A MOBILE AMMUNITION DISTRIBUTION SYSTEM DESIGN ON THE BATTLEFIELD

A DISSERTATION SUBMITTED TO
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FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

By
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January, 2010

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of doctor of philosophy.

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ABSTRACT

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January, 2010

Ammunition has been the most prominent factor in determining the outcome of combat. In this dissertation we study a military logistics problem in which ammunition requirements of the combat units, which are located on the battlefield, are to be satisfied in the right amount when and where they are needed. Our main objective is to provide a decision support tool that can help plan ammunition distribution on the battlefield. We demonstrate through an extensive literature review that the existing models are not capable of handling the specifics of our problem. Hence, we propose a mathematical programming model considering arc-based product-flow with $O(n^4)$ decision variables and constraints. The model is a three-layer commodity-flow location routing formulation that distributes multiple products, respects hard time windows, allows demand points to be supplied by more than one vehicle or depot, and locates facilities at two different layers. We then develop a new mathematical programming model with only $O(n^3)$ decision variables and constraints by considering node-based product-flow. We derive several valid inequalities to speed up the solution time of our models, illustrate the performance of the models in several realistically sized scenarios, and report encouraging results. Based on these mathematical models we propose two three-phase heuristic methods: a routing-first location-second and a location-first routing-second heuristic. The computational results show that complex real world problems can effectively be solved in reasonable times with the proposed heuristics. Finally, we introduce a dynamic model that designs the distribution system in consecutive time periods for the entire combat duration, and show how the static model can be utilized in dynamic environments.

Keywords: Location routing, logistics, distribution, network design.

ÖZET

MUHAREBE SAHASINDA MOBIL BIR MÜHIMMAT DAĞITIM SISTEMI TASARIMI

Hünkâr Toyoğlu
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Tez Yöneticileri: Doç. Dr. Bahar Yetiş Kara ve
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Ocak, 2010

Mühimmat savaşın sonucunu belirleyen en önemli etmendir. Bu doktora çalışmasında muharebe sahasında bulunan birliklerin mühimmat ihtiyaçlarının istenen yer ve zamanda ve doğru miktarda karşılanması gereken askeri bir lojistik problem incelenmiştir. Esas amacımız muharebe sahasında mühimmat dağıtımının planlanmasına yardımcı olabilecek bir karar destek sistemi geliştirmektir. Kapsamlı bir yazılı eser taramasından sonra önceden geliştirilmiş olan modellerin problemimizi çözmeye yeterli olmadıkları gösterilmiştir. Bu sebeple ayrıt tabanlı ürün akışına dayanan, $O(n^4)$ sayıda karar değişkeni ve kısıt içeren bir matematiksel model önerilmiştir. Söz konusu model üç katmanlı ürün akış modeli olup birden fazla ürün dağıtmakta, esnek olmayan zaman pencerelerini içermekte, ihtiyaç noktalarının birden fazla araç veya depo tarafından desteklenmesine izin vermekte ve iki farklı katmana tesis yerleştirmektedir. Daha sonra düğüm tabanlı ürün akışına dayanan ve sadece $O(n^3)$ sayıda karar değişkeni ve kısıt içeren bir matematiksel model geliştirilmiştir. Modellerin çözüm zamanının iyileştirilmesi maksadıyla birçok geçerli eşitsizlik geliştirilmiş, bazı gerçek boyutlu senaryolar üzerinde modellerin performansları denenmiş ve iyi sonuçlar elde edildiği gösterilmiştir. Bu modeller temel alınarak yol atama-yerleştirme ve yerleştirme-yol atama olmak üzere üç aşamalı iki ayrı bulgusal yöntem geliştirilmiştir. Hesaplama sonuçları önerilen yöntemler sayesinde karmaşık gerçek problemlerin makul zamanlar içerisinde çözülebildiğini göstermektedir. Son olarak, muharebe boyunca birbirini izleyen zaman aralıklarında dağıtım ağını tasarlayan dinamik bir model geliştirilmiş ve statik modelden dinamik ortamlarda nasıl faydalanılabileceği gösterilmiştir.

Anahtar sözcükler: Ağ tasarımı, yer seçimi ve yol atama, lojistik, dağıtım.

Sabır ve özveriyle her zaman yanımda olan
sevgi dolu
eşim Arzu'ya

Varlıkları ile hayatımıza anlam veren
mutluluk kaynağı
çocuklarımız Atakan ve Duru'ya

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Chapter 1

Introduction and Motivation

A soldier on the battlefield will be; hungry without food, thirsty without water but dead without ammunition.

The success of any type of combat operation depends on the availability of ammunition (henceforth called ammo), because a combat unit can fight so long as it receives ammo in the proper quantity, when and where it is needed. Hence, ammo is the dominant factor in determining the outcome of combat and any failure to supply the required amount of ammo may result in tactical defeat.

Previously, land forces of most countries relied upon heavy forces that were equipped with large number of heavy weapons for their lethality. The more heavy weapons a land force has, the more fire power it has and the more lethal it is. However, when equipped with such numbers and types of weapons, land forces lose their ability to move fast, and they need to keep enormous ammo inventories stocked at huge depots. Hence, ammo distribution system of these land forces is designed to support a heavy and slow moving force. It usually consists of different types of various depots most of which are underground storage facilities, bunkers or fortified storage areas.

