A NEW APPROACH TO AGE AND RISK TAKING BEHAVIOR OF AGENTS

A Master's Thesis

by
Zeynep KOZAN

Department of
Economics
İhsan Doğramacı Bilkent University
Ankara
September 2012

To My Family

A NEW APPROACH TO AGE AND RISK TAKING BEHAVIOR OF AGENTS

Graduate School of Economics and Social Sciences
of
İhsan Doğramacı Bilkent University

by

ZEYNEP KOZAN

In Partial Fulfillment of the Requirements For the Degree
of
MASTER OF ARTS

in

THE DEPARTMENT OF
ECONOMICS
İHSAN DOĞRAMACI BILKENT UNIVERSITY
ANKARA

September 2012

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assist. Prof. Dr. Emin Karagözoğlu

Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assist. Prof. Dr. Tarık Kara

Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assoc. Prof. Dr. Hüseyin Merdan

Examining Committee Member

Approval of the Graduate School of Economics and Social Sciences

---

Prof. Dr. Erdal Erel

Director

ABSTRACT

A NEW APPROACH TO AGE AND RISK TAKING BEHAVIOR OF AGENTS

KOZAN, Zeynep

M.A., Department of Economics

Supervisor: Assist. Prof. Dr. Emin Karagözoğlu

September 2012

In this thesis, we use evolutionary game theory techniques to analyze the relation between risk taking behavior of agents and their ages. We suppose that risk aversion is the stable pattern for the old agents and risk seeking is the stable pattern for the young agents as it is commonly assumed so in economics literature. First, we solve a benchmark model without heterogeneity in terms of age differentiations. In such a case, we observe that mutation either increases or decreases with respect to the payoff levels, depending on the initial fitness levels of the population groups. In the second step, we introduce heterogeneous population frontier. The anticipated level of the initial mutant proportion provides incentives to triger the evolution. Then, we analyze numerically the effects of the initial level of fitness, initial risk averse and risk seeking proportions on the pattern of the evolution process. Finally, we studied the intertemporal effects of different risk averse and risk seeking population proportions on mutation.

Keywords: Risk Aversion, Risk Seeking, Risk Dominant Equilibrium, Evolutionary Game Theory, Age and Risk, Coordination Games

ÖZET

YAŞ VE BİREYLERİN RİSK ALMA DAVRANIŞLARINA YENİ BİR YAKLAŞIM

KOZAN, Zeynep

Yüksek Lisans, Ekonomi Bölümü

Tez Yöneticisi: Assist. Prof. Dr. Emin Karagözoğlu

Eylül 2012

Bu tezde, bireylerin risk alma davranışları ile yaşları arasındaki ilişkiyi analiz etmek için evrimsel oyun teorisi tekniklerini kullanıyoruz. Ekonomi literatüründe kabul edildiği üzere riskten kaçınmanın yaşlılar için, risk arayışının ise gençler için durağan davranış biçimi olduğunu varsayıyoruz. Öncelikle, yaş farklılaşması kapsamında heterojenlik içermeyen temel bir model çözüyoruz. Böyle bir durumda, popülasyon gruplarının başlangıç uyum seviyeleri durumuna bağlı olarak mutasyonun kazanç seviyelerine göre arttığını ya da azaldığını gözlemliyoruz. İkinci aşamada, modele popülasyon heterojenliği ekliyoruz. Böyle bir modelde, öngörülen başlangıç mutant oranları, evrimin tetiklenmesini sağlıyor. Daha sonra, başlangıç uyum seviyesinin, başlangıç riskten kaçınma ve başlangıç risk arayışı oranlarının, evrim sürecine etkilerini nümerik olarak inceliyoruz. Son olarak, popülasyondaki farklı riskten kaçınma ve risk arayışı oranlarının mutasyon üzerindeki zamanlararası etkilerini araştırıyoruz.

Anahtar Kelimeler: Risten Kaçınma, Risk Arayışı, Risk Baskın Denge, Evrimsel Oyun Teorisi, Yaş ve Risk, Kordinasyon Oyunları

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my supervisor, Assist. Prof. Emin Karagözoğlu, who has supported me throughout my thesis with his patience, knowledge and exceptional guidance in all of the stages of my study. I am indebted to him. I would like to thank Assist. Prof. Tarık Kara, as one of my thesis examining committee members, who gave his time and provided worthy guidance. I also would like to thank Assoc. Prof. Hüseyin Merdan for his interest and helpful suggestions as an examining committee member and for his great support throughout my academic life.

Thanks to Bilkent University for its financial support during my graduate study.

Special thanks to my undergraduate professors, who have always been there and continued on giving me the moral support during my graduate study.

I would like to thank my family for being always there with all their love and continuous support, during my graduate study and entire life.

However, while grateful to them, I bear the sole responsibilities for all the mistakes made in the thesis.

TABLE OF CONTENTS

ABSTRACT . . . . **iii**

ÖZET . . . . **iv**

ACKNOWLEDGMENTS . . . . **v**

TABLE OF CONTENTS . . . . **vi**

LIST OF TABLES . . . . **vii**
LIST OF FIGURES . . . . **viii**

CHAPTER 1: INTRODUCTION . . . . **.1**

CHAPTER 2: THE BENCHMARK MODEL **5**

2.1 The Evolutionary Game. 5
2.1.1 The Young Agents 8
2.1.2 The Old Agents **10**
2.1.3 Heterogeneity **11**
2.1.3.1.1 Case. 17
2.1.3.1.2 Case. 18
2.1.3.1.3 Case. 18
2.1.3.2.1 Case. 21
2.1.3.2.2 Case. 21
2.1.3.3.3 Case. 21

CHAPTER 3: NUMERICAL ANALYSIS **24**

CHAPTER 4: CONCLUSION **30**

BIBLIOGRAPHY **32**

LIST OF TABLES

1. General payoff tables of the model 7
2. Two player game between young agents 10
3. Two player game between old agents. 11
4. Two player game between young and old agents 13
5. The values of the difference between the average fitness of the risk aversion of young population and the average fitness of the entire population for different proportion levels of risk averse youngs and risk averse olds 22
6. The values of the difference between the average fitness of the risk aversion of young population and the average fitness of the entire population for different proportion levels of risk averse youngs and only risk seeking olds, $q_{ra} = 0$ or only risk averse olds, $q_{ra} = 1$ 24
7. Intertemporal analysis of risk dominant strategy frequency and payoff dominant strategy frequency permanence for different scenarios 25

LIST OF FIGURES

1. Average Fitness of Risk Averse Youngs 20

2. Average Fitness of Risk Seeking Olds 23

CHAPTER 1

INTRODUCTION

There are numerous studies in economics literature which studied the effect of age on risk-taking. These studies show that, the well-known conclusion, risk taking decreases with age (Morin and Suarez, 1983; Holmstrom and Milagrom, 1987; Kanodia et al., 1989; Riley and Chow, 1992). Using different measures such as observed portfolio allocations of wealth (Jianakoplos and Bernasek, 2006) or large scale survey studies analyzing the whole population (Barsky et al., 1997; Donkers et al., 2001; Dohmen et al., 2006), these studies show that willingness to take risk is decreasing with age. Using the lowa Gambling Task, a task to measure ambiguity, various studies also find a negative correlation between risk taking and age (Fein et al., 2007; Denburg et al., 2005; Zamarin et al., 2008). Further studies found that violations of expected utility theory are decreasing with age (Kume and Suzuki, 2010; Harbaugh et al., 2002).

On the other hand there are various studies in psychology literature which show that older adults may be more risk seeking than younger adults. Considering the framing effects on young and old agents they conclude that old agents are more likely to be risk averse (i.e., to move away from a risky option) when questions are framed as gains (i.e., positively) and more risk seeking (i.e., to move toward a risky option) when questions are framed as losses (i.e., negatively), (Hasher et al., 2005; Lauriola and Levin, 2001). By considering such a psychological result, our motivation is that under negative conditions such as the last financial crises the world experienced, could the old agents evaluate these conditions as a negative frame? For instance, also Weber at al. (2004) did a meta analysis of studies involving decision outcomes described to study participants and found that increasing age (age ranges were not specified in their paper) was associated with greater risk seeking (more choices of a gamble over a sure thing) in losses; they did not, however, find a link between increasing age and risk aversion (more choices of a sure thing over a gamble) in gains. Another study which was presented by Arkes and Ayton (1999), displays an empirical evidence from studies on sunk-cost effects. There is an interesting difference in test results between adults and children with regard to the sunk-cost effect. Children under 10 years of age seem to be less susceptible to the sunk-cost effect than humans of older ages. Arkes and Ayton explain this by the fact that young humans have more modest cognitive abilities. These cognitive abilities are suggested to be the main explanation for sunk-cost effects, because humans, especially adults, tend to define complex strategies (Janssen and Scheffer, 2004).

