SEVENTH GRADE STUDENTS' CONCEPTUAL AND PROCEDURAL UNDERSTANDING OF FRACTIONS: COMPARISON BETWEEN SUCCESSFUL AND LESS SUCCESSFUL STUDENTS

A MASTER'S THESIS

BY

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

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ABSTRACT

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The aim of the study was to find the correlation between conceptual knowledge (CK) and procedural knowledge (PK) and the difficulties that less successful students have regarding fractions.

The study was conducted with a mixed-methods approach using explanatory design which consisted of two phases. In the first phase of the study, a conceptual and procedural knowledge test (CPKT) was administered to 33 seventh grade students. In the second phase, interviews were conducted with two successful students (ST33 and ST24) and two less successful students (ST01 and ST03).

The results of the CPKT showed a strong positive correlation between students' conceptual knowledge (CK) and procedural knowledge (PK), $r = 0.66$ ($p < .01$). In addition, it was found that students' school mathematics grade (mathematics GPA:

grade point average) at the end of the sixth year was strongly related to both conceptual and procedural knowledge. In the second phase, the interview results showed that while successful students had combined conceptual and procedural knowledge, less successful students had orphaned procedural knowledge. It was concluded that students can benefit from having both conceptual and procedural knowledge in order to develop a good knowledge base in mathematics.

Key words: Conceptual knowledge, procedural knowledge, combined conceptual and procedural knowledge, orphaned procedural knowledge, fractions

ÖZET

7. SINIF ÖĞRENCİLERİNİN KESİRLER KONUSUNDA KAVRAMSAL VE İŞLEMSEL ANLAYIŞLARI

Şakire Örmeci

Yüksek Lisans, Eğitim Programları ve Öğretim

Tez Yöneticisi: Yrd. Doç. Dr. Minkee Kim

Mayıs 2012

Bu araştırmanın esas amacı başarılı ve daha az başarılı öğrencilerin kesirler konusunda kavramsal ve işlemsel bilgilerini karşılaştırmaktır. Araştırmada ayrıca kavramsal bilgi ile işlemsel bilgi arasında ilişki olup olmadığı ve daha az başarılı öğrencilerin kesirler konusunda yaşadıkları zorluklar incelenmiştir.

Bu çalışmada iki basamaklı açıklayıcı karma araştırma metodu kullanılmıştır. Araştırmanın birinci basamağında, toplam 33 yedinci sınıf öğrencisine kavramsal ve işlemsel bilgi testi uygulanmıştır. Araştırmanın ikinci aşamasında ise iki tane başarılı (ST33 ve ST24) ve iki tane daha az başarılı öğrenci (ST01 ve ST03) ile mülakat yapılmıştır.

Kavramsal ve işlemsel bilgi testinin sonuçları, kavramsal ve işlemsel bilgi arasında pozitif bir korelasyon olduğunu göstermiştir. Ayrıca öğrencilerin 6. sınıf yıl sonu matematik notu ile hem kavramsal hem de işlemsel bilgileri arasında pozitif yönde

bir ilişki bulunmuştur. Kavramsal ve işlemsel bilgi testini takiben yapılan mülakatlarda sonucunda başarılı öğrencilerin kesirler konusunda kavramsal ve işlemsel bilgilerinin birleşik olduğu bulunurken daha az başarılı öğrencilerin salt işlemsel bilgiye sahip oldukları tespit edilmiştir. Araştırmanın bütün bu sonuçları gösteriyor ki öğrencilerin matematiksel konuları tam anlamıyla öğrenebilmeleri için kavramsal ve işlemsel bilginin her ikisine sahip olmaları gerekir.

Anahtar kelimeler: Kavramsal bilgi, işlemsel bilgi, birleşik kavramsal ve işlemsel bilgi, salt işlemsel bilgi, kesirler.

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CHAPTER 1: INTRODUCTION

Introduction

Many primary and middle school students have difficulty understanding certain mathematical topics and problem solving in these topics. In order to understand students' learning mathematical concepts, many researchers have analyzed how students acquire mathematical knowledge (Hallett, Nunes, & Bryant, 2010; Hiebert & Lefevre, 1986; Rittle-Johnson & Alibali, 1999). These studies have focused on two main types of knowledge: conceptual knowledge and procedural knowledge.

One of the problematic topics for primary school students is the concept of fractions. Students have difficulties with the concept of fractions and the operations related to them (İpek, Işık, & Albayrak, 2010). Hence, this study analyzed successful and less successful seventh grade students' conceptual and procedural understanding of fractions.

Background

Conceptual knowledge and procedural knowledge

For three decades, conceptual and procedural knowledge has been a popular educational research topic in literature. The research on this topic has attempted to answer the question of how students acquire mathematical knowledge (Hiebert & Lefevre, 1986). Since Hiebert and Lefevre (1986) defined conceptual knowledge and procedural knowledge, these two types of knowledge have been accepted as the main types of knowledge by educators and researchers. Especially in mathematics and science, conceptual and procedural knowledge constitute essential parts of student understanding and the learning of mathematical and scientific concepts (Haapasalo, 2003; Heyworth, 1999; Mccormick, 1997; Rittle-Johnson & Alibali, 1999; Schneider & Stern, 2005). Conceptual and procedural knowledge are interrelated (Baki & Kartal, 2004; Hiebert & Lefevre, 1986). Hence, for mathematical competency, students should iteratively develop both conceptual and procedural knowledge (Birgin & Gürbüz, 2009; Rittle-Johnson & Alibali, 1999). However, due to the nature of mathematics, the conceptual aspect of mathematical concepts is sometimes ignored and the procedural aspects are highlighted during instruction. This often leads to rote-learning of procedures without understanding their meanings.

Recently, conceptual and procedural knowledge have become a popular concern in Turkish education. There are studies investigating the conceptual and procedural achievements of Turkish students at different grade levels (Aksu, 1997; Birgin & Gürbüz, 2009), at the high school level (Baki & Kartal, 2004) and at the university level (İpek et al., 2010; Soylu & Aydın, 2006). The results of these studies show that Turkish students from different grade levels have gained predominantly procedural knowledge and inadequate conceptual knowledge. In addition, these studies attempt to explain the reason for students' failures in mathematics regarding their inadequate conceptual knowledge (Aksu, 1997; Baki & Kartal, 2004; Soylu & Aydın, 2006). In 2004, Turkey made changes in the national curricula from being teacher-centered towards being student-centered. The earlier educational system in Turkey emphasized procedural knowledge and mostly skipped the conceptual knowledge of mathematical topics; hence this led students not to comprehend topics deeply but to learn by rote. On the other hand, the new curricula has a conceptual approach to provide students to comprehend mathematical topics by using their experience (MoNE, 2005). In addition, new curricula with the constructivist approach aims for students to acquire conceptual knowledge and then link it with procedural knowledge in order to communicate with mathematical language (Alkan, 2008). Despite the reform of the Turkish national curricula, university and high school entrance exams with multiple choice questions cause students to learn by rote and to apply certain procedures rather than to conceptualize the topics. This stands as a handicap for the Turkish educational system (Argün, Arıkan, Bulut, & Sriraman, 2010).

Fractions in mathematics education

Fractions are a fundamental topic in mathematics education and constitutes a basis for several topics, such as rational numbers, ratio and measurements (İpek et al., 2010). Fractions are also one of the complicated mathematical topics that often produce difficulty with students (Aksu, 1997; Alacaci, 2010; İpek et al., 2010). In addition students deal with fractions through their education life from primary education. Hence teaching fractions holds an important place in mathematics education.

In Turkey, fractions are taught from the first grade to the eighth grade with appropriate difficulties for each grade level (MoNE, 2009a, 2009b). In the Turkish mathematics curriculum, fractions are introduced with part-whole meaning and different sub-concepts (e.g. compound fraction, proper fraction) are taught to students. The topic is also related to topics such as ratio, decimal numbers, percentage, and is generalized to the topic of rational numbers from primary school to secondary school (Alacaci, 2010).

Problem

The literature shows that there are many aspects to be addressed regarding students' conceptual and procedural understanding of fractions. Some of these studies focused on the acquisition of conceptual and procedural knowledge with an experimental approach (Hallett et al., 2010; Schneider & Stern, 2005), some focus on pre-service teachers' conceptual and procedural knowledge of the fractions (İpek et al., 2010; Rayner, Pitsolantis, & Osana, 2009), and others measured students' conceptual and procedural knowledge levels in fractions (Aksu, 1997; Birgin & Gürbüz, 2009). These studies pointed out the general picture rather than analyzing students' conceptual and procedural understanding deeply with regard to a mathematical topic such as fractions. Although there are studies that examine students' understanding of fractions (Mitchell & Clarke, 2004; Wong & Evans, 2007), there is little descriptive research on successful and less successful students' conceptual and procedural understanding of fractions.

Purpose

The main purpose of this study was to compare successful and less successful seventh grade students with respect to their conceptual and procedural understanding of fractions. As a mixed-methods study, firstly this study determined students' conceptual knowledge and procedural knowledge of fractions with a paper-pencil test. This test revealed information about the correlation between students' conceptual knowledge and procedural knowledge of the topic of fractions and it was used to determine successful and less successful students. Secondly, to examine in detail the differences and similarities between successful and less successful students' usage of conceptual and procedural knowledge of fractions, the study conducted a follow-up interview with four students.

Research questions

The main question of the study: What are the differences and similarities between successful and less successful seventh grade students with respect to their conceptual and procedural understanding of fractions?

In the light of the main question, the sub-questions being examined were:

1. What is the relative strength of conceptual knowledge (CK) and procedural knowledge (PK) in fractions among participating students?
2. Is there any relation between conceptual knowledge (CK) and procedural knowledge (PK)?
3. What kinds of difficulties do less successful students have about the concept of fractions?

Significance

Examining conceptual knowledge and procedural knowledge of successful and less successful seventh grade students in the domain of fractions could provide valuable information about the achievement of successful students and the inefficiency of less successful students with the topic of fractions. Hence, this study can aid educators and researchers to resolve problems that middle school students encounter when they learn fractions.

Definitions of key terms

Conceptual knowledge (CK) is defined by Hiebert and Lefevre (1986) as "a connected web of knowledge, a network in which the linking relationships are as crucial as the discrete pieces of information" (p.3) and stated that the main characteristic of conceptual knowledge is "being rich in relationships" (p. 4). In line with this, Rittle-Johnson and Alibali (1999) described conceptual knowledge as "explicit or implicit understanding of principles that govern the interrelations between pieces of knowledge in a domain" (p.175).

Procedural knowledge (PK) is characterized as knowledge that consists of two parts: "the official language or symbol representation system of mathematics", and "the algorithms or rules for completing mathematical tasks" (Hibert & Lefevre, 1986, p.6).

CHAPTER 2: REVIEW OF LITERATURE

Introduction

To analyze students' conceptual and procedural knowledge of fractions, this literature review gives information about the relation between conceptual and procedural knowledge, characterization of conceptual and procedural knowledge, conceptual and procedural aspects of learning fractions, and learning difficulties and misconceptions about fractions.

Relationships between conceptual and procedural knowledge

For three decades, many mathematics educators have examined the relation between conceptual knowledge and procedural knowledge to understand how students' learning processes develop. There are different views related to the primacy of conceptual knowledge and procedural knowledge while children learn mathematics. Rittle-Johnson and Siegler (1998) described theories about acquiring conceptual and procedural knowledge with regard to their direction: concepts-first, procedure-first, and iterative model.

Concept-first view

Rittle-Johnson and Siegler (1998) proposed the concept-first theory. According to this theory, students firstly acquire conceptual knowledge then derive procedural knowledge from it. In other words conceptual knowledge comes before procedural knowledge. Schneider and Stern (2005) explained this theory by saying "students firstly listen to verbal statements of the concept and then by practicing, derive procedural knowledge." (p. 1)

Procedure-first view

On the other hand, procedure-first theory suggests that students initially learn procedural knowledge than gradually acquire conceptual knowledge (Schneider & Stern, 2005). In addition, Rittle-Johnson and Siegler (1998) explained the procedure-first theory that "procedural knowledge develops before conceptual knowledge" (p.77). For instance, students firstly learn procedural rules and by trial and error methods then practice the procedure a few times and obtain insight about the meaning of the topic which is conceptual knowledge.

Iterative model

Another theory suggested by Rittle-Johnson and Siegler (1998) is that conceptual knowledge and procedural knowledge develop iteratively and increasing in one kind of knowledge leads to increasing the other. Schneider and Stern (2005) later called this method the iterative model that has become the most common view. In this light, Silver (1986) claims that procedural knowledge can be quite limited unless it is connected to conceptual knowledge. Similarly, Schneider and Stern (2005) state that there may be bi-directional relations between conceptual and procedural knowledge after children have prior conceptual knowledge to learn new procedural and conceptual knowledge iteratively. For example, when a student comprehends what a fraction means, she/he can solve operational problems easily and then learn new procedural knowledge such as division of fractions. Learning new procedural knowledge also helps students develop conceptual knowledge. In other words, gaining one type of knowledge (conceptual or procedural) in a topic leads to gaining the other type of knowledge (procedural or conceptual).

Individual differences

Recently, Hallett et al. (2010) suggest that the variety of different views about the relationship between conceptual and procedural knowledge is due to individual differences in learning conceptual and procedural knowledge. They propose that children may have different learning profiles with regard to connecting conceptual knowledge and procedural knowledge. Therefore to understand what affects the learning style, such as developmental processes, facilities or interest, Hallett et al. ask the question: How could there be a child with a predominantly procedural profile if concepts are supposed to be learned first or vice versa? With the help of cluster-analysis technique, they concluded that there were subgroups of children who used conceptual and procedural knowledge differently to solve fraction problems.

The literature has revealed that a mathematical topic is learned by the help of both conceptual knowledge and procedural knowledge. Conceptual knowledge helps students learn a topic by the construction of rich links between pieces of previous knowledge and subtopics (Hiebert & Lefevre, 1986). Students need to understand the meaning of concepts before learning the related algorithms in order to internalize the procedural knowledge of the topic (Saenz-Ludlow, 1995). In other words, the conceptual knowledge leads students to know the meaning of the procedures and solve mathematical problems in a logical way rather than by rote. Therefore, it can be inferred that conceptual knowledge is important in helping students to construct understanding of a topic and also its related procedures.