In general, this distribution system is a continuous replenishment system in which ammo flows from 1^{st} level depots to combat units. Ammo, which is produced or procured, is first received by 1^{st} level depots. From there it is transported to 2^{nd} level depots typically by rail networks. From 2^{nd} level depots ammo is shipped to 3^{rd} level depots mostly by trucks and if possible by rail. Then combat units draw their ammo requirements from 3^{rd} level depots with their own trucks. In general, this system is a push system from 1^{st} level depots to 3^{rd} level depots and a pull system from 3^{rd} level depots to combat units, i.e. it is pushed down to 3^{rd} level depots and pulled from there by combat units.

With the end of Cold War, most of these traditional heavy land forces have moved towards a smaller, more agile, deployable and lethal force. Such a force does not depend solely on its firepower, but also on its mobility. This characteristic enables newly structured forces to move further and faster on the battlefield. To supply such a fast moving force, an effective and efficient distribution system is needed.

Current distribution system of most countries' land forces may face some problems in supporting newly structured agile forces in their variety of missions and rapidly changing combat environments. Therefore, as land forces of most countries change their structure, their ammo distribution systems should be converted to a more mobile and flexible distribution process to provide more effective support.

To realize this request for an effective and flexible support system, we propose Mobile Ammunition Distribution System (Mobile-ADS). Our main objective is to deliver ammo as close to the combat units as possible, and do this in a timely manner. To do so, we suggest Fixed Transfer Points (Fixed-TPs) and Mobile Transfer Points (Mobile-TPs), that – after proper positioning – will cease the need for the remaining depots. Fixed-TPs are either railheads where the rail network ends or suitable locations on rail network where ammo can be transported safely as far as possible on the battlefield. Ammo is transferred from trains to commercial trucks at Fixed-TPs. Mobile-TPs are mostly forward staging areas where ammo trucks or stocks of ammo are kept for a short period of time before being moved further forward to support front line combat units. They are located as close to combat units as possible to provide the least supply time. Ammo is transferred from commercial trucks to ammo trucks at Mobile-TPs. With their small and mobile structure Mobile-TPs can support agile land forces by moving with them accordingly.

The rest of this dissertation is organized as follows: Chapter 2 describes Mobile-ADS design problem, reviews the related literature and compares the characteristics of our problem with those of the majority of the literature. Chapter 3 demonstrates the 4-index static arc-based product flow mixed integer programming formulation of the design problem and derives several valid inequalities to improve the solution time. Chapter 4 presents the static 3-index node-based product flow mixed integer programming formulation of the design problem and derives several valid inequalities. Chapter 5 analyzes the effectiveness of the valid inequalities for both 4-index and 3-index formulations in some test problem instances and determines the ones that help reduce the solution time of the models. Chapter 6 tests the 4-index and 3-index formulations in several realistic size problem instances. Chapter 7 introduces two heuristic approaches of which the first is a "VRP first-LRP second" and the second is a "LRP first-VRP second" heuristic. Chapter 8 evaluates the performance of the two heuristics in the same realistic scenarios. Chapter 9 extends the static 4-index formulation over time and presents a dynamic formulation to cover entire battle duration. Chapter 10 discusses how the static model can assist in a multi-period combat operation, and also how it can help the logistics planners when faced with unplanned combat situations. Chapter 11 presents the summary and the conclusions.

Chapter 2

Problem Definition and Related Literature

In this chapter, we first define the Mobile-ADS design problem. We, then, develop a classification scheme with 17 problem characteristics and classify 78 articles from the literature. Next, we give the classification of the problem we study. Finally, we highlight how our Mobile-ADS design problem differs from the existing studies in the literature.

2.1 Problem definition

Mobile-ADS is a continuous replenishment and a true push system. Highest level depots are the first to receive ammo that is produced or procured. Ammo is then moved forward as far as possible with rail networks. Where and how far we can carry ammo depends on the available rail network structure. We will assume those locations, where the rail network ends (rail heads), as potential Fixed-TP locations.

Within the context of this study we do not analyze the flow from the highest level depots to Fixed-TPs. We assume that the required amount of ammo can

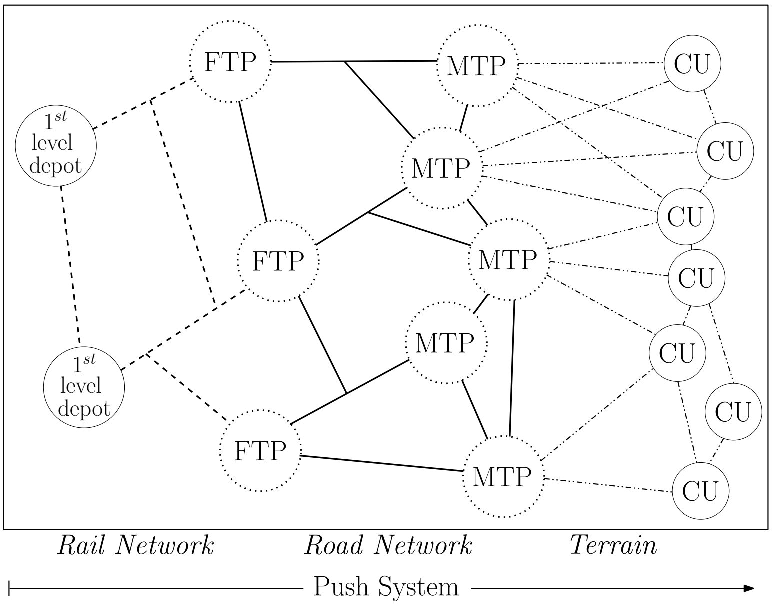

Figure 2.1: Mobile ammo distribution system on the battlefield

be carried from the highest level depots to Fixed-TPs on time by rail. This assumption imposes no constraint on the system since there is enough ammo at the highest level depots and current rail network structure and equipment is sufficient to handle that amount.

