ANALYZING THE EFFECT OF CONSUMER RETURNS IN A MULTI-PERIOD INVENTORY SYSTEM

A THESIS

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MASTER OF SCIENCE

by

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July, 2012

I certify that I have read this thesis and that in my opinion it is full adequate, in scope and in quality, as a dissertation for the degree of Master of Science.

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ABSTRACT

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M.S. in Industrial Engineering

Supervisor: Prof. Dr. Nesim K. Erkip

July 2012

Return of a sold item by a customer becomes tremendously common situation in many industries. Increase in the amount of returned items promotes return information to be a critical factor for inventory control. Undoubtedly another critical parameter for an inventory system is the length of the review period. Effect of the review period or length of the time-bucket is amplified with returned items, because available return information at a decision point is related to the frequency of the review. In this study, we analyze the effects of these two parameters over a multi-period inventory system where the length of a time horizon is fixed. Dynamic programming approach is used to calculate the optimal inventory positions. In dynamic programming, it is assumed that a fixed proportion of sold items are returned. Computational results are obtained to compare the effects of return information under different return proportions and period lengths. These results are used to conduct various analyses to explore the level of the advantage gained by using return information.

Keywords: Consumer returns, dynamic programming, periodic review inventory system

ÖZET

MÜŞTERİ İADELERİNİN ÇOK PERİYOTLU ENVANTER SİSTEMLERİNDEKİ ETKİSİNİN ANALİZİ

İsmail Erikçi
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Tez Yöneticisi: Prof. Dr. Nesim K. Erkip
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Bir çok endüstride satılan ürünün müşteri tarafından iade edilmesi sıkça rastlanan bir durum haline gelmiştir. İade edilen ürünlerin miktarındaki artış iade bilgisinin envanter kontolü için kullanılmasını önemli hale getirmiştir. Şüphesiz ki envanter sistemi için diğer bir kritik parametre gözden geçirme periyodunun uzunluğudur. Gözden geçirme noktaları arasındaki zaman azaldıkça iade bilgisinin daha sıklıkla kullanılması olanaklı hale gelmektedir. Bu çalışmada, ürün iadesi (ve bilgisinin) olması ve envanteri gözden geçirme sıklığının değişmesinin etkisi analiz edilmektedir. En iyi envanter pozisyonları dinamik programlama yaklaşımı kullanılarak hesaplanmıştır. Dinamik programlamada, satılan ürünün sabit bir oranının iade edildiği varsayılmıştır. İade bilgisinin farklı iade oranları ve period uzunlukları altındaki etkisinin karşılaştırılması için sayısal sonuçlar elde edilmiştir. Bu sonuçlar iade bilgisi kullanılarak elde edilen avantajın seviyesinin bulunması için yürütülen analizlerde kullanılmıştır.

Anahtar sözcükler: Müşteri iadeleri, dinamik programlama, periyodik envanter sistemi

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To my family…

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Chapter 1

Introduction

Over the last decades, consumer return policies have become one of the vital fields in industry, especially in apparel and electronics. The value of returned products exceeds $100 billion each year in U.S and $13.8 billion are spent in electronic industry to rebox and resell returned products. (Stock, 2002 and Lawton, 2008). Even though the products are not defective, consumers have a tendency to return the products. Only 5% of returns are truly defective (Lawton 2008). The proportion of returns can be ranged from 11% to 20% in electronics and up to 35% in the apparel industry (Guide 2006). Internet sales and rapid progresses in technology are the main triggers of consumer returns. In other words, higher anticipation from product features and lack of understanding of the product by consumer lead to increase in product returns. Some companies have enhanced new strategies to cut down the returns such as TV makers who put extra notices and the computer companies engrave customers' name on a computer.

Some extra risks arise for consumers due to the advances in high-level technological products and remote purchases. The leniency of return policy is one way of minimizing the inherent consumer risk. Companies are commonly adapted to "no question asked" 100% money-back guarantees to create their competitive advantage because return polices has profound effects on consumer demand, and consumers' buying behavior. Around 90% of consumers highlight that return policies become much more crucial while buying new or unknown product by online or by catalog retailer. Moreover, consumers have a tendency not to shop from a retailer, if the retailer's return policy is inconvenient (Market Wire 2007). Even if return policies affect consumer demand in a positive manner, there is also a side effect of lenient return policies. Firstly, lenient return policies are costly to operate because of stocking, refurbishing or reboxing of returned items. Secondly, lenient return policies give consumer an opportunity to abuse the policy such as borrowing an item for special purposes. Some of the known examples that can be named as an abuse are returning TV sets after Super Bowl or return of camcorders after a wedding (Wood 2001).

The positive and negative impacts on return policies make managers found themselves in dilemma. That is, different return policies are introduced by different managers for the same type of product (Davis et al. 1998). Another study shows that permissible period for returned items fluctuate as well, in a large interval (Stiner 2004). There are mainly 4 parameters which differentiate return policies from each other, these are:

1. Return price
2. Return period
3. Assumption about return decision of customer
4. Options for handling of return

In this study, we consider that the retailer operates in an environment as if the time frame was multi-period and there was a lead-time between the retailer and supplier. It is assumed that the retailer adapts no return policy in a base case, and in alternative cases the retailer adapts a return policy that gives full refund for the returned items. Basically, there is no fixed ordering cost; but there are salvage, backorder, holding and penalty costs. Therefore, in a base case it is backordered multi-period Newsboy problem with lead-time. In alternative cases, predetermined proportions of sold items are returned by the customer who gets a full refund. Returned items can be sold in the next period. There is no remanufacturing or restocking cost for returned item. Therefore, the alternative cases can be called as multi-period backordered Newsboy problem with returns and lead-time.

This study tries to identify the benefit of using information of returns by looking at the inventory system more frequently. To make this identification, we have fixed the length of time horizon but change the period length while the length of lead-time is constant. By this approach, the benefit of looking at the inventory system more frequently and use of return information can be identified. Comparisons among base case and alternative cases provide managerial insights for setting return policy and reviewing frequency as well.

In classical periodic inventory systems, period length is not considered as a decision variable. Commonly in the literature, period length is assumed to be fixed and the analysis is made over cost parameters. However, in a finite time horizon period length has a profound effect over the system because the period length is a parameter for deciding the review frequency of inventory. Treating period length as a decision variable provides a chance to analyze the effect of the period length over total costs, optimal inventory levels etc. To analyze this effect, we define base time unit called time bucket. Length of time horizon and period lengths are defined in terms of time bucket. In this study, we investigate the effect of period length over system by changing the number of time buckets in the period.

In this study, the problem that is briefly mentioned above is solved by dynamic programming. Base case formulation is quite similar to Porteus' formulation and alternative cases are derived from the base case. In the rest of this study, when Porteus work is mentioned, we are referring to Porteus' book chapter 4.2 that was published in 2002.

The rest of this thesis is organized as follows. In Chapter 2, the related literature is reviewed. In Chapter 3, the detail problem definition is given and the mathematical model is presented. The computational results are given and the comparisons are summarized in Chapter 4. The conclusion and possible further research areas can be found in Chapter 5.

Chapter 2

Literature Review

In this study, we consider finite periodic review inventory system with stochastic demand, deterministic lead-times for orders and consumer returns. We utilize dynamic programming approach to solve the problem.

Utilization of dynamic programming approach in the inventory management literature dates back to 1960s. Karlin (1960) formulates dynamic inventory model where demand distribution can vary from period to period. He derives optimal policy for both convex purchase and linear purchase cost and there is a delivery lead-time. Veinott (1965) considers multi-product, dynamic non-stationary inventory problem. Under special conditions, he derives a policy for minimizing the expected discount cost over infinite time horizon. Kaplan (1970) considers dynamic inventory problem with stochastic lead-time where there is a fixed setup, linear order, convex holding and shortage cost. Porteus (2002) shows that base stock policy is optimal for each period of finite-horizon problem assuming that the terminal value function is convex.

Heyman (1978), Simpson (1978), Cohen (1980) and Inderfurth (1997) utilize dynamic programming approach in their model where they consider returns.

There is considerable research on stochastic inventory models with product returns. Simpson (1978) considers a finite periodic review inventory system where supplies and returns are instantaneous. The other assumption of his study is that demand and return streams are joint random variables. The lead-time for returned item is fixed whereas it is zero for the orders. Kelle and Silver (1989) study on the model where fixed proportion of satisfied demand is returned and fixed proportion of returned items can be used. They also regard fixed order cost and stochastic sojourn time of returned item. Muckstadt and Isaac (1981) model the problem by considering single item, multi-echelon environment using continuous review policy. They assume that demands and returns are independent streams, returned and serviceable items are only at retailer level. They assume that system return rate is less than system demand rate. Yuan and Cheung (1998) model the problem for similar environment with an (s,S) order policy via Markovian formulation. Their policy is based on the number of items on-hand and the number of items that are sold in the market. In their model, return stream depend on demand stream. They consider Poisson demand and exponentially distributed market sojourn time. Fleischmann et al (2002) derive optimal control policy for infinite horizon system where the order cost and procurement lead-time are fixed; streams for demand and return are independent.

Another branch of research examines single period problem by considering consumer returns. Vlachos and Dekker (2003) elaborate on various return handling options. They investigate an environment such that order decision is made before the beginning of the selling period and returns can be used to satisfy new orders. They list return handling options as in Figure 2.1 and they derive optimal order quantity for each option.

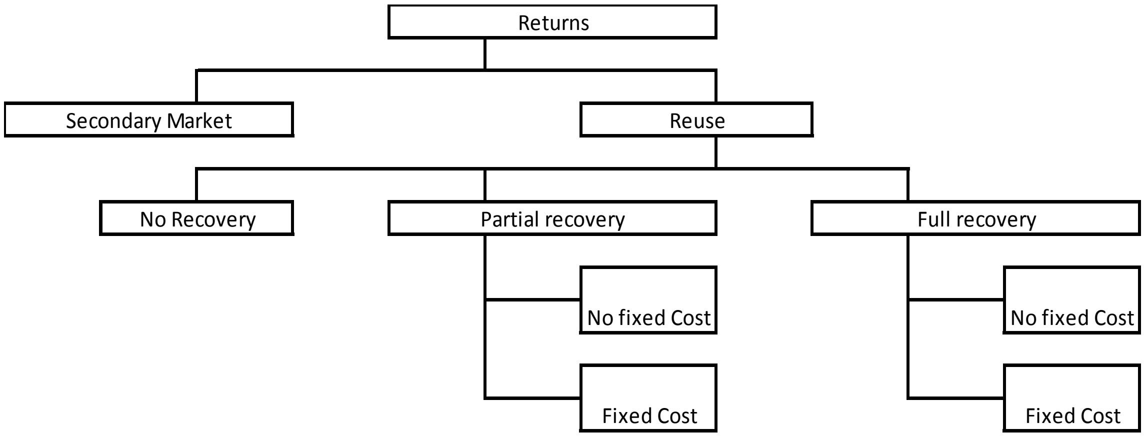

Figure 2.1 Options for Handling Returns from Vlachos and Dekker (2003)

Mostard et al (2005) investigate catalogue/internet case where returned item can be resold if there is a sufficient demand. They assume that order is given before the selling season. What makes the study different than previous studies in the literature is the assumption of unknown distribution for demand. They derive expression for the order quantity which is the obtained distribution-free setting. Mostard and Teunter (2006) consider that a product cannot be resold twice because returned products are generally sold at the end of season. Therefore, there is not enough time to return the sold product. Under this assumption, optimal order quantity is higher than the order quantity where returned item can be resold more than once. Ketzenberg and Zuidwijk (2009) divide one selling season into two periods to formulate the case where returns at the first period can be resold at the second period but returns at the second period may only be salvaged. They also consider return policy and price effects over the demand in their model. They derive optimal decision strategy for deterministic model but in the stochastic model they perform the experimental design.

Davis et al (1995) present a model which assumes that the consumer can only evaluate product after purchase. The model formulates consumer valuation as a Bernoulli random variable in which the product may either match consumer needs or may not. Che (1996) does not restrict consumer valuation with any distribution but consider risk aversion. Su (2009) consider the case where the customer realizes his valuation over the product only after the purchase. They investigate impacts of full return policies and partial return policies. They propose contracts to coordinate supply chain with consumer returns.

Cohen et al (1980) assume that a fixed proportion of satisfied demand is returned and only fixed proportion of these returned items can be added to the serviceable inventory. Presentation of this environment is given in Figure 2.2. In our model, we consider a similar environment but we assume that all returned item can be resold whereas in their model some fraction of returned items can leave the system forever. They consider zero lead-time for orders while our model considers fixed lead-time for orders.

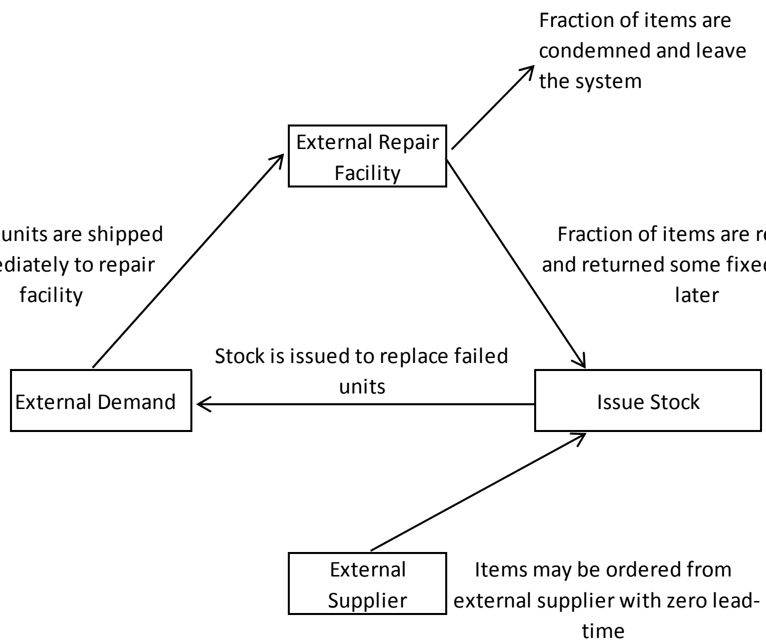

Figure 2.2 The reparable item system that is defined in Cohen et al (1980)

Inderfurth (1997) addresses the environment such that there is finite periodic review system with stochastic demand and stochastic returns. In his model, demand stream and returns streams are independent from each other. He also assumes that disposal of returned item inventory is available at each period. He considers that the supplies and returns have lead-time greater than zero but they are not necessarily equal. Represantation of his environment is given in Figure 2,3. In our study, we disregard disposal of returned item inventory, except in the terminal period left over inventory may be salvaged. The other difference of our study from Inderfurt (1997) is we assume fixed proportion of sold products return after fixed market time.

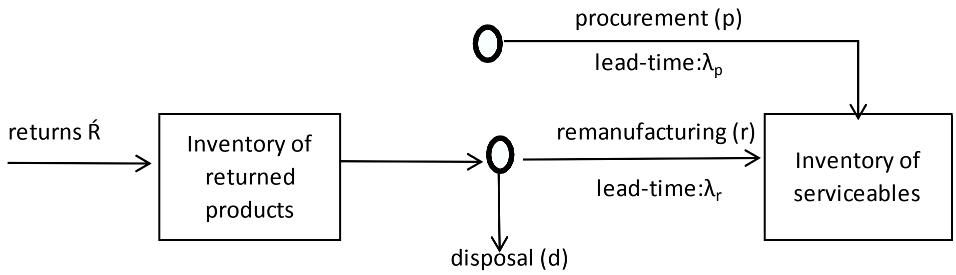

Figure 2.3 The recovery management environment that is defined in Inderfurth (1997)

This study deals with single product inventory system over finite time horizon where orders have fixed lead-time and fixed proportion of sold products return after fixed market time. The representation of our model is given in Figure 4.3. We assume that there is no fixed setup cost, but there is a linear purchase cost, convex holding and shortage cost. We disregard disposal of inventory by assuming that the leftover inventory on hand of previous period can never be greater than the optimal level of current period. Another assumption we make is that all returned items can be used to satisfy demand. We investigate a reduction in the total cost while increasing the frequency of inventory review. Increasing the frequency of inventory review means, shortening period lengths while length of a time horizon is fixed. As we mentioned in the previous chapter, by treating period length as a decision variable we analyze the effect of period length over the inventory system. Different values of period length affect available information about inventory system status. Hence by changing period lengths, we also try to elaborate on the benefit that can be gained by using information of returned item. This type of analysis has not been addressed in the literature before.

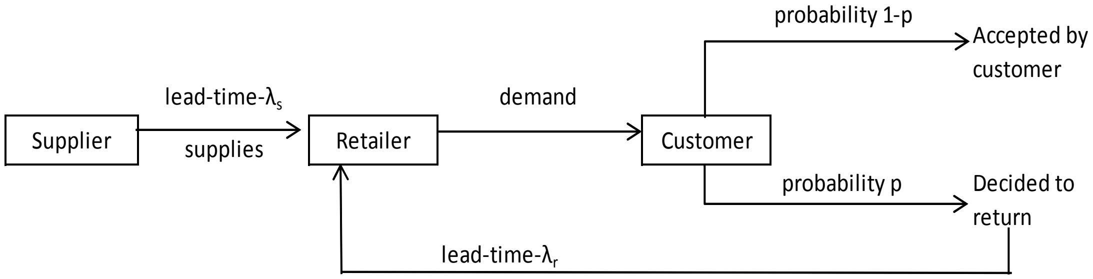

Figure 2.4 The structure of the environment of our problem

Chapter 3

Problem Definition and Mathematical Model

In this chapter, we elaborate on environmental assumptions, model formulation and lastly properties of the model.

3.1. Environmental Assumptions

We define time buckets as a smallest time unit in our setting. Each parameter in the model such as holding cost, shortage cost, discount rate is defined for a time bucket (unit time). Length of period, time horizon, order and return lead-times are also defined in terms of time buckets. The aim of using time buckets is preserving consistency between problems as much as possible. Time buckets is the mechanism that provide consistency in cost calculation of different setting of problem.

Time horizon consists of periods and periods consist of time buckets. The length of a time bucket is fixed and also the length of a time horizon is fixed, hence the number of time buckets in the time horizon is fixed. A time bucket cannot be divided into any smaller part; that is, period length is a multiple of a time bucket. The number of period in the time horizon is adjusted with period lengths. Let time horizon be equal to $z * N$ time-buckets, and let the length of a period be defined as z time-buckets, hence we have N period problem setting. In this example, let holding and shortage costs for time buckets be c_h and c_p respectively; discount rate is α, then for a period consisting of z time bucket holding and shortage costs are $z * c_h$ and $z * c_p$, discount rate for a period is equal to $(\alpha^z + \alpha^{z-1} \ldots + \alpha)/z$.

We assume that order lead-time is fixed and independent from period length. Let order lead-time be equal to $2 * j$ time buckets and period length be equal to j time buckets then order-lead time is equal to 2 periods. In another setting if period length was equal to $j/2$ time buckets then order lead-time is equal to 4 periods. Return lead-time totally depends on the period length. Since a sold item can be added to the inventory only at the beginning of the next period therefore, return lead-time is equal to period length.