A contrary to the well-known results (i.e., risk aversion increases with age) in economics is Wang and Hanna's (1997) research which is consistent with the results in psychology literature we mentioned above. Using the 1983-1989 panel of the survey of consumer finances they find out that relative risk aversion decreases as people age (i.e., the proportion of net wealth invested in risky assets increases as people age). They show that risk tolerance increases with age which is contrary to constant life-cycle risk aversion hypothesis. Their conclusion is young people may appear more risk averse since it is hard for them to endure any short-term investment losses with limited financial resources. Future human wealth cannot be applied to pay present bills, car loans, mortgage debts, etc. Besides, William B. Riley Jr. and K. Victor Chow (1992), who studied asset allocation and individual risk aversion in their research, showed that risk aversion decreases with age- but only up to a point. After age 65 (retirement), risk aversion increases with age.

In this thesis, the psychological concept on risk taking behavior with respect to age is extended by using insight from research on human decision making. Although the common belief in economics lays out that the old humans are more risk averse, while the young ones are more risk seeking, "why" there does not exist coherence with the studies in psychology literature. There is no any research in the literature which tries to present an explanation to this mismatching we observe in these two different disciplines. This is the main motivation for this research to be able to introduce a link between both disciplines and discard this dualism by a theoretical model. One of the questions we address is "why" humans of older ages may invest on risky options while the humans of young ages are not subject to invest on them. In our view, an important factor that might explain these is the "sunk-cost effect"—where human decision making is typically influenced by the level of prior investments. Janssen and Scheffer (2004:6) claim that "According to conventional economic theory, only the incremental costs and benefits of the current options should be included in decision making. However, numerous examples show that humans do take into account prior investment when they consider what course of action to follow." Hence, the learning procedure of agents will be an important input in our work to study the analysis of repositioning of them according to their risk attitudes under evolutionary dynamics.

We will form a model as an evolutionary game which examines the feasible strategies that is known in the literature (old people are more risk-averse than young ones) versus the deviation from these strategies (old can choose to act in a more risk seeking manner). At this point, we will consider the risk aversion as the stable pattern of behavior among the older agents and risk seeking is stable among the younger ones in a population, since the common belief is so in economics literature. In other words, the literature supposes that the agents are programmed to play particular strategies. Such an assumption supports that a stable pattern of behavior in a population should be able to eliminate any invasion by a "mutant", and to do so it must have a higher fitness than the mutant in the population that results from the invasion. Here, the payoff of an agent by playing a particular strategy is interpreted as "fitness". By introducing two equilibrium into the model which are constituted by payoff dominant and risk dominant strategies set, the deviations between these strategies are possible to be examined.

Finally, we will use "replicator dynamics" as the evolutionary dynamic, which are first called by Taylor and Jonker (1978) and Zeeman (1979), to investigate the dynamic properties of an evolutionary stable strategy (ESS). This dynamic specifies that the proportional rate of growth in a strategy's representation in the population, p, is given by the extent to which that strategy does better than the population average.

The main perspective of this work is taking agents as heterogeneous among their life-cycles. For this issue we will work on their decision taking processes by investigating the results of such a heterogeneity. By using evolutionary game framework, one of the main purposes of the model is to find the proportions of risk aversion and risk seeking among different populations at which level they work as a threshold for evolution. There exists limited number of studies focused on what population aging would mean for economic decisions that are sensitive to risk taking characteristics of a population. Then, the solidity of the models used in economics, which have left such a possible behavioral diversification among agents out of account, should be questioned. In this thesis, we question in spite of the fact that there exist some researches about the risk taking behavior of agents and how it changes among them with respect to their ages, why such a new approach is not used in existing models.

The organization of the paper is as follows. In Chapter 2, the benchmark model will be introduced and solved by using evolutionary game which is constituted by both a risk dominant equilibrium and a payoff dominant equilibrium. Since the games we introduced for youngs and olds are symmetric games, we will present the results for youngs without loss of generality. In Chapter 3, numerical analysis and comparative statics for the parameters of the game will take place. Finally, Chapter 4 concludes the paper.

CHAPTER 2

THE BENCHMARK MODEL

2.1 The Evolutionary Game

In this section we construct an explicit model of the process by which the frequency of strategies changes in the population and study properties of the evolutionary dynamics within the model. Thus, once the model of the population dynamics has been specified, all of the standard stability concepts used in the analysis of dynamical system can be brought to bear.

In this research, each game is played between (and among) the young and older agents. Recalling the Riley and Chow's framework, older agents will be taken as the ones under the age of 65, and the ones above are excepted as retired. We will examine each game, which are played by only youngs, played by only olds, and finally played between youngs and olds, in detail section by section.

The basic model is of a repeated game played in periods $t = 1, 2, 3, ...$.The population is large enough. In each period, individuals choose one of two possible actions, "Risk Aversion" and "Risk Seeking" which are denoted by RA and RS. That is $a_{it} \in \{RA, RS\}$. Formally, it is required that $A > B$ and $D > C$ so that (RS, RS) and (RA, RA) are both Nash equilibria. In addition, we assume that $(D - C) \geq (A - B)$ for the younger agents so that (RA, RA) is the "risk dominant" equilibrium. Since the economics literature claims that risk seeking is more common among the young agents, it is consistent with Harsanyi and Selten (1988) taking the strategy set (RS, RS) as payoff dominant for the young individuals. Note that when the strategies have equal security levels $(B = C)$, (RA, RA) is also the Pareto optimum. Similarly, (RS, RS) will be the "risk dominant" equilibrium for the game which is played by old agents. Hence, we will assume that $A > B$ and $D > C$ for that game. In addition, we will assume that $(A - B) > (D - C)$ for the olds, and when $(B = C)$, (RS, RS) will also be Pareto optimum of that game.

Table 1. General payoff tables of the model

		Younger Agent	
		RS	RA
Younger	RS	A, a	C, b
Agent	RA	B, c	D, d

		Older Agent	
		RS	RA
Older	RS	A, a	C, b
Agent	RA	B, c	D, d

		Older Agent	
		RS	RA
Younger	RS	A, a	C, b
Agent	RA	B, c	D, d

How a population in which these plays are repeatedly played will evolve in terms of its risk taking characteristics is the main question. First, assume that the population is quite large. In this case, we can represent the state of the population by simply keeping track of what proportion follows the strategies RA and RS. Let p_{ra} and p_{rs} (without loss of generality, q_{ra} and q_{rs} for the old population) denote these proportions. Furthermore, let the average fitness of risk aversion and risk seeking be denoted by W_{RA} and W_{RS}, respectively, and let $\overline{W}$ be the average fitness of the entire population. The values of W_{RA}, W_{RS}, and $\overline{W}$ can be expressed in terms of the population proportions and payoff values as follows:

$$
\begin{aligned}
W_{RA} &= F_0 + p_{ra}\Delta F(RA, RA) + p_{rs}\Delta F(RA, RS), &(1)\\
W_{RS} &= F_0 + p_{ra}\Delta F(RS, RA) + p_{rs}\Delta F(RS, RS), &(2)\\
\overline{W} &= p_{ra}W_{RA} + p_{rs}W_{RS}. &(3)
\end{aligned}
$$

Second, let us assume that the proportion of the population following the strategies RA and RS in the next generation is related to the proportion of the population following the strategies RA and RS in the current generation according to the rule:

$$
\begin{aligned}
p'_{ra} &= \frac{p_{ra}W_{RA}}{\overline{W}}, &(4)\\
p'_{rs} &= \frac{p_{rs}W_{RS}}{\overline{W}}. &(5)
\end{aligned}
$$

We can rewrite these expressions in the following form:

$$
\begin{aligned}
p'_{ra} - p_{ra} &= \frac{p_{ra}(W_{RA} - \overline{W})}{\overline{W}}, &(6)\\
p'_{rs} - p_{rs} &= \frac{p_{rs}(W_{RS} - \overline{W})}{\overline{W}}. &(7)
\end{aligned}
$$

If we assume that the change in the strategy frequency from one generation to the next are small, then the replicator dynamics:

$$
\begin{aligned}
\frac{dp_{ra}}{dt} &= \frac{p_{ra}(W_{RA} - \overline{W})}{\overline{W}}, &(8)\\
\frac{dp_{rs}}{dt} &= \frac{p_{rs}(W_{RS} - \overline{W})}{\overline{W}}. &(9)
\end{aligned}
$$

The replicator dynamics may be used to model a population of individuals playing the game we introduced above. For this game, the expected fitness of "Risk aversion" and "Risk seeking" are expressed as follows:

$$\begin{aligned} W_{RA} &= F_0 + p_{ra}\Delta F(RA, RA) + p_{rs}\Delta F(RA, RS) \quad (10)\\ &= F_0 + p_{ra}D + p_{rs}B \quad (11)\end{aligned}$$

and

$$\begin{aligned} W_{RS} &= F_0 + p_{ra}\Delta F(RS, RA) + p_{rs}\Delta F(RS, RS) \quad (12)\\ &= F_0 + p_{ra}C + p_{rs}A \quad (13)\end{aligned}$$

By looking at the values of utility levels, we will analyze whether the following indicators are positive or not:

$$\frac{W_{RS} - \overline{W}}{\overline{W}} \quad (14)$$

and

$$\frac{W_{RA} - \overline{W}}{\overline{W}}. \quad (15)$$

If an action a_i taken by some individuals does better than average, its representation in the population grows ($dp_{a_i}/dt > 0$), and if another strategy is even better, then its growth rate is also higher than that of strategy a_i.

