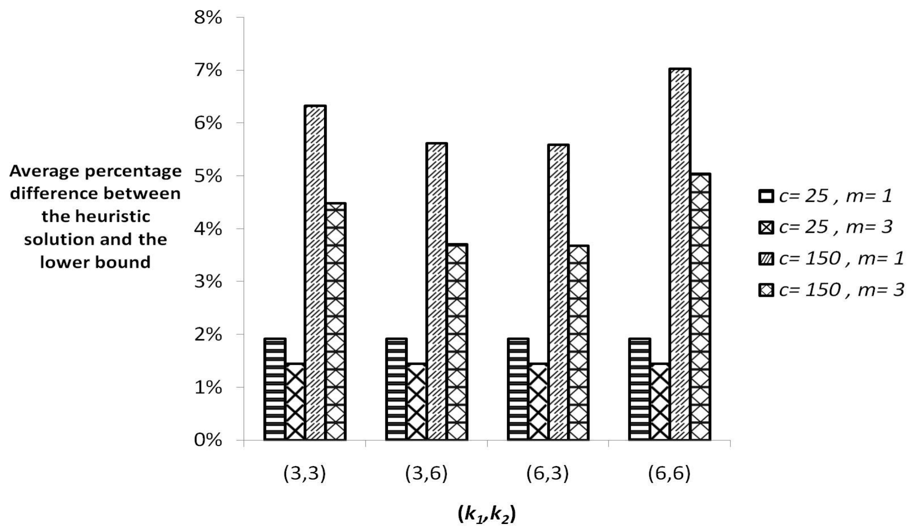

Figure 3.10: Effect of vehicle capacities on the heuristic performance for varying c and m values.

The analysis in this section shows that the overall performance of the proposed heuristic is quite promising. However, its performance changes depending on the parameters of the problem. One of the factors that affect its performance is how the inventory holding cost rate of unprocessed jobs (i.e., h_1) compare to that of the processed jobs (i.e., h_2). Specifically, as the difference between the two increases, the quality of the heuristic improves. The performance of the heuristic also depends on the tour time and the number of vehicles. High values of tour time combined with small number of vehicles leads to lower performance of the heuristic. Similarly, high values of tour cost combined with low vehicle capacities results in lower performance of the heuristic, especially when the vehicle capacities in the inbound and outbound are close to one another.

Chapter 4

Production-Delivery Problem

Multi Period Production Planning and Outbound Transportation: Utilization of Inbound Vehicles

In this chapter, we study a manufacturers production planning and outbound transportation problem. The manufacturer in this setting has to schedule a certain number orders over a single machine. Production and delivery of orders to the customers has to be completed before deadlines. Deliveries can be made using a combination of two types of vehicles, differing in availability and cost. The first type of vehicle (*type I*) is available in unlimited number, but expensive. The availability of *type II* vehicles, on the other hand, is limited and changes over time. The manufacturer decides the composition of vehicles to be used for each delivery after a production plan is made and given the availability of type II vehicles. The manufacturer can utilize type II vehicles when they become available or hold them at the facility for future deliveries. When the manufacturer resorts to the latter option, an additional fee is paid in proportion to the holding time of a vehicle. The problem studied in this chapter is to give production and outbound transportation decisions so that two types of vehicles used for outbound transportation activities, while minimizing inventory and transportation costs.

In the detailed analysis of the problem, we identify three main operating policies that affect the structure of the problem: (i) Consolidation, (ii) splitting, and

(iii) size of the deliveries. The descriptions of the policies are followed by examples of practice of the appliance manufacturer which the problem is motivated.

Consolidation arises when multiple orders are delivered with the same vehicle to save transportation costs. In practice, this corresponds to the setting where a single facility receives multiple customer orders in relatively small amounts. For example, the order size of a typical home appliance retailer may not be enough to occupy a full vehicle capacity. In this case, total demand of customers in the same region are combined (consolidated) and shipped using a single vehicle. In some cases, however, customers may not desire their orders to be delivered together with other orders. This is referred to as *NoConsolidation* case. International customers of the appliance manufacturer which the problem is motivated, generally do not want their orders to be consolidated with others. For domestic retailers on the other hand, the manufacturing company consolidates the orders of the retailers in the same region. Thus, both settings exist in practice.

The relationship between the customers and manufacturers is getting stronger and demands are often defined by long term contracts to be delivered within a time range. In such cases, customers accept partial deliveries of their orders at different time periods. This allows the decision makers to *Split* the orders and deliver throughout the planning horizon. If splitting is not allowed, all products that belong to the same order must be delivered in a single shipment. This is denoted by *NoSplit* in this dissertation. For the same appliance manufacturer, domestic customers accept partial deliveries, whereas some of the international customers require their orders to be delivered in a single shipment.

A significant portion of the non-bulk cargo worldwide transferred by containers. In order to utilize the containers at full capacity, companies try to enforce the orders to be integer multiples of container capacity due to economies of scale. In some industry applications, it may even be infeasible to deliver less than a full truck capacity. The aforementioned appliance manufacturer uses containers for international deliveries and if the containers are not fully utilized (i.e., there are empty spaces), the products do not support each other and they may break or corrupt due to concussion. If all deliveries are to be in full truck loads (i.e., all vehicles are utilized at full capacity), then the size of all orders are integer multiples of the vehicle capacity, we call this special case as $FTL - Delivery$ (Full-Truck-Load Delivery). The unit of delivery as well as demand can be considered as "vehicle capacity" in this case.

All possible combinations of these three operating policies lead to six different problem settings. For general delivery structure, there are four possible cases ($Consolidate - Split$, $NoConsolidate - Split$, $Consolidate - NoSplit$, and $NoConsolidate - NoSplit$). If delivery sizes for all orders are integer multiples of the vehicle capacity, there is no need to consolidate multiple orders into the same vehicle. Therefore, consolidation factor is not relevant and two cases arise in the presence of $FTL-Delivery$ structure ($Split$ and $NoSplit$). Each problem setting (simply called problem) is studied in this chapter, considering both general and linear cost structures (Table 4.1).

For each problem, we either provide a pseudo-polynomial algorithm for general costs case or prove that no such algorithm exists even for linear cost structures. All these theoretical developments are discussed in the following sections.

Table 4.1: Classification of Problems

	$General\ Delivery$		$FTL - Delivery$
	$Consolidate$	$NoConsolidate$	
$Split$	Problem 1	Problem 2	Problem 5
$NoSplit$	Problem 3	Problem 4	Problem 6

The rest of the chapter is organized as follows: notation and a generic formulation that represents a collection of models for all problem settings is presented in Section 4.1. General optimality conditions that are valid for all problems are developed in Section 4.2. The problems with general delivery structure are explored in Section 4.3. Section 4.4 lays down theoretical developments for the problems with a special delivery structure in which all deliveries and order sizes are integer multiples of the vehicle capacity. Computational experiments are discussed in Section 4.5. In Section 4.6, we provide a brief analysis of the problem variant where delivery of orders required to take place within a time window.

4.1 Notation and Generic Model Formulation

In this section, notation and a generic model which can be used to formulate the six problems listed in Table 4.1, is presented. For this problem, we propose a finite horizon lot-sizing model considering T periods and a set N of orders. Without loss of generality, we assume that $N = \{1, \ldots, n\}$. Production capacity of the facility is P_t for period t ($t = 1, \ldots, T$). This capacity is defined in terms of the number of products produced per period independent of the type of products.

The facility produces according to make to order policy. Each order i ($i \in N$), has a deadline D_i and a size S_i. Orders are delivered to the customers at the expense of the manufacturer. The manufacturer uses capacitated vehicles for outbound transportation. Each vehicle holds upto K units of the finished products. There are two types of vehicles (type I and type II). Any number of type I vehicles can be utilized at a cost of \$$C_{1,t}(x)$ per x vehicles in period t. However, in period t, a limited number (i.e., A_t) of type II vehicles is also available at a lower cost (i.e., $C_{2,t}(x)$). It is assumed that A_t number of type II vehicles arrive at the facility at period t ($t = 1, \ldots, T$).

We assume two conditions on transportation cost functions: (i) $0 < C_{2,t}(x) < C_{1,t}(x)$ and (ii) $C_{1,t}(n-x)+C_{2,t}(x) > C_{1,t}(n-x-1)+C_{2,t}(x+1)$ for all $t = 1, \ldots, T$, $x < n$, and $x, n \in \mathcal{Z}^+ \cup \{0\}$. The first condition clearly states that utilizing x type II vehicles costs less than utilizing x type I vehicles for every period. The second condition states that for any combination of type I and type II vehicles, keeping the total number of vehicles the same, utilizing more type II vehicles is always less costly.

Utilization of a type II vehicle for an outbound delivery is possible only if the delivery is ready (or about to be ready) upon availability. Since a type II vehicle may need to be held at the facility for outbound transportation, a waiting cost $W_t(w_t)$ is incurred for period t if w_t number of vehicles are held from period t to

$t+1$. Note that, holding type I vehicles is not cost justified as the the cost is realized in the delivery period. The inventory holding cost for finished goods is $H_t(I_t)$ for period t when the inventory level at the end of period t is I_t. $W_t(w_t)$ and $H_t(I_t)$ are assumed to be increasing functions. There is no inventory holding cost for raw materials due to the fact that the raw materials are common for all product types and they have considerably lower value than end products.

Below is a list of our main assumptions, the parameters and the decision variables.

Assumptions

- Unit inventory holding cost is the same for all jobs
- Production capacity is independent of the type of orders
- Order acceptance/rejection decisions have been already made and there exists a feasible solution
- Cost functions satisfy the following inequalities
  - $0 < C_{2,t}(x) < C_{1,t}(x)$ for all $t = 1, \ldots, T$, and $x \in \mathcal{Z}^+$
  - $C_{1,t}(n-x) + C_{2,t}(x) > C_{1,t}(n-x-1) + C_{2,t}(x+1)$ for all $t = 1, \ldots, T$, $x < n$, and $x, n \in \mathcal{Z}^+ \cup \{0\}$
  - $C_{1,t}(x), C_{2,t}(x), W_t(x)$, and $H_x(x)$ are increasing in x for all $t = 1, \ldots, T$, and $x \in \mathcal{Z}^+$

Parameters

N	:	Set of orders	
T	:	Number of periods	
K	:	Capacity of a vehicle	
$H_t(I)$	:	Cost of holding I units of inventory at period t	$t = 1, \ldots, T$
$C_{1,t}(x)$	:	Cost of hiring x type I vehicles for transportation at period t	$t = 1, \ldots, T$
$C_{2,t}(x)$	:	Cost of using x type II vehicles for transportation at period t	$t = 1, \ldots, T$
$W_t(x)$	:	Cost of holding x type II vehicles from period t to $t+1$	$t = 1, \ldots, T$
A_t	:	Number of type II vehicles become available in period t	$t = 1, \ldots, T$
P_t	:	Production capacity of the facility for period t	$t = 1, \ldots, T$
S_i	:	Size of order i (number of items to produce)	$\forall i \in N$
D_i	:	Deadline to deliver all items for order i	$\forall i \in N$

Decision Variables

π_t	:	Total production amount in period t	$t = 1, \ldots, T$
$\pi_{t,i}$	:	Number of items of order i produced in period t	$\forall i \in N, t = 1, \ldots, T$
$I_{t,i}$	:	Inventory level for items of order i at the end of period t	$\forall i \in N, t = 1, \ldots, T$
I_t	:	Total inventory at the end of period t	$t = 1, \ldots, T$
x_t	:	Number of type II vehicles utilized in period t	$t = 1, \ldots, T$
w_t	:	Number of type II carried from period t to period $t+1$	$t = 1, \ldots, T$
$\sigma_{t,i}$	:	Number of items of order i, delivered in period t	$\forall i \in N, t = 1, \ldots, T$
$\tilde{\sigma}_{t,i}$	:	$\begin{cases} 1, & \text{if order } i \text{ is delivered in period } t \\ 0, & \text{otherwise} \end{cases}$	$\forall i \in N, t = 1, \ldots, T$
θ_t	:	Number of vehicles used for outbound transportation in period t	$t = 1, \ldots, T$
$\theta_{t,i}$	:	Number of vehicles used for outbound transportation in period t for order i	$\forall i \in N, t = 1, \ldots, T$

We use the variable θ_t for the problems where consolidation is allowed. If consolidation is not allowed, we use variable $\theta_{t,i}$ to denote the number of vehicles used for each order i in period t. In the $NoSplit$ case, we use the variable $\tilde{\sigma}_{t,i}$ to represent if the delivery of an order i takes place in period t or not.

Using this notation, a generic model formulation is given below. The model consists of an objective function (Equation (4.1)) and nineteen constraint sets grouped in four categories. The objective function and the first group of constraint sets (Equations (4.2) - (4.6)) are valid for all problem settings. For that reason, these constraint sets are not labeled in the formulation. Notice that other constraint sets in the proposed formulation are labeled by S, nS, C, and nC, corresponding to $Split(S), NoSplit(nS), Consolidate(C)$ and $NoConsolidate(nC)$ cases, respectively. For example, $nC - S$ in Equation (4.13) means that the inequality is valid for the models where Consolidation is $n$ot allowed but Splitting is allowed. In the $FTL - Delivery$ case, inequality (4.13) needs to be converted to equality and some of the variables and constraints can be eliminated during the solution procedure (e.g., $\sigma_{t,i}$ can be replaced by $K\theta_{t,i}$).

For $Split$ case with $FTL - Delivery$, constraint sets (4.13) and (4.14) are used whereas for $NoSplit$ case (4.15) and (4.16) are used.

Model 1: Generic Formulation

Minimize

$$\sum_{t=1}^{T}\{C_{1,t}(\theta_t - x_t) + C_{2,t}(x_t) + W_t(w_t)\} + \sum_{t=1}^{T} H_t(I_t) \tag{4.1}$$

Subject to

$$x_t + w_t \leq A_t + w_{t-1} \qquad t = 1,\ldots,T \tag{4.2}$$

$$\sum_{i\in N} \pi_{t,i} = \pi_t \qquad t = 1,\ldots,T \tag{4.3}$$

$$\sum_{i\in N} I_{t,i} = I_t \qquad t = 1,\ldots,T \tag{4.4}$$

$$x_t \leq \theta_t \qquad t = 1,\ldots,T \tag{4.5}$$

$$\pi_t \leq P_t \qquad t = 1,\ldots,T \tag{4.6}$$

$$I_{t,i} = I_{t-1,i} + \pi_{t,i} - \sigma_{t,i} \qquad t = 1,\ldots,T, \forall i \in N \qquad (S) \tag{4.7}$$

$$\sum_{t=1}^{D_i} \sigma_{t,i} = S_i \qquad \forall i \in N \qquad (S) \tag{4.8}$$

or

$$I_{t,i} = I_{t-1,i} + \pi_{t,i} - \tilde{\sigma}_{t,i} S_i \qquad t = 1,\ldots,T, \forall i \in N \qquad (nS) \tag{4.9}$$

$$\sum_{t=1}^{D_i} \tilde{\sigma}_{t,i} = 1 \qquad \forall i \in N \qquad (nS) \tag{4.10}$$

$$\sum_{i\in N} \sigma_{t,i} \leq \theta_t K \qquad t = 1,\ldots,T \qquad (C-S) \tag{4.11}$$

or

$$\sum_{i\in N} \tilde{\sigma}_{t,i} S_i \leq \theta_t K \qquad t = 1,\ldots,T \qquad (C-nS) \tag{4.12}$$

or

$$\sigma_{t,i} \leq \theta_{t,i} K \qquad t = 1,\ldots,T, \forall i \in N \qquad (nC-S) \tag{4.13}$$

$$\sum_{i\in N} \theta_{t,i} = \theta_t \qquad t = 1,\ldots,T \qquad (nC-S) \tag{4.14}$$

or

$$\tilde{\sigma}_{t,i} \lceil S_i/K \rceil = \theta_{t,i} \qquad t = 1,\ldots,T, \forall i \in N \qquad (nC-nS) \tag{4.15}$$

$$\sum_{i\in N} \theta_{t,i} = \theta_t \qquad t = 1,\ldots,T \qquad (nC-nS) \tag{4.16}$$

$$w_0 = I_{0,i} = 0 \qquad \forall i \in N \tag{4.17}$$

$$\tilde{\sigma}_{t,i} \in \{0,1\} \qquad t = 1,\ldots,T, \forall i \in N \tag{4.18}$$

$$I_{t,i}, \sigma_{t,i}, \pi_{t,i}, \theta_{t,i} \in \mathcal{Z}^+ \cup \{0\} \qquad t = 1,\ldots,T, \forall i \in N \tag{4.19}$$

$$I_t, w_t, x_t, \theta_t \in \mathcal{Z}^+ \cup \{0\} \qquad t = 1,\ldots,T \tag{4.20}$$

The objective function is simply the sum of transportation, vehicle holding and inventory holding costs. Constraint set (4.2), we call it vehicle balance constraints, ensures that the number of type II vehicles that can be utilized or held for the next period is less than the number of type II vehicles available at that period plus the number of vehicles carried from the previous period. Equations (4.3) and (4.4) define total production and inventory quantities. Constraint set (4.5) enforces the number of vehicles used for outbound transportation to be larger than the number inbound vehicles utilized in that period. Production capacities are modeled with the constraint set (4.6). Inventory balance is maintained by either equation set (4.7) or (4.9) depending on whether splitting is allowed or not. Similarly, deadlines are enforced by either constraint set (4.8) or (4.10). Again, the vehicle capacities are modeled by using one of the following constraint sets: (4.11), (4.12), (4.13) or (4.15). The constraint sets (4.14) and (4.16) are used to establish the link between the number of vehicles for individual orders and the total number of vehicles. Finally, equation sets (4.17) - (4.20) are included to set initial conditions and provide nonnegativity and integrality constraints.