We assume order of events in any period is determined as follows. Firstly, items returned from previous period are added to inventory, secondly, if order is given then the order given a lead-time ago is received. Afterwards demand for this period occurs and items are sold. Lastly, cost is evaluated. Figure 3.1 (3.2) represent example of a setting where length of time horizon is 12 time buckets and period lengths is 4 (2) time buckets. Length of lead-time is 1 (2) period at Figure 3.1 (3.2).

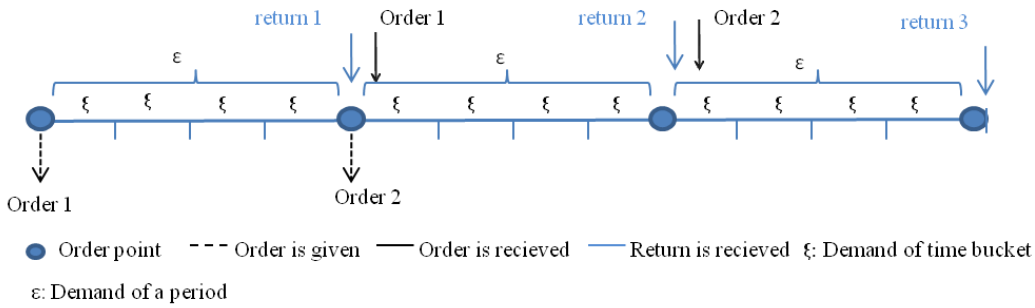

Figure 3. 1 Length of Period = 4*Length of Time Bucket

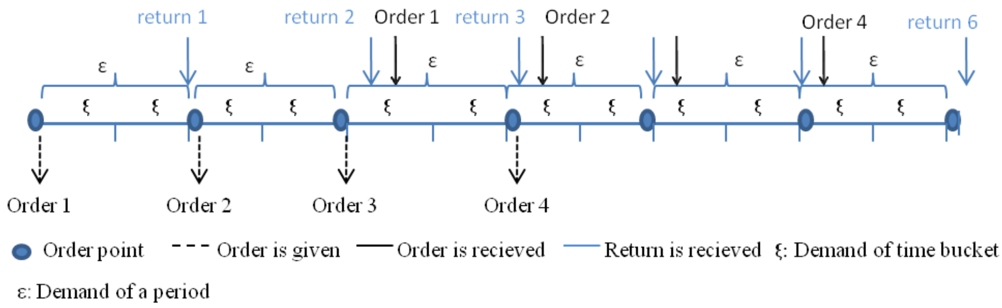

Figure 3. 2 Length of Period = 2*Length of Time Bucket

In the literature, there are some methods that we mentioned in Chapter 2 on how to make consumer return decision, such as using probability function dependent or independent from demand, valuation functions, proportion of sales etc. We use fixed proportion of expected demand as returned assumption but there is a critical point to emphasize; if we disregard this assumption then Markovian property cannot be preserved when an order lead-time is greater than one period. Because in the case where order lead-time is greater than one period, amount of sales and amount of returned items during lead-time depends on past decisions. Therefore, when order lead-time is greater than one period, the decision maker can act optimally only if he/she knows the previous decisions. Because the amount of returned items depends on past decisions, Markovian property cannot be preserved. To overcome this challenge we assume that the amount of returned item is a fixed proportion of expected demand. Hence, the amount of returned items is known for the next periods and independent from past decisions then Markovian property is preserve.

We assume that remanufacturing cost and remanufacturing time for returned items are negligible. There is no difference between returned items and ordered items in a sense that both can be used to satisfy demand. After an item is sold, the beginning of the next period is the only available point for a customer to return this item. If a customer does not return the item at that point, it can be assumed that the customer keeps the item. Since time horizon is finite, there is a special case at the last period for returned items. That is, at the end of last period whether excess inventory is salvaged or backlogged demand is satisfied with paying extra penalty cost in case of shortage. Cost evaluation is made and time horizon is ended. Proportion of sold items at the last period is returned after time horizon ended; however, because they cannot be salvaged or cannot be used to satisfy backlogged demand, they are disregarded. It can be interpreted as if items that are sold at the last period could not be returned.

3.2. Model Formulation

In this section we construct a model by considering environment that is defined above. We use notation in Table 3.1.

Notation	**Definition**
c	Unit purchasing cost
c_p	Unit penalty cost of backlogged demand
c_h	Unit holding cost of leftover inventory
g	Unit salvage value for leftover inventory at the end of time horizon
b	Unit penalty cost of unsatisfied demand at the end of time horizon
τ	Order lead-time in terms of period
r	Return proportion
m	Mean demand
σ	Standard deviation of demand

α	Discount rate

Table 3. 1 Notation Table

We use backward dynamic programming approach to model the problem. Let inventory at the end of the last period N be x, then terminal cost is $v_T(x)$ where

$$v_T(x) = -g\max(0, x) - b\min(0, x) \qquad (3.1)$$

$v_T(x)$ is a convex function when $0 \le g \le b$. There is a graph of $v_T(x)$in Figure 3.3 when $g = 80$ and $b = 120$.

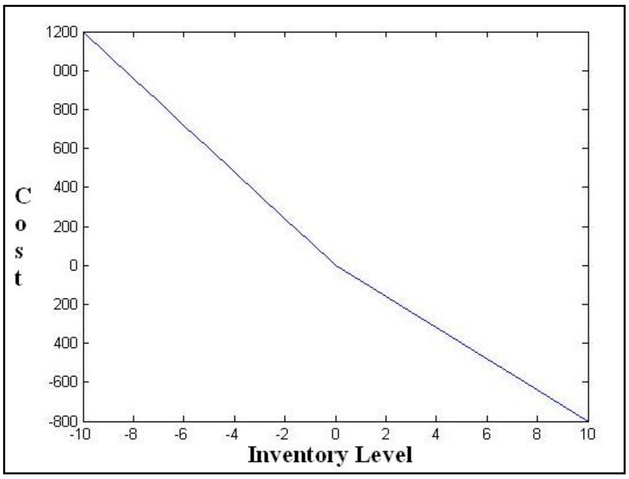

Figure 3.3 Example graph of $v_T(x)$

Holding and shortage cost for periods is a bit different than Porteus' case in terms of returned items and lead-time. Let consider order is given and inventory position is increased to y, then expected holding and shortage cost L(y) is incurred.

$$L(y) = \int_0^\infty \int_0^{\frac{y-\epsilon}{1-r}} c_h(y - \epsilon^\tau + r\epsilon^\tau - \epsilon)\varphi^\tau(\epsilon^\tau)\varphi(\epsilon)d\epsilon^\tau d\epsilon + \int_0^\infty \int_{\frac{y-\epsilon}{1-r}}^{\infty} c_p(\epsilon^\tau - r\epsilon^\tau + \epsilon - y)\varphi^\tau(\epsilon^\tau)\varphi(\epsilon)d\epsilon^\tau d\epsilon \qquad (3.2)$$

ϵ^τ is demand that occurs during lead-time and ϵ is demand during one period. The first term in L(y) is expected holding cost and second term is expected shortage cost. y is the inventory position after order is given. Order can be added to inventory after τ periods. Hence, effect of order can be seen $\tau + 1$ period later. In the cost calculation, $\tau + 1$ period demand is subtracted from inventory position and items that are returned during lead-time is added. We make important assumption about the amount of returned item that we mentioned in the previous section. Number of returned items is proportion of sales and sales depend on demand and inventory on hand. Therefore orders that are given before period t are also critical because they have impact on inventory on hand. Impact of order decision at period t depends on past decisions therefore, Markovian property cannot be preserved. We assume demand is fully satisfied in the past and fixed proportion of demand is returned. Instead of sales, we use demand to calculate returns therefore, we can make decisions independent from past decisions. Hence, L(y) is not a function of sales but a function of demand.

Now, we can write the optimality equations for this environment for 1< t < N-1 where N is the number of periods:

$$f_t(x) = \min_{y \geq x} \left\{ c(y - x) + L(y) + \alpha \int_0^{\infty} f_{t+1}(y - \epsilon + r\max(0,\ \min(y - \tau m + r\tau m, \epsilon)))\, \varphi(\epsilon) d\epsilon \right\} \quad (3.3)$$

x is a state in period t before giving an order, y is an inventory position after giving an order. $c(y - x)$ is order cost, $L(y)$ is expected holding and shortage cost and $\alpha \int_0^{\infty} f_{t+1}(y - \epsilon + r\max(0, \min(y - \tau m + r\tau m, \epsilon)))\, \varphi(\epsilon) d\epsilon$ is expected present value of future cost such that starting at period t+1 in a state $y - \epsilon + r\max(0, \min(y - \tau m + r\tau m, \epsilon)$ and acting optimally for the rest of the time horizon. y is the inventory position and ϵ is the current period demand so it is subtracted from y. To decide the next period starting state, the returns that are related to sales of current period should be added to $y - \epsilon$. Amount of returned items is calculated with $r\max(0, \min(y - \tau m + r\tau m, \epsilon)$ .The term $y - \tau m + r\tau m$ can be interpreted as the expected inventory on hand at the beginning of the period. Then the term $r\max(0, \min(y - \tau m + r\tau m, \epsilon)$ can be interpreted as expected return.

When $t = N$ we use following equation,

$$f_N(x) = \min_{y \geq x}\left\{c(y - x) + L(y) + \alpha \int_0^{\infty} f_{N+1}(y - \epsilon)\, \varphi(\epsilon) d\epsilon\right\} \qquad (3.4)$$

For period $t = N + 1$,

$$f_{N+1}(x) = v_T(x) \text{ for each x.} \qquad (3.5)$$

We can rewrite optimality equations as

$$f_t(x) = \min_{y \geq x}\{G_t(y) - cx\} \qquad (3.6)$$

where

$$G_t(y) = cy + L(y) + \alpha \int_0^{\infty} f_{t+1}(y - \epsilon + r\max(0, \min(y - \tau m + r\tau m, \epsilon)))\, \varphi(\epsilon) d\epsilon \qquad (3.7)$$

$$G_t(y) = cy + L(y) + \alpha \int_0^{\infty} f_{t+1}(y - \epsilon)\, \varphi(\epsilon) d\epsilon \quad \text{if } t = N \qquad (3.8)$$

We construct the model using backward dynamic programming approach; we elaborate on the properties of model in the next section.

3.3. Properties of the Model

Porteus' model is a special case of our model. If the following parameter set is chosen then our model can be reduced to Porteus' model.

$$r = 0; g = b = c \qquad (3.9)$$

Using this parameter set we can rewrite equations such as;

$$f_{t+1}(x) = v_T(x) = -cx \qquad (3.10)$$

$$L(y) = \int_0^{\infty}\int_0^{y-\epsilon} c_h(y-\epsilon^{\tau}-\epsilon)\varphi^{\tau}(\epsilon^{\tau})\varphi(\epsilon)d\epsilon^{\tau}d\epsilon + \int_0^{\infty}\int_{y-\epsilon}^{\infty} c_p(\epsilon^{\tau}+\epsilon-y)\varphi^{\tau}(\epsilon^{\tau})\varphi(\epsilon)d\epsilon^{\tau}d\epsilon \tag{3.11}$$

$$f_t(x) = \min_{y\geq x}\left\{c(y-x) + L(y) + \alpha\int_0^{\infty} f_{t+1}(y-\epsilon)\,\varphi(\epsilon)d\epsilon\right\} \tag{3.12}$$

When we convolve ϵ^{τ} and ϵ, $L(y)$ can be re-written as follows;

$$L(y) = \int_0^{y} c_h(y-\epsilon^{\tau+1})\varphi^{\tau+1}(\epsilon^{\tau+1})d\epsilon^{\tau+1} + \int_y^{\infty} c_p(\epsilon^{\tau+1}-y)\varphi^{\tau+1}(\epsilon^{\tau+1})d\epsilon^{\tau+1} \tag{3.13}$$

The structure that is defined by Porteus and our structure become the same. From this point, by following the same steps of Porteus starting from Lemma 4.3, the same results can be reached.

For other parameter sets, we use the same approach as Porteus did in Lemma 4.3 and Theorem 4.2 to show convexity of f_t and optimality of base stock policy in each period of finite-horizon problem.

Lemma 1. If f_{t+1} is convex, then following hold,

a) G_t is convex
b) f_t is convex.

Proof:

a) To prove that G_t is convex, let firstly examine whether $L(y)$ is convex or not. $L(y)$ can be re-written as follows;

$$L(y) = \int_0^{\infty}\left\{\int_0^{\frac{y-\epsilon}{1-r}} c_h(y-\epsilon^{\tau}+r\epsilon^{\tau}-\epsilon)\varphi^{\tau}(\epsilon^{\tau})d\epsilon^{\tau} + \int_{\frac{y-\epsilon}{1-r}}^{\infty} c_p(\epsilon^{\tau}-r\epsilon^{\tau}+\epsilon-y)\varphi^{\tau}(\epsilon^{\tau})d\epsilon^{\tau}\right\}\varphi(\epsilon)d\epsilon$$

$$L(y) = \int_0^{\infty} z(y)\varphi(\epsilon)d\epsilon$$

(3.14)

where $z(y)$ is

$$z(y) = \int_0^{\frac{y-\epsilon}{1-r}} c_h(y - \epsilon^\tau + r\epsilon^\tau - \epsilon)\varphi^\tau(\epsilon^\tau)d\epsilon^\tau + \int_{\frac{y-\epsilon}{1-r}}^{\infty} c_p(\epsilon^\tau - r\epsilon^\tau + \epsilon - y)\varphi^\tau(\epsilon^\tau)d\epsilon^\tau$$

(3.15)

Since convexity is preserved under integration, proving the convexity of $z(y)$ is sufficient. First derivative of $z(y)$ is equal to

$$\frac{dz(y)}{dy} = z'(y) = -c_p + (c_p + c_h)\Phi^\tau((y - \epsilon)/(1 - r))$$

(3.16)

where Φ^τ is cumulative density function of ϵ^τ . $z'(y)$ is monotonically non-decreasing so $z(y)$ is a convex function, hence $L(y)$ is a convex function.

To sum up, proving the convexity of $L(y)$ is sufficient to show that G_t is a convex function because all other terms are convex.

b) Since convexity preserve under minimization and $G_t$is a convex function

$$f_t(x) = \min_{y \geq x}\{G_t(y) - cx\}$$

hence, f_t is convex.

■

Theorem 1. A base stock policy is optimal in each period of a finite-horizon problem.

Proof: By assumption, the terminal cost function $v_T(x)$ is convex. Then, by Lemma 1 G_N is convex hence $f_N$is convex. Therefore, base stock policy is optimal for period N. For other periods it can be shown that $f_t$is convex by iterating backward through the periods in the sequence t=N,N-1,….1.

■

In this chapter, first, the assumptions that are made about the problem environment are explained. We begin to construct our model by defining terminal cost function v_T. In equation 3.2, the expected holding and shortage cost is defined. By the help of these two functions, optimality equations are derived. The boundary conditions are given in equation 3.4 and 3.5. The equations are re-written in 3.6, 3.7 and 3.8 in a way that they provide the setup for Lemma1 and Theorem 1. In section 3.3, it is shown that under specific conditions Porteus' problem environment and our problem environment are equivalent. By using equations 3.10, 3.12 and 3.13, the optimality of base stock policy can be proven like in Porteus' Theorem 4.2. In Theorem 4.2 Porteus proves that base stock policy is optimal in each period for finite horizon problem. In Lemma 1 and Theorem 1, we follow the steps of Porteus' except we consider more general case. In Lemma 1, to show convexity of f_t , since convexity of purchasing cost and convexity of expected future cost are trivial, we need to show that $L(y)$ is a convex function. Using equations 3.14, 3.15 and 3.16, it is shown that $L(y)$ is convex therefore f_t is convex. Theorem 1 shows that base stock policy is optimal in each period for finite horizon problem using Lemma 1 and by iterating backward through the periods.

Chapter 4

Computational Results and Comparison

In this section, we elaborate on the verification steps and analyze results of the model for varying value of parameters. We mainly analyze the effects of return proportion and length of period over total cost.

4.1. Verification of Computer Program

To explain the assumptions that were used during coding level, the results of the models that were developed in MATLAB and ARENA were compared and comparing MATLAB results with Porteus' result are steps of our verification method.

4.1.1 Assumptions

In this section, we explain the assumptions that were made during the implementation phase. We implemented our model by using MATLAB. The code of the model can be found in Appendix E. To verify our model, we use Porteus' results and we developed another model by using ARENA. The details of the verification are explained in the next sections.

We first assume that demand, the amount of returned items and inventory position are all discrete. This assumption is made because in a coding stage, we use matrix for demand, amount of returned items and inventory position where we need to define a base unit to assign matrix indices. Hence we decided to round decimal points and assume that demand, amount of returned items and inventory position are all integers.

The second assumption is made on the number of returned items. In the previous sections, we mention that a fixed proportion of demand is returned. By multiplying the satisfied demand with that proportion, the results are likely to be decimal numbers. We use this number to decide the next period state and inventory position that is not allowed to be decimal, therefore we need to round up the amount of returned items. Rounding is performed in a way that the amount of returned items is rounded up to the nearest integer.

The last assumption is made for demand and inventory position for first τ (lead-time) periods. We assume that the demand that occurs during the first τ period is equal to the inventory on hand. Demand is fully satisfied for the first τ periods and there is no excess inventory. This assumption was made because of the following reason; in ARENA, it is possible to have a simulation environment that uses sales to calculate the amount of returned items. However in MATLAB, we use demand to calculate the amount of returned items to preserve the Markovian property. Let us consider the following scenario; lead-time is four periods and starting inventory position is zero. Order is given and then demand occurs but demand is backlogged to satisfy later periods because the inventory on hand is zero. According to model that is implemented in ARENA, there is no return for second, third and fourth period because the first order would come in fifth period. Since all the backlogged demand is satisfied at the beginning of the fifth period, it means there is an extreme sale in the fifth period and it will cause an extreme amount of returned item for the next period. In model that is implemented in MATLAB, the return flow is smoother because demand is used to calculate the amount of returned item. By assuming demand that occurs during the first τ period is equal to the inventory on hand, we can guarantee that we also have a smooth return flow in the ARENA model.

4.1.2 Comparison of MATLAB results and ARENA results

To verify the fact that the solution found by preserving the Markovian property is not significantly different from the solution without preserving the Markovian assumption, an ARENA model that could simulate the problem environment was built. The ARENA model uses inventory level as an input in addition to the inputs used in MATLAB model. Basically, there are two differences between the ARENA model and MATLAB model. The first difference is that, in MATLAB we calculate the optimal order up to levels by dynamic programming approach whereas in ARENA we simulate the problem environment with specific order up to levels that are already found in MATLAB model. The second difference is that, the ARENA model calculates the amount of returned items based on sales of the previous period whereas in MATLAB the amount of returned items are calculated based on expected demand. The view of interfaces of the ARENA model is found in Appendix C and the codes are in appendix D. Since we are planning to analyze the results based on the return proportion and review frequency the table of results is divided into 12 parts. For each cell of Table 4.1, we have 72 results that are calculated in MATLAB. We pick 10 out of these 72 cases for each cell and run the ARENA model for cases individually. Hence we get the results of the ARENA model for 120 different cases. Average, minimum and maximum differences are calculated for these 10 different cases. The results can be seen in Tables 4.1, 4.2, 4.3 in terms of percentages.

