OPTIMAL TIMING OF AN ENERGY SAVING TECHNOLOGY ADOPTION

A Master's Thesis

by
Mehmet Fatih HARMANKAYA

Department of
Economics
İhsan Doğramacı Bilkent University
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To My Family

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Graduate School of Economics and Social Sciences
of
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by

MEHMET FATİH HARMANKAYA

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assist. Prof. Dr. Hüseyin Çağrı Sağlam
Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assist. Prof. Dr. Tarık Kara
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

Assoc. Prof. Dr. Levent Akdeniz
Examining Committee Member

Approval of the Graduate School of Economics and Social Sciences

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Prof. Dr. Erdal Erel
Director

ABSTRACT

OPTIMAL TIMING OF AN ENERGY SAVING TECHNOLOGY ADOPTION

HARMANKAYA, Mehmet Fatih

M.A., Department of Economics

Supervisor: Assist. Prof. Dr. Hüseyin Çağrı Sağlam

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In this thesis, we use two stage optimal control techniques to analyze the optimal timing of energy saving technology adoptions. We assume that the physical capital goods sector is relatively more energy intensive than consumption goods sector. First, we solve a benchmark problem without exogenously growing energy saving technology frontier. In such a case, the economy sticks either to the initial technology or immediately switches to a new technology, depending on the growth rate advantage compared to the obsolescence and adjustment costs. In the second step, we introduce exogenously growing energy saving technology frontier. The anticipated level of the technology provides incentives to delay the adoption and generates an interior switching time. Finally, we analyze numerically the effects of the speed of adjustment to the new technology, growth rate of technology, subjective time preference and planning horizon on the optimal timing of technology adoption.

Keywords: Optimal Control, Technology Adoption, Energy Saving Technical Progress, Embodiment

ÖZET

ENERJİ TASARRUFLU TEKNOLOJİ ADAPTASYONUNUN OPTİMAL ZAMANLAMASI

HARMANKAYA, Mehmet Fatih

Yüksek Lisans, Ekonomi Bölümü

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Bu tezde, enerji tasarruflu teknoloji adaptasyonunun optimal zamanlamasını analiz etmek için iki aşamalı optimal kontrol tekniklerini kullanıyoruz. Fiziksel sermaye malları sektörünün, tüketim malları sektörüne oranla enerjiye daha fazla bağımlı olduğunu varsayıyoruz. Öncelikle, dışsal gelişen enerji tasarruflu teknoloji içermeyen temel bir model çözüyoruz. Böyle bir durumda, büyüme oranı avantajının eskime ve ayarlama maliyetleri karşısındaki durumuna bağlı olarak ekonominin başlangıç seviyesindeki teknolojide kaldığını ya da daha yüksek enerji tasarrufu sağlayan yeni teknolojiye başlangıçta geçtiğini gözlemliyoruz. İkinci aşamada, modele dışsal gelişen enerji tasarruflu teknoloji ekliyoruz. Böyle bir modelde, öngörülen teknoloji seviyesi, adaptasyonun gecikmesini teşvik edip bu adaptasyonunun dahili zamanlarda gerçekleşmesini sağlıyor. Son olarak, yeni teknoloji ayarlama hızının, teknoloji gelişme oranının, öznel zaman tercihinin ve planlama süresinin teknoloji adaptasyonu optimal zamanlamasına etkilerini nümerik olarak inceliyoruz.

Anahtar Kelimeler: Optimal Kontrol, Teknoloji Adaptasyonu, Enerji Tasarruflu Teknik Gelişme, Somutlaşma

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CHAPTER 1

INTRODUCTION

Adoption of cleaner technologies has become one of the most important topics in environment and growth fields. This stems from the fact that due to scarcity and the pollutant property of energy resources, consumers may shift their demands to goods that are produced with less energy resources and firms can switch technologies which are using resources more efficiently.

The adoption of cleaner technologies establishes the connection between technology switching and the environmental protection. Cunha-e-sa and Reis (2007) study the optimal timing of adopting a cleaner technology and its effects on the growth rate of the economy in the context of an AK endogenous growth model. They introduce environmental quality to their utility function which increases the utility of the consumption. Boucekkine, Krawczyk and Vallee (2010) study the trade off between economic and environmental benefits where the agent can switch to a cleaner technology that is economically inefficient. They introduce pollution to the utility which negatively effects the total welfare. Differently, this thesis examines energy saving technology adoption which is not previously considered and includes the adoption of a new technology in capital goods sector which uses energy more efficiently.

However, technology adoptions are costly due to the efficiency losses and associated costs of new technologies. Parente (1994) claims that, technology adoptions induce efficiency losses in human and physical capital. There exists a slow learning process in which the economy is unable to produce at its best level. Moreover, when the economy switches to a new technology, the adoption costs occur via different mechanisms. Such costs associated with technology adoption are called learning, obsolescence and adjustment costs in the literature. When these are considered together with learning and other costs of new technologies, the following question may emerge: Is it optimal to switch to the new technology or continue with the older one? While trying to answer this question from an economic perspective, the timing of this adoption is also taken into consideration.

The optimal timing of technology adoptions depends on the growth advantages, the speed of learning and the obsolescence costs (see Boucekkine, Saglam and Vallee (2004)). Real income, human capital, the trade between countries and the type of government are among the other determinants of technology adoption according to Comin and Hobjin (2003) which includes their empirical analysis on cross-country technology adoption in the time period from 1788 to 2001. We investigate the effects of some of these determinants on the optimal switching time.