4.2 Optimality Properties

Although different problems are formulated using different constraint sets, a number of constraints are common for all problems. In this section, we provide a series of theorems and lemmas building on one another that apply to all the problems with a general cost structure. These theorems and lemmas also give important and useful insights about the structure of optimal solutions. Moreover, the results of this section can be used to identify polynomially solvable cases and improve the quality of the heuristic solutions.

In all these models, we allow decision maker to hold only the type II vehicles from one period to another. Each type II vehicle is either used or held again for the next period. If the vehicle balance constraint is not binding for some period t, then there is no need to carry that much of type II vehicles from the previous period. The following theorem states that, if the number of vehicles held from the previous period is positive ($w_{t-1} > 0$), then the vehicle balance constraint is binding for period t, in every optimal solution. In other words, at least one of the following constraints is binding for all periods: $x_t + w_t \le A_t + w_{t-1}$ or $w_{t-1} \ge 0$.

Theorem 4.1 *$\{A_t + w_{t-1} - (x_t + w_t)\}w_{t-1} = 0$ for $t = 2, 3, \ldots, T$, in every optimal solution.*

Proof: Proof is by contradiction. We know by Equation (4.2) that $A_t + w_{t-1} - (x_t + w_t) \ge 0$ and by Equation (4.20) that $w_{t-1} \ge 0$ for $t = 2, 3, \ldots, T$. Thus, $\{A_t + w_{t-1} - (x_t + w_t)\}w_{t-1} \ge 0$. Assume, to the contrary of the hypothesis, that there is an optimal solution S and a period t with $\{A_t + w_{t-1} - (x_t + w_t)\}w_{t-1} > 0$. This is possible only if $\{A_t + w_{t-1} - (x_t + w_t)\} > 0$ and $w_{t-1} > 0$. Now consider another solution S' with everything being same except $w'_{t-1} = w_{t-1} - 1$. Clearly, $w'_{t-1} \ge 0$ and $\{A_t + w'_{t-1} - (x_t + w_t)\} \ge 0$. S' is feasible and the objective function value of S' is smaller then that of S by an amount of $W_{t-1}(w_{t-1}) - W_{t-1}(w_{t-1} - 1)$ as w_{t-1} is reduced by one. Thus, S is not an optimal solution as $W_t(x)$ is an increasing function. ■

Note that if $w_t > 0$ for some period t, by Theorem 4.1, the vehicle balance constraint is binding for the next period. The vehicles that are held from the previous period will eventually be utilized in the future periods. The following lemma states that the number of type II vehicles utilized is positive at the period in which no more vehicles are carried to the next period.

Lemma 4.1 *If $w_t > 0$, then $\exists \tau > t : w_\tau = 0, x_\tau > 0, w_{t'} > 0$ for $t \le t' < \tau$, in every optimal solution.*

Proof: Proof is by construction. Let $w_t > 0$, by Theorem 4.1, as $w_t > 0$, we know that $A_{t+1} + w_t = x_{t+1} + w_{t+1}$. Let

$$\tau = \min_k \{k : w_k = 0, k > t\}$$

Note that, in an optimal solution $w_T = 0$, thus, such τ exists. For $t \le t' < \tau$, $w_{t'} > 0$. Applying Theorem 4.1 to period $\tau - 1$,

$$x_\tau + w_\tau = A_\tau + w_{\tau-1}$$

We know $w_\tau = 0$ and $w_{\tau-1} > 0$ by definition of τ. Hence, $x_\tau > 0$. ■

If a number of vehicles held form one period to the next, the held vehicles are eventually be utilized by Lemma 4.1. Note that, within those periods, Theorem 4.1 applies and vehicle balance constraints are binding. Hence, the next corollary follows by applying Theorem 4.1 for all periods with $w_t > 0$ in Lemma 4.1.

Corollary 4.1 *If* $w_t > 0$, *then* $\exists \tau > t : \sum_{k=t+1}^{\tau} x_k = \sum_{k=t+1}^{\tau} A_k + w_t$.

If the vehicle balance constraint is not binding for some period t, some of the type II vehicles are neither utilized nor held for the next period. In this case, no type I vehicles must be hired. Thence, as the following theorem states, at least one of the constraints is binding: $x_t + w_t \leq A_t + w_{t-1}$ or $\theta_t \geq x_t$.

Theorem 4.2 $\{A_t + w_{t-1} - (x_t + w_t)\}(\theta_t - x_t) = 0$ *for* $t = 1, \ldots, T$, *in every optimal solution.*

Proof: Proof is by contradiction. Let S be an optimal solution and t be a period such that, $(\theta_t - x_t)\{A_t + w_{t-1} - (x_t + w_t)\} > 0$. Consider another solution S' such that $x'_t = x_t + 1$ and everything else being the same. Note that $(\theta_t - x_t) > 0$ and $A_t + w_{t-1} > x_t + w_t$. Then, $(\theta_t - x'_t) \geq 0$ and $A_t + w_{t-1} \geq x'_t + w_t$. Note that, in solution S', the number of type I vehicles is smaller and S' has an objective function value smaller than that of S by an amount equal to $C_{1,t}(\theta_t - x_t) + C_{2,t}(x_t) - C_{1,t}(\theta_t - x_t - 1) + C_{2,t}(x_t + 1) > 0$. Thus, S is not an optimal solution. ■

In the lot-sizing models, the well-known property for the uncapacitated versions is that at each period either there is a positive inventory or a positive production, but not both (Wagner and Whitin [26]). In this study, however, the production amount of the facility is bounded by a capacity. The next theorem states a similar property for the capacitated version: for each period, either there is inventory carried from the previous period or the facility does not produce at full capacity, but not both.

Theorem 4.3 $(P_t - \pi_t)I_{t-1} = 0$ *for* $t = 2, 3, \ldots, T$*, in every optimal solution.*

Proof: Proof is by contradiction. Assume that there exists an optimal solution S and a period t such that in S, $(P_t - \pi_t)I_{t-1} \neq 0$. We know by Equations (4.6) and (4.20) that, $P_t \geq \pi_t$ and $I_{t-1} \geq 0$. Thus, $P_t > \pi_t$ and $I_{t-1} > 0$. Then, $\exists i \in N : I_{t-1,i} = \{\sum_{k=1}^{t-1} \pi_{k,i} - \sum_{k=1}^{t-1} \sigma_{k,i}\} > 0$. Let $\tau = \max\{k : \pi_{k,i} > 0, k < t\}$, we know that such τ exists as $\sum_{k=1}^{t-1} \pi_{k,i} > 0$. Note that, $\sum_{k=\tau+1}^{t-1} \pi_{k,i} = 0$, by selection of τ. $\sum_{k=1}^{t-1} \pi_{k,i} = \sum_{k=\tau+1}^{t-1} \pi_{k,i} + \sum_{k=1}^{\tau} \pi_{k,i} > \sum_{k=1}^{t-1} \sigma_{k,i}$ and $\sum_{k=1}^{\tau} \pi_{k,i} > \sum_{k=1}^{t-1} \sigma_{k,i}$ which implies that $I_{t',i} > 0$ and $I_{t'} > 0, \forall t' = \tau, \tau+1, \ldots, t-1$. Now, consider another solution S' such that

$$\begin{aligned}
\pi'_{t,i} &= \pi_{t,i} + 1 \\
\pi'_{\tau,i} &= \pi_{\tau,i} - 1 \\
I'_{t',i} &= I_{t',i} - 1 \quad \forall t' = \tau, \tau+1, \ldots, t-1 \\
I'_{t'} &= I_{t'} - 1 \quad \forall t' = \tau, \tau+1, \ldots, t-1
\end{aligned}$$

Observe that, $\pi'_t \leq P_t$ as $\pi_t < P_t$. $I_{t',i} \geq 0$, $I_{t'} \geq 0$ and $\sum_{k=1}^{\tau} \pi'_{k,i} \geq \sum_{k=1}^{t-1} \sigma_{k,i}$ as $I_{t',i} > 0$, $I_{t'} > 0, \forall t' = \tau, \tau+1, \ldots, t-1$ and $\sum_{k=1}^{\tau} \pi_{k,i} > \sum_{k=1}^{t-1} \sigma_{k,i}$. Note that, in solution S', there is less inventory held and S' has an objective function value smaller than that of S by an amount equal to $\sum_{k=\tau}^{t-1}\{H_k(I_k) - H_k(I_k - 1)\} > 0$. Therefore S is not an optimal solution. ■

Even though the above theorem is stated for the total inventory and production levels, it can be used for individual inventory levels of each order. If the ending inventory level of an order is positive for some period $t-1$, then the production level of the next period is at the capacity P_t, according to Theorem 4.3. The intuition of this theorem is if the production facility is not producing at its full capacity for a period, then there is no need to carry any inventory from the previous period.

Similar to Lemma 4.1, if the inventory level is positive for a period t, all this inventory will eventually be depleted. The following lemma states that delivery amount is greater than the production amount at the period in which the inventory is depleted.

Lemma 4.2 *If $I_t > 0$, then $\exists \tau > t : I_\tau = 0, \sigma_\tau > \pi_\tau, I_{t'} > 0$ for $t \leq t' < \tau$, in every optimal solution.*

Proof: Proof is by construction. Let $I_t > 0$, and

$$\tau = \min\{k : I_k = 0, k > t\}$$

Note that, in an optimal solution $I_T = 0$, thus such τ exist and for $t \leq t' < \tau$, $I_{t'} > 0$. Note that, $I_{\tau-1} > 0$ and $I_\tau = 0$ by definition of τ which implies that $\sigma_\tau > \pi_\tau$. ∎

For all the periods with positive inventory defined in Lemma 4.2, the use of Theorem 4.3 implies the following corollary.

Corollary 4.2 *If $I_t > 0$, then $\exists \tau > t : \sum_{k=t+1}^{\tau} \sigma_k = \sum_{k=t+1}^{\tau} \pi_k + I_t = \sum_{k=t+1}^{\tau} P_k + I_t$.*

Although the above theorems are valid for any type of cost function, the following one is valid for linear and stationary cost functions in which the cost function is the same for all periods. The following theorem states that if the number of type I vehicles utilized in period t is positive ($\theta_t > x_t$), then, no type II vehicles are held for future periods .

Theorem 4.4 *$(\theta_t - x_t)w_t = 0$ for $t = 1, \ldots, T$, in every optimal solution, if $C_{1,t}(x) = C_1 x, C_{2,t}(x) = C_2 x, H_t(x) = Hx$, and $W_t(x) = Wx$.*

Proof: Proof is by contradiction. Let S be an optimal solution and t be a period such that in S, $(\theta_t - x_t)w_t \neq 0$. We know by Equations (4.5) and (4.20) that $\theta_t - x_t \geq 0$ and $w_t \geq 0$, thus $(\theta_t - x_t)w_t \geq 0$. $(\theta_t - x_t)w_t \neq 0$ is possible only if $\theta_t - x_t > 0$ and $w_t > 0$. By Lemma 4.1, as $w_t > 0$, $\exists \tau > t : w_\tau = 0, x_\tau > 0, w_{t'} > 0$ for $t \leq t' < \tau$. Now construct another solution S' by:

$$x'_t = x_t + 1 \tag{4.21}$$

$$w'_{t'} = w_{t'} - 1, \qquad \forall t' : t \leq t' < \tau \tag{4.22}$$

$$x'_\tau = x_\tau - 1 \tag{4.23}$$

Note that, $x_t + w_t = x'_t + w'_t$, thus, the t^{th} equation in Constraint set (4.2) does not change. Moreover for $t' = t+1, t+2, \ldots, \tau - 1$, both sides of the equations decrease by one for the Equation set (4.2). As $x_t < \theta_t$, $x'_t \leq \theta_t$ and as $x_\tau > 0$, $x'_\tau \geq 0$. Then, S' is a feasible solution. The objective function value of S' is smaller then that of S by an amount of $(\tau - t)W > 0$ as all $w_{t'}$ is reduced by 1 for all $t' = t, t+1, \ldots, \tau - 1$. Therefore, S is not an optimal solution. ■

Note that this theorem can be valid for many other cases except for some unusual cost structures.

The theorems and lemmas stated in this section can be used in the future for the development of implicit enumeration and heuristic procedures. We now continue with the detailed analysis of the problems with general delivery structure.

4.3 Problems with General Delivery Structure

In this section, we study the four problems (given in Table 4.1) for a general delivery structure. We start with the case where both consolidation and splitting are allowed.

4.3.1 Problem 1: Consolidate-Split

This problem setting constitutes a base case for all the problems in this chapter. In this setting, the decision maker has an option of consolidating multiple orders to deliver them within the same vehicle. Moreover, the orders can be split and delivered at different periods.

The following theorem states that this problem can be reduced to a much simpler problem where the production and delivery sequences can be optimally determined. Even though this significantly alleviates the difficulties of the original problem, the problem of deciding the production and delivery quantities in each period still needs to be solved.

Theorem 4.5 *There is an optimal solution to Problem 1, in which the orders are produced and delivered according to Earliest Due Date (Deadline) first (EDD) order.*

Proof: Proof is by construction. Given an optimal solution, the idea is to obtain another solution where the production and delivery sequences are EDD without changing total production and delivery quantities for each period. Consider an optimal solution, define $\sigma_t = \sum_{i \in N} \sigma_{t,i}$ and let the total production and delivery by time t be $TP(t)$ and $TS(t)$, respectively. Without loss of generality assume that $D_1 \leq D_2 \leq \ldots \leq D_{|N|}$, and let total demand of the first i orders be $TD(i)$. (i.e., $TP(t) = \sum_{k=1}^{t} \pi_k, TS(t) = \sum_{k=1}^{t} \sigma_k, TD(i) = \sum_{j=1}^{i} S_j$)

Consider another solution where the first S_1 units produced and delivered are assigned to order 1, the next S_2 units are assigned to order 2, and so on. One can obtain the new solution as follows:

$$\pi'_{t,i} = \min\{S_i, \pi_t, \max\{TD(i) - TP(t-1), 0\}, \max\{TP(t) - TD(i-1), 0\}\}$$
$$\sigma'_{t,i} = \min\{S_i, \sigma_t, \max\{TD(i) - TS(t-1), 0\}, \max\{TS(t) - TD(i-1), 0\}\}$$

The first two terms of the first equality state that the production quantity of order i at period t is limited by the size of the order (S_i) and the total production at that period (π_t). There are two more conditions for the production quantity of order i at period t to be positive: The order i must not be completed before t, and previous orders must be completed by time t. These conditions are satisfied with the last two terms of the above equality. If $TD(i)$ is less than or equal to $TP(t-1)$, order i must have been completed by period $t-1$, thus $\pi'_{t,i} = 0$. If $TD(i)$ is greater than $TP(t-1)$, order i is not completed, and $TD(i) - TP(t-1)$ more units must be produced to complete order i. Thus, $\pi'_{t,i}$ must be less then

$\max\{TD(i) - TP(t-1), 0\}$. If the cumulative production capacity by time t is less than or equal to the total size of first $i-1$ orders (i.e., $TP(t) \leq TD(i-1)$), all production until time t must be dedicated to orders $1, \ldots, i-1$, thus $\pi'_{t,i} = 0$. If $TP(t)$ is greater than $TD(i-1)$, all orders before i are completed by time t and $TP(t) - TD(i-1)$ units can be dedicated for production of order i at period t. Thus, $\pi'_{t,i}$ must be less then $TS(t) - TD(i-1)$.

At each period, delivery size of an order is less than the size of order (S_i) and delivery amount (σ_t). Similar to production quantities, delivery of an order is positive for period t, if it is not completed by time $t-1$ (employed via $\max\{TD(i) - TS(t-1), 0\}$) and all the previous orders are delivered by time t (employed via $\max\{TS(t) - TD(i-1), 0\}$).