Ammo is moved from Fixed-TPs to Mobile-TPs by commercial trucks on road networks. Then Mobile-TPs issue ammo to their attached units with ammo trucks which have the capability to move on terrain and to load and unload themselves with their own crane. In such a system, combat units will not take the logistic burden of drawing their ammo by themselves, on the contrary, ammo will be pushed down to them. Figure 2.1 shows an example of a Mobile-ADS on the battlefield. Solid (dotted) circles represent fixed (potential) locations, respectively. Ammo is distributed from the highest level depots to Fixed-TPs (denoted by FTP in the figure) on rail networks by trains, from Fixed-TPs to Mobile-TPs (denoted by MTP in the figure) on road networks by commercial trucks, and from Mobile-TPs to combat units (denoted by CU in the figure) on terrain by ammo trucks.

Consider a battlefield containing Fixed-TPs, Mobile-TPs and combat units.

In order to model Mobile-ADS we will take a snapshot of the battlefield and freeze the location of combat units at a particular point in time. Hence, the location of combat units will be fixed and the remaining decisions will be the locations of Fixed-TPs and Mobile-TPs in order to provide a bridge between the highest level depots and combat units.

A Mobile-TP can not be located anywhere on the battlefield. Such a site should possess characteristics to allow technical support operations as well as tactical defense against enemy threats. Logistics planners consider these characteristics and perform on-site or map reconnaissance to determine potential Mobile-TP locations before battle commences. As already explained, potential Fixed-TP locations are the rail heads that are close to battlefield.

In light of above explanations our problem is to supply combat units with correct types and quantities of ammo when and where it is needed. To do so, we need to consider the following planning requirements; (a) number and location of Fixed-TPs and Mobile-TPs, (b) vehicle routes and schedules to distribute ammo from Fixed-TPs to combat units via Mobile-TPs.

Solving above problems separately may lead to suboptimal decisions (see, for example, [75] for interdependency between location and routing). Therefore, these decisions must be made simultaneously. Hence, Mobile-ADS design problem, which combines the location, routing and scheduling problems into a single model, is a Location Routing Problem (LRP).

2.2 Literature review

As stated in [59], LRPs solve the combined problem of (1) determining the optimal number and location of facilities that serve more than one demand point and (2) finding the optimal set of vehicle routes and schedules. In an LRP some location(s) must be decided among potential locations otherwise the problem becomes a sole routing problem. Likewise, tours must be allowed among facilities/demand points otherwise the problem would be reduced to a location problem.

If we follow the framework of [41] we can represent the distribution system in an LRP as layers. In this study, Fixed-TPs, Mobile-TPs and combat units exist at the first, second and third layers, respectively. Moreover, in this framework a tour is a round trip through several Mobile-TPs and/or combat units, making multiple deliveries.

There are several earlier studies that introduce LRP or review LRP literature, see for example [13], [41], [53], [59] and most recently [64]. An unpublished study, [1], also deserves considerable attention. In this study; authors review the LRP literature extensively including all problem variants (structure location or extensive facility location problems, etc.) according to a three-characteristic classification scheme: (1) deterministic or stochastic demand and supply, (2) central (pull type) or anticentral (push type) facilities and (3) single or multiple objectives. They explain some milestone studies that have longer lasting impact on LRP research, and state some untouched areas that require further study.

In [59] 12 (including solution methods) problem characteristics, and in [64] 9 (including solution methods) problem characteristics are used to classify the literature. Both classifications have 5 problem characteristics in common: (1) deterministic or stochastic location-routing parameters (demand, supply size, etc.), (2) single or multiple facilities, (3) single or multiple periods and (4) single or multiple objectives (5) exact and heuristic methods ([59] investigates this characteristic separately).

In addition to above characteristics [59] has 7 distinct characteristics: (1) single-stage (only delivery routes) or two-stage (delivery and pick up routes), (2) single or multiple vehicles, (3) capacitated or uncapacitated vehicles, (4) capacitated or uncapacitated facilities, (5) primary (origin or destination of a vehicle) or secondary (intermediate or transshipment node) facilities, (6) none or loose or strict time deadlines and (7) hypothetical or real data.

Likewise, [64] uses 4 distinct characteristics: (1) standard (no routes between facilities) or non-standard hierarchical structure, (2) exact or heuristic solution methods, (3) discrete or network or continuous solution space and (4) single or homogeneous or heterogeneous vehicle fleet.

While problem characteristics of [59] and [64] cover most of the key elements of the LRP framework, they do not fully address some elements that we believe are important. In addition, recent developments in logistics systems necessitate the alteration of some of their elements and employment of the new dimensions of distribution logistics into the classification. These alterations and additions of problem characteristics are described next.