2.1.1 The Young Agents

The payoff matrix of the game which is played between young agents can be taken as follows:

Table 2. Two player game between young agents

		Younger Agent	
		RS	RA
Younger	RS	A, a	C, b
Agent	RA	B, c	D, d

where the equilibrium (RS, RS) is Pareto dominant and the equilibrium (RA, RA) is risk dominant. Then,

$$W_{RA} = F_0 + p_{ra}D + p_{rs}B, \tag{16}$$

$$W_{RS} = F_0 + p_{ra}C + p_{rs}A, \tag{17}$$

$$\overline{W} = p_{ra}W_{RA} + p_{rs}W_{RS}. \tag{18}$$

Hence,

$$\begin{aligned}
\frac{W_{RA} - \overline{W}}{\overline{W}} &= \frac{F_0 + p_{ra}D + p_{rs}B - p_{ra}(F_0 + p_{ra}D + p_{rs}B) - p_{rs}(F_0 + p_{ra}C + p_{rs}A)}{p_{ra}W_{RA} + p_{rs}W_{RS}} \\
&= \frac{F_0 + p_{ra}D + p_{rs}B - p_{ra}F_0 - p_{ra}^2 D - p_{ra}p_{rs}B - p_{rs}F_0 - p_{rs}p_{ra}C - p_{rs}^2 A}{p_{ra}W_{RA} + p_{rs}W_{RS}} \\
&= \frac{(1 - p_{ra} - p_{rs})F_0 + (p_{ra} - p_{ra}^2)D + (p_{rs} - p_{ra}p_{rs})B - p_{rs}p_{ra}C - p_{rs}^2 A}{p_{ra}F_0 + p_{ra}^2 D + p_{ra}p_{rs}B + p_{rs}F_0 + p_{rs}p_{ra}C + p_{rs}^2 A} \\
&= \frac{(p_{ra} - p_{ra}^2)D + p_{rs} - p_{ra}p_{rs})B - p_{rs}p_{ra}C - p_{rs}^2 A}{F_0 + p_{ra}^2 D + p_{ra}p_{rs}B + p_{rs}p_{ra}C + p_{rs}^2 A} \\
&= \frac{p_{ra}(1 - p_{ra})D + p_{rs}(1 - p_{ra})B - p_{rs}p_{ra}C - p_{rs}^2 A}{F_0 + p_{ra}^2 D + p_{ra}p_{rs}B + p_{rs}p_{ra}C + p_{rs}^2 A} \\
&= \frac{(p_{ra}D + p_{rs}B)(1 - p_{ra}) - p_{rs}(p_{ra}C + p_{rs}A)}{F_0 + p_{ra}^2 D + p_{ra}p_{rs}B + p_{rs}p_{ra}C + p_{rs}^2 A} \\
&= \frac{p_{rs}\left[p_{ra}D + p_{rs}B - p_{ra}C - p_{rs}A\right]}{F_0 + p_{ra}^2 D + p_{ra}p_{rs}B + p_{rs}p_{ra}C + p_{rs}^2 A} \\
&= \frac{p_{rs}\left[p_{ra}(D - C) + p_{rs}(B - A)\right]}{F_0 + p_{ra}^2 D + p_{ra}p_{rs}B + p_{rs}p_{ra}C + p_{rs}^2 A}.
\end{aligned}$$

Therefore, since $p_{rs} > 0$, $p_{ra} > 0$, and $D > C$, if $p_{ra}(D-C) > p_{rs}(A-B)$, then $\frac{W_{RA}-\overline{W}}{\overline{W}} > 0$. Thus, the representation of the action RA in the young population grows. That is, if the following condition for the proportion of mutants in the young population is satisfied, then we expect a rise in the representation of the mutant strategy among the young agents:

$$\begin{aligned}
\frac{p_{ra}}{p_{rs}} &> \frac{A-B}{D-C} \Longrightarrow \frac{p_{ra}}{1-p_{ra}} > \frac{A-B}{D-C} \\
&\Longrightarrow p_{ra}D - p_{ra}C > A - B - p_{ra}A + p_{ra}B \\
&\Longrightarrow p_{ra}(D - C + A - B) > A - B \\
&\Longrightarrow p_{ra} > \frac{A-B}{(A-B)+(D-C)}.
\end{aligned}$$

This result is consistent with Kandori, Mailath, and Rob (1993) and Ellison (1993) who studied the dynamics of a model constituted by a 2x2 coordination game with uniform matching.

2.1.2 The Old Agents

The payoff matrix of the game which is played between young agents can be taken as follows:

Table 3. Two player game between old agents

		Older Agent	
		RS	RA
Older	RS	A, a	C, b
Agent	RA	B, c	D, d

where the equilibrium (RA, RA) is Pareto dominant and the equilibrium (RS, RS) is risk dominant. Then,

$$W_{RA} = F_0 + q_{ra}D + q_{rs}B, \quad (19)$$

$$W_{RS} = F_0 + q_{ra}C + q_{rs}A, \quad (20)$$

$$\overline{W} = q_{ra}W_{RA} + q_{rs}W_{RS}. \quad (21)$$

Therefore, since $q_{rs} > 0$, $q_{ra} > 0$, and $A > B$, if $q_{rs}(A - B) > q_{ra}(D - C)$, then $\frac{W_{RS} - \overline{W}}{\overline{W}} > 0$. Thus, the representation of the action RS in the old population grows. That is, whenever the following condition as of the mutants' proportion in the old population is satisfied, we expect a rise in the representation of the mutant strategy among the old agents:

$$\begin{aligned} \frac{q_{rs}}{q_{ra}} &> \frac{D-C}{A-B} \Longrightarrow \frac{q_{rs}}{1-q_{rs}} > \frac{D-C}{A-B} \\ &\Longrightarrow q_{rs}A - q_{rs}B > D - C - q_{rs}D + q_{rs}C \\ &\Longrightarrow q_{rs}(A - B + D - C) > D - C \\ &\Longrightarrow q_{rs} > \frac{D-C}{(D-C)+(A-B)}. \end{aligned}$$

2.1.3 Heterogeneity

In a large population it is reasonable to assume that population is constituted by different kinds of agents belonging to various age groups. Here we suggest that this difference among the agents creates a kind of heterogeneity in the population. Hence, it is necessary probing into a case of matching process of players as of different age groups.

The payoff matrix of the game which is played between young agents and old agents can be taken as follows:

Table 4. Two player game between young and old agents

		Older Agent	
		RS	RA
Younger	RS	A, a	C, b
Agent	RA	B, c	D, d

Assume that the population size is N. Let the number of the youngs as of this population be n, hence the number of olds will be $N - n$. Thus, $(p_{RA_{young}})n$ gives the share of the risk averse youngs and $(p_{RS_{young}})n$ gives the share of the risk seeking youngs in the population. Similarly, $p_{RA_{old}}(N - n)$ gives the share of the risk averse olds and $p_{RS_{old}}(N - n)$ gives the share of the risk seeking olds in the population.

The probability of matching of a young with an old can be taken as $\frac{N-n}{N-1} = \pi$. Then, the probability of matching of a young with another young agent will be $\frac{n-1}{N-1} = 1 - \pi$.

Therefore, the average fitness for the young population can be written as follows:

$$
\begin{aligned}
W_{RA} &= \left[F_0 + (p_{RA_{young}} D + p_{RS_{young}} B)\right](1 - \pi) + \left[F_0 + (p_{RA_{old}} D + p_{RS_{old}} B)\right]\pi. \\
W_{RS} &= \left[F_0 + (p_{RA_{young}} C + p_{RS_{young}} A)\right](1 - \pi) + \left[F_0 + (p_{RA_{old}} C + p_{RS_{old}} A)\right]\pi.
\end{aligned}
$$