Note that for the original solution,

$$\sum_{k=1}^{t} \pi_t = \sum_{i \in N} \sum_{k=1}^{t} \pi_{t,i} \geq \sum_{i \in N} \sum_{k=1}^{t} \sigma_{t,i} = \sum_{i:D_i \leq t} S_i + \sum_{i:D_i > t} \sum_{k=1}^{t} \sigma_{k,i} \geq \sum_{i:D_i \leq t} S_i$$

In the above equality, the set N is divided into two disjoint sets depending on D_i. If $D_i \leq t$, then the number of items delivered by time t is the size S_i of the order i. This constitutes the first part of the equality. The second part of the equality represents the remaining orders with $D_i > t$. To summarize;

$$\sum_{k=1}^{t} \pi_t \geq \sum_{k=1}^{t} \sigma_t \geq \sum_{i:D_i \leq t} S_i \text{ for } t = 1, \ldots, T$$

This indicates that there is sufficient production and delivery quantities to fulfill all demand with deadline less than or equal to t for $t = 1, \ldots, T$. This ensures the feasibility of the new solution. Since total production and delivery sizes remain the same, the cost of the new solution is also the same as the original solution. This proves that the new solution is also optimal.

■

The above theorem is valid only if splitting and consolidation are allowed. If consolidation is not allowed, then the number of outbound vehicles depend on the number of the orders to be delivered at that period, as well. Rescheduling the delivery of orders may result in necessity for different number of outbound vehicles, even if the total delivery size of two solutions (original and updated) are the same. Then, the above attempt to reschedule the deliveries may not result in equal total costs, thus, the above theorem will not be valid if consolidation is not allowed. Note also that, the above equality for calculation of σ'_{ti}, allows orders to be split and delivered in different periods. The third term in the above equality for the calculation of σ'_{ti} is positive only if order i is not completely delivered by time $t-1$. In other words, partial deliveries are allowed in the updated solution, thus, the above theorem is not valid for the cases where splitting is not allowed.

An important implication of Theorem 4.5 is that the orders whose due dates are equal, can be consolidated. Furthermore, without loss of generality, we assume that $D_1 \leq D_2 \leq D_3 \leq ... \leq D_{|N|}$ and define a new parameter δ_t as follows:

$$\delta_t = \sum_{i:D_i=t} S_i \quad \forall t = 1, \ldots, T.$$

With this parameter and results of Theorem 4.5, the generic multi item model discussed in Section 4.1 can be converted to a much simpler model (single item model) given below. Note that, an optimum solution of the new model (Model 2) requires a post-processing to convert single item solution which consists of total production and delivery amounts to multi item solution by assigning the first S_1 items to order 1, the next S_2 items to order 2, and so on.

Model 2: Single Item Formulation

Minimize

$$\sum_{t=1}^{T} \{C_{1,t}(\theta_t - x_t) + C_{2,t}(x_t) + W_t(w_t)\} + \sum_{t=1}^{T} H_t(I_t)$$

Subject to

$$x_t + w_t \leq A_t + w_{t-1} \qquad t = 1, \ldots, T \tag{4.24}$$

$$I_t = I_{t-1} + \pi_t - \sigma_t \qquad t = 1, \ldots, T \tag{4.25}$$

$$\sum_{k=1}^{t} \sigma_k \geq \sum_{k=1}^{t} \delta_k \qquad t = 1, \ldots, T \tag{4.26}$$

$$\pi_t \leq P \qquad t = 1, \ldots, T \tag{4.27}$$

$$\theta_t K \geq \sigma_t \qquad t = 1, \ldots, T \tag{4.28}$$

$$x_t \leq \theta_t \qquad t = 1, \ldots, T \tag{4.29}$$

$$w_0 = I_0 = 0 \tag{4.30}$$

$$I_t, \sigma_t, \pi_t, w_t, x_t, \theta_t \in \mathcal{Z}^+ \cup \{0\} \qquad t = 1, \ldots, T \tag{4.31}$$

Here, we propose a dynamic programming formulation to solve this problem which runs in Pseudo polynomial time. Existence of a pseudo-polynomial algorithm proves that the problem may be $\mathcal{NP}$-Hard but not $\mathcal{NP}$-Hard in the strong sense. The following dynamic programming formulation solves this problem in $O(TD^6W^2/K^2)$ time, where D is the cumulative demand and $W := \min(D/K, \sum_{i=1}^{T} A_i)$.

Algorithm 4.1 *Define total demand size until time t by $TD(t)$.*

$$TD(t) = \sum_{k=1}^{t} \delta_t = \sum_{i:D_i \leq t} S_i \quad \forall t = 1, \ldots, T$$

Define $C(t, \pi, \sigma, w)$ as the minimum total cost accumulated at the end of period t, when the total production and delivery quantities in the first t periods are π and σ respectively and the number of vehicles hold for period $t+1$ is w.

$$\mathcal{X}(t,\pi,\sigma,w) \;=\; \{(\pi_t,\sigma_t,x_t,\theta_t,w_t)|\pi_t \leq P_t, w_t \leq w, \sigma_t \leq K\theta_t, x_t \leq \theta_t\}$$

$$C(t,\pi,\sigma,w) = \begin{cases} \infty & ,\pi < \sigma \\ \infty & ,\sigma < TD(t) \\ \min\limits_{\substack{\mathcal{X}(t,\pi,\sigma,w) \\ x_t+w_t \leq A_t+w_{t-1}}} \left\{ \begin{array}{l} C(t-1,\pi-\pi_t,\sigma-\sigma_t,w_{t-1}) \\ +C_{1,t}(\theta_t - x_t) + C_{2,t}(x_t) \\ +H_t(\pi-\sigma) + W_t(w_t) \end{array} \right\} & ,ow \end{cases}$$

initial conditions:

$$\begin{aligned} C(0,0,0,0) &= 0 \\ C(t,\pi,\sigma,w) &= \infty \quad \forall t,\pi,\sigma,w : \min(t,\pi,\sigma,w) < 0 \end{aligned}$$

In the above algorithm, the set $\mathcal{X}$ together with the constraint $x_t + w_t \leq A_t + w_{t-1}$ constitute the feasible region for the amount of production (π_t), delivery (σ_t), number of vehicles used (x_t and θ_t) and number of type II vehicles held for the next period (w_t) in period t. The first two rows of the recursion correspond to infeasible solutions. The total size deliveries can neither be greater than total production amount, nor less than total delivery size by time t. Thus, the cost of such infeasible solutions is set to infinity.

Consider a feasible solution for period t (i.e., $\pi_t, \sigma_t, x_t, \theta_t, w_t$), the calculation of $C(t,\pi,\sigma,w)$ is as follows: The total production and delivery quantities for until period $t-1$ must be $\pi - \pi_t$ and $\sigma - \sigma_t$, respectively. Similarly, in order for x_t and w_t to be feasible, at least w_{t-1} type II vehicles must be held from the previous period. Hence, the minimum total cost accumulated until period $t-1$ is $C(t-1,\pi-\pi_t,\sigma-\sigma_t,w_{t-1})$. The transportation cost incurred at period t is $C_{1,t}(\theta_t - x_t) + C_{2,t}(x_t)$ as the number of type I and type II vehicles utilized at period t is $\theta_t - x_t$ and x_t, respectively. Note that, the difference between the total production and delivery quantities (i.e., $\pi - \sigma$) is equal to the amount of inventory at the end of period t, incurring a cost of $H_t(\pi - \sigma)$. An additional

$W_t(w_t)$ is incurred since w_t vehicles are held at the facility for the later periods. Recall that this calculation must be repeated for every feasible solution for period t.

For any feasible solution, $\sigma \leq \pi \leq D$, where D is the cumulative demand (i.e., $D := TD(T)$) and $w \leq W := \min(D/K, \sum_{i=1}^{T} A_i)$, and $\pi_t \leq \pi$, $\sigma_t \leq \sigma$, $x_t \leq \theta_t \leq D/K$, $w_t \leq w$ for all $t = 1, \ldots, T$. For each possible value of $C(t, \pi, \sigma, w)$ the minimization is done over $O(D) \cdot O(D) \cdot O(D/K) \cdot O(D/K) \cdot O(W)$ different values $\pi_t, \sigma_t, x_t, \theta_t, w_t,$, respectively. Thus, the number of elementary operation needed to calculate each $C(t, \pi, \sigma, w)$ is $O(D^4W/K^2)$.

The cost of an optimal production and transportation plan over the entire interval is equal to $C(T, D, D, 0)$. This is calculated according to the above forward recursion and the corresponding production, delivery and vehicle holding decisions of the optimal plan can be made by standard backtracing techniques. A backward recursion could be formulated just as easily. For each period, at most $O(D) \cdot O(D) \cdot O(W)$ different π, σ, w triples must be calculated. Thus, a total of $O(TD^2W)$ different $C(t, \pi, \sigma, w)$ values are needed to find an optimal solution to Problem 1. Hence, the total complexity of Algorithm 4.1 is $O(TD^6W^2/K^2)$.

For this problem, existence of a pseudo-polynomial algorithm proves that the problem is not NP-Hard in the strong sense. However, the status of the problem is still open. Either there exists a polynomial time algorithm or the problem is NP-Hard in the ordinary sense, which can be proven in further studies.

4.3.2 Problem 2: NoConsolidate-Split

This problem is a version of Problem 1 where consolidation is not allowed due to customer preferences, i.e., customers would not like their products to be shipped together with other orders. In this case, the vehicles cannot deliver more than one order at a time. The following theorem states that this version of the problem is $\mathcal{NP}$-Hard in the strong sense even for the linear cost structure.

Theorem 4.6 *Problem 2 is $\mathcal{NP}$-hard in the strong sense.*

Proof: Proof is done by a reduction from 3-Partition(3P) problem and note that, Problem 2 is clearly in $\mathcal{NP}$. 3P is defined as follows:

INSTANCE: Set $\mathcal{G}$ of $3t$ elements, a bound $B \in Z^+$, and a size $s(a) \in Z^+$ for each $a \in \mathcal{G}$ such that $B/4 < s(a) < B/2$ and such that $\sum_{a \in \mathcal{G}} s(a) = tB$.

QUESTION: Can $\mathcal{G}$ be partitioned into t disjoint sets $\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_t$ such that $\sum_{a \in \mathcal{G}_\tau} s(a) = B$ for $\tau = 1, 2, .., t$(note that each $\mathcal{G}_\tau$ must therefore contain exactly three elements from $\mathcal{G}$)?

REDUCTION: Take an arbitrary instance of 3P. The corresponding instance of Problem 2 is constructed as follows: $N = \mathcal{G}$, i.e., for each element a in set $\mathcal{G}$ define an order $a \in N$ with size $S_a = s(a)$. Define $T = t, K = P = B$, and for each a in N define $D_a = T$ and for $\tau = 1, 2, ..., T$: define $A_\tau = 3$ and $C_{1,\tau}(x) = 2x, C_{2,\tau}(x) = H_\tau(x) = W_\tau(x) = x$. There is a solution to 3P if and only if there is a solution to Problem 2 with cost less than or equal to $z^* = 3t$.

Assume that there is a solution to Problem 2 with cost z which is less than or equal to $z^* = 3t$. Since there are $3t$ orders to be satisfied with a total size of tK, total cost of transportation is at least $3t$. Thus, all type II vehicles are utilized and no inventory or vehicle holding cost is incurred. As a result, exactly three orders are completed and delivered at each period. Moreover, total production capacity of the facility is equal to total demand ($tP = tB$). Thus, the total number of items produced at each period is equal to P. This means that, three orders with total size equal to P are completed and delivered at each period. Now construct a solution to 3P as follows: for all orders produced and delivered in period τ, put the corresponding element in set $\mathcal{G}$ into $\mathcal{G}_\tau$. As the size of orders $S_a = s(a)$, for each disjoint set $\mathcal{G}_\tau$, $\sum_{a \in \mathcal{G}_\tau} s(a) = B$ $(\tau = 1, 2, ..., t)$.

If there is a solution to 3P, construct a solution to Problem 2 as follows: for each disjoint set $\mathcal{G}_\tau, \tau = 1, 2, ..., t$, produce and deliver all the items of order $a \in \mathcal{G}_\tau$ in period τ. Similar reduction with the previous case imply that the solution has a cost of $z = 3t \leq z^*$. ■

4.3.3 Problem 3: Consolidate-NoSplit

This case is applicable for customers who would like to receive the entire order in one shipment rather than receiving partial orders at different time periods. This is because receiving partial shipments might create additional burden in material handling and information system of the company. In some cases, it may also cause confusion as well. This problem is $\mathcal{NP}$-Hard in the strong sense even for the linear cost structure as the following theorem states.

Theorem 4.7 *Problem 3 is $\mathcal{NP}$-Hard in the strong sense.*

Proof: Similar to the proof of Theorem 4.6 with $A_\tau = 1$ for each $\tau = 1, 2, .., T$ and $z^* = t$. ■

4.3.4 Problem 4: NoConsolidate-NoSplit

In this problem, neither consolidation nor splitting is allowed. As stated in the following theorem, the problem is $\mathcal{NP}$-Hard in the strong sense even for the linear cost structure.

Theorem 4.8 *Problem 4 is $\mathcal{NP}$-Hard in the strong sense.*

Proof: Similar to the proof of Theorem 4.6 with $A_\tau = 3$ for each $\tau = 1, 2, .., T$ and $z^* = 3t$. ■

For the last three problems that are proven to be $\mathcal{NP}$-Hard in the strong sense, researchers can only develop some heuristics for realistic sizes. In Chapter 5, we provide a very efficient tabu search heuristic for Problem 4. Some efficient algorithms can be provided for special cases of these problems, as well.

4.4 Problems with FTL-Delivery Structure

For the problems discussed in this section, vehicles are required to be fully utilized in outbound transportation and therefore size of orders must be integer multiples of vehicle capacity. In other words, the number of items in each vehicle is either 0 or K.

Recall that consolidation is a reasonable way of reducing transportation costs when there are multiple orders of small sizes. In the $FTL - Delivery$ case, since order sizes are already integer multiples of vehicle capacity, consolidation (whether allowed or not) does not affect the structure of the problem. Thus, there are only two problem settings for the $FTL - Delivery$ case with respect to whether splitting is allowed or not.

We first state two theorems that are valid for both *Split* and *NoSplit* cases. The result of these theorems can be utilized as a part of optimization algorithms to solve the problems. The following theorem states that if the production capacity is an integer multiple of vehicle capacity, then the total production in each period is an integer multiple of vehicle capacity.

Theorem 4.9 *If* $\exists n_t \in \mathcal{Z}^+ \cup \{0\} : P_t = n_t K$, *for* $t = 1, 2, .., T$, *then in every optimal solution* $\exists m_t \in \mathcal{Z}^+ \cup \{0\} : \pi_t = m_t K$ *for* $t = 1, 2, .., T$.

Proof: Proof is by contradiction. Suppose there exists an optimal solution S such that total production quantity is not an integer multiple of vehicle capacity for some periods. Let t be the latest such period (i.e., $\forall m \in \mathcal{Z}^+$, $\pi_t \neq mK$). Note that $\frac{\sum_{k=1}^{T} \pi_k}{K}$ is integer as total production is equal to total demand, and $\frac{\sum_{k=t+1}^{T} \pi_k}{K}$ is integer by selection of t. As $\frac{\pi_t}{K}$ is not an integer, $\frac{\sum_{k=1}^{t-1} \pi_k}{K}$ is neither. On the other hand, $\frac{\sum_{k=1}^{t-1} \sigma_k}{K}$ is integer as vehicles are utilized at full capacity. Note that, $\sum_{k=1}^{t-1} \pi_k \geq \sum_{k=1}^{t-1} \sigma_k$, and $\frac{\sum_{k=1}^{t-1} \pi_k}{K}$ is not integer but $\frac{\sum_{k=1}^{t-1} \sigma_k}{K}$ is integer, thus, $\sum_{k=1}^{t-1} \pi_k > \sum_{k=1}^{t-1} \sigma_k$. In other words, there is at least $\lceil \frac{\pi_t}{K} \rceil K - \pi_t$ units of inventory carried from period $t-1$ to period t. Let $i = argmax_j\{I_{t-1,j}\}$, and $\tau = argmax_{k<t}\{\pi_{k,i} > 0\}$.

Now, consider another solution S' with the following modification on solution S:

$$\begin{aligned} \pi'_{\tau,i} &= \pi_{\tau,i} - 1 \\ \pi'_{t,i} &= \pi_{t,i} + 1 \\ I'_{t',i} &= I_{t',i} - 1, \qquad \text{for } t' = \tau, \tau+1, \ldots, t-1. \\ I'_{t'} &= I_{t'} - 1, \qquad \text{for } t' = \tau, \tau+1, \ldots, t-1. \end{aligned}$$

Note that, S' is feasible as

$$\sum_{k=1}^{\tau} \pi_{k,i} = \sum_{k=1}^{t-1} \pi_{k,i} > \sum_{k=1}^{t-1} \sigma_{k,i}$$

the new solution S' has a lower objective function value than S by an amount $\sum_{k=\tau}^{t-1} H_k(I_k) - \sum_{k=\tau}^{t-1} H_k(I_k - 1) > 0$, as $H_t(x)$ is an increasing function for $t = 1, \ldots, T$. Therefore, S is not an optimal solution. ■

If the production capacity of the facility is an integer multiple of vehicle capacity for all periods, the following theorem states that there is an optimal solution in which the production quantity for each order is also an integer multiple of the vehicle capacity for every period.