		Return proportion			
		0	0.1	0.2	0.3
Number of Review Periods	12	0.0021	-0.0137	-0.0009	-0.0015
	6	0.0016	-0.0003	-0.0005	-0.0014
	3	0.0027	0.0001	-0.0006	-0.0013

Table 4. 1 Average of % cost difference result of models that are developed in MATLAB and ARENA

		Return proportion			
		0	0.1	0.2	0.3
Number of Review Periods	12	0	-0.0152	-0.0027	-0.0033
	6	0.0009	-0.0007	-0.0028	-0.0046
	3	0.0007	-0.0006	-0.0022	-0.0039

Table 4. 2 Minimum of % cost difference result of models that are developed in MATLAB and ARENA

		Return proportion			
		0	0.1	0.2	0.3
Number of Review Periods	12	0.0032	-0.0130	0.0004	0.0006
	6	0.0028	0.0006	0.0048	0.0017
	3	0.0062	0.0009	0.0011	0.0009

Table 4. 3 Maximum of % cost difference result of models that are developed in MATLAB and ARENA

As it can be seen from the tables above, the difference between the two models is insignificant. The results support the assumption which was made earlier to preserve the Markovian property does not always yield any significant difference in cost calculation. However the assumption provides us the chance of utilizing dynamic programming approach with less number of states.

4.1.3 Verification of MATLAB results by Porteus' results

As a second method for verification, we use theorem 4.3 that Porteus states. The Porteus' model and our model is equivalent when we set $r = 0$ and $c = g = b$. Theorem 4.3 states that the optimal order up to the levels for inventory positions are equal for every period in the specified environment. Let order up to levels be equal to S where S satisfies;

$$\Phi(S) = \frac{c_p - (1 - \alpha)c}{c_p + c_h} \tag{4.1}$$

Φ represents the cumulative distribution function of demand. In the table of results, there are 36 cases that are equivalent to Porteus' problem environment. The results of all of these cases satisfy Porteus' Theorem 4.3. Hence, it can be said that our model was correctly built and implemented.

4.2. Results

In this section we present results that are calculated in MATLAB. We calculate optimal order up to levels according to the values of parameters that are given in Table 4.4.

Parameters	Values
c	100, 120
c_p	5, 10, 15
c_h	1, 5
g	80, 100
b	100, 120
α	0.99, 1
r	0, 0.1, 0.2, 0.3

Table 4. 4 Values of parameters that are used to generate different cases

These values are considered for one time-bucket. They are recalculated with respect to period length in Appendix A. Let period length be equal to 4 time-buckets then c_p gets the values 20, 40 and 60 and c_h gets the values 4, 20. α is also recalculated with respect to period length. Parameters $c,\ g,\ b$ do not vary with respect to period length.

We consider a time-horizon with a length of 12 time-buckets. A review period can consist of 1, 2 or 4 time-buckets. In other words, we consider 12, 6 or 3 period problem respectively.

The demand for a time bucket is discretizing from normal distribution with mean 20 and standard deviation 2. The probability distribution of a demand for a period is calculated using determined distribution of time bucket. For instance, if a period consist of 4 time bucket then demand for this period has a probability distribution with mean 40 and standard deviation 4.

As we explained in the previous section in first τ periods, there is an enough inventory to satisfy demand and also there is no excess inventory for first τ periods. Let us consider a 12 period problem where a period length is equal to a time bucket length. Let r be equal to 0.2. Inventory on hand for the first period is equal to mean of demand which is 20, and 16 for other periods. Demand that occurs for first τ periods is 20 for each period. Therefore the first period sales is equal to demand and it is 20 and that means 4 items will be returned and can be resold at the second period. Inventory on hand in the second period is increased from 16 to 20. This procedure continues in the same manner until the first order arrives. After first order arrives, demand is random variable with mean 20 and standard deviation 2. 6 period and 3 period problems also have the similar scenarios. The results that are taken from MATLAB runs can be seen in Appendix A.

4.3. Discussion of Results

In this section, we interpret the results of the model for varying values of the parameters, review frequency and return proportion. We conduct a set of analysis to explore the relations between cost, review frequency and return proportion. In the analyses, comparisons are made over expected total cost. Expected total costs are calculated according to optimality equations that are defined in Chapter 3.

Since the unsatisfied demand is backordered to satisfy next period, at the end of time horizon all the demand is satisfied. This situation is valid for all cases independent from period length and return proportion. Hence the total cost includes a part that inevitably occurs because of cost of purchasing product to satisfy demand. This cost was treated as sunk cost and the analyses are conducted second time by disregarding sunk cost. Purchasing time can vary with period length, therefore holding and shortage cost is a function of review frequency. Disregarding sunk cost enables us to identify the relation between review frequencies and return proportion with remaining costs more clearly. The sunk cost can take different values in order to return proportion and discount rate. The discounted sunk cost can be calculated but using formula 4.1. In the analyses we disregard the minimum value of the sunk cost which is calculated according to formula 4.1. Sunk cost takes its minimum value when $\mathrm{r} = 0.3$.

$$\text{Sunk cost}_{\mathrm{r},\alpha} = \mathrm{c} * (\text{expected horizon demand} * (1 - \mathrm{r}) - \text{order lead time demand}) * \alpha^{\omega} \tag{4.1}$$

where $\omega = number\ of\ time\ buckets\ in\ (time\ horizon - order\ lead\ time)$

Firstly, we analyze the effect of frequency of review over total cost for each return proportion. We compare the cost of two cases where they only differentiate from each other by the frequency of review. The details of analysis can be found in 4.3.1.

Secondly, we investigate the relation between purchase cost and advantage of high review frequency. Two cases are chosen and the only difference between them is the purchase cost and the costs of these two cases are calculated for different period lengths. We compare costs and try to identify which purchase costs are more advantageous in terms of cost under what conditions. The details of the analysis can be found in 4.3.2.

Lastly, the advantage of switching return proportion from zero to other values is analyzed. In this analysis, cases where there are no returned items are taken as base cases. We investigate in which period length is more advantageous to switching no return case to return cases. We repeat this analysis for each return probability. Details and figures can be found in section 4.3.3

The figures and tables in section 4.3.1, 4.3.2 and 4.3.3 belongs to cases where $\alpha = 1$. Figures and tables for cases $\alpha = 0.99$ can be found in Appendix B.

As a notification, we use "*realistic* saving" as a term in the analyses where sunk cost is disregarded. When total cost that contains sunk cost is compared we use "saving" only.

4.3.1 Analysis of savings of switching between period lengths for each return proportion

The first analysis is conducted to identify the relation between period length and cost. We observe a change in cost in terms of percentage for each case. In other words, we investigate the percentage of savings of switching from one period length to another. The formula that is used to calculate this percentage can be written as follows;

$$\% \text{ savings of switching from X review periods to Y review periods} =$$

$$100 * \frac{\text{Expected Total Cost(X period)} - \text{Expected Total Cost(Y period)}}{\text{Expected Total Cost(X period)}} \tag{4.2}$$

Let us consider case number 1 in Table A.10 as an example where the number of review periods is 3 and the total cost is 14515. Now consider case number 1 in Table A.6 where the number of review period is 6 and the total cost is 14480. These two cases have same parameters but only differ from each other by period length. Then savings of switching 3 review period problem to 6 review period problem for case 1 when $r = 0.1$ is

$$100 * \frac{14515 - 14480}{14515} = 0{,}24$$

The percentages for switching from 3 review periods to 6 review periods and 3 review periods to 12 review periods are calculated. The summary of results can be found in Table 4.5.

Let first focus on cases where $r = 0$; the reason of saving is controlling inventory position and increasing frequency of orders. Since there is no return for these cases, cost advantage comes solely from frequency of controlling inventory position. When r is not equal to zero there is a chance to use return information in decision points. As a verification of this claim, from Table 4.5, it can be seen that for all return proportion the average saving is higher than no return situation.

Same analysis is conducted with cost such that sunk cost is disregarded. In this case the effect of switching between numbers of period can be explored more explicitly. Formula 4.2 is modified in a way that sunk cost is disregarded.

$$\%\, realistic \text{ savings of switching from X review periods to Y review periods} =$$

$$100 * \frac{\text{Expected Total Cost(X period)} - \text{Expected Total Cost(Y period)}}{\text{Expected Total Cost(X period)} - \text{Sunk Cost}} \tag{4.3}$$

By disregarding sunk cost we extract the inevitable cost from total cost. Hence we focus on the cost that can be influenced by change in number of period. Summary of results can be found in Table 4.6.

In both analyses, the percentage of saving is largest for all cases when $r = 0.3$ for both switching from 3 review periods to 6 review periods and 3 review periods to 12 review periods. From Figure 4.1 and 4.2, it can be concluded that the magnitude of the percentage of realistic savings increases with return proportion.

		r = 0	r = 0.1	r = 0.2	r = 0.3
% savings of switching 3 reviews to 6 reviews	Minimum	0.06	0.08	0.07	0.12
	Maximum	0.41	0.47	0.52	0.61
	Average	0.23	0.25	0.28	0.35
% savings of switching 3 review to 12 review	Minimum	0.09	0.11	0.12	0.16
	Maximum	0.68	0.75	0.85	1.00
	Average	0.37	0.41	0.46	0.57

Table 4. 5 Summary of % savings of switching 3 reviews to 6 reviews or 12 reviews for all return proportions

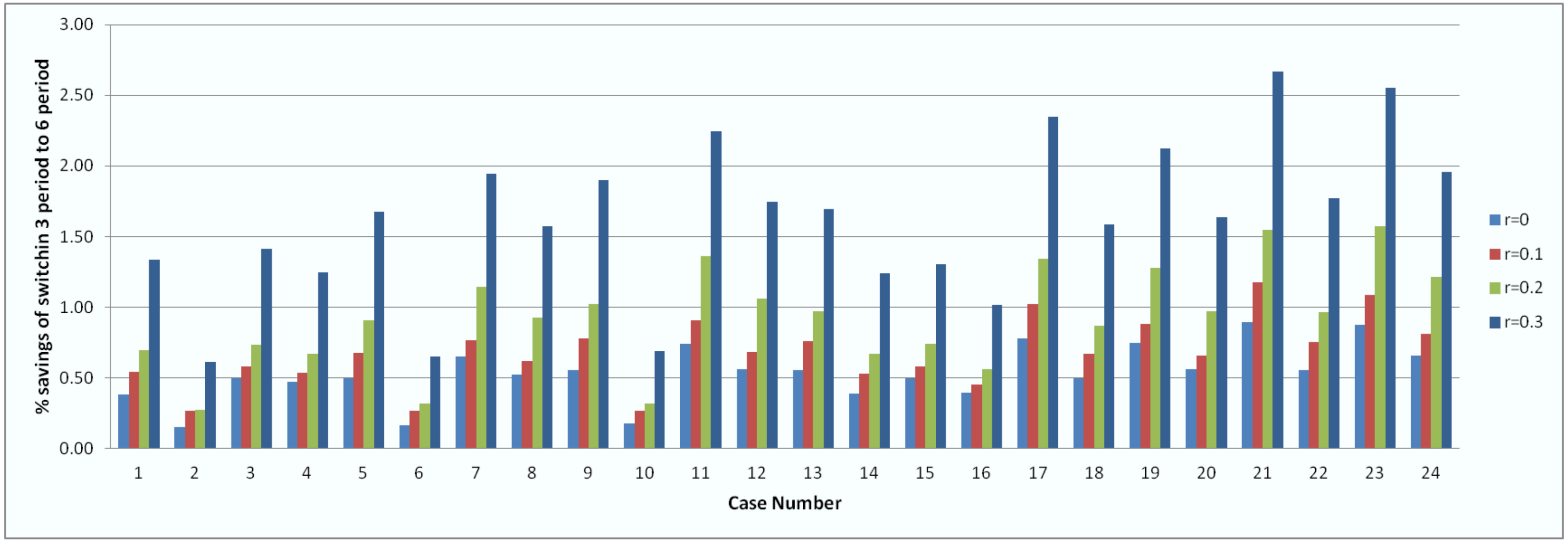

Figure 4.1Realistic savings of switching 3 reviews to 6 reviews in terms of percentage for all return proportions

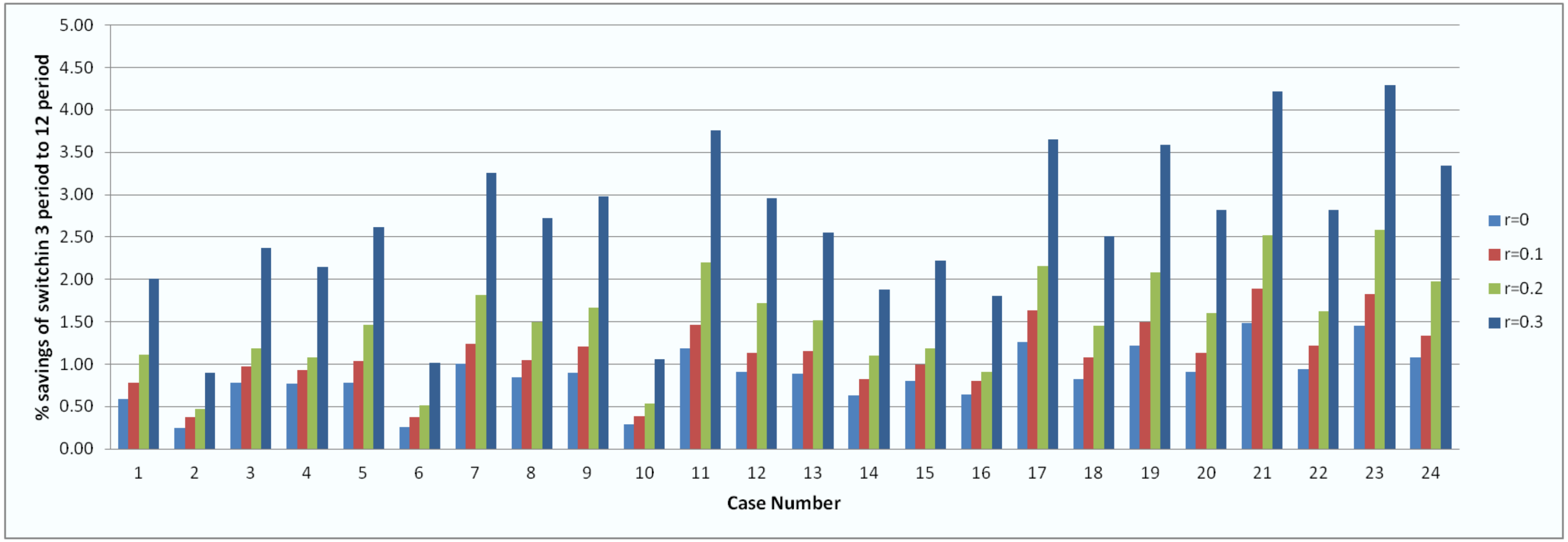

Figure 4.2 Realistic savings of switching 3 reviews to 12 reviews in terms of percentage for all return proportions

		r = 0	r = 0.1	r = 0.2	r = 0.3
% realistic savings of switching 3 reviews to 6 reviews	Minimum	0.14	0.21	0.22	0.53
	Maximum	0.90	1.18	1.57	2.67
	Average	0.50	0.64	0.87	1.54
% realistic savings of switching 3 reviews to 12 reviews	Minimum	0.21	0.28	0.37	0.73
	Maximum	1.48	1.89	2.58	4.29
	Average	0.82	1.04	1.42	2.52

Table 4.6 Summary of % realistic savings of switching 3 reviews to 6 reviews or 12 reviews for all return proportions

4.3.2 Analysis of the advantage of operating inventory system with more review periods for various unit cost items

In this analysis, we try to explore whether operating with more review periods with consumer returns in a fixed time horizon is advantageous for units with a high unit purchase cost or units with low unit purchase cost. This advantage is defined with following formula;

$$\text{Z} = \text{The advantage of operating with more review periods with high unit purchase cost instead of low unit purchase cost items}$$

$$\text{Z} = \text{\% savings of switching from X periods to Y periods (high c)} - \text{\% savings of switching from X periods to Y periods (low c)} \tag{4.4}$$

Let us consider Case 2 in Table A.11, we formulate a way to calculate the percentage of savings of switching from 3 review period to 6 review period in the previous section, formula 4.2. We calculate the same percentage for Case 25 in Table A.11. These two cases have the same parameter sets except they differentiate from each other by unit purchase cost. Case 25 has higher purchase cost than Case 2. We have percentages of savings of these two cases; hence we apply the formula 4.4 to get the results. The comparison of operating 6 review periods instead of 3 review period problem for items that have a unit purchase cost of 120 with items that have a unit purchase cost of 100 can be found in Figure 4.3. In Figure 4.4, the same analysis is conducted for operating 12 review periods instead of 3 review period setting. The summary of result can be found in Table 4.7.