Boucekkine, Saglam and Vallee (2004) studies various adoption problems in the optimal growth framework. They study the optimal timing of switching to new technologies with and without learning behavior. Two stage optimal control techniques are used to determine the switching time. When learning behavior does not exist, the solution will be immediate or never adoption. However, when it is introduced, the economy will switch immediately or choose the delay option. Saglam (2010) also studies the optimal pattern of technology adoption with multiple switches instead of a single switch. They introduce technology-specific adjustment cost on the depreciation parameter to explain the loss of expertise caused by newer technologies. We simply consider the adjustment cost of the new technology similar to Saglam (2010).

The usage of energy resources have been extensively analyzed in the optimal growth literature. Most of these analyses are based on the assumption that physical capital and consumption good use the same technology in production. This assumption implies that the energy intensities of these goods are same. However, as Perez-Barahona (2007) states, physical capital accumulation usually involves the transformation of raw materials into iron, steel and non-ferrous metals. Transport and storage of goods are also included in physical capital whereas consumption good sector is more related to food, clothes and construction. Azamahoau et al. (2006) shows that energy intensities of physical capital goods are much more higher than consumption goods. They find that the ratio between energy consumption and the value added is 0.809 for iron and steel, 0.85 for storage and transport whereas 0.134 in food and tobacco, 0.082 in textile production. Perez-Barahona (2007) uses a type of setting where physical capital accumulation is more energy intensive than consumption good. They consider a general equilibrium model consisting of final good, physical capital and resource extraction sectors. Within these sectors, they study the implications of assuming different technologies for physical capital accumulation and consumption.

In this thesis, we use a simple optimal growth model to solve the technology adoption problems in continuous time. Technological progress is assumed to be embodied in capital good production, specifically in energy usage. In addition to switching to the new technology, our problem involves obsolescence costs and learning costs integrated in depreciation term. The economy starts with a given initial technological menu and level of embodied technical change in energy saving technology. New technological menu is also available starting from the beginning of the planning horizon. The agent may switch to the new technology or continue to use the current one at any instant of the time. However, new technology is costly as more embodiment in capital sector, specifically in energy sector, implies a decrease in the relative price of capital. This decrease induces a rise in the level of resources used in investment which drops the consumption level. The welfare cost of this drop is referred to obsolescence costs as stated in Saglam (2010). In addition to these obsolescence costs, switching to the new technology induces accelerated erosion effect on physical capital which can be considered as the learning cost of new technology. The loss of expertise after switching is expressed by using associating accelerated depreciation to the new technology. We want to examine under which conditions and when, the economy would switch to a more efficient energy saving technical progress knowing the obsolescence and learning costs of the switching.

We consider a simple AK type production function in consumption good and Cobb-Douglas type function in capital good sector. Due to AK production function, long term growth is no longer exogenous. Boucekkine et al. (2004) assume that in the new technology case, disembodied technological progress is lower in order to represent the loss of expertise after switching. Differently, we introduce costs of the new technology in the depreciation parameter. In our benchmark model, we assume that the higher level of embodied technical progress is available with a higher depreciation rate because of learning costs. In the extended model, we assume that there exists an anticipated technology adoption and time-varying embodied energy saving technological progress.

In the capital good sector, we consider a capital accumulation rule similar to Perez-Barrahona (2007) which implies that energy intensity of capital good is higher than consumption good. In contrast with Perez-Barrahona (2007), we do not include the extraction sector of energy resource. Instead, energy is assumed to be purchased directly with a given cost function in order to simplify the model. We are able to derive the paths followed by the decision variables analytically which allows us to use two-stage optimal control techniques proposed by Tomiyama and Rossana (1989). By using this approach, we can generate three possible decisions related to optimal timing: immediate adoption, technological sclerosis, i.e., sticking to the initial technology through the planning horizon implying corner solutions and delayed adoption as an interior solution.

The organization of the paper is as follows. In Chapter 2, the benchmark model will be introduced and solved by using two-stage optimal control technique for both finite and infinite horizon. The procedure of the two-stage approach and how it works will also be presented. In Chapter 3, we will extend our model by allowing the energy saving technology frontier level to increase throughout the time. We apply same procedure as benchmark case in order to reach optimal timing of adoption. However, as in many optimal timing problems, we are unable to reach open form analytic solutions. Thus, in Chapter 4, numerical analysis and comparative statics for the parameters of the will take place. Finally, Chapter 5 concludes the paper.

CHAPTER 2

THE BENCHMARK MODEL

2.1 The Model

In this section, we consider an economy inhabited by a representative agent who deals with the problem of technology adoption which tries to maximize the following inter-temporal utility function:

$$\int_0^T u(C(t))e^{-\rho t}dt$$

where C is the aggregate consumption and $u(.)$ is assumed to be increasing and concave. We do not analyze any labor dynamics throughout the paper, so population is assumed to be normalized to one and there is no population growth. Time horizon is taken as finite[1] in order to illustrate the sensitivity of optimal adoption timing to the optimization horizon and ρ denotes the subjective time preference.

For the consumption good sector, we use AK technology which uses physical capital as only input.

$$Y(t) = A(t)K(t) \tag{1}$$

where A denotes marginal productivity of capital and K denotes physical capital used to produce consumption good. The final good is either consumed or invested

[1] Infinite time horizon is considered separately in subsection 2.3

in physical capital or used for purchasing energy, which is used in production of capital good, satisfying the budget constraint:

$$Y(t) = C(t) + I(t) + f(R(t)) \tag{2}$$

where I and R denotes investment and energy usage respectively and f is a convex cost function of energy.