Characterization scale for conceptual and procedural knowledge

Baki and Kartal (2004) developed a scale for characterization of conceptual and procedural knowledge in the light of definitions and classification of conceptual and procedural knowledge in the literature. Although this scale was developed to evaluate students' answer about the topic algebra, it can be applied to other mathematical topics such as fractions. The criteria consist of general features of conceptual and procedural knowledge. Table 1 presents this characterization scale:

Table 1
Characterization scale for conceptual knowledge, procedural knowledge, and coupled conceptual and procedural knowledge (Baki & Kartal, 2004)

Knowledge type	Characterization Criteria
Procedural knowledge (P)	P1.Solving procedures step by step P2.Using mathematical knowledge which was learnt before (theorem, definitions, property, rules) in the level of knowledge (the bottom level of Bloom's taxonomy) P3.Being able to use algebraic relations and to conduct basic procedures
Conceptual knowledge (C)	C1.Knowing basic concepts and meaning of these concepts C2.Finding solution way by grasping the meaning of question and correlating given information with intended result C3.Using mathematics knowledge which was learnt before (theorems, definitions, and postulates) in the level of comprehension and application C4.By perceiving question as whole, appreciating hints properly and relevantly C5.Dividing problem into easy sub-steps C6.Making generalization and drawing shape and figures to back up a complex and hard problem C7.Matching the problem with given figure and graph C8.Matching problem with the features of this problem after determining these features
Coupled conceptual and procedural knowledge (C-P)	C-P1.Understanding, using, writing, retrenchment, and simplifying the symbols and statements which constitute of language of mathematic C-P2.Solving equation after converting problem into equation and checking rationality of solutions. C-P3.Converting given relation into another relation by associating these relations among themselves.

Fractions in mathematics

Fractions is the topic that presents a framework for many topics in secondary school, high school mathematics, and even further mathematics concepts such as rational numbers, ratio, measurement of quantities, and algebraic fractions. Thus fractions have an important place in mathematics education (Alacaci, 2010; İpek et al., 2010). In addition to being the framework of many mathematical topics, fractions are among the most complex and rich mathematical concepts that students encounter in primary education (Alacaci, 2010; Charalambous & Pitta-Pantazi, 2007). There is a consensus on the reason for the complexities of learning fractions which states that fractions have an interrelated construct (Kieren, 1993; Lamon, 1999). The interrelated sub-constructs of fractions are identified as the five meanings of fractions: part-whole meaning, ratio meaning, operator meaning, quotient meaning, and measure meaning. These meanings are represented in four different models: region, area, number line and set (Alacaci, 2010; Charalambous & Pitta-Pantazi, 2007; Lamon, 1999).

Meanings of fractions

Part-whole meaning

Part-whole meaning of a fraction is defined as a situation where the fraction represents one or more parts of a quantity or a set of objects that is partitioned into equal parts (Lamon, 1999). Hence the denominator of a fraction expresses the whole and how many parts it was divided. In addition, part-whole meaning is the most used meaning in teaching fractions and presents a base for other meanings of fractions (Alacaci, 2010; Charalambous & Pitta-Pantazi, 2007). The most important feature of part-whole meaning is equal partitioning so that students can comprehend sufficiently this meaning and its features to learn and not to confuse this with other meanings.

Ratio meaning

Lamon (1999) defines the ratio as the comparison of two quantities of the same type. For example, comparison of the number of girls in a class to the number of boys represents a ratio. In addition it is not necessary for comparison parts to represent together the whole (Alacaci, 2010). Ratio meaning comprises a conceptual base for equivalent fractions.

Operator meaning

Operator meaning of fractions is regarded as shrinker or stretcher; duplicator or partitioning quantities (Alacaci, 2010; Charalambous & Pitta-Pantazi, 2007). For instance, the question “after shrinking a number by $\frac{1}{4}$, which fraction should be multiplied by the new number to obtain original number?” is related to operator meaning. Charalambous and Pitta-Pantazi (2007) emphasized the conceptual relation between fraction multiplication and operator meaning of the fractions. Hence students can learn the fraction multiplication concept with operator meaning.

Quotient meaning

Quotient meaning of fractions mostly deals with sharing problems of one or more quantities to individuals or something (Alacaci, 2010; Kieren, 1993). Unlike part-whole meaning, within the quotient meanings two different measures are considered (Charalambous & Pitta-Pantazi, 2007). For example, three apples are shared among five people with the question: “How much apple does each person get?” represents the quotient meaning of fractions. In this meaning equal partitioning is important.

Measure meaning

Measure meaning represents measurement quantities such as length, area and volume that could not be defined by integers (Alacaci, 2010). For example, to explain altitudes above the sea, it can be said that the mountain has $1\frac{2}{5}$ km altitudes. This meaning helps students to accept fractions as a number and lets them gain insight about the addition of fractions (Alacaci, 2010; Charalambous & Pitta-Pantazi, 2007; Lamon, 1999).

Charalambous and Pitta-Pantazi (2007) suggested a model of the links between meanings of fractions and also sub-concepts and operations with fractions as shown in the Figure 1. The part-whole meaning provides base for other meanings and it is correlated with sub-concepts and operations of fractions through other meanings. Because of the interconnected structure of fractions, it is necessary to be careful and act responsibly during the teaching of fractions with all aspects of fractions should be taught gradually (Alacaci, 2010).

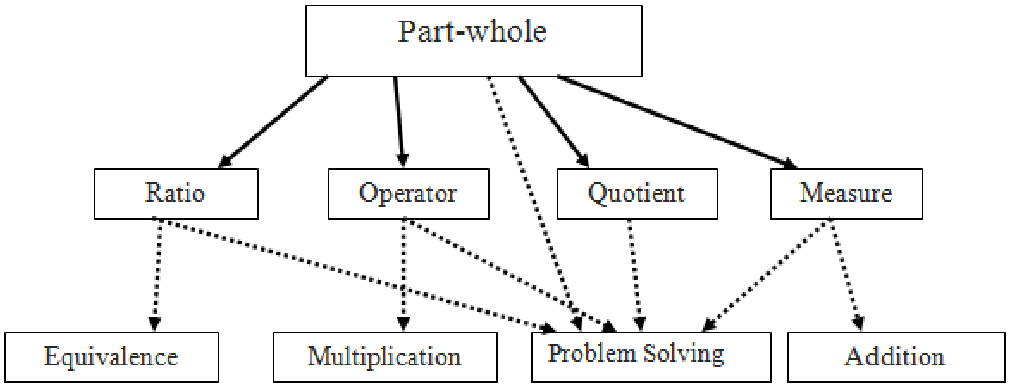

Figure 1. The relation between meanings of fractions and related fraction concept (Charalambous & Pitta-Pantazi, 2007)

Representation models of fractions

There are four types of representation models for fractions: region, area, number line, and set models. In some resource for fractions, region model and area model are accepted as same models (Petit, Laird, & Marsden, 2010), but region and area models differ to a point.

In the region model, the whole is partitioned into equal parts and chosen parts are highlighted (see A, Figure 2). The important feature of the region representation model is that parts should be equal in size and shape. This model is strongly related to the part-whole meaning of fractions and is commonly used for the representation model in the teaching of fractions (Alacaci, 2010).

The area representation model has a minor difference from the region model. This difference is that the parts should not be same shapes. However, the parts should have equal areas. Figure 2 B is an example of area representation and both lined area and highlighted area represent $\frac{1}{3}$ as fraction.

Furthermore, fractions are represented on number lines as numbers. Units on the number line are divided into equal lengths and so the location of points indicates the fraction (see Figure 2 C). This representation model is a useful model for teaching the measurement the meaning of fractions (Alacaci, 2010).

The last representation model is the set model. In this model, fractions are represented by a subset of a set of objects. The objects comprising the set cannot be divided into small parts since the parts represent the unit of the fraction (Alacaci, 2010; Lamon, 1999). The example for set representation model is Figure 2 D which represents the fraction $\frac{3}{8}$ which shows the comparison of lined triangles to all triangles. This model can be used to teach the ratio meaning of fractions (Alacaci, 2010; Petit et al., 2010). For example, in Figure 2, the ratio of lined triangles to highlighted triangles is $\frac{3}{5}$. Features of representation models for fractions are summarized in the Table 2.

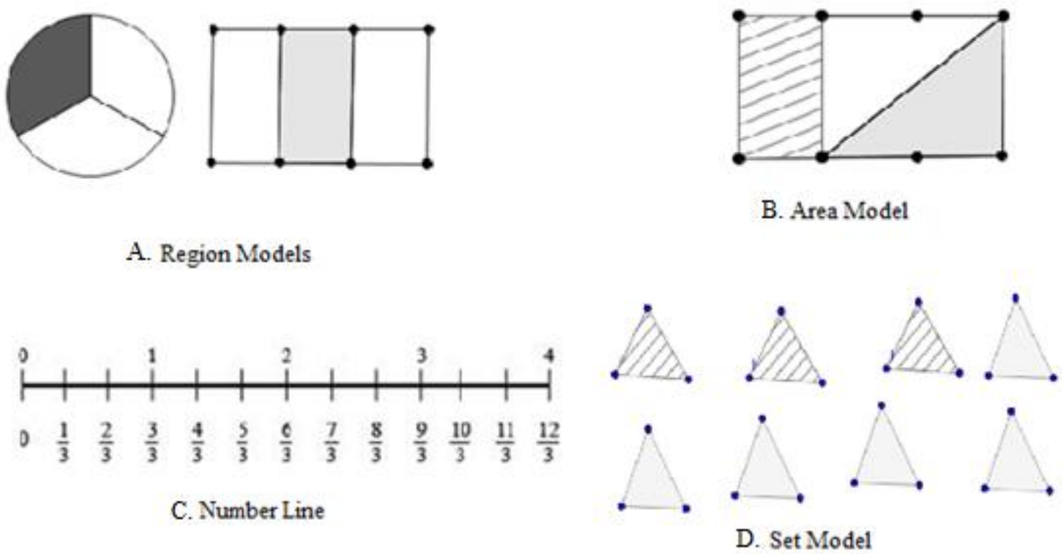

Figure 2. Examples for representation models of fractions

Table 2
Representation models of fractions (Alacaci, 2010; Petit et al., 2010)

	The Whole	"Equal parts" are defined by	What the fraction indicates
Region model	The whole is determined by total area of a defined region	Equal in size and shape	The part covered of whole unit of area
Area Model	The whole is determined by total area of defined region	Equal in size (area) not necessary equal in shape	The part covered of whole unit of area
Set model	The whole is determined by daefinition (set of objects)	Equal number of object	The count of objects in the subset of the defined set of objects
Number line	Unit of distance or length	Equal distance	The location of a point in relation to the distance from zero with regard to the defined unit

Conceptual knowledge and procedural knowledge of fractions

Various measurements in many studies indicate that students' procedural skills are higher than their conceptual competency in solving fraction problems (Aksu, 1997). Baki and Kartal (2004) argue that the reason for students' predominance procedural knowledge is learning fractions based on procedural knowledge. So that, most students tend to forget how to solve fraction problems after a while, even after years of practice. Lamon (1999) defends that the main reason for students having difficulty with fractions is the gaps in conceptual understanding of fractions. Hence to better understand students' problems with the topic of fractions, it is necessary to determine conceptual knowledge and procedural knowledge of fractions. Conceptual knowledge of fractions is mainly related to its meaning, the perception of proportion and whole numbers. On the other hand, procedural knowledge is related to computation with fractions and it depends on certain rules. For example, the rule for dividing fractions is inverting the second fraction and multiplying it with the first one. Hence students can easily solve procedural problems by using the relevant rules. Some conceptual and procedural problems about fractions that were used in previous research are given in the Table 3:

Table 3
Example questions for measuring conceptual and procedural knowledge

Conceptual Items:	**Procedural Items:**
In a whole or 1, how many thirds are there? How many times as much as ¼ is three quarters? Which is greater ½ or ¼ ? (Aksu, 1997)	5 + 3/8= ? 30* 3/5=? 1/8 – 1/9=? 7 ½ ÷1 ¼ =? (Aksu, 1997)
Circle the fraction CLOSER in size to 1/2: 5/8 or 1/5.	
Erin won 5/8 of the games he played; Pat won ¾; Val won 9/16; and Kelly won 2/3. Which of the players had the best record?	Circle the correct answer for: 5+ ½ + 0.5 a. It cannot be done; b. 5; c. 5.5; d. 6; e. 1 (Sowder et al., 1998)
Jane said that 12 ÷1/2 is 6, but Summy said no, it is 24. Which is right? How do you know? (Sowder, Philipp, Armstrong, & Schappelle, 1998)	One pizza has to be shared equally between 5 girls. What fraction of pizza does each girl get? (Hallett et al., 2010)

Learning difficulties and misconception in fractions

Although students have little difficulty in learning natural numbers, most of them are challenged with fractions and develop misconceptions because of its complex nature. There are many researches presenting reasons for difficulties and misconceptions about this topic of fractions. Alacaci (2010) mentioned that most of the misconceptions about fractions are caused from students generalizing the rules of integers to fractions. On the other hand, Amato (2005) stated that one of the reasons for students' learning difficulties about fractions was seeing fractions as only part of a shape or a whole not as a number. These are the general reason for difficulties and misconceptions about fractions. There are also studies about common misconceptions and difficulties that students have related to specific sub-concepts of fractions.

One of the misconceptions about fractions is related to the equal partitioning. Petit et al. (2010) state that when students were asked whether the shaded area as in the Figure 3 represents $\frac{1}{4}$ of whole shape or not, some of them answered that it represents by just focusing on the number of divided pieces. The reason of this misconception is that teaching fractions based on symbols (e.g., $\frac{3}{4}$) and rules rather than letting students comprehend conceptual knowledge of fractions (Alacaci, 2010; Litwiller & Bright, 2002).

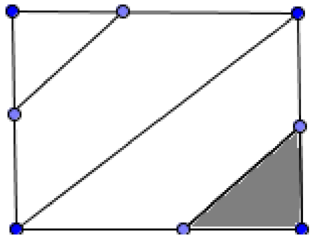

Figure 3. Is 1/4 of rectangle shaded?