Theorem 4.10 *If $\exists n_t \in \mathcal{Z}^+ \cup \{0\} : P_t = n_t K$, for $t = 1, 2, .., T$, then there is an optimal solution in which $\exists m_{t,i} \in \mathcal{Z}^+ \cup \{0\} : \pi_{t,i} = m_{t,i} K \; \forall i \in N$ for $t = 1, 2, .., T$.*

Proof: Proof is by construction. Consider an optimal solution S in which some orders have production quantity which is not an integer multiple of vehicle capacity. Note that the total production at each period is an integer multiple of the vehicle capacity due to Theorem 4.9. Let i be the smallest indexed order with this property and let t and τ $(t < \tau)$ be the last two periods where production of order i is not an integer multiple of vehicle capacity (i.e., $\frac{\pi_{\tau,i}}{K}$ and $\frac{\pi_{t,i}}{K}$ are not integer). Note that, $\frac{\sum_{k=1}^{t} \pi_{k,i}}{K} > \lfloor \frac{\sum_{k=1}^{t} \pi_{k,i}}{K} \rfloor \geq \frac{\sum_{k=1}^{t} \sigma_{k,i}}{K}$. This means that a portion of production quantity for order i at period t can be moved to period τ. As total production quantity for all periods is an integer multiple of vehicle capacity, $\exists j \in N : \pi_{\tau,j} - \lfloor \frac{\pi_{\tau,j}}{K} \rfloor K > 0$. Also note that, $j > i$ (as i is the smallest indexed order with production not being an integer multiple of vehicle capacity).

Let

$$\Delta = \min\{(\pi_{\tau,j} - \lfloor\frac{\pi_{\tau,j}}{K}\rfloor K), (\lceil\frac{\pi_{\tau,i}}{K}\rceil K - \pi_{\tau,i})\}$$

and set

$$\pi_{\tau,i} \leftarrow \pi_{\tau,i} + \Delta$$
$$\pi_{\tau,j} \leftarrow \pi_{\tau,j} - \Delta$$
$$\pi_{t,i} \leftarrow \pi_{t,i} - \Delta$$
$$\pi_{t,j} \leftarrow \pi_{t,j} + \Delta$$

Repeat the same argument until $\frac{\pi_{\tau,i}}{K}$ is integer. Note that it takes at most $|N| - j$ steps. Then select different t and τ and repeat the same arguments until $\frac{\pi_{t,i}}{K}$ is integer for all $t = 1, 2, \ldots, T$. Note that during this process, no orders with index less than i is altered. Repeating the same procedure for all $i \in N$, results in a solution that the production quantity for each order is an integer multiple of vehicle capacity, for each period. ■

We now continue with analyzing the Split and Nosplit cases separately.

4.4.1 Problem 5: Split with FTL-Delivery

This problem is similar to Problem 1 (Consolidate-Split) in the sense that $FTL-Delivery$ can be considered as a special case of *General* delivery structure. Hence, the EDD property stated in Theorem 4.5 in Section 4.3.1 is also valid for this problem. Moreover, the model developed for $Consolidate - Split$ case (Model 2) can also be used for $FTL - Delivery$ case with appropriate modifications (by replacing inequality in constraint set (4.28) by equality). For the modified version of Model 2, the decision variables and parameters with a superscript K are defined as follows: $\pi_t^K = \frac{\pi_t}{K}, P_t^K = \frac{P_t}{K}, I_t^K = \frac{I_t}{K}, \delta_t^K = \frac{\delta_t}{K}$ for $t = 1, \ldots, T$.

Model 3: Single Item Formulation with *FTL* Delivery

Minimize

$$\sum_{t=1}^{T} \left\{C_{1,t}(\theta_t - x_t) + C_{2,t}(x_t) + W_t(w_t) + H_t(KI_t^K)\right\} \tag{4.32}$$

Subject to

$$x_t + w_t \leq A_t + w_{t-1} \qquad t = 1, \ldots, T \tag{4.33}$$

$$I_t^K = I_{t-1}^K + \pi_t^K - \theta_t \qquad t = 1, \ldots, T \tag{4.34}$$

$$\sum_{k=1}^{t} \theta_k \geq \sum_{k=1}^{t} \delta_k^K \qquad t = 1, \ldots, T \tag{4.35}$$

$$\pi_t^K \leq P_t^K \qquad t = 1, \ldots, T \tag{4.36}$$

$$x_t \leq \theta_t \qquad t = 1, \ldots, T \tag{4.37}$$

$$w_0 = I_0^K = 0 \tag{4.38}$$

$$I_t^K, \pi_t^K, w_t, x_t, \theta_t \in \mathcal{Z}^+ \cup \{0\} \qquad t = 1, \ldots, T \tag{4.39}$$

A modified version of Algorithm 4.1 developed for $Consolidate - Split$ case can also be used to solve this problem in $O(TD^5W^2/K^2)$ time, where D is the cumulative demand and $W := \min(D/K, \sum_{i=1}^{T} A_i)$. Note that the only difference is the new feasible region $\mathcal{X}^K$ is smaller than $\mathcal{X}$ because $\sigma_t \leq K\theta_t$ is replaced by $\sigma_t = K\theta_t$. With this new feasible region, the problem can be solved in $O(TD^5W^2/K^2)$ time. This indicates that Problem 5 is a little bit simpler than Problem 1. However, the status of this problem is still open. It needs to be proven that either it is NP-Hard in the ordinary sense or there is a polynomial time algorithm.

4.4.2 Problem 6: NoSplit with FTL

This version of problem is similar to $Consolidate - NoSplit$ and $NoConsolidate - NoSplit$. This problem is not simpler than Problems 2 or 4 as stated in the following theorem.

Theorem 4.11 *Problem 6 is $\mathcal{NP}$-Hard in the strong sense even for the linear cost structure.*

Proof: Similar to the proof of Theorem 4.6 with $A_\tau = P = B$ for each $\tau = 1, 2, .., T$, $K = 1$ and $z^* = Bt$. ■

After providing complexity results, we continue with computational experiments to evaluate the performance of the proposed generic model and identify the effects of consolidation and splitting policies on the system performance.

4.5 Computational Experiments

Since both the *Consolidation* and the *Splitting* policies are relaxations of *NoConsolidation* and *NoSplitting* cases, any *NoConsolidation* solution is also a feasible solution for *Consolidation* case. Since the feasible region is enlarged, reduced total costs are probably obtained in the *NoConsolidation* policy. The same is true for *Splitting*. The percentage improvement in the total cost depends on various input parameters (factors) such as production capacities, size of orders, availability of type II vehicles and transportation and inventory holding costs. We discuss the results of computational experiments to investigate the effects of operating policies (*Consolidation* vs. *NoConsolidation* and *Splitting* vs. *NoSplitting*) under the experimental conditions defined by the input parameters, outlined above.

In the experiments, the cost functions are selected to represent the real behavior of the aforementioned appliance manufacturer. We use a monthly production planning horizon of T =30 days. We assume six working days in a week and the production capacity is the same throughout the month. In other words, the production capacity is P_t for six consecutive days and zero for the seventh day. In order to investigate the effect of production capacity on the system performance, we consider two levels of production capacities: 1000 units/day and 1500 units/day. In the low capacity case, average load (total size of orders / total production capacity) is around 90%, which represents high utilization environment. In the high capacity case, which corresponds to low utilization, average load is about 60%. The deadlines of the orders are uniformly distributed so that weekly average loads are about the same as the monthly average loads. The vehicle capacity is 100 units/vehicle.

In setting experimental conditions, three different levels (low, medium and high) of order sizes are used even though the total size of the orders is kept fixed at 24000 units. The low order size case corresponds to the less than truck load case and related input data is randomly generated from Uniform(10,99). For the medium size orders, the required data is generated from Uniform(110,999), in which case, order size is between one truck load and one day production capacity. For the high order size case, data is generated from Uniform(1100,4000) and order sizes vary between one and four days of production capacity.

Recall that, for the $FTL-Delivery$ case, order sizes must be integer multiples of vehicle capacity. In this case, medium and high order sizes are generated from $100 \times$ Uniform(2,10) and $100 \times$ Uniform(12,40), respectively. Since all order sizes must be integer multiples of vehicle capacity, the low order sizes can not be considered for the $FTL - Delivery$ case.

Two levels (high and low) are designed for the availability of type II vehicles. At the low level, the number of type II vehicles is selected from Uniform(0,6) which leads to situation where the capacity of the inbound vehicles is approximately 30% and 20% of the daily production capacities corresponding to low and high production rates, respectively. At the high level of for the number of type II vehicles, data is generated from Uniform(4,10) that results in approximately 70% and 45% of the daily production capacities for the low and high production capacity cases, respectively.

In summary, experimental conditions are defined such that there are two levels for consolidation, splitting policies, three levels of order sizes, two levels of production capacities and two levels for the availability of type II vehicles. Hence, there are totally 48 experimental points for the general delivery case (See Table 4.2). In the FTL-Delivery case, since the consolidation factor dropped out and the low order sizes can not be considered, there are 16 experimental points. In each experimental point, 10 randomly generated problem instances are used.

The generic model discussed in Section 4.1 is solved using GAMS version 22.6 running on a 2.6 GHz dual core AMD Opteron 252 server running Linux version 2.6.9 with 2 GBs of physical memory. For all the problems, the maximum solution

Table 4.2: Experimental Design

Consolidation*	Allowed (C), not allowed (nC)
Splitting	Allowed (S), not allowed (nS)
Production capacity	High (1 500), Low (1 000)
Availability of type II vehicles	High ∼U(4,10), Low ∼U(0,6)
Order sizes	High ∼U(1100, 4000), Medium ∼U(110, 1000), Low* ∼U(10,99)
Cost Parameters	$C_{1,t}(x) = 700x, C_{2,t}(x) = 100x$, $H_t(x) = x, W_t(x) = 150x\ \forall t$
Vehicle capacity	$K = 100$
* :	Not considered for FTL Delivery

Table 4.3: Average Solution Times (in CPU seconds)

	Consolidate	*NoConsolidate*
Split	69.80	527.08
NoSplit	414.66	119.63

time for GAMS is limited to 600 seconds. The average solution times for different problems are given in Table 4.3.

Optimality gap is calculated by subtracting the final lower bound from the best integer solution, divided by the lower bound. The average optimality gaps for the general delivery case are given in Table 4.4. In general, low optimality gaps are obtained in shorter solution times for *NoConsolidation-NoSplit* case or when both *Consolidation* and *Splitting* are allowed. This counter-intuitive result is explained as follows. The first case (*Consolidate-Split*) is the base case which has been shown to be easier than others as a pseudo polynomial algorithm is proposed in Section 4.1. In the second and third cases (either *Consolidation* or *Splitting* is allowed) the problem is relatively more difficult due to additional constraints. In the last case (*NoConsolidation-NoSplit*), however, the gap values are low and the average solution times are smaller because the vehicle capacity constraints are all in form of equalities (Equations sets (4.15) and (4.16)). Since optimality gaps are generally very low, we safely assume that effects of operating policies on the system performance can be analyzed with the proposed optimization models.

Table 4.4: Average Gap Values

	Order size		
	High	*Medium*	*Low*
Consolidate – Split	0.03%	0.00%	0.14%
Consolidate – NoSplit	0.00%	7.28%	3.41%
NoConsolidate – Split	3.58%	13.56%	0.20%
NoConsolidate – NoSplit	0.00%	0.27%	0.00%

In order to understand the effects of consolidation and splitting policies, we use a differential approach, in which for each instance, we define Δ*Splitting* as the percentage cost difference between the *Split* and *NoSplit* cases, keeping the remaining factors as the same. In other words, Δ*Splitting* = (Total cost with *NoSplit* - Total cost with *Split*)/(Total cost with *NoSplit*). Δ*Consolidation* is defined in a similar way. Since the effects of these operating policies depend highly on the availability of type II vehicles (A_t) and production capacities (P_t), the results are presented with respect to these parameters, in Tables 4.5 and 4.6, respectively.

The rows of Table 4.5 correspond to effects of operating policies, whereas the columns correspond to order sizes. Each column is further divided into two levels (high and low) for the availability of type II vehicles. Note that, the first value at the upper left of each cell is the percentage cost reduction when the availability of type II vehicles is high and the other value at the lower right of each cell is the percentage cost reduction when the availability of type II vehicles is low. The effect of both consolidation and splitting policies are relatively high when the availability of type II vehicles is high.

Observe that, the effect of splitting increases as the order sizes increase. This is due to the fact that, when the order size is higher than the production capacity (P_t), the required production takes more than one period. This means that a portion of an order must be produced earlier and kept in inventory, this leads an increase in the inventory holding cost. When splitting is allowed, however, there is no need to keep inventory unless it is cost effective. Hence, it is intuitive that the effect of splitting increases as the order sizes increase.

Table 4.5: Percentage Cost Reduction by Allowing Consolidation and Splitting

	Order size					
	High		*Medium*		*Low*	
availability of type II vehicles	*High*	*Low*	*High*	*Low*	*High*	*Low*
$\Delta Splitting$	54.33%	22.45%	7.47%	2.23%	1.76%	0.95%
$\Delta Consolidation$	2.82%	1.27%	16.91%	9.00%	75.54%	55.49%

As can be seen in Table 4.5, the effect of consolidation is relatively high when the order sizes are low. For the low order size case, there are 450 orders with sizes being less than one vehicle capacity. When consolidation is not allowed, the total number of vehicles that is needed to deliver all orders is 450. On the other hand, the total size of all orders is equal to the total capacity of 240 vehicles. Thus, the total number of vehicles to deliver all orders can be reduced down to 240 when consolidation is allowed. For the high order cases, the effect of consolidation is insignificant. This is because the number of orders is very low. As there are 10 orders, even if consolidation is allowed, the number of outbound vehicles needed to deliver all orders can be reduced by at most 10. Thus, as expected, the effect of consolidation increases when the size of the orders are low, especially less than one vehicle capacity.

Note that, the effect of consolidation depends on the transportation costs whereas the effect of splitting is related to inventory holding costs. Thus, the consolidation policy is more effective when the transportation costs are high whereas splitting policy is more critical when inventory holding costs are higher.

The effects of operating policies with respect to production capacities are summarized in Table 4.6. The first number at the upper left of each cell is the average percentage cost reduction when the production capacity is high, whereas the second number is the average percentage cost reduction for low production capacity. As expected, effect of $Consolidation$ is more significant when order sizes are low. Observe that for each order size level, $\Delta Consolidation$ is approximately the same irrespective of the production capacities. Hence, we conclude that the

Table 4.6: Percentage Cost Reduction with Respect to Production Capacity

	Order size					
	High		*Medium*		*Low*	
Production Capacity	*High*	*Low*	*High*	*Low*	*High*	*Low*
Δ*Splitting*	34.68%	42.11%	4.13%	5.57%	1.11%	1.65%
Δ*Consolidation*	2.15%	1.93%	13.84%	12.06%	65.82%	65.73%

Table 4.7: Percentage Cost Reduction for FTL-Delivery

	Order Size	
Availability of type II vehicles	*High*	*Medium*
High	52.43%	1.57%
Low	19.70%	0.15%

effect of consolidation does not depend on production capacities. For splitting, however, the effect depends on production capacity. The first numbers in each cell is lower than the second numbers for the first row of Table 4.6. When the production capacities are high, it takes less number of periods to produce high size orders. Thus, the effect of splitting is relatively more at the low production capacities.

For the FTL-Delivery case, the effect of splitting and availability of type II vehicles is presented in Table 4.7. Results indicate that the effect of splitting increases with the order sizes similar to the general delivery case, as discussed before.

For the FTL-Delivery case, the average optimality gaps and solution times are given in Tables 4.8 and 4.9, respectively. Observe that, average optimality gaps are relatively smaller than the general delivery case. Furthermore, all the FTL-Delivery problems with *Split* are solved to optimality in less then 0.1 CPU seconds. This result is interesting and needs further investigation.

In conclusion, both the consolidation and the splitting policies have significant effects on the system performance. The effect of the consolidation policy is magnified when the order sizes are low and transportation costs are high. The

Table 4.8: Average Optimality Gap Values for FTL-Delivery

	Order Size	
	High	*Medium*
Split	0.00%	0.00%
NoSplit	0.00%	0.27%

Table 4.9: Average Solution Times for FTL-Delivery (in CPU seconds)

	Order Size	
	High	*Medium*
Split	0.02	0.07
NoSplit	28.78	297.48

effect of the splitting policy is more when the order sizes and inventory holding costs are high and the production capacities are low. The computation time for FTL-Delivery is less than 0.1 CPU seconds when splitting is allowed which motivates further research.