Energy saving technological progress is special for our model. Increasing the efficiency of energy usage is the fundamental aim of the problem. By improving energy efficiency, same level of capital good can be produced by low level of energy. The energy intensity of physical capital is higher with respect to the consumption good. To imply this intensity, we assume that physical capital accumulation is a function of energy and investment.[2] Energy is purchased directly with a given cost function. The technology for physical capital uses Cobb-Douglas function with the following accumulation rule:

$$\dot{K}(t) = (q(t)R(t))^{\alpha} I(t)^{1-\alpha} - \delta(t)K(t)$$

Here, $q(t)$ denotes the energy saving technological progress, which is assumed to be constant for this section, $\delta(t)$ denotes the depreciation function and $K(0) = K_0 > 0$ is taken as given. Accordingly, we assume that the energy and the investment are substitutes in physical capital production. Energy is purchased at a price of p relative to the consumption good to be used in capital good production.

There are two technological menus for the economy. The economy starts with (δ_1, q_1) and another option (δ_2, q_2) is available starting from $t = 0$ where $q_2 > q_1$ and $\delta_2 > \delta_1$. The economy can switch to a new technological regime with a more efficient energy usage in capital production at any instant of time. In contrast to increase in A the rise in q will only affect the capital goods. This rise will decrease the relative price of physical capital which induces a drop in consumption which

[2] See Perez-Barrahona (2007)

is referred as obsolescence cost. Moreover, switching to a more efficient energy saving technology incurs a loss in expertise expressed as an increased depreciation. Switching to a new technology induces accelerated erosion in physical capital and a slow adjustment process for reaching the best productivity level of the technology. Due to this erosion effect the depreciation function after switching is composed of two parts including the technology specific adjustment cost of the adoption. The second part is eroded with the speed of θ as time passes. Put differently, θ can be expressed as the speed of learning the usage of the new technology. More precisely, we have:

$$\delta_2(t) = \delta + \eta e^{-\theta(t-t_1)} \quad \forall t \in [t_1, T] \tag{3}$$

where η and θ are positive parameters and adjustment costs are eliminated with a speed measured by the parameter, θ.

Now, assume that the economy switches to a new technology regime at a date t_1. The state equation of capital differs after and before t_1 due to the technological menu change. Before the adoption, i.e., $0 \leq t < t_1$, the evolution of physical capital can be written as:

$$\dot{K}(t) = (q_1 R(t))^{\alpha} I(t)^{1-\alpha} - \delta_1(t) K(t) \tag{4}$$

After the adoption, i.e., $t_1 \leq t < T$, the evolution of capital is:

$$\dot{K}(t) = (q_2 R(t))^{\alpha} I(t)^{1-\alpha} - \delta_2(t) K(t) \tag{5}$$

where $\delta_1(t) = \delta$ and $\delta_2(t)$ is given as in equation (3). Note that, there is a trade off between two consecutive regimes. The productivity parameter of energy, q is higher in new technology regime, however, as the depreciation rate increases in the switching time, the capital accumulation is negatively effected. To solve the problem, we can move to the two stage optimal control approach.

2.2 Two Stage Optimal Control Approach

Our optimal control problem can be written as

$$\max_{R,C,t_1} \int_0^T u(C(t))e^{-\rho t}dt$$

subject to the constraints (1), (2), (4) and (5) and given $K_0 > 0$. Due to its dynamic structure, the problem can be rewritten as:

$$U(C,t_1) = \int_0^{t_1} u(C(t))e^{-\rho t}dt + \int_{t_1}^T u(C(t))e^{-\rho t}dt$$

Here, $t_1 \in [0,T]$ denotes the optimal switching time to new technologic regime. Since, two stage optimal control technique is well suited to our problem, we use this approach to find the value of the optimal t_1. This approach needs to divide the problem into two stages and operates in the following way:

2.2.1 The Second Stage Problem

We first assume that switching realizes at t_1 and the initial capital stock at t_1 is given, namely $K(t_1) = K_1$. For this stage we use logarithmic utility function and try to maximize:

$$U_2(K_1,t_1) = \int_{t_1}^T \ln(C(t))e^{-\rho t}dt$$

subject to the state equation (5) and free $K(T)$. In order to simplify the model we take linear cost function for energy. The corresponding Hamiltonian can be defined as:

$$H_2 = e^{-\rho t}\ln(C(t)) + \lambda_2(t)[(q_2R(t))^{\alpha}(A(t)K(t) - C(t) - pR(t))^{1-\alpha} - \delta_2(t)K(t)]$$

To simplify notation, we will not use time index after this point unless it is necessary. First order conditions can be written as:

$$H_C^2 = \frac{e^{-\rho t}}{C} - \lambda_2[(1-\alpha)(q_2R)^{\alpha}(AK - C - pR)^{-\alpha}] = 0,$$

$$H_R^2 = \lambda_2[\alpha q_2(q_2R)^{\alpha-1}(AK - C - pR)^{1-\alpha} - p(1-\alpha)(q_2R)^{\alpha}(AK - C - pR)^{-\alpha}] = 0,$$

$$H_K^2 = \lambda_2\left[(1-\alpha)A(q_2R)^{\alpha}(AK - C - pR)^{-\alpha} - \delta_2\right] = -\dot{\lambda}_2$$

where H_C^2, H_R^2 and H_K^2 are the first order conditions with respect to consumption, energy and capital respectively and K_1 is given. By using first order condition for energy usage we reach:

$$R(t) = \frac{\alpha}{p}AK(t) - C(t)$$

for every $t \in [t_1, T]$. Replacing this value on the first order condition for consumption we get the value of co-state variable as:

$$\lambda_2(t) = \frac{e^{-\rho t}}{C(t)}(1-\alpha)^{\alpha-1}(\frac{p}{\alpha q_2})^{\alpha}$$