Furthermore, students have difficulties comparing quantities of fractions since they are usually taught the concept of fractions related to the whole number concept (Mitchell & Clarke, 2004). For example, when students were asked which fraction was bigger, 1/3 or ¼, they often say that ¼ is bigger since they think as a natural number 4 is bigger than 3 (Nunes, Bryant, Hurry, & Pretzlik, 2006). Another misconception about the comparison of fractions is related to just focusing on denominator and ignoring numerator-denominator relation (Haser & Ubuz, 2000; McLeod & Newmarch, 2006). For instance, when students were asked to compare fractions $\frac{5}{8}$ and $\frac{1}{2}$, they say $\frac{1}{2}$ is bigger than $\frac{5}{8}$ since they think that a bigger denominator makes a fraction smaller and so ignored the relation between denominator and numerator (Alacaci, 2010).

Another misconception is incorrect fraction addition (Alacaci, 2010). In this misconception students add numerators and denominators between each other (e.g. $\frac{1}{2} + \frac{3}{4} = \frac{4}{6}$). The reason for this misconception involves the applying rules of integer (Haser & Ubuz, 2000; McLeod & Newmarch, 2006).

In addition, students have misconception about fractions, which arises from the ignoring reference whole. Many students consider that a fraction represents same quantity for different shapes (Alacaci, 2010). This shows that students do not have knowledge that the quantity represented by a fraction depends on the reference whole. For example, the fact that $\frac{1}{2}$ of a big pizza is bigger than $\frac{1}{2}$ of a small pizza is often misunderstood, see Figure 4.

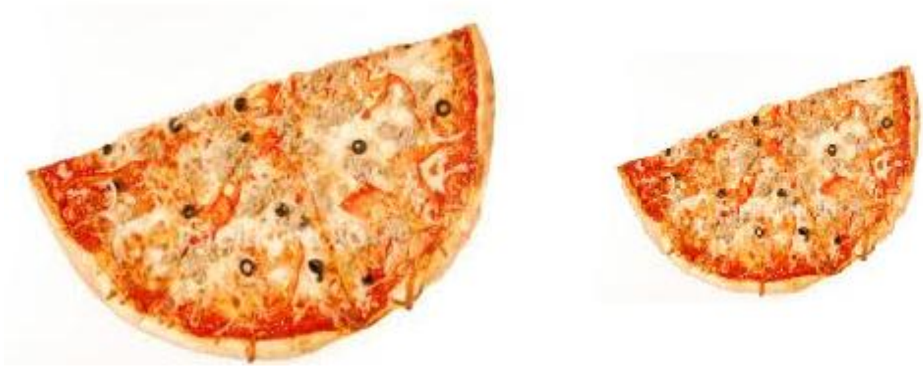

Figure 4. Half of a big pizza and half of a small pizza

The last type of students’ misconception is related to fraction division and multiplication (Aksu, 1997; Fredua-Kwarteng & Ahia, 2006). Despite the ease of applying rules for division and multiplication of fraction, students find it hard to explain why dividing a fraction produces a bigger value since dividing numbers creates smaller value for natural numbers. According to Liping (1999), division by fraction is the most complicated operation to understand during elementary school years. In his book, he compared Chinese and US teachers’ understanding of fraction division and performance on calculation, and found significant differences. His study indicated that US teachers’ conceptual understanding of fraction division is weaker than the Chinese teachers’. Furthermore, the study found that all of the Chinese teachers solved the same division problem correctly whereas only 43% of US teachers performed the correct calculation. This study indicates that not only students but also teachers have some difficulties about the division of fractions. Hence this variation shows that for total comprehension of a topic, it is necessary to learn the meaning of the concept and its related procedure.

Summary

Throughout this review, conceptual and procedural knowledge was found to form two main aspects of knowledge for the learning of mathematical topics. Conceptual knowledge is critical to the construct of fundamental knowledge of the topics by linking relevant mathematical concepts and to make meaningful the procedural knowledge. The topic in mathematics education, fractions, represents frameworks for various important concepts such as rational numbers, proportions, and decimal numbers. Many students have difficulties with the understanding of fractions. Especially, students find it challenging to understand the meaning of fractions, or conceptual knowledge of fractions because of its complex structure. The gap in the conceptual knowledge of fractions leads students to a restricted procedural knowledge so that they apply the procedures by rote without understanding their meanings.

CHAPTER 3: METHOD

Research design

This study was conducted with a mixed-methods approach using explanatory design with a follow-up explanation procedure (Creswell & Clark, 2007). Explanatory design consists of two sequential phases in the order of quantitative to qualitative. The follow-up procedure was applied while researcher used qualitative data to explain and expand the quantitative data collected in the first phase (Creswell & Clark, 2007). Consequently, this study was conducted into two phases; starting with a conventional paper-pencil test to collect quantitative data and continued with an interview to collect qualitative data. The conventional paper-pencil test was a useful instrument to analyze students' performance and to determine their level of success relative to other students (Heyworth, 1999). The follow-up interview provided deeper information about how students obtain answers for a specific question and their mental process (Heyworth, 1999). Hence, how students use conceptual knowledge (CK) and procedural knowledge (PK) in the paper-pencil test was analyzed in more detail in this current study.

Context

This study was conducted in a private secondary school in Ankara, Turkey with students of a seventh grade class. Since fractions are taught towards the end of the first semester in sixth grade, this study was applied to seventh grade students in the first semester of the school year.

Participants

This study was applied to seventh grade students and conducted with two randomly selected seventh grade classes sampling a total of 33 students. In order to provide for students' confidentiality and to perform data analysis without prejudice, a code was given to each student before the test. This had been explained to the students and the researcher wanted students to write the code on the test instead of their name. However, these codes were changed to make the findings understandable for readers. Hence students' codes are like ST01 to ST33 which were given according to students' mathematics GPA in ascending sort (e.g. ST01 with lowest GPA and ST33 with highest GPA).

Furthermore, the paper-pencil test was administered to all students in these classes. Results in conceptual knowledge and procedural knowledge test (CPKT) was used to select two successful students (ST24, ST33) who earned higher scores from both CK and PK tests and two less successful students (ST01, ST03) who received lower scores from both CK and PK tests for the follow-up interview. Parents of students participating in the study were informed and their permission was obtained via a letter (see Appendix D).

Instrumentation

Conceptual knowledge and procedural knowledge test (CPKT)

The paper-pencil test was developed by the researcher and contained conceptual and procedural questions to analyze students' conceptual and procedural understanding of fractions. The questions were prepared according to MoNE's objectives for fractions (MoNE, 2009b) and the related literature (MoNE, 2009c; Pesen, 2007;

Sowder et al., 1998; Van de Walle, 2007). In addition the test items were designed to find out the students' understanding of the seven areas in fractions: (1) meaning of fractions, (2) comparison fractions, (3) reference whole, (4) concept of addition and subtraction of fractions, (5) concept of fraction multiplication and division, (6) representation of fractions and (7) rules of operations. These contents are related to difficulties which students generally had and which were highlighted in previous studies (Alacaci, 2010; Charalambous & Pitta-Pantazi, 2007; Lamon, 1999).

In order to provide the validity and reliability of this test, the opinion of two secondary school teachers and one academician were collected. The expert opinion form was added in the appendix (see Appendix E). Necessary changes in the context and format of the test were conducted after consulting the expert and the teachers. The final version of the test contained 13 conceptual questions and 8 procedural questions (see Appendix B). This test was given to students during two lesson hours (totally 80 minutes) and observed by the researcher to prevent possible problems during the examination. Since the teacher of the classes emphasized the importance of the test at the beginning of the exam, it was observed that all students tried to solve test problems seriously.

Follow-up interview

The follow-up interview aimed to identify differences and similarities between successful and less successful students with regard to their understandings of fractions. With respect to the students' answers on the paper-pencil test, the interview questions were designed referring to the seven contents of fractions mentioned above (see Appendix C). The main questions utilized in the interview and their focus content are shown in Table 4.

Table 4
Interview protocols concerning the seven contents

Contents	Interview Question
Meaning of fractions	1 and 2
Comparison of fractions	3
Reference whole	4
Addition & subtraction	5
Fraction multiplication & division	6
Representation of fractions	7
Rules of operations	8

The questions were asked in order to find out students' conceptual and procedural understanding. Since the interview was semi-structured, extra questions rather than the ones in the Appendix C were asked to allow students to explain their understanding and to find out what they know and what they do not know. The extra questions depended on students' answers to main questions and related to the seven contents. Some of the questions were as follows:

- This is one of the examples for the meaning of fractions. According to you, is there any other meaning of fractions? Do you know?
- If you did not do any calculation, without finding common denominator, how could you have compared fractions?
- If I want to add these fractions and ask you add them together by modeling, what should we do?
- Why do we find the common denominator? Why is it necessary?
- What does fraction multiplication do to make number smaller?

Method of data collection

Background form was given to the students to collect demographic information just before the test (see Appendix A). In addition, another purpose of this form was to gather information about the students' general disposition towards mathematics and their overall mathematics GPA. However, most of the students did not remember their GPA. Hence students' mathematics GPAs of the previous year were found through their mathematics teachers.

In the first phase of study, students’ answers for the conceptual and procedural questions in CPKT were the initial data regarding students’ understanding of fractions. The data collected in the first phase was also provided a framework for the interview. Hence in the second phase, focusing on students’ verbal response and their understanding of the fraction concept was analyzed in detail.

Method of data analysis

In the first phase of the study, the students’ results on the conceptual and procedural knowledge test were determined by an answer key prepared by the researcher. The answer key was also checked and approved by the classroom teachers and was approved. Each question in the conceptual part of the test was graded using 0, 0.5, 1 points where the 0 point was an incorrect answer and wrong explanations, the 0.5 point was an incorrect answer with some reasonable correct explanations, and the 1 point was the correct answer with explanations. Similarly each procedural question was graded with 0 or 1 where 0 was the incorrect solution and 1 was the correct solution. Since there were 13 conceptual and eight procedural questions with further sub-questions, the conceptual part of the test was evaluated out of 20 and procedural part was evaluated out of 11.

In order to determine the students’ level of conceptual and procedural knowledge, z-scores were calculated for the students’ total scores in each part of the test. Correlation analysis was performed to determine whether there was a relationship between students’ conceptual knowledge and procedural knowledge. The analysis of the test was performed by the correlational statistics software SPSS package 18.

In the second phase of the study, the follow-up interview was conducted one week after the first phase. It was carried out with two successful (ST24, ST33) who received higher scores in both conceptual knowledge and procedural knowledge tests and two less successful students (ST01, ST03) who received lower scores from both parts of the test. Interviews were made in Turkish and recorded by a voice recorder. The researcher transcribed the interviews into English. The interview data was analyzed with respect to the seven areas of fractions (see instrumentations). After transcription, students' answers to the interview questions were coded according to knowledge types, for conceptual knowledge (CK), procedural knowledge (PK), coupled conceptual and procedural knowledge (C-P), and misconceptions (MC). Students' knowledge was determined by a characterization scale for conceptual and procedural knowledge which was developed by Baki and Kartal (2004) and students' misconceptions were determined by related literatures (Alacaci, 2010; Pesen, 2007) .

CHAPTER 4: RESULTS

Introduction

A conceptual knowledge and procedural knowledge test (CPKT) was given to 33 students from two seventh grade classes in a private school in Ankara, Turkey. The test items were designed to find out students' conceptual knowledge (CK) and procedural knowledge (PK) levels of fractions regarding seven contents of : meaning of fractions, comparison fractions, reference whole, the concept of addition and subtraction with fraction, the concept of fraction multiplication, and representation of fractions and rules of operations.

In the light of the research question and sub-questions, this study investigated during the follow-up interviews what the differences and similarities were between successful and less successful students' conceptual and procedural understandings of fractions, and what type of difficulties less successful students had about the concept of fractions.

Findings from CPKT

With regard to the research questions and the sub-questions, students' levels of CK and PK were determined in order to highlight their relationship. According to test results, students' levels were determined by z-scores of total CK (*M*=12.73, *SE*= 3.18) and PK (*M*=7.06, *SE*=2.54) scores. Table 5 indicates students' level of CK and PK, and last year's GPA in mathematics.

Table 5
Levels of students according to GPA, and z-scores of CK and PK

Students	GPA	Z-score of CK	Z-score of PK	Students	GPA	Z-score of CK	Z-score of PK
ST01	44.5	-2.38	-1.21	ST17	79	0.24	-0.02
ST04	54.7	-2.23	-1.60	ST18	79.1	0.24	-0.81
ST06	59.2	-1.61	-1.60	ST28	88.9	0.24	0.76
ST03	52.4	-1.46	-1.21	ST19	79.2	0.39	0.76
ST07	59.8	-1.15	-1.21	ST27	87.5	0.39	0.76
ST12	70.7	-0.84	-0.42	ST29	89.8	0.39	1.16
ST14	72.9	-0.84	-0.42	ST13	70.7	0.55	-0.81
ST05	55.8	-0.53	-1.21	ST15	74.1	0.55	-0.42
ST10	67	-0.38	0.37	ST16	78.4	0.55	-1.21
ST08	60.3	-0.22	1.55	ST24	84.7	0.70	0.76
ST26	86.2	-0.22	-0.42	ST30	90.1	0.70	0.76
ST32	94.9	-0.22	1.55	ST22	82.6	0.85	1.55
ST02	49.1	-0.07	0.37	ST33	95.9	1.16	0.37
ST09	63.1	-0.07	-1.21	ST20	79.7	1.32	0.37
ST21	81.8	-0.07	-0.42	ST25	84.8	1.78	1.16
ST23	84.6	0.08	0.37	ST31	91.4	1.93	1.55
ST11	67.1	0.24	-0.02				

The scatter plot of students' z-score in CK and PK presents a strong correlation between PK and CK (see Figure 5). According to Pearson test results, there was a big positive correlations between conceptual knowledge (CK) score and procedural knowledge (PK) score, $r = 0.66$ ($p < .01$). This shows that 43 % of variance in CK score was related to the variance in the PK score.