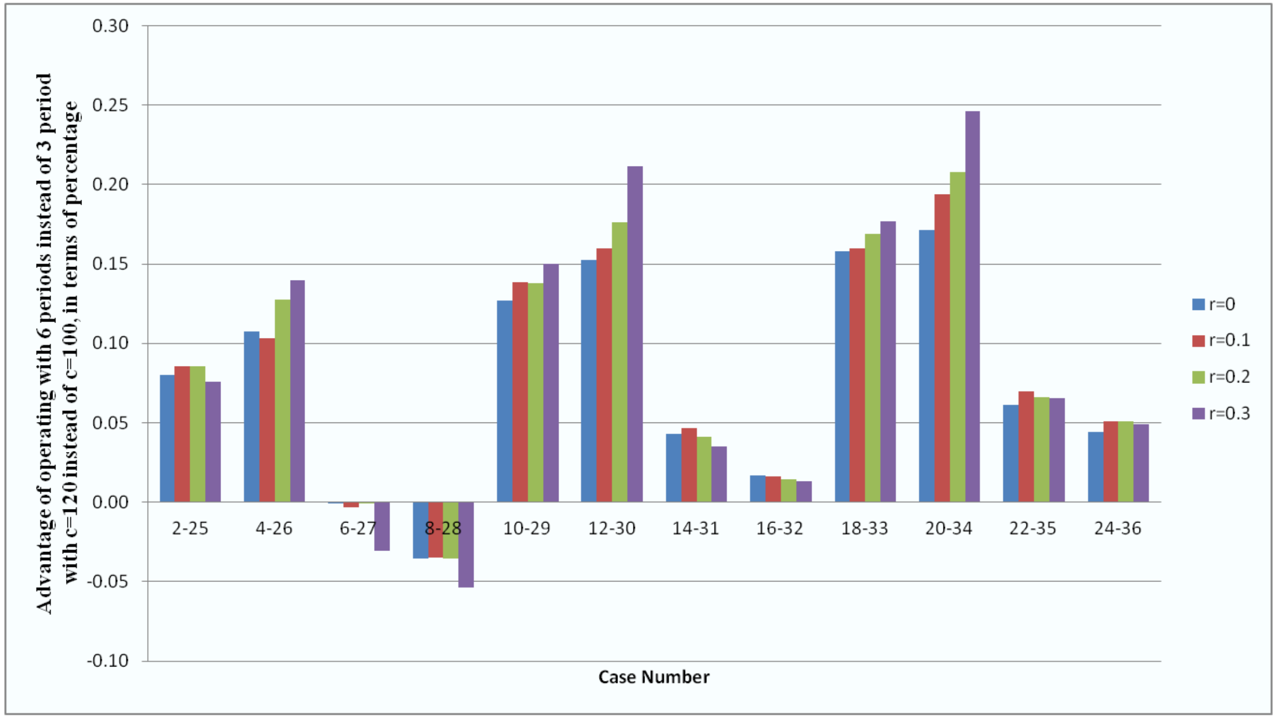

Figure 4.3 Advantage of operating with 6 reviews instead of 3 reviews with c=120 instead of c=100, in terms of percentage

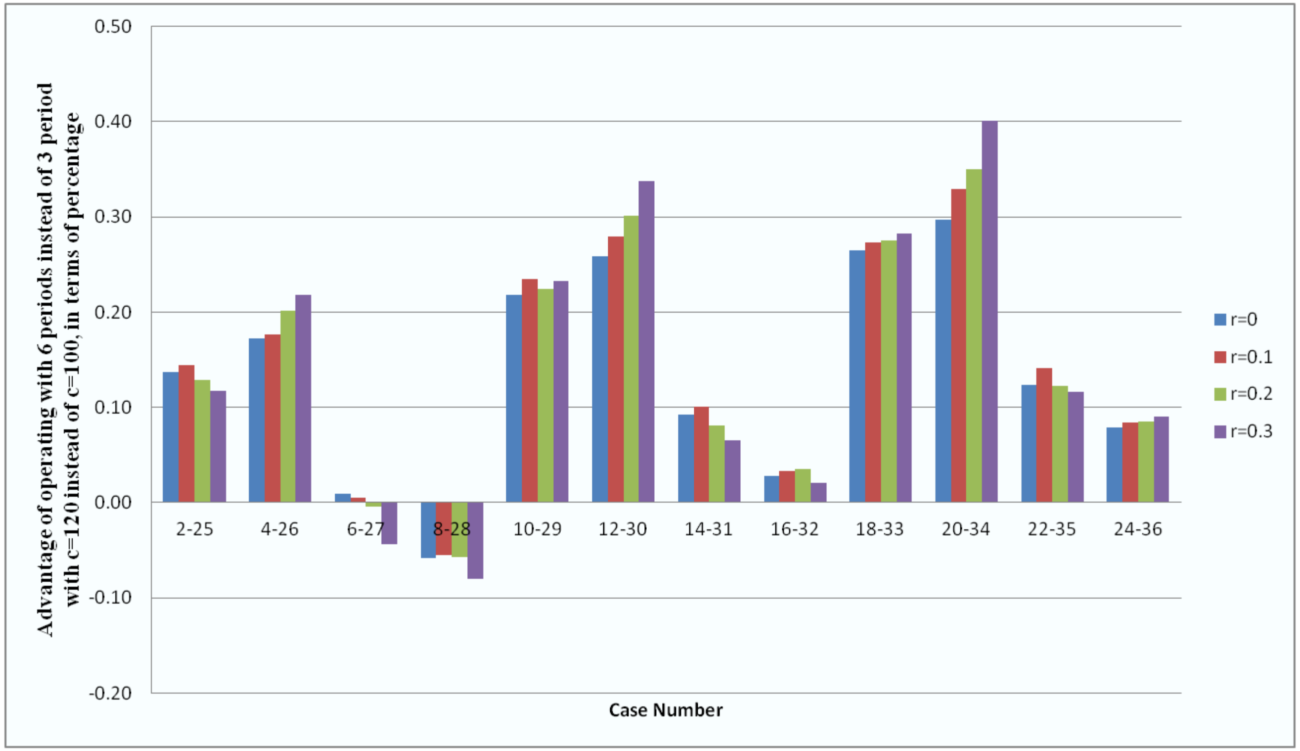

Figure 4.4 Advantage of operating with 12 reviews instead of 3 reviews with c=120 instead of c=100, in terms of percentage

		r = 0	r = 0.1	r = 0.2	r = 0.3
Advantage of operating with 6 reviews instead of 3 reviews with c=120 instead of c=100, in terms of percentage	Minimum	-0.04	-0.04	-0.04	-0.05
	Maximum	0.17	0.19	0.21	0.25
	Average	0.08	0.08	0.09	0.09
Advantage of operating with 6 reviews instead of 3 reviews with c=120 instead of c=100, in terms of percentage	Minimum	-0.06	-0.06	-0.06	-0.08
	Maximum	0.30	0.33	0.35	0.40
	Average	0.14	0.15	0.15	0.15

Table 4. 7 Extra advantage of shortening period length of items when purchased cost $\mathbf{c_p = 120}$ instead of $\mathbf{c_p = 100}$, in terms of percentage

From Figure 4.3 and 4.4 it can be concluded that the advantage depends on unit holding and unit shortage cost. The advantage of operating with high unit cost item is lost as the difference between unit holding and unit shortage cost decreases. The reason can be explained with value of return information. The return information is used to prevent backordered cost. When the difference between shortage cost and holding cost decreases, the value of the return information also decreases. In these cases value of return information takes its minimum value and the advantage that can be gained by using return information is insignificant. There are some cases that the advantage of operating with low unit cost item is higher. Because the value of return information can be assumed insignificant and lower unit cost yields lower total cost, hence the advantage in terms of percentage is greater for lower unit cost items.

The same claims are valid for the analyses that are conducted by disregarding sunk cost. The summary of results can be found in Table 4.8.

		r = 0	r = 0.1	r = 0.2	r = 0.3
Advantage of operating with 6 reviews instead of 3 reviews with c=120 instead of c=100, in terms of percentage	Minimum	-0.08	-0.09	-0.11	-0.23
	Maximum	0.38	0.49	0.65	1.08
	Average	0.17	0.21	0.27	0.41
Advantage of operating with 6 reviews instead of 3 reviews with c=120 instead of c=100, in terms of percentage	Minimum	-0.12	-0.14	-0.17	-0.34
	Maximum	0.65	0.83	1.09	1.77
	Average	0.30	0.37	0.46	0.66

Table 4.8 Extra advantage of shortening period length of items when purchased cost $c_p = 120$ instead o f$c_p = 100$, in terms of percentage

4.3.3 Analyses related to switching from no return case to return cases

4.3.3.1 Analysis of percentage of savings with switching from no return case to return cases for all period lengths

Finally, we explore the relation between return proportion and advantage of operating with more review periods. In this analysis, the cases where $r = 0$ are used as a base case to identify which number of period is most advantageous for switching no return case to return cases. Then the formula for calculating the percentage of savings of switching no return case to return cases for a defined period length is as follows;

$$\% \text{ savings of switching from } r = 0 \text{ to } r = k \text{ for a X review period problem} =$$

$$\frac{100 * (\text{Expected Total Cost(X period and } r = 0\text{)} - \text{Expected Total Cost(Y period and } r = k))}{\text{Expected Total Cost(X period and } r = 0\text{)}} \qquad (4.5)$$

Let us consider Case 1 in Table A.9, the total cost is 16134 where $r = 0$ and number of period is 3. Case 1 in Table A.10 has the same parameter set except $r = 0.1$ rather than 0 and its total cost is 14515. Then percentage of savings of switching from $r = 0$ to $r = 0.1$ for 3 review period problem environment with $\alpha = 1$ is

$$100 * \frac{(16134) - (14515)}{16134}$$

In Table 4.9, the summary of results can be found for all period numbers and return proportions.

Switching no return case to return case always yields positive gain, independent from return proportion. The percentage of realistic savings of switching from $r = 0$ to $r = 0.3$ is the most advantageous cases for all period numbers. This percentage is proportional to the number of review periods hence the percentage of saving is maximized when $r = 0.3$ and period number is equal to 12.

In previous analysis expected total costs contain sunk cost. In Table 4.10, the summary of results that disregards sunk cost can be found for all period and return proportion. The formula to calculate percentages is as follow.

$$\frac{100 * (\text{Expected Total Cost}(\text{X period and r} = 0\ ) - \text{Expected Total Cost}(\ \text{Y period and r} = \text{k}))}{\text{Expected Total Cost}(\text{X period and r} = 0\ ) - \text{Sunk Cost}} \quad (4.6)$$

The characteristic of results in Table 4.10 are the same with results in Table 4.9 Therefore previous claims are valid for these results. Only difference between two analyses that is worth to be mentioned is by disregarding sunk cost we eliminate the cost which is inevitably part of the total cost. Therefore the effect of switching no return case to return case is magnified.

		3 reviews	6 reviews	12 reviews
% savings of switching from r=0 to r=0.1	Minimum	10.00	10.05	10.06
	Maximum	10.15	10.17	10.18
	Average	10.07	10.09	10.11
% savings of switching from r=0 to r=0.2	Minimum	19.90	19.94	19.96
	Maximum	20.00	20.05	20.09
	Average	19.96	20.00	20.02
% savings of switching from r=0 to r=0.3	Minimum	29.78	29.91	29.97
	Maximum	30.00	30.04	30.06
	Average	29.89	29.97	30.03

Table 4. 9 Summary of results for all period lengths and for all return proportions.

		3 reviews	6 reviews	12 reviews
% realistic savings of switching from r=0 to r=0.1	Minimum	21.84	21.98	22.07
	Maximum	22.25	22.30	22.31
	Average	22.03	22.14	22.21
% realistic savings of switching from r=0 to r=0.2	Minimum	43.02	43.37	43.60
	Maximum	44.22	44.26	44.31
	Average	43.65	43.86	44.00
% realistic savings of switching from r=0 to r=0.3	Minimum	64.28	64.82	65.21
	Maximum	66.31	66.44	66.49
	Average	65.38	65.74	65.98

Table 4. 10 Summary of results without sunk cost for all period lengths and for all return proportions.

4.3.3.2 Analysis of switching from no return case and 3 review periods to return cases with 12 review periods

In this sub-section, we make a comparison very similar to the analysis in 4.3.3.1. We investigate the effects of using advantage of return information together with increasing review frequency. Therefore, the cases with 3 periods with no returns are taken as a base case and the cases with 12 periods with returns are used for comparison. The results can be found in Table 4.11.The formulation that is given in 4.5 is used as follow to calculate the percentages of savings.

$$= \frac{\mathbf{100} * (\mathbf{Exp.\,Total\,Cost}(\mathbf{3\ period\ and\ r = 0}) - \mathbf{Exp.\,Total\,Cost}(\mathbf{12\ period\ and\ r = k}))}{\mathbf{Exp.\,Total\,Cost}(\mathbf{3\ period\ and\ r = 0})} \quad (4.7)$$

The same analysis is conducted by disregarding sunk cost. The summary of results can be found in Table 4.12. In this case formula 4.7 is modified as follow.

$$= \frac{\mathbf{100} * (\mathbf{Exp.\,Total\,Cost}(\mathbf{3\ period\ and\ r = 0}) - \mathbf{Exp.\,Total\,Cost}(\mathbf{12\ period\ and\ r = k}))}{\mathbf{Exp.\,Total\,Cost}(\mathbf{3\ period\ and\ r = 0}) - \mathbf{Sunk\ Cost}} \quad (4.8)$$

For both tables Table 4.8 and Table 4.12, the percentage of saving is maximum on the average when switching is from 3 period to 12 period and $r = 0$ to $r = 0.3$. The result that is represented in Table 4.11 (4.12) can be compared with the result in column 3 of Table 4.9 (4.10). Table 4.7 (4.10) solely represents the advantage of switching from no return case to return cases. In Table 4.11 (4.12), there is an additional factor that increases the percentage of savings and this is the increase in frequency of the review.

		12 period
% savings of switching from 3 reviews r=0 to 12 reviews r=0.1	Minimum	10.16
	Maximum	10.75
	Average	10.44
% savings of switching from 3 reviews r=0 to 12 reviews r=0.2	Minimum	20.09
	Maximum	20.60
	Average	20.32
% savings of switching from 3 reviews r=0 to 12 reviews r=0.3	Minimum	30.11
	Maximum	30.50
	Average	30.29

Table 4. 11 Summary of results % savings of switching from 3 reviews with $\mathbf{r = 0}$ to 12 reviews with various return proportions

		12 period
% realistic savings of switching from 3 reviews r=0 to 12 reviews r=0.1	Minimum	22.47
	Maximum	23.33
	Average	22.85
% realistic savings of switching from 3 reviews r=0 to 12 reviews r=0.2	Minimum	44.08
	Maximum	44.88
	Average	44.46
% realistic savings of switching from 3 reviews r=0 to 12 reviews r=0.3	Minimum	65.62
	Maximum	66.67
	Average	66.26

Table 4. 12 Summary of results % realistic savings of switching from 3 reviews with r=0 to 12 reviews with various return proportions

4.3.4 Analysis of using optimal order up to levels of no return case in a return case

In this analysis is conduct to identify the increase in total cost if optimal order up to levels of no return case is used in a system where consumer returns are occurred. In other words, the order up to levels are decided under the assumption of there are not consumer returns, but consumers return some items that can be resold at the next period. By not taking returns into consideration in decision level, system is operated with order up to levels that are not optimal for the system. In this analysis, we investigate the increase in total cost, in other words loss of not operating optimally. The summary of results can be found in Table 4.13. The formula of loss of not operating optimally is as follow;

$$\mathrm{Q} = \text{Expected Total Cost}(\text{optimal order up to levels of } \mathrm{r} = 0 \text{ in a system where } \mathrm{r} = \mathrm{k})$$

$$\%\ \text{loss of not operating optimally when } \mathrm{r} = \mathrm{k} \text{ in X review period problem} =$$

$$\frac{100 * (\mathrm{Q} - \text{Expected Total Cost}(\text{X period and } \mathrm{r} = \mathrm{k}))}{\text{Expected Total Cost}(\text{X period and } \mathrm{r} = \mathrm{k})} \tag{4.9}$$

The same analysis is conducted by disregarding sunk cost. The summary of results can be found in Table 4.14. In this case formula 4.7 is modified as follow;

$$\mathrm{Q} = \text{Expected Total Cost}(\text{optimal order up to levels of } \mathrm{r} = 0 \text{ in a system where } \mathrm{r} = \mathrm{k})$$

$$\%\ realistic \text{ loss of not operating optimally when } \mathrm{r} = \mathrm{k} \text{ in X review period problem} =$$

$$\frac{100 * (\mathrm{Q} - \text{Expected Total Cost}(\text{X period and } \mathrm{r} = \mathrm{k}))}{\text{Expected Total Cost}(\text{X period and } \mathrm{r} = \mathrm{k}) - \text{Sunk Cost}} \tag{4.10}$$

From Table 4.13 ve Table 4.14, it can be concluded that percentage of loss is maximum when $r = 0.3$, because the amount of returned that is ignored during calculation of optimal order up to levels is highest when $r = 0.3$.

These results can be interpreted as what is the penalty or extra cost of not acting optimally. When order up to points are decide according to $r = 0$ where system actually has return proportion 0.1, the loss is around %1.3 of total cost for all period lengths. This percentage increases with return proportion and it is around %6 for $r = 0.2$ and %11 for $r = 0.3$.

		3 reviews	6 reviews	12 reviews
% loss of not operating optimally when r=0.1	Minimum	0.39	0.39	0.37
	Maximum	1.86	2.17	2.39
	Average	1.23	1.32	1.33
% loss of not operating optimally when r=0.2	Minimum	0.94	0.88	0.93
	Maximum	5.75	6.22	6.50
	Average	3.82	3.75	3.97
% loss of not operating optimally when r=0.3	Minimum	1.65	1.67	1.45
	Maximum	10.82	11.39	11.79
	Average	7.24	7.44	7.10

Table 4.13 % loss of not operating optimally

		3 reviews	6 reviews	12 reviews
% realistic loss of not operating optimally when r=0.1	Minimum	0.99	0.99	0.94
	Maximum	4.65	5.44	6.00
	Average	3.09	3.33	3.38
% realistic loss of not operating optimally when r=0.2	Minimum	2.98	2.78	2.97
	Maximum	17.54	19.13	20.07
	Average	11.84	11.70	12.42
% realistic loss of not operating optimally when r=0.3	Minimum	7.59	7.72	6.60
	Maximum	46.58	49.62	51.87
	Average	31.97	33.21	31.96

Table 4. 14 % realistic loss of not operating optimally

4.4. Importance with respect to return policy application

In this section, we sum up the results of the analyses in terms of return policy. In the analyses that are conducted over total costs including sunk cost percentage of savings and advantage of operating with more review periods is smaller than the cases which exclude sunk cost. The result of both cost structures implies same outcomes. The percentages of savings are interpreted as upper bounds because in the problem environment remanufacturing costs are disregarded. Hence, in the environment that considers remanufacturing cost there is a chance to get smaller percentages.

There is another point that needs to be emphasized. That is when number of review period is increased, the number of returned item that are resold increases as well. Because returns that are related to last review period sales are disregarded so when the length of the review period is longer, sales is higher which means amount of returned items is higher. By disregarding last period returns, the amount of returns that are disregarded in long review period environment is higher than short review period environment.

The first outcome of our analyses is that if an inventory system already adopts return policy then increasing frequency of review yields higher savings in terms of percentage.

The second outcome is operating with more review periods in a fixed time horizon is generally more advantageous for items that have high unit purchase cost instead of items that have low unit purchase costs. This advantage is lost as the difference between unit holding and unit shortage cost decreases.

The last outcome is that allowing consumer returns in an inventory system yield positive savings in terms of cost. The magnitudes of savings depend on the number of period, review frequency, in time horizon where time horizon is assumed to be a fixed length. The percentage of savings is higher when review frequency takes its maximum value. The percentage of savings is maximized when review frequency and return proportion is high.

Chapter 5

Conclusion

In this thesis, we conducted a study on determining the optimal order up to levels for an inventory system where return of an item is allowed. We assume that a fixed proportion of items sold are returned by customers. There are lead-times for returned items and orders. Optimal orders up to levels are calculated for systems with different return proportions and different review frequency of inventory. We verify our results using a simulation and Porteus' theorem.

We compare the percentage of savings of switching between various review frequencies and return proportions to know whether some managerial insights under the assumptions of change in review frequency is feasible and there is no extra cost for handling returned items. Some critical insights are as follows. Firstly, increasing frequency of controlling inventory position is more advantageous in terms of percentage savings when the return proportion is high. Secondly, increasing review frequency yields more advantage in terms of percentage of savings when purchase cost is high. This savings proportionally increases with the increase in return proportion. Lastly, we observe that switching from no return cases to return cases yields positive savings. This advantage is increase with return proportion and period lengths.