A distribution system may consist of two layers (facilities and customers) or three/four layers (primary facilities, secondary facilities and customers). In todays just-in time environment a distribution system should address the time restrictions of customers. Customers may invoke no time restrictions (very unlikely), soft time restrictions (for example, one-sided time windows or time limits on driving times) or hard time restrictions (for example, two-sided time windows). Note that both loose and strict time deadlines of [59] are in the soft time restriction category. In a two-layer LRP (assuming customers are located at fixed and known locations at the second layer) we need to locate facilities at the first layer. In a three-layer LRP (with the same fixed customer location assumption at the third layer) we need to locate facilities at the first and/or second layers. In general, in a two or three/four-layer LRP locational decisions may exist at a single layer or at two different layers. LRPs may seek to distribute either a single product or multiple products. In an LRP with single sourcing each customer is to be supplied by exactly one vehicle or depot. On the contrary, customers may be supplied by more than one vehicle or depot which is referred to as multi sourcing. In an LRP there may exist inventories at the facilities that are to be located or there may not exist any inventory.

In light of above explanations we use a classification scheme consisting of 17 problem characteristics a summary of which is depicted in Table 2.1. Briefly, our classification shares the same 4 common characteristics with [59] and [64]: (1) deterministic or stochastic location-routing parameters (demand, supply size, etc.), (2) single or multiple facilities, (3) single or multiple periods and (4) single or multiple objectives.

We have 5 common characteristics with [59]: (1) single-stage (only delivery

Table 2.1: Classification scheme

1. Hierarchical level	7. Number of layers	13. Locational decision
a. Delivery or pickup	a. Two	a. At one layer
b. Delivery and pickup	b. Three/Four	b. At two layers
2. Nature of demand	8. Planning period	14. Product
a. Deterministic	a. Single	a. Single
b. Stochastic	b. Multiple	b. Multiple
3. Number of facilities	9. Time restriction	15. Sourcing
a. Single	a. No	a. Single
b. Multiple	b. Soft	b. Multiple
4. Vehicle fleet	c. Hard	16. Inventory
a. Single	10. Objective function	a. Exist
b. Homogeneous	a. Single	b. Not exist
c. Heterogeneous	b. Multiple	17. Solution method
5. Vehicle capacity	11. Data	a. Exact
a. Capacitated	a. Real	b. Heuristic
b. Uncapacitated	b. Hypothetical	
6. Facility capacity	12. Solution space	
a. Capacitated	a. Continuous	
b. Uncapacitated	b. Discrete	

routes) or two-stage (delivery and pick up routes), (2) capacitated or uncapacitated vehicles, (3) capacitated or uncapacitated facilities, (4) none or loose or strict time deadlines and (5) hypothetical or real data. Three of our characteristics exist in the classification of [64]: (1) exact or heuristic solution methods, (2) discrete or network or continuous solution space and (3) single or homogeneous or heterogeneous vehicle fleet.

In addition to above characteristics, we have 5 distinct characteristics that are not used by [59] or [64]: (1) two or three-four layers, (2) location at one layer or two layers, (3) single or multiple products, (4) single or multiple sourcing and (5) inventory exists or not exist.

We classify 78 studies according to the explained scheme. The details of our classification are presented in Tables 2.2 through 2.4 in chronological order.

Table 2.2: Classification of the literature (*Part 1: Articles from 1976 to 1989*)

	1		2		3		4			5		6		7		8		9			10		11		12		13		14		15		16		17	
	a	b	a	b	a	b	a	b	c	a	b	a	b	a	b	a	b	a	b	c	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
[22]	✓			✓	✓		✓				✓		✓	✓		✓		✓			✓			✓	✓		✓		✓		✓			✓	✓	
[26]	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
[27]	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓			✓	✓		✓		✓		✓			✓	✓	
[35]	✓			✓		✓		✓			✓		✓	✓		✓		✓			✓		✓			✓	✓		✓		✓			✓		✓
[67]		✓	✓			✓		✓		✓			✓	✓		✓		✓			✓		✓			✓	✓		✓		✓			✓		✓
[37]	✓		✓			✓			✓	✓			✓		✓	✓		✓			✓			✓		✓	✓		✓		✓			✓		✓
[44]	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
[65]	✓		✓			✓		✓		✓		✓		✓		✓			✓		✓		✓			✓	✓		✓		✓			✓		✓
[46]	✓		✓			✓		✓			✓		✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
[56]	✓		✓			✓		✓		✓		✓			✓	✓			✓		✓			✓		✓	✓		✓		✓			✓		✓
[70]	✓		✓			✓		✓		✓		✓			✓	✓		✓			✓		✓			✓	✓		✓		✓			✓		✓
[39]	✓		✓			✓		✓			✓		✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
[71]	✓		✓			✓		✓		✓		✓			✓	✓		✓			✓		✓			✓	✓		✓		✓			✓		✓
[14]	✓			✓	✓		✓			✓			✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
[45]	✓		✓			✓		✓		✓		✓		✓		✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
[15]	✓			✓	✓		✓				✓		✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓		✓
[18]	✓		✓			✓			✓	✓		✓			✓	✓		✓			✓			✓		✓	✓			✓	✓			✓		✓
[47]	✓		✓			✓		✓		✓		✓		✓		✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
[81]	✓			✓	✓		✓				✓		✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓		✓
[16]	✓			✓	✓		✓				✓		✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓		✓
[42]	✓		✓			✓		✓			✓		✓		✓		✓	✓			✓			✓		✓	✓		✓		✓			✓	✓	✓
[43]	✓			✓		✓		✓		✓			✓		✓	✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
[49]	✓		✓			✓		✓		✓			✓	✓		✓			✓		✓		✓			✓	✓		✓		✓			✓		✓
[66]	✓		✓			✓		✓		✓		✓		✓		✓	✓		✓		✓		✓			✓	✓		✓		✓			✓		✓
[89]	✓		✓			✓	✓				✓	✓		✓		✓		✓				✓		✓		✓	✓		✓		✓			✓	✓	