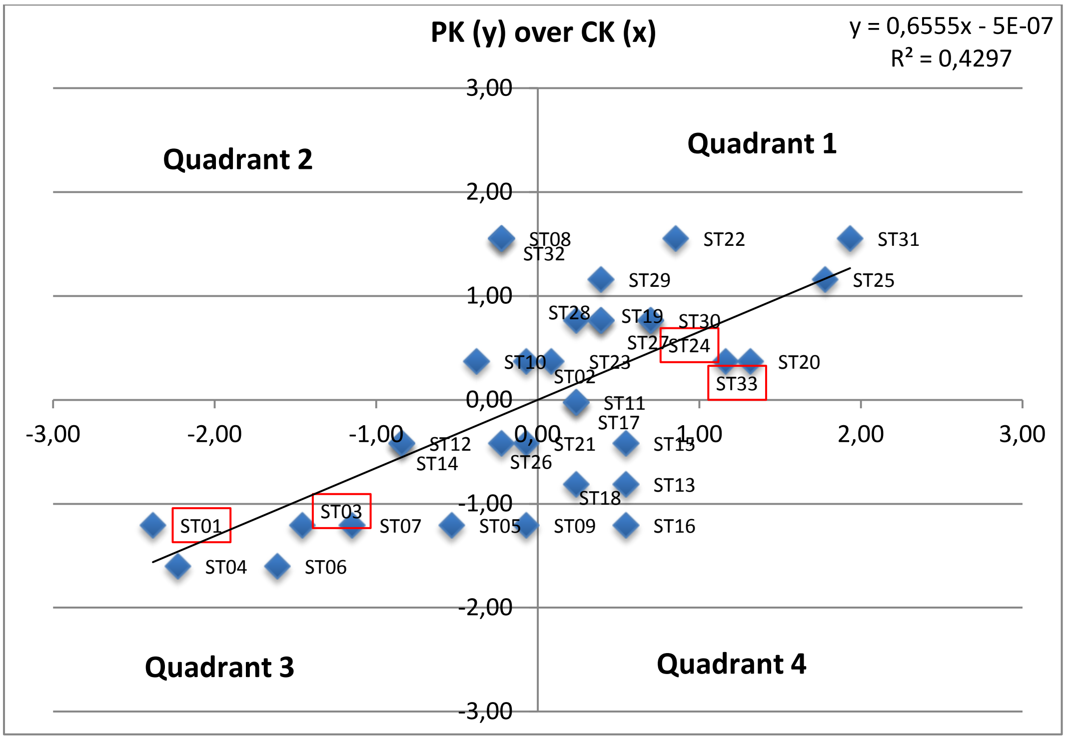

Figure 5. Z-scores scatter plots of procedural knowledge (PK) and conceptual knowledge (CK)

Furthermore, Figure 5 describes the information about distribution of students' performance in CPKT. The Figure 5 was analyzed into four quadrants; Quadrant 1, Quadrant 2, Quadrant 3 and Quadrant 4 (see Table 6). Without the students ST11 and ST17 which are on the border, the distribution of students' scores according to quadrants is summarized in Table 6. The distribution reveals that the majority of the students were in Quadrant 1 and Quadrant 3 which was the expected result of the Pearson test. That is, if a student was higher in one type of knowledge, she or he also tended to be higher in the other knowledge or vice versa.

Table 6
Four quadrants of the student distribution with regards to CK and PK

Quadrant	Student performance	Number of students
Quadrant 1	Having higher scores in both CK and PK	12
Quadrant 2	Having lower score in CK and higher score in PK	4
Quadrant 3	Having lower scores in both CK and PK	11
Quadrant 4	Having higher score in CK and lower score in PK	4

What to note about the quadrants and students' GPA in mathematics is that 11 of 12 students in Quadrant 1 had higher mathematics GPA scores and 9 of 11 students in Quadrant 3 had lower mathematics GPA scores. This distribution indicates that students' general achievement in mathematics was also related to students' competency in their conceptual knowledge and procedural knowledge. Furthermore, the Pearson correlation test also revealed that there was a big positive correlation between the GPA of mathematics and the CK score, $r = 0.73$ ($p < .01$). This correlation revealed that 53 % of the variance in GPA was related to the variance in the CK score. Similarly, there was a big positive correlation between the GPA of mathematics and the PK score, r =0 .832 ($p < .01$). It shows that 69% of variance in the GPA was related to the variance in the PK score.

Findings from follow-up interviews with target students

The successful students ST33 and ST24 were chosen from Q1 where the students were higher in both the conceptual knowledge and procedural knowledge parts of the test. The less successful students ST03 and ST01 were chosen from Q3 where students had lower in both parts of the test. Meanwhile, the successful students had higher GPA scores and less successful students had lower GPA scores in mathematics.

Students' profiles of conceptual knowledge and procedural knowledge regarding the content knowledge

Students' answers to interview questions were coded according to knowledge types, for conceptual knowledge (CK), procedural knowledge (PK) and coupled conceptual and procedural knowledge (C-P) and misconceptions (MC). Table 7 describes the students' understanding profiles to the contents of fractions.

Table 7
Student knowledge profiles regarding the content knowledge

Student	Know-ledge type	Contents in fraction							
		Meaning of fractions	Comparison of fraction	Reference whole	Addition & subtrac-tion	Multiplication & division	Representa tion	Rules of operation	Total
ST33	CK	X	X	X	X		X		5
	PK		X			X		X	3
	C-P	X			X				2
	MC			X					1
ST24	CK	X		X		X	X		4
	PK		X			X		X	3
	C-P	X			X				2
	MC		X	X					2
ST03	CK		X						1
	PK	X	X		X	X		X	5
	C-P	X							1
	MC	X		X			X		3
ST01	CK								0
	PK	X			X	X		X	4
	C-P								0
	MC	X		X					2

Note: X indicates existence of the knowledge.

Conceptual knowledge (CK). Understanding meanings of fractions represented in different contexts and expressing proportional reasoning (e.g. giving clear explanation to comparison fraction) are indicators of student's level of CK. For example, both successful students knew the part-whole meaning and its related features. They were aware of the fact that the fundamental feature of part-whole meaning was partitioning into equal parts. The following explanation of ST24 represents her correct CK about the part-whole meaning of fractions:

> **ST24:** I thought that $\frac{1}{4}$ pieces would be simplified or expanded for the questions like second one but I realized that it is really given $\frac{1}{4}$ in one question, in other one it is given as $\frac{1}{5}$. In addition, in other figures, the pieces were not equal. Since they are not equally shared, I understood that they are not $\frac{1}{4}$. In order to represent a fraction, a whole should be divided into equal parts (Appendix F, ST24-2).

In addition, ST33 and ST03 had the conceptual knowledge that fractions become smaller while their denominators increase and so they compared the fractions with the same numerator using this knowledge. They associated the increase in denominator with the decrease in the size of parts. ST03 explained that "$\frac{5}{6}$ was bigger than $\frac{5}{8}$. When it is divided by 6, the share is bigger." (Appendix F, ST03-18)

Procedural Knowledge (PK). Knowing rules and algorithms about fractions, knowing order of operations and solving problem based correct calculations demonstrates that a student has PK. For example, all students know the order of operations as a rule. ST24 explains the order of operation well by the following sentence:

> **ST24:** Firstly it is started to solve from the operation in the parenthesis, then since there is no multiplication or division in the parenthesis, subtraction is done. (Appendix F, ST24-51)

Although a few questions were asked to find out their CK about the content, some students gave answers depending on the rule without knowing the meaning of this rule. Hence this knowledge was also accepted as PK. When the reason of finding common denominator was asked, the less successful students stated the reason for finding the common denominator that "this is rule" and they needed to find it to add the fraction together. This indicated that the students' knowledge about addition is PK-based.

Coupled Conceptual and Procedural Knowledge (C-P). Using CK and PK simultaneously, and modeling an operational question are pointers of coupled CK and PK. When the reason for finding the common denominator in fraction addition and subtraction was asked to ST33, she explained the reason by modeling one addition problem. She associated her operation knowledge with models and shapes. This indicated that she had a CK of fraction addition and regarding her modeling example her CK is not separated from PK. Hence, ST33 had C-P about addition. The conversation between the researcher and ST33 is about the student' C-P:

> **ST33:** Yes I see. If I model for addition $\frac{1}{3}+\frac{1}{2}$, there are 3 parts and 1 part is taken, also in here there are 2 parts and 1 part is taken. I could not add them like this. Hence I should make parts equal.
> **Researcher:** What do you mean?
> **ST33:** We don't know how big this part is in that whole so we need to find common denominator and make parts equal.
> **Researcher:** Do you mean that we try to find equal size parts?
> **ST33:** Yes. (Appendix F, ST33-23:ST33-27)

Misconceptions (MC). During the interviews, it was observed that students developed several misconceptions about fraction contents such as the meaning of fraction, reference whole, and the representation of fractions. ST24's answer for the question of comparison fractions is an example of MC related to fractions. When the researcher asked students to compare fractions without calculations, it was seen that while comparing $\frac{4}{5}$ and $\frac{7}{8}$, ST24 just focused on denominators and ignored the numerator-denominator relation and compared the fractions incorrectly.

> **ST24:** $\frac{4}{5}$ is bigger one because dividing one pizza into 5 parts and taking 4 parts give bigger parts taking 7 parts from the pizza divided into 8 parts. (Appendix F, ST24-24)

Comparison between successful and less successful students' conceptual and procedural understanding of fractions

It is difficult to determine any straight forward differences and similarities between the successful and less successful students with respect their CK and PK understanding from Table 7. In addition since C-P contains both conceptual and procedural knowledge, the amount of C-P that are the components of students' PK and CK added to PK and CK. Therefore, the frequency of student PK and CK was presented in Figure 6.

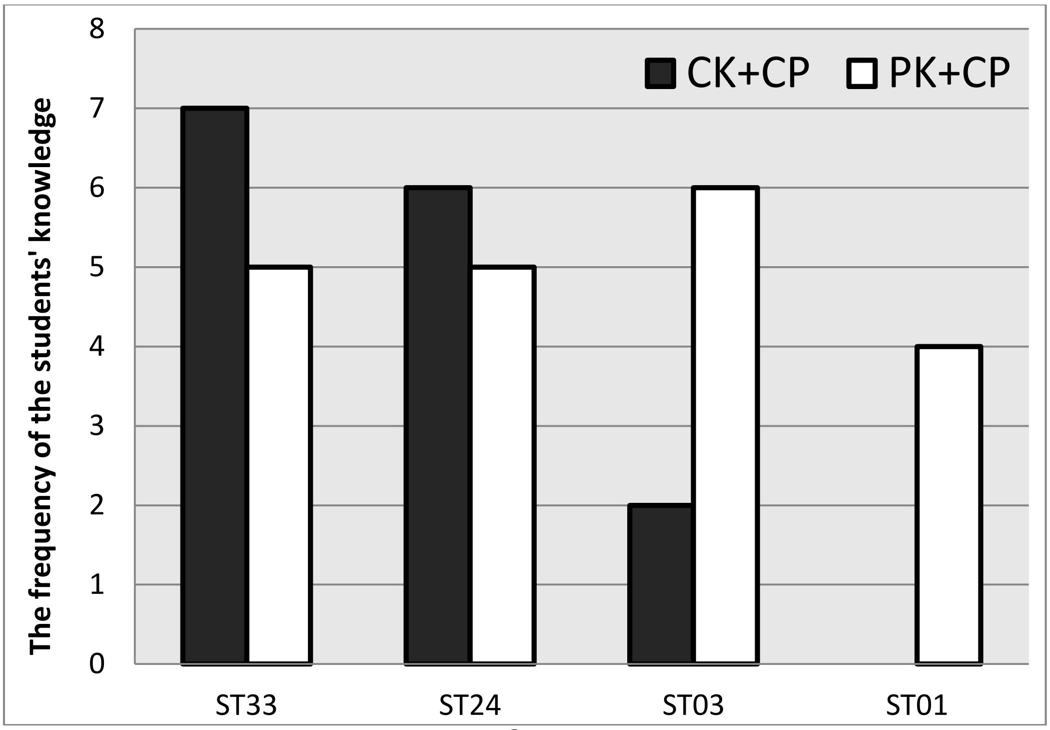

Figure 6. Similar procedural levels (x^2 (3) = 0.4, p > .05) and different conceptual levels (x^2(3)= 8.73, p < .05).

As it is seen from Figure 6, both successful and less successful students had the similar level of procedural knowledge (PK). According to Chi-square test result, there was not a statistically significant difference between the students' PK (x^2 (3) = 0.4, p > 0.05). On the other hand, the difference between the students' CK was statically significant (x^2(3) = 8.73, p < 0.05). It is clearly measured that the successful students had more conceptual knowledge of fractions than the less successful students did.

In addition, the successful students had combined conceptual and procedural knowledge, since the number of their CK and PK were close to each other. The combined knowledge could have originated from the fact that the successful students answered the interview questions by combining procedural knowledge with conceptual knowledge, whereas the less successful students answered the same questions by merely recalling rules of fractions. For instance, when it was asked why a common denominator should be found in the addition and subtraction problems of fractions, the successful students said that it is used to equalize parts in quantity to be able to add. This shows that the successful students are able to link their procedural knowledge of fraction addition with conceptual knowledge (Appendix F, ST24-34:ST24-39; ST33-23:ST33-27)

On the other hand, the less successful students had *orphaned procedural knowledge* failing to combine PK with CK. They were measured to have lower levels of CK although they had the same level of PK as their counterparts did. In other words, the less successful students memorized the rules and algorithms about faction concepts and tried to use these rules without understanding the meanings of each concept. For example, in interview the less successful students stated the reason of finding common denominator was "this is the rule." (Appendix F, ST01-50, ST03-36). This example shows that the less successful students had predominantly procedural knowledge.

Difficulties and misconceptions about fractions among the less successful students

The less successful students had difficulties and misconceptions about fractions. Table 7 indicates that the less successful students had larger gaps in CK, compared with the successful students. With excerpts from the interviews, this section highlights the differences between the less successful and the successful students' understanding of fractions and in terms of their misconceptions.