A possible extension of this study could be done by transforming our model and assuming that the returns spend stochastic sojourn times in the market and after they are returned there is remanufacturing cost and time. In this case, lead-time for returns is uncertain and if there is a lead-time for orders then, it is expected that Markovian property cannot hold. Hence, calculating optimal order up to levels becomes more complex. Another extension could be done by finding the optimal return proportion level by assuming there is a relation between demand and return proportion. An example of this kind of relation is defined by Ketzenberg and Zuidwijk (2009).

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Appendix

Appendix A

In this section, we provide optimal order up to levels and total costs for different cases.

Case Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	15	15
c_h	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5	1	1
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	104	104	104	104	100	100	100	100	106	106	106	106	102	102	102	102	107	107
	104	104	104	104	100	100	100	100	106	106	106	106	102	102	102	102	107	107
	104	104	104	104	100	100	100	100	106	106	106	106	102	102	102	102	107	107
	104	104	104	104	100	100	100	100	106	106	106	106	102	102	102	102	107	107
	104	104	104	104	100	100	100	100	106	106	106	106	102	102	102	102	107	107
	104	104	104	104	100	100	100	100	106	106	106	106	102	102	102	102	107	107
	104	104	104	104	100	100	100	100	106	106	106	106	102	102	102	102	107	107
	96	100	104	108	96	100	100	104	98	101	106	108	97	101	102	105	99	101
Total Cost	16079	16130	16054	16057	16159	16215	16144	16159	16107	16146	16065	16067	16226	16270	16196	16207	16125	16158

Case Number	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	15	15	15	15	15	15	5	5	5	5	10	10	10	10	15	15	15	15
c_h	1	1	5	5	5	5	1	1	5	5	1	1	5	5	1	1	5	5
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	107	107	103	103	103	103	104	104	100	100	106	106	102	102	107	107	103	103
	107	107	103	103	103	103	104	104	100	100	106	106	102	102	107	107	103	103
	107	107	103	103	103	103	104	104	100	100	106	106	102	102	107	107	103	103
	107	107	103	103	103	103	104	104	100	100	106	106	102	102	107	107	103	103
	107	107	103	103	103	103	104	104	100	100	106	106	102	102	107	107	103	103
	107	107	103	103	103	103	104	104	100	100	106	106	102	102	107	107	103	103
	107	107	103	103	103	103	104	104	100	100	106	106	102	102	107	107	103	103
	107	109	99	101	103	105	94	96	94	96	96	98	96	97	97	99	97	99
Total Cost	16071	16072	16269	16305	16229	16237	19286	19279	19365	19359	19320	19307	19437	19426	19345	19325	19486	19469

Table A. 1 Total Costs for 12 reviews when $r = 0$ and $\alpha = 1$

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	15	15
c_h	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5	1	1
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	94	94	94	94	92	92	92	92	95	95	95	95	93	93	93	93	95	95
	95	95	95	95	92	92	92	92	96	96	96	96	93	93	93	93	96	96
	95	95	95	95	92	92	92	92	96	96	96	96	93	93	93	93	97	97
	96	96	96	96	92	92	92	92	97	97	97	97	94	94	94	94	98	98
	97	97	97	97	92	92	92	92	98	98	98	98	94	94	94	94	98	98
	97	97	97	97	92	92	92	92	98	98	98	98	94	94	94	94	98	98
	97	97	97	97	92	92	92	92	98	98	98	98	94	94	94	94	98	98
	88	92	96	99	88	92	92	96	90	93	98	100	90	92	94	96	91	93
Total Cost	14459	14507	14436	14438	14524	14577	14509	14524	14483	14520	14444	14446	14579	14621	14551	14561	14500	14530

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	15	15	15	15	15	15	5	5	5	5	10	10	10	10	15	15	15	15
c_h	1	1	5	5	5	5	1	1	5	5	1	1	5	5	1	1	5	5
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	95	95	93	93	93	93	94	94	92	"	95	95	93	93	95	95	93	93
	96	96	94	94	94	94	95	95	92	92	96	96	93	93	96	96	94	94
	97	97	94	94	94	94	95	95	92	92	96	96	93	93	97	97	94	94
	98	98	95	95	95	95	96	96	92	92	97	97	94	94	98	98	95	95
	98	98	95	95	95	95	97	97	92	92	98	98	94	94	98	98	95	95
	98	98	95	95	95	95	97	97	92	92	98	98	94	94	98	98	95	95
	98	98	95	95	95	95	97	97	92	92	98	98	94	94	98	98	95	95
	98	100	91	93	95	97	87	88	87	88	88	90	88	90	89	91	89	91
Total Cost	14449	14450	14615	14649	14577	14584	17344	17338	17409	17403	17374	17362	17469	17458	17397	17378	17510	17494

Table A. 2 Total Costs for 12 reviews when $\mathbf{r = 0.1}$ and $\boldsymbol{\alpha = 1}$

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	15	15
c_h	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5	1	1
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	86	86	86	86	84	84	84	84	87	87	87	87	85	85	85	85	87	87
	87	87	87	87	84	84	84	84	87	87	87	87	85	85	85	85	88	88
	87	87	87	87	84	84	84	84	88	88	88	88	85	85	85	85	89	89
	87	87	87	87	84	84	84	84	89	89	89	89	85	85	85	85	89	89
	88	88	88	88	87	87	87	87	89	89	89	89	87	87	87	87	90	90
	88	88	88	88	87	87	87	87	89	89	89	89	87	87	87	87	90	90
	88	88	88	88	87	87	87	87	89	89	89	89	87	87	87	87	90	90
	81	84	88	91	80	84	84	88	82	85	89	91	82	84	86	88	83	85
Total Cost	12861	12904	12840	12842	12932	12979	12918	12932	12883	12916	12848	12849	12972	13010	12948	12957	12898	12925

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	15	15	15	15	15	15	5	5	5	5	10	10	10	10	15	15	15	15
c_h	1	1	5	5	5	5	1	1	5	5	1	1	5	5	1	1	5	5
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	87	87	85	85	85	85	86	86	84	84	87	87	85	85	87	87	85	85
	88	88	86	86	86	86	87	87	84	84	87	87	85	85	88	88	86	86
	89	89	86	86	86	86	87	87	84	84	88	88	85	85	89	89	86	86
	89	89	86	86	86	86	87	87	84	84	89	89	85	85	89	89	86	86
	90	90	87	87	87	87	88	88	87	87	89	89	87	87	90	90	87	87
	90	90	87	87	87	87	88	88	87	87	89	89	87	87	90	90	87	87
	90	90	87	87	87	87	88	88	87	87	89	89	87	87	90	90	87	87
	90	91	83	85	87	88	79	81	79	80	81	82	81	82	82	83	81	83
Total Cost	12852	12853	13002	13033	12968	12975	15427	15421	15497	15492	15454	15443	15542	15532	15475	15458	15577	15562

Table A. 3 Total Costs for 12 reviews when $\mathbf{r = 0.2}$ and $\boldsymbol{\alpha = 1}$

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	15	15
c_h	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5	1	1
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	78	78	78	78	76	76	76	76	79	79	79	79	77	77	77	77	79	79
	78	78	78	78	76	76	76	76	79	79	79	79	77	77	77	77	80	80
	79	79	79	79	76	76	76	76	80	80	80	80	77	77	77	77	80	80
	79	79	79	79	76	76	76	76	80	80	80	80	77	77	77	77	81	81
	81	81	81	81	78	78	78	78	81	81	81	81	78	78	78	78	81	81
	81	81	81	81	78	78	78	78	81	81	81	81	78	78	78	78	81	81
	81	81	81	81	78	78	78	78	81	81	81	81	78	78	78	78	81	81
	73	76	79	82	73	76	76	79	74	77	81	82	74	76	78	80	75	77
Total Cost	11250	11289	11231	11233	11313	11355	11300	11312	11270	11299	11237	11238	11353	11387	11330	11338	11283	11307

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	15	15	15	15	15	15	5	5	5	5	10	10	10	10	15	15	15	15
c_h	1	1	5	5	5	5	1	1	5	5	1	1	5	5	1	1	5	5
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	79	79	77	77	77	77	78	78	76	76	79	79	77	77	79	79	77	77
	80	80	78	78	78	78	78	78	76	76	79	79	77	77	80	80	78	78
	80	80	78	78	78	78	79	79	76	76	80	80	77	77	80	80	78	78
	81	81	78	78	78	78	79	79	76	76	80	80	77	77	81	81	78	78
	81	81	78	78	78	78	81	81	78	78	81	81	78	78	81	81	78	78
	81	81	78	78	78	78	81	81	78	78	81	81	78	78	81	81	78	78
	81	81	78	78	78	78	81	81	78	78	81	81	78	78	81	81	78	78
	81	83	75	77	78	80	72	73	71	73	73	74	73	74	74	75	74	75
Total Cost	11241	11242	11383	11411	11352	11358	13494	13489	13557	13552	13518	13508	13601	13592	13537	13522	13636	13622

Table A. 4 Total Costs for 12 reviews when $r = 0.3$ and $\alpha = 1$

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	30	30
c_h	2	2	2	2	10	10	10	10	2	2	2	2	10	10	10	10	2	2
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	125	125	125	125	120	120	120	120	127	127	127	127	122	122	122	122	128	128
	125	125	125	125	120	120	120	120	127	127	127	127	122	122	122	122	128	128
	125	125	125	125	120	120	120	120	127	127	127	127	122	122	122	122	128	128
	118	121	125	128	117	120	120	123	120	122	127	128	119	121	122	124	121	123
Total Cost	16100	16145	16059	16064	16181	16236	16157	16181	16136	16167	16071	16074	16257	16297	16215	16231	16159	16184

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	30	30	30	30	30	30	10	10	10	10	20	20	20	20	30	30	30	30
c_h	2	2	10	10	10	10	2	2	10	10	2	2	10	10	2	2	10	10
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	128	128	123	123	123	123	125	125	120	120	127	127	122	122	128	128	123	123
	128	128	123	123	123	123	125	125	120	120	127	127	122	122	128	128	123	123
	128	128	123	123	123	123	125	125	120	120	127	127	122	122	128	128	123	123
	128	129	120	122	123	125	116	118	115	117	118	120	117	119	119	121	118	120
Total Cost	16078	16080	16307	16339	16251	16263	19315	19300	19392	19381	19363	19336	19479	19457	19397	19359	19539	19507

Table A. 5 Total Costs for 6 reviews when $\mathbf{r = 0}$ and $\boldsymbol{\alpha = 1}$

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	30	30
c_h	2	2	2	2	10	10	10	10	2	2	2	2	10	10	10	10	2	2
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	115	115	115	115	112	112	112	112	116	116	116	116	113	113	113	113	116	116
	116	116	116	116	112	112	112	112	117	117	117	117	114	114	114	114	118	118
	117	117	117	117	112	112	112	112	118	118	118	118	114	114	114	114	119	119
	110	113	116	119	109	112	112	115	112	114	118	120	111	113	114	116	113	114
Total Cost	14480	14521	14440	14444	14547	14600	14525	14547	14511	14541	14450	14452	14612	14649	14572	14586	14532	14555

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	30	30	30	30	30	30	10	10	10	10	20	20	20	20	30	30	30	30
c_h	2	2	10	10	10	10	2	2	10	10	2	2	10	10	2	2	10	10
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	116	116	114	114	114	114	115	115	112	112	116	116	113	113	116	116	114	114
	118	118	115	115	115	115	116	116	112	112	117	117	114	114	118	118	115	115
	119	119	117	117	117	117	117	117	112	112	118	118	114	114	119	119	117	117
	119	120	112	113	115	116	108	110	107	109	110	112	109	111	111	113	110	112
Total Cost	14455	14457	14652	14682	14599	14610	17371	17358	17438	17427	17416	17390	17512	17491	17447	17410	17562	17531

Table A. 6 Total Costs for 6 reviews when r = 0.1 and α = 1

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	30	30
c_h	2	2	2	2	10	10	10	10	2	2	2	2	10	10	10	10	2	2
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	107	107	107	107	104	104	104	104	108	108	108	108	105	105	105	105	108	108
	108	108	108	108	104	104	104	104	109	109	109	109	106	106	106	106	110	110
	108	108	108	108	107	107	107	107	110	110	110	110	107	107	107	107	111	111
	102	105	108	111	101	104	104	107	104	106	110	111	103	105	106	108	105	106
Total Cost	12882	12921	12846	12850	12949	12998	12929	12949	12912	12939	12855	12857	13003	13039	12967	12981	12931	12952

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	30	30	30	30	30	30	10	10	10	10	20	20	20	20	30	30	30	30
c_h	2	2	10	10	10	10	2	2	10	10	2	2	10	10	2	2	10	10
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	108	108	106	106	106	106	107	107	104	104	108	108	105	105	108	108	106	106
	110	110	106	106	106	106	108	108	104	104	109	109	106	106	110	110	106	106
	111	111	107	107	107	107	108	108	107	107	110	110	107	107	111	111	107	107
	111	112	104	105	107	108	100	102	100	101	102	104	102	103	103	105	103	104
Total Cost	12860	12862	13041	13069	12993	13003	15454	15442	15519	15509	15495	15472	15583	15564	15524	15491	15629	15601

Table A. 7 Total Costs for 6 reviews when r = 0.2 and α = 1

Case Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	30	30
c_h	2	2	2	2	10	10	10	10	2	2	2	2	10	10	10	10	2	2
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	99	99	99	99	96	96	96	96	100	100	100	100	97	97	97	97	100	100
	99	99	99	99	96	96	96	96	101	101	101	101	98	98	98	98	101	101
	101	101	101	101	98	98	98	98	101	101	101	101	98	98	98	98	102	102
	94	97	100	102	93	96	96	99	96	97	101	103	95	97	98	99	97	98
Total Cost	11270	11306	11236	11240	11336	11380	11317	11335	11298	11323	11244	11247	11389	11422	11355	11368	11316	11335

Case Number	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	30	30	30	30	30	30	10	10	10	10	20	20	20	20	30	30	30	30
c_h	2	2	10	10	10	10	2	2	10	10	2	2	10	10	2	2	10	10
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	100	100	98	98	98	98	99	99	96	96	100	100	97	97	100	100	98	98
	101	101	98	98	98	98	99	99	96	96	101	101	98	98	101	101	98	98
	102	102	99	99	99	99	101	101	98	98	101	101	98	98	102	102	99	99
	102	103	96	97	99	100	92	94	92	93	94	96	94	95	95	97	95	96
Total Cost	11249	11251	11426	11452	11381	11390	13520	13508	13585	13575	13558	13536	13647	13629	13585	13554	13692	13666

Table A. 8 Total Costs for 6 reviews when $\mathbf{r = 0.3}$ and $\boldsymbol{\alpha = 1}$

Case Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	60	60
c_h	4	4	4	4	20	20	20	20	4	4	4	4	20	20	20	20	4	4
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	166	166	166	166	160	160	160	160	168	168	168	168	162	162	162	162	169	169
	159	162	166	168	158	160	160	162	162	163	168	169	160	161	162	164	163	164
Total Cost	16134	16173	16069	16075	16216	16273	16182	16216	16180	16204	16082	16086	16307	16346	16250	16270	16208	16225

Case Number	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	60	60	60	60	60	60	20	20	20	20	40	40	40	40	60	60	60	60
c_h	4	4	20	20	20	20	4	4	20	20	4	4	20	20	4	4	20	20
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	169	169	164	164	164	164	166	166	160	160	168	168	162	162	169	169	164	164
	169	170	161	162	164	165	157	159	156	158	160	162	159	160	161	163	160	161
Total Cost	16090	16093	16366	16395	16290	16305	19364	19334	19436	19416	19432	19380	19546	19507	19477	19408	19618	19566

Table A. 9 Total Costs for 3 reviews when $\mathbf{r} = \mathbf{0}$. and $\boldsymbol{\alpha} = \mathbf{1}$

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	60	60
c_h	4	4	4	4	20	20	20	20	4	4	4	4	20	20	20	20	4	4
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	156	156	156	156	152	152	152	152	157	157	157	157	154	154	154	154	158	158
	151	154	157	159	150	152	152	154	154	155	159	160	152	153	154	156	155	156
Total Cost	14515	14552	14452	14459	14579	14634	14546	14578	14557	14580	14464	14467	14657	14694	14602	14622	14583	14600

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	60	60	60	60	60	60	20	20	20	20	40	40	40	40	60	60	60	60
c_h	4	4	20	20	20	20	4	4	20	20	4	4	20	20	4	4	20	20
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	158	158	155	155	155	155	156	156	152	152	157	157	154	154	158	158	155	155
	160	161	153	154	156	157	149	151	148	150	152	154	151	152	153	155	152	153
Total Cost	14470	14472	14709	14736	14636	14650	17423	17394	17478	17458	17487	17436	17574	17537	17529	17462	17639	17588

Table A. 10 Total Costs for 3 reviews when $\mathbf{r = 0.1}$. and $\boldsymbol{\alpha = 1}$

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	60	60
c_h	4	4	4	4	20	20	20	20	4	4	4	4	20	20	20	20	4	4
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	148	148	148	148	144	144	144	144	149	149	149	149	146	146	146	146	150	150
	143	146	149	151	142	144	144	146	146	147	151	152	144	145	146	147	147	148
Total Cost	12915	12950	12855	12861	12977	13029	12947	12977	12955	12977	12866	12870	13053	13088	13001	13020	12979	12995
Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	60	60	60	60	60	60	20	20	20	20	40	40	40	40	60	60	60	60
c_h	4	4	20	20	20	20	4	4	20	20	4	4	20	20	4	4	20	20
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	150	150	147	147	147	147	148	148	144	144	149	149	146	146	150	150	147	147
	152	153	145	146	147	148	141	143	141	142	144	146	143	144	145	147	144	145
Total Cost	12872	12875	13103	13128	13035	13048	15502	15475	15556	15537	15562	15515	15648	15613	15602	15539	15710	15663

Table A. 11 Total Costs for 3 reviews when $\mathbf{r = 0.2.}$ and $\boldsymbol{\alpha = 1}$

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	60	60
c_h	4	4	4	4	20	20	20	20	4	4	4	4	20	20	20	20	4	4
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	140	140	140	140	136	136	136	136	141	141	141	141	138	138	138	138	142	142
	135	138	141	143	134	136	136	138	138	139	143	144	136	137	138	139	139	140
Total Cost	11306	11340	11249	11255	11367	11417	11338	11367	11345	11366	11260	11263	11441	11474	11391	11409	11369	11384
Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	60	60	60	60	60	60	20	20	20	20	40	40	40	40	60	60	60	60
c_h	4	4	20	20	20	20	4	4	20	20	4	4	20	20	4	4	20	20
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Order up to levels	142	142	139	139	139	139	140	140	136	136	141	141	138	138	142	142	139	139
	144	144	137	138	139	140	134	135	133	134	136	138	135	136	137	139	136	137
Total Cost	11266	11268	11489	11513	11424	11436	13571	13545	13625	13607	13630	13584	13714	13680	13668	13608	13774	13728