Table 2.3: Classification of the literature (*Part 2: Articles from 1990 to 1999*)

	1		2		3		4			5		6		7		8		9			10		11		12		13		14		15		16		17	
	a	b	a	b	a	b	a	b	c	a	b	a	b	a	b	a	b	a	b	c	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
[19]	✓		✓			✓		✓			✓	✓	✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓		✓
[83]	✓		✓			✓			✓	✓			✓	✓		✓			✓		✓			✓		✓	✓		✓		✓			✓		✓
[52]	✓		✓			✓		✓			✓		✓	✓		✓		✓				✓	✓			✓	✓			✓	✓			✓	✓	
[72]	✓		✓			✓		✓			✓		✓	✓		✓		✓				✓	✓			✓	✓		✓		✓			✓	✓	
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[13]	✓			✓	✓		✓				✓		✓	✓		✓		✓			✓			✓		✓	✓		✓		✓			✓	✓	
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Table 2.4: Classification of the literature (*Part 3: Articles from 2000 to 2009*)

	1		2		3		4			5		6		7		8		9			10		11		12		13		14		15		16		17	
	a	b	a	b	a	b	a	b	c	a	b	a	b	a	b	a	b	a	b	c	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
[28]	✓			✓		✓		✓		✓			✓	✓		✓			✓		✓		✓			✓	✓		✓		✓			✓		✓
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[87]	✓		✓			✓			✓	✓		✓		✓		✓		✓			✓			✓		✓	✓		✓		✓			✓		✓
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[2]	✓		✓			✓		✓		✓			✓		✓	✓			✓		✓			✓		✓	✓		✓		✓			✓		✓
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[3]	✓		✓			✓		✓			✓	✓		✓		✓		✓			✓			✓		✓	✓		✓		✓			✓		✓
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[5]	✓		✓			✓		✓			✓	✓			✓	✓		✓				✓	✓			✓		✓		✓	✓			✓	✓	
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[23]	✓		✓			✓		✓		✓			✓	✓		✓			✓			✓	✓			✓	✓		✓		✓			✓		✓
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[4]	✓			✓		✓		✓			✓	✓		✓		✓		✓			✓			✓		✓	✓		✓		✓			✓		✓
[79]	✓			✓		✓		✓		✓			✓		✓	✓		✓			✓			✓		✓	✓		✓		✓		✓			✓
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[6]	✓		✓			✓			✓	✓		✓			✓	✓		✓			✓		✓	✓		✓	✓		✓		✓			✓		✓

2.3 Comparison

To better reflect the characteristics of our Mobile-ADS design problem, to express where it stands in the LRP literature and to highlight how it distinguishes from the previous studies, we further explain some specifics pertaining to Mobile-ADS design problem and compare its classification with that of the majority of the LRP literature. The classification of Mobile-ADS design problem can be stated as follows:

- 1a. In Mobile-ADS we only consider the delivery of ammo to combat units.
- 2a. All parameters (travel times, capacities, depot and transportation costs, etc.) in the problem are assumed to be fixed and known. Here, the most problematic issue is daily ammo demand of combat units. Today, military services of almost all countries generally use three approaches to estimate the amount of ammo expected to be consumed daily (consumption rate) in combat. In the first method, we predict the number of targets a weapon will encounter on a daily basis, and multiply it with the required amount of ammo to destroy each target. In the second method, we predict the life of a weapon in combat before it is destroyed by enemy. Then, we predict the number of engagements in its lifetime, and multiply it with the expected ammo expenditure per engagement. In the third method, we use mathematical programming models. We define a combat scenario consisting of friendly and enemy weapons. We input several weapon and target characteristics, such as probability of hit, probability of kill, etc. Then the model gives the amount of ammo expended by each friendly weapon to defeat allocated enemy weapons. Most parameters used in these three methods (expected target number, expected ammo expenditure per engagement, etc.) are based on historical data and actual field experiments or tests. However, they are constantly adjusted as new data is collected. In addition, to make the predictions more accurate, these parameters change depending on the type of mission (offense, defense, etc.), terrain (desert, forest, etc.), day of the combat (first day, second day, etc.) and anticipated operational tempo. Although, there is no visible way to predict daily requirements certainly

ahead of time, we are not totally in the dark either. Hence, we consider anticipated daily consumption rates, calculated as explained above, and treat them as fixed demands for the sake of our model's tractability. Therefore, we assume that demand is fixed and known. Hence, Mobile-ADS design problem is a deterministic LRP.