4.6 Demand Time Windows

In this section, we analyse the problem variant with demand time windows, in which delivery of an order has to take place within a time window. For each order i, we define $E_i \leq D_i$ as the earliest time that an order can be delivered. Hence, delivery of an order has to take place within $[E_i, D_i]$. This version of the problem is at an extension since setting $E_i = 0, \forall i \in N$ is a special case for the problem with demand time windows. Thus, this version of the problem is at least as hard as the original versions. In order the employ demand time windows, the following changes on Constraint sets (4.8) and (4.10) must be made on the Generic Model Formulation defined in Section 4.1, respectively;

$$\sum_{t=E_i}^{D_i} \sigma_{t,i} = S_i \qquad \forall i \in N \tag{4.40}$$

$$\sum_{t=E_i}^{D_i} \tilde{\sigma}_{t,i} = 1 \qquad \forall i \in N \tag{4.41}$$

We would like to also note that, Theorems 4.1,4.2, 4.3, and 4.4; Lemmas 4.1 and 4.2; and Corollaries 4.1 and 4.2 are remain valid with the inclusion of demand time windows. The EDD property defined in Theorem 4.5 remains valid only if earliest delivery times for all orders are agreeable with their deadlines, i.e., $E_i \leq E_j \rightarrow D_i \leq D_j; \forall i, j \in N$.

For two of the problems studied in this chapter, although we provide pseudo-polynomial algorithms for a general cost structure, the status of these problems is still open. Four of the six problems considered in this study are proven to be intractable, which motivates the need to design heuristic solution procedures for these problems. Further research may also needed to develop either heuristic or enumerative solution procedures which employ the optimality conditions in this chapter. In Chapter 5, we propose a tabu-search algorithm for Problem 4 defined in Section 4.3.4.

Chapter 5

Hierarchical vs. Central Coordination

Hierarchically versus Centrally-Coordinated Decisions for A Joint Production and Transportation Planning Problem

We consider a manufacturer's planning problem to schedule the production of orders and arrange for their transportation to respective destinations. Motivated by some industry practices, we present formulations for three different solution approaches, which we refer to as the *uncoordinated solution*, the *hierarchically-coordinated solution* and the *centrally-coordinated solution*. In both the uncoordinated solution and the hierarchically-coordinated solution, production planning decisions are made first, followed by outbound transportation decisions. In the uncoordinated solution, planning efforts for transportation are limited, often made using a heuristic and without giving explicit consideration to transportation costs and constraints. In the hierarchically-coordinated solution, transportation planning is done in more detail in an effort to optimize the related costs. Finally, in the centrally-coordinated solution, production and transportation decisions are made jointly, aiming to minimize overall costs.

In this chapter, we first present mathematical formulations for solving the problem of interest using the three approaches. The mathematical formulations for the uncoordinated and hierarchically-coordinated solutions are based on an identification of two subproblems, those are the production subproblem and the transportation subproblem. In the production subproblem, the objective is to find a schedule of jobs on a single machine to minimize inventory holding costs without any job being tardy. In the transportation subproblem, a plan is made to deliver the completed orders with the least cost considering the different vehicle availabilities. We show that solving the production subproblem is $\mathcal{NP}$-hard in the strong sense, however, we come up with polynomial algorithms for solving the two subproblems given the delivery times of orders. This structure of the problem enables us to propose a novel application of the tabu-search method as a heuristic to minimize the sum of inventory holding and transportation costs.

It is obvious that integration of production scheduling and transportation decisions reduces the total costs as opposed to making the related decisions in a sequential fashion. Similar to the studies (Chen and Vairaktarakis [6], Pundoor and Chen [19]) investigating this issue, we conclude that the savings due to integration can in fact be significant. By comparing the hierarchically-coordinated solution to the centrally-coordinated solution, we quantify the savings due to integration. Furthermore, by comparing the uncoordinated solution to the hierarchically-coordinated solution, we quantify the savings that can be achieved by optimal usage of the transportation choices.

Although this problem is a special case of Problem 4 in Chapter 4, for the sake of completeness we begin with a detailed description of the problem and the notation in the next section. We continue in Section 5.2 with the explanation and the modeling of the three solution approaches. In Section 5.3, we provide a further analysis of the underlying subproblems. In Section 5.4, a heuristic based on tabu search is proposed for the joint problem of minimizing inventory holding and transportation costs. This is followed by the results of an extensive numerical analysis on the comparison of the three solution approaches and the performance of the heuristic.

5.1 Problem Definition and Notation

We consider a manufacturer's production-planning and delivery-scheduling problem, which concerns N orders to be satisfied in T periods. Production capacity of the manufacturer is limited by P_t units in period t, indepedent of the type of items to be produced. The production for each order i, which has size S_i, must be completed and the order must be delivered before its deadline D_i. Late deliveries are not allowed. In this setting, order acceptance and rejection decisions have been already made, and there exists a feasible production plan that makes every order ready for delivery before its deadline. Cost of carrying one unit of inventory from one period to the next amounts to $\$H$ for all orders.

Orders are delivered to the customers at the expense of the manufacturer. The manufacturer uses capacitated vehicles for outbound transportation. Each vehicle holds upto K units of the finished product. Any number of these vehicles can be utilized at a cost of $\$C_1$ per vehicle in each period. However, in period t, a limited number (i.e., A_t) of them is also available at a lower cost (i.e., C_2) and we refer to the vehicles with cost C_1 as *type I* and to those with cost C_2 as *type II*. The latter type of vehicles can be held at the facility at an additional cost of $\$W$ per vehicle per period. Note that, this problem is a special case of Problem 4 in Chapter 4, with linear and stationary cost functions. Following restrictions exist on outbound shipments: i) customers do not accept partial deliveries (NoSplit), ii) different orders cannot be shipped in the same vehicle (NoConsolidation). Therefore, the number of vehicles needed for delivery of order i is given by $\lceil S_i/K \rceil$. The problem is to find a production plan that minimizes the sum of transportation and inventory holding costs. The plan must imply the delivery schedule of orders, the number of both types of vehicles used in outbound transportation and the production quantity in each period. Different approaches may be used to solve the production planning problem in this setting. Before proceeding with a detailed discussion of these approaches in the next section, we summarize below some of the notation used in this chapter. Additional notation will be defined when it is necessary.

N: Number of orders.
T: Number of periods.
P_t: Production capacity in period t.
S_i: Size of order i.
D_i: Deadline of order i.
H: Cost of carrying one unit of inventory from one period to the next.
K: Capacity of a truck in number of units.
C_1: Cost of utilizing a type I vehicle.
C_2: Cost of utilizing a type II vehicle.
W: Cost of holding a type II vehicle for a period.
A_t: Number of type II vehicles available in period t.
$Cost_u$: Total cost of the uncoordinated solution.
$Cost_h$: Total cost of the hierarchical solution.
$Cost_c$: Total cost of the centralized solution.

5.2 Solution Approaches

In this section, we discuss the three approaches briefly introduced in the beginning of this chapter (those are the *centrally-coordinated solution*, the *uncoordinated solution* and the *hierarchically-coordinated solution*) for solving the problem of interest. In the centrally-coordinated solution, production-planning and transportation decisions are made jointly in a single step. The other two approaches follow a two-step process which relies on solving the underlying subproblems, those are the production subproblem and the transportation subproblem, sequentially. The production subproblem is mainly finding the production quantity in each period and the delivery schedule of orders to minimize inventory holding costs. Since this problem is solved independently, without giving any consideration to the outbound shipment costs, its optimal solution does not foresee the savings from transportation costs if the completed orders are held in inventory. Therefore, a plan that minimizes inventory holding costs delivers the orders as soon as they are completed. The transportation subproblem is, given the delivery schedule of orders, determining the number of type I and type II vehicles to be used over time to minimize transportation costs.

The first steps of the uncoordinated and the hierarchically-coordinated solutions are the same and are mainly solving the production subproblem optimally. The two solutions differ in their second steps where the transportation subproblem is solved. In the hierarchically-coordinated solution, this subproblem is also solved optimally whereas in the uncoordinated solution it is not. More specifically, in the uncoordinated solution, transportation arrangements are made to deliver the completed orders in each period using only the vehicles which are available in that period. Since type II vehicles are less costly, they are preferred over the type I vehicles. If there is no type II vehicle, outbound shipments are made using type I vehicles. As an implication of this difference, hierarchically-coordinated solution allows for holding type II vehicles over periods to satisfy future delivery requirements while the uncoordinated-solution does not.

In the remaining parts of this section, we present these approaches in more detail. For the sake of simplicity, we use a notation similar to the previous chapter with some modifications. The following is a list of decision variables that are common to all three approaches:

π_t	:	Total production amount in period t	$t = 1, \ldots, T$
I_t	:	Inventory carried from period t to $t+1$	$t = 1, \ldots, T$
y_t	:	Number of type I vehicles used in period t	$t = 1, \ldots, T$
x_t	:	Number of type II vehicles used in period t	$t = 1, \ldots, T$
w_t	:	Number of type II vehicles carried from period t to $t+1$	$t = 1, \ldots, T$
$\tilde{\sigma}_{ti}$	:	$\begin{cases} 1 & \text{If order } i \text{ is delivered in period } t \\ 0 & \text{otherwise} \end{cases}$	$t = 1, \ldots, T; i = 1, \ldots, N$

Recall that θ_t defined in the previous chapter was the number of vehicles used in period t. In this chapter, however, y_t and x_t are defined to be the number of type I and type II vehicles used in period t, respectively. The modification on the decision variables is simply replacing $\theta_t - x_t$ by y_t.

5.2.1 Centrally-Coordinated Solution:

In presenting the details of the different solution approaches, we start with the centrally-coordinated solution. The following integer programming formulation, modified from the generic model in Chapter 4, models all aspects of outbound transportation in obtaining a production plan. In this chapter, we refer to this model as the *Integrated Model* and its optimal objective function value for a problem instance using the notation $Cost_c$.

Integrated Model:

$$Min \quad \sum_{t=1}^{T} (C_1 y_t + C_2 x_t + W w_t + H I_t)$$

$$I_t = I_{t-1} + \pi_t - \sum_{i=1}^{N} \tilde{\sigma}_{ti} S_i \qquad t = 1, \ldots, T \tag{5.1}$$

$$x_t + w_t \leq A_t + w_{t-1} \qquad t = 1, \ldots, T \tag{5.2}$$

$$\pi_t \leq P_t \qquad t = 1, \ldots, T \tag{5.3}$$

$$\sum_{i=1}^{N} \lceil S_i / K \rceil \tilde{\sigma}_{ti} = x_t + y_t \qquad t = 1, \ldots, T \tag{5.4}$$

$$\sum_{t=1}^{D_i} \tilde{\sigma}_{ti} = 1 \qquad i = 1, \ldots, N \tag{5.5}$$

$$x_t, y_t, w_t, \pi_t, I_t \in \{0\} \cup \mathcal{Z}^+ \qquad t = 1, \ldots, T \tag{5.6}$$

$$\tilde{\sigma}_{ti} \in \{0, 1\} \qquad t = 1, \ldots, T; i = 1, \ldots, N \tag{5.7}$$

$$w_0 = 0, I_0 = 0 \tag{5.8}$$

The objective function in the above formulation is the sum of transportation and inventory holding costs. The first constraint set represents the inventory balance equations. Inequality (5.2) corresponds to the balance constraints for type II vehicles. Note that, x_t may include type II vehicles that have been carried from earlier periods as well as those that become recently available in period t. Constraint set (5.2) is in the form of an inequality because some of the type II vehicles may not be utilized. Inequality (5.3) ensures that production capacity is not exceeded in any period. Equation (5.4) implies that the total demand for vehicles to be used in outbound shipment in a period is satisfied through either type I or type II vehicles. Equation (5.5) guarantees that every order is delivered before its deadline. Constraint sets (5.6) - (5.8) refer to nonnegativity, integrality and initial conditions of some variables, respectively. Here, $\mathcal{Z}^+$ is the set of positive integers.

The mathematical formulation introduced above considers the transportation costs and capacities explicitly in making the production planning decisions. Recall that, we show that the problem of interest as modeled herein is $\mathcal{NP}$-Hard in the strong sense in Chapter 4. In the next section, we present the other two approaches in detail.

5.2.2 Other Solution Approaches: Uncoordinated and Hierarchically-coordinated

Recall that, both the uncoordinated and the hierarchically-coordinated solutions rely on the production subproblem and the transportation subproblem. The formulations of these subproblems are decomposed from the Integrated Model and presented below.

Production Subproblem:

$$
\begin{aligned}
Min \quad & \textstyle\sum_{t=1}^{T} HI_t \\
I_t = I_{t-1} + \pi_t - \textstyle\sum_{i=1}^{N} \tilde{\sigma}_{ti} S_i & \quad t = 1, \ldots, T \\
\pi_t \leq P_t & \quad t = 1, \ldots, T \\
\textstyle\sum_{t=1}^{D_i} \tilde{\sigma}_{ti} = 1 & \quad i = 1, \ldots, N \\
\pi_t, I_t \in \{0\} \cup Z^+ & \quad t = 1, \ldots, T \\
I_0 = 0 & \\
\tilde{\sigma}_{ti} \in \{0, 1\} & \quad t = 1, \ldots, T; i = 1, \ldots, N
\end{aligned}
$$

Transportation Subproblem:

$$
\begin{aligned}
Min \quad & \textstyle\sum_{t=1}^{T} (C_1 y_t + C_2 x_t + W w_t) \\
x_t + w_t \leq A_t + w_{t-1} & \quad t = 1, \ldots, T \\
x_t + y_t = \textstyle\sum_{i=1}^{N} \lceil S_i / K \rceil \hat{\tilde{\sigma}}_{ti} & \quad t = 1, \ldots, T \\
x_t, y_t, w_t \in \{0\} \cup Z^+ & \quad t = 1, \ldots, T \\
w_0 = 0 &
\end{aligned}
$$

In the production subproblem, issues related to transportation are not considered. Similarly, the transportation subproblem does not have the production and inventory related costs and constraints. Note also that, the indicator variable showing whether a delivery is to be made for order i in period t, i.e., $\tilde{\sigma}_{ti}$, is a decision variable in the production subproblem whereas its value is an input to the transportation subproblem. In the transportation subproblem, $\hat{\tilde{\sigma}}_{ti}$ denotes a given value of $\tilde{\sigma}_{ti}$.

Now, we are ready to provide the detailed descriptions of the uncoordinated and the hierarchically-coordinated solutions. Before doing so, we define further piece of notation. Let $Cost_b$ and $\tilde{\sigma}^*_{ti}$ be the optimal values of the objective function and $\tilde{\sigma}_{ti}$, respectively, as an output of the production subproblem. This solution implies that the total vehicle requirement for deliveries in period t is

$\sum_{i=1}^{N} \lceil S_i/K \rceil \tilde{\sigma}^*_{ti}$. Therefore, the following description applies to the uncoordinated solution.

Description of The Uncoordinated Solution:

1. Solve the production subproblem.
2. Set $x_t = \min\left(\sum_{i=1}^{N} \lceil S_i/K \rceil \tilde{\sigma}^*_{ti}, A_t\right)$, $y_t = \sum_{i=1}^{N} \lceil S_i/K \rceil \tilde{\sigma}^*_{ti} - x_t$ and $w_t = 0$. Compute the resulting costs as follows:

$$Cost_u = Cost_b + \sum_{t=1}^{T} \left(C_1 y_t + C_2 x_t\right).$$

In comparison to the uncoordinated solution, the second step of the hierarchically-coordinated solution exploits the possibility of carrying type II vehicles from one period to the next to get better advantage of the cheaper transportation alternative. More specifically, holding a type II vehicle for a delivery that has to take place within the next β periods is less costly than using a type I vehicle for the same delivery, where

$$\beta = max\left\{b : C_2 + bW < C_1,\ b \in \{0\} \cup Z^+\right\}. \tag{5.9}$$

As it will be discussed in Section 5.3, the value of β is critical as an input to our proposed algorithm for the optimal solution of the transportation subproblem. Therefore, it is also utilized by the following algorithm for obtaining the hierarchically-coordinated solution and the resulting cost.

Description of The Hierarchically-Coordinated Solution:

1. Solve the production subproblem and do the following initialization of variables.

   (a) For $t = 1$ to $t = T$ and for $i = 1$ to $i = N$, set $\hat{\tilde{\sigma}}_{ti} = \tilde{\sigma}^*_{ti}$.

(b) Compute the value of β using Expression (5.9).

2. Solve the transportation subproblem given $\hat{\tilde{\sigma}}_{ti}$ and β. Compute $Cost_h$ as the summation of the optimal costs of the two subproblems (i.e., $Cost_h = Cost_b + Cost_t(\hat{\tilde{\sigma}}_{ti})$).

Here, $Cost_t(\hat{\tilde{\sigma}}_{ti})$ refers to the optimal objective function value of the transportation subproblem given the delivery dates of orders as implied by the optimal solution of the production subproblem.