Table A. 12 Total Costs for 3 reviews when $\mathbf{r = 0.3.}$ and $\boldsymbol{\alpha = 1}$

Case Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	15	15
c_h	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5	1	1
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Order up to levels	102	102	102	102	99	99	99	99	104	104	104	104	101	101	101	101	105	105
	102	102	102	102	99	99	99	99	104	104	104	104	101	101	101	101	105	105
	102	102	102	102	99	99	99	99	104	104	104	104	101	101	101	101	105	105
	102	102	102	102	99	99	99	99	104	104	104	104	101	101	101	101	105	105
	102	102	102	102	99	99	99	99	104	104	104	104	101	101	101	101	105	105
	102	102	102	102	99	99	99	99	104	104	104	104	101	101	101	101	105	105
	102	102	102	102	99	99	99	99	104	104	104	104	101	101	101	101	105	105
	95	100	102	106	95	100	99	104	98	101	104	107	97	100	101	104	99	101
Total Cost	15544	15594	15527	15533	15596	15652	15585	15604	15584	15623	15552	15556	15675	15720	15653	15666	15610	15642
Case Number	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	15	15	15	15	15	15	5	5	5	5	10	10	10	10	15	15	15	15
c_h	1	1	5	5	5	5	1	1	5	5	1	1	5	5	1	1	5	5
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Order up to levels	105	105	102	102	102	102	102	102	99	99	104	104	101	101	105	105	102	102
	105	105	102	102	102	102	102	102	99	99	104	104	101	101	105	105	102	102
	105	105	102	102	102	102	102	102	99	99	104	104	101	101	105	105	102	102
	105	105	102	102	102	102	102	102	99	99	104	104	101	101	105	105	102	102
	105	105	102	102	102	102	102	102	99	99	104	104	101	101	105	105	102	102
	105	105	102	102	102	102	102	102	99	99	104	104	101	101	105	105	102	102
	105	105	102	102	102	102	102	102	99	99	104	104	101	101	105	105	102	102
	105	107	98	101	102	105	94	95	94	95	96	97	96	97	97	99	97	98
Total Cost	15566	15569	15726	15763	15694	15703	18640	18636	18688	18684	18690	18680	18776	18766	18723	18706	18834	18819

Table A. 13 Total Costs for 12 reviews when $\mathbf{r = 0}$ and $\boldsymbol{\alpha = 0.99}$ per time bucket

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	15	15
c_h	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5	1	1
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Order up to levels	93	93	93	93	91	91	91	91	94	94	94	94	93	93	93	93	94	94
	93	93	93	93	91	91	91	91	94	94	94	94	93	93	93	93	95	95
	93	93	93	93	91	91	91	91	95	95	95	95	93	93	93	93	96	96
	94	94	94	94	91	91	91	91	95	95	95	95	93	93	93	93	96	96
	94	94	94	94	91	91	91	91	97	97	97	97	93	93	93	93	97	97
	94	94	94	94	91	91	91	91	97	97	97	97	93	93	93	93	97	97
	94	94	94	94	91	91	91	91	97	97	97	97	93	93	93	93	97	97
	87	92	92	97	87	92	90	95	89	93	95	97	89	92	92	96	91	93
Total Cost	13973	14025	13963	13973	14014	14070	14007	14028	14008	14047	13983	13989	14080	14123	14062	14076	14029	14060

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	15	15	15	15	15	15	5	5	5	5	10	10	10	10	15	15	15	15
c_h	1	1	5	5	5	5	1	1	5	5	1	1	5	5	1	1	5	5
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Order up to levels	94	94	93	93	93	93	93	93	91	91	94	94	92	92	94	94	93	93
	95	95	93	93	93	93	93	93	91	91	94	94	93	93	95	95	93	93
	96	96	94	94	94	94	93	93	91	91	95	95	93	93	96	96	94	94
	96	96	94	94	94	94	93	93	91	91	95	95	93	93	96	96	94	94
	97	97	94	94	94	94	93	93	91	91	97	97	93	93	97	97	94	94
	97	97	94	94	94	94	93	93	91	91	97	97	93	93	97	97	94	94
	97	97	94	94	94	94	93	93	91	91	97	97	93	93	97	97	94	94
	96	98	90	93	94	96	85	87	85	86	88	89	87	89	89	90	89	90
Total Cost	13993	13997	14122	14158	14094	14105	16756	16754	16793	16791	16800	16791	16867	16860	16827	16814	16916	16904

Table A. 14 Total Costs for 12 reviews when $\mathbf{r = 0.1}$ and $\boldsymbol{\alpha = 0.99}$ per time bucket

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	15	15
c_h	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5	1	1
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Order up to levels	85	85	85	85	83	83	83	83	86	86	86	86	85	85	85	85	86	86
	85	85	85	85	83	83	83	83	86	86	86	86	85	85	85	85	87	87
	85	85	85	85	83	83	83	83	87	87	87	87	85	85	85	85	88	88
	85	85	85	85	83	83	83	83	87	87	87	87	85	85	85	85	88	88
	87	87	87	87	87	87	87	87	87	87	87	87	87	87	87	87	88	88
	87	87	87	87	87	87	87	87	87	87	87	87	87	87	87	87	88	88
	87	87	87	87	87	87	87	87	87	87	87	87	87	87	87	87	88	88
	79	84	84	89	79	84	82	87	82	84	86	89	81	84	84	87	83	85
Total Cost	12428	12474	12418	12428	12487	12538	12480	12500	12460	12495	12437	12443	12533	12573	12517	12530	12481	12509

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	15	15	15	15	15	15	5	5	5	5	10	10	10	10	15	15	15	15
c_h	1	1	5	5	5	5	1	1	5	5	1	1	5	5	1	1	5	5
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Order up to levels	86	86	85	85	85	85	85	85	83	83	86	86	84	84	86	86	85	85
	87	87	85	85	85	85	85	85	83	83	86	86	85	85	87	87	85	85
	88	88	86	86	86	86	85	85	83	83	87	87	85	85	87	87	86	86
	88	88	86	86	86	86	85	85	83	83	87	87	85	85	88	88	86	86
	88	88	87	87	87	87	87	87	87	87	87	87	87	87	88	88	87	87
	88	88	87	87	87	87	87	87	87	87	87	87	87	87	88	88	87	87
	88	88	87	87	87	87	87	87	87	87	87	87	87	87	88	88	87	87
	87	89	82	84	85	88	78	79	78	79	80	81	80	81	81	83	81	82
Total Cost	12448	12452	12566	12599	12541	12552	14903	14901	14960	14958	14942	14935	15013	15007	14969	14957	15051	15040

Table A. 15 Total Costs for 12 reviews when $r = 0.2$ and $\alpha = 0.99$ per time bucket

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	5	5	5	5	5	5	5	5	10	10	10	10	10	10	10	10	15	15
c_h	1	1	1	1	5	5	5	5	1	1	1	1	5	5	5	5	1	1
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Order up to levels	77	77	77	77	75	75	75	75	78	78	78	78	77	77	77	77	78	78
	77	77	77	77	75	75	75	75	78	78	78	78	77	77	77	77	79	79
	77	77	77	77	75	75	75	75	79	79	79	79	77	77	77	77	79	79
	77	77	77	77	75	75	75	75	79	79	79	79	77	77	77	77	80	80
	78	78	78	78	78	78	78	78	81	81	81	81	78	78	78	78	81	81
	78	78	78	78	78	78	78	78	81	81	81	81	78	78	78	78	81	81
	78	78	78	78	78	78	78	78	81	81	81	81	78	78	78	78	81	81
	72	76	76	80	72	76	74	78	74	76	78	80	73	76	76	79	75	77
Total Cost	10873	10915	10864	10873	10921	10967	10915	10934	10904	10936	10883	10888	10968	11003	10952	10964	10921	10946

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	15	15	15	15	15	15	5	5	5	5	10	10	10	10	15	15	15	15
c_h	1	1	5	5	5	5	1	1	5	5	1	1	5	5	1	1	5	5
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
Order up to levels	78	78	77	77	77	77	77	77	75	75	78	78	76	76	78	78	77	77
	79	79	77	77	77	77	77	77	75	75	78	78	77	77	79	79	77	77
	79	79	77	77	77	77	77	77	75	75	78	78	77	77	79	79	77	77
	80	80	78	78	78	78	77	77	75	75	79	79	77	77	79	79	78	78
	81	81	78	78	78	78	78	78	78	78	79	79	78	78	81	81	78	78
	81	81	78	78	78	78	78	78	78	78	79	79	78	78	81	81	78	78
	81	81	78	78	78	78	78	78	78	78	79	79	78	78	81	81	78	78
	79	81	74	76	77	79	70	72	70	71	72	74	72	73	73	75	73	74
Total Cost	10891	10895	11000	11030	10978	10986	13038	13036	13084	13082	13076	13069	13137	13131	13099	13087	13174	13164

Table A. 16 Total Costs for 12 reviews when $r = 0.3$ and $\alpha = 0.99$ per time bucket

Case Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	30	30
c_h	2	2	2	2	10	10	10	10	2	2	2	2	10	10	10	10	2	2
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505
Order up to levels	123	123	123	123	119	119	119	119	125	125	125	125	121	121	121	121	126	126
	123	123	123	123	119	119	119	119	125	125	125	125	121	121	121	121	126	126
	123	123	123	123	119	119	119	119	125	125	125	125	121	121	121	121	126	126
	117	121	123	126	116	120	119	123	119	122	125	127	118	121	121	124	121	122
Total Cost	15754	15801	15724	15733	15814	15871	15796	15823	15800	15833	15748	15753	15901	15942	15867	15884	15829	15853

Case Number	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	30	30	30	30	30	30	10	10	10	10	20	20	20	20	30	30	30	30
c_h	2	2	10	10	10	10	2	2	10	10	2	2	10	10	2	2	10	10
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505
Order up to levels	126	126	123	123	123	123	122	122	119	119	125	125	121	121	126	126	123	123
	126	126	123	123	123	123	122	122	119	119	125	125	121	121	126	126	123	123
	126	126	123	123	123	123	122	122	119	119	125	125	121	121	126	126	123	123
	126	127	120	121	123	124	115	117	115	116	117	119	117	118	119	121	118	120
Total Cost	15760	15764	15956	15988	15908	15921	18895	18885	18949	18941	18956	18933	19049	19031	18996	18963	19115	19088

Table A. 17 Total Costs for 6 reviews when $r = 0$ and $\alpha = 0.99$ per time bucket

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	30	30
c_h	2	2	2	2	10	10	10	10	2	2	2	2	10	10	10	10	2	2
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505
Order up to levels	114	114	114	114	111	111	111	111	115	115	115	115	113	113	113	113	115	115
	114	114	114	114	111	111	111	111	116	116	116	116	113	113	113	113	117	117
	117	117	117	117	111	111	111	111	117	117	117	117	113	113	113	113	118	118
	108	112	113	117	108	111	110	114	111	113	115	117	110	112	113	115	112	114
Total Cost	14163	14212	14142	14155	14212	14270	14199	14229	14204	14237	14162	14169	14287	14328	14258	14277	14231	14256

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	30	30	30	30	30	30	10	10	10	10	20	20	20	20	30	30	30	30
c_h	2	2	10	10	10	10	2	2	10	10	2	2	10	10	2	2	10	10
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505
Order up to levels	115	115	114	114	114	114	113	113	111	111	115	115	113	113	115	115	114	114
	117	117	114	114	114	114	114	114	111	111	116	116	113	113	117	117	114	114
	118	118	115	115	115	115	117	117	111	111	117	117	113	113	117	117	114	114
	117	118	111	113	114	116	107	108	106	107	109	111	109	110	110	112	110	111
Total Cost	14174	14179	14336	14367	14294	14308	16985	16978	17030	17024	17040	17022	17118	17102	17079	17050	17176	17153

Table A. 18 Total Costs for 6 reviews when $\mathbf{r} = \mathbf{0.1}$ and $\boldsymbol{\alpha} = \mathbf{0.99}$ per time bucket

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	30	30
c_h	2	2	2	2	10	10	10	10	2	2	2	2	10	10	10	10	2	2
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505
Order up to levels	106	106	106	106	103	103	103	103	107	107	107	107	105	105	105	105	107	107
	106	106	106	106	103	103	103	103	108	108	108	108	105	105	105	105	109	109
	107	107	107	107	107	107	107	107	108	108	108	108	107	107	107	107	109	109
	101	104	105	108	100	103	102	106	103	105	107	109	102	104	105	107	104	106
Total Cost	12599	12644	12580	12592	12655	12709	12643	12671	12639	12670	12600	12607	12717	12755	12690	12708	12665	12688

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	30	30	30	30	30	30	10	10	10	10	20	20	20	20	30	30	30	30
c_h	2	2	10	10	10	10	2	2	10	10	2	2	10	10	2	2	10	10
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505
Order up to levels	107	107	106	106	106	106	105	105	103	103	107	107	105	105	107	107	106	106
	109	109	106	106	106	106	106	106	103	103	107	107	105	105	108	108	106	106
	109	109	107	107	107	107	107	107	107	107	108	108	107	107	109	109	107	107
	108	110	104	105	106	107	99	100	99	100	101	103	101	102	103	104	102	103
Total Cost	12612	12617	12760	12788	12721	12734	15109	15102	15162	15156	15163	15145	15234	15220	15198	15172	15286	15264

Table A. 19 Total Costs for 6 reviews when $r = 0.2$ and $\alpha = 0.99$ per time bucket

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	30	30
c_h	2	2	2	2	10	10	10	10	2	2	2	2	10	10	10	10	2	2
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505
Order up to levels	98	98	98	98	95	95	95	95	99	99	99	99	97	97	97	97	99	99
	98	98	98	98	95	95	95	95	99	99	99	99	97	97	97	97	100	100
	101	101	101	101	98	98	98	98	101	101	101	101	98	98	98	98	101	101
	93	96	97	100	92	95	94	98	95	97	99	101	94	96	97	99	96	98
Total Cost	11026	11067	11008	11018	11077	11127	11066	11091	11061	11089	11025	11031	11138	11172	11113	11129	11085	11106

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	30	30	30	30	30	30	10	10	10	10	20	20	20	20	30	30	30	30
c_h	2	2	10	10	10	10	2	2	10	10	2	2	10	10	2	2	10	10
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505	0.98505
Order up to levels	99	99	98	98	98	98	97	97	95	95	99	99	97	97	99	99	98	98
	100	100	98	98	98	98	98	98	95	95	99	99	97	97	100	100	98	98
	101	101	98	98	98	98	101	101	98	98	101	101	98	98	101	101	98	98
	100	101	96	97	98	99	91	93	91	92	94	95	93	94	95	96	94	95
Total Cost	11035	11039	11180	11206	11144	11156	13222	13215	13270	13264	13270	13253	13341	13328	13302	13277	13392	13372

Table A. 20 Total Costs for 6 reviews when r = 0.3 and α = 0.99 per time bucket

Case Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	60	60
c_h	4	4	4	4	20	20	20	20	4	4	4	4	20	20	20	20	4	4
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525
Order up to levels	164	164	164	164	159	159	159	159	166	166	166	166	162	162	162	162	167	167
	159	161	164	166	157	160	159	162	161	163	166	167	160	161	162	163	163	164
Total Cost	15941	15984	15892	15903	16006	16067	15980	16018	15997	16023	15916	15923	16108	16147	16058	16081	16031	16050

Case Number	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	60	60	60	60	60	60	20	20	20	20	40	40	40	40	60	60	60	60
c_h	4	4	20	20	20	20	4	4	20	20	4	4	20	20	4	4	20	20
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525
Order up to levels	167	167	163	163	163	163	163	163	159	159	166	166	162	162	167	167	163	163
	167	168	161	162	163	164	157	158	156	157	159	161	158	160	161	163	160	161
Total Cost	15930	15934	16172	16202	16106	16122	19126	19102	19181	19164	19206	19161	19302	19269	19257	19196	19382	19335

Table A. 21 Total Costs for 3 reviews when $\mathbf{r} = \mathbf{0}$ and $\boldsymbol{\alpha} = \mathbf{0.99}$ per time bucket

Case Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	60	60
c_h	4	4	4	4	20	20	20	20	4	4	4	4	20	20	20	20	4	4
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525
Order up to levels	154	154	154	154	151	151	151	151	156	156	156	156	153	153	153	153	157	157
	150	153	154	156	148	151	150	153	153	154	157	158	151	153	153	155	154	155
Total Cost	14335	14381	14298	14315	14383	14445	14362	14403	14391	14418	14324	14333	14475	14516	14433	14458	14425	14444

Case Number	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	60	60	60	60	60	60	20	20	20	20	40	40	40	40	60	60	60	60
c_h	4	4	20	20	20	20	4	4	20	20	4	4	20	20	4	4	20	20
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0,97525	0.97525	0.97525	0.97525	0.97525	0.97525
Order up to levels	157	157	154	154	154	154	154	154	151	151	156	156	153	153	157	157	154	154
	158	159	153	154	155	156	148	150	147	148	151	153	150	151	152	154	151	153
Total Cost	14338	14344	14535	14564	14476	14494	17194	17177	17234	17222	17275	17237	17346	17318	17326	17272	17421	17380

Table A. 22 Total Costs for 3 reviews when r = 0.1 and α = 0.99 per time bucket

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	60	60
c_h	4	4	4	4	20	20	20	20	4	4	4	4	20	20	20	20	4	4
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525
Order up to levels	146	146	146	146	143	143	143	143	148	148	148	148	145	145	145	145	149	149
	142	145	146	148	141	143	142	145	145	146	148	150	143	145	145	146	146	147
Total Cost	12754	12798	12720	12736	12801	12860	12782	12822	12808	12834	12745	12754	12891	12929	12851	12876	12840	12858

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	60	60	60	60	60	60	20	20	20	20	40	40	40	40	60	60	60	60
c_h	4	4	20	20	20	20	4	4	20	20	4	4	20	20	4	4	20	20
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525
Order up to levels	149	149	146	146	146	146	146	146	143	143	148	148	145	145	149	149	146	146
	150	150	145	146	147	147	140	142	139	140	143	145	142	143	144	146	143	145
Total Cost	12759	12765	12948	12976	12894	12910	15298	15282	15337	15326	15374	15339	15445	15419	15423	15373	15517	15478

Table A. 23 Total Costs for 3 reviews when $r = 0.2$ and $\alpha = 0.99$ per time bucket

Case Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**
c	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
c_p	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	60	60
c_h	4	4	4	4	20	20	20	20	4	4	4	4	20	20	20	20	4	4
g	80	80	100	100	80	80	100	100	80	80	100	100	80	80	100	100	80	80
b	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120	100	120
α	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525
Order up to levels	138	138	138	138	135	135	135	135	140	140	140	140	137	137	137	137	141	141
	134	137	138	140	133	135	134	137	137	138	140	141	135	137	137	138	138	139
Total Cost	11166	11208	11133	11148	11212	11269	11194	11231	11218	11243	11158	11166	11300	11336	11261	11284	11249	11266

Case Number	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**
c	100	100	100	100	100	100	120	120	120	120	120	120	120	120	120	120	120	120
c_p	60	60	60	60	60	60	20	20	20	20	40	40	40	40	60	60	60	60
c_h	4	4	20	20	20	20	4	4	20	20	4	4	20	20	4	4	20	20
g	100	100	80	80	100	100	80	100	80	100	80	100	80	100	80	100	80	100
b	100	120	100	120	100	120	120	120	120	120	120	120	120	120	120	120	120	120
α	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525	0.97525
Order up to levels	141	141	138	138	138	138	138	138	135	135	140	140	137	137	141	141	138	138
	141	142	137	138	138	139	132	134	131	132	135	137	134	135	136	138	135	137
Total Cost	11171	11176	11355	11381	11302	11318	13392	13377	13431	13420	13466	13433	13536	13511	13513	13465	13605	13568

Table A. 24 Total Costs for 3 reviews when $\mathrm{r} = 0.3$ and $\alpha = 0.99$ per time bucket

Appendix B

In this section, we provide the same figures in Chapter 4 for cases where $\alpha = 0.99$ per time bucket.

