INTEGRATED MACHINE-SCHEDULING AND INVENTORY PLANNING OF DOOR MANUFACTURING OPERATIONS AT OYAK RENAULT FACTORY

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by

Nurcan Bozkaya

July, 2012

I certify that I have read this thesis and that in my opinion it is full adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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I certify that I have read this thesis and that in my opinion it is full adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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ABSTRACT

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A car passes through press, body shell, painting and assembly stages during its manufacturing process. Due to the increased competition among car manufacturers, they aim to continuously advance and improve their processes. In this study, we analyze planning operations for the production of front/back and left/right doors in body shell department of Bursa Oyak-Renault factory and propose heuristic algorithms to improve their planning processes. In this study, we present four different mathematical models and two heuristics approaches which decrease the current costs of the company particularly with respect to inventory carrying and setup perspectives. In the body shell department of the company, there are two parallel manufacturing cells which produces doors to be assembled on the consumption line. The effective planning and scheduling of the jobs on these lines requires solving the problem of integrated machine-scheduling and inventory planning subject to inclusive eligibility constraints and sequence independent setup times with job availability in flexible manufacturing cells of the body shell department. The novelty in the models lie in the integration of inventory planning and production scheduling decisions with the aim of streamlining operations of the door manufacturing cells with the consumption line. One of the proposed heuristic approaches is Rolling Horizon Algorithm (RHA) which divides the planning horizon into sub-intervals and solves the problem by rolling the solutions through sub-intervals. The other proposed algorithm is Two-Pass Algorithm which divides the planning horizon into sub-intervals and solves each sub-problem in each sub-interval to optimality for two times by maintaining the starting and ending inventory levels feasible. These approaches are implemented with Gurobi optimization software and Java programming language and applied within a decision support system that supports daily planning activities.

Keywords: Decision Support System, Integrated Manufacturing System, production planning and scheduling, car manufacturing.

ÖZET

OYAK RENAULT FABRİKASI KAPI ÜRETİM HATLARINDA ENTEGRE ENVANTER PLANLAMA VE MAKİNE ÇİZELGELEME OPERASYONLARI

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Bir otomobil, üretimi sırasında, özetle pres, kaporta, boya ve montaj aşamalarından geçmektedir. Otomotiv üreticileri arasındaki artan rekabet koşullarında firma, süreçlerini sürekli olarak geliştirmek ve iyileştirmek istemektedir. Bu çalışma kapsamında, Bursa Oyak-Renault fabrikasının kaporta atölyesindeki ön/arka ve sağ/sol kapı üretiminin planlama operasyonları analiz edilmiş ve planlama süreçlerini iyileştirmek için kesin ve sezgisel algoritmalar önerilmiştir. Bu çalışmada dört farklı matematiksel model ve firmanın özellikle envanter taşıma ve kurulum maliyetleri açısından maliyetlerini düşüren iki sezgisel yaklaşım önerilmiştir. Firmanın kaporta atölyesinde tüketim hattında araç gövdesine monte edilen kapıların üretimini yapan iki paralel üretim hücresi bulunmaktadır. Bu hücrelerdeki işlerin etkin bir şekilde planlanması ve çizelgelenmesi, kaporta atölyesindeki esnek üretim hücrelerinde kapsayan makine atama kısıtlarını, iş elverişliliği ve sıra-bağımsız kurulum zamanlarını gözönünde bulunduran entegre makine çizelgeleme ve envanter planlama probleminin çözülmesini gerektirmektedir. Modellerdeki yenilik, tüketim hattı ile kapı üretim hücrelerindeki operasyonların uygun hale getirilmesi amacı ile envanter planlama ve üretim çizelgeleme kararlarını entegre olarak alabilmesinde yatmaktadır.

Önerilen sezgisel yaklaşımlardan biri planlama ufkunu alt-aralıklara bölen ve her bir alt-aralığı yuvarlayarak çözen “Yuvarlanan Planlama Ufku”dur. Diğer bir yaklaşım ise, planlama ufkunu alt-aralıklara bölerek her bir alt problemi başlangıç ve bitiş envanter seviyelerini koruyarak iki kere çözen “İki-Aşamalı Algoritma”dır. Geliştirdiğimiz bu yaklaşımlar bilgisayar ortamında Gurobi optimizasyon yazılımı ve Java programlama dili kullanılarak çözüm üretmek üzere işlenmiş, günlük kullanıma elverişli şekilde bir karar destek sistemi çerçevesinde uygulanmıştır.

Anahtar Kelimeler: Karar Destek Sistemi, Bütünleşik Üretim Sistemi, üretim planlama ve çizelgeleme, araç imalatı.

To my family…

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Chapter 1

Introduction

Car manufacturing sector is a highly competitive environment in which every company has to adapt and increase productivity while reducing their expenses. To this end, planning takes an important role.

In this study, we focus on the scheduling and planning operations of the manufacturing cells in Oyak-Renault factory in Bursa. This study focuses on the planning operations of the body shell department. Due to increased competition, Renault aims to continuously improve its production processes. As a step in this direction, it wishes to apply the RIMS-Renault Integrated Manufacturing System and achieve high production flexibility in all facilities. RIMS application is considered particularly crucial when integrating new models to existing manufacturing cells. In line with the RIMS approach, Renault aims to install flexible door manufacturing cells in the body shell department of the plant. In this new system, each manufacturing cell can produce the specified models which may result in machine eligibility restrictions. Setup operations are expensive in time and cost, hence there is a trade-off between inventory holding and setup costs. In addition, it is necessary to streamline the production schedules with the pace of the downstream consumption line so that the continuing operations do not experience unwanted disruption due to lack of part availability. Currently, the company holds excessively high levels of inventory for body parts of different models of cars to ensure sufficient availability of parts to continuously feed the consumption lines in accordance with the demand schedule. Accordingly, we develop an integrated optimization of the planning and scheduling methods of these flexible cells with a special consideration for the integration of new models. The novelty in the models lies in the integration of inventory planning and production scheduling decisions with the aim of streamlining operations of the door manufacturing cells with the consumption line. With this streamlined approach, it is desired to satisfy downstream demand in a just-in-time manner to the extent possible and in turn, reduce inventory levels to a possible minimum.

This thesis is structured as follows. In Chapter 2, we first introduce the problem environment and then provide the definition of the problem. In Chapter 3, we present the review of the literature. It consists of the studies related to parallel machine scheduling with eligibility restrictions and scheduling under the sequence independent setup times. In Chapter 4, we present the mathematical models that we formulated to solve the problem. In Chapter 5, the details of the proposed solution methods are explained. In Chapter 6, we explain the data set, the test environment, and the comparison methods. Then, we report the test results and give a discussion of the results. Finally in Chapter 7, we conclude with final remarks and the future search directions.

Chapter 2

Problem Definition

A car typically passes through press, body shell, painting, and assembly stages during its manufacturing process as seen in Figure 2.1. These stages are highly interdependent and therefore planning and scheduling of the jobs in each stage are important for an effective production. In this thesis, planning operations of the body shell department at Oyak-Renault's Bursa plant are examined. In particular, we investigate the door manufacturing environment at the body shell department and tackle with the problem of integrated machine-scheduling and inventory planning subject to inclusive eligibility constraints and sequence independent setup times with job availability in flexible manufacturing cells of the body shell department.

The general stages of the manufacturing process can be explained as follows. Firstly, body parts are formed in the press department and sent to the body shell department, where they are welded together to produce the body shell of a car. The body shell is then subjected to the painting operation, after which car doors are removed from the body to be assembled later again. The unassembled door interiors are subjected to the trim operations, after which they are assembled back on the body. The next stage is the assembly, during which the electronic and mechanic components are assembled on the body.

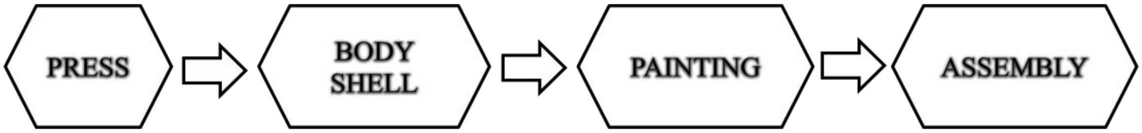


Figure 2.1 The main stages of car manufacturing environment

As for the car door manufacturing environment at Oyak-Renault’s Bursa plant, there are two cells producing car doors. The first cell produces front doors while the other one produces rear doors. There are also manufacturing cells for bonnet and trunk doors. Manufacturing cells of bonnet and trunk doors are out of the scope of our study. The manufacturing operations are determined based on whether a door belongs to the front or the rear. The operations for left and right doors of either the front or the rear are very similar. Thus, the right and left car door operations are run in a symmetric and simultaneous manner. Incidentally, it may be sufficient to explain one of these cells to present the general structure of car door manufacturing. Since the manufacturing cells for left and right doors of either the front or the rear are located in parallel and consist of identical sequence of operations, the schedule obtained for one of the doors can be implemented for the others as well. Figure 2.2 illustrates operations and the general structure of these cells in the existing situation.

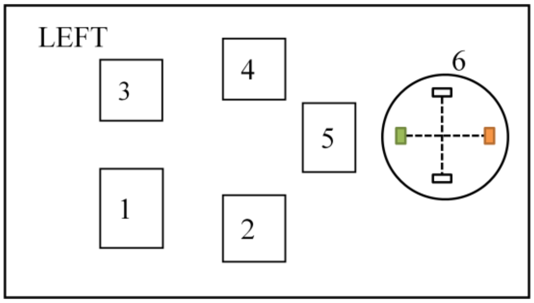


Figure 2.2 The schematic illustration of the car door manufacturing cell at Oyak-Renault

The operations numbered as 1-6 in Figure 2.2 in a car door manufacturing cell can be stated as follows:

1. Initial unification of the interior frame.
2. Final unification of the interior frame.
3. Riveting of the interior frame.
4. Gluing the exterior cover.
5. Assembly of the interior and exterior frames to each other.
6. Robotic curling of the interior and exterior frames to each other.

In the car door manufacturing cells, the bottleneck operation is the robotic operation, which is the curling of the interior and exterior pressed door parts to each other, shown as operation 6 in Figure 2.2. The other operations can be paced in accordance with this operation. The robot has two heads in the current situation. Each head has the die of certain type of a car model door's production. The heads are represented as colored boxes in Figure 2.2 which represents one of the current manufacturing cells in Renault plant. As shown in this figure, two types of car doors can be produced with the colored heads and the other two heads are not installed in the current situation. The empty slots are reserved for the new models' dies that will be produced in the future. Setup operations include the head turns of these robots.

Additionally, the storage area allocated for keeping inventories is an important issue for the company because of the need for extra space in the facility. To this end, it is desired to decline the inventory levels. In the body shell department, the produced car doors are stored in special unitizing vessels, each of which consist of eight door parts, in a storage area. They are stored in the storage area to be transferred into the consumption line which can be seen on Figure 2.3. The ‘consumption line’ (referred to as the “ferage” line in Oyak-Renault) is the line where the body of the cars flow with a specified pattern. During this flow, the doors are assembled to the body of the cars. Currently, doors are assembled to seven different car models in the consumption line. Two of these seven different car models’ doors are produced in the body shell department. In the existing condition, the other models’ doors are produced in the same plant, but in different buildings and they are transferred to the same consumption line. This causes an unnecessary cost to the company. For that reason, company wants to produce all types of doors in the body shell department.

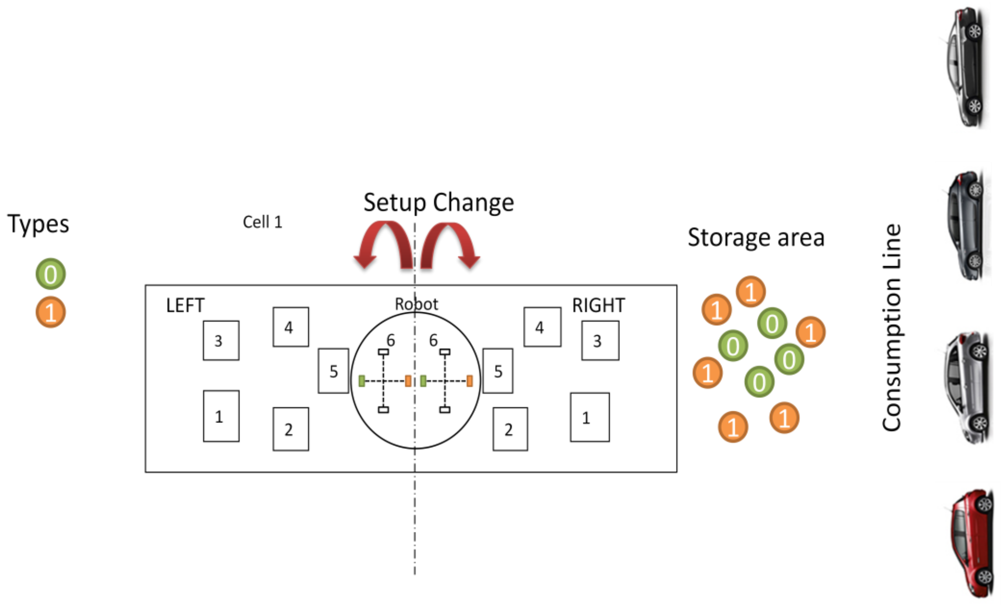


Figure 2.3 The existing system of the car door manufacturing cells

Oyak-Renault is willing to make the car door manufacturing cells more flexible and efficiently planned and scheduled. To this end, their goal is to apply the RIMS-Renault

Integrated Manufacturing System which makes the manufacturing cells flexible and the production efficient for achieving high production flexibility in all facilities. RIMS application is considered particularly crucial when integrating new models to existing manufacturing cells. In line with the RIMS approach, Oyak-Renault plans to convert the current car door manufacturing cells to flexible cells. Additionally, they also plan to install one additional flexible cell which can produce four different types of doors. Thus, they will have two flexible door manufacturing cells in the body shell department of the plant which is able to produce four different types of doors, and in the future, cells in the body shell department may produce up to eight different types. As mentioned above, in the new system there will be two cells (Cell 1 and Cell 2). Cell 1 is the currently existing cell and Cell 2 is the cell that will be installed in the future. In the new system, each manufacturing cell may produce only the specified car types resulting in an inclusive kind of machine eligibility restrictions. In inclusive kind of eligibility restrictions, both cells are able to produce some specified types of doors. To explain the inclusive kind eligibility property in our problem, we present the four different car door types as 0, 1, 2, and 3. Cell 1 can process four different types (0, 1, 2, and 3) of car doors while the other cell can process only two (types 2 and 3), which results in an inclusive eligibility restriction property.

In our thesis, we consider the problem of scheduling and planning of the door manufacturing cells which arises from the integration of the new system in the car door manufacturing cells. As we stated earlier, the cells have several operations and the bottleneck operation is the robotic operation in each cell. Since the other operations can be paced in accordance with the robotic operation, robots can be considered in the form of two parallel machines. The problem in the car door manufacturing environment can be stated as follows:

There are two parallel machines with the same speeds but each machine can process jobs belonging to a certain subset of car models which means that machines have eligibility restrictions. These machines produce door parts for the downstream operation. The jobs are carried with special unitizing vessels from the storage area of the door manufacturing cells to the consumption line. Thus, a number of door frames are transferred together to the consumption line in the form of a single transit batch. A full transit batch carries eight doors of a given type. Due to the limited number of unitizing vessels (transit batches), only fully loaded vessels are authorized for transfer. Therefore, we model eight of the same type of doors as a single job in our problem. The jobs are arranged into families based on the door types. Doors in the same family are identical in the sense that a due date for a given job can be satisfied from the inventory of jobs belonging to the same family. A batch can be defined as a set of jobs between two consecutive setups. The problem has the job availability property due to the fact that a job's start and completion times are different from other jobs in the same batch (Allahverdi, 2008). A sequence-independent setup time is required when switching between jobs belonging to different families because of the need of changing the robot's head. Since setup operations are expensive in time and cost, there is a trade-off between inventory holding and setup costs. Thus, we consider both the inventory holding and setup costs while modeling the problem. Jobs need to be finished before their due dates imposed by the consumption line. The time instant that a car body requiring a certain type of a door arrives the door assembly station sets the demand time of that particular door type. Since a job consisted of eight of the same type of doors, due date of a job is set to the earliest demand time at the consumption line of a door in the corresponding transit batch. We do not allow for late jobs since it is highly crucial not to stop the consumption line in our problem context. Ultimate aim is to satisfy the consumption line just-in-time without keeping any inventories however operating with zero inventory may not be possible due to the setup time on the robot operation. Therefore, a "contingency stock" level for the body parts of the different types of cars must be kept in the buffer space in order to ensure sufficient availability of parts to continuously feed the consumption line in accordance with the demand schedule. However, the company has a limited buffer space. This causes a high unit storage area cost. Hence, the company is willing to reduce inventory levels in the buffer space. Therefore, it is justified to seek academic solutions to handle an integrated optimization of the planning and scheduling operations of these flexible cells with a special consideration for the integration of new models.

For practical reasons, the company desires to have no idle time between successive operations in the body shell department. This leads us to consider two scenarios. The first scenario allows idle time between successive jobs where the second forces consecutive jobs to be processed immediately one after another. Additionally, the demand of the consumption line is known six days before and last minute changes are negligible, therefore the problem is solved with deterministic perspective.

To sum up, we can briefly state the following factors that should be considered while planning the production in these flexible cells:

- Setup times/costs,
- Storage area restrictions,
- Inventory and storage area costs,
- Unitizing vessels costs,
- Demand rate of the downstream operations.

In the next chapter, we provide a literature review of the studies in the current literature.

Chapter 3

Literature Review

In this chapter, we provide a brief literature review of the studies which are closely related to both parallel machine scheduling problems under sequence independent setup times and eligibility restrictions. In the following sections, Lawler et al.'s (1993) standard three-field notation is used for describing the scheduling problem.

2.1. Parallel Machine Scheduling with Eligibility Restrictions

Scheduling with eligibility constraints have been studied in the context of computer science and operation research under different names. Two of these names are scheduling with processing set restrictions and scheduling with eligibility constraints. We call this problem as the scheduling problem with eligibility restrictions.

Leung and Li (2008) provide a comprehensive survey on scheduling with processing set restrictions. They covered offline and online algorithms for both non preemptive and preemptive scheduling environments with different performance criteria such as makespan, maximum lateness, total (weighted) completion time, total (weighted) number of tardy jobs, as well as total (weighted) tardiness. Lee et al. (2010a) also provided a survey in online scheduling in parallel machine scheduling subject to eligibility constraints while minimizing the makespan. Two basic online scheduling paradigms (online over list and online over time) are considered by Lee et al. (2010a). They reviewed all the results in the literature related with eligibility constraints for these two paradigms and provided extensions. Furthermore they pointed out the open problems in this area.

In the problem of parallel machine scheduling with eligibility restrictions, the machines can process specified groups of jobs. There are two special cases of parallel machine scheduling with eligibility restrictions. These cases are nested and inclusive eligibility set restrictions.

Let $\{M_1, M_2, .., M_m\}$ be the arbitrary subsets of machine set M and $\{J_1, J_2, .., J_n\}$ be the subsets of jobs where $m \leq n$. In the case of nested eligibility restrictions, M_j and M_k are either disjoint sets, $M_j \subseteq M_k$ or $M_k \subseteq M_j$. The inclusive eligibility set restriction is a special case of the nested eligibility set restrictions where for every pair of M_j and M_k, either $M_j \subseteq M_k$ or $M_k \subseteq M_j$.

Pinedo (1995) showed for the parallel machine scheduling problem with equal processing time and nested machine eligibility restrictions subject to the objective of minimizing makespan, the least flexible first (LPT) dispatching rule gives optimal solution.

Centeno and Armacost (1997) considered the problem of parallel machine scheduling under machine eligibility restrictions with equal due dates and release dates plus a constant. Their objective is to minimize the maximum lateness. They present an efficient algorithm for the problem and use a real data set from a semiconductor manufacturing firm.

Centeno and Armacost (2004) consider the parallel machine scheduling problem with machine eligibility restrictions and release time under the objective of minimizing makespan. They propose online algorithms to solve the problem and show that the longest processing time (LPT) rule outperforms the least flexible job (LFJ) rule in the absence or presence of job release times.

Lee et al. (2011) studied the parallel machine scheduling where jobs have different release times and equal processing times under the machine eligibility restrictions. Their objective is to minimize makespan. They presented algorithms for both online and offline scheduling problem.

Lin and Li (2004) consider both identical and uniform parallel machine scheduling problem with unit processing time under the objective of minimizing makespan. They develop $O(n^3 log_n)$ and $O(m(m+n))$ time algorithms, respectively for the above mentioned problems. Li (2006) extends their work and provides extensions of their models with respect to other objective criterion. The author improves the computational complexities of Lin and Li's (2004) algorithms.

Ou et al. (2008) consider the problem of loading and unloading cargoes of a vessel. Their problem is assigning a set of jobs to the identical parallel machines with inclusive machine eligibility restrictions subject to minimizing the makespan of the schedule. They provide an efficient approximation algorithm and a polynomial time $(1+\varepsilon)$ - approximation scheme (PTAS) to solve the problem. They present that the proposed approximation algorithm has a worst-case bound of 4/3. However, the polynomial time $(1+\varepsilon \in)$ - approximation scheme (PTAS) is not computationally efficient when ε is close to zero.

Glass and Mill (2006) provides efficient algorithms for the parallel machine scheduling problem with identical processing times under nested eligibility restrictions on a food processing plant. Their algorithms are provided for standard regular objective functions.

Li (2006) studies the problem of parallel machine scheduling with unit-length jobs under machine eligibility restrictions. He provides efficient algorithms for various objectives.

Huo and Leung (2010a) study the parallel machine scheduling problem with nested eligibility restrictions under the minimizing makespan objective. They improve a given approximation algorithm for the nested eligibility restriction problem with a worst case bound of 7/4. They propose an algorithm that gives a better worst case bound of 5/4 for two machines and 3/2 for three machines. Huo and Leung (2010b) study the same problem and provided a worst-case bound of 5/3 which is better than the best known algorithm whose worst-case bound is 7/4.

Biró and McDermid (2011) study the matching problems on bipartite graphs and they survey the relationship of this problem and parallel machine scheduling problem under the machine eligibility restrictions with the objective of minimizing makespan. They provide approximation algorithms for those problems' variations where the sizes of the jobs are restricted. They also showed that under the nested processing set restrictions case the two problems become polynomial-time solvable.