Let $p_{RA_{young}} = p_{ra}$ and $p_{RS_{young}} = p_{rs}$ for a simplier notation. Similarly, let $p_{RA_{old}} = q_{ra}$ and $p_{RS_{old}} = q_{rs.}$ Then the average fitness of the whole population is obtained as follows:

$$
\overline{W} = p_{ra} q_{ra} W_{RA} + p_{rs} q_{rs} W_{RS}.
$$

Therefore,

$$\begin{aligned}
\frac{W_{RA}-\overline{W}}{\overline{W}} &= \frac{(F_0+p_{ra}D+p_{rs}B)(1-\pi)+(F_0+q_{ra}D+q_{rs}B)\pi}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}} \\
&\quad -\frac{p_{ra}q_{ra}\left[(F_0+p_{ra}D+p_{rs}B)(1-\pi)+(F_0+q_{ra}D+q_{rs}B)\pi\right]}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}} \\
&\quad -\frac{p_{rs}q_{rs}\left[(F_0+p_{ra}C+p_{rs}A)(1-\pi)+(F_0+q_{ra}C+q_{rs}A)\pi\right]}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}. \\
\frac{W_{RA}-\overline{W}}{\overline{W}} &= \frac{(1-\pi)F_0+\pi F_0-(1-\pi)p_{ra}q_{ra}F_0-(1-\pi)p_{rs}q_{rs}F_0}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}} \\
&\quad -\frac{\pi p_{ra}q_{ra}F_0+\pi p_{rs}q_{rs}F_0}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}} \\
&\quad +\frac{(1-\pi)(p_{ra}D+p_{rs}B)-(1-\pi)p_{ra}q_{ra}(p_{ra}D+p_{rs}B)}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}} \\
&\quad +\frac{\pi(q_{ra}D+q_{rs}B)-\pi p_{ra}q_{ra}(q_{ra}D+q_{rs}B)}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}} \\
&\quad -\frac{(1-\pi)p_{rs}q_{rs}(p_{ra}C+p_{rs}A)-\pi p_{rs}q_{rs}(q_{ra}C+q_{rs}A)}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}.
\end{aligned}$$

Finally we have

$$\begin{aligned}
\frac{W_{RA}-\overline{W}}{\overline{W}} &= \frac{\left[p_{ra}(1-q_{ra})+q_{ra}(1-p_{ra})\right]F_0}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}} \\
&\quad +\frac{(1-p_{ra})(1-q_{ra})[(\pi-1)(1-p_{ra})-\pi(1-q_{ra})]}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}A \\
&\quad +\frac{\left[(1-\pi)(1-p_{ra})+\pi(1-q_{ra})\right](1-p_{ra}q_{ra})}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}B \\
&\quad +\frac{(1-p_{ra})(1-q_{ra})\left[(\pi-1)p_{ra}-\pi q_{ra}\right]}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}C \\
&\quad +\frac{\left[(1-\pi)p_{ra}+\pi q_{ra}\right](1-p_{ra}q_{ra})}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}D.
\end{aligned}$$

Since $p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS} > 0$, it is sufficient to investigate whether the condition below is satisfied or not:

$$
\begin{aligned}
W_{RA} - \overline{W} &= [p_{ra}(1-q_{ra}) + q_{ra}(1-p_{ra})]\, F_0 \\
&\quad +(1-p_{ra})(1-q_{ra})[(\pi-1)(1-p_{ra}) - \pi(1-q_{ra})]A \\
&\quad +(1-p_{ra}q_{ra})\,[(1-\pi)(1-p_{ra}) + \pi(1-q_{ra})]\, B \\
&\quad +(1-p_{ra})(1-q_{ra})\,[(\pi-1)p_{ra} - \pi q_{ra}]\, C \\
&\quad +(1-p_{ra}q_{ra})\,[(1-\pi)p_{ra} + \pi q_{ra}]\, D \\
&> 0.
\end{aligned}
$$

The equality above also enable us to examine the necessary conditions for related payoffs to observe an increase in mutation behavior in a population.[1] That is to say that for a given population, by determining the proper payoff levels which are presented to the agents, a population can be directed to a particular behavior. Consequently, when the inequalities below hold, this guarantees that the risk dominant strategy, RA, will dominate the young population:

$$
\begin{aligned}
A &> \frac{p_{ra}(q_{ra}-1) + q_{ra}(p_{ra}-1)}{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra}-1)} F_0 \\
&\quad - \frac{(p_{ra}q_{ra}-1)(p_{ra}-1) - \pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra}-1)} B \\
&\quad - \frac{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra})}{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra}-1)} C \\
&\quad - \frac{p_{ra}+\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra}-1)} D
\end{aligned}
$$

[1] The mathematical computing program cannot enable us to solve for the payoff level D explicitly.

$$
\begin{aligned}
B \quad > \quad & \frac{p_{ra}(q_{ra}-1)+q_{ra}(p_{ra}-1)}{(p_{ra}q_{ra}-1)(p_{ra}-1)-\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}F_0 \\
& -\frac{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra}-1)}{(p_{ra}q_{ra}-1)(p_{ra}-1)-\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}A \\
& -\frac{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra})}{(p_{ra}q_{ra}-1)(p_{ra}-1)-\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}C \\
& -\frac{p_{ra}+\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}{(p_{ra}q_{ra}-1)(p_{ra}-1)-\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}D
\end{aligned}
$$

$$
\begin{aligned}
C \quad > \quad & -\frac{p_{ra}(q_{ra}-1)+q_{ra}(p_{ra}-1)}{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra})}F_0 \\
& +\frac{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra}-1)}{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra})}A \\
& +\frac{(p_{ra}q_{ra}-1)(p_{ra}-1)-\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra})}B \\
& +\frac{p_{ra}+\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra})}D
\end{aligned}
$$

Similarly, for a given population and given game, under particular payoff levels if the condition below holds for the initial fitness of the young population, it is guaranteed that an increase in mutation among the young agents will be observed:

$$
\begin{aligned}
F_0 \quad > \quad & \frac{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra}-1)}{p_{ra}(q_{ra}-1)+q_{ra}(p_{ra}-1)}A \\
& +\frac{(p_{ra}q_{ra}-1)(p_{ra}-1)-\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}{p_{ra}(q_{ra}-1)+q_{ra}(p_{ra}-1)}B \\
& -\frac{(p_{ra}-1)(q_{ra}-1)(p_{ra}-\pi p_{ra}+\pi q_{ra})}{p_{ra}(q_{ra}-1)+q_{ra}(p_{ra}-1)}C \\
& +\frac{p_{ra}+\pi(p_{ra}-q_{ra})(p_{ra}q_{ra}-1)}{p_{ra}(q_{ra}-1)+q_{ra}(p_{ra}-1)}D
\end{aligned}
$$

Similarly, the probability of matching of an old with a young can be taken as $\frac{n}{N-1} = 1-\pi+\epsilon$, where $\epsilon = \frac{1}{N-1}$. Then, the probability of matching of an old with another old agent will be $\frac{N-n-1}{N-1} = \pi-\epsilon$. By following the same steps above for the old agents, the average fitness for the old population can be written as follows:

$$\begin{aligned} W_{RA} &= \left[F_0 + (p_{RA_{young}}D + p_{RS_{young}}B)\right](1-\pi+\epsilon) + \left[F_0 + (p_{RA_{old}}D + p_{RS_{old}}B)\right](\pi-\epsilon) \\ W_{RS} &= \left[F_0 + (p_{RA_{young}}C + p_{RS_{young}}A)\right](1-\pi+\epsilon) + \left[F_0 + (p_{RA_{old}}C + p_{RS_{old}}A)\right](\pi-\epsilon) \end{aligned}$$

and

$$\overline{W} = p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}.$$

Therefore,

$$\begin{aligned} \frac{W_{RS}-\overline{W}}{\overline{W}} &= \frac{(F_0 + p_{ra}C + p_{rs}A)(1-\pi+\epsilon) + (F_0 + q_{ra}C + q_{rs}A)(\pi-\epsilon)}{p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}} \\ &\quad - \frac{p_{ra}q_{ra}\left[(F_0 + p_{ra}D + p_{rs}B)(1-\pi+\epsilon) + (F_0 + q_{ra}D + q_{rs}B)(\pi-\epsilon)\right]}{p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}} \\ &\quad - \frac{p_{rs}q_{rs}\left[(F_0 + p_{ra}C + p_{rs}A)(1-\pi+\epsilon) + (F_0 + q_{ra}C + q_{rs}A)(\pi-\epsilon)\right]}{p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}}. \\ \frac{W_{RS}-\overline{W}}{\overline{W}} &= \frac{(1-\pi+\epsilon)F_0 - (1-\pi+\epsilon)p_{ra}q_{ra}F_0 - (1-\pi+\epsilon)p_{rs}q_{rs}F_0}{p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}} \\ &\quad + \frac{(\pi-\epsilon)F_0 - (\pi-\epsilon)p_{ra}q_{ra}F_0 - (\pi-\epsilon)p_{rs}q_{rs}F_0}{p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}} \\ &\quad + \frac{(1-\pi+\epsilon)(p_{ra}C + p_{rs}A) - (1-\pi+\epsilon)p_{rs}q_{rs}(p_{ra}C + p_{rs}A)}{p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}} \\ &\quad + \frac{(\pi-\epsilon)(q_{ra}C + q_{rs}A) - (\pi-\epsilon)p_{rs}q_{rs}(q_{ra}C + q_{rs}A)}{p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}} \\ &\quad - \frac{(1-\pi+\epsilon)p_{ra}q_{ra}(p_{ra}D + p_{rs}B) + (\pi-\epsilon)p_{ra}q_{ra}(q_{ra}D + q_{rs}B)}{p_{ra}q_{ra}W_{RA} + p_{rs}q_{rs}W_{RS}}. \end{aligned}$$