- 3b. We locate multiple fixed and mobile transfer points.
- 4c. We have two different groups of vehicles, namely commercial and ammo trucks. In addition, in each group we have various types of trucks that have different capacities which in turn have different acquisition and operation costs. For example, there exist 20, 30 and 40-ton commercial trucks and 5, 8 and 10-ton ammo trucks.
- 5a. Both commercial and ammo trucks are capacitated.
- 6a. Due to man power, terrain, enemy threat and fire safety considerations, all transfer points have capacities.
- 7b. Three layers exist and Fixed-TPs/Mobile-TPs/combat units are located at the first/second/third layers, respectively.
- 8ab. The problem of designing Mobile-ADS is very complex if we want to capture all realities at once. Hence, to be able to better explain the problem and its formulation, we introduce the following limitation. Since battles generally continue for days, weeks or months, we need to consider several consecutive planning periods in our problem. However, due to the complexity, the dynamic version of the problem is presented following the consideration of the single planning period. Overall, we consider consecutive 24-hour planning periods since each combat unit possesses a specific amount of ammo on hand to initiate and continue combat operations for 24 hours until it is supplied from the rear.
- 9c. Battlefield is open to unexpected circumstances. At different times of combat, depending on the combat type and enemy threat, some ammo types may become more valuable and combat units may require them more urgently than other types of ammo. Therefore, there are different time

deadlines for each type of ammo and for each combat unit. Not supplying a unit by this deadline with the necessary ammo means the unit continues combat without the required ammo type – a potentially lethal situation for a unit engaged with the enemy in combat – between the unit's time deadline and the supply time. In addition, as combat continues, units change their locations and resupply becomes even more difficult. Supplying a unit with ammo requires the unit to halt for some time and take the required precautions such as perimeter security, etc. A combat unit can not always halt in battle especially when it is actively engaged with the enemy. Hence, after combat starts, units need some time to gain a position that renders them available for supply and this time constitutes the earliest time that a unit can be supplied. In summary, we have two-sided time windows, i.e. hard time restrictions in our problem.

- 10a. Total Mobile-ADS cost consists of three separate components, namely, fixed cost of opening transfer points, acquisition cost of vehicle fleet and transportation cost of ammo. Hence, we have a single objective function that unifies multiple cost components.

- 11b. Since this is a military application on a sensitive topic all data we present in this dissertation is hypothetical.

- 12b. As already explained, before battle starts, logistics planners determine potential locations for each type of transfer point on the battlefield. Therefore, transfer points can only be located at these predefined potential locations and hence the solution space of our problem is discrete.

- 13b. In Mobile-ADS design problem our aim is to locate Fixed-TPs at the first layer and Mobile-TPs at the second layer properly to supply combat units on time. Hence, locational decisions exist at two different layers in our problem.

- 14b. On the battlefield, combat units need and use several types of ammo. They need them at different locations, in different times and at different rates. Therefore, in Mobile-ADS we distribute multiple ammo types.

- 15b. In an LRP with single sourcing, each customer is to be supplied by exactly one vehicle or depot. In our problem, even the same ammo type may be brought to a unit by two or more different trucks. Hence, multi sourcing exists in our model.
- 16b. We do not hold inventory at transfer points.
- 17ab. We solve the model exactly by a commercial solver and heuristically by two methods that are developed in this dissertation.

Table 2.5 summarizes the classification of the 78 studies we examined and compares the classification of the proposed Mobile-ADS design problem with that of the majority of the literature. We utilize a capacitated heterogeneous vehicle fleet whereas the studies in the literature generally use capacitated homogeneous fleet. Majority of the previous studies consider uncapacitated facilities but we consider capacitated ones. In the literature majority of the models are two layers whereas Mobile-ADS has three layers.

Table 2.5: Summary of LRP studies

1. Hierarchical level		7. Number of layers		13. Locational decision	
a. Delivery or pickup	73	a. Two	61	a. At one layer	75
b. Delivery and pickup	5	b. Three/Four	17	b. At two layers	3
2. Nature of demand		8. Planning period		14. Product	
a. Deterministic	61	a. Single	72	a. Single	74
b. Stochastic	17	b. Multiple	7	b. Multiple	4
3. Number of facilities		9. Time restriction		15. Sourcing	
a. Single	19	a. No	63	a. Single	77
b. Multiple	59	b. Soft	14	b. Multiple	1
4. Vehicle fleet		c. Hard	1	16. Inventory	
a. Single	12	10. Objective function		a. Exist	4
b. Homogeneous	52	a. Single	72	b. Not exist	74
c. Heterogeneous	14	b. Multiple	7	17. Solution method	
5. Vehicle capacity		11. Data		a. Exact	25
a. Capacitated	47	a. Real	23	b. Heuristic	55
b. Uncapacitated	31	b. Hypothetical	57		
6. Facility capacity		12. Solution space			
a. Capacitated	31	a. Continuous	4		
b. Uncapacitated	48	b. Discrete	74		

Majority: 1a, 2a, 3b, 4b, 5a, 6b, 7a, 8a, 9a, 10a, 11b, 12b, 13a, 14a, 15a, 16b, 17b.
Mobile-ADS: 1a, 2a, 3b, 4c, 5a, 6a, 7b, 8a,b, 9c, 10a, 11b, 12b, 13b, 14b, 15b, 16b, 17a,b.

Except for 7 studies, the literature solve static LRP problems, while we solve both a single-period static and a multi-period dynamic case. Time restriction issue is rarely incorporated within the context of LRP models in the literature. 14 studies include soft time restrictions and only one study (time windows are used in [78] however no mathematical formulation is given) includes hard time restrictions. We utilize hard time restrictions in Mobile-ADS design problem. Locational decisions exist at only one layer in all studies except three. In other words, almost all three layer models locate facilities only at the second layer and facility locations at the first layer are assumed to be known a priori. We locate facilities at two different layers.