First, although part-whole meaning and region representation were taught in primary school years, the less successful students still had difficulties in determining fractions related to part-whole meaning and region representations. They tried to find fractions by counting parts and ignored whether the parts were equal in size or not. The following excerpt between the less successful student (ST03) and the researcher shows that the student had little knowledge about the equal partitioning and had developed a misconception related to the meaning of part-whole:

> **RESEARCHER:** Hi. In the first question, I asked you to determine whether the shaded area shows $\frac{1}{4}$ of whole or not. How do you decide this? Can you explain again?
> **ST03:** The first one was divided into 4 parts and 1 part was taken. So it is $\frac{1}{4}$.
> **RESEARCHER:** Humm Humm. For second one?
> **ST03:** It was divided 5 parts and it shows $\frac{1}{5}$. So the answer is no. For this one (3rd question, 1c) it is divided into 4 parts, so it also indicates $\frac{1}{4}$.
> **RESEARCHER:** However this shape was not divided into equal parts. Does it still indicate a fraction? Is just dividing 4 parts enough to representing a fraction?
> **ST03:** I am not sure.
> **RESEARCHER:** Ok. Continue.
> **ST03:** This is true. It is also divided into 4 parts. (Shape is divided unequal parts!) The others are true because of the same reason.
> **RESEARCHER:** Again these are not divided into equal parts (1e). Is it enough to divide just four parts? It is not important to be equal parts, is not it?
> **ST03:** I do not know.

Second, comparing fractions is a problematic concept for less successful students. They had difficulty in comparing fractions without calculations. They preferred to find common denominator and then compare the fractions (Appendix G, ST01-29:ST01-33; ST03-22:ST03-24).

Third, they developed misconception about comparison fraction by ignoring reference wholes. Referring to the pizza questions in CPKT (Appendix B, C6), students' answers for the questions on the test were discussed again and it was observed that less successful students compared fractions independently from their reference whole and accepted fractions as just numbers. They said it was impossible since " $\frac{1}{2}$ is bigger than $\frac{1}{3}$" (Appendix F, ST01-35; ST03-26). Moreover the less successful students could not deduce that a fraction represents the same quantities in different wholes and the wholes should be equal. The reason for this might have been originated from the difficulty to correlate fractions with quantity for more than one different shape.

Another difficulty among the less successful students was determining fractions represented by area representations. While the explanation of the answer of conceptual question C11, ST03 said "In that case we can't say anything since it is not equal." (Appendix F, ST03-50). He meant that the shape was not divided into equal parts and so nothing can be said for parts. This misconception could have originated from his confusion of area representation with region representation. This could be a reason why students have difficulties determining fractions represented from area representation.

Finally, the less successful students had computational difficulties while solving procedural knowledge based problem. They made computational mistakes, especially while finding common denominators and enlarging fractions. In addition, ignoring sign rule led students to solve the problem incorrectly.

CHAPTER 5: DISCUSSION

Introduction

Conceptual (CK) and procedural knowledge (PK) are the knowledge types needed in order to have a well-developed knowledge base in mathematics. The present study investigated the differences and similarities between successful and less successful students' conceptual and procedural understanding of the topic of fractions in mathematics.

Discussions of the findings

Strong positive correlation between CK and PK

In the present study, it was found that there was a large positive correlation between students' CK and PK of fractions, $r = 0.66$ ($p < 0.01$). That is, having a higher level in one type of knowledge was associated with having a higher level in the other type of knowledge or having a low level in one type of knowledge was related to having a low level in the other. This finding was consistent with the finding of previous researches which pointed that CK and PK are interrelated knowledge (Baki & Kartal, 2004; Hiebert & Lefevre, 1986). However, there are still discussions about how knowledge develop and which knowledge leads the increase in the other knowledge (Hallett et al., 2010; Rittle-Johnson & Siegler, 1998; Schneider & Stern, 2005). Rittle-Johnson and Sigler (1998) mentioned in their research the concept-first and procedure-first views that claim one type of knowledge is acquired first and it leads to the development of the other type of knowledge. On the other hand, Schneider and Stern (2005) suggested that conceptual and procedural knowledge developed iteratively and an increase in one kind leads to an increase in the other. Hallett et al. (2010) suggested that the reason of occurrence such different views about the relationship between conceptual and procedural knowledge was individual differences in learning conceptual and procedural knowledge. Despite these different theories, the important fact is that students do not comprehend the whole mathematical topic using only one type of knowledge since mathematics is a discipline of not only algorithms and rules but also relations, reasoning and concepts. Without conceptual knowledge, students cannot understand the meaning of mathematical concepts and related procedures (Birgin & Gürbüz, 2009; Lamon, 1999). In addition without procedural knowledge, students cannot apply rules and algorithms and solve computational problems (Sáenz-Ludlow, 1995; Silver, 1986). Aydın and Soylu (2006) found that when conceptual knowledge and procedural knowledge are not equilibrated, students do not have a full facility with the topics. The literature and the findings in this study address the issue that two knowledge types should be learned in balance.

Orphaned procedural knowledge of the less successful students to combined knowledge of the successful students

After determining the successful and less successful students using CPKT, the differences and similarities between successful students and less successful students' conceptual and procedural understandings of fractions were investigated by an interview. According to the interview results, the successful students had more CK than the less successful students. Furthermore, there was no significant difference between successful and less successful students' PK. They had similar amount of PK. However, the less successful students were measured to have orphaned PK whereas the successful students had the combined CK and PK. This finding indicates that while the successful students correlate their PK with CK, the less successful students did not. Furthermore, in interpreting these results, with successful students' high mathematics GPAs, the reason for their success in the topic of fractions is having the combined CK and PK. This can be also explanation for why the less successful students have unsatisfactory mathematics GPA with their orphaned PK.

The origin of predominant procedural knowledge could have been procedural based teaching (Baki & Kartal, 2004). PK alone is not enough for students to be successful in mathematics; it should be supported by CK. The orphan PK often leads students to rote-learning and after a while computational mistakes (Aksu, 1997). Therefore students need to understand the meaning of concepts while learning related algorithms in order to internalize the procedural knowledge of the topic (Sáenz-Ludlow, 1995). In line with this, these results are also consistent with the finding "CK and PK is interrelated knowledge"; CK and PK should be developed in tandem.

Difficulties and misconceptions of less successful students about fractions

Answering the last research question "What kind of difficulties do less successful students have about the concept of fractions?", this study indicated that the less successful students had some difficulties and misconceptions about fractions. These difficulties and misconceptions could be presented under the headings: misconceptions related to part-whole meaning, reference whole, and difficulties while comparing fractions, determining fraction represented by area representation and computation. These findings lend support to other research about difficulties and misconceptions related to the topic of fractions (Alacaci, 2010; Baki & Kartal, 2004; Haser & Ubuz, 2000; Lamon, 1999; McLeod & Newmarch, 2006; Petit et al., 2010).

Part-whole meaning is the basic concept of fractions and provides the bases for the other meanings of fractions (Charalambous & Pitta-Pantazi, 2007). In Turkey, this meaning is taught in the introduction of fractions at first grade and all fractional topics are associated with this meaning in the following years (MoNE, 2009a). That is, students deal with this concept continuously while learning fractions. However, the present study indicates that less successful students do not comprehend the part-whole meaning and its related feature "equal partitioning". When they were asked to determine whether the shaded area of given shapes represents $\frac{1}{4}$ of given shapes, they showed faultily that all shapes containing four parts although some are not divided into equal parts. They ignored equal partitioning and just counted divided parts to determine the fraction. This shows that students did not conceptualize the part-whole meaning of fractions. Even being in seventh grade, the less successful students still had misconception about part-whole meaning and this indicates that their teachers might not have emphasized the equal partitioning feature of fractions adequately to allow all students comprehend the part-whole meaning (Petit et al., 2010).

While comparing fractions, most of the students ignored the quantities represented by these fractions and reference wholes. The quantity represented by a fraction depends on the quantity of reference whole; the fraction can represent different quantities in different wholes. However, students tended to use rules related to comparison fractions (Petit et al., 2010). These comparison rules were developed by accepting fractions as numbers and independent from their reference whole. When students were asked whether it is possible to be Elif's pizza slice big, less successful students gave the response "Yes, $\mathbf{\frac{1}{2}}$ is bigger than $\mathbf{\frac{1}{3}}$**".** The misconception might have originated from the mathematics lessons in which teacher taught the rules of fraction comparison by giving examples for the number meaning of fractions and did not emphasize reference whole. Therefore students ignored the reference whole while comparing fraction.

The present study also indicated that less successful students not only have misconception related to reference whole with comparing fractions, they also have difficulties with general comparison of fractions. It was found that students tended to compare fractions after finding a common denominator and without computation they could not compare fractions. The less successful students did not have proportional reasoning and they just apply rules for comparison of fractions. This difficulty also comes from on the same reason “procedural based teaching”. Teachers teach comparison of fractions depending on some rules and algorithms. For example, if two fractions have the same numerator, the fraction with smaller denominator is bigger than the other one. Focusing on the rules leads students to learn the rules by rote. Since students do not reinforce their procedural knowledge with meanings behind the rules, they often confuse rules and make mistakes (Aksu, 1997; Soylu & Aydın, 2006).

Another finding of the study about less successful students’ difficulties related to the topic of fractions was that the students could not determine fractions represented by area representation and they confused area representation with region representation. This representation model is the most conceptually complex one for students (Alacaci, 2010). This clarifies why less successful students could not determine the fractions presented by area representation in CPKT whereas successful students could. The most important reason for the less successful students’ difficulty of determining a fraction presented by area representation might have originated from teachers who did not emphasize other representation models except to region representation during teaching fractions. Therefore, examples of different fraction representation models should be included in lessons of fraction.

In summary, CPKT and interview results showed that less successful students had computational difficulties in fraction topics. Although they had procedural knowledge, they still make computational mistakes. The reason for less successful students still having computational mistakes is probably their orphaned PK. Since they did not reinforce their PK with CK and learn the rules by rote, they forget or confused the rules and solved procedural problems incorrectly (Aksu, 1997; Soylu & Aydın, 2006). Since CK and PK are related to each other and students' general achievement is also related to their CK and PK. Combining the two types of knowledge is critical for students' success in mathematics.

Implications for practice

The results of the present study showed the importance of combining CK and PK for students' success with fractions and in general mathematics. Therefore CK and PK should be taught in balance and students should be let combine CK and PK. According to the results of the present study, the reason of less successful students having orphaned procedural knowledge and still having difficulties about the concept of fractions was that the fractions were taught by a procedural approach. Although in 2004 the changes were made in mathematics curricula in Turkey towards the new approach of providing students to acquire CK and combine CK with PK, the results of the present study indicate that there still exists the need to change mathematics education from PK- dominant teaching to balanced CK and PK teaching. Therefore, it should be focused on the reasons why procedural based teaching is still continuing and these reasons should be discontinued.

One of the main reasons for continuing procedural-based teaching could be due to common exams containing multiple choice questions and measuring students' PK such as high school and university entrance exams (SBS, YGS, and LYS). Therefore, the current examination system should be reconsidered to include open-ended questions to measure students not only PK but also CK. Thus the pressure on teachers to assist students in obtaining higher scores on these exams is removed and teachers can change their teaching styles to emphasize the meaning and operation of fractions.

Another reason was the intense mathematics curriculum that rushes teachers to finish many mathematical topics in a short time. The mathematics curricula have unnecessary details for many topics which teachers have to teach. Hence due to time constraint, teachers skip conceptual knowledge and prefer to primarily give rules and algorithms. Therefore the intensity of topics in the mathematics curriculum should be reduced and curriculum should be redesigned such that PK and CK are given in balance.

In addition, teacher training program should be changed so that trainee teachers learn how to teach CK and PK in balance. Because of the teachers' educational background, many teachers insist on teaching predominantly PK. Therefore they present students with generally one type of learning which the application of rules and algorithms is. If teachers learn how to teach CK and PK in balance, they can change their teaching styles towards teaching CK and PK together.

Implication for future research

In the present study, the differences and similarities between successful and less successful students' understanding of fractions was investigated. The successful and less successful students were chosen from Quadrant 1 where students had higher scores on both CK and PK test, and Quadrant 3 where students had lower scores in both tests respectively. Although most students were in Quadrant 1 and Quadrant 3, there exist students who had higher scores in PK test and lower scores in CK test (in Quadrant 2) and lower in PK test and higher in CK test (in Quadrant 4). This shows that there are individual differences among students' understanding of fractions, which is theoretically surprising. In future research, these knowledge patterns of students in Quadrant 2 and Quadrant 4 should be investigated more in detail.

Limitations

This study was conducted with two seventh grade classes in a school in Ankara. Hence the findings of this study may not be generalized to all seventh grade students in Turkey. Data were collected in two phases: a conventional paper-pencil test and a follow-up interview. Since the paper-pencil test which is the conceptual and procedural knowledge test (CPKT) was prepared by the researcher and the interview was conducted based on students' answers in the test, there were a few constraints during data collection and analysis. The questions in the paper-pencil test may not be appropriate for all students' learning levels; the language may be ambiguous and lead to misunderstanding. In order to minimize these constraints, opinions of experts and teachers regarding the test were collected and necessary modifications were made (see Appendix E). Hence, CPKT was validated by one academician and two middle school teachers. Furthermore, researchers who wish to use CPKT may consider applying item analysis such as difficulty, discrimination, item-total correlation, and alpha if item deleted.

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APPENDICES

Appendix A: Background form

Let's recognize you.

1. ___ Girl ____ Boy

2. I am _______ years old.

3. Last year, my mathematics grade was _______ (written or in figures).

4.If I rank the courses that I take in school according to those I like most:

(The most liked should be "1")

1- ....

2- ....

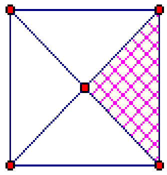
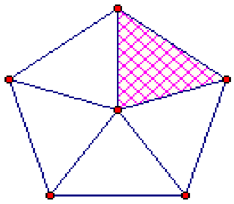
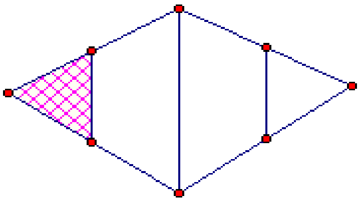
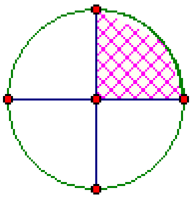
3- ....