Finally we have

$$
\begin{aligned}
\frac{W_{RS}-\overline{W}}{\overline{W}} &= \frac{\left[(1-p_{rs})q_{rs}+(1-q_{rs})p_{rs}\right]F_0}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}} \\
&+\frac{\left[(1-\pi+\epsilon)p_{rs}+(\pi-\epsilon)q_{rs}\right](1-p_{rs}q_{rs})}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}A \\
&+\frac{(1-p_{rs})(1-q_{rs})\left[(\pi-1-\epsilon)p_{rs}-(\pi-\epsilon)q_{rs}\right]}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}B \\
&+\frac{\left[(1-\pi+\epsilon)(1-p_{rs})+(\pi-\epsilon)(1-q_{rs})\right](1-p_{rs}q_{rs})}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}C \\
&+\frac{(1-p_{rs})(1-q_{rs})[(\pi-1-\epsilon)(1-p_{rs})-(\pi-\epsilon)(1-q_{rs})]}{p_{ra}q_{ra}W_{RA}+p_{rs}q_{rs}W_{RS}}D.
\end{aligned}
$$

Hence, if the condition below holds, then the mutant strategy risk seeking, RS, will dominate the old population:

$$
\begin{aligned}
W_{RS}-\overline{W} &= \left[(1-p_{rs})q_{rs}+(1-q_{rs})p_{rs}\right]F_0 \\
&+(1-p_{rs}q_{rs})\left[(1-\pi+\epsilon)p_{rs}+(\pi-\epsilon)q_{rs}\right]A \\
&+(1-p_{rs})(1-q_{rs})\left[(\pi-1-\epsilon)p_{rs}-(\pi-\epsilon)q_{rs}\right]B \\
&+(1-p_{rs}q_{rs})\left[(1-\pi+\epsilon)(1-p_{rs})+(\pi-\epsilon)(1-q_{rs})\right]C \\
&+(1-p_{rs})(1-q_{rs})[(\pi-1-\epsilon)(1-p_{rs})-(\pi-\epsilon)(1-q_{rs})]D \\
&> 0.
\end{aligned}
$$

Case 2.1.3.1.1: $p_{ra}=q_{ra}=1$,

$$
W_{RA}-\overline{W}=0.
$$

Conclusion 1 *If there exist only risk averse olds and risk averse youngs in a population, then any representation of the mutant strategy among the youngs will not be observed.*

Case 2.1.3.1.2: $p_{ra} \in [0,1]$ **and** $q_{ra} = 1$,

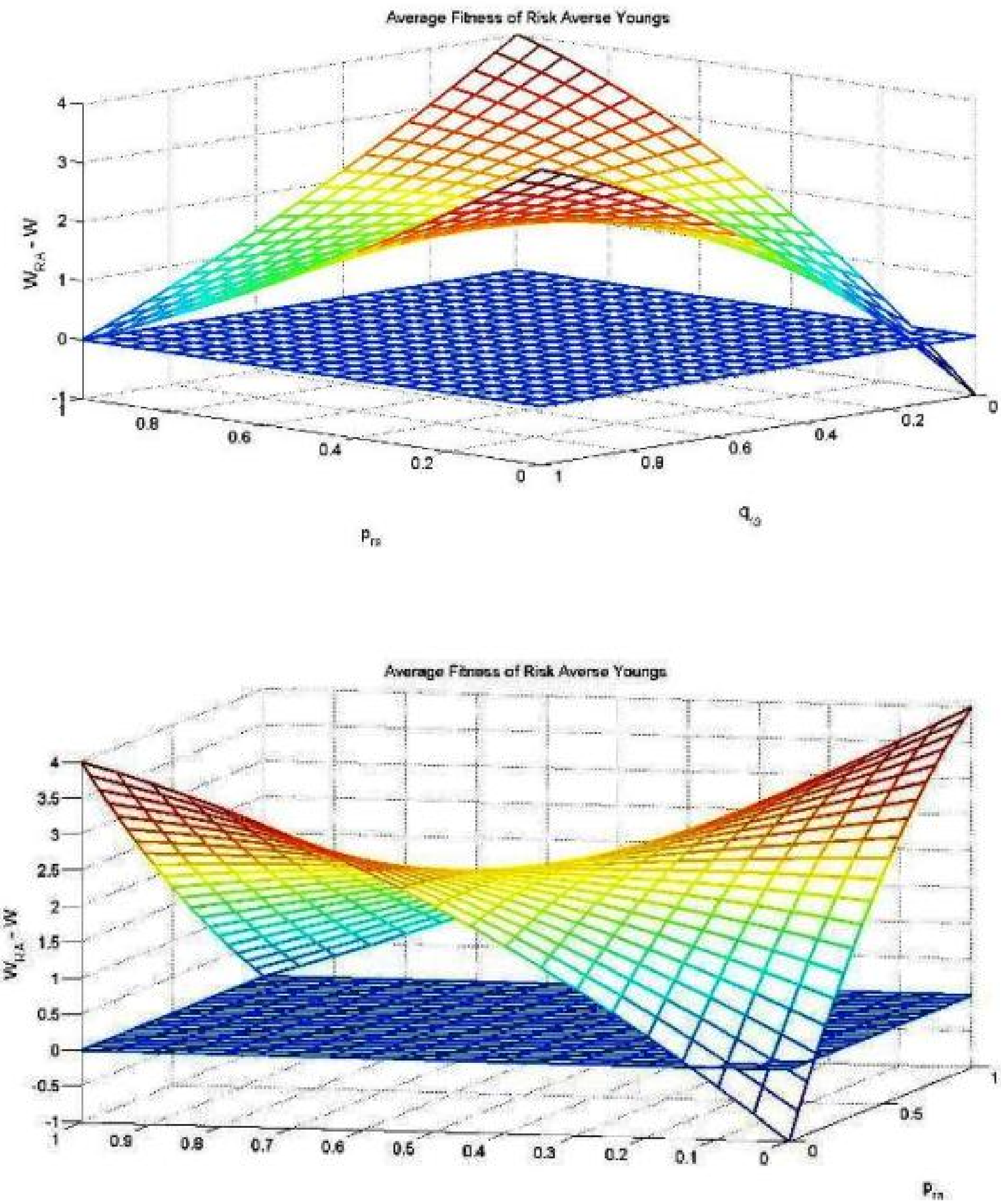
$$W_{RA} - \overline{W} = (1-p_{ra})F_0 + (1-\pi)(1-p_{ra})^2 B + \left[(1-\pi)p_{ra} + \pi\right](1-p_{ra})D > 0.$$

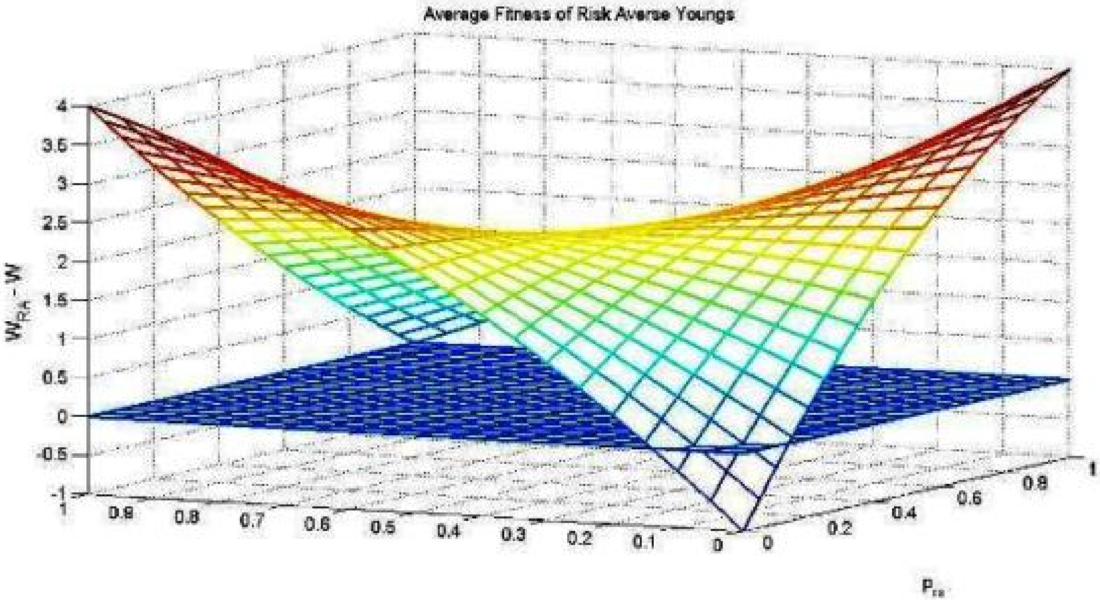
Conclusion 2 *If there exist only risk averse old agents in a population, then the representation of the action RA in the young population grows. That is to say that the representation of the mutant strategy among the youngs arises.*