To the best of our knowledge, there are only 4 studies that distribute multiple products and the rest deals with single product. However, single-product formulations can hardly help model complex real world distribution systems with many number of types of products to be distributed. In Mobile-ADS design problem we distribute multiple products. Last of all, except one study all previous models compel single sourcing and no study allows multiple sourcing (a customer is allowed to be served by at most two vehicles in the model of [63], but one for pickups and one for deliveries). We utilize multiple sourcing which may help to reach better solutions in large distribution systems.

This brief analysis illustrates that some characteristics of the Mobile-ADS design problem are rarely included in the previous models. This dissertation is aimed directly to handle these aspects and to incorporate them into a single model. To the best of our knowledge this is the first attempt to construct such an inclusive real world LRP model.

Chapter 3

Static 4-Index Model Development

In this chapter we present the mathematical formulation of Mobile-ADS design problem for a fixed period and derive several valid inequalities to speed up the solution time.

We model the battlefield as a network of three types of nodes, i.e. potential Fixed-TP and Mobile-TP locations and fixed combat unit locations. With this representation we consider the Mobile-ADS, shown in Figure 2.1, as a directed and connected network $G = (N, A)$ that is defined by a set N of nodes and a set A of arcs. N is partitioned into three mutually exclusive subsets such that $N = N_F \bigcup N_M \bigcup N_C$ where N_F (N_M) is the set of potential Fixed-TP (Mobile-TP) locations respectively and N_C is the set of combat unit locations. Moreover, we let $N_{FM} = N_F \bigcup N_M$ and $N_{MC} = N_M \bigcup N_C$. A consists of two types of road networks that is $A = A_1 \bigcup A_2$. A_1 is the two-way road network, on which commercial trucks can travel between Fixed-TPs and Mobile-TPs and among Mobile-TPs. A_2 is the two-way trace network on the battle terrain, on which ammo trucks can travel between Mobile-TPs and combat units and among combat units. Figure 3.1 shows an example of a route of a commercial and ammo truck.

V is the set of all vehicles consisting of two subsets, $V = V_F \bigcup V_M$, where V_F

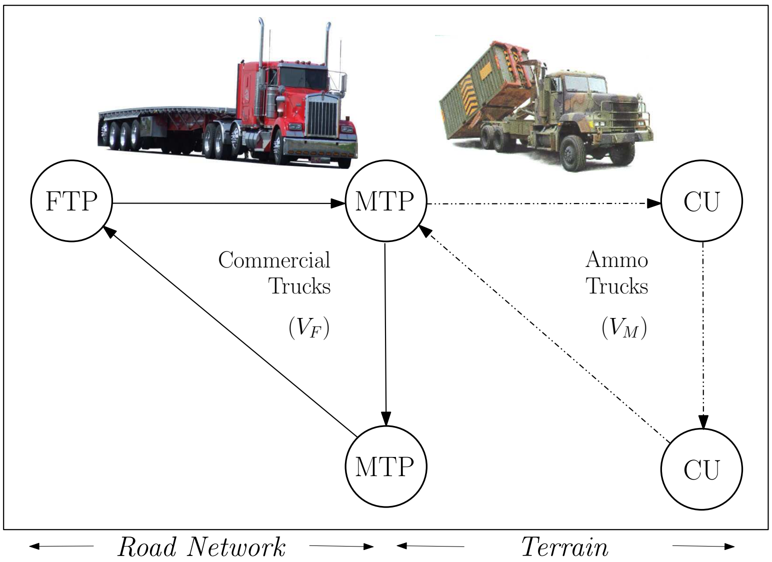


Figure 3.1: Routes of commercial and special ammo trucks

is the set of commercial trucks that are stationed at Fixed-TPs, and V_M is the set of ammo trucks that are stationed at Mobile-TPs. P is the ammo type set.

$D = [D_{ij}]$ is the distance matrix where the distance D_{ij} between two nodes i and j is the length of the shortest path from i to j on G. We assume that the matrix D of shortest distances is known in advance. We also assume that all travel on G occur on the shortest paths and at two different constant speeds. $speed_c$ and $speed_a$ denote the constant speeds of commercial and ammo trucks, respectively where $speed_c > speed_a$. Then $T = [TI_{ij}]$ is the travel time matrix where (1) if $i \in N_F$ and $j \in N_M$ or $i, j \in N_M, i \neq j$ then $TI_{ij} = D_{ij}/speed_c$ and (2) if $i \in N_M$ and $j \in N_C$ or $i, j \in N_C, i \neq j$ then $TI_{ij} = D_{ij}/speed_a$.

We also present all sets, parameters and decision variables that are used throughout this study in the Appendix.

We introduce a 4-index commodity flow mixed integer programming formulation for the Mobile-ADS design problem. Note that only the nonnegative commodity flow variable has 4 indices and all binary variables have three or less indices. We use the following parameters in the formulation. Q_{ip} is the demand of combat unit i for ammo type p. Each transfer point i has a nonnegative capacity represented by CD_{ip} for ammo type p. Each vehicle v has a nonnegative capacity CV_{vp} for ammo type p and in addition all vehicles also have total capacities represented by CT_v. As explained above there are different time deadlines for each type of ammo and for each combat unit and each unit has a different earliest time after which that unit can be supplied. TE_{ip} is the earliest and TL_{ip} is the latest time that combat unit i be supplied with ammo type p. TM_p is the maximum latest arrival time of ammo type p among combat units, that is $TM_p = \max_{i \in N_C}\{TL_{ip}\}$. $TM = \max_{p \in P}\{TM_p\}$ represents the maximum of the latest arrival times of all ammo types. TC_{vp} is the cost of transporting one unit of ammo type p on vehicle v per hour. VC_v is the cost of acquiring vehicle v. FC_i is the fixed cost of establishing/opening transfer point i.