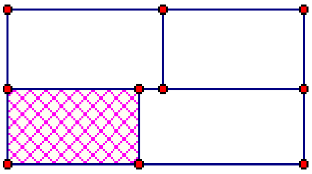
THANK YOU VERY MUCH. YOUR PARTICIPATION IS VERY IMPORTANT FOR US ☺.

Appendix B: Conceptual and procedural knowledge test (CPKT)

Conceptual questions:

1) State whether the highlighted part shows $\frac{1}{4}$ of figures given in the table or not. If not, why?

	Explanation Shaded area shows $\frac{1}{4}$ of the whole …
	◯ Yes ◯ No Because;
	◯ Yes ◯ No Because;
	◯ Yes ◯ No Because;
	◯ Yes ◯ No Because;

	○ Yes ○ No Because;

2) You are expected to explain one of the 4th grade friends what denominator and numerator of a fraction are. How do you explain it? Give examples.

3) Ali ate $\frac{3}{4}$ of a chocolate bar. What is the left of the chocolate is given the below. Make a drawing of how big the chocolate bar was before Ali ate any.

4) Which of the fractions bellow is bigger? Why? Explain your answer.

a) $\frac{5}{6}$ or $\frac{5}{8}$

b) $\frac{7}{8}$ or $\frac{4}{5}$

c) $\frac{6}{5}$ or $\frac{7}{6}$

5) Estimate the following addition without any calculation $\frac{12}{13} + \frac{7}{8}$. Which integer is it close to? Explain your answer convincingly.

6) Ali and his friend Elif decided to buy a slice of pizza. Pizza boy said that there were only half of a pizza and $\frac{1}{3}$ of another pizza, and added that they should wait for a time for new pizza. Since Ali was hungry, he wanted the half pizza and so Elif bought $\frac{1}{3}$ pizza. However, they realized that Elif's pizza slice was bigger than Ali's. Do you think this is possible? Explain your reasoning.

7) Erdem solved the following addition operation like this $\frac{3}{6} + \frac{2}{3} = \frac{(3+2)}{(6+3)} = \frac{5}{9}$

Whereas, Ömer solved the same operation and found $\frac{3}{6} + \frac{2}{3} = \frac{(3+4)}{6} = \frac{7}{6}$.

Which of the solution is correct? Why? Explain your reasoning.

8)

$\frac{4}{5}$ **x** $\frac{1}{3} = \frac{4}{15}$

Develop a problem with respect to operation given left.

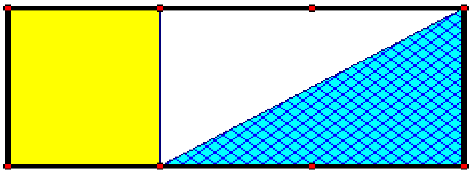
9) Estimate the following addition without any calculation $2\frac{1}{2}$ **x** $\frac{3}{4}$. Which integer is it close to? Explain your answer convincingly.

10) Tuğba follows the following operation steps to find how many bottles are needed to fill $1\frac{3}{4}$ L fruit juice into $\frac{1}{2}$ L bottle:

$$1\frac{3}{4} : \frac{1}{2} = \frac{7}{4} : \frac{1}{2} = \frac{7}{4} \text{ x } 2 = \frac{7}{2}$$

Tuğba applied invert and multiply rule. How can the problem be solved other than using invert-multiply rule? If there is, explain solution steps.

11) Which fractions of the whole shape does each shaded and lined areas represent? Explain.



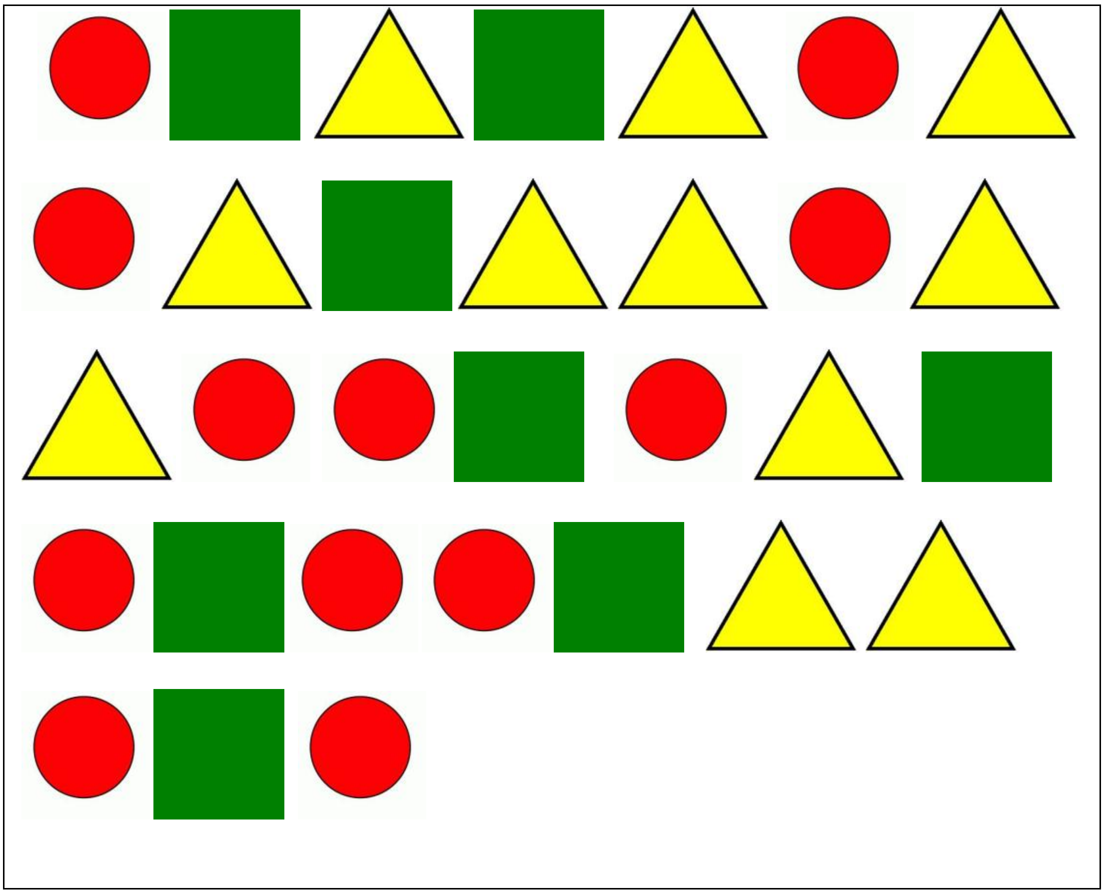
12) 2 apples are shared by 6 friends equally. How much apple does each child get? Explain your reasoning.

13) There is a box containing different figures below. Hence,

a) What fraction of the figures in the box are triangles?

b) What is the fraction representing the ratio of circles to squares?

Explain your reasoning.



Procedural questions:

Solve questions given below by showing steps.

1) What is the solution of addition $\frac{1}{3}+\frac{1}{6}+\frac{1}{12}$?

2) Find the fractions represented by letters in the operations below.

   a) $1\frac{1}{8}+\ a = 2\ \frac{3}{8}$

   b) $\frac{9}{10}\ -\ b = \frac{4}{5}$

3) What is the solution of subtraction $1\frac{7}{10}-\ \frac{2}{4}$?

4) What is the solution of multiplication $\frac{5}{6} \text{ x } \frac{3}{4}$?

5) Find the value of *a*, *b*, and *c* given in the below fractions.

$$\frac{1}{3} = \frac{2}{a} = \frac{b}{15} = \frac{4}{c}$$

6) Solve multiplication operation given below. Simplify the solution.

a) $\frac{6}{13}$ x $\frac{13}{12}$ =?

7) What is the solution of following operation $(3 - 1\frac{1}{3}) : \frac{2}{5}$?

8) What is the solution of following operation $\left(\frac{3}{5} - \frac{2}{10}\right)$ x 3?

Appendix C: Semi-structured ınterview questions

1) How do you determine whether the shaded area show $\frac{1}{4}$ of figures in the first question of the conceptual part of the test? You can show again.
2) Are there other meanings of numerator and denominator of a fraction? If there are, what?
3) In the 4th question of conceptual part, It is wanted from you to determine which fraction is bigger. If you recall the answer that you gave, which ways do you follow to determine bigger fraction? Are there other way to determine it?
4) For a fraction represents same quantity for differents wholes, how should wholes be?

5) We find firstly common denominator while doing addition and subtruction of fraction with different denominators. What is the reason for finding common denominator?

6)When we multiply positive integers, we acquire bigger number than multipliers. However when we multiply fractions, we can obtain smaller number than multipliers as in the following example: $2\frac{1}{2} \times \frac{3}{4} = 1\frac{7}{8}$ What is the reason?

7) In the 11th question of conceptal part, why do shaded and lined region in the figure represent same fraction, $\frac{1}{3}$ of whole as if they show different quantities of whole?

8) Solve the following operation by showing each steps $\left(\frac{3}{4} - \frac{5}{6}\right) \times \left(1\frac{2}{3} + \frac{7}{3}\right)$.

Appendix D: Parent informing and permission letter

Dear parent,

I am a masters student in Master at Curriculum and Instruction with Teaching Certificate program in Bilkent University. In addition, I am a student-teacher in the field of mathematics. Since this year is my second year, I am working on my thesis. With your permission and help, I will conduct my study.

One of the main goals of the program which I attend is to improve teachers and their teaching styles by locating problems in education and producing solutions to these problems.

The main goal of the study that I will conduct is analyzing similarities and differences between seventh grade students' conceptual and procedural understandings of fractions. This study facilitates to understand students' learning process of fractions and produce solutions of problems which students encounter while learning fractions. Because of these reasons, students' participations and your contribution is very important for this study.

In the first part of the study, my purpose is to learn students' general attitudes towards mathematics by using the background form that I attached to the letter. In the main part of the study, I will administer a test to determine students' conceptual and procedural knowledge levels in the content of fractions and problems that students encounter while learning fractions. In the light of the data acquired from the test, I plan to have an interview with four students in the second part of study.

In the study the names of students will be kept confidential. Moreover, every type of data will be kept confidential and information about students will not be shared with other students, participants and parents of students. At the end of the study, the results will be presented without giving any personal information about participation.

I hope that you will contribute to this study. For further information, you can contact with me by the mail address below. Thank you for your support in advance.

Sincerely,

Şakire ÖRMECİ

sakire@bilkent.edu.tr

Appendix E: Expert opinion form

Dear ……..

In 2011-2012 academic year, I will conduct an educational research with 7th grade students. In the study, it is planned to investigate conceptual and procedural understanding of successful and less successful students in the content of fractions. Hence as researcher, I developed an instrument named "Conceptual and Procedural Test" to determine and analyze 7th grade students conceptual and procedural understanding of fractions.

This test has two parts; the first part includes conceptual questions and the second part includes procedural questions. The questions are based on the following definitions:

> *Conceptual knowledge:* A connected web of knowledge, a network in which the linking relationships are as crucial as the discrete pieces of information (Hiebert & Lefevre, 1986).

> *Procedural knowledge:* The official language or symbol representation system of mathematics and the algorithms or rules for completing mathematical tasks (Hiebert & Lefevre, 1986).

Furthermore, the conceptual and procedural questions in this test is developed according to MoNEs'(2009b) objectives in the program of the 6th grade mathematics and related literature (Alacaci, 2010; MoNE, 2009c; Van de Walle, 2007).It is noted for each question which type of knowledge it measured and which objectives it refers to. The objectives are stated in the program of 6th grade mathematics course booklet of MoNE (2009b) is as in the following:

O1. Students compare fractions and show fractions on numerical axis.
O2. Students make addition and subtraction with fractions
O3. Students make multiplication with fractions.
O4. Students make division with fractions.
O5. Students estimate the results of computation with fractions
O6. Students construct and solve fraction problems.

The detail information about each question in the test is given following tables.

Conceptual questions are developed according to different concepts and representation of fractions to measure students' conceptual knowledge of fractions.

Conceptual question #	Measured conceptual knowledge	References
C1	part-whole concept with respect to region representation	(Van de Walle, 2007)
C2	the concept of numerator and denominator	(Van de Walle, 2007)
C3	part-whole meaning, finding whole from given fractional part	(Sowder et al., 1998)
C4	quantity meaning of fractions and referring to O1	(MoNE,(2009b), (MoNE, 2009c)
C5	quantity meaning of fractions and referring to O5	(MoNE,(2009b), (MoNE, 2009c)
C6	The referenced whole concept	(Alacaci, 2010)
C7	Concept of subtractions and addition, common denominator referring to O2	(MoNE,(2009b), (MoNE, 2009c)
C8	constructing fractions problem, fractions multiplication concept and referring to O6	(MoNE,(2009b), (MoNE, 2009c)
C9	fractions multiplication concept and referring to O3 and O5	(MoNE,(2009b), (MoNE, 2009c)
C10	division of fractions and referring to O4	(MoNE,(2009b), (MoNE, 2009c)
C11	part-whole meaning of fraction, area representation of fraction	(Alacaci, 2010)
C12	quotient meaning of fractions	(Alacaci, 2010), (MoNE, 2009c)
C13	set representation of fractions, in a) partial-whole meaning of fractions in b) ratio meaning of fractions	(Alacaci, 2010), (MoNE, 2009c)

The procedural questions are designed to measure students' procedural knowledge about fractions. By these questions it will be analyzed whether students follow operations in the correct order.

Procedural question #	Measured procedural knowledge	References
P1	Addition with fractions and referring to O2	(MoNE,(2009b), (MoNE, 2009c)
P2	Addition and subtractions with fractions and referring to O2	(MoNE,(2009b), (MoNE, 2009c)
P3	Subtraction with fractions, mixed fractions and referring to O2	(MoNE,(2009b), (MoNE, 2009c)
P4	Multiplication with fractions and referring to O3	(MoNE,(2009b), (MoNE, 2009c)
P5	Equivalent fractions and referring to O1	(MoNE,(2009b), (MoNE, 2009c)
P6	Multiplication with fractions, simplification and referring to O3	(MoNE,(2009b), (MoNE, 2009c)
P7	Subtraction and division with fractions, mixed fractions and referring to O2 and O4	(MoNE,(2009b), (MoNE, 2009c)
P8	Subtraction and multiplication with fractions, mixed fractions and referring to O2 and O4	(MoNE,(2009b), (MoNE, 2009c)

In the light of the above explanations, please analyse the questions as an expert.