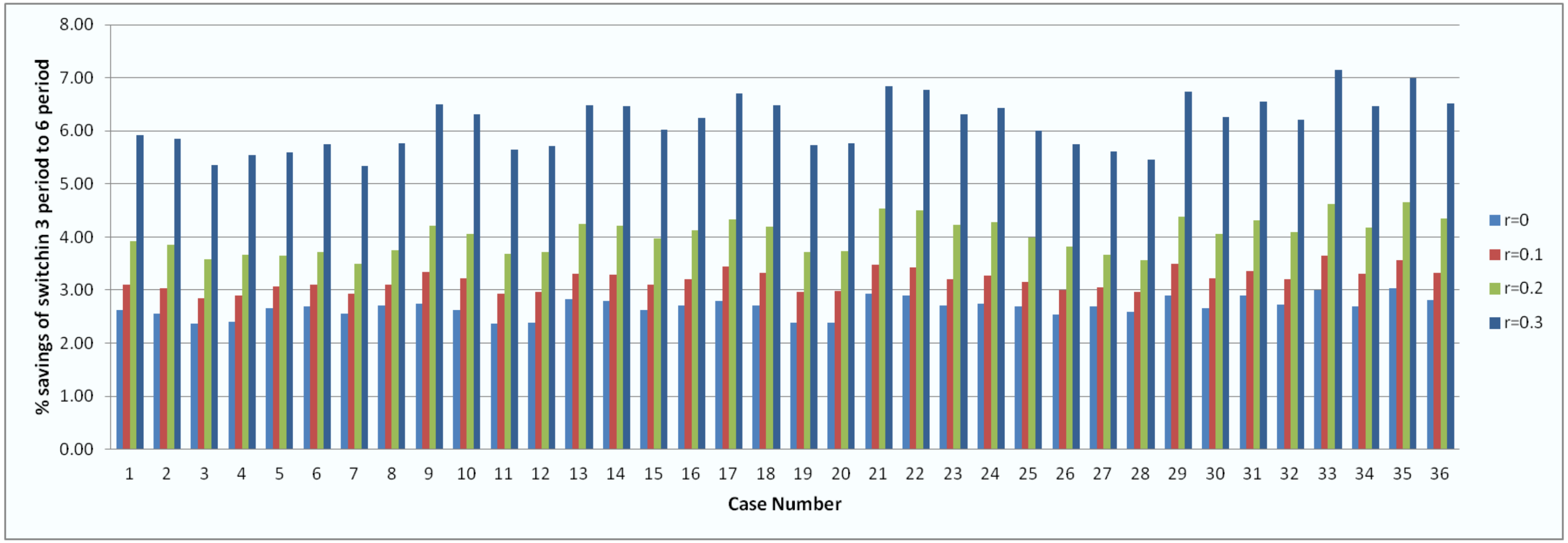

Figure B. 1 % realistic savings of switching 3 reviews to 6 reviews in terms of percentage for all return proportions for $\alpha = 0.99$

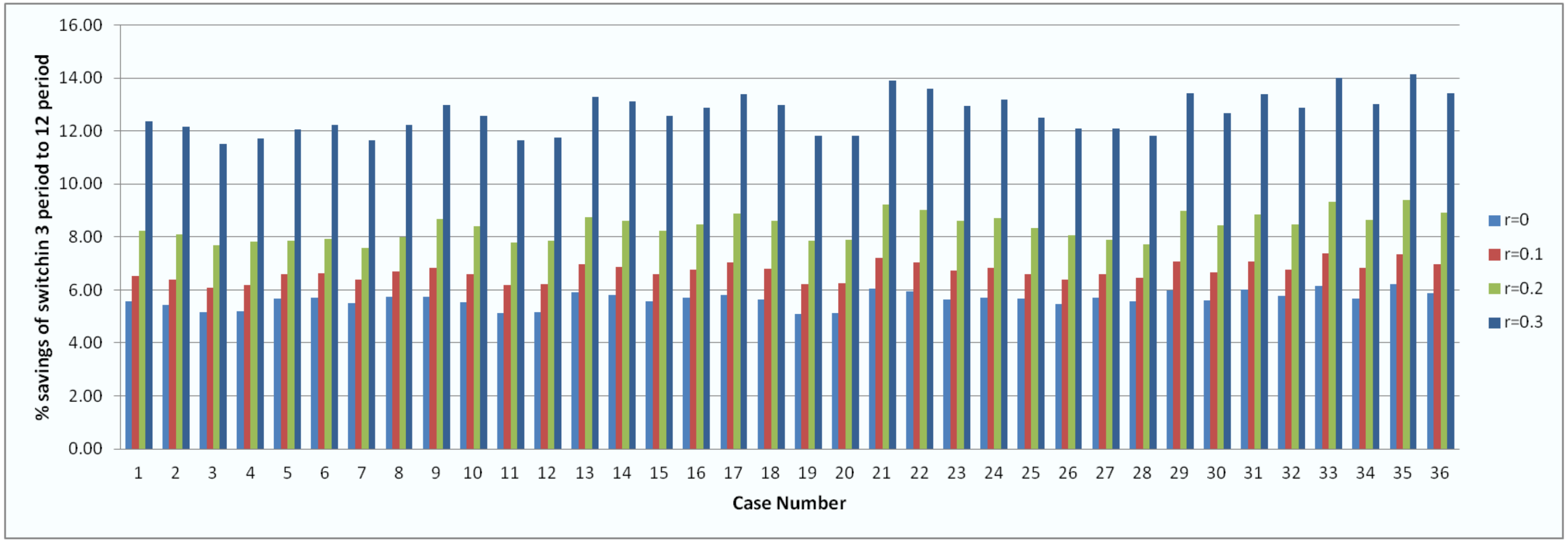

Figure B. 2% realistic savings of switching 3 reviews to 12 reviews in terms of percentage for all return proportions for $\boldsymbol{\alpha = 0.99}$.

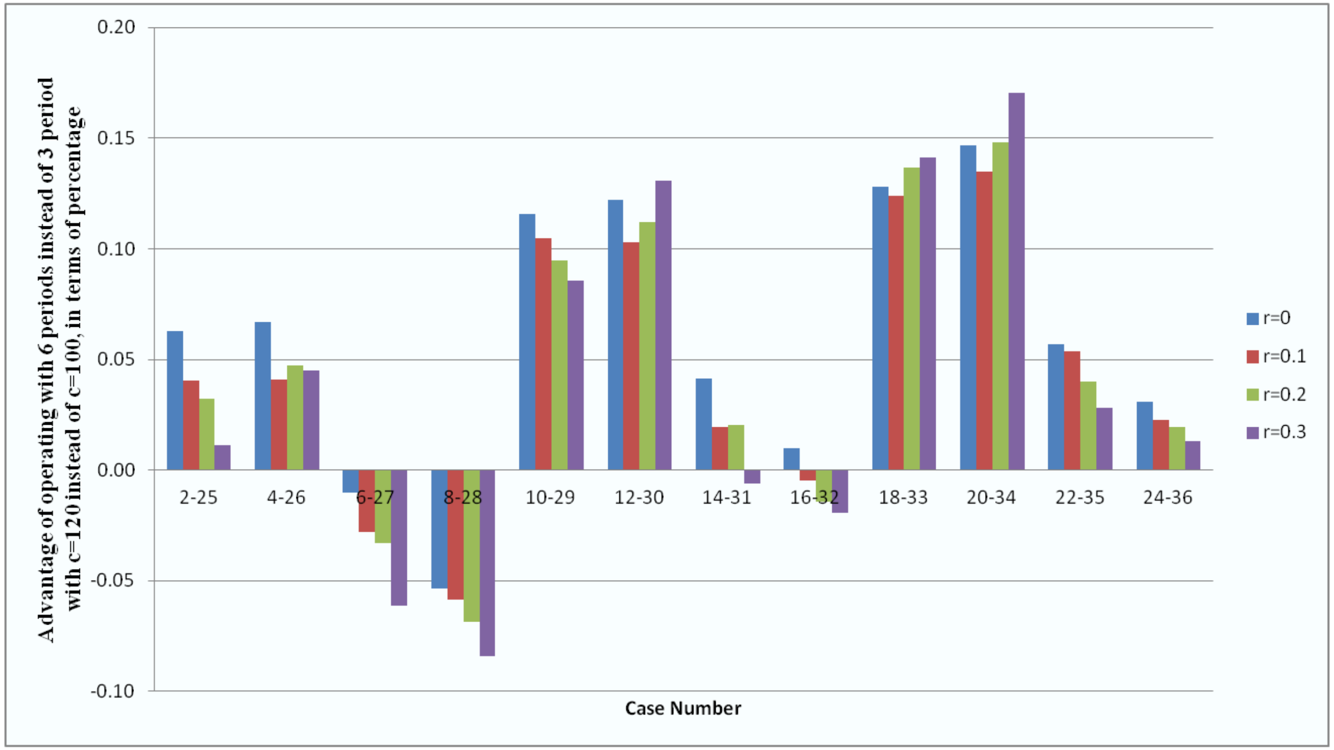

Figure B. 3 Advantage of operating with 6 reviews instead of 3 reviews with c=120 instead of c=100, in terms of percentage

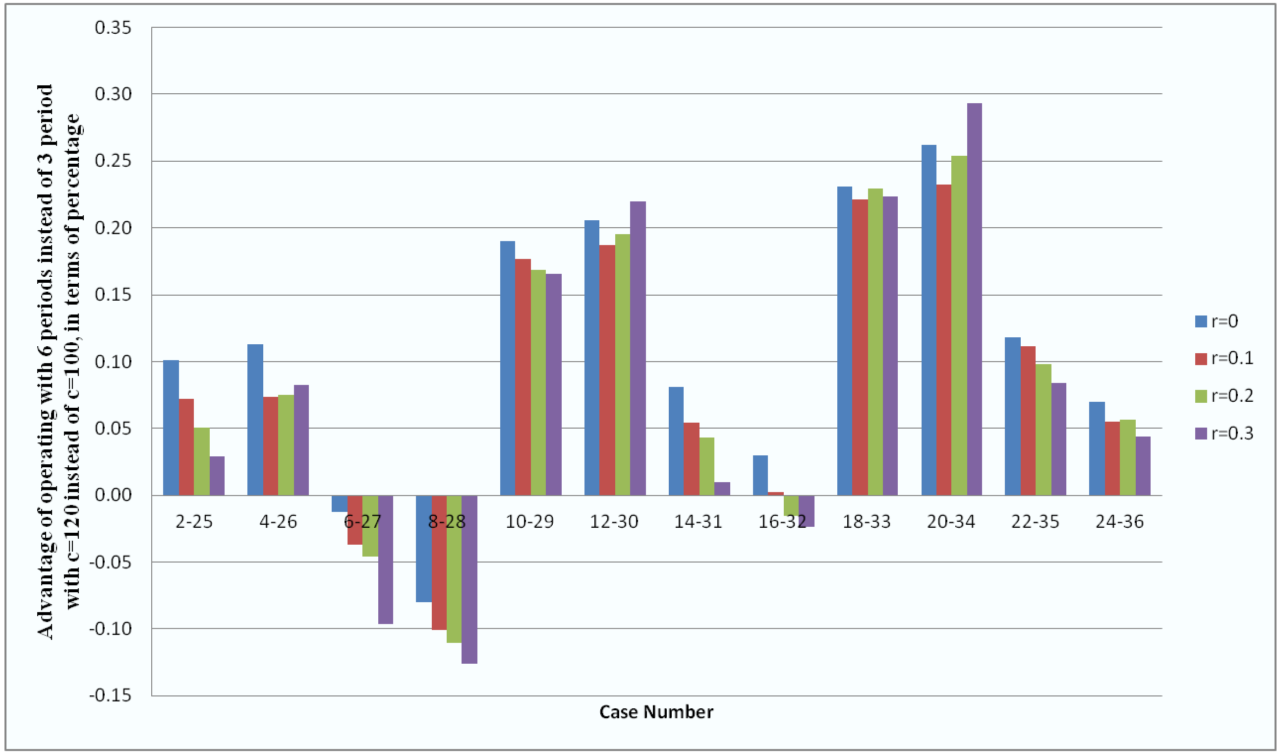

Figure B. 4 Advantage of operating with 12 reviews instead of 3 reviews with c=120 instead of c=100, in terms of percentage

Appendix C

In this section, the preview of simulation model that is developed in ARENA is provided.

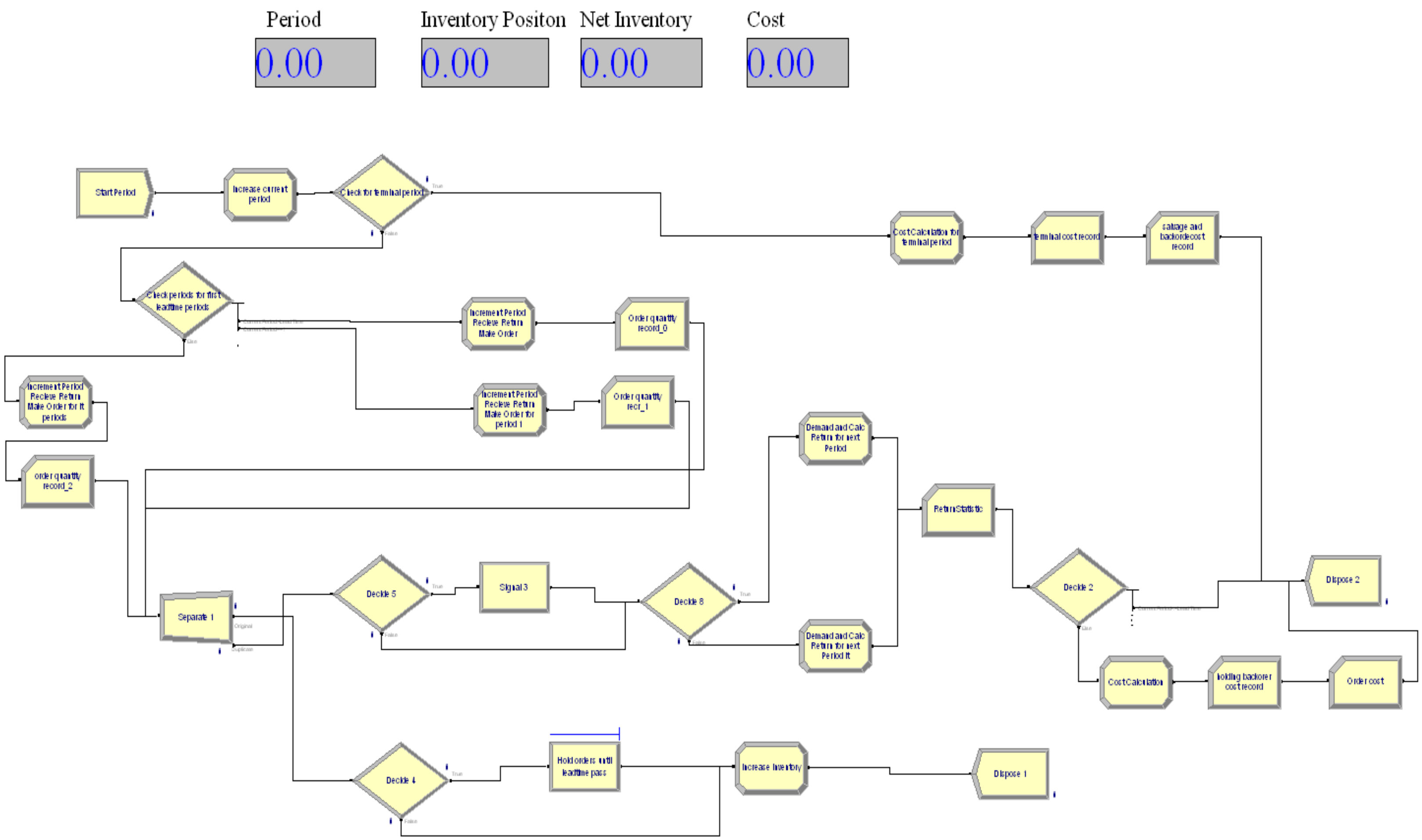

Figure C. 1 Preview of model that is developed in ARENA

Appendix D

In this section, the code of simulation model that is developed in ARENA is provided.