Epstein and Levin (2011) study one of the open problems that are proposed by Leung and Li (2008). They provide three polynomial time approximation schemes for the parallel machine scheduling problem with eligibility restrictions under the objective of makespan minimization.

2.2. Scheduling under Sequence Independent Setup Times

Allahverdi et al. (1999, 2008) and Potts and Kovalyov (2000) provide an extensive literature review related to scheduling problems involving setup considerations with batching decisions. There are generally two problem types about the problems with setup considerations which can be classified as sequence independent setup times and sequence dependent-setup times. Setups can also be classified as batch setup times or non-batch setup times. Moreover, setup times can be classified as minor or major setup times. When different types of jobs belong to the same family, a minor setup time is required. A major setup time is required between different job families. We are dealing with the parallel machine scheduling problem under sequence independent setup times with batching decisions. Thus, we focus on the studies that consider the setup operations with batch setup times.

So (1990) study the identical parallel machine scheduling problem with minor or major setup times between types. The problem is finding a feasible schedule which maximizes the total reward under the fixed machine capacity. They assume that the rewards has inverse ratio with processing times, i.e. the rewards are decreased while the processing times are increased. They propose three heuristics and compare their performances. Wittrock (1990) also study the identical parallel machine scheduling problem with minor or major setup times under the objective of minimizing makespan. They develop a heuristic that uses the binary search approach of the MULTIFIT heuristics and compare the results with an earlier approach described by Tang and Wittrock (1985) and Tang (1990).

Monma and Potts (1989) consider the two identical parallel machine scheduling problem with batch setup times. They propose pseudo-polynomial algorithms for the maximum completion time, maximum lateness, total weighted completion time and weighted number of late jobs for a fixed number of batches on a specified number of machines.

They show that when the batch size is arbitrary, two identical parallel machine problems are NP-hard for both preemptive and non-preemptive cases under the objective of maximum completion time, number of late jobs, total weighted completion time problems. Cheng and Chen (1994) also study the problem of scheduling several batches of jobs on identical two parallel machines with minimizing the total completion time of jobs. They show that even for the case of the sequence independent setup times and equal processing times, the problem is NP-hard. Monma and Potts (1993) extend their earlier studies for the problem of preemptive scheduling with batch setup times on *m* identical parallel machines with minimizing the maximum completion time. They propose two heuristics.

Schutten and Leussink (1996) consider the problem of *m* identical parallel machine scheduling of *n* independent jobs with release dates, due dates, and batch setups under the objective of minimizing maximum lateness. They provided a branch and bound algorithm to solve the problem.

Brucker et al. (1998) study the parallel machine batch scheduling problem with deadlines. They showed that the problem of two identical machines is NP-hard even with the case of common deadline, unit processing times and setup times.

Liaee and Emmons (1997) review the scheduling problem of several families of the jobs on single or parallel machines with setup time under the group technology assumption. They prove that unless all the families contain the same number, the problem of minimizing the total completion time on parallel machines with sequence independent setup times under the group technology assumption is NP-hard. Liu et al. (1999) study the group sub-lotting problem on two identical parallel machines with common setup times and unit processing times. They establish that the problem is NP-hard in the ordinary sense, and propose a pseudo polynomial-time algorithm for the problem of minimizing the total completion time on two identical parallel machines with batch sequence independent common setup times and equal processing times.

Leung et al. (2008) study the batch scheduling problem on m parallel machines where the processing time of each job is used as a step function of its waiting time. For each job i, if its waiting time is less than a specified threshold D, then it requires a basic processing time $p_i = a_i$; otherwise, it requires an extended processing time $p_i = a_i + b_i$. The objective is to minimize the total completion time. They showed that even if there is a single machine and $b_i = b$ for all $i = 1..n$, the problem is *NP*-hard in the strong sense. They also provide an approximation algorithm for the case of $b_i < D$ for all $i = 1..n$ with a performance guarantee of 2.

Yi and Wang (2003) address a parallel machine scheduling problem, which involves both batch setup times and earliness-tardiness penalties for the jobs, have a common due date. They present a fuzzy logic embedded genetic algorithm to solve the problem. Yi et al. (2004) present also a fuzzy logic embedded genetic algorithm for solving the problem of parallel machine scheduling with setup times. The objective of their problem is to minimize the total flow time of grouped jobs. Webster and Azizoglu (2001) and Azizoglu and Webster (2003), study the same problem with the objective of minimizing total weighted flow time. Webster and Azizoglu (2001) present backward and forward dynamic programming algorithms and derived two properties to improve the computational performance of the algorithms. Azizoglu and Webster (2003) design branch-and-bound algorithms to solve the problem. Since the problem is unary NP-hard, there are difficulties with solving the problem optimally with large sized problems. Their algorithms are solving the problem with 25 jobs on two or three machines and 15 jobs on five machines in a reasonable amount of time. Dunstall and Wirth (2005a) provided branch-and-bound algorithms which use a least loaded-processor (LLP) branching scheme for the same problem of parallel machine scheduling with family setup times.

Dunstall and Wirth (2005b) also study the same problem. Heuristics based on a combination of list-scheduling, improvement phases and the solution of single machine sub-problems are presented.

Chen and Powell (2003) provide a column generation based branch-and-bound exact solution algorithms for the parallel machine scheduling problem with sequence independent batch setup times under the objective of minimizing the weighted number of tardy jobs. Their algorithms found optimal solutions for problems up to 40 jobs, 4 machines, and 6 job families.

Chen and Wu (2006) study the unrelated parallel machine scheduling problem with auxiliary equipment constraints under the objective of minimizing the total tardiness. They proposed a heuristic based on threshold accepting methods, tabu lists and improvement procedures. According to the computational results of their heuristic, it outperforms the basic simulated annealing heuristic with respect to the solution quality and run time.

Gambosi and Nicosia (2000) study a parallel machine scheduling problem with sequence independent batch setup times where the objective is to minimize the maximum completion time. They analyze a suitable version of the classical list scheduling algorithm and propose an on-line algorithm for the problem.

Lin and Jeng (2004) study the parallel machine batch scheduling problem to minimize the maximum lateness and the number of tardy jobs. They propose two dynamic programming algorithms to solve the problems optimally. The algorithms need exponential computational times for optimal solutions. For a fixed number of machines the computational complexities become pseudo polynomial.

Wilson et al. (2004) address the problem of parallel machine scheduling with sequence independent setup times and job release times under the objective of makespan minimization for cut and sew operations of upholstered furniture manufacturing.

Yang (2004) considers the parallel machine scheduling problem of component fabrication for *N* two-component products under the objective of minimizing the total completion time. Yang (2004) presents two heuristics to solve the problem near-optimal.

The most recent study on identical parallel machine scheduling with family setup times is conducted by Liao et al. (2012). The objective of their study is to minimize the total weighted completion time. They extend the work of Dunstall and Wirth (2005b) and improve their heuristics. Liao et al. (2012) show that their heuristics outperforms Dunstall and Wirth's heuristics in terms of both computationally efficiency and solution quality. A brief summary of the related literature on parallel machine scheduling problems with sequence independent setup times is given in the Table 3.1.

To the best of our knowledge, there is no paper that covers both machine eligibility constraints and sequence independent batch setup times on parallel machine scheduling problems. In this thesis, we are dealing with both sequence independent batch setup times under the job availability property and machine eligibility restrictions in this study in an automotive firm's door manufacturing cells.

Table 3.1 A Summary of the problems on parallel machine scheduling with sequence independent setup times

References	**Criterion**
Tang and Wittrock (1985)	C_{max}
Monma and Potts (1989)	C_{max}, L_{max}, NLJ, WTCT (two-machine, prmp and non-prmp)
So (1990)	Total reward (minor and major setups, fixed processing capacity)
Tang (1990)	C_{max} (minor and major setups)
Wittrock (1990)	C_{max} (minor and major setups)
Monma and Potts (1993)	C_{max}
Cheng and Chen (1994)	TCT (two machines)
Schutten and Leussink (1996)	L_{max} (r_j)
Liaee and Emmons (1997)	$\sum C_j$ (group technology)
Brucker et al. (1998)	L_{max} (r_j, deadlines)
Liu et al. (1999)	$\sum C_j$ ($p_j = p_i$, common setup time)
Gambosi and Nicosia (2000)	C_{max} (online scheduling)
Webster and Azizoglu (2001)	$\sum w_j f_j$
Azizoglu and Webster (2003)	$\sum w_j f_j$
Chen and Powell (2003)	$\sum w_j C_j$, $\sum w_j U_j$
Yi and Wang (2003)	$\sum w_j E_j + \sum w_j T_j$
Wilson et al. (2004)	C_{max} (r_j, common setup time)
Yi et al. (2004)	$\sum f_j$
Lin and Jeng (2004)	L_{max}, $\sum U_i$
Chen and Wu (2006)	$\sum T_j$ (R, jobs restricted to be processed on certain machines)
Dunstall and Wirth (2005a)	$\sum w_j C_j$
Dunstall and Wirth (2005b)	$\sum w_j C_j$
Liao et al. (2012)	$\sum w_j C_j$, $\sum w_j f_j$

Chapter 4

Model Development

In this chapter, we present our mathematical model which is developed to solve the integrated scheduling and inventory planning problem under sequence independent setup times and eligibility restrictions. After describing problem characteristics, we provide models for two different versions of the problem.

As explained in details in the previous chapter, we consider the problem of scheduling *K* independent jobs belonging to *f* different families on *m* parallel machines with eligibility constraints. The eligibility constraints indicate that a subset of machines can process a specified subset of job families and a subset of job families can be processed by a specified subset of machines. The objective is to minimize the total setup and inventory carrying costs while respecting the storage area availability constraints and the pull rate of the downstream operation.

While modeling the problem, the following system characteristics are observed:

1. Each job is available at time zero.
2. The processing times are equal for all families of jobs.
3. Each job is required to be processed on one of the *m* identical machines in parallel according to the machine eligibility restrictions based on families.
4. Each machine can process only one task at any time and also each job can be processed by one machine at any time.
5. Preemption is not allowed.
6. Jobs have the job availability property which means a job's start and completion times are different from other jobs in the same batch.
7. Setup times are independent of the job sequences and are equal for all job families.

In this part, we explain how the system works at the door manufacturing cells. Suppose that different families of jobs are demanded from the consumption line in a specified planning horizon. These demanded jobs have due dates imposed by the consumption line. We plan to produce the total requirement of a given planning horizon while guaranteeing the fulfillment of the consumption line on time so that all jobs are delivered exactly when demanded on their respective due dates. The planning horizon starts with a pre-specified contingency stock level for each family. We also wish this plan to retain the stability of the contingency stock levels at the start and at the end of the planning horizon. The objective is to minimize the total setup and inventory holding costs.

We illustrate the dynamics of the system by using the following simple example. Suppose that there are two families of jobs, red and blue, where one job of each family is demanded within a specified planning horizon. Suppose that the planning horizon ends at time 60 and the job of family blue at time 21 while the job of family red is demanded at time 30 (see Figure 4.1). Moreover, contingency stock is two for red (R1, R2) and zero for blue. Since the system dynamics encourage maintaining the contingency stock, two red family inventories should be maintained at the end. First, a blue family job is produced so that the demand at 21 is satisfied right on time (see Figure 4.1). Job R1 has a due date at time 30 from the consumption line. As there is an available red job, this demand is satisfied from contingency stock, but a red family production is scheduled to start at time 39 so that the contingency stock is maintained at the end of the horizon. Note that, this schedule is the optimal one when the objective is to meet the demand on time with minimum setup and inventory carrying costs. The unused contingency stock is carried until the end of the horizon.

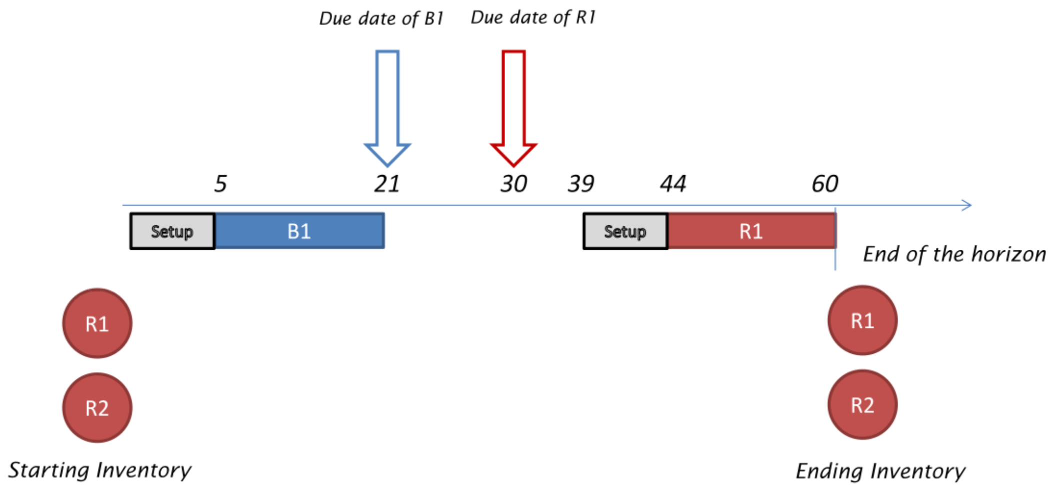


Figure 4.1 The first part of a small example of the problem

In the following sections, we provide mathematical models for the two different cases of our problem. The first case allows idle time between successive jobs. We refer to this case as IT which is short for Idle Time. Due to the operational restrictions (such as the workers' tendency to finish the on hand job as possible as they can, the company's efficiency concerns etc.) the company does not prefer to ask their workers to wait idle between successive jobs. Therefore, we also develop another version of the model where insertion of idle times is not allowed between successive jobs. We denote this latter case by NIT which is short for No Idle Time. The example of Figure 4.1 exhibits IT case. If we were to solve this example as a NIT problem, then the setup for the red item would start at 29 and the production would stop at time 45.

In Sections 4.1 and 4.2 we present the IT and NIT models, respectively. In these models, we enforce the stability of the contingent stock as explained above. In Sections 4.1 and 4.2, we also provide the relaxed versions of these models (denoted by IT-R and NIT-R) where this requirement is relaxed. The relaxed models are utilized in the heuristic solution algorithms presented in Chapter 5.

4.1. Case with Idle Times

In this subsection, we explain our base model which is the case where we allow idle time between successive jobs. First, we present indices, parameters and decision variables of the model. Then, model formulations and explanations are provided.

4.1.1. Model IT

The following notation is used in our models.

Indices and Parameters:

Sets:

J: number of jobs in the planning horizon

K: $|E_m|$

L: number of door type

Indices:

k: position index, $k = 1 \ldots K$

j: job index, $j = 1 \ldots J$

m: machine index, $m = 1,2$

l: family index, $l = 1 \ldots L$

Parameters:

c: setup cost

h: inventory holding cost per unit per unit time

S: setup time

d_j: due date of job *j*

p_j: processing time of job *j*

f_j: family of job *j*

E_m: set of jobs that can be processed on machine *m*

I_l: contingency stock of type *l*

H: length of the planning horizon

M: a large positive number

Decision Variables

$$X_{kjm} = \begin{cases} 1 \text{ if job } j \text{ is produced in position } k \text{ on machine } m \\ 0 \text{ otherwise} \end{cases}$$

$$q_{kjm} = \begin{cases} 1 \text{ if job } j \text{ is produced in position } k \text{ on machine } m \text{ to replenish inventor} y \\ 0 \text{ otherwise} \end{cases}$$

$$y_{km} = \begin{cases} 1 \text{ if setup is required after position } k \text{ on machine } m \\ 0 \text{ otherwise} \end{cases}$$

W_{km} : starting time of the job in position *k* on machine *m*

g_{km} : waiting time of the job in position *k* on machine *m*

We propose to model the problem as follows.

IT:

Min

$$\sum_{m=1}^{2} \sum_{k=1}^{K} (c \times y_{km} + h \times g_{km})$$

$$+ h \left\{ \sum_{k=1}^{K} \sum_{j \in E_m} \sum_{m=1}^{2} (d_j \times q_{kjm}) + \left(\sum_{l:f[j]=l} I_l - \sum_{k=1}^{K} \sum_{j \in E_m} \sum_{m=1}^{2} q_{kjm} \right) \times H \right\}$$

Subject to

$\sum_{m=1}^{2}\sum_{k=1}^{K} X_{kjm} = 1 \quad , \forall j \in E_m$ (1)

$\sum_{j \in E_m} X_{kjm} \leq 1, \; \forall k = 1,2,..,K \,, \; m = 1,2$ (2)

$\sum_{j \in E_m} X_{kjm} \; \geq \sum_{j \in E_m} X_{(k+1)jm} \;, \; \forall k = 1,2,..,K \,, \; m = 1,2$ (3)

$W_{km} + \sum_{j \in E_m}(p_j \times X_{kjm}) + y_{(k+1)m} \times S \leq \; W_{(k+1)m} \; + M \times (1 - \sum_{j \in E_m} X_{(k+1)jm}) \,,$
$\forall k = 1,2,..,K-1 \,, m = 1,2$ (4)

$M \; \times \; y_{(k+1)m} \geq \sum_{j \in E_m} f_j \; \times X_{kjm} - \; \sum_{j \in E_m} f_j \; \times X_{(k+1)jm} \,, \; \forall k = 1,2,..,K-1 \,,$
$m = 1,2$ (5)

$M \; \times \; y_{(k+1)m} \geq -\sum_{j \in E_m} f_j \; \times X_{kjm} + \; \sum_{j \in E_m} f_j \; \times X_{(k+1)jm} \;, \forall k = 1,2,..,K -$
$1 \,, m = 1,2$ (6)

$W_{km} + \; \sum_{j \in E_m} p_j \times X_{kjm} + g_{km} \; = \; \sum_{j \in E_m}(d_j \times \; X_{kjm} \; + (H - d_j) \times q_{kjm}) \;,$
$\forall k \; 1,2,..,K, m = 1,2$ (7)

$\sum_{k=1}^{K} S \times y_{km} \; + \sum_{k=1}^{K} \sum_{j \in E_m} p_j \times X_{kjm} \; \leq H, \forall m = 1,2$ (8)

$q_{kjm} \; \leq X_{kjm} \,, \; \forall k = 1,2,..,K, \; j = 1,2,..J, \; m = 1,2$ (9)

$\sum_{m=1}^{2}\sum_{k=1}^{K}\sum_{j:f(j)=l} q_{kjm} \; \leq I_l \,, \; \forall l$ (10)

$W_{km} \; \leq H \,, \; \forall k = 1,2,..,K, m = 1,2$ (11)

$X_{kjm} \in \; \{0,1\}$ (12)

$y_{km} \in \; \{0,1\}$ (13)

$q_{kjm} \in \; \{0,1\}$ (14)

$W_{km} \; \geq 0$ (15)

$g_{km} \; \geq 0$ (16)

The objective of our problem is to minimize the total holding cost of the inventory, setup costs, and unitizing vessel costs. Constraint set (1) ensures that each job is assigned to exactly one position and one machine. Constraint set (2) restricts the maximum number of jobs that can be processed in a given position on each machine to one. Constraint set

(3) indicates that if no job is assigned to a given position on a machine, other jobs cannot be assigned to the following positions on the same machine. Constraint set (4) ensures that the starting time of the subsequent job on a given machine cannot be earlier than the finishing time of the former job. Constraint sets (5) and (6) determine the required setups between the different types of jobs. Constraint set (7) determines the waiting time of the jobs. Note that, constraint sets (7) and (16) together satisfy the demand. If there is only direct production, the sum of the start time of the production of a job with the processing time has to be smaller than the due date of the job. If the job is fulfilled from inventory at first and then replenished, the sum of the start time of the production of a job with processing time has to be less than the planning horizon. Also we do not allow decreasing the contingency stock level, so the demand is satisfied in this way. Constraint set (8) restricts the total setup and processing time of all jobs so that they do not exceed the length of the planning horizon. Constraint set (9) allows only those jobs consumed earlier from the existing inventory to be produced to replenish inventory. Constraint set (10) restricts the total number of jobs consumed from the inventory to be less than the contingency stock. Constraint set (11) limits the starting time of any job to be no late than the end of the planning horizon. Finally, constraint sets (12) - (14) define the binary restrictions followed by (15) and (16) which are the non-negativity constraints.

4.1.2. Model IT-R

When there is a feasible solution with the current contingency stock level, relaxed version of IT finishes with the same stock level. However, when there is no feasible solution with that level, it decreases the contingency stock level in an attempt to find a feasible solution. This flexibility is provided by constraint sets (17) - (34).