5.3 Analysis of the Subproblems

The solutions of the production subproblem and/or the transportation subproblem are utilized in the uncoordinated and the hierarchically-coordinated solutions. Furthermore, the tabu search heuristic that will be described in Section 5.4 is based on solving these two problems optimally for given delivery dates. Therefore, we analyze these subproblems further in this section. We start with establishing the status of the production subproblem in the next theorem. Then, we present polynomial time algorithms for obtaining optimal solutions of the two subproblems given the delivery dates of orders.

Theorem 5.1 *The production subproblem (production planning problem without transportation considerations), is $\mathcal{NP}$-Hard in the strong sense.*

Proof: Proof is done by a reduction from 3-Partition (3P) problem, and note that, the production planning problem without transportation considerations is clearly in $\mathcal{NP}$. 3P is defined as follows:

INSTANCE: Set $\mathcal{G}$ of $3t$ elements, a bound $B \in Z^+$, and a size $s(a) \in Z^+$ for each $a \in \mathcal{G}$ such that $B/4 < s(a) < B/2$ and such that $\sum_{a\in\mathcal{G}} s(a) = tB$.

QUESTION: Can $\mathcal{G}$ be partitioned into t disjoint sets $\mathcal{G}_1, \mathcal{G}_2, ..., \mathcal{G}_t$ such that $\sum_{a \in \mathcal{G}_\tau} s(a) = B$ for $\tau = 1, 2, .., t$ (note that each $\mathcal{G}_\tau$ must therefore contain exactly three elements from $\mathcal{G}$)?

REDUCTION: Take an arbitrary instance of 3P. The corresponding instance of our problem is constructed as follows: for each element a in set $\mathcal{G}$ define an order with size $S_a = s(a)$ (i.e., $N = |\mathcal{G}|$) . Set $T = t, H = 1, P_t = B$, and, for each $a = 1, 2, \ldots, N$ set $D_a = T$. We will show that there is a solution to 3P if and only if there is a solution to our problem with cost less than or equal to $z^* = 0$.

Assume that there is a solution to our problem with cost z that is less than or equal to 0. Thus, no inventory holding cost is incurred. Since there are $3t$ orders to be satisfied with a total size of tB, and total production capacity of the facility is equal to total demand ($\sum_{t=1}^{T} P_t = tB$), the total number of items produced at each period is equal to B. This means that, three orders with total size equal to P are completed and delivered at each period, without any inventory held at the facility. Now construct a solution to 3P as follows: for all orders produced and delivered in period τ, put the corresponding element in set $\mathcal{G}$ into $\mathcal{G}_\tau$. As the size of orders $S_a = s(a)$, for each disjoint set $\mathcal{G}_\tau$, $\sum_{a \in \mathcal{G}_\tau} s(a) = B$ $(\tau = 1, 2, ..., t)$.

If there is a solution to 3P, construct a solution to our problem instance as follows: for each disjoint set $\mathcal{G}_\tau, \tau = 1, 2, ..., t$, produce and deliver all the items of order $a \in \mathcal{G}_\tau$ in period τ. Similar reduction with the previous case implies that the solution has a cost of $z = 0$. ■

Now, let us consider the two subproblems given the delivery dates of all orders. Note that, it is always possible to obtain a feasible solution to the transportation subproblem simply by using the type I vehicles, which are plentiful. The production subproblem, on the other hand, may not be feasible depending on the delivery dates given. More specifically, if the total size of orders that must be completed and sent by time t is greater than the cumulative production capacity until that period, the production subproblem is infeasible. We propose the following algorithm for finding the optimal solution to the production subproblem given that $\tilde{\sigma}_{ti} = \hat{\tilde{\sigma}}_{ti}$. With a slight change of notation, $Cost_b(\hat{\tilde{\sigma}}_{ti})$ is used to refer to the optimal costs of the production subproblem under given delivery dates of orders.

Algorithm 5.1 Optimal Solution of the Production Subproblem Given the Delivery Dates:

1. *Do the following initialization of variables.*
   (a) *Set* $Cost_b(\hat{\tilde{\sigma}}_{ti}) = 0$.
   (b) *For* $t = 1$ *to* $t = T$, *set* $F_t = \sum_{i=1}^{N} \hat{\tilde{\sigma}}_{ti} S_i$.
2. *For* $t = T$ *down to* $t = 1$
   (a) *Determine the production amount in period* t *using* $\pi_t = \min\{F_t, P_t\}$.
   (b) *If* $F_t > \pi_t$,
      i. *If* $t = 1$, *then there is no feasible solution. Stop and exit.*
      ii. *If* $t \neq 1$, *do the following:*
         A. $F_{t-1} = F_{t-1} + F_t - \pi_t$.
         B. *Update the optimal costs using* $Cost_b(\hat{\tilde{\sigma}}_{ti}) = Cost_b(\hat{\tilde{\sigma}}_{ti}) + (F_t - \pi_t) \times H$.

In the above algorithm, F_t is the amount that has to be produced within $[1, t]$ for the deliveries that will take place within $[t, T]$. The algorithm follows a backwards recursive path to find the production quantity in each period and the resulting cost. If $F_t \leq P_t$, then there is enough capacity in the current period to produce for the deliveries in $[t, T]$. Therefore, F_t amount of this capacity is utilized right away to make timely deliveries without increasing inventory holding costs. If $F_t > P_t$ and $t = 1$, then the production capacity in the first period is not enough to make timely deliveries within $[1, T]$, and therefore, the production subproblem is infeasible for the given delivery dates. Otherwise, if $F_t > P_t$ and $t \neq 1$, the production capacity in the current period t is not enough to satisfy the delivery amount within $[t, T]$, however, there is possibility to satisfy $F_t - P_t$ of this quantity with the production in earlier periods. In this case, F_{t-1} is increased by as much as $F_t - P_t$. Since, at this point, we know that $F_t - P_t$ number of items will be held in the inventory for at least a period, the total costs are updated to incorporate the holding cost of this much inventory for one period. It can be observed that the above algorithm runs in $O(T)$.

The following algorithm solves the transportation subproblem optimally for given delivery dates $\hat{\hat{\sigma}}_{ti}$.

Algorithm 5.2 Optimal Solution of the Transportation Subproblem:

1. *Do the following initialization of variables.*

   (a) *Set* $Cost_t(\hat{\hat{\sigma}}_{ti}) = 0$.

   (b) *For* $t = 1$ *to* $t = T$, *set* $x_t = 0$, $w_t = 0$, $G_t = \sum_{i=1}^{N} \lceil S_i/K \rceil \hat{\hat{\sigma}}_{ti}$ *and* $z_t = A_t$.

2. *For* $b = 0$ *to* β

   For $t = 1$ *to* $T - b$

   (a) *Determine the number of type II vehicles among those that become available in period* t, *to be used in period* $t + b$. *That is, compute* $\Delta_t = min\{G_{t+b}, z_t\}$.

   (b) *Update the number of vehicles needed for deliveries in period* $t + b$ *using* $G_{t+b} = G_{t+b} - \Delta_t$.

(c) Decrease the number of type II vehicles that are available in period t by Δ_t (that is, set $z_t = z_t - \Delta_t$).

(d) Increase the number of type II vehicles utilized in period $t+b$ by Δ_t (that is, set $x_{t+b} = x_{t+b} + \Delta_t$).

(e) If $b > 0$, for $\tau = 0$ to $(b-1)$ set $w_{t+\tau} = w_{t+\tau} + \Delta_t$.

3. For $t = 1$ to $t = T$,

(a) Set $y_t = G_t$.

(b) Update the optimal costs using $Cost_t(\hat{\hat{\sigma}}_{ti}) = Cost_t(\hat{\hat{\sigma}}_{ti}) + w_t \times W + y_t \times C_1 + x_t \times C_2$.

In the above algorithm, b represents the number of periods that a type II vehicle is held. Expression (5.9) implies that it is not optimal to hold a type II vehicle for more than β number of periods. Therefore, b ranges from 0 to β. Within steps 2.(a)–2.(e) of the algorithm, first, among the type II vehicles that have been on hold for the last b periods, the number that will be used in period $t + b$ is found. Later, the overall need for vehicles in period $t + b$ (i.e., G_{t+b}), the number of type II vehicles that are available in period t (i.e., z_t), the number of type II vehicles used in period $t + b$ (i.e., x_{t+b}), and the inventory of vehicles throughout periods t to $t+b-1$ (i.e., $w_{t+\tau}$ for $\tau = 0, \ldots, b-1$) are updated. The algorithm runs steps 2.(a)–2.(e) in such a sequence of t and b values that type II vehicles are used in the most immediate period that a need for vehicles arises. This way, holding cost of vehicles is minimized along with the total transportation costs. In the last step of the algorithm, a plan is made to satisfy the remaining need for vehicles in any period using type I vehicles, and the cost is updated.

We conducted an extensive numerical analysis to compare the three solution approaches introduced in Section 5.2. The results, which are discussed in more detail in Section 5.5, show that the total costs of the centrally-coordinated solution can be less than that of the uncoordinated solution by as much as its 75% and less than that of the hierarchically-coordinated solution by as much as its

58%. Due to such results derived from the computational analysis, we conclude that significant savings can be achieved if the centrally-coordinated solution is used instead of the other two approaches. Furthermore, all the three approaches rely on solving problems that are $\mathcal{NP}$-Hard in the strong sense. Therefore, the uncoordinated solution and the hierarchically-coordinated solution do not provide a computational advantage over the centrally-coordinated solution. These results establish a need for a heuristic that can be used in practice to make the production planning and transportation decisions jointly as in the centrally-coordinated solution. In the next section, we propose a meta-heuristic that utilizes the tabu-search technique for this purpose.

5.4 Tabu Search

The uncoordinated and the hierarchically-coordinated solutions are based on our observation that production planning decisions are made prior to transportation decisions in many real-life practices. In these two approaches, first the production subproblem is solved optimally. Then, transportation arrangements are put together to comply with the production plan that minimizes inventory holding costs. In the hierarchically-coordinated solution, the transportation subproblem is also solved optimally. As both the approaches focus on sequentially minimizing the two cost components, total costs are not necessarily optimized. The tabu-search heuristic that we propose is also based on the two subproblems. However, as opposed to the uncoordinated and the hierarchically-coordinated solutions, it makes use of the solutions of these subproblems simultaneously rather than sequentially, and aims to minimize the total costs rather than individual cost components.

Recall that, the joint production and transportation planning problem defined in Section 5.2 requires the determination of the following: production amount in each period, delivery times of orders and the number of both types of vehicles to be used for deliveries. If the delivery dates of orders are known, the production amounts and the vehicles used in each period can be determined optimally using Algorithm I and Algorithm II, respectively. This structure of the joint problem enables us to define a solution by an array of size N, where the i^{th} element stores the information regarding the delivery period of order i. The tabu search begins with an initial seed solution in which the delivery time of each order is set to its deadline. At each iteration, a neighborhood of the current seed is generated and all the solutions in the neighborhood are evaluated for their costs. The cost of a solution is simply the summation of the optimal objective function values of the two subproblems. The solution with the least cost in the neighborhood and is not tabu, is selected as the new seed, and a new iteration begins. The search for the best solution continues until the stopping criterion is met.

The neighborhood of a seed is generated by changing the delivery dates of all orders one-order-at-a-time, keeping the delivery dates of the remaining orders as they currently are. In changing the delivery date of order i, we consider a feasible range of values, that is $[E_i, D_i]$. E_i here represents the earliest feasible delivery date of order i. Its value is computed by taking into account the production capacity over time and the sizes of all orders that have to be completed before D_i. Defining $\delta_{t,i}$ as the total size of all orders apart from order i, that have to be completed in or before period t, we propose the following procedure to obtain values for E_i for all i.

Algorithm 5.3 Computing Values for Earliest Delivery Dates (E_i):

For $i = 1$ to $i = N$, do the following:

(a) For $t = 1$ to $t = T$, initialize $\delta_{t,i}$ as $\delta_{t,i} = \sum_{j:j \neq i, D_j \leq t} S_j$.

(b) For $t = T - 1$ down to $T = 1$, update the value of $\delta_{t,i}$ using the following:

$$\delta_{t,i} = \max\{\delta_{t+1,i} - P_{t+1}, \delta_{t,i}\}. \quad (5.10)$$

(c) *Set* $E_i = \min\left\{t : \sum_{k=1}^{t} P_k - \delta_{t,i} \geq S_i\right\}$.

In order to describe why E_i, as found in the above algorithm, is the earliest delivery date for order i, let us start elaborating from the last step of the algorithm. $\sum_{k=1}^{t} P_k - \delta_{t,i}$ is the remaining of the total production capacity in periods $1, \ldots, t$ that can be reserved for order i. If $\sum_{k=1}^{t} P_k - \delta_{t,i} < S_i$ for some period t, then it is not possible to finish the production of order i before or in period t. If order i can be delivered before or in period t, then it must be true that $\sum_{k=1}^{t} P_k - \delta_{t,i} \geq S_i$, and therefore, in order to find the earliest delivery date, we choose the smallest among all such t. The $\delta_{t,i}$ values for all t are found in the first and the second steps of the algorithm. Initially, $\delta_{t,i}$ is set to $\sum_{j:j\neq i, D_j \leq t} S_j$, that is the total size of all orders other than i that have deadlines smaller than or equal to t. Then, $\delta_{t,i}$ values are updated by tracing backwards from $t = T - 1$ to all periods $T-2, \ldots, 1$. The update is done using Equation (5.10). In this equation, if the maximum is given by $\delta_{t+1,i} - P_{t+1}$, then–given that only $\delta_{t,i}$ units are produced within the first t periods for orders other than i–the production capacity in period $t+1$ is not enough to make timely future deliveries. Therefore, the excess requirement (i.e., $\delta_{t+1,i} - P_{t+1} - \delta_{t,i}$) also needs to be satisfied through the production in the first t periods.

The job of which delivery date has been changed to form the newly selected seed at each iteration, is added to the tabu list. Therefore, a solution in a neighborhood is considered as tabu if this solution is constructed by changing the delivery date of a job that is in the tabu list. However, we use the following rule as an aspiration criterion: If the best solution in the neighborhood has less cost than that of the best solution so far, then it is taken as the new seed even if it is tabu. In the next section, we present our numerical experimentation with the three solution approaches and the tabu search heuristic. As it will be discussed in this section, we use varying tabu lengths for instances with different order sizes.

5.5 Computational Analysis

In this section, we first report the results of a computational analysis to quantify the savings due to the centrally-coordinated solution and to examine how the resulting costs of the three approaches differ under varying problem parameters. Then, we present some results by a comparison of the tabu search heuristic with the centrally-coordinated solution, that is the optimal solution of the Integrated Model. More specifically, we seek answers to the following questions:

- How do the inventory holding cost (i.e., H) and the vehicle holding cost (i.e., W) affect the outcomes of the three solution approaches? How do the results change with varying order sizes?
- How does the availability pattern of the type II vehicles affect the differences in costs? Here, we consider both the average number of type II vehicles that are available in each period during the planning horizon (that is the *average number of type II vehicles per period*) and the degree of changes in their availability from one period to another (that is the *period-to-period variability of the number of type II vehicles*).
- What is the impact of the production capacity on the outcomes of the different solutions?
- What is the worst case and the average performance of the tabu search heuristic as compared to the centrally-coordinated solution? How do these results change under varying problem parameters?

As discussed in Section 5.2, centrally-coordinated solution leads to the optimal costs and the hierarchically-coordinated solution is an improvement over the uncoordinated solution. Therefore, it is true for any instance that $Cost_u \geq Cost_h \geq Cost_c$. However, in light of the first three questions above, our objective is to examine the magnitudes of the differences between the cost values under relevant combinations of parameter settings. With this objective, we define the following measures for a problem instance:

$$\Delta_{u,h} = \frac{Cost_u - Cost_h}{Cost_u} \times 100\%$$

$$\Delta_{h,c} = \frac{Cost_h - Cost_c}{Cost_h} \times 100\%$$

$$\Delta_{u,c} = \frac{Cost_u - Cost_c}{Cost_u} \times 100\%$$

Note that, each of the $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ values refers to the percentage cost improvement of one solution approach over another. Given that the mathematical models for the production subproblem and the centrally-coordinated solution are solved optimally, we have $\Delta_{u,h} \geq 0$, $\Delta_{h,c} \geq 0$ and $\Delta_{u,c} \geq 0$. In order to test the performance of the heuristic, we consider how the resulting cost for an instance compares to the lower bound provided by GAMS. Before we proceed with a detailed discussion of these results, we first present the experimental design.

5.5.1 Experimental Design

Considering the questions highlighted at the beginning of this section, we use the following six parameters as factors: vehicle holding cost (W), inventory holding cost (H), production capacity, order sizes, average number of type II vehicles per period, and period-to-period variability of the number of type II vehicles. We do not take the length of the planning horizon, vehicle costs and capacities as factors of analysis, and therefore, we keep their values fixed as $T = 1$ month, $C_1 = 1000$, $C_2 = 100$ and $K = 100$. In what follows, we describe the factor levels used in experimentation and how they are generated.