Code of the simulation model that is developed in ARENA:

```
;
;
;     Model statements for module:  BasicProcess.Create 1 (Start Period)
;

29$          CREATE,        1,DaysToBaseTime(0.0),Entity
1:DaysToBaseTime(30):NEXT(30$);

30$          ASSIGN:        Start Period.NumberOut=Start Period.NumberOut +
1:NEXT(21$);

;
;
;     Model statements for module:  BasicProcess.Assign 6 (Assign 6)
;
21$          ASSIGN:        Current Period=Current Period +1:NEXT(9$);
```

```
;
;
;     Model statements for module:  BasicProcess.Decide 3 (Decide 3)
;
9$           BRANCH,        1:
                        If,Current Period==MaxPeriods+1,33$,Yes:
                        Else,34$,Yes;
33$          ASSIGN:        Decide 3.NumberOut True=Decide 3.NumberOut
True + 1:NEXT(10$);


34$          ASSIGN:        Decide 3.NumberOut False=Decide 3.NumberOut
False + 1:NEXT(22$);



;
;
;     Model statements for module:  BasicProcess.Assign 5 (Cost Calculation for
terminal period)
;
10$          ASSIGN:        SalvageCost=MX(InventoryLevel,0)*SalvCost:
                        BackorderCost=MN(InventoryLevel,0)*BackCost*(-1):
                        TerminalCost=(1/alpha)* TotalCost-
SalvageCost+BackorderCost:NEXT(14$);
```

```
;
;
;     Model statements for module:  BasicProcess.Record 2 (Record 2)
;
14$           TALLY:         Terminal,TerminalCost,1:NEXT(19$);


;
;
;     Model statements for module:  BasicProcess.Record 8 (BC)
;
19$           TALLY:         BC,BackorderCost-SalvageCost,1:NEXT(6$);


;
;
;     Model statements for module:  BasicProcess.Dispose 2 (Dispose 2)
;
6$            ASSIGN:        Dispose 2.NumberOut=Dispose 2.NumberOut + 1;
35$           DISPOSE:       Yes;


;
;
;     Model statements for module:  BasicProcess.Decide 6 (Decide 6)
```

```
;
22$           BRANCH,        1:
                      If,Current Period>Lead Time,0$,Yes:
                      If,Current Period==1,23$,Yes:
                      Else,25$,Yes;


;
;
;     Model statements for module:  BasicProcess.Assign 10 (Increment Period
Recieve Return Make Order for lt periods)
;
25$           ASSIGN:        Period=Current Period:
                      InventoryLevel=InventoryLevel + Return(Current Period):
                      Inventory=32:
                      Inventory=Inventory + Return(Current Period):
                      OrderCost(Current Period + Lead
Time)=MX(OrderUpTo(Current Period) - InventoryLevel,0):
                      OrderQty(Current Period)=MX(OrderUpTo(Current Period)
- InventoryLevel,0):
                      OrderQty(MaxPeriods+1)=MX(OrderUpTo(MaxPeriods+1)
- MX(InventoryLevel,0),0):
                      OrderUpTo(MaxPeriods+1)=InventoryLevel:
                      InventoryLevel=MX(InventoryLevel, OrderUpTo(Current
Period)):
                      Md=MOD(Current Period,Lead Time):NEXT(26$);
```

```
;
;
;     Model statements for module:  BasicProcess.Record 11 (Record 11)
;
26$           TALLY:         QTY(Current Period),OrderQty(Current
Period),1:NEXT(1$);



;
;
;     Model statements for module:  BasicProcess.Separate 1 (Separate 1)
;
1$            DUPLICATE,     100 - 50:
                             1,40$,50:NEXT(39$);


39$           ASSIGN:        Separate 1.NumberOut Orig=Separate 1.NumberOut
Orig + 1:NEXT(16$);


40$           ASSIGN:        Separate 1.NumberOut Dup=Separate 1.NumberOut
Dup + 1:NEXT(20$);



;
;
;     Model statements for module:  BasicProcess.Decide 4 (Decide 4)
;
16$           BRANCH,        1:
```

```
                  If,Lead Time>=1,41$,Yes:
                  Else,42$,Yes;
41$       ASSIGN:      Decide 4.NumberOut True=Decide 4.NumberOut
True + 1:NEXT(11$);


42$       ASSIGN:      Decide 4.NumberOut False=Decide 4.NumberOut
False + 1:NEXT(2$);



;
;
;   Model statements for module:  AdvancedProcess.Hold 2 (Hold 2)
;
11$       QUEUE,       Hold 2.Queue;
          WAIT:        1,1:NEXT(2$);



;
;
;   Model statements for module:  BasicProcess.Assign 2 (Assign 2)
;
2$        ASSIGN:      Return(Current Period +2)=ANINT(-
1*MN(MN(Inventory,0),OrderQty(Current Period))*Prob):
                  Inventory=Inventory + OrderQty(Period):NEXT(4$);



;
```

```
;
;     Model statements for module:  BasicProcess.Dispose 1 (Dispose 1)
;
4$           ASSIGN:       Dispose 1.NumberOut=Dispose 1.NumberOut + 1;
43$          DISPOSE:      Yes;



;
;
;     Model statements for module:  BasicProcess.Decide 5 (Decide 5)
;
20$          BRANCH,       1:
                           If,Current Period>=Lead Time,44$,Yes:
                           Else,45$,Yes;
44$          ASSIGN:       Decide 5.NumberOut True=Decide 5.NumberOut
True + 1:NEXT(13$);


45$          ASSIGN:       Decide 5.NumberOut False=Decide 5.NumberOut
False + 1:NEXT(27$);



;
;
;     Model statements for module:  AdvancedProcess.Signal 3 (Signal 3)
;
13$          SIGNAL:       1,1:NEXT(27$);
```

```
;
;
;     Model statements for module:  BasicProcess.Decide 8 (Decide 8)
;
27$           BRANCH,        1:
                             If,Current Period>Lead Time,46$,Yes:
                             Else,47$,Yes;
46$           ASSIGN:        Decide 8.NumberOut True=Decide 8.NumberOut
True + 1:NEXT(3$);


47$           ASSIGN:        Decide 8.NumberOut False=Decide 8.NumberOut
False + 1:NEXT(28$);



;
;
;     Model statements for module:  BasicProcess.Assign 3 (Demand and Calc
Return for next Period)
;
3$            ASSIGN:        Demand=ANINT(NORM(Mean, Std)):
                             Return(Current Period +
1)=ANINT(MX(MN(Demand,Inventory),0)*Prob)+ Return(Current Period
+1):
                             InventoryLevel=InventoryLevel - Demand:
                             Inventory=Inventory - Demand:NEXT(7$);
```

```
;
;
;     Model statements for module:  BasicProcess.Record 1 (ReturnStatistic)
;
7$           TALLY:         ReturnStat(Current Period + 1),Return(Current Period
+ 1),1:NEXT(8$);



;
;
;     Model statements for module:  BasicProcess.Decide 2 (Decide 2)
;
8$           BRANCH,        1:
                          If,Current Period<=Lead Time,6$,Yes:
                          Else,5$,Yes;


;
;
;     Model statements for module:  BasicProcess.Assign 4 (Cost Calculation)
;
5$           ASSIGN:        OCost=UnitCost*OrderCost(Period):
                          HCost=MX(Inventory,0)*UnitHoldingCost:
                          BCost=MX(-Inventory,0)*UnitBackorderCost:
```

```
TotalCost=(1/alpha )* TotalCost + OCost + HCost +
BCost:NEXT(17$);

;
;
;     Model statements for module:  BasicProcess.Record 6 (HB)
;
17$          TALLY:        hold(Current Period),HCost+BCost,1:NEXT(18$);

;
;
;     Model statements for module:  BasicProcess.Record 7 (Order)
;
18$          TALLY:        Order,OCost,1:NEXT(6$);

;
;
;     Model statements for module:  BasicProcess.Assign 11 (Demand and Calc
Return for next Period lt)
;
28$          ASSIGN:       Demand=40:
                           Return(Current Period +
1)=ANINT(MX(MN(Demand,Inventory),0)*Prob)+ Return(Current Period
+1):
```

```
                    InventoryLevel=InventoryLevel - Demand:
                    Inventory=Inventory - Demand:NEXT(7$);



;
;
;     Model statements for module:  BasicProcess.Assign 1 (Increment Period Recieve Return Make Order)
;
0$          ASSIGN:     Period=Current Period:
                    InventoryLevel=InventoryLevel + Return(Current Period):
                    Inventory=Inventory + Return(Current Period):
                    OrderCost(Current Period + Lead Time)=MX(OrderUpTo(Current Period) - InventoryLevel,0):
                    OrderQty(Current Period)=MX(OrderUpTo(Current Period) - InventoryLevel,0):
                    OrderQty(MaxPeriods+1)=MX(OrderUpTo(MaxPeriods+1) - MX(InventoryLevel,0),0):
                    OrderUpTo(MaxPeriods+1)=InventoryLevel:
                    InventoryLevel=MX(InventoryLevel, OrderUpTo(Current Period)):
                    Md=MOD(Current Period,Lead Time):NEXT(15$);



;
;
;     Model statements for module:  BasicProcess.Record 3 (Record 3)
```

;

15$ TALLY: QTY(Current Period),OrderQty(Current Period),1:NEXT(1$);

;

;

; Model statements for module: BasicProcess.Assign 9 (Increment Period Recieve Return Make Order for period 1)

;

23$ ASSIGN: Period=Current Period:

InventoryLevel=InventoryLevel + Return(Current Period):

Inventory=Inventory + Return(Current Period):

OrderCost(Current Period + Lead Time)=MX(OrderUpTo(Current Period) - InventoryLevel,0):

OrderQty(Current Period)=MX(OrderUpTo(Current Period) - InventoryLevel,0):

OrderQty(MaxPeriods+1)=MX(OrderUpTo(MaxPeriods+1) - MX(InventoryLevel,0),0):

OrderUpTo(MaxPeriods+1)=InventoryLevel:

InventoryLevel=MX(InventoryLevel, OrderUpTo(Current Period)):

Md=MOD(Current Period,Lead Time):NEXT(24$);

;

;

; Model statements for module: BasicProcess.Record 10 (Record 10)

;

24$ TALLY: QTY(Current Period),OrderQty(Current Period),1:NEXT(1$);

Appendix E

In this section,the code of model that is developed in MATLAB is provided. We write different M-files for each return proportion. The below code is wirttten for the cases where $r = 0.1$

Code of the MATLAB model for the cases where $r = 0.1$:

```
mean_b=input('enter mean for normal: ');
std=input('enter standard deviation for normal : ');
no= input('enter number of period : ');
iter= input('enter number of iterations : ');
m=input('number of base period in the one period length  : ');

%initialize variables
dd=0;
gra=zeros(2*no+7,384);
gra1=zeros(2*no+7,384);
gra2=zeros(2*no+7,384);
gra3=zeros(2*no+7,384);
mean=mean_b
pro= pro_n(mean,std,iter,m);
```

```
holdarr=hold_arr(pro,mean,std,iter,m);
c_arr=[100,120];
cp_arr=[5,10,15];
ch_arr=[1,5];
g_arr=[80,100];
B_arr=[100,120];
a_arr=[0.99];
xx=0;
tic
for ij=1:1:2
    c = c_arr(ij);
    for iij=1:1:3
        cp_b=cp_arr(iij);
        cp=cp_b*m;
        for ijj=1:1:2
            ch_b=ch_arr(ijj);
            ch=ch_b*m;
            for iig=1:1:2
                g=g_arr(iig);
                for iib=1:1:2
                    B=B_arr(iib);
                    for iia=1:1:1
                        a_b = a_arr(iia);
                        xx=0;
                        for i=1:1:m
```

```
            xx=xx + a_b^i;
          end
          a=xx/m;
          if B >= c
            dd=dd+1

it = 10000;
if m==4
  k=5*(2*mean+2*std); % maximum amunt of order
elseif m==2
  k=4*(2*mean+2*std);
else
  k=3*(2*mean+2*std);
end
p=0.1; % p proportion of return
z=10;
  K=zeros(no+1,12000);
  F=inf(no+1,12000);

  Y=zeros(no+1,12000);
  Z=zeros(no+1,12000);
  W=zeros(no+1,12000);
  A=inf(1,12000);
  % AA=inf(1,12000);
  alt=0;
```

```
    ust=k;
    pre=10*std/iter;
    q=pre;
    ooo=0;
    if m==1
        ss=4;
    elseif m==2
        ss=2;
    elseif m==4;
        ss=1;
    end


            for IN= q*(round((alt-10*mean)/q)):q:q*(round((ust+10*mean)/q))
% loop for inventory position
                y=round(IN/q+it); %transform inventory position to index
                A(1,y)= -g*max(0,IN)-B*min(0,IN); %assigning cost for last
period
            end


          for u= 1:1:no-ss %loop for period
            t=round(no-ss+1-u);


            if t== no-ss
```

```
        for IN= q*(round((alt-5*mean)/q)):q:q*(round((ust+5*mean)/q))
% loop for inventory position
          y=round(IN/q+it); %transform inventory position to index
          tcos= inf(1,10000);
          j=0;
          for i=0:q:k  % loop for order quantities
            j=j+1; % transform order point to index
            hold=0;
            pen=0;
            x=IN+i; %inventory position after order

            if m==1

              t_hp= hpcost_nv8(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);

            elseif m==2;

              t_hp= hpcost_nv9(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);

            else
              t_hp= hpcost_nv10(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);
            end
```

```
purc=c*i; % order cost

exp=0;

% Expectation for future cost

kk=0;

 for e1 = 4*(mean-5*std): pre: (mean+ 5*std)*4

     kk=kk+1;

     ee=0;

     for e2 = m*(mean-5*std): pre: (mean+ 5*std)*m

         ee=ee+1;

         exp= exp +(a) *A(1,round(it+(x-e1-
e2+(p*e1)))/pre)*pro(ee)*holdarr(kk);

     end

 end


    t_cost = t_hp + purc + a*exp  ;

    t_cos(1,j) =t_cost; %total cost

    t_cos_hp(1,j)=t_hp;

    t_cos_exp(1,j)=a*exp;

    t_cos_purc(1,j)=purc;

end;



[C,I] = min(t_cos);

C;

I;

opt_or= (I-1)* pre;
```

```
          K(t,y)=opt_or;
          F(t,y)= C ;
          Y(t,y)= t_cos_hp(1,I);
          Z(t,y)= t_cos_exp(1,I);
          W(t,y)= t_cos_purc(1,I);

    end

elseif t==1
        if m==1
            IN1=74;
        elseif m==2
            IN1=76;
        else
            IN1=80;
        end

        y=round(IN1/q+it); %transform inventory position to index
        tcos= inf(1,10000);
        j=0;
        for i=0:q:k  % loop for order quantities
            j=j+1; % transform order point to index
            hold=0;
            pen=0;
            x=IN1+i; %inventory position after order
```

```
% Expectation over two period demand for holding and
% backorder costs
if m==1

    t_hp= hpcost_nv8(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);

elseif m==2;

    t_hp= hpcost_nv9(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);

else
    t_hp= hpcost_nv10(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);
end
purc=c*i; % order cost
exp=0;
% Expectation for future cost
kk=0;
    if m==1
        exp= exp + F(t+1,round(it+(x-20+(p* max(0,
min(x,20))))))/pre);
    elseif m==2
        exp= exp + F(t+1,round(it+(x-40+(p* max(0,
min(x,40))))))/pre);
    else
```

```
                exp= exp + F(t+1,round(it+(x-80+(p* max(0,
min(x,80)))))/pre);
            end

            t_cost = t_hp + purc + a*exp  ;
            t_cos(1,j) =t_cost; %total cost
            t_cos_hp(1,j)=t_hp;
            t_cos_exp(1,j)=a*exp;
            t_cos_purc(1,j)=purc;
        end
         [C,I] = min(t_cos);
         C;
         I;
         opt_or= (I-1)* pre;
         K(t,y)=opt_or;
         F(t,y)= C ;
         Y(t,y)= t_cos_hp(1,I);
         Z(t,y)= t_cos_exp(1,I);
         W(t,y)= t_cos_purc(1,I);

    else

    for IN1= q*(round((alt-mean)/q)):q:q*(round((ust+mean)/q)) % loop
for inventory position
```

```
y=round(IN1/q+it); %transform inventory position to index
    tcos= inf(1,10000);
    j=0;
    for i=0:q:k  % loop for order quantities
        j=j+1; % transform order point to index
        hold=0;
        pen=0;
        x=IN1+i; %inventory position after order
        if m==1
            t_hp= hpcost_nv8(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);

        elseif m==2;


            t_hp= hpcost_nv9(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);


        else
            t_hp= hpcost_nv10(ch,cp, x, mean, std, pre,z,p,
pro,holdarr,m,t);
        end
        purc=c*i; % order cost
        exp=0;
        % Expectation for future cost
        kk=0;


        if m==1
```

```
            if  t<5
                exp= exp + F(t+1,round(it+(x-20+(p*(20))))/pre);
            else
                for e1 = m*(mean-5*std): pre: (mean+ 5*std)*m
                      kk=kk+1;
                          if IN < 0
                             exp= exp + F(t+1,round(it+(x-e1+(p*( max(0,
min(x-4*mean+4*mean*p,e1))))))/pre)*pro(kk);
                          else
                             exp= exp + F(t+1,round(it+(x-e1+(p*( max(0,
min(x-4*mean+4*mean*p,e1))))))/pre)*pro(kk);
                          end


                end
            end


         else
            if  t<3
                exp= exp + F(t+1,round(it+(x-40+(p*(40))))/pre);
            else
                for e1 = m*(mean-5*std): pre: (mean+ 5*std)*m
                      kk=kk+1;
                          if IN < 0
                             exp= exp + F(t+1,round(it+(x-e1+(p*( max(0,
min(x-4*mean+4*mean*p,e1))))))/pre)*pro(kk);
                          else
```

```
                    exp= exp + F(t+1,round(it+(x-e1+(p*( min(x-4*mean+4*mean*p,e1)))))/pre)*pro(kk);
                   end

                end
              end
            end
              t_cost = t_hp + purc + a*exp  ;
              t_cos(1,j) =t_cost; %total cost
              t_cos_hp(1,j)=t_hp;
              t_cos_exp(1,j)=a*exp;
              t_cos_purc(1,j)=purc;
           end
            [C,I] = min(t_cos);
            C;
            I;
            opt_or= (I-1)* pre;
            K(t,y)=opt_or;
            F(t,y)= C ;
            Y(t,y)= t_cos_hp(1,I);
            Z(t,y)= t_cos_exp(1,I);
            W(t,y)= t_cos_purc(1,I);
         end
       end
     end
%       [C,I]=min(AA);
```

```
%       C
    for i=1:1:2*no+1
        if i==1
            gra(1,dd)=c;
            gra(2,dd)=cp;
            gra(3,dd)=ch;
            gra(4,dd)=g;
            gra(5,dd)=B;
            gra(6,dd)=a;

        elseif i== no
            for k=2:1:no+1
                if k==2
                    if m==1
                        gra(k+6,dd)= K(k-1,10074)+74;
                    elseif m==2
                        gra(k+6,dd)= K(k-1,10076)+76;
                    else
                        gra(k+6,dd)= K(k-1,10080)+80;
                    end

                else
                    gra(k+6,dd)= K(k-1,10000);
                end
            end
```

```
else
    for k=no+1:1:2*no-ss
        rr=k-no;
        if rr==1
            if m==1
                gra(k+6,dd)= F(rr,10074);
                gra1(k+6,dd)= Y(rr,10074);
                gra2(k+6,dd)= Z(rr,10074);
                gra3(k+6,dd)= W(rr,10074);
            elseif m==2
                gra(k+6,dd)= F(rr,10076);
                gra1(k+6,dd)= Y(rr,10076);
                gra2(k+6,dd)= Z(rr,10076);
                gra3(k+6,dd)= W(rr,10076);
            else
                gra(k+6,dd)= F(rr,10080);
                gra1(k+6,dd)= Y(rr,10080);
                gra2(k+6,dd)= Z(rr,10080);
                gra3(k+6,dd)= W(rr,10080);
            end

        else
            gra(k+6,dd)= F(rr,10000);
            gra1(k+6,dd)= Y(rr,10000);
            gra2(k+6,dd)= Z(rr,10000);
```

```
                gra3(k+6,dd)= W(rr,10000);
              end
            end
          end
        end
    %
              end
            end
          end
        end
      end
    end
  end
  toc
```