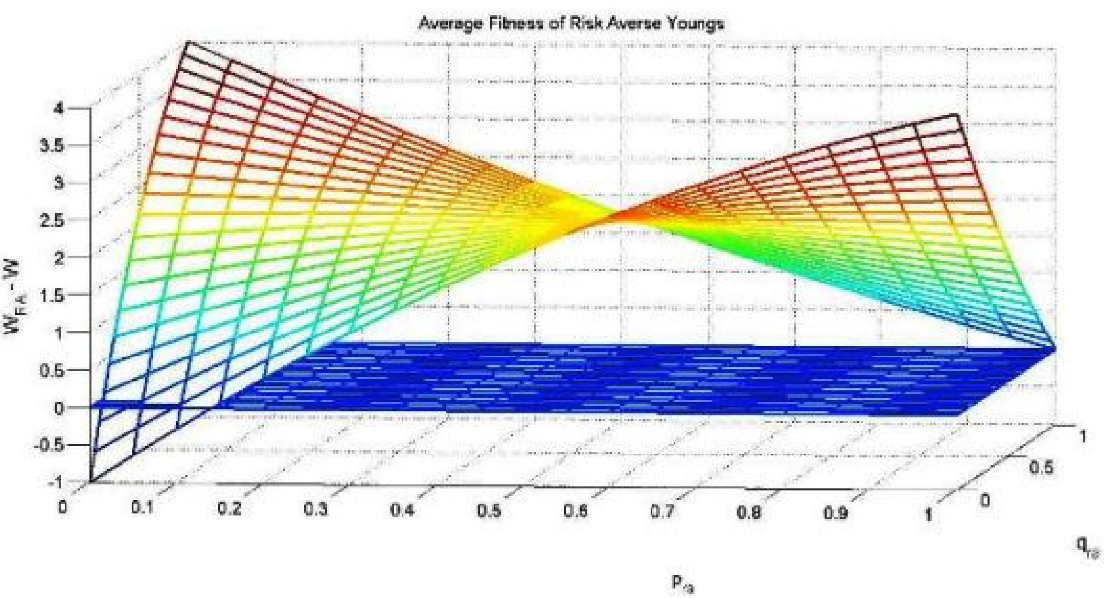
Case 2.1.3.1.3: $p_{ra} \in [0,1]$ **and** $q_{ra} = 0$,

$$\begin{aligned} W_{RA} - \overline{W} &= p_{ra}F_0 + (1-p_{ra})[(\pi-1)(1-p_{ra}) - \pi]A \\ &\quad +[(1-\pi)(1-p_{ra}) + \pi]B \\ &\quad +(1-p_{ra})[(\pi-1)p_{ra} - \pi q_{ra}]C \\ &\quad +p_{ra}(1-\pi)D \end{aligned}$$

Figure 1. Average Fitness of Risk Averse Youngs




Average Fitness of Risk Averse Youngs
W_RA - W
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1


Average Fitness of Risk Averse Youngs
W_RA - W
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1
0.5
0

Case 2.1.3.2.1: $p_{rs} = q_{rs} = 1$,

$$W_{RS} - \overline{W} = 0.$$

Conclusion 3 *If there exist only risk seeking youngs and risk seeking olds in a population, then any representation of the mutant strategy among the olds will not be observed.*

Case 2.1.3.2.2: $q_{rs} \in [0, 1]$ **and** $p_{rs} = 1$,

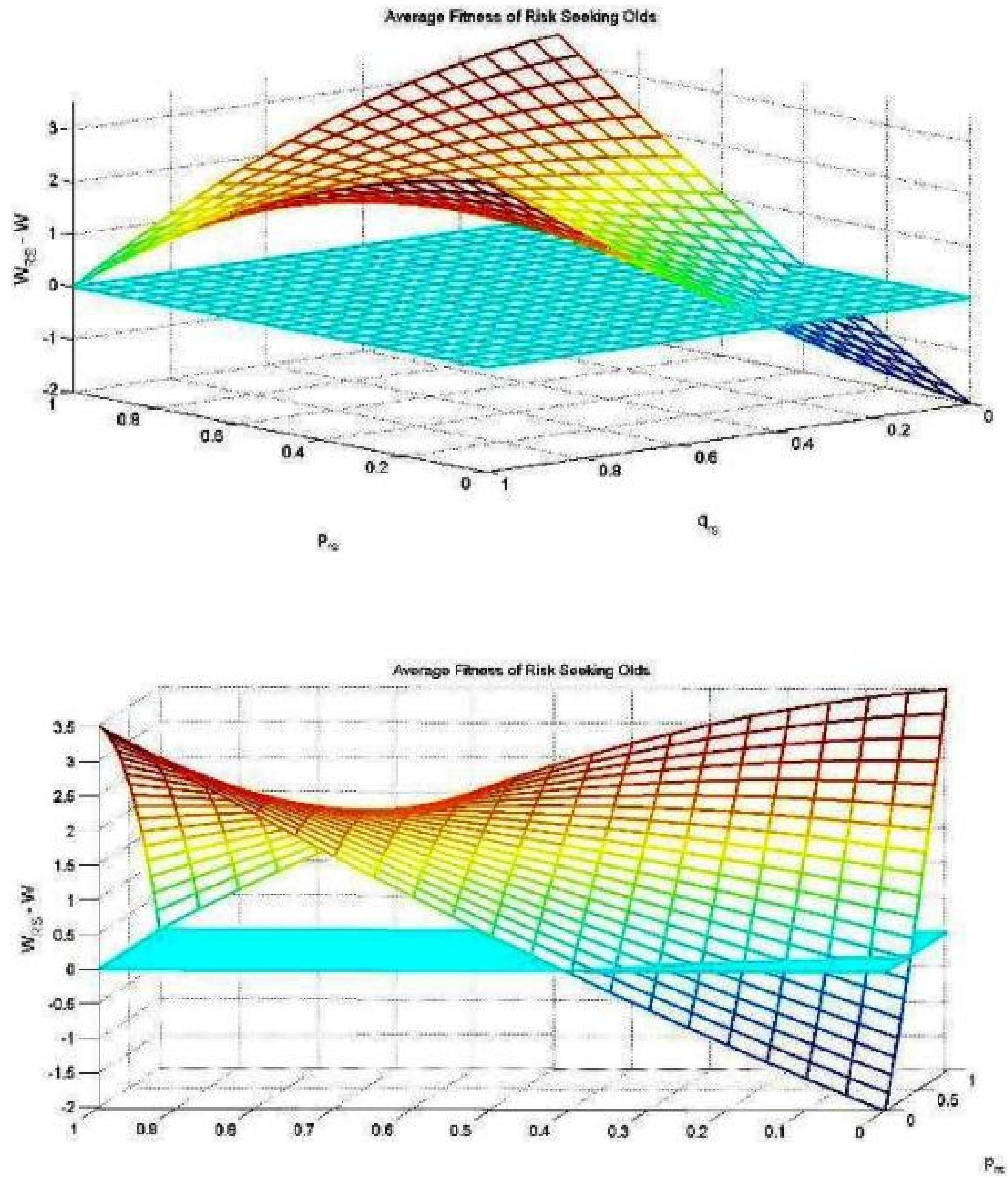
$$W_{RS} - \overline{W} = (1 - q_{rs})F_0 + (1 - q_{rs})(\pi - \epsilon)q_{rs}A + (1 - q_{rs})^2(\pi - \epsilon)C > 0.$$

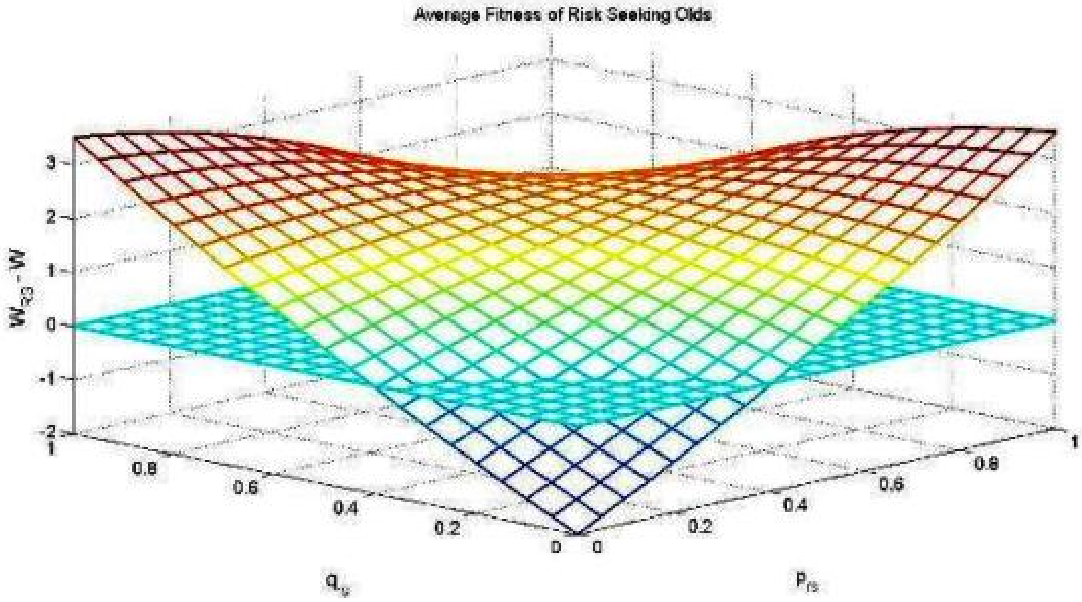
Conclusion 4 *If there exist only risk seeking young agents in a population, then the representation of the action RS in the old population grows. That is to say that the representation of the mutant strategy among the olds arises.*