In this relaxed model, the indices and the parameters are the same as IT but we need the following new decision variables:

Decision Variables

$$X_{kjm} = \begin{cases} 1 \text{ if job } j \text{ is produced in position } k \text{ on machine } m \text{ and used to} \\ \qquad\qquad\qquad \text{satisfy the demand immediately} \\ 0 \text{ otherwise} \end{cases}$$

$$R_{kjm} = \begin{cases} 1 \text{ if job } j \text{ is supplied from inventory and produced on machine } m \\ 0 \text{ otherwise} \end{cases}$$

$$q_j = \begin{cases} 1 \;\text{ if job } j \text{ is supplied from inventory} \\ 0 \text{ otherwise} \end{cases}$$

$$y_{km} = \begin{cases} 1 \text{ if setup required after position } k \text{ on machine } m \\ 0 \text{ otherwise} \end{cases}$$

W_{km}: starting time of the job in position *k* on machine *m*

g_{km} : waiting time of the job in position *k* on machine *m*

<u>IT-R:</u>

Min

$$\sum_{m=1}^{2}\sum_{k=1}^{K}(c \times y_{km} + h \times g_{km}) + h\left\{\sum_{j\in E_m}(d_j \times q_j) + \left(\sum_{l:f[j]=l} I_l - \sum_{j\in E_m} q_j\right) \times H\right\}$$

$$+ \; M \sum_{k=1}^{K}\sum_{j\in E_m}\sum_{m=1}^{2}(1 - X_{kjm} - R_{kjm})$$

Subject to

$q_j + \sum_{m=1}^{2}\sum_{k=1}^{K} X_{kjm} = 1, \forall\, j \in\; E_m$ (17)

$\sum_{m=1}^{2}\sum_{k=1}^{K}(X_{kjm} + R_{kjm}) \;\le 1\,, \forall\, j \in\; E_m$ (18)

$\sum_{m=1}^{2}\sum_{k=1}^{K} R_{kjm} \;\le q_j,\;\; \forall\, j \in\; E_m$ (19)

$\sum_{j\in E_m}(X_{kjm} +\; R_{kjm}) \le 1\;,\;\forall\, k = 1,2,..,K\;,\; m = 1,2$ (20)

$\sum_{j\in E_m}(X_{kjm} +\; R_{kjm}) \;\ge \sum_{j\in E_m}(X_{(k+1)jm} +\; R_{(k+1)jm})\;,\;\forall\, k = 1,2,..,K-1\;,\; m =$
$1,2$ (21)

$W_{km} + \sum_{j\in E_m} p_j \times (X_{kjm} + R_{kjm}) + y_{(k+1)m} \times S \leq W_{(k+1)m} + M \times (1 - \sum_{j\in E_m}(X_{(k+1)jm} + R_{(k+1)jm}))$, $\forall k = 1,2,..,K-1, m = 1,2$ (22)

$M \times y_{(k+1)m} \geq \sum_{j\in E_m} f_j \times (X_{kjm} + R_{kjm}) - \sum_{j\in E_m} f_j \times (X_{(k+1)jm} + R_{(k+1)jm})$, $\forall k = 1,2,..,K-1$, $m = 1,2$ (23)

$M \times y_{(k+1)m} \geq -\sum_{j\in E_m} f_j \times (X_{kjm} + R_{kjm}) + \sum_{j\in E_m} f_j \times (X_{(k+1)jm} + R_{(k+1)jm})$, $\forall k = 1,2,..,K-1$, $m = 1,2$ (24)

$W_{km} + \sum_{j\in E_m} p_j \times \left(X_{kjm} + R_{kjm}\right) + g_{km} = \sum_{j\in E_m}(d_j \times X_{kjm} + H \times R_{kjm}), \forall k = 1,2,..,K, m = 1,2$ (25)

$\sum_{k=1}^{K} S \times y_{km} + \sum_{k=1}^{K}\sum_{j\in E_m} p_j \times (X_{kjm} + R_{kjm}) \leq H,\ m = 1,2$ (26)

$\sum_{j:f(j)=l} q_j \leq I_l$, $\forall l$ (27)

$W_{km} \leq H, \forall k = 1,2,..,K,\ m = 1,2$ (28)

$X_{kjm} \in \{0,1\}$ (29)

$y_{km} \in \{0,1\}$ (30)

$q_j \in \{0,1\}$ (31)

$R_{kjm} \in \{0,1\}$ (32)

$W_{km} \geq 0$ (33)

$g_{km} \geq 0$ (34)

The objective of Model IT-R is the same with Model IT as both of their aim is to minimize the total inventory holding and setup costs. In this model, constraint set (17) corresponds to constraint set (1) in Model IT. It differs from Model IT as it can replenish the job from the inventory. Different than Model IT, Model IT-R consists of constraint sets (18), (19), (31) and (32). Constraint set (18) provides that if a job is produced, it is either immediately produced or supplied from inventory and then replenished. Constraint set (19) indicates that a job, which is supplied from inventory, can either be produced or not. Constraint set (31) is a binary decision variable for jobs which are supplied from inventory. Lastly constraint set (32) is a binary decision variable for replenishing the contingency stock. Constraint sets (20), (21), (22), (23), (24), (25), (26), (27), (28), (29), (30), (33), (34) correspond to constraint sets (2), (3), (4), (5), (6), (7), (8), (10), (11), (12), (13), (15), (16), respectively. The objective function of this model is slightly different from Model IT. In this model, we add a new term to the objective function of Model IT which always maintains the contingency stock level. In this term, we penalize not replenishing the contingency stock with a sufficiently large number M so as to ensure feasibility. We compute the minimum value of M by taking the difference of the maximum and minimum possible objective function value of producing versus not producing the same job.

M used in objective function can be deduced as follows;

$$M \geq c + 2\,H \;\times h$$

If M is selected larger than $c + 2\,H \;\times h$ than it is enough large to make the model effectively work.

4.2. Case with No Idle Times

Recall that, in No Idle Times case we do not allow to insert idle time between successive jobs. In this case, the same notations and parameters are used with the IT case. In the following subsections the different parts of the models from the NIT case is explained.

4.2.1. Model NIT

NIT:

Min

$$\sum_{m=1}^{2}\sum_{k=1}^{K}(c \times y_{km} \;+ h \;\times g_{km}\,)$$

$$+\,h\left\{\sum_{k=1}^{K}\sum_{j\in E_m}\sum_{m=1}^{2}(d_j \times\; q_{kjm}) + \left(\sum_{l:f[j]=l} I_l - \sum_{k=1}^{K}\sum_{j\in E_m}\sum_{m=1}^{2} q_{kjm}\right)\times H\right\}$$

Subject to

(1) - (16)

$$W_{km} + \sum_{j\in E_m}(p_j \times X_{kjm}) + y_{(k+1)m} \times S \geq W_{(k+1)m} - M \times (1 - \sum_{j\in E_m} X_{(k+1)jm}),$$
$$\forall\, k = 1,2,..,K-1\,, m = 1,2 \qquad (35)$$

In this model, constraint sets (1) - (16) are the same with the IT case. Additionally, we add the following constraint to the previous model. This constraint set (35) ensures to schedule the jobs on machines successively without idle time between jobs.

4.2.2. Model NIT-R

NIT-R:

Min

$$\sum_{m=1}^{2}\sum_{k=1}^{K}(\, c \times y_{km} \; + h \; \times g_{km}\,) + h\left\{\sum_{j\in E_m}(d_j \times q_j) + \left(\sum_{l:f[j]=l} I_l - \sum_{j\in E_m} q_j\right) \times H\right\}$$
$$+\; M \sum_{k=1}^{K}\sum_{j\in E_m}\sum_{m=1}^{2}(1 - X_{kjm} - R_{kjm})$$

Subject to

(17) - (34)

$$W_{km} + \; \sum_{j\in E_m} p_j \times \left(X_{kjm} + \; R_{kjm}\right) + y_{(k+1)m} \times S \geq \; W_{(k+1)m} - M \times (1 -$$
$$\sum_{j\in E_m}(X_{(k+1)jm} + \; R_{(k+1)jm})\,)\;,\; \forall\, k = 1,2,..,K-1\,, m = 1,2 \qquad (36)$$

In this model, constraint sets (17) - (34) are the same with IT case. Additionally, we add the following constraint to the previous model. With the addition of constraint set (36), our model is modified to the version of no idle time between successive jobs is allowed on machines.

In the next chapter, two different algorithms about the problem are expressed.

Chapter 5

Solution Approaches

Models developed in Chapter 4 solve the problems optimally, but due to exponential time requirements, they are not convenient to use for practical purposes when the problem size is large. For instance, Model IT is used to solve problem instances with two-, four- and eight-hour planning horizons including approximately 9, 16, 32 jobs, respectively. The problems with two-hour data (approximately 9 jobs) take approximately 1 minute to solve, whereas the four-hour (approximately 16 jobs) data takes more than 6 hours and the eight-hour (approximately 32 jobs) data could not be solved with IT model in a reasonable time due to memory problems on a computer with a 3.7 GHz Intel i7 processor running with a 16 GB of RAM. Although the relaxed model (IT-R) response is faster than the IT model, it still fails to solve the practical problems in reasonable time limits. Thus, we propose two computationally efficient heuristic algorithms: Rolling Horizon Algorithm (RHA) and Two Pass Algorithm (TPA). The aim of these heuristic algorithms is to schedule the jobs with the lowest inventory level. These heuristic algorithms split the planning horizon into smaller periods and solve each period with a proposed exact model. Thus, they can be used to plan for longer horizons and they also offer the advantage of providing a longer term perspective which allows flagging potential infeasibility issues in satisfying demand in further periods in time.

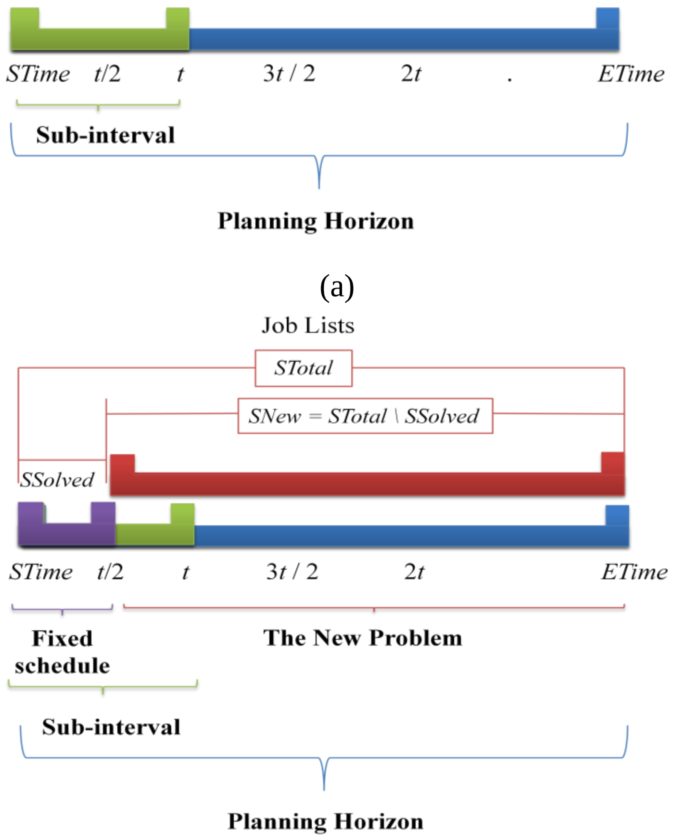
Note that, as we mentioned in the previous chapter in details, our problem has two different versions, which are IT and NIT cases. The structure of the heuristics is the same for both cases, only the corresponding model is different in the heuristics. So, we explain the proposed algorithm for only one case in this chapter.

In the proposed solution approaches, the general working principle is to divide the planning horizon into sub-intervals so that each of these sub-intervals can be solved optimally in a reasonable time by the mathematical models proposed in Chapter 4. In the following subsections, we provide a description of the general working principles of these solution approaches.

5. 1. Rolling Horizon Algorithm (RHA)

In this algorithm, first an incumbent planning horizon with a length of t is determined for which the optimal solution can be found by IT or NIT in a reasonable time. The first half of the optimal plan obtained for the incumbent horizon is settled as a final plan, and a new incumbent horizon of length t starting from the ending point of the currently settled optimal plan is determined. The algorithm proceeds in the same manner and stops when all of the original planning horizon with length H is exhausted. Let *STime* and *ETime* be the starting and ending time of the planning horizon, respectively and *STotal* be the set of jobs in [*STime*, *ETime*] time interval. Let t be an interval length where $t \leq$ *ETime* – *STime* (see Figure 5.1-(a)). Firstly, the algorithm takes the first sub-interval and assumes [*STime*, *STime* + t] as the incumbent planning horizon to solve the corresponding problem optimally with IT-R model. After obtaining the optimal plan for the incumbent horizon, the algorithm settles the schedule obtained for the first half [*STime*, *STime* + t/2]. In Figure 5.1-(b) the set of jobs that are settled in a final plan is represented as *SSolved*. The planning horizon is shown with the blue line. The green line represents the incumbent planning horizon while the purple line shows the settled plan. After the first sub-interval is solved, the job list of the planning horizon (*STotal*) is updated by subtracting the scheduled jobs. This process actually truncates the original

[*STime*, *ETime*] with *STotal* jobs problem into a new problem (*STime*, *ETime*) with *SNew* = *STotal* \ *SSolved* jobs, which is the incumbent job list consisting of the unscheduled jobs. Since the proposed mathematical model allows the due dates to be first fulfilled from inventory and later puts back the jobs to inventory, there might be jobs that are used from inventory in [*STime*, *STime* + *t*/2] and produced in [*STime* + *t*/2, *STime* + *t*] time intervals. Note that if we update *STime*, RHA does not consider these jobs in the incumbent planning horizon so not updating *STime* always makes RHA to reconsider those jobs in the incumbent planning interval. For this reason, the truncated problem's time interval starts from *STime* instead of *STime* + *t*/2. This process repeats by shifting sub-interval *t* to 3*t*/2, 2*t*, 5*t*/2 … until it reaches to the end of the planning horizon *ETime*.



(a)

(b)

Figure 5.1 A representation which shows how Rolling Horizon Algorithm works

In the following part, we give the notation that is used throughout the algorithm (see Table 5.1) and provide the pseudo-code of the algorithm. In the pseudo-code, *CTime* is set to *STime* at the beginning and increased *t*/2 amount in each iteration. The iterations continue until *STime* exceeds the *ETime*. $STime_m$ and $Type_m$ are used to feed the mathematical model so that the model keeps track of the available time of the machine *m* and the family of the last job produced, so that it decides when to schedule the job in the new iteration or to incur a setup time or not. $Type_m$ is set to – 1 which corresponds to a non existing family so that the model incurs a setup time for the first job in the schedule and $Type_m$ and $STime_m$ are updated afterwards (see Lines 20 – 22 and 24 – 26, respectively). At each iteration, the algorithm finds jobs that are in sub-interval [*STime*, *CTime* + t] (see Lines 6 - 8), solves the corresponding optimization problem and then updates the job list *STotal*. The model solves the corresponding sub-problem (see Lines 10 – 12) and updates the total job list *STotal* (see Lines 16 – 18).

Table 5.1 Notations Used in Rolling Horizon Algorithm

Notations	
m	machine index, *m* = 1,2
j	job index, *j* = 1,..,*J*
t	length of the interval
ETime	ending time of the planning horizon
STime	starting time of the planning horizon
$Type_j$	family of job *j*
$Type_m$	most recent setup family on machine *m*
d_j	due date of job *j*
$Start_j$	starting time of job j's processing operation
q_j	Boolean variable showing if the due date for job *j* is fulfilled from inventory

M_j	The machine on which job *j* is scheduled
$STotal$	set of jobs in (*STime*, *ETime*) time interval
$SSolved$	list of scheduled jobs where each scheduled job is a tuple $< j, Start_j, q_j, M_j >$
$STime_m$	starting time of the operations on each machine *m*
$CTime$	current time of the system
$S_{x,y}$	job set between time *x* to time *y*
p	processing time of a job

Algorithm - RHA Rolling Horizon

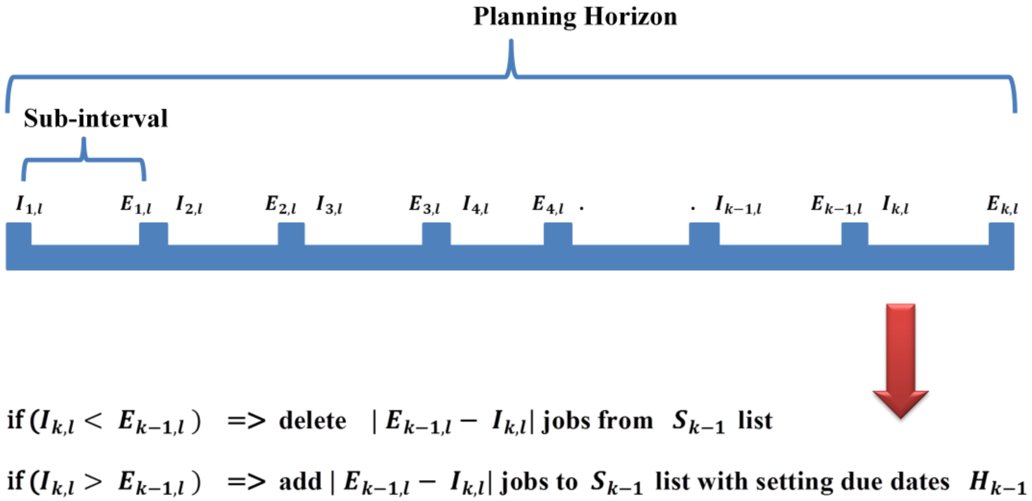
```
set $STime_m$ to $STime$
set $Type_m$ to -1 for each $m$
set $CTime$ to $STime$
while $CTime < ETime$
    $S_{STime,CTime+t} = \emptyset$
    for each job $j \in S_{STime,ETime}$
        if $STime \le d_j$ and $d_j < (CTime + t)$
            $S_{STime,CTime+t} = S_{STime,CTime+t} \cup \{ j \}$

    for each job $j \in \{S_{STime,CTime+t}\}$
        solve IT-R to obtain $Start_j,\ q_j,\ M_j$ using $Stime_m$
        and $type_m$
    $SSolved = \emptyset$
    for each job $j \in \{S_{STime,CTime+t}\}$

        if $start_j \le CTime + t/2$
            $SSolved = SSolved \cup < j, Start_j, q_j, M_j >$
            $Stotal$ = Stotal \ { $j$ }

        if $q_j \ne 1$ and $M_j = 0$ and $(Start_j + p) > Stime_0$
            $Stime_0 = Start_j + p$
            $Type_0 = Type_j$

```

```
24                  if $q_j \neq 1$ and $M_j = 1$ and $Start_j + p > Stime_1$
25                      $Stime_1 = Start_j + p$
26                      $Type_1 = Type_j$
27
28          $CTime = CTime + {}^{t}/_{2}$
```

The algorithm tries to schedule the jobs in the incumbent planning horizon in line 11 and in case of infeasibility for the solution in the incumbent planning horizon, the algorithm increments the contingency stock levels for each family and solves the problem from scratch. We have not included this procedure to keep the algorithm concise.

5. 2. Two-Pass Algorithm (TPA)

This algorithm has three stages. At the very beginning of this algorithm, the planning horizon is divided into manageable sub-intervals which can be optimally solved by the proposed MIP models in a reasonable time. In the first pass stage, the optimal schedules are found for successive sub-problems by using the proposed IT-R or NIT-R model, also taking into consideration the last job's finishing time and the current setup on each machine. If the finishing time of the last job in a sub-interval exceeds the ending time of the interval, the excess amount of the processing time on its assigned machine is added to the starting time of the next sub-interval's starting time. Continuity between successive sub-intervals is attained in this way. With this procedure, we find the required contingency stock and ending inventory level of each sub-interval and compare these for successive sub-intervals. If ending inventory level of one of the sub-intervals is different from starting inventory level of the following sub-interval, this will cause a problem while applying the plans continuously. For that reason, there must be consistency between the successive sub-intervals' inventory levels and this is ensured in the second pass of the algorithm. In the update stage, the job lists of each sub-problem are updated by comparing the successive sub-problems' inventory levels. If the consecutive sub-interval's ending and contingency stock levels are not equal, the algorithm deletes the excessive number of jobs from the previous sub-interval's job list or adds the lacking number of jobs to the previous sub-interval's job list. We refer to this procedure as "Update Stage". The details of this stage can be seen on Figure 5.2. In the second pass, all of the sub-problems with updated job lists are solved with the IT model, which is based on the principle of retaining the contingency stock levels at the end of each sub-interval. In order to ensure the continuity of the inventory levels between intervals for the updated job list, it is important to ensure the same set of jobs produced in the first pass and new jobs added in update stage are produced in the second pass. Hence, the algorithm uses IT model which replenishes the contingency stock level.



if $(I_{k,l} < E_{k-1,l})$ **=> delete** $|E_{k-1,l} - I_{k,l}|$ **jobs from** S_{k-1} **list**

if $(I_{k,l} > E_{k-1,l})$ **=> add** $|E_{k-1,l} - I_{k,l}|$ **jobs to** S_{k-1} **list with setting due dates** H_{k-1}

Figure 5.2 The details of the "Update Stage" of TPA

The details of the aforementioned stages can be seen in the following pseudo-code. The proposed Two-Pass Algorithm finds the contingency stock and ending inventory levels for each interval indexed with *k*. The set of jobs in each interval *k* is denoted by S_k. In order to find the contingency stock level of interval *k*, the proposed algorithm initializes contingency stock to 0 for each family and attempts to solve the set of jobs S_k. If a feasible solution cannot be found, it then increments the contingency stock level, until a feasible solution becomes available for interval *k* (see Line 8). The ending inventory for interval k is calculated by decrementing the contingency stock for the jobs that are used from the inventory in the computed feasible solution which is done in line 14. In the update stage, the algorithm provides continuity between the contingency stock and ending level of intervals. In this stage, the algorithm compares the contingency stock level of interval k and ending inventory level of interval k – 1. If the contingency stock level of interval k is less than the ending inventory level of interval k – 1 for some family l, then the algorithm deletes from the set of jobs which have the earliest due date in family l and uses it from the inventory (see Line 18). For the other case when the contingency stock level k is larger than the ending inventory level of k – 1 for some family l, the algorithm introduces new jobs of family l which have the due date of the previous interval's horizon as shown in line 20. Finally, the second pass solves the new problem with the updated job set S_k for each interval k to ensure the feasibility of the intervals when new jobs are introduced or existing jobs are removed from the production. In Line 24, second pass schedules jobs in S_k, in case of infeasibility in that line, the algorithm increments the contingency stock level at the first interval by removing jobs according to their due date sequence that are produced in the second pass and the algorithm starts to the second pass from scratch. This process is not shown in the pseudo-code to keep the exposition simple.

In addition to the notation in section 5.1 we define the following notations for this algorithm.

Table 5.2 Notations used in Two-Pass Algorithm

Notations	
k	interval index, k = 1,..,K
$I_{k,l}$	initial inventory requirement of family l at the beginning of the k^{th} interval
$E_{k,l}$	ending inventory of family l at the end of the k^{th} interval
S_k	set of jobs that must be processed in the k^{th} interval

D_k	set of jobs that are used from inventory and not produced in the k^{th} interval
R_k	solution set that consists of the scheduled jobs at the k^{th} interval
$Q_{k,l}$	used from inventory and not produced jobs of family l in the k^{th} interval
H_k	planning horizon of the k^{th} interval

Algorithm - TPA Two-Pass

```
First pass
set $STime_m$ to $STime$ for each $m$
set $Type_m$ to -1 for each $m$
set $CTime$ to $STime$
for each $S_k \in STotal$
    for each $k$ initialize $I_{k,l} = 0$
    solve $S_k$ using IT model with $Stime_m$ and $Type_m$
    while $S_k$ is infeasible
        $I_{k,l} = I_{k,l} + 1$ for all $l$
        solve $S_k$ and obtain $I_{k,l}$ for all $l$
    endwhile
    compute $E_{k,l} = (I_{k,l} - Q_{k,l})$ for each $l$
    for each job $j \in R_k$
        if job $j \in D_k$
            $S_k = S_k - \{$ job $j \}$
            $D_k = D_k + \{$ job $j \}$
Update stage
    for each $S_k \in \{ S_2 , \ldots, S_K \}$
        if ( $I_{k,l} < E_{k-1,l}$)
            delete jobs $|E_{k-1,l} - I_{k,l}|$ from $S_{k-1}$ list
        else
            add jobs $|E_{k-1,l} - I_{k,l}|$ with due date $H_{k-1}$ to $S_{k-1}$ list
Second pass
    set $STime_m$ as the start time of each $m$
    for each $S_k \in S$
        update $I_{k,l}$ by using inventory for each job in $D_k$
    solve $S_k$ with $STime_m$ and $I_{k,l}$ obtained from the first pass using IT
    model
```

In the next chapter, we first present the test instances and then analyze the solutions obtained by using the proposed heuristics.