Using this equation with the first order condition for physical capital, we have:

$$\frac{\dot{C}(t)}{C(t)} = \frac{A}{(1-\alpha)^{\alpha-1}(\frac{p}{\alpha q_2})^{\alpha}} - \delta_2 - \rho$$

From this equation we reach the paths followed by consumption, capital and co-state variable.

$$C(t) = a_1 e^{At(1-\alpha)^{1-\alpha}(\frac{p}{q_2\alpha})^{-\alpha} + \frac{e^{-(t+t_1)\theta}\eta}{\theta} - t(\delta+\rho)}$$

where a_1 is the constant of integration which is unknown.

$$
\begin{aligned}
K(t) \;=\; & \frac{1}{\rho} e^{-A(t_1-t(-1+\alpha))(1-\alpha)^{\alpha}\left(\frac{p}{q_2\alpha}\right)^{-\alpha}+\frac{\left(-1+e^{(-t+t_1)\theta}\right)\eta}{\theta}-t_1\rho-t(\delta+\rho)}(1-\alpha)^{-\alpha} \\
& \left(\frac{p}{q_2\alpha}\right)^{-\alpha}[a_1 e^{At_1(1-\alpha)^{\alpha}\left(\frac{p}{q_2\alpha}\right)^{-\alpha}+\frac{\eta}{\theta}}\left(e^{t\rho}-e^{t_1\rho}\right)(-1+\alpha) \\
& +e^{t\rho+t_1\left(A(1-\alpha)^{-\alpha}\left(\frac{p}{q_2\alpha}\right)^{-\alpha}\alpha+\delta+\rho\right)} K_1(1-\alpha)^{\alpha}\left(\frac{p}{q_2\alpha}\right)^{-\alpha}\rho],
\end{aligned}
$$

$$
\lambda_2(t) = \frac{e^{-At_1(1-\alpha)^{1-\alpha}(\frac{p}{q_2\alpha})^{-\alpha}+t\delta-\frac{e^{(-t+t_1)\theta}\eta}{\theta}}(1-\alpha)^{\alpha-1}(\frac{p}{q_2\alpha})^{\alpha}}{a_1}.
$$

By using the limit condition for the economy, $\lim_{t\to T}\lambda_2(t)K(t)=0$, we can reach the value of a_1 as:

$$
a_1 = \frac{e^{-\frac{\eta}{\theta}+T\rho+t_1\left(-A(1-\alpha)^{1-\alpha}(\frac{p}{q_2\alpha})^{-\alpha}+\delta+\rho\right)}K_1(1-\alpha)^{\alpha}\left(\frac{p}{q_2\alpha}\right)^{\alpha}\rho}{e^{T\rho}-e^{t_1\rho}}.
$$

By incorporating these values into the integration, we get the value for the optimal welfare in the second stage as $U_2^*(K_1,t_1)$ which is twice differentiable both with respect to K_1 and t_1.

2.2.2 The First Stage Problem

After solving the second stage problem, we now turn to the original problem and rewrite it as:

$$
\max_{C,R,t_1} U(C,t_1) = \int_0^{t_1} u(C(t))e^{-\rho t}dt + U_2^*(K_1,t_1)
$$

subject to the constraint (4) with given K_0 and free K_1 values. To solve this problem, by using Pontryagin maximum principle and taking K_1 and t_1 as fixed, we can write corresponding Hamiltonian as:

$$
H_1 = e^{-\rho t}\ln(C(t)) + \lambda_1(t)[(q_1R(t))^{\alpha}(A(t)K(t)-C(t)-pR(t))^{1-\alpha}-\delta_1K(t)]
$$

and get the paths for consumption, capital and co-state variables as follows:

$$C(t) = a_0 e^{At(1-\alpha)^{1-\alpha}(\frac{p}{q_1\alpha})^{-\alpha}-t(\delta+\rho)}$$

where a_0 is the constant of integration which is unknown.

$$\begin{aligned} K(t) \quad &= \quad \frac{1}{\rho} e^{-t\left(-A(1-\alpha)^{1-\alpha}\left(\frac{p}{q_1\alpha}\right)^{-\alpha}\right)}(1-\alpha)^{-\alpha}\left(\frac{p}{q_1\alpha}\right)^{-\alpha} \\ &\left[a_0\left(e^{t\rho}-1\right)(-1+\alpha)+e^{t\rho}K_0(1-\alpha)^{\alpha}\left(\frac{p}{q_1\alpha}\right)^{\alpha}\rho\right], \end{aligned}$$

$$\lambda_1(t) = \frac{e^{t_1\left((1-\alpha)^{1-\alpha}(\frac{p}{q_1\alpha})^{-\alpha}+\delta\right)}(1-\alpha)^{\alpha-1}(\frac{p}{q_1\alpha})^{\alpha}}{a_0}.$$

By using the continuity condition $\lambda_2(t_1^*) = \lambda_1(t_1^*)$ for co-state variable at t_1^* we find a_0 as:

$$a_0 = \frac{e^{T\rho+t_1\left(-A(1-\alpha)^{1-\alpha}(\frac{p}{q_1\alpha})^{-\alpha}+\delta+\rho\right)}K_1(1-\alpha)^{-1+\alpha}\left(\frac{p}{q_1\alpha}\right)^{\alpha}\rho}{e^{T\rho}-e^{t_1\rho}}.$$

and by using the continuity condition for capital stock we solve for K_1:

$$K_1 = \frac{e^{-t_1(\delta+\rho)}\left(-e^{t_1\left(A(1-\alpha)^{1-\alpha}(\frac{p}{q_1\alpha})^{-\alpha}+\rho\right)}+e^{At_1(1-\alpha)^{1-\alpha}(\frac{p}{q_1\alpha})^{-\alpha}+T\rho}\right)K_0}{e^{T\rho}-1}.$$