1. Is there a question which is not appropriate for seventh grade students? If there is, which one? Why? How can it be modified?

2. Is there any question which does not match the explanations given in the above table? If there is, which one? How can it be modified?

3. Is there any question which is unclear? If there is, which one? How can it be modified?

4. Is there any question which should be deleted from the test? If there is, which one?

Any other suggestion and comments about conceptual and procedural test.

Appendix F: Transcriptions

Transcription 1: Interview with ST24 (high in GPA, 84.7)

ST24-1.RESEARCHER: In the first question, I asked you to determine whether the shaded area shows $\frac{1}{4}$ of whole or not. How do you decide this? Can you explain again?

ST24-2.ST24 I thought that $\frac{1}{4}$ pieces would be simplified or expanded for the questions like second one but I realized that it is really given $\frac{1}{4}$ in one question, in other one it is given as $\frac{1}{5}$. In addition, in other figures, the pieces were not equal. Since they are not equally shared, I understood that they are not $\frac{1}{4}$. In order to representing a fraction, a whole should be divided into equal parts.

ST24-3.RESEARCHER: Ok. In the second question I want you to explain your friends what denominator and numerator mean. Can you tell how you explained?

ST24-4.ST24 Firstly I draw a figure and then divide it into equal part. All parts show whole and I said this is denominator. The parts that I take or give show the numerator.

ST24-5.RESEARCHER: Is this a sharing?

ST24-6.ST24 Yes.

ST24-7.RESEARCHER: This is one of the examples for the meaning of fractions. According to you, is there any other meaning of fractions? Do you know?

ST24-8.ST24 Actually**,** in any example the meaning is the same. If I share my friends a cake or if I give some of my pencil, the meaning is the same.

ST24-9.RESEARCHER: Is not there any other meanings? What did you do in your lessons? Do you remember?

ST24-10.ST24 We dealt with examples such as for cooking a cake, $\frac{1}{2}$ kg flour is needed. I think this has also same meaning since I take half of 1 kg flour.

ST24-11.RESEARCHER: Ok. Let me ask a question. The number of girls in our class is divided to number of the boys………..

ST24-12.ST24 Humm. Ratio !

ST24-13.RESEARCHER: Does the ratio represent a fraction?

ST24-14.ST24 Yes.

ST24-15.RESEARCHER: Is this another meaning of the fractions?

ST24-16.ST24 Yes.

ST24-17.RESEARCHER: Is there any other meanings?

ST24-18.ST24: I don't know.

ST24-19.RESEARCHER: Ok. In the fourth question, I wanted to you compare fractions, what did you do while comparing fraction?

ST24-20.ST24 Firstly I found common denominators by expansion. After that I wrote the one which is with smallest numerator smaller than the other fraction, for example $\frac{40}{48}$ is bigger than$\frac{30}{48}$.

ST24-21.RESEARCHER: Ok, if you did not do any calculation, without finding common denominator, how could you have compared fractions?

ST24-22.ST24 It is difficult to compare without finding common denominator because for $\frac{7}{8}$ we take 7 parts from 8 parts or for $\frac{4}{5}$ we take 4 parts from 5 parts However if we approach the problem from this point of view, there is no difference between these fractions since when we take parts, 1 part is left behind.

ST24-23.RESEARCHER: Humm. You can make comment on this point. Are the left parts in same amount?

ST24-24.ST24 No. $\frac{4}{5}$ is bigger one because dividing one pizza into 5 parts and taking 4 parts give bigger parts taking 7 parts from the pizza divided into 8 parts. (Understanding wrongly, may be misconception)

ST24-25.RESEARCHER: I see. Do you remember I asked pizza question which claims Elif's pizza is bigger than Ali's pizza although Elif's is $\frac{1}{3}$ and Ali's is $\frac{1}{2}$. You said this is impossible.

ST24-26.ST24 Yes.

ST24-27.RESEARCHER: However it is possible. Let me explain you. As you know, there are pizza sizes; small, middle and big. Let say Ali's slice is taken from small pizza and Elif's slice is taken from big pizza. If I draw for- you…. Elif's pizza slice is bigger, is not it?

ST24-28.ST24 Yes, I see.

ST24-29.RESEARCHER: So, for a fraction represents same quantity for different shapes, how should shapes be? At first she had difficulty to understand what the question asks. Hence I explained in several time, I draw shapes as whole.

ST24-30.RESEARCHER: I think you haven't understood. Let me explain another way. There is a fraction and there are two different shapes. In order that this fraction represents same quantity for these shapes, how should the shapes be relatively?

ST24-31.ST24 They have to be in equal size.

ST24-32.RESEARCHER: Ok. Thank you again. You know, we find common denominator for adding problems if fractions' denominators are not equal. Why do we find common denominator? Why is it necessary?

ST24-33.ST24 Because when we find common denominator, they represent same whole. Then we look for which one has big numerator since fraction with bigger numerator is biggest one.

ST24-34.RESEARCHER: But we are talking about addition not comparison. Let me show you what I mean on figures. This is $\frac{1}{2}$ and this is $\frac{1}{3}$. If I want to add these fractions and ask you add them together by modeling, what should we do?

ST24-35.ST24 I divide the fraction which has smaller numerator one more time (not exact answer).

ST24-36.RESEARCHER: So you divide $\frac{1}{2}$ with 3 and $\frac{1}{3}$ with 2, is this what you mean? (I tried to make clear what she meant)

ST24-37.ST24 Yes.

ST24-38.RESEARCHER: The aim is here to find same quantity, isn't it?

ST24-39.ST24 Yes. For this reason we find common denominator.

ST24-40.RESEARCHER: Ok. You know, when we multiply two positive integer we obtain bigger number than multipliers. However when we multiply two fractions, sometime we can obtain smaller number than the multipliers. What could be the reason? Do you have any idea?

ST24-41.ST24 Because when we multiply two fractions, we multiply denominators. It is not finding common denominator like in addition. When we multiply denominators, we make fractions smaller since we divide whole into more parts and obtain smaller pieces. For example at first we divided a whole into 2, now we divide it into 8 parts.

ST24-42.RESEARCHER: Ok. I see. That is we divide it smaller parts.

ST24-43.ST24 Yes

ST24-44.RESEARCHER: Do you remember 11th question. I asked which the fraction each shaded and lined areas represent according to whole shape and you answered both shapes represent $\frac{1}{3}$ of the whole shape. How did you find it? They seem representing different fractions.

ST24-45.ST24 In this shape, there are 3 parts and these 3 parts are equal. That is to say, if I divide from this and add that small part into here I found that the shaded and lined area represent $\frac{1}{3}$ of the whole shape separately.

ST24-46.RESEARCHER: Very nice. Can you draw again for me to remember later?

ST24-47.A6 (draw again)

ST24-48.RESEARCHER: I want you to solve this procedural question. While you are solving problem, please tell loudly what you are doing.

ST24-49.ST24 Ok.

ST24-50.RESEARCHER: What is the result of $(\frac{3}{4} - \frac{5}{6}) * (1\frac{2}{3} + \frac{7}{3})$?

ST24-51.ST24 Firstly it is started to solve from the operation in the parenthesis, then since there is no multiplication or division in the parenthesis, subtraction is done. For this subtraction, we firstly find common denominator and which is 24 when 4 and 6 are multiplied. When we multiplied 3 by 6, it is 18 and 4 by 5, it is 20. Then we find the common denominator of this but first we should convert complex fraction to compound fraction. It becomes$\frac{5}{3}$. They already have common denominator so we add them together. We subtract these and positive, negative sign rule is valid for also fraction. Hence 18-20 becomes -2. So it is -$\frac{2}{24}$. If we add this it is $\frac{12}{3}$. When we multiply them, since positive and negative are multiplied, it is negative and the result - $\frac{1}{3}$. (she made simplifications.)

Transcription 2: Interview with ST01 (low in GPA, 44.5)

ST01-1.**RESEARCHER:** In the first question, I asked you to determine whether the shaded area shows $\frac{1}{4}$ of whole or not. What did you think while solving this problem? How do you decide this? Can you explain again?

ST01-2.**ST01:** I counted the parts of the shape. For first one, since there are 4, this is $\frac{1}{4}$. For second one, from the same way; one, two, three, four and five, (silently counting). There are five. This is not $\frac{1}{4}$. (there is a misconception)

ST01-3.**RESEARCHER:** Why do you call it $\frac{1}{4}$? It is divided into four parts and….. (waiting for student to answer)

ST01-4.**ST01:** 1 part is taken.

ST01-5.**RESEARCHER:** Ok. In 3rd one, does it represent $\frac{1}{4}$? Why?

ST01-6.**ST01:** Yes. Same, there are four parts. (misconception)

ST01-7.**RESEARCHER:** But the parts are not equal, are they?

ST01-8.**ST01:** Yes.

ST01-9.**RESEARCHER:** In previous questions they are divided into equal parts and represent $\frac{1}{4}$. Does this still represent $\frac{1}{4}$.

ST01-10.**ST01:** No.

ST01-11.(When I asked A7 what is necessary for a shape to represent a fraction, he had difficulty to answer although we had just commented on it.)

ST01-12.**RESEARCHER:** The 4th one ?

ST01-13.**ST01:** It represents. And the last one does not represent. I solved wrongly.

ST01-14.**RESEARCHER:** Why?

ST01-15.**ST01:** Since it is not divided into equal parts. (he learnt equal parts are needed to represent a fraction.)

ST01-16.**RESEARCHER:** In the second question I want you to explain your friends what denominator and numerator mean. Can you tell how you explained?

ST01-17.**ST01:** I explained to my friend like this; if I draw a fraction line, upper number is numerator and lower number is denominator. For example in $\frac{1}{4}$, 1 is numerator and 4 is denominator. (just recalling, learning by rote)

ST01-18.**RESEARCHER:** Is there any meaning of numerator and denominator?

ST01-19.**ST01:** No.

ST01-20.**RESEARCHER:** Are you sure? For example we share $\frac{3}{4}$ of a cake. Sharing is a meaning of fraction. Is there any other meaning of fraction?

ST01-21.**ST01:** ……….. (He did not give any answer. He was quite.)

ST01-22.**RESEARCHER:** Ok. You don’t know. In the fourth question, I wanted to you compare fractions, but you could not give any proper answer. I asked you to solve this problem and explain your solution way loudly.

ST01-23.**ST01:** Three of them?

ST01-24.**RESEARCHER:** Yes.

ST01-25.**ST01:** It gives $\frac{5}{6}$ and $\frac{5}{8}$. $\frac{5}{6}$ is bigger.

ST01-26.**RESEARCHER:** Why?

ST01-27.**ST01:** Because …..ııııı. (There is no answer again) I could not solve such problems.

ST01-28.**RESEARCHER:** What is difficult for you while solving this problem?

ST01-29.**ST01:** I solved but I could not explain. I solved it by finding common denominator.

ST01-30.**RESEARCHER:** But there must be a reason. What did you think while solving?

ST01-31.**ST01: …………(** There is no clear answer. He could not give proper answer.)
ST01-32.**RESEARCHER:** When do you have difficulty while solving such question? What is the problem?
ST01-33.**ST01:** I could not explain how one of them is bigger.
ST01-34.**RESEARCHER:** Ok. I see. Do you remember I asked pizza question which claims Elif's pizza is bigger than Ali's pizza although Elif's is $\frac{1}{3}$ and Ali's is$\frac{1}{2}$. You said this is impossible.
ST01-35.**ST01:** Yes. Ali took more pizza because $\mathbf{\frac{1}{2}}$ is bigger than $\mathbf{\frac{1}{3}}$.
ST01-36.**RESEARCHER:** However it is possible. Let me explain you. As you know, there are pizza sizes; small, middle and big. Let say Ali's slice is taken from small pizza and Elif's slice is taken from big pizza. If I draw for you…. Elif's pizza slice is bigger, is not it?
ST01-37.**ST01:** Yes.
ST01-38.**RESEARCHER:** Ok. Let me ask you in another way. For a fraction represents same quantity for different shapes, how should shapes be?
ST01-39.(Since student did not understood at first, I explained question again by drawing shapes. I draw two shapes and ask how shapes should be in order that $\frac{1}{2}$ represents same quantity in each shape.)
ST01-40.**ST01:** They should be equal.
ST01-41.(But student answered with unsure tone so I ask again)
ST01-42.**RESEARCHER:** Should they be equal?
ST01-43.**ST01:** No. one of them should be bigger and the other small (!)
ST01-44.(So I drew again and tried to explain clearly.)
ST01-45.**RESEARCHER:** Let me show again. This is $\frac{1}{2}$ of this shape and this is $\frac{1}{2}$ of that shape, ok? How should both whole shapes be relatively for these $\frac{1}{2}$s represent equal quantities?
ST01-46.**ST01:** ……………….. (he could not gave answer.)
ST01-47.**RESEARCHER:** Ok. Let's pass another question. You know, we find common denominator for adding problems if fractions' denominators are not equal. Why do we find common denominator? Why is it necessary?
ST01-48.**ST01:** To make denominators equal.
ST01-49.**RESEARCHER:** Why do we make denominators equal? Why do we need?
ST01-50.ST01: ……………. We need to find common denominator, this is rule.
ST01-51.…………………..
ST01-52.**RESEARCHER:** Do not you know?
ST01-53.**ST01:** Yes.
ST01-54.**RESEARCHER:** Ok. I also pass this question. You know, when we multiply two positive integer we obtain bigger number than multipliers. However when we multiply two fractions, sometime we can obtain smaller number than the multipliers. What could be the reason? Do you have any idea? (I explained the question by giving example and I provide student understand the question. I saw that student was quite at procedure but he could not interpret and tell the conceptual idea behind the answer.)
ST01-55.**ST01:** Multiplied numbers …….Humm (thinking process)
ST01-56.**RESEARCHER:** What does fraction do to make number smaller?
ST01-57.**ST01:** ………………… (silence, there is no answer again)
ST01-58.**RESEARCHER:** Ok. You don't know. What does fraction mean for you? Do you remember that your teacher explain something about fractions yesterday?
ST01-59.**ST01:** Something division…(with unsure tone)
ST01-60.**RESEARCHER:** Can it be sharing?
ST01-61.**ST01:** Yes. Sharing.