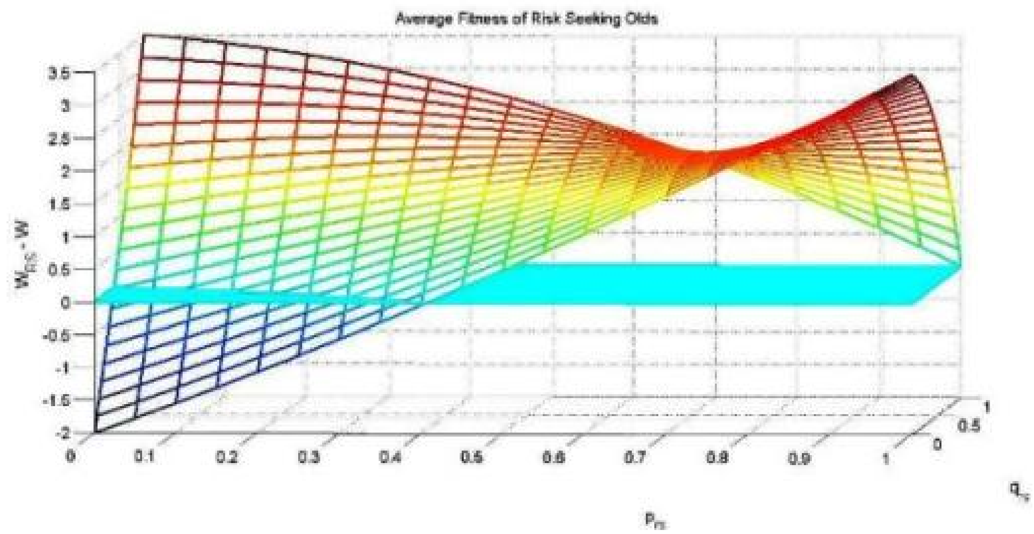
Case 2.1.3.2.3: $q_{rs} \in [0, 1]$ **and** $p_{rs} = 0$,

$$\begin{aligned} W_{RS} - \overline{W} &= q_{rs}F_0 + (\pi - \epsilon)q_{rs}A \\ &\quad -(\pi - \epsilon)(1 - q_{rs})q_{rs}B \\ &\quad +[(1 - \pi + \epsilon) + (\pi - \epsilon)(1 - q_{rs})]C \\ &\quad +(1 - q_{rs})[(\pi - 1 - \epsilon) - (\pi - \epsilon)(1 - q_{rs})]D. \end{aligned}$$

Figure 2. Average Fitness of Risk Seeking Olds




Average Fitness of Risk Seeking Olds
W_RS - W
q_rs
p_rs


Average Fitness of Risk Seeking Olds
W_RS - W
p_rs
q_rs

CHAPTER 3

NUMERICAL ANALYSIS

In this section, we perform the numerical analysis and comparative statics. We examine how risk aversion will roll among the young agents for some sample values of initial proportions of the risk averse youngs and risk averse olds for the game which is played between youngs and olds. Under heterogeneity, we present the mutation behavior among the youngs as of raise and fall tendency and the final circumstance of the mutant invasion. Thus, we are able to obtain the necessary proportion conditions of the agent groups playing the mutant strategy to observe an increase in mutation, thus an invasion of the entire population. That is, we analyze how the fitness of risk aversion of young people position itself in different heterogeneous agent groups.

Without loss of generality in this section we will make our analsis for some reasonable sample values for the payoffs of each game such that $A = 5$, $B = 4$, $C = 0$, and $D = 2$. These payoffs are consistent with our assumption that each game has one payoff dominant equilibrium and one risk dominant equilibrium.

Moreover, we also make our analysis for different initial fitness levels, F_0, which work as initial endowment for the agents in the economy. Hence, the results we found by encoding different levels of F_0 into the model will be an important indicator of analyzing the mutation behavior.

Table 5. The values of the difference between the average fitness of the risk aversion of young population and the average fitness of the entire population for

different proportion levels of risk averse youngs and risk averse olds when $F_0 = 1, 5, 10, 50, 0, -1, -3, -4$ respectively ($\pi = 0.5$)

p_{ra}	q_{ra}	$W_{RA} - \overline{W}$	$W_{RA} - \overline{W}$	$W_{RA} - \overline{W}$	$W_{RA} - \overline{W}$
0.00	0.00	−1.000	−1.000	−1.000	−1.000
0.05	0.05	−0.301	0.078	0.553	4.353
0.10	0.10	−0.297	1.017	1.917	9.117
0.15	0.15	0.801	1.821	3.096	13.30
0.20	0.20	1.216	2.496	4.096	16.90
0.25	0.25	1.547	3.047	4.922	19.92
0.30	0.30	1.799	3.479	5.579	22.38
0.35	0.35	1.978	3.798	6.073	24.27
0.40	0.40	2.088	4.008	6.408	25.61
0.45	0.45	2.135	4.115	6.590	26.39
0.50	0.50	2.125	4.125	6.625	26.63
0.55	0.55	2.062	4.042	6.517	26.32
0.60	0.60	1.952	3.872	6.272	25.47
0.65	0.65	1.800	3.620	5.895	24.09
0.70	0.70	1.611	3.291	5.391	22.19
0.75	0.75	1.391	2.891	4.766	19.77
0.80	0.80	1.144	2.424	4.024	16.82
0.85	0.85	0.876	1.896	3.171	13.37
0.90	0.90	0.593	1.313	2.213	9.413
0.95	0.95	0.299	0.679	1.154	4.954
1.00	1.00	0.000	0.000	0.000	0.000

p_{ra}	q_{ra}	$W_{RA} - \overline{W}$	$W_{RA} - \overline{W}$	$W_{RA} - \overline{W}$	$W_{RA} - \overline{W}$
0.00	0.00	−1.000	−1.000	−1.000	−1.000
0.05	0.05	−0.396	−0.491	−0.681	−0.776
0.10	0.10	0.117	0.000	−0.423	−0.603
0.15	0.15	0.546	0.291	−0.218	−0.473
0.20	0.20	0.896	0.576	−0.064	−0.384
0.25	0.25	1.172	0.796	0.046	−0.328
0.30	0.30	1.379	0.959	0.119	−0.301
0.35	0.35	1.523	1.068	0.157	−0.297
0.40	0.40	1.608	1.128	0.168	−0.312
0.45	0.45	1.640	1.145	0.155	−0.339
0.50	0.50	1.625	1.125	0.125	−0.375
0.55	0.55	1.567	1.072	0.082	−0.412
0.60	0.60	1.472	0.992	0.032	−0.448
0.65	0.65	1.345	0.889	−0.020	−0.475
0.70	0.70	1.191	0.771	−0.069	−0.489
0.75	0.75	1.016	0.640	−0.109	−0.484
0.80	0.80	0.824	0.504	−0.136	−0.456
0.85	0.85	0.621	0.366	−0.143	−0.398
0.90	0.90	0.413	0.233	−0.127	−0.307
0.95	0.95	0.204	0.109	−0.080	−0.175
1.00	1.00	0.000	0.000	0.000	0.000

We conclude that for lower levels of the initial fitness, the probability of observing mutation will be higher. That is to say that, if F_0 increases, then the risk aversion among the young agents decreases.

Now, we consider how risk aversion evolves among the youngs for a given population if the population of olds is only constituted by risk seeking ones. Hence, the young agents will match only with risk seeking olds. Similarly, we present the results of the condition if there exist only risk averse olds in the population. Thus, the young agents now will only match with risk averse olds.

Table 6. The values of the difference between the average fitness of the risk aversion of young population and the average fitness of the entire population for different proportion levels of risk averse youngs and only risk seeking olds, $q_{ra} = 0$ or only risk averse olds, $q_{ra} = 1$

p_{ra}	$W_{RA} - \overline{W}$ *when* $q_{ra} = 0$	$W_{RA} - \overline{W}$ *when* $q_{ra} = 1$
0.00	−1.000	4.000
0.05	−0.631	3.753
0.10	−0.275	3.510
0.15	0.068	3.273
0.20	0.400	3.040
0.25	0.718	2.813
0.30	1.025	2.590
0.35	1.319	2.372
0.40	1.600	2.160
0.45	1.869	1.952
0.50	2.125	1.750
0.55	0.000	1.553
0.60	2.600	1.360
0.65	2.819	1.173
0.70	3.025	0.990
0.75	3.219	0.812
0.80	3.400	0.640
0.85	3.569	0.472
0.90	3.725	0.310
0.95	3.869	0.152
1.00	4.000	0.000

The effect of changes in the structure of the matching process shows that if there exist only risk averse olds in a population, then risk aversion exactly arises among the youngs. Similarly, if there exist only risk seeking olds in a population, then the evolution of risk aversion among the youngs depends on the level of the proportion of risk averse youngs; the level of playing the mutant strategy would increase, decrease or stay the same.