Chapter 6

Computational Results

In this chapter, our aim is to show the effectiveness of the heuristic algorithms for the planning and scheduling problem under study. We compare the heuristics and exact methods in terms of contingency stock levels, average inventory carrying levels, setup numbers and CPU times. To this end, we first explain the characteristics of the test problems and environment we used to compare the heuristics and the exact solution methods. Afterwards, we give the test results and present their detailed comparisons. Finally, we summarize the results.

6. 1. Test Instances

We use a test bed which includes both real and random problems. Real problems are gathered directly from the operations of the body shell department of Oyak-Renault and the realized demand generated by the consumption line in a specific time period in year 2011. Whereas, random problems are randomly generated with different characteristics which are appropriate to future goals of the company and suitable to understand the behavior of the solution methods at different cases.

For real problems, data for three consecutive workdays are used to test the heuristic algorithms and the exact methods. The planning horizon is set to eight hours, therefore from three work days, we generate nine problem instances since the company is working for three shifts and 24 hours every day. There are approximately 35 jobs in each instance. In the current operations of Renault, there are eight different types of doors demanded by the consumption line, but four of these types are produced by the existing door manufacturing cells. Therefore, we only include the demand of these four types in our problem instances. There are seven parameters of the test bed namely, the planning horizon length, the sub-interval length, the setup times, the processing time, the inventory holding cost and the setup cost. These parameters can be seen in Table 6.1. The sub-interval length is used in heuristics. The setup time, holding cost, and setup cost are given in Table 6.1 by Renault engineers. The processing times depend on the number of workers allocated to the body shop and assume one of the values given in Table 6.1. This allocation might change depending on the tactical decisions made by planners. Hence, we analyze two scenarios for each problem instance, one with a slower operation (30 vehicles per hour) and the other with a faster operation (20 vehicles per hour). We denote slow operations by P1 and faster operations by P2.

Table 6.1 The parameters of solution methods for the real problems

Parameters	Values
Planning Horizon	28800 sec. (8 hours)
Sub-Interval Length	7200 sec. (2 hours)
Setup Time	300 sec. (5 min.)
Processing Time - 1	960 sec. (16 min.)
Processing Time - 2	1440 sec. (24 min.)
Holding Cost	1.48×10^{-5} €/sec.
Setup Cost	3 €

Since Renault aims to increase the flexibility of the manufacturing cells and therefore the type of the doors manufactured on these cells, we generate the random problems according to their future plans. Thus, in random problems there are six different types of doors with approximately 30 jobs in total in each instance. For practical purposes, we set the planning horizon to four hours for random instances.

We employ two different generation methods for the random problems. In the first generation method GM-S1, there are eight types of doors (which are represented by A, B, C, D, E, F, G and H) demanded by the consumption line. They are demanded in a prespecified manner. The details can be seen in Table 6.2. In Table 6.2 where, “Prob.” represents the probability rate of each type and “Num.” represents the number of jobs of a given type to be requested consecutively. For instance, in GM-S1, one piece of ‘E’, ‘F’, ‘G’, and ‘H’ door types are demanded and ‘C’ and ‘E’ are demanded twice as many as them. Likewise, ‘A’ and ‘B’ door types are demanded three times as many ‘D’ and ‘E’. Second generation method GM-S2 is similar to GM-S1 however it has different probability rates for each type as given in Table 6.2. In order to generate random problems for a given generation method, we first select a door type among the six alternatives using a probability distribution for the generation method in Table 6.2 and place a given number of selected type jobs consecutively (see Table 6.2). The difference between the due dates of every two consecutive doors demanded by the consumption line is set as 1 minute and this process is repeated until the time of the last door reaches to the end of the planning horizon.

Table 6.2 The characteristics of Generation Method

	GM			
	S1		**S2**	
Type	**Prob.**	**Num.**	**Prob.**	**Num.**
A	1/8	6	6/20	6
B	1/8	6	6/20	6
C	1/8	2	2/20	2
D	1/8	2	2/20	2
E	1/8	1	1/20	1
F	1/8	1	1/20	1
G	1/8	1	1/20	1
H	1/8	1	1/20	1

The parameters that are used by solution methods are different for the random problems than real problems. These parameters are listed in Table 6.3. Although the planning horizon is set to 8 hours in real problems, it is set to 4 hours in random problems because the number of jobs in random problems is more than that in real problems. The setup time, holding cost, and setup cost are given the same values that are used in for real problems as given in Table 6.3. Since the jobs are more frequent in the random problems than real problems, the company has to work with higher capacity so the processing time is shorter. However, similar to real problems, there are still two different processing times, slow (40 vehicles per hour) or fast (60 vehicles per hour). In both test instances, the parameter M that is used in constraint sets of Model IT and IT-R is set as $M = H + S + p$ which is always greater than the right hand side (RHS) of the constraints.

Recall that, we have two different versions of the exact methods which are idle time - IT Model that allows idle time between consecutive jobs and the no idle time - NIT Model that does not allow such idle time insertion. Throughout the chapter we represent the solutions with the same abbreviations, so IT represents the solution of the problem version which allows idle time between jobs and NIT represents the solution of the problem version which does not allow idle time between consecutive jobs. Besides, we use P1 (P2) for referring whether the solution is for Processing Time - 1 or Processing Time - 2. For example, IT-P1 indicates idle time solution with Processing Time -1.

Table 6.3 The parameters for random problems

Parameters	Values
Planning Horizon	14400 sec. (4 hours)
Sub-Interval Length	7200 sec. (2 hours)
Setup Time	300 sec. (5 min.)
Processing Time - 1	480 sec. (8 min.)
Processing Time - 2	720 sec. (12 min.)
Holding Cost	1.48×10^{-5} €/sec.
Setup Cost	3 €

We use JAVA programming language to implement the solution methods and run our experiments on a computer with a 3.7 GHz Intel i7 processor and 16 GB memory.

6. 2. Computational Results

We solve real and random problems with the heuristic algorithms and exact solution methods. We present the results of the exact methods in section 6.2.1. We give the comparison of the exact methods with the heuristics in section 6.2.2. Finally, we compare the heuristics with each other in section 6.2.3. In the comparisons, we report average results. The detailed results for each instance of these test problems can be found in Appendix 1 - 32.

6.2.1. Exact Methods

In this section, we give the results for the exact methods. Note that these methods attempt to solve test problems with minimum contingency stock levels and increment them if they cannot find a solution. Additionally, exact solution methods do not allow decreasing the contingency stock levels in the end of the planning horizon. In Table 6.4, we show the results for average CPU time and average number of setups for real problems with the NIT-P2 and IT-P2 exact solutions. We present the results for the average inventory level and maximum inventory level for each job type among all real problems in Table 6.5. In general, NIT-P2 solutions have larger average inventory levels than IT-P2 solutions; because in NIT-P2 solutions, idle times between the production of jobs are not allowed and this leads to compressed production schedule of the demanded jobs, therefore in the no idle time case, the production of a job starts earlier than that in the idle time case. This causes carrying more inventories in the no idle time case with respect to the idle time case. Differently from the solutions for the no idle time case, idle time solutions shift the production of the jobs forward in the planning horizon as much as possible. This leads to less number of setups in idle time (IT) solutions. The maximum inventory level for a job type gives us a good idea about the maximum number of fully loaded unitizing vessels needed for that type among all problems. The results can be seen from Table 6.4 and Table 6.5.

Table 6.4 Average CPU time and average number of setups found by exact methods with the same contingency stock and ending inventory levels for real problems with processing time 2

		Avg. CPU time (sec.)	Avg. # of Setup		
			M/C. 1	M/C. 2	Total
P2	NIT	28,164	6.6	4.6	11.1
	IT	9,139	6.6	3.2	9.8

Table 6.5 The average and maximum inventory levels obtained by exact methods for real problems with processing time 2

		Exact								
		Average Inv. Levels					Max.Inv.			
	Type	0	1	2	3	Total	0	1	2	3
P2	NIT	1.01	1.19	1.26	0.79	4.24	4	4	4	3
	IT	0.78	0.36	0.64	0.69	2.46	5	2	3	2

6.2.2. Comparison of Heuristics with Exact Methods

TPA and RHA are two heuristics we proposed. Unlike the exact methods, TPA and RHA may decrease the contingency stock levels at the end of the solution. In order to compare their results with the exact method, we solve the same test instances with the exact method using the same contingency stock and ending inventory levels that heuristics found. To this end, we modify Model IT-R (or similarly NIT-R) and refer to them as IT-R-M (or NIT-R). In the modified model, we change the objective function for IT-R (or NIT-R) and add the following parameters:

Parameters:

e_l: ending inventory level of family *l*

J_l: the number of jobs belonging to family *l*

The details of IT-R-M and NIT-R-M models can be found in the following subsections.

IT-R-M:

Min

$$\sum_{m=1}^{2}\sum_{k=1}^{K}(c \times y_{km} + h \times g_{km}) + h\left\{\sum_{j\in E_m}(d_j \times q_j) + \left(\sum_{l:f[j]=l} I_l - \sum_{j\in E_m} q_j\right) \times H\right\}$$

Subject to

(17) - (34)

$$\sum_{k=1}^{K}\sum_{j:f[j]=l}\sum_{m=1}^{2}(X_{kjm} + R_{kjm}) = J_l + e_l - I_l,\ \forall\ l = 1,2..L \qquad (37)$$

In this model, constraint sets (17) - (34) are the same as those in the IT - R case discussed in Chapter 4 and we change the objective function as seen above. With the addition of constraint set (37), our model is modified so that it finds solutions with given contingency levels and finishes with given ending inventory levels.

NIT-R-M:

Min

$$\sum_{m=1}^{2}\sum_{k=1}^{K}(c \times y_{km} + h \times g_{km}) + h \left\{ \sum_{j \in E_m} (d_j \times q_j) + \left(\sum_{l:f[j]=l} I_l - \sum_{j \in E_m} q_j \right) \times H \right\}$$

Subject to

(17) - (34)

(36)

$$\textstyle\sum_{k=1}^{K} \sum_{j:f[j]=l} \sum_{m=1}^{2} (X_{kjm} + R_{kjm}) = J_l + e_l - I_l \text{ , } \forall\, l = 1{,}2..L \qquad (38)$$

In this model, constraint sets (17) - (34) and (36) are the same as that in the NIT - R case given in Chapter 4 and we change the objective function and add constraint set (38) as seen above.

6.2.2.1 Comparison of TPA with Exact Methods

We report the results of TPA and exact methods for the real problems in Table 6.6 and Table 6.7. Table 6.6 shows the average CPU time and average number of setups for each machine while Table 6.7 lists the average and maximum inventory levels. When we consider the average running time of the algorithms in Table 6.7-(a), exact methods take more than 10 hours on the average to solve real problems while TPA solves them in less than one minute. Obviously, exact methods are not suitable for practical purposes. If we examine the number of setups, TPA has approximately 10.45 setups on average while the exact methods have approximately 6.85 setups. TPA schedules the jobs in sub-intervals and combines them to produce the final schedule, hence it does not consider the whole planning horizon at once. Therefore, it aims to produce jobs as early so it tends to carry less inventory than the exact methods. Consequently, the average inventory levels of TPA are less than the exact methods. Besides, TPA has no higher maximum inventory levels than exact methods for 14 of 16 different combinations of type, processing time and idle/no idle times factors as seen in Table 6.7-(b). This can explain one of the reasons for TPA carrying less inventory than exact methods.

Table 6.6 Comparison of average CPU time and average number of setups obtained by TPA and exact methods for real problems with processing time 1 and 2

		Avg. CPU time (sec.)		Avg. # of Setup					
				Exact			TPA		
		Exact	TPA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P1	NIT	47,778	13	4.4	1.9	6.3	6.2	3.0	9.2
	IT	36,153	31	4.2	2.3	6.6	7.6	3.1	10.7
P2	NIT	18,385	16	4.8	2.7	7.4	7.2	2.9	10.1
	IT	17,538	25	4.6	2.6	7.1	7.3	4.4	11.8

Table 6.7 Comparison of average and maximum inventory levels obtained by TPA and exact methods for real problems with processing time 1 and 2

		Average Inventory Levels									
		Exact					TPA				
	Type	0	1	2	3	Total	0	1	2	3	Total
P1	NIT	1.74	3.19	1.81	0.70	7.44	1.63	2.48	1.87	0.51	6.49
	IT	1.01	1.30	0.89	1.03	4.23	0.54	0.56	0.50	0.60	2.19
P2	NIT	1.16	1.51	1.17	0.60	4.43	1.07	1.56	1.07	0.48	4.19
	IT	0.93	0.87	1.07	0.84	3.70	0.84	0.77	0.65	0.81	3.07

(a)

		Maximum Inventory Levels							
		Exact				TPA			
	Type	0	1	2	3	0	1	2	3
P1	NIT	5	7	6	2	6	9	6	2
	IT	5	7	5	3	3	3	2	2
P2	NIT	4	5	4	2	3	5	4	2
	IT	4	5	5	3	3	5	3	2

(b)

We list the results of TPA and exact methods for GM-S1 random problems in Tables 6.8 and 6.9. We present average running time and average number of setups in Table 6.8; and average and maximum inventory levels in Table 6.9. When we consider the average CPU time of the algorithms, TPA is quite faster than the exact methods. If we compare average CPU times of the TPA for random problems with average CPU times of TPA with real problems, we see that TPA requires more time for random problems than the real problems. This may indicate relative difficulty of random problems. For the number of setups, maximum (1.8 setup) and minimum difference (1.1 setup) among average number of setups indicate that the exact methods are slightly better than the TPA. Considering the differences between TPA and exact methods in Table 6.6 and 6.8, TPA is more successful in random problems than it is in real problems with respect to its number of setups. The data in random problems are uniformly distributed and more demanding than the real problems; hence this characteristic of the data restricts exact methods to utilize more empty intervals in the planning horizon. If we examine Table 6.9-(a), TPA has lower average inventory levels than exact methods except GM-S1-NIT-P2 case. For that case, the exact method has an average inventory level that is 0.1 door/sec. less than TPA's. In Table 6.9-b, the maximum inventory levels of TPA are less than those of exact methods for 22 of 24 different combinations of type, processing time and idle/no idle times factors. The comparison of these combinations shows the heuristics' efficiency in terms of the resulting storage area requirements.

Table 6.8 Comparison of average CPU time and number of setups obtained by TPA and exact methods for random problems (GM-S1) with processing time 1 and 2

			Avg. CPU time (sec.)		Avg. # of Setup					
					Exact			TPA		
			Exact	TPA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
GM S1	P1	NIT	20,846	327	3.2	3.0	6.2	4.6	3.0	7.6
		IT	25,401	2,256	3.3	3.0	6.3	4.2	3.2	7.4
	P2	NIT	38,819	113	3.0	3.0	6.0	4.4	3.0	7.4
		IT	49,537	355	3.0	2.4	5.4	4.6	2.6	7.2

Table 6.9 Comparison of average and maximum inventory levels obtained by TPA and exact methods for random problems (GM-S1) with processing time 1 and 2

			Average Inventory Levels													
			Exact							TPA						
		Type	0	1	2	3	4	5	Total	0	1	2	3	4	5	Total
GM S1	P1	NIT	2.1	2.7	1.4	1.6	1.2	0.9	10.1	1.9	2.2	1.2	1.0	1.4	1.1	8.8
		IT	0.8	1.7	1.4	1.1	0.8	0.7	6.6	1.6	0.7	0.7	0.8	1.0	0.7	5.5
	P2	NIT	1.8	1.8	1.4	1.1	1.4	1.2	8.7	2.1	1.6	1.3	1.3	1.3	1.2	8.8
		IT	1.2	1.0	0.9	1.4	1.0	0.7	6.1	0.6	0.5	0.8	0.8	1.1	0.8	4.6

(a)

			Maximum Inventory Levels											
			Exact						TPA					
		Type	0	1	2	3	4	5	0	1	2	3	4	5
GM S1	P1	NIT	7	8	4	4	3	2	6	7	4	3	3	3
		IT	5	6	4	3	2	2	5	4	2	2	2	2
	P2	NIT	6	5	5	3	3	3	7	5	4	4	3	2
		IT	5	5	3	4	2	2	2	2	2	2	2	2

(b)

6.2.2.2 Comparison of RHA and Exact Methods

Similar to TPA, we report the results of RHA and exact methods for real problems in Tables 6.10 and 6.11. Table 6.10 contains average CPU time and number of setups information. If we look at the average CPU times in Table 6.10, RHA finds solutions in less than 15 seconds on the average while the exact methods find solutions approximately in 7 hours. If we compare RHA's and TPA's CPU times (Table 6.6) for real problems, RHA is also faster than TPA, because TPA solves the problem in two passes. The average number of setups is 7.15 for RHA and 11.05 for the exact methods. Table 6.11-(a) contains average inventory levels for RHA and exact methods. For the average inventory levels, RHA is better than the exact methods for the idle time (IT) and no idle time (NIT) P2 cases, but the exact method is better than RHA for the no idle time (NIT) P1 case. Maximum inventory levels are given in Table 6.11-(b) and RHA has no higher maximum inventory levels for 15 of 16 different combinations of type, processing time and idle/no idle times factors.

Table 6.10 Comparison of CPU time and number of setups obtained by RHA and exact methods for real problems with processing time 1 and 2 for average

		Avg. CPU time (sec.)		Avg. # of Setup					
				Exact			RHA		
		Exact	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P1	NIT	48,752	6	4.2	2.2	6.4	10.7	1.6	12.2
	IT	27,641	22	4.1	2.3	6.4	7.1	2.0	9.1
P2	NIT	21,852	5	5.4	3.8	9.2	8.6	3.6	12.1
	IT	25,143	11	4.3	2.2	6.6	7.6	3.2	10.8

Table 6.11 Comparison of average and maximum inventory levels obtained by RHA and exact methods for real problems with processing time 1 and 2

		Average Inventory Levels									
		Exact					RHA				
	Type	0	1	2	3	Total	0	1	2	3	Total
P1	NIT	1.82	3.39	2.46	1.15	8.82	2.08	3.72	1.83	0.76	8.40
	IT	1.05	0.67	1.12	1.11	3.95	0.64	0.39	0.73	0.35	2.11
P2	NIT	1.17	1.45	1.29	0.73	4.64	1.16	1.97	1.17	0.50	4.80
	IT	0.99	0.59	1.03	0.83	3.44	0.72	0.69	0.74	0.57	2.73

(a)

		Maximum Inventory Levels							
		Exact				RHA			
	Type	0	1	2	3	0	1	2	3
P1	NIT	5	8	7	3	5	8	4	2
	IT	5	7	5	3	3	3	3	2
P2	NIT	4	6	6	3	5	6	4	2
	IT	4	4	4	2	4	3	3	2

(b)

We also compare our algorithms with exact methods for random problems as well and give the results for the GM-S1 cases in Tables 6.12 and 6.13. In terms of CPU times, RHA is faster than the exact methods for all cases as given in Table 6.12, however its average setup numbers (approximately 6.75 on average) are greater than the exact methods (approximately 8.90 on average) for all cases. Average inventory levels for RHA and exact methods are given in Table 6.13-(a). RHA is better than the exact methods for all cases except GM-S1-P2 idle time (IT) case. We give results for maximum inventory levels in Table 6.13-(b). The results indicate that RHA has no higher maximum inventory levels for 22 of 24 different combinations of type, processing time and idle/no idle times factors.

Table 6.12 Comparison of average CPU time and number of setups obtained by RHA and exact methods for random problems (GM-S1) with processing time 1 and 2

			Avg. CPU time (sec.)		Avg. # of Setup					
					Exact			RHA		
			Exact	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
GM S1	P1	NIT	19,243	22	3.6	3.2	6.8	4.8	5.0	9.8
		IT	42,668	39	3.2	3.0	6.2	4.2	4.2	8.4
	P2	NIT	30,386	30	3.8	3.8	7.6	4.4	4.2	8.6
		IT	25,194	26	3.6	2.8	6.4	4.4	4.4	8.8

Table 6.13 Comparison of average and maximum inventory levels obtained by RHA and exact methods for random problems (GM-S1) with processing time 1 and 2

			Average Inventory Levels														
			Exact							RHA						
		Type	0	1	2	3	4	5	Total	0	1	2	3	4	5	Total
GM S1	P1	NIT	2.7	3.1	2.1	1.6	1.6	1.2	12.4	2.5	3.0	2.1	2.1	1.4	1.0	12.2
		IT	1.0	2.0	1.1	1.2	0.9	0.7	7.0	1.7	1.5	0.9	0.9	0.9	0.8	6.6
	P2	NIT	2.6	2.8	1.5	1.6	1.4	1.1	11.0	2.4	2.8	1.3	1.8	1.5	1.1	10.9
		IT	1.2	1.2	0.7	1.4	1.0	0.9	6.4	1.6	1.7	0.8	1.1	1.3	0.7	7.1

(a)

			Maximum Inventory Levels											
			Exact						RHA					
		Type	0	1	2	3	4	5	0	1	2	3	4	5
GM S1	P1	NIT	9	8	5	4	3	3	6	7	4	4	3	2
		IT	5	6	5	4	2	2	8	6	2	3	2	2
	P2	NIT	7	7	5	4	3	3	7	7	4	4	3	3
		IT	4	5	2	4	2	2	4	5	2	3	3	2

(b)

6.2.3. Comparison of Heuristics

In this section, we compare the results of heuristic algorithms both for real problems and random problems. We first give the comparison of real problems in section 6.2.3.1. Then we provide the comparison of random problems for the heuristic algorithms in section 6.2.3.2.