Vehicle holding cost: We consider five levels for this factor and generate them around the value of β, which is the maximum number of periods that holding a vehicle is justified. It can be observed from Expression (5.9) that, there exists a unique value of β that corresponds to every value of W. Furthermore, the hierarchically-coordinated solution explicitly utilizes this value. A commonly used value of β by the industry practice that has motivated this study, is equal to 4. Therefore, we take low, medium and high values of β as 2, 4 and 8, respectively. As β is an important parameter for the purposes of this study, our analysis also considers its extreme values, those are $\beta = 0$ and $\beta = 32$. Note that, the values of W that correspond to the different levels of β are reported in Table 5.2.

Inventory holding cost: Five levels of H are generated around a factor that we refer to as α and define as follows:

$$\alpha = max\left\{a : C_1 > C_2 + a * H * K,\ a \in \{0\} \cup Z^+\right\}. \qquad (5.11)$$

In our setting, when an order is ready to be delivered, there clearly exists a tradeoff between delivering it right away or holding it in the inventory so that a less costly delivery option that will be available in a future period can be used. α shows the maximum number of periods that a full truck load of items can be stocked at the expense of inventory holding costs, and yet, the savings from transportation costs exceed these extra costs. Expression (5.11) implies that there exists a unique value of α for each H. We consider 10, 4, 2, 1 and 0.25 as different levels of H, which correspond to α values of 0, 2, 4, 8 and 32, respectively.

Production capacity: The length of the production planning horizon is taken as one month, equivalent to T =30 days. A day is considered as a period and it is assumed that there are six working days followed by a no-production day. Therefore, there are 26 production periods within the planning horizon. Although there is no production in the remaining 4 days, costs are incurred for carrying inventories of items and inventories of vehicles over these periods. The production capacity over the production periods, is constant. We consider two levels for the production capacity, those are 1000 units/day and 1500 units/day. As it will be discussed later, we generate the order sizes in such a way that the sizes of all orders to be produced sum up to 24000 units. This being said, the average load of the system–defined as total size of all orders/total production capacity–is approximately 90% in the low production-capacity case (i.e., $24000/(26 \times 1000)$), and is approximately 60% in the high production-capacity case (i.e., $24000/(26 \times 1500)$).

Order sizes: Three different sets of orders are used in combination with other factors. All orders in a set have small, medium or large sizes. An order's size is identified as one of these depending on how it compares to the vehicle capacity (i.e., $K = 100$) and to the low level of the daily production capacity (i.e., $P_t = 1000$). Mainly, small-size orders have less than 100 items, medium-size orders have more than 100 items but less than 1000 items, and large-size orders have more than 1000 items. The number of items in a low-size order is taken as a uniformly distributed random variable between 10 and 100. The number of items in a medium-size order is generated from a uniform distribution ranging from 100 to 1000. The sizes of orders in the third set are generated using a uniformly distributed random variable between 1000 and 4000. The total number of items over all orders in a set is kept at 24000 units. This sum is maintained by reducing the number of items in the first order that makes the total size greater than 24000. As a result, the number of orders in the sets of low-size, medium-size and large-size orders turns out to be 450, 45 and 10, respectively.

Availability pattern of the type II vehicles: Average number of type II vehicles per period and its period-to-period variability describe the pattern of arrivals. These two attributes are taken as factors of analysis and two levels are considered for each. The number of type II vehicles in each period is generated using a discrete uniform distribution, and the availability pattern of the type II vehicles is controlled using the mean and coefficient of variation (CV) of this random variable. The average number of type II vehicles per day assumes either a value of 2.5 vehicles/day or 7.5 vehicles/day. The bounds of the uniformly distributed random variable corresponding to the number of type II vehicles per day, are chosen in such a way that the coefficient of variation is either 0.2 or 0.6. The parameters of the uniformly distributed random variable used to create different availability patterns are reported in Table 5.1. For example, an average number of 2.5 vehicles/day combined with 0.2 as the coefficient of variation represents a case where type II vehicles are less available but arrive in a steady stream. Similarly, an average number of 7.5 vehicles/day combined with 0.6 as the coefficient of variation represents a case where type II vehicles are more available in number, but their availability shows more variability among different days.

Table 5.1: Parameter settings for arrival patterns of type II vehicles

	Period-to-period variability	
Average number of vehicles per day	$CV = 0.2$	$CV = 0.6$
Low (2.5 vehicles/day)	[2,3]	[0,5]
High (7.5 vehicles/day)	[5,10]	[0,15]

The factor levels used in the analysis and described above in detail, are summarized in Table 5.2. In total, there are 600 different experimental settings. For each combination of factor levels, 10 random instances are solved.

Table 5.2: Experimental design

Design Parameter	Levels
Vehicle holding cost	$\beta = (0, 2, 4, 8, 32)$ or $W = (1000, 400, 200, 100, 25)$
Inventory holding cost	$\alpha = (0, 2, 4, 8, 32)$ or $H = (10, 4, 2, 1, 0.25)$
Production capacity	High (1500), Low (1000)
Average # of type II vehicles per day	Low (2.5), High (7.5)
Variability of the # of type II vehicles per day	$CV = 0.2, CV = 0.6$
Order sizes	Low ∼U(10,100), Medium ∼U(100, 1000), High ∼U(1000, 4000)

5.5.2 Comparison of the Three Solution Approaches

In order to make a comparison of the three solution approaches, all the mathematical models discussed in Section 5.2 were coded using GAMS version 22.6 (using CPLEX 11.0 as solver) and run on a Linux box with 8 GBs of physical memory, running Debian Lenny (5.0.7) on 8 x Intel Xeon E5430 processors at 2.66 GHz. The solution time of each model for an instance was limited to 36000 CPU seconds (10 hours of CPU time). The model for the production subproblem was solved with less than 0.02% optimality gap in 5960 out of 6000 instances. The Integrated Model was solved with less than 0.11% optimality gap for 5986 out of 6000 instances. In the remaining instances, GAMS failed to provide a solution due to memory interrupt. For four instances, neither Integrated Model nor production subproblem can be solved. Thus a total of 50 instances are not solved. These instances are modified and re-run so that 0.2% optimality gap is accepted as termination criterion. As a result, all instances are solved with at most 0.2% optimality gap.

The average solution time for the centrally coordinated solution procedure is 496 CPU seconds. The average solution times for the uncoordinated and hierarchically-coordinated procedures are 355 CPU seconds and the difference between the solution times of these procedures is insignificant. This is mainly because both solution procedures start with solving the production subproblem optimally, which is NP-Hard in the strong sense. Note that the difference between the uncoordinated and hierarchically-coordinated solution procedures is how the underlying transportation subproblem is solved, i.e., it is solved heuristically in the former whereas optimally in the latter. Note also that, the transportation subproblem can be solved in polynomial time using Algorithm 5.2. Thus, the solution times for solving the transportation problem either heuristically or optimally requires approximately the same time. Although the solution times for both uncoordinated and hierarchically-coordinated procedures are approximately the same, and the optimal objective function value of the hierarchically-coordinated solution procedure is always less than or equal to that of the uncoordinated solution procedure, we still include the uncoordinated solution procedure in our analysis for two reasons: i) to compare its solution with hierarchically-coordinated solution and assess the value of optimal usage of transportation opportunities and ii) the manufacturing company which the problems are inspired, use the uncoordinated solution procedure in practice. We start our analysis with the effects of inventory and vehicle holding cost.

5.5.2.1 The Effects of Inventory Holding Cost and the Vehicle Holding Cost

In this section, we report our observations about how the inventory holding cost and the vehicle holding affect the outcomes of the three solution approaches and how the results change with varying order sizes. In order to perform this analysis, we look into the averages of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ over all instances of the same size orders. The results for small-size, medium-size and large-size orders are summarized in Table 5.3, Table 5.4 and 5.5, respectively. The values of α and β change along the rows and the columns of these tables. In each cell, the averages of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ over all instances with the corresponding α and β values, are noted. For example, the entries in the second row, second column of Table 5.3 show that, over all instances with small-size orders, $\alpha = 0$ and $\beta = 0$, the averages of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ amount to 0.00%, 10.58% and 10.58%, respectively.

It can be observed from Table 5.3, Table 5.4 and Table 5.5 that, $\Delta_{u,h} = 0$ when $\beta = 0$. This is because the first steps of the uncoordinated solution and the hierarchically-coordinated solution are the same, but the hierarchically-coordinated solution entails the type II vehicles to be carried to future periods as long as the savings justify the increase in vehicle holding costs. In case of $\beta = 0$, it is less costly to use a type I vehicle in any period instead of carrying a type II vehicle from an earlier period. Therefore, the hierarchically-coordinated solution

Table 5.3: Average of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ values in case of small-size orders

	$\beta = 0$	$\beta = 2$	$\beta = 4$	$\beta = 8$	$\beta = 32$
$\alpha = 0$	0.00% 10.58% 10.58%	2.21% 8.60% 10.58%	3.19% 7.73% 10.64%	3.69% 7.49% 10.86%	4.06% 7.68% 11.38%
$\alpha = 2$	0.00% 11.28% 11.28%	2.23% 9.30% 11.28%	3.18% 8.42% 11.28%	3.66% 7.97% 11.29%	4.02% 7.84% 11.49%
$\alpha = 4$	0.00% 11.45% 11.45%	2.20% 9.50% 11.45%	3.15% 8.63% 11.45%	3.63% 8.18% 11.45%	3.99% 7.87% 11.48%
$\alpha = 8$	0.00% 11.43% 11.43%	2.20% 9.48% 11.43%	3.16% 8.60% 11.43%	3.65% 8.14% 11.43%	4.01% 7.79% 11.43%
$\alpha = 32$	0.00% 11.52% 11.52%	2.26% 9.52% 11.52%	3.25% 8.60% 11.52%	3.75% 8.13% 11.52%	4.12% 7.77% 11.52%

reduces to the uncoordinated solution, and hence $\Delta_{u,h} = 0$.

Examining Table 5.3, Table 5.4 and Table 5.5, we observe that the maximum of the average $\Delta_{u,h}$ values is 4.12%, 16.77% and 40.30% in case of small-size, medium-size and large-size orders, respectively. These values are realized when β attains its highest value. Excluding the values when $\beta = 0$, the minimums are 2.20%, 5.91%, 6.90%, and these values are realized when $\beta = 2$. Furthermore, the average $\Delta_{u,h}$ values increase as β increases in each row of Table 5.3, Table 5.4 and Table 5.5. This implies that, as it becomes less costly to carry type II vehicles over periods, the hierarchical solution uses this opportunity to reduce the costs of the uncoordinated solution, and the potential of improvement is the highest when the order sizes are the largest.

The maximum of the average $\Delta_{h,c}$ values is 11.52%, 25.50% and 9.23% in case of small-size, medium-size and large-size orders, respectively. These values are realized when α attains its highest value. The minimums are 7.49%, 10.74%, 2.42%, and they coincide with the cases having $\alpha = 0$. Furthermore, the average $\Delta_{h,c}$

Table 5.4: Average of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ values in case of medium-size orders

	$\beta = 0$	$\beta = 2$	$\beta = 4$	$\beta = 8$	$\beta = 32$
$\alpha = 0$	0.00% 11.20% 11.20%	5.91% 10.97% 15.97%	10.42% 11.22% 20.04%	13.75% 10.93% 22.66%	16.39% 10.74% 24.94%
$\alpha = 2$	0.00% 18.62% 18.62%	6.12% 14.96% 19.78%	10.74% 13.12% 21.88%	14.13% 11.97% 23.79%	16.77% 11.51% 25.68%
$\alpha = 4$	0.00% 22.10% 22.10%	6.20% 17.97% 22.60%	10.75% 14.83% 23.32%	14.03% 12.89% 24.41%	16.54% 12.22% 26.00%
$\alpha = 8$	0.00% 23.78% 23.78%	6.12% 19.84% 24.25%	10.74% 16.57% 24.85%	14.13% 13.57% 24.96%	16.77% 12.20% 26.25%
$\alpha = 32$	0.00% 25.50% 25.50%	6.11% 21.53% 25.74%	10.75% 18.08% 25.91%	14.02% 15.30% 26.23%	16.53% 13.07% 26.70%

values increase as α increases in each row of Table 5.4 and Table 5.5, and in most of the rows of Table 5.3. This implies that, the performance of the hierarchically-coordinated solution approaches to that of the centrally-coordinated solution as the inventory holding cost decreases. It is worthwhile to note that Table 5.3 exhibits some exceptions. For example, average $\Delta_{h,c}$ is 9.50% when $\alpha = 4$ and $\beta = 2$ whereas it is equal to 9.48% when $\alpha = 8$ and $\beta = 2$. We believe this is because inventory holding costs constitute a lesser portion of the total costs in comparison to transportation costs in case of small-size orders. Therefore, average $\Delta_{h,c}$ is not much sensitive to changes in α, and hence these exceptions are not representative of the general behavior.

Table 5.3, Table 5.4 and Table 5.5 suggest that the maximum of the average $\Delta_{u,c}$ values is 11.52%, 26.70% and 44.59% in case of small-size, medium-size and large-size orders, respectively. These values are realized when both α and β are at their highest values. The minimums are 10.58%, 11.20%, 2.70%, and these values are realized when both α and β assume their smallest values. Furthermore,

Table 5.5: Average of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ values in case of large-size orders

	$\beta = 0$	$\beta = 2$	$\beta = 4$	$\beta = 8$	$\beta = 32$
$\alpha = 0$	0.00% 2.70% 2.70%	6.90% 2.42% 9.07%	13.51% 2.62% 15.64%	18.23% 2.85% 20.34%	22.11% 3.11% 24.22%
$\alpha = 2$	0.00% 4.02% 4.02%	9.47% 3.66% 12.65%	18.55% 4.04% 21.60%	24.88% 4.66% 27.98%	30.06% 5.58% 33.32%
$\alpha = 4$	0.00% 5.36% 5.36%	10.92% 5.06% 15.22%	21.41% 5.35% 25.25%	28.70% 5.83% 32.32%	34.62% 6.98% 38.37%
$\alpha = 8$	0.00% 6.74% 6.74%	11.79% 6.55% 17.28%	23.12% 6.76% 27.83%	31.02% 7.08% 35.24%	37.43% 8.20% 41.61%
$\alpha = 32$	0.00% 8.59% 8.59%	12.60% 8.54% 19.70%	24.81% 8.52% 30.57%	33.37% 8.48% 38.11%	40.30% 9.23% 44.59%

average $\Delta_{u,c}$ values are nondecreasing in β at all order sizes, and increasing in α when orders are medium or large size. As it can be seen in Table 5.3, they are predominantly increasing in α when orders are small size, but there are some exceptions. We again attribute this to the fact that not many inventories are held in case of small-size orders, and therefore, the behavior of average $\Delta_{u,c}$ with respect to α is not well observed. As a result of these observations, we conclude that the savings due to the centrally-coordinated solution are in fact significant, and the percentage savings over the uncoordinated solution increases as inventory holding cost and vehicle holding cost become smaller.

5.5.2.2 The Effects of the Availability Pattern of the Type II Vehicles

In this section, we discuss the results of our computational study within the context of the second objective, that is to determine how the availability pattern of the type II vehicles affect the differences in costs. For this purpose, we look into the averages of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ over all instances with the same arrival pattern. Recall that, the arrival pattern of the type II vehicles is identified by two attributes, the mean and the variability of the number of type II vehicles per day. The results for four different availability patterns are summarized in Table 5.6. The values of the two attributes change along the rows and the columns of these tables. In each cell, the averages of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ over all instances with the same availability pattern of type II vehicles, are noted.

Table 5.6: Average of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ values under different arrival patterns of type II vehicles

Average # of Vehicles/Day	Day-to-day variability	
	$CV = 0.2$	$CV = 0.6$
Low (2.5 vehicles/day)	4.26% 2.02% 6.23%	4.81% 2.97% 7.68%
High (7.5 vehicles/day)	14.90% 15.38% 28.40%	16.38% 19.46% 33.12%

It can be observed from Table 5.6 that percentage improvements of both the hierarchically-coordinated solution and the centrally-coordinated solution over the uncoordinated solution, as represented by $\Delta_{u,h}$ and $\Delta_{u,c}$, respectively, increase in the average number of type II vehicles available. This implies that the value of coordination is higher when the opportunity of savings due to effective utilization of the different transportation options is higher. Observe also that $\Delta_{h,c} = 2.02\%$ when $CV = 0.2$, and $\Delta_{h,c} = 2.97\%$ when $CV = 0.6$. This suggests; although the opportunity of savings is limited at low levels of the average number of type II vehicles per day, the hierarchically-coordinated solution performs almost as well as the centrally-coordinated solution in capturing this opportunity. When the results in Table 5.6 are examined for the variability in number of type II vehicle arrivals, we observe that the value of coordination becomes higher as the dispersion increases. Also, the discrepancy between the performances of the centrally-coordinated solution and the hierarchically-coordinated solution grows with increased variability.