2.2.3 Value of the Optimal t_1

Since t_1 exists in the one of the state equation, namely in equation (5), we need to satisfy the following equations in order to have the interior solution:

$$\frac{\partial U_2^*(K_1,t_1)}{\partial t_1} = H_1^*(K_1,t_1) + \int_0^{t_1}\frac{\partial H_1^*}{\partial t_1}dt$$

This equation is the same as in Tomiyama and Rossana (1989):

$$H_2^*(K_1, t_1^*) - H_1^*(K_1, t_1^*) = \int_0^{t_1} \frac{\partial H_1^*}{\partial t_1} dt + \int_{t_1}^{T} \frac{\partial H_2^*}{\partial t_1} dt \tag{6}$$

In this case, the sufficient condition for maximum can be written as:

$$\frac{\partial H_2^*(K_1, t_1)}{\partial t_1} - \frac{\partial H_1^*(K_1, t_1)}{\partial t_1} < \frac{\partial}{\partial t_1} \left[\int_0^{t_1} \frac{\partial H_1^*}{\partial t_1} dt + \int_{t_1}^{T} \frac{\partial H_2^*}{\partial t_1} dt \right]$$

Corner solutions may also arise in this situation:

(i) Immediate switching: $t_1^* = 0$ if

$$H_1^*(K_1, t_1^*) - H_2^*(K_1, t_1^*) \geq \int_0^{t_1^*} \frac{\partial H_1^*}{\partial t_1} dt + \int_{t_1^*}^{T} \frac{\partial H_2^*}{\partial t_1} dt \text{ when } t_1^* = 0 \tag{7}$$

(ii) Technological sclerosis: The economy will never switch to new technology on $[0, T]$ if

$$H_1^*(K_1, t_1^*) - H_2^*(K_1, t_1^*) \leq \int_0^{t_1^*} \frac{\partial H_1^*}{\partial t_1} dt + \int_{t_1^*}^{T} \frac{\partial H_2^*}{\partial t_1} dt \text{ when } t_1^* = T \tag{8}$$

Using the first and the second stage problems together, we characterize the consumption, capital, energy and co-state variables paths for given K_0 and t_1. Finally, in order to determine optimal t_1, we use the equation (6) and get the equation that optimal t_1 should satisfy. However, for this case we have no interior solution for t_1 such that it belongs to the interval $(0, T)$. As stated above, corner solutions which are immediate switching or technological sclerosis may arise in this case. If the expression (7) holds at $t_1 = 0$, then the economy will switch immediately to the new technology. Otherwise, the expression (8) holds at $t_1 = T$, the technology will never switch to the newer one, i.e. technological sclerosis occurs.

Note that, there is no incentive to switch to the new technological regime in the time interval $(0, T)$. If the associated costs are to be eliminated sufficiently to increase the total welfare during the planning horizon, the option of delaying adoption cannot be optimal. In this case, the economy switches to the new technology immediately. Otherwise, if the costs cannot be eliminated sufficiently and total welfare is less than no switching case, the economy will never switch to new technology. In this type of model, there is no delaying option of new regime, i.e., delaying adoption will have no benefit.

The increase in the rate of the energy saving technological progress is associated with an increase in the depreciation rate of the physical capital, which is decreasing with the time up to the initial level. The change in this rate is combined with the costs induced by the loss in expertise, namely learning and obsolescence costs. If the economy switches to new regime and resulting improvement in efficiency is enough to compensate the loss in expertise, the economy will face a higher growth rate and will not delay the adoption.

2.3 The Infinite Horizon Case

In this section, we study the infinite horizon extension of our benchmark model, i.e., $T = \infty$. For this case, we follow the same steps as in the solution of the benchmark model. The optimization horizon enters to the model in the second stage optimization, so-called new technology problem. Now the limit conditions are replaced by the transversality conditions as when T goes to infinity:

$$\lim_{t \to \infty} \lambda_2^*(\mathrm{t})\mathrm{K}^*(\mathrm{t}) = 0$$

As one can easily check this is the unique departure from the benchmark finite horizon model. In the new technology problem, on $[t_1, \infty)$, paths for consumption, capital and co-state variable remains same as in finite case except for the coefficient a_1. In this case, a_1 can be written as:

$$a_1 = e^{-\frac{\eta}{\theta} + t_1\left(-A(1-\alpha)^{1-\alpha}\left(\frac{p}{q_2\alpha}\right)^{-\alpha} + \delta + \rho\right)} K_1(1-\alpha)^{\alpha-1}\left(\frac{p}{q_2\alpha}\right)^{\alpha}\rho.$$

In the old technology problem, the situation is similar to the new technology problem. On $[0, t_1)$, paths for consumption, capital and co-state variable remains same as in finite case except for the coefficient a_0. By using continuity of co-state variable at t_1, we get a_0 as:

$$a_0 = e^{t_1\left(-A(1-\alpha)^{1-\alpha}(\frac{p}{q_1\alpha})^{-\alpha}+\delta+\rho\right)} K_1(1-\alpha)^{\alpha-1}\left(\frac{p}{q_1\alpha}\right)^{\alpha}\rho.$$

Also, using the continuity for the physical capital leads

$$K_1 = e^{At_1(1-\alpha)^{1-\alpha}(\frac{p}{q_1\alpha})^{-\alpha}+-t_1(\delta+\rho)}K_0.$$

Now, following the same steps as in the finite case and by using equation (6) for our problem, we reach the equation that t_1 should satisfy. For this case, we have no interior solution for t_1 such that it belongs to the interval $(0, \infty)$. Corner solutions which are immediate switching or technological sclerosis also arise in this case. If the expression (7) holds at $t_1 = 0$, then the economy will switch immediately to the new technology. Otherwise, the expression (8) holds at $t_1 = \infty$, the technology will never switch to the newer one, i.e., the technological sclerosis occurs. Similar to the previous case, there is no incentive to delay the adoption of the new technology.