6.2.3.1. Comparison of RHA vs. TPA with Real Problems

TPA and RHA results for real problems are given in Table 6.14, 6.15 and 6.16. These tables contain average contingency stock and ending inventory levels, CPU time, number of setups, average and maximum inventory levels. If we consider the average contingency stock and ending inventory levels given in Table 6.14, both algorithms start with closer contingency stock levels, but RHA decreases contingency stock levels with a small degree compared to TPA. RHA is faster than TPA in terms of average CPU time as given in Table 6.15, and TPA has less average number of setups compared to RHA except P2 idle time (IT) case. The average number of setups are directly oriented with the number of produced jobs in real problems, so we can say TPA and RHA are similar to each other in terms of average number of setups although TPA has less average number of setups in general. The results for the average inventory levels are given in Table 6.16-(a), RHA has less average inventory levels than TPA for P1 idle time (IT) and P2 idle time (IT) cases. When we look at the maximum inventory levels at Table 6.16-(b), both algorithms more or less have the same maximum inventory levels, TPA has no higher maximum inventory levels for 12 of 16 different combinations of type, processing time and idle/no idle times factors. Similar to TPA, RHA have no higher maximum inventory levels for 12 of 16 different combinations of type, processing time and idle/no idle times factors.

Table 6.14 Comparison of average contingency stock and ending inventory levels obtained by TPA and RHA for real problems with processing time 1 and 2

		CS & EI															
		TPA								RHA							
		CS				EI				CS				EI			
	Type	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
P1	NIT	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.8
	IT	1.0	1.0	1.0	1.0	0.8	0.8	0.8	0.9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.9
P2	NIT	1.0	1.6	1.2	1.2	0.1	0.1	0.1	0.1	1.2	1.2	1.2	1.2	0.6	1.0	0.9	0.6
	IT	1.3	2.1	1.7	1.3	1.0	1.0	1.0	0.7	1.6	1.6	1.6	1.6	1.1	1.2	0.7	0.4

Table 6.15 Comparison of average CPU time and number of setups obtained by TPA and RHA for real problems with processing time 1 and 2

		Avg. CPU time (sec.)		Avg. # of Setup					
				TPA			RHA		
		TPA	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P1	NIT	13	6	6.2	3.0	9.2	10.7	1.6	12.2
	IT	31	22	7.6	3.1	10.7	7.1	2.0	9.1
P2	NIT	16	5	7.2	2.9	10.1	8.6	3.6	12.1
	IT	25	11	7.3	4.4	11.8	7.6	3.2	10.8

Table 6.16 Comparison of average and maximum inventory levels obtained by TPA and RHA for real problems with processing time 1 and 2

		Average Inventory Levels									
		TPA					RHA				
	Type	0	1	2	3	Total	0	1	2	3	Total
P1	NIT	1.63	2.48	1.87	0.51	6.49	2.08	3.72	1.83	0.76	8.40
	IT	0.54	0.56	0.50	0.60	2.19	0.64	0.39	0.73	0.35	2.11
P2	NIT	1.07	1.56	1.07	0.48	4.19	1.16	1.97	1.17	0.50	4.80
	IT	0.84	0.77	0.65	0.81	3.07	0.72	0.69	0.74	0.57	2.73

(a)

		Maximum Inventory Levels							
		TPA				RHA			
	Type	0	1	2	3	0	1	2	3
P1	NIT	6	9	6	2	5	8	4	2
	IT	3	3	2	2	3	3	3	2
P2	NIT	3	5	4	2	5	6	4	2
	IT	3	5	3	2	4	3	3	2

(b)

6.2.3.2. Comparison of RHA vs. TPA with Random Problems

The results of TPA and RHA for GM-S1 random problems are reported in Table 6.17-(a) and (b) , 6.18 and 6.19. We give the results for average contingency stock levels and ending inventory levels in Table 6.17. The results in Table 6.17-(a-b) show that RHA does not decrease contingency stock levels much compared to TPA. These results are coherent with the results for real problems in Table 6.15. In Table 6.18, we report results for average CPU time and average number of setups. RHA has CPU time that is less than one minute in average, but TPA's CPU time average is more than 10 minutes. In case of average number of setups, RHA plans more setups than TPA, this is normal since RHA produces more jobs than TPA. We present results for average inventory levels in Table 6.19-(a) which shows that RHA has higher average inventory levels than TPA. Since RHA finishes the planning horizon with higher ending inventory levels than TPA, the results are reasonable. We give the results for maximum inventory levels in Table 6.19-(b), TPA has less or equal maximum inventory levels than RHA for all cases.

Table 6.17 Comparison of average contingency stock and ending inventory levels obtained by TPA and RHA for random problems (GM-S1) with processing time 1 and 2

		Average CS & EI											
		TPA											
		CS						EI					
	Type	0	1	2	3	4	5	0	1	2	3	4	5
P1	NIT	1.6	1.6	1.6	1.6	1.6	1.6	0.0	0.0	0.4	0.4	0.8	0.6
	IT	1.6	1.6	1.6	1.6	1.6	1.6	1.0	1.0	1.0	0.8	0.6	0.8
P2	NIT	1.8	1.8	1.8	1.8	1.8	1.8	0.2	0.2	0.6	0.4	0.8	0.8
	IT	1.8	1.8	2.0	1.8	1.8	1.8	1.0	1.0	0.4	0.6	0.6	0.8

(a)

		Average CS & EI											
		RHA											
		CS						EI					
	Type	0	1	2	3	4	5	0	1	2	3	4	5
P1	NIT	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.4	1.6	1.2	0.8
	IT	1.6	1.6	1.6	1.6	1.6	1.6	1.2	1.6	1.2	1.6	1.2	1.4
P2	NIT	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.4	1.4	1.6	1.2
	IT	1.8	1.8	1.8	1.8	1.8	1.8	1.6	1.6	1.0	1.8	1.2	1.0

(b)

Table 6.18 Comparison of TPA vs. RHA for random problems (GM-S1) with processing time 1 & 2 for average CPU time and number of setups

			Avg. CPU time (sec.)		Avg. # of Setup					
					TPA			RHA		
			TPA	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
GM S1	P1	NIT	327	22	4.6	3.0	7.6	4.8	5.0	9.8
		IT	2,256	39	4.2	3.2	7.4	4.2	4.2	8.4
	P2	NIT	113	30	4.4	3.0	7.4	4.4	4.2	8.6
		IT	355	26	4.6	2.6	7.2	4.4	4.4	8.8

Table 6.19 Comparison of average and maximum of maximum inventory levels obtained by TPA and RHA for random problems (GM-S1) with processing time 1 and 2

			Average Inventory Levels													
			TPA							RHA						
		Type	0	1	2	3	4	5	Total	0	1	2	3	4	5	Total
GM S1	P1	NIT	1.9	2.2	1.2	1.0	1.4	1.1	8.8	2.5	3.0	2.1	2.1	1.4	1.0	12.2
		IT	1.6	0.7	0.7	0.8	1.0	0.7	5.5	1.7	1.5	0.9	0.9	0.9	0.8	6.6
	P2	NIT	2.1	1.6	1.3	1.3	1.3	1.2	8.8	2.4	2.8	1.3	1.8	1.5	1.1	10.9
		IT	0.6	0.5	0.8	0.8	1.1	0.8	4.6	1.6	1.7	0.8	1.1	1.3	0.7	7.1

(a)

			Maximum Inventory Levels											
			TPA						RHA					
		Type	0	1	2	3	4	5	0	1	2	3	4	5
GM S1	P1	NIT	6	7	4	3	3	3	6	7	4	4	3	2
		IT	5	4	2	2	2	2	8	6	2	3	2	2
	P2	NIT	7	5	4	4	3	2	7	7	4	4	3	3
		IT	2	2	2	2	2	2	4	5	2	3	3	2

(b)

The results for TPA and RHA for GM-S2 random problems are reported in Table 6.20, 6.21, 6.22. In Table 6.20, we report average contingency stock and ending inventory levels in Table 6.20. The results show that although TPA and RHA start from the same average contingency stock, TPA consumes the contingency stock levels more than RHA does. If we look at the average CPU time and number of setups comparison of TPA and

RHA in Table 6.21, TPA is slower than RHA in terms of average CPU time, and it schedules less number of setups than RHA. We give results for average and maximum inventory levels in Table 6.22. Since TPA decreases the ending inventory levels, it has less average inventory than the RHA as given in Table 6.22-(a). Results for the maximum inventory levels in Table 6.22-(b) indicate that TPA has less or equal maximum inventory levels than RHA for 21 of 24 different combinations of type, processing time and idle/no idle times factors. This can be explained also with the decreased contingency stock levels in TPA.

Table 6.20 Comparison of average contingency stock & ending inventory levels obtained by TPA and RHA for random problems (GM-S2) with processing time 1 and 2

			CS & EI											
			TPA											
			CS						EI					
		Type	0	1	2	3	4	5	0	1	2	3	4	5
GM-1 S2	P1	NIT	1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.2	0.0	0.0	0.0	0.0
		IT	1.0	1.0	1.0	1.0	1.0	1.0	0.6	0.6	0.4	0.6	0.6	0.6
	P2	NIT	1.8	1.8	1.8	1.8	1.8	1.8	0.4	0.6	0.2	0.4	0.2	0.0
		IT	1.8	1.8	1.8	1.8	1.8	1.8	1.0	1.2	1.0	1.0	1.0	1.0

(a)

			CS & EI											
			RHA											
			CS						EI					
		Type	0	1	2	3	4	5	0	1	2	3	4	5
GM-1 S2	P1	NIT	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
		IT	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.8	1.0	1.0	1.0	1.0
	P2	NIT	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.6	1.6
		IT	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.6

(b)

Table 6.21 Comparison of average CPU time and number of setups obtained by TPA and RHA for random problems (GM-S2) with processing time 1 and 2

			Avg. CPU time (sec.)		Avg. # of Setup					
					TPA			RHA		
			TPA	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
GM S2	P1	NIT	40	18	4.2	4.2	8.4	5.8	5.0	10.8
		IT	257	32	4.8	4.6	9.4	5.4	4.2	9.6
	P2	NIT	30	15	4.4	2.8	7.2	5.4	4.6	10.0
		IT	116	17	4.4	4.2	8.6	4.2	4.6	8.8

Table 6.22 Comparison of TPA vs. RHA for random problems (GM-S2) with processing time 1 & 2 for average and maximum inventory levels

			Average Inventory Levels													
			TPA							RHA						
		Type	0	1	2	3	4	5	Total	0	1	2	3	4	5	Total
GM S2	P1	NIT	0.9	1.1	1.2	1.4	1.1	0.9	6.6	1.5	1.4	1.7	2.0	1.7	1.5	9.9
		IT	0.5	0.7	0.7	0.5	0.5	0.5	3.3	0.7	0.5	0.7	0.6	0.7	0.5	3.8
	P2	NIT	1.2	1.4	1.4	1.6	1.6	1.0	8.2	2.0	1.9	2.2	2.0	1.8	1.9	11.8
		IT	1.1	1.0	0.9	0.8	0.7	0.7	5.2	1.7	1.4	1.4	1.1	0.8	1.5	7.8

(a)

			Maximum Inventory Levels											
			TPA						RHA					
		Type	0	1	2	3	4	5	0	1	2	3	4	5
GM S2	P1	NIT	3	4	5	4	5	4	4	4	4	5	5	4
		IT	2	3	3	2	2	2	2	3	2	3	3	2
	P2	NIT	3	4	5	5	5	4	4	3	5	5	5	4
		IT	2	2	2	2	2	3	5	4	4	3	3	4

(b)

6.3. Summary

Exact methods are better than the heuristics, they optimize number of setups and average inventory levels. However, they are impractical because of their CPU time and it is not convenient to use them. The heuristics generally have better or close average inventory levels compared to exact methods, this is important due to the storage area restrictions. We decrease approximately 70% of the current inventory levels in the company. As we mentioned before, maximum inventory levels are also important when we consider the number of unitizing vessels. Our heuristic algorithms decreased the current maximum inventory levels of the company by about 75%. RHA and TPA obtain their best results for the random data case, which is appropriate for the near future goal of the company in which they want to increment the number of door types and want to solve scheduling problems with more frequent demand of jobs as simulated in random problems. Additionally, our algorithms are flexible such that they adapt to cases of infeasibility and they always give the best solution they can find by starting from the lowest minimal contingency stock and incrementing until they have a feasible solution. This is also important for practical purposes. When we compare RHA and TPA, we see that RHA conserves contingency stock levels better than TPA and it is faster than TPA. In terms of average and maximum inventory levels and number of setups, TPA is better than RHA. Renault company can use either of the solutions according to their purposes.

Chapter 7

Conclusion & Future Directions

This thesis proposes two mixed integer programming based solution approaches to make integrated inventory carrying and scheduling decisions in a parallel machine environment subject to machine eligibility restrictions.

Both heuristics provide effective results and are computationally efficient for practical purposes. The algorithms are general enough to be of value to other plants or departments and applications that involve the use of robotic/flexible manufacturing cells feeding a downstream operation with a steady demand pattern. Both algorithms (RHA and TPA) are programmed in Java programming language using the Gurobi library. We obtain solutions that synchronize the operations in the door manufacturing processes with the speed of the consumption line. In this way, we obtain significant reductions in the inventory levels with respect to the current levels. In comparison to the exact methods, the two algorithms have significantly modest CPU time requirements. Still, they lead to a reduction of about 70% in the present inventory levels at Oyak-Renault. These results and other potential benefits offered by our solution approaches to the company are verified and validated based on real data collected at real and random problems test scenarios. Renault engineers and planners also attest to the practical applicability of the resulting production plans. Currently, the algorithms proposed in this thesis are being implemented at Renault in the form a decision support system and pilot studies are expected to commence in the end of 2012.

One future research direction is to investigate other heuristic mechanisms to better capture the trade-offs between doing setup or keeping inventory. Additionally, design of the solutions for multiple machines and different shop structures is another direction that can increase the generality of possible applications.

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APPENDIX

				CPU Time	# of Setup			Average Mean and Max. Inventory Levels									CS				EI			
								Type 0		Type 1		Type 2		Type 3		Total	Type				Type			
		Day	Shift	(sec.)	M/C. 1	M/C. 2	Total	Mean	Max	Mean	Max	Mean	Max	Mean	Max		0	1	2	3	0	1	2	3
P2	NIT	1	1	16,701	5	5	10	0.88	3	1.05	3	1.39	4	1.19	3	4.5	1	1	1	1	1	1	1	1
			2	50,164	5	3	8	0.86	3	1.61	4	1.47	3	1.16	2	5.1	1	1	1	1	1	1	1	1
			3	15,839	8	4	12	0.72	2	1.67	4	1.11	3	0.73	2	4.2	1	1	1	1	1	1	1	1
		2	1	36,896	10	5	15	1.09	3	1.23	3	0.76	2	0.23	1	3.3	1	1	1	1	1	1	1	1
			2	65,771	7	6	13	0.69	2	0.75	2	1.52	4	0.40	1	3.4	1	1	1	1	1	1	1	1
			3	15,353	6	4	10	1.49	3	1.05	3	1.01	3	0.63	2	4.2	1	1	1	1	1	1	1	1
		3	1	27,452	10	6	16	0.44	2	0.96	2	0.70	2	0.51	1	2.6	1	1	1	1	1	1	1	1
			2	18,686	4	4	8	1.81	4	1.05	3	1.63	4	1.19	3	5.7	1	1	1	1	1	1	1	1
			3	6,620	4	4	8	1.13	3	1.31	3	1.71	4	1.06	2	5.2	1	1	1	1	1	1	1	1
		Average.		28,165	6.6	4.6	11.1	1.0		1.2		1.3		0.8		4.2								
	IT	1	1	13,024	6	3	9	0.56	2	0.30	2	0.61	2	0.72	2	2.2	1	1	1	1	1	1	1	1
			2	48,151	5	3	8	0.60	2	0.42	2	0.64	2	0.74	2	2.4	1	1	1	1	1	1	1	1
			3	2,416	6	3	9	0.65	2	0.22	1	0.67	3	1.03	2	2.6	1	1	1	1	1	1	1	1
		2	1	1,885	8	3	11	0.57	2	0.49	2	0.83	3	0.26	1	2.1	1	1	1	1	1	1	1	1
			2	5,913	9	3	12	0.74	2	0.44	2	0.56	2	0.42	2	2.2	1	1	1	1	1	1	1	1
			3	2,621	6	3	9	0.54	2	0.35	2	0.90	3	0.81	2	2.6	1	1	1	1	1	1	1	1
		3	1	1,650	10	5	15	0.38	2	0.30	2	0.48	2	0.43	1	1.6	1	1	1	1	1	1	1	1
			2	2,016	4	3	7	2.15	5	0.25	2	0.46	2	0.88	2	3.7	1	1	1	1	1	1	1	1
			3	4,584	5	3	8	0.80	3	0.45	2	0.60	2	0.89	2	2.7	1	1	1	1	1	1	1	1
		Average.		9,140	6.6	3.2	9.8	0.8		0.4		0.6		0.7		2.5								

Appendix 1 The solutions found by exact methods with the same contingency stock and ending inventory levels for real problems with processing time 2 based on CPU time and number of setups

				CPU Time		# of Setup					
				(sec.)		Exact			TPA		
		Day	Shift	Exact	TPA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P1	NIT	1	1	14,389	7	5	1	6	3	5	8
			2	23,798	12	5	1	6	11	1	12
			3	164,957	12	5	2	7	4	5	9
		2	1	3,831	12	4	2	6	3	5	8
			2	66,569	19	4	2	6	6	2	8
			3	81,489	21	5	2	7	7	1	8
		3	1	4,966	16	4	3	7	3	5	8
			2	47,390	9	4	2	6	10	1	11
			3	22,607	9	4	2	6	9	2	11
	Average			47,777	13	4	2	6	6	3	9
	IT	1	1	3,075	18	4	2	6	7	3	10
			2	32,659	22	4	1	5	6	3	9
			3	72711	25	4	2	6	6	4	10
		2	1	7,539	37	5	2	7	8	3	11
			2	95,425	34	5	2	7	10	2	12
			3	39,631	41	4	3	7	8	3	11
		3	1	18,184	40	5	3	8	8	4	12
			2	32,235	37	4	3	7	9	2	11
			3	23,916	22	3	3	6	6	4	10
	Average			36,153	31	4	2	7	8	3	11

Appendix 2 1 Comparison of CPU time and number of setups obtained by TPA and exact methods for real problems with processing time 1

				CPU Time		# of Setup					
				(sec.)		Exact			TPA		
		Day	Shift	Exact	TPA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P2	NIT	1	1	19,950	6	6	2	8	7	2	9
			2	25,849	14	4	3	7	6	4	10
			3	4,447	9	6	3	9	9	2	11
		2	1	40,469	7	5	2	7	7	2	9
			2	31,512	19	5	2	7	5	4	9
			3	3,132	17	4	2	6	6	4	10
		3	1	11,174	39	4	3	7	8	2	10
			2	9,068	9	5	3	8	8	2	10
			3	19,859	18	4	4	8	9	4	13
	Average			18,384	15	5	3	7	7	3	10
	IT	1	1	23,013	14	6	2	8	7	4	11
			2	33,381	15	4	3	7	7	5	12
			3	14,188	18	4	3	7	8	4	12
		2	1	17,693	18	6	2	8	8	6	14
			2	11,217	15	4	3	7	9	4	13
			3	22,241	102	3	2	5	6	3	9
		3	1	13,285	15	4	2	6	8	4	12
			2	9,068	8	5	3	8	5	4	9
			3	13,753	15	5	3	8	8	6	14
	Average			17,538	25	5	3	7	7	4	12

Appendix 3 2 Comparison of CPU time and number of setups obtained by TPA and exact methods for real problems with processing time 2

				CS & EI															
				Exact								TPA							
				CS (Type)				EI (Type)				CS (Type)				EI (Type)			
		Day	Shift	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
P1	NIT	1	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
			2	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
			3	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
		2	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
			2	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
			3	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
		3	1	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
			2	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
			3	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
	Average			1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00
	IT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			3	1	1	1	1	0	0	0	1	1	1	1	1	0	0	0	1
		2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			2	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
			3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Average			1.00	1.00	1.00	1.00	0.78	0.78	0.78	0.89	1.00	1.00	1.00	1.00	0.78	0.78	0.78	0.89

Appendix 4 Comparison of contingency stock & ending inventory levels obtained by TPA and exact methods for real prolems with processing time 1

				CS & EI															
				Exact								TPA							
				CS (Type)				EI (Type)				CS (Type)				EI (Type)			
		Day	Shift	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
P2	NIT	1	1	1	1	1	2	0	0	0	0	1	1	1	2	0	0	0	0
			2	1	2	1	1	0	0	0	0	1	2	1	1	0	0	0	0
			3	1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0
		2	1	1	2	2	1	0	0	0	0	1	2	2	1	0	0	0	0
			2	1	2	2	1	0	0	0	0	1	2	2	1	0	0	0	0
			3	1	2	1	1	0	0	0	0	1	2	1	1	0	0	0	0
		3	1	1	2	1	2	0	0	0	0	1	2	1	2	0	0	0	0
			2	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0
			3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Average			1.00	1.56	1.22	1.22	0.22	0.22	0.22	0.11	1.00	1.56	1.22	1.22	0.11	0.11	0.11	0.11
	IT	1	1	1	1	1	2	1	1	1	1	1	1	1	2	1	1	1	1
			2	1	2	1	1	1	1	1	1	1	2	1	1	1	1	1	1
			3	1	2	2	1	1	1	1	0	1	2	2	1	1	1	1	0
		2	1	1	2	2	1	1	1	1	1	1	2	2	1	1	1	1	1
			2	1	2	2	1	1	1	1	0	1	2	2	1	1	1	1	0
			3	3	5	3	2	1	1	1	1	3	5	3	2	1	1	1	1
		3	1	2	3	2	2	1	1	1	1	2	3	2	2	1	1	1	1
			2	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
			3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Average			1.33	2.11	1.67	1.33	1.00	1.00	1.00	0.67	1.33	2.11	1.67	1.33	1.00	1.00	1.00	0.67