ST01-62.RESEARCHER: Ok. Do you remember 11th question. I asked which the fraction each shaded and lined areas represent according to whole shape and you find $\frac{2}{4}$. How could you find it? Can you explain?

ST01-63.**ST01:** Because this is square and I divided it into 2 part and this is also divided into 2. If there are four parts and 2 of them are shaded, it is $\frac{2}{4}$.

ST01-64.**RESEARCHER:** But I am asking what the fractions representing this shaded area in whole shape and this lined area in whole shapes are separately.

ST01-65.**ST01:** HUmm $\frac{1}{3}$

ST01-66.**RESEARCHER:** Yes. It is $\frac{1}{3}$ but how?

ST01-67.**ST01:** For example, if this is a whole.

ST01-68.**RESEARCHER:** It is not a whole. Whole figure is this but this part is taken.

ST01-69.**ST01:** ıııııı…………(there is no answer)

ST01-70.**RESEARCHER:** Ok. One more question and it will finish after this question. Now I want you to solve tis problem by telling what you are doing.

ST01-71.**ST01:** $\frac{3}{4}$ - $\frac{5}{6}$, when I subtract them, I firstly found common denominator. 6 time 3 is 18 over 4 time 6 is 24, so $\frac{18}{24}$. 6 times 5 is 30.

ST01-72.**RESEARCHER:** But this should be 4.

ST01-73.**ST01:** Sorry I am wrong. 4 times 5 is 20 and 4 times 6 is again 24 and so $\frac{20}{24}$. $\frac{18}{24}$ - $\frac{30}{24}$ It becomes $\frac{12}{24}$.

ST01-74.**RESEARCHER:** Is there any mistake here? You said $\frac{18}{24}$ - $\frac{30}{24}$ and find $\frac{12}{24}$. I think there is a little problem.

ST01-75.**ST01:** 18 -30 is 12.

ST01-76.RESEARCHER: You have just learnt in subtraction result takes the sign of bigger number, here is bigger number is-30 and its sign is negative so answer is - $\frac{12}{24}$. In addition you also made mistake in finding common denominator and multiplication. Do you have difficulty to solve such problems.

ST01-77.**ST01:** I was excited.

ST01-78.**RESEARCHER:** Humm you were excited. So do you have problem in the exam solving procedural questions like this one?

ST01-79.**ST01:** Sometimes when I was excited.

Transcription 3: Interview with ST03 (low in GPA, 52.4)

ST03-1.**RESEARCHER:** Hi. In the first question, I asked you to determine whether the shaded area shows $\frac{1}{4}$ of whole or not. How do you decide this? Can you explain again?

ST03-2.**ST03** The first one was divided into 4 parts and 1 part was taken. So it is $\frac{1}{4}$.

ST03-3.**RESEARCHER:** Humm humm. For second one?

ST03-4.**ST03** It was divided 5 parts and it shows $\frac{1}{5}$. So the answer is no. For this one (3rd question, 1c) it is divided into 4 parts, so it also indicates $\frac{1}{4}$. (That student has also misconception about the fraction concept.)

ST03-5.**RESEARCHER:** However this shape was not divided into equal parts. Does it still indicate a fraction? Is just dividing 4 parts enough to representing a fraction?

ST03-6.**ST03** I am not sure.

ST03-7.**RESEARCHER:** Ok. Continue.

ST03-8.**ST03** This is true. It is also divided into 4 parts. (Shape is divided unequal parts !) The others are true because of the same reason.

ST03-9.**RESEARCHER:** Again these are not divided into equal parts (1e). Is it enough to divide just four parts? It is not important to be equal parts, is not it?

ST03-10.**ST03** I do not know.

ST03-11.**RESEARCHER:** Ok. In the second question I want you to explain your friends what denominator and numerator mean. Can you tell how you explained?

ST03-12.**ST03** Above line is numerator, below line is denominator. This is the number of shared pieces. (Student meant the sharing meaning of fractions)

ST03-13.**RESEARCHER:** Can you explain what you mean by an example?

ST03-14.**ST03** For example, assume we were 5 and went a birthday party. Birthday cake was divided into 5 slices. My 3 friends ate birthday cake. Since 3 slices were eaten out of 5, it is $\frac{3}{5}$.

ST03-15.**RESEARCHER:** Humm. Sharing is meaning of a fraction. Is there any other meaning of fractions? Do you know?

ST03-16.**ST03** There is not. I know like this.

ST03-17.**RESEARCHER:** Thank you. In the fourth question, I wanted to you compare fractions, what did you do while comparing fraction?

ST03-18.**ST03** $\mathbf{\frac{5}{6}}$ is bigger than $\mathbf{\frac{5}{8}}$**.** When it is divided by 6, the share is bigger. (Good reasoning)

ST03-19.**RESEARCHER:** Ok. What about second one?

ST03-20.**ST03** ……………… (Student thought for a while.)

ST03-21.**RESEARCHER:** Your answer is true but how did you solve it?

ST03-22.**ST03** May be I made calculation, I found common denominator.

ST03-23.**RESEARCHER:** In the first one you compare shares; in the second one you found common denominator. Can you solve this problem by using another method?

ST03-24.**ST03 Hu**mmm. Other methods…… I don't know.

ST03-25.**RESEARCHER:** Ok. I see. Do you remember I asked pizza question which claims Elif's pizza is bigger than Ali's pizza although Elif's is $\frac{1}{3}$ and Ali's is $\frac{1}{2}$. You said this is impossible.

ST03-26.**ST03:** Yes. Because $\mathbf{\frac{1}{2}}$ is bigger than $\mathbf{\frac{1}{3}}$.

ST03-27.**RESEARCHER:** However it is possible. Let me explain you. As you know, there are pizza sizes; small, middle and big. Let say Ali's slice is taken from small pizza and Elif's slice is taken from big pizza. If I draw for you…. Elif's pizza slice is bigger, is not it?

ST03-28.**ST03:** Yes.

ST03-29.**RESEARCHER:** Ok. Let me ask you in another way. For a fraction represents same quantity for different shapes, how should shapes be?

ST03-30.(Since student did not understood at first, I explained question again by drawing shapes. I draw two shapes and ask how shapes should be in order that $\frac{1}{2}$ represents same quantity in each shape.)

ST03-31.**ST03:** I don't know.

ST03-32.(So I drew again and tried to explain clearly.)

ST03-33.**RESEARCHER:** Let me show again. This is $\frac{1}{2}$ of this shape and this is $\frac{1}{2}$ of that shape, ok? How should both whole shapes be relatively for these $\frac{1}{2}$s represent equal quantities?

ST03-34.**ST03:** ……………….. (he could not gave answer.)

ST03-35.**RESEARCHER:** Ok. You know, we find common denominator for adding problems if fractions' denominators are not equal. Why do we find common denominator? Why is it necessary?

ST03-36.**ST03** This is rule**.**I can solve more easily. When I make denominator equal, I just add or subtract above ones.

ST03-37.**RESEARCHER:** Do you find common denominator just since it is easy?

ST03-38.**ST03** Yes.

ST03-39.**RESEARCHER:** Is not there any other reason of finding common denominator?

ST03-40.**ST03** I think so.

ST03-41.**RESEARCHER:** Ok. You know, when we multiply two positive integer we obtain bigger number than multipliers. However when we multiply two fractions, sometime we can obtain smaller number than the multipliers. What could be the reason? Do you have any idea?

ST03-42.**ST03** I am not sure. But when we multiply $\frac{5}{2}$ with $\frac{3}{4}$, it becomes $\frac{15}{8}$. Then we convert it complex fraction, we find it is smaller.

ST03-43.**RESEARCHER:** Ok I see but what does a fraction do to make a number smaller?

ST03-44.**ST03** There should be a reason but I don't know.

ST03-45.**RESEARCHER:** Ok. Do you remember 11th question. I asked which the fraction each shaded and lined areas represent according to whole shape and you said $\frac{2}{3}$. How did you find it?

ST03-46.**ST03** I also solved it wrongly.

ST03-47.**RESEARCHER:** Yes. What is the answer?

ST03-48.**ST03** It can be $\frac{1}{2}$.

ST03-49.**RESEARCHER:** But whole shape is this. (I show the whole shape by drawing the boundaries of the shape.)

ST03-50.**ST03** In that case we can't say anything since it is not equal. (He meant that the shape was not divided into equal parts.) (Area representation can contradict the region meaning and students developed misconception.)

ST03-51.**RESEARCHER:** As last question, I want you to solve this procedural problem loudly. Explain what you are doing in each step.

ST03-52.**ST03** Firstly I found common denominator since they are 4 and 6. 6 times 4 is 24, 6 times 3 is 18 and 4 times 5 is 20. It is 18 mines 20 over 24. This is equal to $\frac{2}{24}$. Then it is multiplied by… I am passing this part. Firstly I convert complex fraction to compound fraction. It is $\frac{5}{3}$. Since their denominators are equal, I directly add them together. Hence it is $\frac{12}{3}$. Then I multiply $\frac{2}{24}$ by $\frac{12}{3}$ and I found $\frac{1}{6}$. [There are a few procedural mistakes but I think student is good at procedures.]

ST03-53.**RESEARCHER:** Thank you.

Transcription 4: Interview with ST33 (high in GPA, 95.9)

ST33-1.**RESEARCHER:** In the first question, I asked you to determine whether the shaded area shows $\frac{1}{4}$ of whole or not. How do you decide this? Can you explain again?

ST33-2.**ST33** That square was divided into 4 parts. All parts are equal and 1 of them is taken. So it is true. (for 1a) This one was divided into 5 parts. Hence it does not represent $\frac{1}{4}$ (for 1b) This one was again divided into 4 parts but parts are not equal. So it is wrong. This one is true since all parts are equal and 1 is taken. This is not true because parts are not equal.

ST33-3.(She has good conceptual knowledge about fractions)

ST33-4.**RESEARCHER:** Ok. Very good. In the second question I want you to explain your friends what denominator and numerator mean. Can you tell how you explained?

ST33-5.**ST33** Denominator is below part and numerator is above part. Denominator states the number of the parts which a whole is divided into and numerator is the number of parts that we take from the whole.

ST33-6.**RESEARCHER:** Yes. According to you, are there any other meanings of fractions? You said fraction means sharing. Is there any other meaning different than sharing?

ST33-7.**ST33** Mostly sharing examples come to my mind. I don't know.

ST33-8.**RESEARCHER:** I can give an example for an idea to come your mind. For example, the number of girls in our class is divided to number of the boys………..

ST33-9.**ST33** Hahh. Ratio.

ST33-10.**RESEARCHER:** Yes. Any other thing that comes your mind?

ST33-11.**ST33** No.

ST33-12.**RESEARCHER:** Do you remember I asked pizza question which claims Elif's pizza is bigger than Ali's pizza although Elif's is $\frac{1}{3}$ and Ali's is $\frac{1}{2}$. This is possible. Now I want you ask that in order to a fraction represent same quantities in different two shapes, how should the shapes be relatively?

ST33-13.**ST33** Shapes should be equal in size.

ST33-14.**RESEARCHER:** Ok. In the fourth question, I wanted to you compare fractions, what did you do while comparing fraction?

ST33-15.**ST33** In a, I did not find common denominator because numerators are already equal. Since $\frac{5}{6}$ has smaller denominator, taken parts are bigger. (good reasoning)

ST33-16.**RESEARCHER:** Humm. What did you do in others?

ST33-17.**ST33** In b, I found common denominator. The one with bigger denominator is bigger than the other.

ST33-18.**RESEARCHER:** Except a, you solved others by calculation. Without calculation, how can you solve this problem?

ST33-19.**ST33** May be I can look its nearness of half. That's all. (she has good knowledge)

ST33-20.**RESEARCHER:** You know, we find common denominator for adding problems if fractions' denominators are not equal. Why do we find common denominator? Why is it necessary?

ST33-21.**ST33** It provides to solve adding problem more easily.

ST33-22.**RESEARCHER:** Is it just for organizing operations? Can you think in another way? For example modeling?

ST33-23.**ST33** Yes I see. If I model for addition $\frac{1}{3}+\frac{1}{2}$, there are 3 parts and 1 part is taken, also in here there are 2 parts and 1 part is taken. I could not add them like this. Hence I should make parts equal.

ST33-24.**RESEARCHER:** What do you mean?

ST33-25.**ST33** We don’t know how big this part is in that whole so we need to find common denominator and make parts equal.

ST33-26.**RESEARCHER:** Do you mean that we try to find equal size parts?

ST33-27.**ST33** Yes.

ST33-28.**RESEARCHER:** Ok. You know, when we multiply two positive integer we obtain bigger number than multipliers. However when we multiply two fractions, sometime we can obtain smaller number than the multipliers. What could be the reason? Do you have any idea?

ST33-29.**ST33** They are different from integers.

ST33-30.**RESEARCHER:** Yes. What is the feature of fractions?

ST33-31.**ST33** …………………..(there is no answer). Maybe the reason is that fractions are between integers. (student has no clear idea)

ST33-32.**RESEARCHER:** Ok. Let’s pass another question. Do you remember 11th question. I asked which the fraction each shaded and lined areas represent according to whole shape and you solve this problem correctly. How did you solve it? Can you explain it?

ST33-33.**ST33** In first part, 1 of 3 parts is shaded, so it is $\frac{1}{3}$. In the second, there are 2 triangles in here and 1 square but. That part of triangles and other part make together a square, hence this triangle represents $\frac{1}{3}$ of the whole shape.

ST33-34.**RESEARCHER:** Ok. The last question. I want you solve this problem loudly.

ST33-35.**ST33** Firstly we start from the operations in the parentheses and then we convert complex fraction to compound fractions. (The sound is improper and I could not transcript it but student solve problem correctly.)

ST33-36.**RESEARCHER:** Thank you very much.