In this case, we consider the energy saving technology to jump the given constant level. As a result, we find that there is no interior switching option for this setup of the model. Accordingly, we introduce exogenously growing energy saving technology in the next section which guarantees interior solution for optimal timing.

CHAPTER 3

EXOGENOUSLY GROWING ENERGY SAVING TECHNOLOGY FRONTIER

In the benchmark model, we assumed that the technology is constant at switching time. However, the energy specific technology level is continually increasing along with time. This situation leads the representative agent to wait for a jump to a higher level of technology by delaying the adoption. In our model, the agent knows the growth in the technology will continue till the end of the planning horizon and make decisions accordingly. At $t = 0$, the level of the energy saving technological progress is anticipated at any instant of the optimization period.

We consider a linearly increasing technology with a speed of γ. The available level of energy saving technology at time t is given by $q(t) = 1+ \gamma t$.[3] At any t_1, the economy may switch to a more efficient energy using technology effecting the efficiency of capital goods sector positively, where the adopted energy saving technology level will be $q(t_1) = 1+ \gamma t_1$. As explained in the benchmark model, this rise in q will only affect the capital goods, in contrast to an increase in A. The rise will induce a drop in consumption which is referred as obsolescence cost. Moreover, switching to a more efficient energy saving technology incurs a loss in expertise which can be expressed as an accelerated depreciation. As a result, the

[3] See Dogan, Le Van and Saglam (2011). Moreover, exponentially growing technology case, namely $q(t) = e^{\gamma t}$, may also be examined.

accumulation rule for the stock of capital for the second stage is:

$$\dot{K}(t) = [(1+\gamma t_1)R(t)]^{\alpha} I(t)^{1-\alpha} - \delta_2(t)K(t)$$

All other assumptions, equations and the parameters remain as in the benchmark model. We will follow exactly the same steps as in the benchmark model. As defined in the two stage optimal control approach, we start by defining corresponding Hamiltonian for the second stage as:

$$H_2 = e^{-\rho t}\ln(C(t)) + \lambda_2(t)[((1+\gamma t_1)(R(t))^{\alpha}(A(t)K(t) - C(t) - pR(t))^{1-\alpha} - \delta_2(t)K(t)]$$

After writing the first order conditions and making necessary calculations including algebraic operations similar to the benchmark case, we get the paths for consumption, capital and costate variable for second stage are as follows:

$$C(t) = a_1 e^{At(1-\alpha)^{1-\alpha}(\frac{p}{\alpha+t_1\alpha\gamma})^{-\alpha} + \frac{e^{-(t-t_1)\theta}\eta}{\theta} - t(\delta+\rho)},$$

$$\begin{aligned} K(t) &= \frac{1}{\rho} e^{-A(t_1-t(-1+\alpha))(1-\alpha)^{-\alpha}\left(\frac{p}{\alpha+t_1\alpha\gamma}\right)^{-\alpha} + \frac{\left(-1+e^{(-t+t_1)\theta}\right)\eta}{\theta} - t_1\rho - t(\delta+\rho)} (1-\alpha)^{-\alpha} \\ &\left(\frac{p}{\alpha+t_1\alpha\gamma}\right)^{-\alpha} [a_1 e^{At_1(1-\alpha)^{-\alpha}\left(\frac{p}{\alpha+t_1\alpha\gamma}\right)^{-\alpha} + \frac{\eta}{\theta}} \left(e^{t\rho} - e^{t_1\rho}\right)(-1+\alpha) \\ &+ e^{t\rho + t_1\left(A(1-\alpha)^{-\alpha}\left(\frac{p}{\alpha+t_1\alpha\gamma}\right)^{-\alpha}\alpha + \delta + \rho\right)} K_1(1-\alpha)^{\alpha}\left(\frac{p}{\alpha+t_1\alpha\gamma}\right)^{-\alpha}\rho], \end{aligned}$$

$$\lambda_2(t) = \frac{e^{-At_1(1-\alpha)^{1-\alpha}(\frac{p}{\alpha+t_1\alpha\gamma})^{-\alpha} + t\delta - \frac{e^{(-t+t_1)\theta}\eta}{\theta}} (1-\alpha)^{\alpha-1}(\frac{p}{\alpha+t_1\alpha\gamma})^{\alpha}}{a_1}$$

By using the limit condition for the economy, $\lim_{t\to T}\lambda_2(t)K(t) = 0$, we can reach the value of a_1 as:

$$a_1 = \frac{e^{-\frac{\eta}{\theta}+T\rho+t_1\left(-A(1-\alpha)^{1-\alpha}(\frac{p}{\alpha+t_1\alpha\gamma})^{-\alpha}+\delta+\rho\right)} K_1(1-\alpha)^{-1+\alpha}\left(\frac{p}{\alpha+t_1\alpha\gamma}\right)^{\alpha}\rho}{e^{T\rho}-e^{t_1\rho}}$$