Appendix 5 Comparison of contingency stock & ending inventory levels obtained by TPA and exact methods for real prolems with processing time 2

				Average Mean and Max. Inventory Levels																	
				Exact									TPA								
				Type 0		Type 1		Type 2		Type 3		Total	Type 0		Type 1		Type 2		Type 3		Total
		Day	Shift	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.		Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
P1	NIT	1	1	1.46	4	3.71	7	1.41	4	0.60	2	7.18	1.12	3	2.26	5	2.43	5	0.69	2	6.49
			2	1.49	4	3.78	7	1.16	3	1.09	2	7.52	0.59	2	4.58	9	0.88	3	0.26	1	6.31
			3	2.36	5	2.95	6	1.45	4	0.81	2	7.57	1.52	4	1.88	4	2.86	6	0.71	2	6.97
		2	1	1.16	3	3.80	7	1.84	4	0.52	2	7.33	1.21	3	1.98	4	2.48	5	0.68	2	6.35
			2	1.64	4	2.91	7	2.56	6	0.40	1	7.51	1.51	3	3.27	7	2.12	4	0.45	1	7.35
			3	1.99	4	3.74	7	1.53	4	0.60	2	7.87	1.75	4	3.74	7	1.56	4	0.52	1	7.57
		3	1	2.41	5	2.15	5	1.59	4	0.51	2	6.66	1.03	3	1.87	4	2.43	5	0.60	2	5.92
			2	1.88	4	3.07	6	2.06	4	0.78	2	7.79	3.12	6	1.02	3	1.19	4	0.30	1	5.64
			3	1.29	3	2.59	6	2.72	6	0.94	2	7.53	2.81	6	1.70	5	0.92	3	0.37	1	5.81
	Average			1.74		3.19		1.81		0.70		7.44	1.63		2.48		1.87		0.51		6.49
	IT	1	1	0.32	2	2.95	7	0.76	3	1.09	3	5.11	0.68	2	0.56	2	0.55	2	0.68	2	2.46
			2	2.38	5	1.07	4	0.74	3	0.93	2	5.13	0.52	2	0.61	2	0.78	2	0.70	2	2.62
			3	1.70	4	1.19	4	0.62	2	0.95	2	4.46	0.41	2	0.77	2	0.49	2	0.80	2	2.46
		2	1	0.58	3	2.35	5	1.59	4	1.09	2	5.62	0.61	2	0.66	2	0.74	2	0.80	2	2.81
			2	1.46	4	0.16	2	0.27	2	0.39	1	2.28	0.34	2	0.22	2	0.34	2	0.16	1	1.05
			3	0.67	3	2.71	6	0.52	2	1.22	3	5.11	0.66	3	0.48	3	0.38	2	0.38	1	1.90
		3	1	0.87	3	0.29	2	0.67	3	1.26	3	3.09	0.41	2	0.59	2	0.37	1	0.72	2	2.09
			2	1.01	3	0.19	2	1.24	4	0.51	2	2.95	0.72	2	0.45	2	0.47	2	0.40	2	2.03
			3	0.12	1	0.75	3	1.59	5	1.80	3	4.27	0.49	2	0.70	2	0.36	2	0.73	2	2.28
	Average			1.01		1.30		0.89		1.03		4.23	0.54		0.56		0.50		0.60		2.19

Appendix 6 Comparison of average mean and max. inventory levels obtained by TPA and exact methods for real prolems with processing time 1

				Average Mean and Max. Inventory Levels																	
				Exact									TPA								
				Type 0		Type 1		Type 2		Type 3		Total	Type 0		Type 1		Type 2		Type 3		Total
		Day	Shift	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.		Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
P2	NIT	1	1	1.16	3	1.40	3	0.96	3	0.59	2	4.11	1.33	3	0.70	2	1.02	3	0.59	2	3.65
			2	1.47	4	1.20	3	1.36	3	0.81	2	4.84	1.09	2	1.91	4	1.21	3	0.46	1	4.67
			3	0.71	2	1.53	3	1.08	3	0.51	1	3.83	0.98	3	1.28	3	1.07	3	0.16	1	3.49
		2	1	1.38	3	1.62	4	1.00	2	0.80	2	4.79	0.98	2	2.22	5	0.87	2	0.46	1	4.54
			2	1.44	3	1.76	4	1.31	3	0.45	1	4.96	1.11	3	1.06	3	1.88	4	0.56	1	4.60
			3	1.45	4	1.56	4	1.18	3	0.51	2	4.70	0.83	2	1.50	4	1.28	3	0.54	1	4.15
		3	1	1.18	3	1.35	3	1.51	4	0.38	2	4.42	1.18	3	1.23	3	0.68	2	0.57	2	3.65
			2	0.49	2	1.84	5	0.40	2	0.27	1	8.23	1.00	3	2.23	5	0.82	3	0.18	1	8.23
			3	1.13	3	1.31	3	1.71	4	1.06	2	5.21	1.15	3	1.92	5	0.82	2	0.82	2	4.71
	Average			1.16		1.51		1.17		0.60		5.01	1.07		1.56		1.07		0.48		4.63
	IT	1	1	0.61	2	2.16	5	0.88	3	0.41	2	4.06	0.71	2	0.61	2	0.53	2	0.92	2	2.77
			2	0.98	3	0.72	2	2.19	5	0.61	2	4.50	0.56	2	0.85	2	0.38	1	0.72	2	2.52
			3	0.73	2	0.99	3	0.51	2	0.80	2	3.04	0.92	2	0.64	3	0.76	2	0.60	1	2.92
		2	1	1.16	3	0.26	2	0.74	2	1.24	3	3.39	0.55	2	0.56	2	0.83	2	0.87	2	2.81
			2	1.43	4	0.27	2	0.72	3	0.97	2	3.40	0.58	2	0.46	2	0.82	2	0.58	1	2.44
			3	1.41	3	0.83	5	1.96	4	0.57	2	4.76	1.72	3	1.41	5	1.01	3	1.01	2	5.14
		3	1	0.76	2	0.27	3	1.61	4	1.78	3	4.42	1.10	3	1.07	3	0.63	2	1.18	2	3.97
			2	0.49	2	1.84	5	0.40	2	0.27	1	3.00	0.70	2	0.82	2	0.52	2	0.62	2	2.65
			3	0.80	3	0.45	2	0.60	2	0.89	2	2.73	0.75	2	0.54	2	0.39	2	0.75	2	2.42
	Average			0.93		0.87		1.07		0.84		3.70	0.84		0.77		0.65		0.81		3.07

Appendix 7 Comparison of average mean and max. inventory levels obtained by TPA and exact methods for real prolems with processing time 2

						# of Setup					
				CPU Time sec.		Exact			TPA		
			Day	Exact	TPA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
S1	P1	NIT	1	5,773	199	4	3	7	3	5	8
			2	6,946	42	2	4	6	6	1	7
			3	23,625	686	3	3	6	4	3	7
			4	50,119	664	3	3	6	5	3	8
			5	17,768	44	4	2	6	5	3	8
		Average		20,846	327	3.2	3	6.2	4.6	3	7.6
		IT	1	18,045	1,857	3	4	7	5	4	9
			2	22,331	171	3	2	5	5	3	8
			3	25,514	266	3	1	4	5	0	5
			4	23,785	941	3	3	6	3	4	7
			5	34,259	8,044	4	4	8	3	5	8
		Average		24,787	2,256	3.2	2.8	6	4.2	3.2	7.4
	P2	NIT	1	38,468	118	2	4	6	4	3	7
			2	33,958	32	3	2	5	5	2	7
			3	51,782.	167	3	3	6	4	4	8
			4	39,962	199	3	3	6	4	3	7
			5	29,927	47	4	3	7	5	3	8
		Average		38,819	113	3	3	6	4.4	3	7.4
		IT	1	75,231	969	3	3	6	6	4	10
			2	55,420	91	3	2	5	5	2	7
			3	46,508	70	2	1	3	3	1	4
			4	35,915	243	3	2	5	4	2	6
			5	34,609	404	4	4	8	5	4	9
		Average		49,537.	355	3	2.4	5.4	4.6	2.6	7.2

Appendix 8 Comparison of CPU time and number of setups obtained by TPA and exact methods for random problems (GM-S1) with processing time 1 and 2

				CS & EI																							
				Exact												TPA											
				CS (Type)						EI (Type)						CS (Type)						EI (Type)					
			Day	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
S1	P1	NIT	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0
			2	2	2	2	2	2	2	0	0	0	0	1	1	2	2	2	2	2	2	0	0	0	0	1	1
			3	2	2	2	2	2	2	0	0	1	1	2	1	2	2	2	2	2	2	0	0	1	1	2	1
			4	2	2	2	2	2	2	0	0	1	1	1	1	2	2	2	2	2	2	0	0	1	1	1	1
			5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0
		Average		1.60	1.60	1.60	1.60	1.60	1.60	0.00	0.00	0.40	0.40	0.80	0.60	1.60	1.60	1.60	1.60	1.60	1.60	0.00	0.00	0.40	0.40	0.80	0.60
		IT	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1
			2	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	1	1	1	1	1	1
			3	2	2	2	2	2	2	1	1	1	1	1	0	2	2	2	2	2	2	1	1	1	1	1	0
			4	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	1	1	1	1	1	1
			5	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1
		Average		1.60	1.60	1.60	1.60	1.60	1.60	1.00	1.00	1.00	0.80	0.60	0.80	1.60	1.60	1.60	1.60	1.60	1.60	1.00	1.00	1.00	0.80	0.60	0.80
	P2	NIT	1	2	2	2	2	2	2	1	1	1	0	0	1	2	2	2	2	2	2	1	1	1	0	0	1
			2	2	2	2	2	2	2	0	0	0	0	1	1	2	2	2	2	2	2	0	0	0	0	1	1
			3	2	2	2	2	2	2	0	0	1	1	2	1	2	2	2	2	2	2	0	0	1	1	2	1
			4	2	2	2	2	2	2	0	0	1	1	1	1	2	2	2	2	2	2	0	0	1	1	1	1
			5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0
		Average		1.80	1.80	1.80	1.80	1.80	1.80	0.20	0.20	0.60	0.40	0.80	0.80	1.80	1.80	1.80	1.80	1.80	1.80	0.20	0.20	0.60	0.40	0.80	0.80
		IT	1	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	1	1	1	1	1	1
			2	2	2	2	2	2	2	1	1	0	1	1	1	2	2	2	2	2	2	1	1	0	1	1	1
			3	2	2	2	2	2	2	1	1	0	0	1	0	2	2	2	2	2	2	1	1	0	0	1	0
			4	2	2	2	2	2	2	1	1	1	1	0	1	2	2	2	2	2	2	1	1	1	1	0	1
			5	1	1	1	1	1	1	1	1	0	0	0	1	1	1	2	1	1	1	1	1	0	0	0	1
		Average		1.80	1.80	1.80	1.80	1.80	1.80	1.00	1.00	0.40	0.60	0.60	0.80	1.80	1.80	2.00	1.80	1.80	1.80	1.00	1.00	0.40	0.60	0.60	0.80

Appendix 9 Comparison of contingency stock & ending inventory levels obtained by TPA and exact methods for random problems (GM-S1) with processing time 1 and 2

				Average Mean and Max.Inventory Levels																									
				Exact													TPA												
				Type 0		Type 1		Type 2		Type 3		Type 4		Type 5		Total	Type 0		Type 1		Type 2		Type 3		Type 4		Type 5		Total
			Day	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.		Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
S1	P1	NIT	1	1.03	4	2.74	5	1.59	3	1.36	3	0.60	1	0.25	1	7.56	1.13	4	1.43	4	1.93	4	1.18	3	0.39	1	0.23	1	6.29
			2	2.39	7	2.22	4	1.18	3	1.21	3	1.27	2	1.53	2	9.81	1.55	4	2.45	5	0.60	2	0.63	3	1.27	2	1.94	2	8.43
			3	3.11	7	2.30	6	2.02	3	2.16	3	2.12	3	1.55	2	13.25	2.33	6	2.80	7	1.42	2	1.25	2	2.51	3	1.93	3	12.24
			4	2.00	5	2.80	6	2.00	4	2.72	4	1.73	2	1.34	2	12.60	2.34	5	2.24	4	1.64	4	1.62	2	1.73	2	1.63	3	11.20
			5	1.86	4	3.52	8	0.43	2	0.77	2	0.47	1	0.00	1	7.05	2.35	4	2.00	5	0.30	2	0.28	1	1.10	2	0.00	1	6.05
		Average		2.08		2.72		1.44		1.64		1.24		0.93			1.94		2.18		1.18		0.99		1.40		1.15		
		IT	1	0.18	2	2.77	6	2.65	4	0.33	1	0.45	1	0.60	1	6.96	0.65	2	0.21	1	0.49	1	0.98	2	0.61	1	0.58	1	3.53
			2	0.15	2	1.53	4	0.50	2	1.88	3	1.27	2	1.87	2	7.20	2.55	5	0.68	2	1.00	2	0.49	2	1.27	2	1.00	2	6.98
			3	1.49	5	1.61	4	1.75	2	1.92	3	1.56	2	1.04	2	9.38	2.08	5	1.28	4	1.07	2	1.01	2	1.56	2	1.04	2	8.05
			4	0.70	2	2.17	5	0.69	2	1.66	2	0.84	2	0.74	2	6.80	0.79	2	0.83	3	0.90	2	0.98	2	0.93	2	0.74	2	5.18
			5	0.87	2	0.33	2	0.50	2	0.33	1	0.49	1	0.59	1	3.11	2.10	4	0.26	2	0.00	1	0.71	2	0.73	2	0.00	1	3.80
		Average		0.68		1.68		1.22		1.23		0.92		0.97			1.63		0.65		0.69		0.83		1.02		0.67		
	P2	NIT	1	1.22	3	1.38	3	2.98	5	1.21	3	1.06	2	1.00	2	8.83	2.82	6	2.37	5	1.62	4	1.09	2	1.06	2	1.32	2	10.27
			2	2.19	6	2.36	5	0.76	2	1.16	2	1.27	2	1.93	2	9.67	3.15	7	0.95	3	0.84	2	0.58	2	1.27	2	1.93	2	8.71
			3	2.65	6	1.20	3	1.43	2	0.93	2	2.42	3	1.97	3	10.60	1.09	3	1.83	5	2.00	3	2.08	3	2.05	3	1.46	2	10.51
			4	1.75	4	2.63	4	1.20	2	1.83	3	1.72	2	1.28	2	10.40	2.18	4	1.38	3	1.80	4	2.23	4	1.33	2	1.35	2	10.26
			5	1.33	3	1.23	3	0.59	2	0.33	1	0.72	2	0.00	1	4.20	1.38	3	1.23	3	0.35	2	0.45	1	0.72	2	0.00	1	4.14
		Average		1.83		1.76		1.39		1.09		1.44		1.23			2.13		1.55		1.32		1.28		1.29		1.21		
		IT	1	2.18	5	0.68	3	1.38	3	0.81	2	1.06	2	0.73	2	6.83	0.88	2	0.42	2	0.70	2	1.44	2	1.13	2	0.66	2	5.21
			2	0.27	2	2.11	5	0.50	2	1.94	3	1.27	2	1.00	2	7.08	0.35	2	0.93	2	0.83	2	0.54	2	1.27	2	1.00	2	4.91
			3	0.96	2	0.45	2	1.07	2	1.08	2	1.56	2	1.04	2	6.16	0.43	2	0.15	2	1.07	2	0.71	2	1.56	2	1.04	2	4.97
			4	1.91	4	0.86	2	1.33	3	2.67	4	0.79	2	0.74	2	8.30	0.41	2	0.63	2	1.13	2	1.20	2	0.79	2	0.74	2	4.88
			5	0.50	2	0.78	2	0.00	1	0.38	2	0.54	2	0.00	1	2.20	0.92	2	0.40	2	0.42	2	0.26	1	0.50	1	0.63	1	3.13
		Average		1.16		0.98		0.86		1.38		1.04		0.70		6.11	0.60		0.50		0.83		0.83		1.05		0.81		4.62

Appendix 10 Comparison of average mean and max. inventory levels obtained by TPA and exact methods for random problems (GM-S1) with processing time 1 and 2

			CPU Time sec.		# of Setup					
					Exact			RHA		
		Day	Exact	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P1	NIT	1	7,748	3	5	1	6	12	1	13
			22,580	6	2	4	6	13	1	14
			48,287	12	7	1	8	11	1	12
		2	33,403	4	5	1	6	10	2	12
			44,267	6	5	1	6	9	2	11
			73,158	5	2	5	7	10	2	12
		3	10,280	6	5	2	7	9	2	11
			185,278	2	4	2	6	12	1	13
			13,765	6	3	3	6	10	2	12
	Avg.		48,752	6	4	2	6	11	2	12
	IT	1	3,075	14	4	2	6	7	1	8
			32,659	23	4	1	5	8	1	9
			26,066	33	5	1	6	8	2	10
		2	7,539	24	5	1	6	5	3	8
			60,554	14	5	2	7	8	1	9
			44,535	21	2	5	7	9	1	10
		3	18,184	23	5	3	8	7	3	10
			32,235	23	4	3	7	7	2	9
			23,916	22	3	3	6	5	4	9
	Avg.		27,640	22	4	2	6	7	2	9

Appendix 11 Comparison of CPU time and number of setups obtained by RHA and exact methods for real prolems with processing time 1

			CPU Time sec.		# of Setup					
					Exact			RHA		
		Day	Exact	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P2	NIT	1	18,408	3	6	4	10	9	4	13
			7,150	5	5	3	8	7	5	12
			30,622	4	5	5	10	9	3	12
		2	19,358	3	7	5	12	10	4	14
			62,387	6	5	1	6	7	1	8
			23,006	10	3	3	6	8	3	11
		3	5,874	3	10	5	15	10	5	15
			4,951	3	4	4	8	9	4	13
			24,909	5	4	4	8	8	3	11
	Avg.		21,852	5	5	4	9	9	4	12
	IT	1	13,144	3	5	3	8	10	5	15
			17,437	6	4	3	7	11	4	15
			21,954	17	4	1	5	7	1	8
		2	20,490	5	4	1	5	8	3	11
			26,264	15	3	3	6	6	1	7
			31,214	17	5	1	6	7	2	9
		3	33,899	13	5	1	6	6	3	9
			52,333	11	5	3	8	6	5	11
			9,544	8	4	4	8	7	5	12
	Avg.		25,142	11	4	2	7	8	3	11

Appendix 12 Comparison of CPU time and number of setups obtained by RHA and exact methods for real prolems with processing time 2

			CS & EI															
			Exact								RHA							
			CS (Type)				EI (Type)				CS (Type)				EI (Type)			
		Day	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
P1	NIT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		3	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Avg.		1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.78	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.78
	IT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
		3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Avg.		1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.89

Appendix 13 Comparison of contingency stock & ending inventory levels obtained by RHA and exact methods for real prolems with processing time 1

			CS & EI															
			Exact								RHA							
			CS (Type)				EI (Type)				CS (Type)				EI (Type)			
		Day	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
P2	NIT	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1
			1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0
			1	1	1	1	0	1	0	0	1	1	1	1	0	1	0	0
		2	1	1	1	1	0	1	0	0	1	1	1	1	0	1	0	0
			2	2	2	2	1	2	2	1	2	2	2	2	1	2	2	1
			2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	1
		3	1	1	1	1	0	0	0	1	1	1	1	1	0	0	0	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	0	1	0	1	1	1	1	1	0	1	0
	Avg.		1.22	1.22	1.22	1.22	0.67	1.00	0.89	0.56	1.22	1.22	1.22	1.22	0.67	1.00	0.89	0.56
	IT	1	1	1	1	1	0	1	0	1	1	1	1	1	0	1	0	1
			1	1	1	1	1	1	0	0	1	1	1	1	1	1	0	0
			2	2	2	2	1	2	0	0	2	2	2	2	1	2	0	0
		2	2	2	2	2	0	2	2	1	2	2	2	2	0	2	2	1
			2	2	2	2	2	2	0	0	2	2	2	2	2	2	0	0
			2	2	2	2	2	1	2	0	2	2	2	2	2	1	2	0
		3	2	2	2	2	2	2	1	0	2	2	2	2	2	2	1	0
			1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1
			1	1	1	1	1	0	0	1	1	1	1	1	1	0	0	1
	Avg.		1.56	1.56	1.56	1.56	1.11	1.22	0.67	0.44	1.56	1.56	1.56	1.56	1.11	1.22	0.67	0.44

Appendix 14 Comparison of contingency stock & ending inventory levels obtained by RHA and exact methods for real prolems with processing time 2

			Average Mean and Max. Inventory Levels																	
			Exact									RHA								
			Type 0		Type 1		Type 2		Type 3		Total	Type 0		Type 1		Type 2		Type 3		Total
		Day	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.		Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
P1	NIT	1	1.34	3	4.26	8	2.11	5	1.24	3	8.95	1.62	3	4.26	8	1.73	3	0.89	2	8.49
			1.15	3	3.01	7	2.59	6	2.07	3	8.82	1.52	3	4.34	8	1.71	3	1.21	2	8.77
			1.96	4	3.42	6	2.63	6	0.36	2	8.37	1.67	3	3.99	7	2.21	4	0.84	2	8.71
		2	1.38	3	4.33	8	1.93	5	1.09	2	8.73	2.54	5	2.50	5	1.80	3	0.80	2	7.64
			1.96	5	4.36	8	1.72	4	0.53	2	8.57	2.30	5	3.40	6	1.96	4	0.32	1	7.98
			2.07	5	1.58	4	3.99	7	0.94	2	8.58	2.24	4	4.27	8	1.83	4	0.80	2	9.14
		3	1.83	4	3.06	7	2.37	5	0.71	2	7.97	2.59	5	2.68	5	1.60	3	0.21	1	7.08
			2.42	5	3.68	7	1.76	4	1.62	3	8.23	1.92	3	4.12	8	1.71	4	0.90	2	8.23
			2.28	5	2.80	7	3.03	6	1.83	3	9.94	2.31	4	3.95	7	1.95	4	0.89	2	9.11
	Avg.		1.82		3.39		2.46		1.15		8.68	2.08		3.72		1.83		0.76		8.35
	IT	1	0.32	2	2.95	7	0.76	3	1.09	3	4	0.51	2	0.08	1	0.66	2	0.35	1	1.60
			2.38	5	1.07	4	0.74	3	0.93	2	3	0.66	3	0.07	1	0.64	2	0.39	1	1.76
			1.51	4	0.11	2	2.06	5	1.19	2	3	0.96	3	0.54	2	0.77	2	0.27	1	2.54
		2	0.58	3	0.03	1	1.58	4	1.09	2	3	0.97	3	0.40	2	0.74	2	0.39	1	2.50
			0.83	3	0.18	2	1.39	4	1.16	3	4	0.96	3	0.18	2	0.78	3	0.28	1	2.20
			1.82	5	0.45	3	0.03	1	0.97	2	3	0.61	3	0.22	2	0.66	2	0.13	1	1.62
		3	0.87	3	0.29	2	0.67	3	1.26	3	4	0.43	2	0.22	2	0.98	3	0.22	1	1.85
			1.01	3	0.19	2	1.24	4	0.51	2	3	0.06	1	0.68	2	0.71	3	0.55	2	2.00
			0.12	1	0.75	3	1.59	5	1.80	3	5	0.63	3	1.08	3	0.62	2	0.59	2	2.91
	Avg.		1.05		0.67		1.12		1.11		3.56	0.64		0.39		0.73		0.35		2.11