5.5.2.3 The Effects of the Production Capacity

In order to see the effects of the production capacity on the performance of the three solution approaches, we investigate how the averages of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ change at different production levels. The results are summarized in Table 5.7.

Table 5.7: Average of $\Delta_{u,h}$, $\Delta_{h,c}$ and $\Delta_{u,c}$ values at varying production capacities

Production capacity	
Low	High
10.51%	9.66%
4.59%	15.33%
14.64 %	23.07%

The percentage improvement of the hierarchically-coordinated solution over the uncoordinated solution, as represented by $\Delta_{u,h}$, decreases as the production capacity increases. Note that, increase in the production capacity enlarges the feasible region of the production subproblem, reducing inventory holding costs in the first phase. This, however, results in a possible increase in the transportation costs in the second phase. Consider an extreme case where there is no limit on the production capacity, hence, all orders can be produced at the first period and the transportation subproblem becomes trivial. Thus, it is intuitive that the percentage improvement of the hierarchically-coordinated solution over the uncoordinated solution decreases as the production capacity increases.

The percentage improvements of the centrally-coordinated solution over both the hierarchically-coordinated and uncoordinated solutions, as represented by $\Delta_{h,c}$ and $\Delta_{u,c}$, respectively, significantly increase in the production capacity. Recall that, increase in production capacity enlarges the feasible region of the centrally-coordinated solution procedure and production subproblem. Thus, cost of the centrally-coordinated solution and optimal objective function value of the production subproblem tends to decrease as the feasible region enlarges. This implies that the cost reduction in the first phase of the uncoordinated and hierarchically coordinated solutions are much less then cost reduction of the centrally-coordinated solution procedure.

5.5.2.4 Performance of the Tabu Search Heuristic

In light of the fourth question of interest, we tested the performance of the heuristic with respect to the centrally-coordinated solution using the 6000 instances described in Section 5.5.1. Since the Integrated Model for obtaining the centrally-coordinated solution cannot be solved optimally for all instances, we compared the cost of the heuristic solution to the lower bound provided by GAMS. In obtaining heuristic solutions for instances with low, medium and high order sizes, we set the tabu length as 200, 25 and 7, respectively. We also used the following scheme for terminating the search: if the algorithm fails to improve the best solution for 2000 consecutive iterations, the seed is replaced with the best solution so far, however, the tabu list is not changed. The algorithm is terminated if this happens 100 times or total search time exceeds 60 CPU seconds. As the initial seed, we set the delivery times of all orders to their deadlines, in order to guarantee to start from a feasible solution. In the analysis, the solutions provided by tabu search heuristic are compared to lower bounds provided by the IP models. The percentage deviation of the heuristic solution is calculated by subtracting the lower bound from the heuristic solution and dividing the difference by the lower bound.

As a result of our experimentation, we observed that the tabu search performs quite well in general. In more than 37% of the instances (2 256 out of 6 000), tabu search terminated with an optimal solution. In approximately 90% of the instances (5 421 out of 6 000), the deviation between the cost of the heuristic solution and the lower bound was as much as 1% of the lower bound, and in approximately 99.7% of the instances (5 983 out of 6 000), the deviation was at most 5% of the lower bound. The average and the maximum percentage deviations were 0.31% and 10.13%. The maximum deviation was realized at an instance where vehicle holding cost is high (i.e., $\beta = 0$), inventory holding cost is high (i.e., $\alpha = 0$), production capacity is low, orders are of medium size, the number of type II vehicles per day is high on the average but shows variability among different days. In fact, after a detailed analysis of the results, we have observed that only two of the parameters have an impact on the performance of the heuristic, that is worth noting. Those are the order size and the variability in the number of type II vehicles. As Table 5.8 shows, the average and maximum deviation of the heuristic from the lower bounds is the most when orders are of medium-size, the mean and the variability of the number of type II vehicles are high.

Table 5.8: Average and maximum percentage deviation of the heuristic from the lower bounds, under different arrival patterns of type II vehicles and order sizes

Order Size	Day-to-day variability	Average # of Vehicles/Day	
		Low	High
Small-size	$CV = 0.2$	0.03% (0.25%)	0.22% (2.22%)
	$CV = 0.6$	0.07% (0.55%)	0.32% (3.70%)
Medium-size	CV = 0.2	0.08% (1.62%)	0.94% (5.46%)
	$CV = 0.6$	0.11% (1.36%)	1.62% (10.13%)
Large-size	$CV = 0.2$	0.04% (1.30%)	0.15% (3.29%)
	$CV = 0.6$	0.05% (1.19%)	0.15% (4.85%)

Note that, the performance of tabu search may be because of two reasons: the nature of the feasible region and initial solution, or the inherent properties of tabu search. If the performance of the proposed tabu search algorithm is due to the shape of the feasible region or the initial solution, a steepest descent algorithm with the same neighborhood would perform just as good as tabu search. In order to test it, we compared the results of the steepest descent and tabu search algorithms for the same initial solutions. The results are summarized in Table 5.9 where the numbers are the percentage cost improvement of tabu search algorithm with respect to steepest descent. Our analysis show that there is up to 77% cost decrease due to utilization of tabu search over the steepest descent algorithm. The rows of the table represent different order sizes, whereas the columns correspond to either the average number of type II vehicles (columns 2 and 3) or different levels of α (columns 4-8).

Table 5.9: Average percentage cost improvement of tabu search over steepest descent

	Average # of type II vehicles		α				
Order Size	Low	High	0	2	4	8	32
Low	4.73%	6.74%	15.36%	6.97%	3.70%	1.99%	0.66%
Medium	14.53%	40.86%	52.20%	36.71%	28.62%	23.57%	19.30%
High	9.54%	19.72%	33.82%	26.02%	20.64%	16.71%	13.04%

The percentage cost improvement of tabu search over steepest descent increases as the average number of type II vehicles increases. The values in the third column of the table are greater than the values in the second column. It can also be observed from Table 5.9 that the steepest descent algorithm performs almost as good as tabu search when α is high and order sizes are low. Note that, the percentage cost improvement of the tabu search algorithm over the steepest descent increases as α decreases and the orders are of medium size. The incremental performance of tabu search is the smallest for low order sizes and the largest for medium order sizes. This result may be due to the fact that, for medium order sizes, the inventory holding and transportation costs are balanced and steepest descent algorithms is quickly trapped at a local optimum.

As a result, we conclude that, the good performance of the heuristic algorithm is due to the inherent quality of the tabu search algorithm, especially for medium order sizes and when average number of type II vehicles and inventory holding costs are high.

Chapter 6

Conclusion

In this dissertation, we study integration of scheduling decisions in supply chains involving production as well as inbound and outbound transportation. Supply, production and delivery are among the key functions for manufacturing companies. In many traditional systems, these functions are managed independently. However, recent studies in supply chain management show that there is a significant opportunity for savings if the related decisions are made in an integrated manner (Thomas and Griffin [23], Dawande et al. [8]). Integration of decisions among the different stages and functions of the supply chain exists at different phases of planning. Examples include coordination of decisions in the following areas: innovation, pricing at the strategic level; inventory control, lot sizing at the tactical level; scheduling at the operational level. In this dissertation, we consider the tactical and operational levels, separately.

Within this context, we have defined several problem domains, proposed lower bounds, optimality conditions and a variety of solution procedures (sequential versus integrated, exact versus heuristic) for these problems. The main contribution of this thesis to the literature is explicitly modeling utilization of the same vehicles in the inbound and outbound transportation. Specifically, we allow effective utilization of the same vehicles both in the inbound and outbound transportation. We generalize this concept to a setting in which there are two transportation alternatives differing from one another by availability and cost. Efficient utilization of transportation alternatives is a great opportunity in reducing costs, energy consumption and pollution. Although existence of multiple transportation types has been studied in the literature (Chen and Lee [4], Stecke and Zhao [22], Wang and Lee [27]), there is no study considering transportation types with different costs and availabilities. Our studies associated with the research questions raised in the introduction are as follows:

- In Chapter 3 of this dissertation, we develop a model that coordinates production and transportation activities while utilizing a finite number of capacitated vehicles for inbound and outbound transportation activities.
- In Chapter 4, we provide an integrated model for production and outbound transportation problem while utilizing the vehicles used in the inbound transportation in the outbound. We generalize this concept to a setting in which there are two transportation alternatives differing from one another by availability and cost.
- In Chapter 4, we identify three operating policies that affect the structure of the problem: consolidation, splitting and the size of the deliveries. The effects of these policies are also investigated in Chapter 4.
- In Chapter 5, we propose three solution procedures for the integrated production and outbound transportation problem which differ in how the underlying production and transportation subproblems are solved. The benefits of jointly solving production and transportation problems under various problem parameters are analyzed in the same chapter.

Our findings for the corresponding parts are as follows:

6.1 Scheduling-Transportation Problem

The first part of this dissertation studies the joint problem of finding the production and vehicle schedules for inbound and outbound transportation of a single stage in the supply chain. In the specific setting of interest, a certain number of jobs are carried from an origin to a production facility at a distant location and returned back to the origin after their processing. There are multiple vehicles with limited capacities and they can be utilized for both inbound and outbound transportation. Inventory holding costs and transportation costs in this setting are high, therefore, coordination of the schedules for production and transportation is important.

Our study falls into the area of supply chain scheduling with transportation considerations. While many of the studies in this area focus on just the delivery schedule and consider the joint scheduling problem for a scheduling related objective, our study models the shipment related constraints both in the inbound and the outbound, and aims to minimize the sum of inventory holding and transportation costs. In the study, we first show that the problem under consideration is $\mathcal{NP}$-Hard in the strong sense and provide an IP model. We then prove some properties of the solution space and develop lower bounds on the optimal objective function value. Using these properties and lower bounds, we propose a heuristic based on beam-search approach. Over an extensive computational analysis, we demonstrate the performances of the lower bounds and the heuristic. Incorporation of lower bounds and optimality properties into the proposed IP model leads up to 99% reduction in solution times. The proposed lower bounds and heuristic are quite tight, increasing in the difference between the raw material and finished goods inventory holding costs.

6.2 Production-Delivery Problem

In the second part of this thesis, we study the capacitated production planning and outbound transportation problem while utilizing inbound vehicles for outbound transportation. The benefits of utilizing the same vehicle for both inbound and outbound transportation are exploited in the proposed models. We propose a generalization in which inbound vehicles are treated as a different type.

In this part, we study a joint production and transportation planning problem of a manufacturer. The specific problem faced by the manufacturer is to schedule orders with deadlines on a single machine to minimize the sum of inventory holding and outbound transportation costs without allowing any tardiness. An important characteristic of the problem setting is that there are two vehicle types; one in unlimited availability but expensive, and the other in limited and changing availability but cheaper.

We have identified three operating policies that affect the structure of the problem (consolidation, splitting and size of deliveries) and provide a generic mathematical formulation by which every possible combination of operating policies can be solved by using a subset of the constraint sets in the formulation. We develop general optimality conditions valid for all problems and study each problem by either providing a pseudo-polynomial algorithm for a general cost structure or proving that no such algorithm exists even for a linear cost structure. The complexity results are summarized in Table 6.1. Computational experiments indicate that operating policies have considerable effects on the system performance depending on order sizes, availability of vehicles, production capacities, and inventory and transportation cost components.

In general, the computational results indicate that the effect of consolidation is magnified when transportation costs are high and order sizes are low (especially less than the vehicle capacity). When the order sizes are low, there is a cost reduction due to consolidation, which amounts up to 76% (66% on the average)

Table 6.1: Summary of the complexity results

	General Delivery		*FTL − Demand*
	Consolidate	*NoConsolidate*	
Split	Problem 1*	Problem 2+	Problem 5*
NoSplit	Problem 3+	Problem 4+	Problem 6+

*: Pseudo-polynomial algorithm for general cost structure
+: Strongly $\mathcal{NP}$-Complete even for a linear cost structure

of the cost if consolidation is not allowed. Similarly, when the availability of type II vehicles is high, there is a cost reduction due to consolidation which amounts up to 76% (31% on the average) of the cost if consolidation is not allowed. Thus, companies with low order sizes, experiencing high availability of type II vehicles or with high transportation costs should consider negotiating with their customers for allowing consolidation. Companies may make contracts to share savings due to consolidation for the customers who accept consolidation, or share the transportation costs for the customers who does not accept consolidation.

Our results indicate that when inventory holding costs and order sizes are high, especially more than the production capacity, and production capacities are low, the effect of splitting seems to be more crucial. When the order sizes are high, there is a cost reduction due to splitting which amounts up to 59% (38% on the average) of the cost if splitting is not allowed. Similarly, when the availability of type II vehicles is high, the cost reduction due to allowance of splitting amounts up to 54% (13% on the average) of the case if splitting is not allowed. When the production capacities are low (i.e., production takes place in a high utilization environment), there is a cost reduction due to allowing splitting which amounts up to 30% (12% on the average) of the case if splitting is not allowed. Hence, companies with high order sizes, experiencing high availability of type II vehicles or producing in a high utilization environment should consider strengthening their relations with customers by making long term contracts to deliver the orders within a time range (i.e., allow for splitting). The effects are so high that, the savings due to slitting can be shared in these contracts.

6.3 Hierarchical versus Central Coordination

The third part of the dissertation is dedicated to a detailed analysis of one of the problems defined in the second part. In this part, we assume that an order destined to a specific customer cannot be delivered in multiple batches and orders of different customers cannot be delivered in the same vehicle. We have identified two underlying subproblems–production subproblem and transportation subproblem, and provided their mathematical formulations. Motivated by our observations from several industry practices, we have presented three approaches to solve the manufacturer's production and transportation planning problem. Those are the uncoordinated solution, the hierarchically-coordinated solution and the centrally-coordinated solution. The first two approaches are based on solving the production subproblem first, followed by the transportation subproblem. The centrally-coordinated solution aims to minimize the total costs by making all the related decisions in an integrated manner. The difference between the uncoordinated and the hierarchically-coordinated solutions lies in the fact that, given the production decisions, transportation subproblem is solved optimally in the latter.

The problem of making the production and transportation decisions in an integrated manner is $\mathcal{NP}$-hard in the strong sense. We show in this chapter that the production subproblem has similar complexity. However, given the delivery dates of orders, we provide polynomial algorithms for solving the two subproblems. Based on these algorithms, we propose a tabu-search heuristic for minimizing the total costs. The results of an extensive numerical analysis reveal that the heuristic takes less than a minute to find a solution, which deviates from the lower bound by at most 10.13% and by 0.31% on the average. We also make a numerical comparison of the three solution approaches and provide several insights about how the solutions differ under varying problem parameters. Our results mainly demonstrate that the value of integration is particularly high when orders have large sizes, inventory holding and vehicle holding costs are low, and the availability of the lower-cost vehicle shows high variability. Computational results indicate that optimal usage of transportation alternatives saves up to 71% and integration of transportation and production decisions results in up to 58% cost reduction.

6.4 Future Research Directions

In Chapter 3, we focus on integrating scheduling decisions involving production as well as inbound and outbound transportation. The issue of coordinating the schedules for the production and a finite number of capacitated vehicles which can be utilized both in the inbound and outbound, can be extended to other settings as well. Immediate extensions include modeling the production scheduling problem at a more detailed level and/or solving the problem for different objective functions. Conflict and cooperation issues can be investigated in this setting by modeling the existence of a decision maker, i.e., a trucking company, who owns the trucks and makes their scheduling decisions (see Dawande et al. [8] as an example).

In Chapter 4, we consider a tactical level model and study a manufacturer's production planning and outbound transportation problem with production capacities while utilizing alternative transportation opportunities to minimize transportation and inventory holding costs. We provide formulations and complexity results for each combination of operating policies affecting the structure of the problem. Even though, we provide pseudo-polynomial algorithms for the Problems 1 ($Consolidate-Split$) and 5 ($Split$ with $FTL-Delivery$), the complexity status of these problems are still open, and it still needs to be proven that these problems are either $\mathcal{NP}$-Hard (in the ordinary sense) or there is an efficient algorithm. There may also be practical cases for which polynomial algorithms can be developed.

In Chapter 5, we identified three solution approaches regarding the decision making process for planning the production and outbound transportation of orders, which vary in how the underlying production and transportation subproblems are solved. We quantify the savings due to integration and explicit consideration of transportation availabilities for one of the problems defined in Chapter 4. The value of integration can be investigated for the other problems defined in Chapter 4 in the future studies.

In our study, we assume that all vehicles have same capacity. Allowing different types of vehicles with different capacities and cost structures can make the problem more applicable, yet more complicated. We also assume that total production capacity is enough to satisfy all orders on time, i.e., there is a feasible solution. However, if the total size of orders is more than total production capacity, some orders need to be rejected. Incorporating order acceptance and rejection decisions together with the production and transportation decisions would be a realistic extension.

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