Now, we can consider the effects of the mutation on the population structure by means of the agents' intertemporal strategy choices. For this purpose, first suppose that the age groups choose their actions which lead them to the payoff dominant equilibrium as in economics literature argument. Hence, let the population be constituted by risk seeking youngs and risk averse olds at time $t = 0$. Let $p_{rs} = 0.90$ and $q_{ra} = 0.90$. Then, we expect a rise in number of the risk averse youngs and risk seeking olds at $t = 1$. Let take these new higher proportions as $p_{ra} = 0,90$ and $q_{ra} = 0.10$ at $t = 1$. Then, we observe the reverse at $t = 2$, i.e., risk aversion decreases among the youngs. This means that, for this given game, under given initial mutant proportions below, risk dominancy fluctuates.

Table 7. Intertemporal analysis of risk dominant strategy frequency and payoff dominant strategy frequency permanence for different scenarios ($n \in Z_+$)

t	p_{ra}	p_{rs}	q_{ra}	q_{rs}	$W_{RA_{young}} - \overline{W}$	$W_{RS_{old}} - \overline{W}$
0	0.10	0.90	0.90	0.10	3.325	2.825
1	0.90	0.10	0.10	0.90	3.325	2.825
2	0.95	0.05	0.05	0.95	3.644	3.144
n	0.975	0.025	0.025	0.975	3.817	3.317
$n+1$	1.00	0.00	0.00	1.00	4.000	3.500

t	p_{ra}	p_{rs}	q_{ra}	q_{rs}	$W_{RA_{young}} - \overline{W}$	$W_{RS_{old}} - \overline{W}$
0	0.10	0.90	0.90	0.10	3.325	2.825
1	0.125	0.875	0.0125	0.9875	−0.032	0.761
2	0.10	0.90	0.01	0.99	−0.201	0.630
n	0.00	1.00	0.00	1.00	−1.00	0.00

t	p_{ra}	p_{rs}	q_{ra}	q_{rs}	$W_{RA_{young}} - \overline{W}$	$W_{RS_{old}} - \overline{W}$
0	0.05	0.95	0.05	0.95	-0.301	0.548
1	0.03	0.97	0.01	0.99	-0.707	0.232
2	0.01	0.99	0.005	0.995	-0.888	0.088
n	0.00	1.00	0.00	1.00	-1.00	0.00

Hence, if we analyze a population when it is in its general risk taking behavior pattern such that one group leaves playing risk dominant strategy and the other group continues on choosing its risk dominant action, then we conclude that at the end mutation become extinct for the payoff dominant equilibrium biased group.

Thus, this enable us to obtain the threshold levels of population proportions to observe a risk dominancy continuousness in the subject population and vice versa.

CHAPTER 5

CONCLUSION

In this study, we have applied evolutionary game theory techniques to solve a $2x2$ coordination game which has one payoff dominant and one risk dominant equilibrium in a model including youngs and olds populations to analyze their risk taking behaviors. We have first solved a benchmark model by taking the population homogeneous in terms of age. Hence, we made the calculations for the games which are played between only young agents and between only old agents. Then, we derived the necessary conditions depending on the introduced payoff levels for risk dominant strategy invasion for both youngs and olds populations.

In the second step, we have introduced heterogeneity into the benchmark model. We stated the possibilities of matching between different age groups in this setup and reached the equations that promote evolution in terms of risk taking behavior of agents. Then, we derived the open form analytical levels of the payoffs which guarantee that for a given population, members would focus on choosing the risk dominant action. We also provided numerically the effects of the initial fitness level, initial risk averse and risk seeking proportions on the pattern of the evolution process. We find that decrease in initial level of fitness increases the mutation. Moreover, we concluded that in a population the olds of which are only risk averse, mutation increases in time among the youngs. However, in a population the olds of which are only risk seeking, there exists an initial risk aversion threshold for the youngs population to promote risk aversion among them.

Finally, we studied the intertemporal effects of different risk averse and risk seeking population proportions on mutation. We showed that it is possible to find a spesific initial mutation proportion which guarantees the further mutation continuousness or fluctuations on mutation. Moreover, in a population if one group leaves playing risk dominant strategy and the other group continues on choosing its risk dominant action, then we concluded that mutation become extinct for the payoff dominant equilibrium biased group.

Further extensions could be considered by applying different matching processes which allow risk taking attitude switches. In this case, the possibility for matching of players could be taken different for different population groups such that some features like distance, neighborhood or social status would play a deterministic role on the possibilities. This is appropriate to describe situations where players interact only with some spesific players. Also, exogeneous variables that promote strategy deviations could be included covering framing effect and sunk cost fallacy which would have effect on the difference between the payoff levels since people evaluate these payoffs with respect to their own lose understanding. Another important question is how even small changes in Arrow-Pratt measure of relative risk aversion (RRA) do affect the decision process of agents. By using a three-period OLG Model in which agents are heterogeneous with respect to their ages, as well as their risk attitudes, we would work with not constant inter-temporal elasticity of substitution as usual. RRA would vary across the agents such as $\sigma(t)$, $\sigma(t+1)$, $\sigma(t+2)$ where $\sigma(.)$ is the inter-temporal elasticity of substitution (the inverse of relative risk aversion measure). Hence, with respect to this link between time and risk aversion measure, the intertemporal mutation behavior of a population would be studied in detail and would be included in OLG framework endogeneously. Moreover, age distribution could be detailed by adding more age intervals to the analysis. Finally, by increasing the number of agents, the interactions among agents could be analyzed.

BIBLIOGRAPHY

Mohr, P.N.C., Biele, G. and Heekeren, H.R. 2010. "Neural Processing of Risk," *The Journal of Neuroscience* 30(19): 6613-6619.

Strough, J.N. and et al. 2008. "Are Older Adults Less Subject to the Sunk-Cost Fallacy Than Younger Adults?," *Psychological Science* 19(17): 650-652.

Janssen, M.A. and Scheffer, M. 2004. "Overexploitation of Renewable Resources by Ancient Societies and the Role of Sunk-Cost Effects," *Ecology and Society* 9(1): 6.

Stahl, D.O. and Huyck, J.V. 2002. “Learning Conditional Behavior in Similar Stag Hunt Games.” Working Paper. Wharton University of Pennsylvania, Philadelphia.

Halek, M. and Eisenhauer, J.G. 2001. "Demography of Risk Aversion," *The Journal of Risk and Insurance* 68(1): 1-24.

Klaczynski, P.A. 2001. "Framing Effects on Adolescent Task Representations, Analytic and Heuristic Processing, and Decision Making: Implications for the Normative/Descriptive Gap," *Journal of Applied Developmental Psychology* 22(3): 289-309.

Cunha-e-sa, A.M. and Reis, A.B. 2007. “The Optimal Timing of Adoption of a Green Technology,” *Environmental and Resource Economics* 36: 35-55.

Schützeichel, J. and Michl, T. 2010. “A Neuroeconomic Perspective on Age and Risk-taking Behavior.” Paper presented at LabSi Conference on “Neuroscience and Decision-Making,” held in University of Siena, Santa Chiara, Italy, September 20,21.

Weibull, J.W. 1998. “What Have We Learned from Evolutionary Game Theory So Far?.” Working Paper 487. The Research Institute of Industrial Economics IUI, Sweden.

Mailath, G.J. 1998. "Do People Play Nash Equilibrium? Lessons from Evolutionary Game Theory," *Journal of Economic Literature* 36(3): 1347-1374.

Friedman, D. 1998. "On Economic Applications of Evolutionary Game Theory," *Journal of Evolutionary Economics* 8(1): 15-43.

Wang, H. and Hanna, S.D. 1997. "Does Risk Tolerance Decrease With Age?," *Financial Counseling and Planning* 8(2): 1-6.

Jagannathan, R. and Kocherlakota, N.R. 1996. "Why Should Older People Invest Less in Stocks Than Younger People?," *Federal Reserve Bank of Minneapolis Querterly Review* 20(3): 11-23.

Kandori, M., Mailath, G. and Rob, R. 1993. "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica* 61(1): 29-56.

Ellison, G. 1994. "Learning, Local Interaction, and Coordination," *Econometrica* 61(5): 1047-1071.

Harsanyi, J. and Selten, R. 1988. *A General Theory of Equilibrium Selection in Games*. Cambridge, MA: MIT Press.

Morin, R.A. and Suarez, A.F. 1983. "Risk-aversion Revisited," *The Journal of Finance* 38(4): 1201-1216.