Appendix 15 Comparison of average mean and max. inventory levels obtained by RHA and exact methods for real prolems with processing time 1

			Average Mean and Max. Inventory Levels																		
			Exact									RHA								
			Type 0		Type 1		Type 2		Type 3		Total	Type 0		Type 1		Type 2		Type 3		Total
		Day	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.		Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
P2	NIT	1	1.10	3	1.52	4	0.96	3	0.86	2	4.44	0.40	1	2.04	5	1.13	3	0.75	2	4.31
			1.60	4	1.45	4	1.27	3	0.59	2	4.91	1.26	3	1.73	3	1.43	3	0.25	1	4.66
			0.88	2	1.11	3	1.08	3	0.51	1	3.58	0.56	2	2.05	4	0.96	3	0.23	1	3.80
		2	0.67	2	1.12	2	1.03	3	0.08	1	2.90	0.79	2	1.60	4	0.53	2	0.06	1	2.97
			1.74	4	3.61	6	1.59	4	0.73	2	7.67	1.04	2	3.61	6	2.06	4	0.81	2	7.52
			1.41	4	1.81	4	3.25	6	1.93	3	8.40	1.90	4	3.09	5	1.97	4	0.95	2	7.91
		3	0.53	2	0.80	3	0.57	2	0.38	1	2.28	0.61	2	0.94	3	0.44	2	0.35	1	2.34
			1.81	4	1.05	3	1.63	4	1.19	3	8.23	1.90	4	1.89	4	1.07	3	0.68	2	8.23
			0.80	3	0.59	2	0.23	2	0.28	1	1.90	1.97	5	0.77	2	0.98	2	0.45	1	4.17
	Avg.		1.17		1.45		1.29		0.73		4.92	1.16		1.97		1.17		0.50		5.10
	IT	1	0.84	3	0.84	3	0.25	1	0.72	2	2.65	0.24	1	0.09	1	0.35	2	0.37	1	1.05
			1.92	4	0.21	1	0.48	2	0.41	1	3.03	0.39	2	0.56	2	0.25	1	0.27	1	1.46
			0.89	3	0.25	2	1.51	4	1.05	2	3.70	0.90	3	0.90	3	0.82	2	0.85	2	3.47
		2	1.47	4	0.34	2	1.18	4	0.87	2	3.86	0.58	2	0.85	2	1.17	3	1.23	2	3.84
			0.29	2	1.72	4	1.80	4	1.03	2	4.84	1.00	3	0.90	2	0.99	3	0.41	2	3.30
			0.92	2	0.37	2	1.72	4	0.99	2	4.00	1.06	2	1.21	2	1.45	3	0.51	2	4.24
		3	1.21	3	0.27	2	1.02	3	0.50	2	3.00	1.24	4	0.92	2	0.84	2	0.93	2	3.94
			0.53	2	0.72	3	1.02	3	1.03	2	3.30	0.62	2	0.53	2	0.45	2	0.31	1	1.92
			0.80	3	0.60	3	0.28	2	0.89	2	2.57	0.46	2	0.29	2	0.32	2	0.28	1	1.35
	Avg.		0.99		0.59		1.03		0.83		3.44	0.72		0.69		0.74		0.57		2.73

Appendix 16 Comparison of average mean and max. inventory levels obtained by RHA and exact methods for real prolems with processing time 2

						# of Setup					
				CPU Time sec.		Exact			RHA		
			Day	Exact	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
S1	P1	NIT	1	30,189	40	4	3	7	5	7	12
			2	17,993	25	2	4	6	3	5	8
			3	12,143	15	3	3	6	4	5	9
			4	16,769	17	4	3	7	7	3	10
			5	19,123	11	5	3	8	5	5	10
		Average		19,243	22	3.6	3.2	6.8	4.8	5	9.8
		IT	1	50,066	37	3	3	6	5	5	10
			2	30,628	8	3	2	5	4	3	7
			3	26,567	45	3	3	6	3	3	6
			4	71,561	60	3	3	6	3	4	7
			5	34,520	44	4	4	8	6	6	12
		Average		42,668	39	3.2	3	6.2	4.2	4.2	8.4
	P2	NIT	1	20,734	89	4	4	8	4	4	8
			2	18,015	11	4	3	7	4	4	8
			3	40,780	7	3	4	7	4	4	8
			4	39,260	12	3	4	7	4	5	9
			5	33,141	31	5	4	9	6	4	10
		Average		30,386	30	3.8	3.8	7.6	4.4	4.2	8.6
		IT	1	24,407	32	3	4	7	4	8	12
			2	35,769	31	4	2	6	5	2	7
			3	15,672	16	3	2	5	4	4	8
			4	32,798	27	3	3	6	4	4	8
			5	17,325	25	5	3	8	5	4	9
		Average		25,194	26	3.6	2.8	6.4	4.4	4.4	8.8

Appendix 17 Comparison of CPU time and number of setups obtained by RHA and exact methods for random problems (GM-S1) with processing time 1 and 2

				CS & EI																							
				Exact												RHA											
				CS (Type)						EI (Type)						CS (Type)						EI (Type)					
			Day	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
S1	P1	NIT	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0
			2	2	2	2	2	2	2	2	2	2	2	1	1	2	2	2	2	2	2	2	2	2	2	1	1
			3	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	1
			4	2	2	2	2	2	2	2	2	1	2	1	1	2	2	2	2	2	2	2	2	1	2	1	1
			5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		Average		1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.4	1.6	1.2	0.8	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.4	1.6	1.2	0.8
		IT	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1
			2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	1	2
			3	2	2	2	2	2	2	0	2	1	2	2	1	2	2	2	2	2	2	0	2	1	2	2	1
			4	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2
			5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		Average		1.6	1.6	1.6	1.6	1.6	1.6	1.2	1.6	1.2	1.6	1.2	1.4	1.6	1.6	1.6	1.6	1.6	1.6	1.2	1.6	1.2	1.6	1.2	1.4
	P2	NIT	1	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	1
			2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
			3	2	2	2	2	2	2	2	2	1	1	2	1	2	2	2	2	2	2	2	2	1	1	2	1
			4	2	2	2	2	2	2	2	2	1	2	1	2	2	2	2	2	2	2	2	2	1	2	1	2
			5	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	1	0
		Average		1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.4	1.4	1.6	1.2	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.4	1.4	1.6	1.2
		IT	1	2	2	2	2	2	2	2	2	2	2	2	1	2	2	2	2	2	2	2	2	2	2	2	1
			2	2	2	2	2	2	2	2	2	0	2	1	2	2	2	2	2	2	2	2	2	0	2	1	2
			3	2	2	2	2	2	2	2	2	1	2	2	0	2	2	2	2	2	2	2	2	1	2	2	0
			4	2	2	2	2	2	2	2	1	1	2	1	2	2	2	2	2	2	2	2	1	1	2	1	2
			5	1	1	1	1	1	1	0	1	1	1	0	0	1	1	1	1	1	1	0	1	1	1	0	0
		Average		1.8	1.8	1.8	1.8	1.8	1.8	1.6	1.6	1.0	1.8	1.2	1.0	1.8	1.8	1.8	1.8	1.8	1.8	1.6	1.6	1.0	1.8	1.2	1.0

Appendix 18 Comparison of contingency stock & ending inventory levels obtained by RHA and exact methods for random problems (GM-S1) with processing time 1 and 2

				Average Mean and Max. Inventory Levels																									
				Exact													RHA												
				Type 0		Type 1		Type 2		Type 3		Type 4		Type 5		Total	Type 0		Type 1		Type 2		Type 3		Type 4		Type 5		Total
			Day	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.		Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
S1	P1	NIT	1	1.40	4	3.35	6	2.69	4	1.38	2	1.03	2	0.22	1	10.07	2.00	4	2.56	4	1.94	4	1.25	3	1.26	2	0.15	1	9.17
			2	3.81	9	2.06	5	3.06	5	2.45	4	1.27	2	1.50	2	14.15	3.48	6	2.59	5	2.89	4	2.58	4	1.27	2	1.35	2	14.15
			3	4.52	9	3.43	6	2.02	3	1.52	3	2.51	3	1.93	3	15.93	2.12	5	3.26	7	2.61	3	2.72	4	2.19	3	1.29	2	14.19
			4	2.04	4	4.55	8	1.20	2	1.51	3	1.73	2	1.63	3	12.68	3.07	5	3.53	5	1.84	4	2.74	4	1.46	2	1.46	2	14.10
			5	1.97	4	2.08	4	1.66	3	1.12	2	1.37	3	0.73	1	8.93	1.90	4	3.12	6	1.46	3	1.22	3	0.90	2	0.73	1	9.33
		Average		2.75		3.10		2.13		1.60		1.58		1.20			2.51		3.01		2.15		2.10		1.42		1.00		
		IT	1	1.65	5	2.77	6	2.90	5	0.33	1	0.23	1	0.38	1	8.25	0.80	3	1.36	4	0.68	2	1.03	2	0.23	1	0.38	1	4.47
			2	0.38	3	1.39	5	0.53	2	2.73	4	1.27	2	1.03	2	7.33	3.35	8	1.67	3	0.71	2	1.23	3	1.27	2	1.03	2	9.25
			3	0.45	2	2.18	5	1.07	2	0.83	2	1.56	2	1.10	2	7.20	1.63	4	1.27	3	1.41	2	0.73	2	1.56	2	1.10	2	7.68
			4	1.60	3	3.04	6	0.90	2	1.02	2	0.82	2	0.95	2	8.32	1.50	4	2.82	6	1.06	2	1.02	2	0.82	2	1.34	2	8.55
			5	1.08	3	0.77	4	0.32	1	1.10	2	0.42	2	0.00	1	3.68	0.99	3	0.63	2	0.55	2	0.33	1	0.70	2	0.06	1	3.25
		Average		1.03		2.03		1.15		1.20		0.86		0.69			1.66		1.55		0.88		0.86		0.91		0.78		
	P2	NIT	1	2.39	4	3.31	7	2.85	5	1.60	3	1.34	2	0.90	2	12.39	3.08	6	2.62	4	1.75	4	1.91	3	2.03	3	1.05	2	12.43
			2	3.77	7	2.66	4	1.49	3	1.68	3	1.56	2	1.76	3	12.92	3.62	7	2.78	6	1.34	3	2.20	3	1.56	2	1.38	3	12.88
			3	1.48	4	2.62	5	1.41	2	2.22	4	2.20	3	1.75	2	11.68	1.86	4	3.58	7	1.61	2	1.68	3	1.70	2	1.11	2	11.53
			4	3.38	6	3.60	6	1.05	2	2.24	4	1.18	2	1.32	2	12.77	2.13	4	3.09	5	1.20	2	2.39	4	1.25	2	2.20	3	12.27
			5	2.05	4	1.56	3	0.83	2	0.18	1	0.74	2	0.00	1	5.37	1.45	3	2.11	5	0.37	1	0.72	2	0.75	2	0.00	1	5.40
		Average		2.62		2.75		1.53		1.58		1.40		1.15			2.43		2.84		1.25		1.78		1.46		1.15		
		IT	1	2.05	4	1.61	4	0.42	2	2.10	3	1.11	2	1.42	2	8.70	1.45	2	2.47	4	0.62	2	1.86	3	1.95	3	0.85	2	9.21
			2	0.69	3	1.96	5	0.50	2	1.08	2	1.27	2	1.05	2	6.54	1.26	3	2.49	5	0.53	2	1.19	3	1.27	2	1.05	2	7.79
			3	0.82	3	0.32	2	1.07	2	2.41	4	1.56	2	1.04	2	7.22	2.51	4	1.16	4	1.07	2	0.88	2	1.68	2	1.04	2	8.35
			4	2.01	4	0.85	4	0.95	2	1.03	2	0.91	2	0.79	2	6.54	1.64	3	1.92	4	0.95	2	1.03	2	0.98	2	0.79	2	7.31
			5	0.47	2	1.34	3	0.60	2	0.38	2	0.38	1	0.00	1	3.18	0.97	2	0.61	2	0.65	2	0.38	2	0.43	1	0.00	1	3.05
		Average		1.21		1.21		0.71		1.40		1.05		0.86			1.57		1.73		0.76		1.07		1.26		0.75		

Appendix 19 Comparison of average mean and max. inventory levels obtained by RHA and exact methods for random problems (GM-S1) with processing time 1 and 2

			CS & EI															
			TPA								RHA							
			CS (Type)				EI (Type)				CS (Type)				EI (Type)			
		Day	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
P1	NIT	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
			1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
			1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
		2	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
			1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	0
			1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
		3	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	0
			1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
			1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
	Avg.		1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.78
	IT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1
		2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
		3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Avg.		1.00	1.00	1.00	1.00	0.78	0.78	0.78	0.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.89

Appendix 20 Comparison of contingency stock & ending inventory levels obtained by TPA and RHA for real problems with processing time 1

			CS & EI															
			TPA								RHA							
			CS (Type)				EI (Type)				CS (Type)				EI (Type)			
		Day	0	1	2	3	0	1	2	3	0	1	2	3	0	1	2	3
P2	NIT	1	1	1	1	2	0	0	0	0	1	1	1	1	0	1	1	1
			1	2	1	1	0	0	0	0	1	1	1	1	1	1	1	0
			1	1	1	1	0	0	0	0	1	1	1	1	0	1	0	0
		2	1	2	2	1	0	0	0	0	1	1	1	1	0	1	0	0
			1	2	2	1	0	0	0	0	2	2	2	2	1	2	2	1
			1	2	1	1	0	0	0	0	2	2	2	2	2	2	2	1
		3	1	2	1	2	0	0	0	0	1	1	1	1	0	0	0	1
			1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0
	Avg.		1.00	1.56	1.22	1.22	0.11	0.11	0.11	0.11	1.22	1.22	1.22	1.22	0.67	1.00	0.89	0.56
	IT	1	1	1	1	2	1	1	1	1	1	1	1	1	0	1	0	1
			1	2	1	1	1	1	1	1	1	1	1	1	1	1	0	0
			1	2	2	1	1	1	1	0	2	2	2	2	1	2	0	0
		2	1	2	2	1	1	1	1	1	2	2	2	2	0	2	2	1
			1	2	2	1	1	1	1	0	2	2	2	2	2	2	0	0
			3	5	3	2	1	1	1	1	2	2	2	2	2	1	2	0
		3	2	3	2	2	1	1	1	1	2	2	2	2	2	2	1	0
			1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	1
			1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1
	Avg.		1.33	2.11	1.67	1.33	1.00	1.00	1.00	0.67	1.56	1.56	1.56	1.56	1.11	1.22	0.67	0.44

Appendix 21 Comparison of contingency stock & ending inventory levels obtained by TPA and RHA for real problems with processing time 2

			CPU Time sec.		# of Setup					
					TPA			RHA		
		Day	TPA	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P1	NIT	1	7	3	3	5	8	12	1	13
			12	6	11	1	12	13	1	14
			12	12	4	5	9	11	1	12
		2	12	4	3	5	8	10	2	12
			19	6	6	2	8	9	2	11
			21	5	7	1	8	10	2	12
		3	16	6	3	5	8	9	2	11
			9	2	10	1	11	12	1	13
			9	6	9	2	11	10	2	12
	Avg.		13	6	6.2	3.0	9.2	10.7	1.6	12.2
	IT	1	18	14	7	3	10	7	1	8
			22	23	6	3	9	8	1	9
			25	33	6	4	10	8	2	10
		2	37	24	8	3	11	5	3	8
			34	14	10	2	12	8	1	9
			41	21	8	3	11	9	1	10
		3	40	23	8	4	12	7	3	10
			37	23	9	2	11	7	2	9
			22	22	6	4	10	5	4	9
	Avg.		31	22	7.6	3.1	10.7	7.1	2.0	9.1

Appendix 22 Comparison of for CPU time and number of setups obtained by TPA and RHA for real problems with processing time 1

			CPU Time sec.		# of Setup					
					TPA			RHA		
		Day	TPA	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
P2	NIT	1	6	3	7	2	10	9	4	13
			14	5	6	4	8	7	5	12
			9	4	9	2	10	9	3	12
		2	7	3	7	2	12	10	4	14
			19	6	5	4	6	7	1	8
			17	10	6	4	6	8	3	11
		3	39	3	8	2	15	10	5	15
			9	3	8	2	8	9	4	13
			18	5	9	4	8	8	3	11
	Avg.		15	5	7.2	2.9	9.2	8.6	3.6	12.1
	IT	1	14	3	7	4	8	10	5	15
			15	6	7	5	7	11	4	15
			18	17	8	4	5	7	1	8
		2	18	5	8	6	5	8	3	11
			15	15	9	4	6	6	1	7
			102	17	6	3	6	7	2	9
		3	15	13	8	4	6	6	3	9
			8	11	5	4	8	6	5	11
			15	8	8	6	8	7	5	12
	Avg.		25	11	7.3	4.4	6.6	7.6	3.2	10.8

Appendix 23 Comparison of CPU time and number of setups obtained by TPA and RHA for real problems with processing time 2

			Average Mean and Max. Inventory Levels																	
			TPA									RHA								
			Type 0		Type 1		Type 2		Type 3		Total	Type 0		Type 1		Type 2		Type 3		Total
		Day	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.		Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
P1	NIT	1	1.12	3	2.26	5	2.43	5	0.69	2	6.49	1.62	3	4.26	8	1.73	3	0.89	2	8.49
			0.59	2	4.58	9	0.88	3	0.26	1	6.31	1.52	3	4.34	8	1.71	3	1.21	2	8.77
			1.52	4	1.88	4	2.86	6	0.71	2	6.97	1.67	3	3.99	7	2.21	4	0.84	2	8.71
		2	1.21	3	1.98	4	2.48	5	0.68	2	6.35	2.54	5	2.50	5	1.80	3	0.80	2	7.64
			1.51	3	3.27	7	2.12	4	0.45	1	7.35	2.30	5	3.40	6	1.96	4	0.32	1	7.98
			1.75	4	3.74	7	1.56	4	0.52	1	7.57	2.24	4	4.27	8	1.83	4	0.80	2	9.14
		3	1.03	3	1.87	4	2.43	5	0.60	2	5.92	2.59	5	2.68	5	1.60	3	0.21	1	7.08
			3.12	6	1.02	3	1.19	4	0.30	1	5.64	1.92	3	4.12	8	1.71	4	0.90	2	8.65
			2.81	6	1.70	5	0.92	3	0.37	1	5.81	2.31	4	3.95	7	1.95	4	0.89	2	9.11
	Avg.		1.63		2.48		1.87		0.51		6.49	2.08		3.72		1.83		0.76		8.40
	IT	1	0.68	2	0.56	2	0.55	2	0.68	2	2.46	0.51	2	0.08	1	0.66	2	0.35	1	1.60
			0.52	2	0.61	2	0.78	2	0.70	2	2.62	0.66	3	0.07	1	0.64	2	0.39	1	1.76
			0.41	2	0.77	2	0.49	2	0.80	2	2.46	0.96	3	0.54	2	0.77	2	0.27	1	2.54
		2	0.61	2	0.66	2	0.74	2	0.80	2	2.81	0.97	3	0.40	2	0.74	2	0.39	1	2.50
			0.34	2	0.22	2	0.34	2	0.16	1	1.05	0.96	3	0.18	2	0.78	3	0.28	1	2.20
			0.66	3	0.48	3	0.38	2	0.38	1	1.90	0.61	3	0.22	2	0.66	2	0.13	1	1.62
		3	0.41	2	0.59	2	0.37	1	0.72	2	2.09	0.43	2	0.22	2	0.98	3	0.22	1	1.85
			0.72	2	0.45	2	0.47	2	0.40	2	2.03	0.06	1	0.68	2	0.71	3	0.55	2	2.00
			0.49	2	0.70	2	0.36	2	0.73	2	2.28	0.63	3	1.08	3	0.62	2	0.59	2	2.91
	Avg.		0.54		0.56		0.50		0.60		2.19	0.64		0.39		0.73		0.35		2.11

Appendix 24 Comparison of average mean and max. inventory levels obtained by TPA and RHA methods for real problems with processing time 1