Again solving the first stage problem similar to the benchmark case by taking $q(0) = 1$ and t_1 and K_1 fixed, we solve the first stage problem by using Pontryagin maximum principle and write the first order conditions. By using these conditions we get the results for the first stage:

$$C(t) = a_0 e^{At(1-\alpha)^{1-\alpha}(\frac{p}{\alpha})^{-\alpha}-t(\delta+\rho)}$$

$$\begin{aligned} K(t) &= \frac{1}{\rho}e^{-t\left(-A(1-\alpha)^{1-\alpha}\left(\frac{p}{\alpha}\right)^{-\alpha}\right)}(1-\alpha)^{-\alpha}\left(\frac{p}{\alpha}\right)^{-\alpha} \\ &\quad \left[a_0\left(e^{t\rho}-1\right)(-1+\alpha)+e^{t\rho}K_0(1-\alpha)^{\alpha}\left(\frac{p}{\alpha}\right)^{\alpha}\rho\right] \end{aligned}$$

$$\lambda_1(t) = \frac{e^{t_1\left((1-\alpha)^{1-\alpha}(\frac{p}{\alpha})^{-\alpha}+\delta\right)}(1-\alpha)^{\alpha-1}\left(\frac{p}{\alpha}\right)^{\alpha}}{a_0}$$

After making heavy algebraic calculations, we will solve for optimal t_1 by means of the continuity and the optimality conditions. The continuity condition states that the co-state variable for first stage and second stage at the adoption time will yield the same value, i.e. $\lambda_1 \underset{t=t_1}{|} (t) = \lambda_2 \underset{t=t_1}{|} (t)$. By using this continuity condition, one can find a_0 as:

$$a_0 = \frac{e^{T\rho+t_1\left(-A(1-\alpha)^{1-\alpha}(\frac{p}{\alpha})^{-\alpha}+\delta+\rho\right)} K_1(1-\alpha)^{-1+\alpha}\left(\frac{p}{\alpha}\right)^{\alpha}\rho}{e^{T\rho}-e^{t_1\rho}}$$

Also, the capital stock should have same value at t_1 for both stages which yield:

$$K_1 = \frac{e^{-t_1(\delta+\rho)}\left(-e^{t_1\left(A(1-\alpha)^{1-\alpha}(\frac{p}{\alpha})^{-\alpha}+\rho\right)} + e^{At_1(1-\alpha)^{1-\alpha}(\frac{p}{\alpha})^{-\alpha}+T\rho}\right)K_0}{e^{T\rho}-1}$$

Now we can find the optimal value of t_1. To achieve this, we need to apply the optimality condition stated by Tomiyama and Rossana (1989) since our state equation is dependent to t_1. Solving this for our problem leads optimal value of t_1 should satisfy following equation:

$$\begin{aligned}
&e^{-t_1\rho}(\ln[(t_1\gamma)^{\alpha}] + \frac{1}{\rho(\theta+\rho)}e^{-T(\theta+\rho)}(-e^{t_1(\theta+\rho)}\eta\rho + e^{T\theta+t_1\rho}\alpha\gamma(\theta+\rho) + \\
&e^{T(\theta+\rho)}(\eta\rho - \alpha\gamma(\theta+\rho))) - \frac{1}{(1+t_1\gamma)\rho^2}e^{-T(\theta+2\rho)}(1-\alpha)^{-\alpha}Ae^{T(\theta+\rho)}(-1+\alpha) \\
&(\left(\frac{p}{(\alpha+\alpha t_1\gamma)}\right)^{-\alpha}(e^{T\rho}(-\alpha\gamma+\rho+t_1\gamma\rho) - e^{t_1\rho}(-\alpha\gamma+\rho+(t_1-T\alpha+t_1\alpha)\gamma\rho))) \\
&+\left(\frac{p}{\alpha}\right)^{-\alpha}(1+t_1\gamma)(e^{T\rho}-e^{t_1\rho})\rho)) = 0
\end{aligned}$$

Since this equation cannot be solved analytically for t_1, thus we make numerical analysis in the next section.

CHAPTER 4

NUMERICAL ANALYSIS

In this section, we perform the numerical analysis and comparative statics. We analyze how the trade off between technical progress and adjustment costs, i.e., how the optimal value of t_1, responds to an exogenous changes of the parameters of the model. For the benchmark parametrization, we start by taking parameters as given in Saglam (2010). We take $\delta = 0.1$ when there is no adjustment in the depreciation. This value is consistent with the literature as Nadiri and Prucha (1996) estimates this rate between 0.059 and 0.12. Moreover, relative price of the energy is taken as $p = 1.5$ to make the comparative analysis. We should also take the parameter η carefully in order not to make depreciation rate exceed the necessary level for interior switching or not to block the accumulation of capital.

As underlined earlier sections, productivity may not be high at the early stages of the implementation of the new technology. There are many studies examining this inefficiency such as Bahk and Gort (1993). They use panel data from 15 different industries and estimate that adjustment to new technology is realized within 6 years. Consistent with these learning-by-doing models, we assume that adjustment cost is eliminated with a speed of $\theta = 0.7$. The other parameters are taken as in Table 1.

Table 1. Values of the benchmark parametrization

γ	α	ρ	θ	η	A	K_0	T
0.02	0.3	0.03	0.7	0.2	2	1	30

While finding optimal technology adoption timing, it should be examined that this value maximizes the total welfare. For this reason we calculate the total welfare as a function of t_1 by taking the given parameters. Our results show that the value that we find is optimal and maximizes the total welfare. When we take the parameters above, we find optimal t_1 as 11.35 with a total welfare of 167.61 which is its maximum value and the point where the second derivative is negative.

After finding the optimal value of t_1 and calculating optimal welfare we want to see the effects of the changes in the parameters. In table 2, the effect of changes in the technological growth parameter, γ; in table 3, the effect of changes in price of the energy, p; in table 4, the effect of changes in adjustment parameter, θ; in table 5, the effect of changes in the time preference parameter, ρ; in table 6, the effect of changes in planning horizon, T; in table 7, the effect of changes in the Cobb-Douglas parameter for energy usage, α are presented.