			Average Mean and Max. Inventory Levels																	
			TPA									RHA								
			Type 0		Type 1		Type 2		Type 3		Total	Type 0		Type 1		Type 2		Type 3		Total
		Day	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.		Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
P2	NIT	1	1.33	3	0.70	2	1.02	3	0.59	2	3.65	0.40	1	2.04	5	1.13	3	0.75	2	4.31
			1.09	2	1.91	4	1.21	3	0.46	1	4.67	1.26	3	1.73	3	1.43	3	0.25	1	4.66
			0.98	3	1.28	3	1.07	3	0.16	1	3.49	0.56	2	2.05	4	0.96	3	0.23	1	3.80
		2	0.98	2	2.22	5	0.87	2	0.46	1	4.54	0.79	2	1.60	4	0.53	2	0.06	1	2.97
			1.11	3	1.06	3	1.88	4	0.56	1	4.60	1.04	2	3.61	6	2.06	4	0.81	2	7.52
			0.83	2	1.50	4	1.28	3	0.54	1	4.15	1.90	4	3.09	5	1.97	4	0.95	2	7.91
		3	1.18	3	1.23	3	0.68	2	0.57	2	3.65	0.61	2	0.94	3	0.44	2	0.35	1	2.34
			1.00	3	2.23	5	0.82	3	0.18	1	4.23	1.90	4	1.89	4	1.07	3	0.68	2	5.55
			1.15	3	1.92	5	0.82	2	0.82	2	4.71	1.97	5	0.77	2	0.98	2	0.45	1	4.17
	Avg.		1.07		1.56		1.07		0.48		4.19	1.16		1.97		1.17		0.50		4.80
	IT	1	0.71	2	0.61	2	0.53	2	0.92	2	2.77	0.24	1	0.09	1	0.35	2	0.37	1	1.05
			0.56	2	0.85	2	0.38	1	0.72	2	2.52	0.39	2	0.56	2	0.25	1	0.27	1	1.46
			0.92	2	0.64	3	0.76	2	0.60	1	2.92	0.90	3	0.90	3	0.82	2	0.85	2	3.47
		2	0.55	2	0.56	2	0.83	2	0.87	2	2.81	0.58	2	0.85	2	1.17	3	1.23	2	3.84
			0.58	2	0.46	2	0.82	2	0.58	1	2.44	1.00	3	0.90	2	0.99	3	0.41	2	3.30
			1.72	3	1.41	5	1.01	3	1.01	2	5.14	1.06	2	1.21	2	1.45	3	0.51	2	4.24
		3	1.10	3	1.07	3	0.63	2	1.18	2	3.97	1.24	4	0.92	2	0.84	2	0.93	2	3.94
			0.70	2	0.82	2	0.52	2	0.62	2	2.65	0.62	2	0.53	2	0.45	2	0.31	1	1.92
			0.75	2	0.54	2	0.39	2	0.75	2	2.42	0.46	2	0.29	2	0.32	2	0.28	1	1.35
	Avg.		0.84		0.77		0.65		0.81		3.07	0.72		0.69		0.74		0.57		2.73

Appendix 25 Comparison of average mean and max. inventory levels obtained by TPA and RHA methods for real problems with processing time 2

				CS & EI																							
				TPA												RHA											
				CS (Type)						EI (Type)						CS (Type)						EI (Type)					
			Day	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
S1	P1	NIT	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0
			2	2	2	2	2	2	2	0	0	0	0	1	1	2	2	2	2	2	2	2	2	2	2	1	1
			3	2	2	2	2	2	2	0	0	1	1	2	1	2	2	2	2	2	2	2	2	2	2	2	1
			4	2	2	2	2	2	2	0	0	1	1	1	1	2	2	2	2	2	2	2	2	1	2	1	1
			5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
		Avr.		1.60	1.60	1.60	1.60	1.60	1.60	0.00	0.00	0.40	0.40	0.80	0.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.40	1.60	1.20	0.80
		IT	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1
			2	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	1	2
			3	2	2	2	2	2	2	1	1	1	1	1	0	2	2	2	2	2	2	0	2	1	2	2	1
			4	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	2	2	2
			5	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
		Avr.		1.60	1.60	1.60	1.60	1.60	1.60	1.00	1.00	1.00	0.80	0.60	0.80	1.60	1.60	1.60	1.60	1.60	1.60	1.20	1.60	1.20	1.60	1.20	1.40
	P2	NIT	1	2	2	2	2	2	2	1	1	1	0	0	1	2	2	2	2	2	2	2	2	2	2	2	1
			2	2	2	2	2	2	2	0	0	0	0	1	1	2	2	2	2	2	2	2	2	2	2	2	2
			3	2	2	2	2	2	2	0	0	1	1	2	1	2	2	2	2	2	2	2	2	1	1	2	1
			4	2	2	2	2	2	2	0	0	1	1	1	1	2	2	2	2	2	2	2	2	1	2	1	2
			5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	1	0
		Avr.		1.80	1.80	1.80	1.80	1.80	1.80	0.20	0.20	0.60	0.40	0.80	0.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.40	1.40	1.60	1.20
		IT	1	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	1
			2	2	2	2	2	2	2	1	1	0	1	1	1	2	2	2	2	2	2	2	2	0	2	1	2
			3	2	2	2	2	2	2	1	1	0	0	1	0	2	2	2	2	2	2	2	2	1	2	2	0
			4	2	2	2	2	2	2	1	1	1	1	0	1	2	2	2	2	2	2	2	1	1	2	1	2
			5	1	1	2	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	1	1	1	0	0
		Avr.		1.80	1.80	2.00	1.80	1.80	1.80	1.00	1.00	0.40	0.60	0.60	0.80	1.80	1.80	1.80	1.80	1.80	1.80	1.60	1.60	1.00	1.80	1.20	1.00

Appendix 26 Comparison of contingency stock & ending inventory levels obtained by TPA and RHA for random problems (GM-S1) with processing time 1 and 2

				CS & EI																							
				TPA												RHA											
				CS (Type)						EI (Type)						CS (Type)						EI (Type)					
			Day	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
S1	P1	NIT	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0
			2	2	2	2	2	2	2	0	0	0	0	1	1	2	2	2	2	2	2	2	2	2	2	1	1
			3	2	2	2	2	2	2	0	0	1	1	2	1	2	2	2	2	2	2	2	2	2	2	2	1
			4	2	2	2	2	2	2	0	0	1	1	1	1	2	2	2	2	2	2	2	2	1	2	1	1
			5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
		Avr.		1.60	1.60	1.60	1.60	1.60	1.60	0.00	0.00	0.40	0.40	0.80	0.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.60	1.40	1.60	1.20	0.80
		IT	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1
			2	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	1	2
			3	2	2	2	2	2	2	1	1	1	1	1	0	2	2	2	2	2	2	0	2	1	2	2	1
			4	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	1	2	2	2
			5	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
		Avr.		1.60	1.60	1.60	1.60	1.60	1.60	1.00	1.00	1.00	0.80	0.60	0.80	1.60	1.60	1.60	1.60	1.60	1.60	1.20	1.60	1.20	1.60	1.20	1.40
	P2	NIT	1	2	2	2	2	2	2	1	1	1	0	0	1	2	2	2	2	2	2	2	2	2	2	2	1
			2	2	2	2	2	2	2	0	0	0	0	1	1	2	2	2	2	2	2	2	2	2	2	2	2
			3	2	2	2	2	2	2	0	0	1	1	2	1	2	2	2	2	2	2	2	2	1	1	2	1
			4	2	2	2	2	2	2	0	0	1	1	1	1	2	2	2	2	2	2	2	2	1	2	1	2
			5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	1	0
		Avr.		1.80	1.80	1.80	1.80	1.80	1.80	0.20	0.20	0.60	0.40	0.80	0.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.40	1.40	1.60	1.20
		IT	1	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	1
			2	2	2	2	2	2	2	1	1	0	1	1	1	2	2	2	2	2	2	2	2	0	2	1	2
			3	2	2	2	2	2	2	1	1	0	0	1	0	2	2	2	2	2	2	2	2	1	2	2	0
			4	2	2	2	2	2	2	1	1	1	1	0	1	2	2	2	2	2	2	2	1	1	2	1	2
			5	1	1	2	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	1	1	1	0	0
		Avr.		1.80	1.80	2.00	1.80	1.80	1.80	1.00	1.00	0.40	0.60	0.60	0.80	1.80	1.80	1.80	1.80	1.80	1.80	1.60	1.60	1.00	1.80	1.20	1.00

Appendix 27 Comparison of contingency stock & ending inventory levels obtained by TPA and RHA for random problems (GM-S1) with processing time 1 and 2

						# of Setup					
				CPU Time (sec.)		TPA			RHA		
			Day	TPA	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
S1	P1	NIT	1	199	40	3	5	8	5	7	12
			2	42	25	6	1	7	3	5	8
			3	686	15	4	3	7	4	5	9
			4	664	17	5	3	8	7	3	10
			5	44	11	5	3	8	5	5	10
		Avr.		327	22	4.6	2.6	7.6	4.8	5	9.8
		IT	1	1,857	37	5	4	9	5	5	10
			2	171	8	5	3	8	4	3	7
			3	266	45	5	0	5	3	3	6
			4	941	60	3	4	7	3	4	7
			5	8,044	44	3	5	8	6	6	12
		Avr.		2,256	39	4.6	2.6	7.4	4.2	4.2	8.4
	P2	NIT	1	118	89	4	3	7	4	4	8
			2	32	11	5	2	7	4	4	8
			3	167	7	4	4	8	4	4	8
			4	199	12	4	3	7	4	5	9
			5	47	31	5	3	8	6	4	10
		Avr.		113	30	4.6	2.6	7.4	4.4	4.2	8.6
		IT	1	969	32	6	4	10	4	8	12
			2	91	31	5	2	7	5	2	7
			3	70	16	3	1	4	4	4	8
			4	243	27	4	2	6	4	4	8
			5	404	25	5	4	9	5	4	9
		Avr.		355	26	4.6	2.6	7.2	4.4	4.4	8.8

Appendix 28 Comparison of CPU time and number of setups contingency stock & ending inventory levels obtained by TPA and RHA for random problems (GM-S1) with processing time 1 and 2

				Average Mean and Max. Inventory Levels																									
				TPA													RHA												
				Type 0		Type 1		Type 2		Type 3		Type 4		Type 5			Type 0		Type 1		Type 2		Type 3		Type 4		Type 5		Total
			Day	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Total	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
S1	P1	NIT	1	1.1	4	1.4	4	1.9	4	1.2	3	0.4	1	0.2	1	6.3	2.0	4	2.6	4	1.9	4	1.3	3	1.3	2	0.2	1	9.2
			2	1.6	4	2.4	5	0.6	2	0.6	3	1.3	2	1.9	2	8.4	3.5	6	2.6	5	2.9	4	2.6	4	1.3	2	1.3	2	14.2
			3	2.3	6	2.8	7	1.4	2	1.3	2	2.5	3	1.9	3	12.2	2.1	5	3.3	7	2.6	3	2.7	4	2.2	3	1.3	2	14.2
			4	2.3	5	2.2	4	1.6	4	1.6	2	1.7	2	1.6	3	11.2	3.1	5	3.5	5	1.8	4	2.7	4	1.5	2	1.5	2	14.1
			5	2.4	4	2.0	5	0.3	2	0.3	1	1.1	2	0.0	1	6.0	1.9	4	3.1	6	1.5	3	1.2	3	0.9	2	0.7	1	9.3
		Avr.		1.9		2.2		1.2		1.0		1.4		1.1			2.5		3.0		2.1		2.1		1.4		1.0		
		IT	1	0.7	2	0.2	1	0.5	1	1.0	2	0.6	1	0.6	1	3.5	0.8	3	1.4	4	0.7	2	1.0	2	0.2	1	0.4	1	4.5
			2	2.5	5	0.7	2	1.0	2	0.5	2	1.3	2	1.0	2	7.0	3.4	8	1.7	3	0.7	2	1.2	3	1.3	2	1.0	2	9.3
			3	2.1	5	1.3	4	1.1	2	1.0	2	1.6	2	1.0	2	8.0	1.6	4	1.3	3	1.4	2	0.7	2	1.6	2	1.1	2	7.7
			4	0.8	2	0.8	3	0.9	2	1.0	2	0.9	2	0.7	2	5.2	1.5	4	2.8	6	1.1	2	1.0	2	0.8	2	1.3	2	8.5
			5	2.1	4	0.3	2	0.0	1	0.7	2	0.7	2	0.0	1	3.8	1.0	3	0.6	2	0.5	2	0.3	1	0.7	2	0.1	1	3.3
		Avr.		1.6		0.7		0.7		0.8		1.0		0.7			1.7		1.5		0.9		0.9		0.9		0.8		
	P2	NIT	1	2.8	6	2.4	5	1.6	4	1.1	2	1.1	2	1.3	2	10.3	3.1	6	2.6	4	1.8	4	1.9	3	2.0	3	1.0	2	12.4
			2	3.2	7	0.9	3	0.8	2	0.6	2	1.3	2	1.9	2	8.7	3.6	7	2.8	6	1.3	3	2.2	3	1.6	2	1.4	3	12.9
			3	1.1	3	1.8	5	2.0	3	2.1	3	2.1	3	1.5	2	10.5	1.9	4	3.6	7	1.6	2	1.7	3	1.7	2	1.1	2	11.5
			4	2.2	4	1.4	3	1.8	4	2.2	4	1.3	2	1.3	2	10.3	2.1	4	3.1	5	1.2	2	2.4	4	1.3	2	2.2	3	12.3
			5	1.4	3	1.2	3	0.3	2	0.5	1	0.7	2	0.0	1	4.1	1.5	3	2.1	5	0.4	1	0.7	2	0.7	2	0.0	1	5.4
		Avr.		2.1		1.6		1.3		1.3		1.3		1.2			2.4		2.8		1.3		1.8		1.5		1.1		
		IT	1	0.9	2	0.4	2	0.7	2	1.4	2	1.1	2	0.7	2	5.2	1.5	2	2.5	4	0.6	2	1.9	3	2.0	3	0.9	2	9.2
			2	0.3	2	0.9	2	0.8	2	0.5	2	1.3	2	1.0	2	4.9	1.3	3	2.5	5	0.5	2	1.2	3	1.3	2	1.0	2	7.8
			3	0.4	2	0.2	2	1.1	2	0.7	2	1.6	2	1.0	2	5.0	2.5	4	1.2	4	1.1	2	0.9	2	1.7	2	1.0	2	8.3
			4	0.4	2	0.6	2	1.1	2	1.2	2	0.8	2	0.7	2	4.9	1.6	3	1.9	4	1.0	2	1.0	2	1.0	2	0.8	2	7.3
			5	0.9	2	0.4	2	0.4	2	0.3	1	0.5	1	0.6	1	3.1	1.0	2	0.6	2	0.6	2	0.4	2	0.4	1	0.0	1	3.0
		Avr.		0.6		0.5		0.8		0.8		1.1		0.8			1.6		1.7		0.8		1.1		1.3		0.7		

Appendix 29 Comparison of average mean and max. inventory levels obtained by TPA and RHA for random problems (GM-S1) with processing time 1 and 2

						# of Setup					
				CPU Time (sec.)		TPA			RHA		
			Day	TPA	RHA	M/C. 1	M/C. 2	Total	M/C. 1	M/C. 2	Total
S1	P1	NIT	1	31	4	2	6	8	6	3	9
			2	39	25	5	4	9	5	5	10
			3	70	9	2	5	7	4	7	11
			4	43	38	7	3	10	7	5	12
			5	16	15	5	3	8	7	5	12
		Avr.		40	18	4.2	4.2	8.4	5.8	5.0	10.8
		IT	1	49	21	5	4	9	5	3	8
			2	1,044	53	5	6	11	5	5	10
			3	116	19	4	5	9	6	4	10
			4	68	51	5	5	10	5	4	9
			5	10	18	5	3	8	6	5	11
		Avr.		257	32	4.8	4.6	9.4	5.4	4.2	9.6
	P2	NIT	1	46	7	5	2	7	3	6	9
			2	39	26	2	5	7	6	4	10
			3	25	9	4	2	6	7	2	9
			4	33	24	7	3	10	7	5	12
			5	8	7	4	2	6	4	6	10
		Avr.		30	15	4.4	2.8	7.2	5.4	4.6	10.0
		IT	1	31	12	5	3	8	5	3	8
			2	175	10	7	5	12	5	4	9
			3	117	32	2	5	7	4	6	10
			4	247	22	5	4	9	4	6	10
			5	11	10	3	4	7	3	4	7
		Avr.		116	17	4.4	4.2	8.6	4.2	4.6	8.8

Appendix 30 Comparison of CPU time and number of setups obtained by TPA and RHA for random problems (GM-S2) with processing time 1 and 2

				CS & EI																							
				TPA												RHA											
				CS (Type)						EI (Type)						CS (Type)						EI (Type)					
			Day	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5	0	1	2	3	4	5
S1	P1	NIT	1	1	1	1	1	1	1	0	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
			2	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
			3	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
			4	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
			5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
		Avr.		1.0	1.0	1.0	1.0	1.0	1.0	0.0	0.2	0.0	0.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
		IT	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
			3	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
			4	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
			5	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
		Avr.		1.0	1.0	1.0	1.0	1.0	1.0	0.6	0.6	0.4	0.6	0.6	0.6	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.8	1.0	1.0	1.0	1.0
	P2	NIT	1	2	2	2	2	2	2	0	2	0	1	0	0	2	2	2	2	2	2	2	2	2	2	2	2
			2	2	2	2	2	2	2	0	0	1	0	0	0	2	2	2	2	2	2	2	2	2	2	1	2
			3	2	2	2	2	2	2	1	1	0	1	0	0	2	2	2	2	2	2	2	2	2	2	2	2
			4	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0
			5	2	2	2	2	2	2	1	0	0	0	1	0	2	2	2	2	2	2	2	2	2	2	2	2
		Avr.		1.8	1.8	1.8	1.8	1.8	1.8	0.4	0.6	0.2	0.4	0.2	0.0	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.6	1.6
		IT	1	2	2	2	2	2	2	1	2	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
			2	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
			3	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
			4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
			5	2	2	2	2	2	2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2
		Avr.		1.8	1.8	1.8	1.8	1.8	1.8	1.0	1.2	1.0	1.0	1.0	1.0	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.6

Appendix 31 Comparison of contingency stock & ending inventory levels obtained by TPA and RHA for random problems (GM-S2) with processing time 1 and 2

				Average Mean and Max. Inventory Levels																									
				TPA													RHA												
				Type 0		Type 1		Type 2		Type 3		Type 4		Type 5			Type 0		Type 1		Type 2		Type 3		Type 4		Type 5		Total
			Day	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Total	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	Mn.	Mx.	
S1	P1	NIT	1	1.25	3	2.12	3	2.00	4	0.63	2	1.07	3	0.53	1	7.60	1.70	3	1.68	3	2.09	4	1.25	2	2.63	5	1.40	3	10.75
			2	1.15	3	0.58	1	0.96	3	1.96	4	0.76	2	0.98	4	6.38	2.19	4	0.96	2	1.58	4	2.22	4	1.38	2	1.45	4	9.77
			3	0.92	3	1.14	2	1.67	5	1.38	3	1.85	5	0.08	1	7.04	1.48	4	1.49	3	2.07	4	2.17	4	1.44	4	0.90	2	9.55
			4	0.85	2	1.11	4	0.73	2	1.43	3	0.36	3	1.48	3	5.96	1.23	2	1.40	4	1.71	3	2.69	5	1.09	2	2.05	3	10.16
			5	0.22	1	0.57	2	0.89	2	1.39	3	1.47	3	1.43	3	5.97	1.06	2	1.46	3	1.28	2	1.54	3	2.15	4	1.77	3	9.26
		Avr.		0.9		1.1		1.2		1.4		1.1		0.9		6.6	1.5		1.4		1.7		2.0		1.7		1.5		9.9
		IT	1	0.72	1	0.87	1	0.79	2	0.77	2	0.48	2	0.60	2	4.23	1.08	2	0.87	1	0.79	2	0.62	1	0.38	1	0.58	2	4.30
			2	0.70	2	0.89	2	1.05	3	0.73	2	0.90	1	0.23	1	4.50	0.98	2	0.03	1	0.76	1	0.34	1	1.22	3	0.34	1	3.66
			3	0.98	2	1.25	3	0.80	2	0.28	1	0.27	2	0.68	1	4.26	0.89	2	0.07	1	0.72	2	0.26	1	0.58	3	0.68	1	3.20
			4	0.09	1	0.36	2	0.38	1	0.36	1	0.15	1	0.25	2	1.59	0.36	2	0.94	3	0.46	1	1.50	3	0.80	1	0.25	2	4.30
			5	0.22	1	0.10	1	0.33	1	0.30	1	0.50	1	0.61	2	2.05	0.22	1	0.65	1	0.68	2	0.27	2	0.66	2	0.83	2	3.31
		Avr.		0.5		0.7		0.7		0.5		0.5		0.5		3.3	0.7		0.5		0.7		0.6		0.7		0.5		3.8
	P2	NIT	1	1.33	3	2.77	4	1.73	4	1.51	2	2.16	4	0.85	2	10.35	2.35	3	2.13	3	3.00	5	2.34	3	1.68	4	2.37	3	13.87
			2	1.07	3	0.58	2	2.13	4	2.42	5	0.80	2	1.26	4	8.25	2.18	4	2.02	3	1.89	3	2.38	5	1.39	2	1.80	4	11.65
			3	1.68	3	2.40	3	1.73	5	1.87	4	2.01	5	0.25	2	9.95	2.44	4	2.03	3	2.40	4	1.68	3	2.48	5	1.96	3	12.98
			4	0.50	1	0.68	2	0.55	2	0.66	3	0.28	2	1.31	2	3.98	1.15	2	1.31	3	1.48	3	1.33	3	0.68	2	0.87	2	6.82
			5	1.22	2	0.63	2	1.06	2	1.53	3	2.77	4	1.43	3	8.62	1.93	2	2.00	3	2.42	3	2.10	4	2.83	4	2.47	4	13.75
		Avr.		1.2		1.4		1.4		1.6		1.6		1.0		8.2	2.0		1.9		2.2		2.0		1.8		1.9		11.8
		IT	1	1.28	2	1.88	2	0.68	2	1.00	2	0.27	2	0.50	2	5.60	1.63	3	2.26	4	1.80	4	1.72	3	0.43	3	1.90	3	9.74
			2	0.78	2	0.70	2	1.35	2	0.71	2	0.73	2	0.70	2	4.97	2.39	5	0.80	2	1.39	2	1.23	3	0.68	3	2.27	4	8.75
			3	1.41	2	0.99	2	1.26	2	0.73	2	0.68	2	0.78	2	5.84	1.52	3	1.78	3	1.58	3	1.37	2	0.97	3	1.85	3	9.07
			4	0.67	2	0.56	2	0.51	1	1.09	2	0.82	1	0.80	3	4.44	0.85	2	0.20	1	0.61	2	0.49	1	0.81	1	0.50	2	3.46
			5	1.22	2	0.75	2	0.73	2	0.70	2	1.05	2	0.80	2	5.25	2.03	3	1.81	3	1.58	3	0.56	3	1.11	2	1.10	3	8.20
		Avr.		1.1		1.0		0.9		0.8		0.7		0.7		5.2	1.7		1.4		1.4		1.1		0.8		1.5		7.8

Appendix 32 Comparison of average mean and max inventory levels obtained by TPA and RHA for random problems (GM-S2) with processing time 1 and 2