As expected, the increase in the growth rate of energy saving technical progress accelerates the adoption of the new technology. The associated increase in the growth rate advantage reduces the time required for the growth rate advantage to dominate the costs of obsolescence and accelerated depreciation. With the higher values of γ, instead of waiting switching, one may switch to new technology regime before and the total welfare increases with the increase in γ.

Table 2. The effect of changes in the technological growth parameter

γ	t_1^*	$total\ welfare$
0.01	15.23	165.55
0.02	11.35	167.61
0.03	10.27	169.76
0.04	9.71	171.86
0.05	9.35	173.89

The rise in the relative price of energy leads the decrease in the usage of energy in capital good production. In this case, delaying adoption of the new technology will increase the growth rate advantage of the adoption. Thus, when we increase the linear price of the energy, optimal value of t_1 increases, however when it is compared to γ, it has less effect on the optimal timing. Moreover, as one can easily predict, total welfare is decreasing with the increase in price.

Table 3. The effect of changes in the energy price

p	t_1^*	$total\ welfare$
1	11.01	197.32
1.5	11.35	167.61
2	11.63	148.69
2.5	11.88	135.37
3	12.10	125.12

An increase in θ accelerates the adjustment to the new technology so that increase in θ will decrease the adoption time. The rise in θ will eliminate the effect of erosion in capital faster. However, for sufficiently small values of θ, the agent would never switch to new technology and face a technologic sclerosis. Change in θ does not have significant effects on the total welfare.

Table 4. The effect of changes in the adjustment parameter

θ	t_1^*	$total\ welfare$
0.1	*no switch*	165.41
0.3	16.84	165.41
0.5	12.62	166.75
0.7	11.35	167.61
0.9	10.71	168.16
1.1	10.31	168.54

The effect of the time discounting parameter on the pattern of technology adoptions can be seen in Table 5. It is observed that if the impatience rate is higher, the economy tends to delay the adoption. As ρ increases, the delay in the adoption of the more efficient technology occurs due to the obsolescence costs. With our parameter setting, as ρ increases, the advantage of growth rate of new technology is dominated by the obsolescence costs.

Table 5. The effect of changes in the subjective time preference

ρ	t_1^*	*total welfare*
0.01	11.29	252.54
0.02	11.31	205.26
0.03	11.35	167.61
0.04	11.41	137.52
0.05	11.48	113.41

Now, we consider the optimal pattern of technology adoption shifts in response to the changes in the planning horizon. It is clear that longer planning horizons provides an incentive to delay the adoption to have more from the growth rate advantage. As it is proven in Boucekkine et al. (2004), longer planning horizons lead delays in the adoption time of new technology. On the other hand, if the horizon is short enough, the agent would stick to the initial technology and end up with technological sclerosis.

Table 6. The effectof changes in the planning horizon

T	t_1^*	*total welfare*
10	*no switch*	21.82
30	11.35	167.61
50	15.38	354.16
70	19.13	518.10
∞	30.29	891.46

The effect of changes in the share of the energy in capital good production is presented in Table 7. If the share of energy gets higher, optimal adoption time decreases which enables the economy to utilize more from the efficient energy usage.

Table 7. The effect of changes in the share of energy in capital good production

α	t_1^*	$total\ welfare$
0.2	12.28	203.25
0.3	11.35	167.61
0.4	10.87	146.85
0.5	10.55	136.15
0.6	10.30	133.63

Finally, the initial level of capital stock, K_0 and the level of disembodied technology, A do not change the optimal adoption timing whereas any increase in initial capital stock level yields a greater level of total welfare certainly. In addition to these comparative statics, we observed that the main factor that affects the adoption is the obsolescence cost and adjustment cost due to loss in expertise caused by the embodied technological change. The gain from the rate of this change is associated with a reduction in the price of energy, also in capital, so that more resources are supplied to capital production and consumption level drops. If the adoption is delayed too much, the obsolescence cost gets higher and more drop in consumption is realized. Therefore, it is not optimal to devote more time for waiting later technologies in order to utilize the advantages of newer technology.

CHAPTER 5

CONCLUSION

In this study, we have applied two stage optimal control techniques to solve the optimal adoption problem in a model including energy usage and endogenous depreciation. We have first solved a benchmark model without exogenously growing energy saving technology. To do so, we derived necessary conditions of optimality for two stage optimal control problems in which the switching time appears in the state equation. In this case, delaying the adoption is never optimal. If the growth advantage of the technology adoption is higher than the obsolescence costs and adjustment costs associated with the depreciation, the economy switches immediately; otherwise, it sticks to the initial technology and technologic sclerosis occurs.

In the second step, we have introduced exogenously growing energy saving technology to the benchmark model. We stated the optimality conditions in this setup and reached the equations that value of the optimal timing should satisfy. Although, we cannot derive the open form analytical optimal adoption timing, we showed numerically that interior solution for optimal timing is attained. We also provided numerically the effects of the planning horizon, growth rate of technology, discounting parameter, speed of adjustment, share of energy in capital good production and initial level of capital stock on the optimal pattern of the technology adoption. We find that increase in speed of adjustment decreases the optimal adoption time. Moreover, any technology growth rate increase also decreases this time. However, increasing the planning horizon of the model delays the adoption to get more benefits from increasing technology frontier.

Further extensions could be considered by applying multi-stage optimal control which allow technology switches. Also, energy sector could be included covering extraction processes and stock levels. Moreover, the damages and the harmful effects of pollutant energy resources could be added to the analysis. Finally, by increasing the number of agents, the interactions among agents could be analyzed..

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