The Mobile-ADS design problem can be summarized as follows. We have a finite number of combat units engaged with enemy on the battlefield. We also have a finite set of potential Fixed-TP and potential Mobile-TP locations (two sets are disjoint). Ammo flows from Fixed-TPs to Mobile-TPs by commercial trucks and from Mobile-TPs to combat units by special ammo trucks. We have to decide on (1) the number and location of Fixed-TPs and Mobile-TPs to be established and (2) the number, home transfer point and route of commercial and ammo trucks to serve combat units while minimizing transfer point establishment, truck acquisition and ammo transportation costs, such that the following conditions are satisfied;

- Total demand of combat units is satisfied within the given time window,
- Transfer point and truck capacity restrictions are respected,
- Each commercial (ammo) truck is dispatched from its home transfer point and returns to that point after serving the Mobile-TPs (combat units) on its route,
- Each truck is dispatched only once in a planning period.

3.1 Constraints

We employ the following notational rules throughout the formulation. We enumerate separate constraints as usual such as 3.1, 3.2, ... If we write the same constraint several times for disjoint index sets then we enumerate them such as 3.1a, 3.1b, ... or 3.2a, 3.2b, ... and so on.

3.1.1 Product flow balance constraints

We use nonnegative decision variable f_{ijvp} to denote the amount of flow of ammo type p carried from node i to node j by vehicle v.

$$\sum_{v \in V_F} \left(\sum_{\substack{j \in N_{FM} \\ j \neq i}} f_{jivp} - \sum_{\substack{j \in N_M \\ j \neq i}} f_{ijvp} \right) = \sum_{v \in V_M} \sum_{j \in N_C} f_{ijvp} \quad \forall i \in N_M, p \in P \tag{3.1a}$$

$$\sum_{v \in V_M} \left(\sum_{\substack{j \in N_{MC} \\ j \neq i}} f_{jivp} - \sum_{\substack{j \in N_C \\ j \neq i}} f_{ijvp} \right) = Q_{ip} \quad \forall i \in N_C, p \in P \tag{3.1b}$$

$$\sum_{\substack{j \in N_{FM} \\ j \neq i}} f_{jivp} \geq \sum_{\substack{j \in N_M \\ j \neq i}} f_{ijvp} \quad \forall i \in N_M, v \in V_F, p \in P \tag{3.2a}$$

$$\sum_{\substack{j \in N_{MC} \\ j \neq i}} f_{jivp} \geq \sum_{\substack{j \in N_C \\ j \neq i}} f_{ijvp} \quad \forall i \in N_C, v \in V_M, p \in P \tag{3.2b}$$

Ammo enters the network from Fixed-TPs and it is consumed by combat units. Constraints (3.1) ensure that inflow to a Mobile-TP or combat unit is equal to the sum of the total outflow from that node and the demand of that node. Constraints (3.2) guarantee that vehicles can not pick up ammo at intermediate nodes on their routes by forcing that a vehicle does not leave a node with more amount of an ammo type than the amount it was carrying while it was entering that node.

Note that constraints (3.1b) declare the problem infeasible if the total demand can not be satisfied for some reason, such as lack of ammo or trucks or transfer points, tight time windows, etc. In such situations, instead of giving no solution, we may want to provide the best solution that can be obtained with available resources. To do so, we need to allow the demand satisfaction limitation to be overruled at a certain cost by rewriting hard constraints (3.1b) as soft constraints as follows $\sum_{v\in V_M}\left(\sum_{\substack{j\in N_{MC}\\ j\neq i}} f_{jivp} - \sum_{\substack{j\in N_C\\ j\neq i}} f_{ijvp}\right) + u_{ip} = Q_{ip}$ where u_{ip} is a non-negative decision variable, indicating the amount of unmet demand of ammo type p at combat unit i, with a high enough positive cost coefficient in the objective function.

3.1.2 Vehicle flow balance constraints

We use binary decision variable x_{ijv}, where $x_{ijv} = 1$ if vehicle v travels from node i to node j and $x_{ijv} = 0$ otherwise.

$$\sum_{i\in N_F}\sum_{j\in N_M} x_{ijv} \leq 1 \quad \forall v \in V_F \tag{3.3a}$$

$$\sum_{i\in N_M}\sum_{j\in N_C} x_{ijv} \leq 1 \quad \forall v \in V_M \tag{3.3b}$$

$$\sum_{j\in N_M} x_{jiv} = \sum_{j\in N_M} x_{ijv} \quad \forall i \in N_F, v \in V_F \tag{3.4a}$$

$$\sum_{j\in N_C} x_{jiv} = \sum_{j\in N_C} x_{ijv} \quad \forall i \in N_M, v \in V_M \tag{3.4b}$$

$$\sum_{\substack{j\in N_{FM}\\ j\neq i}} x_{jiv} = \sum_{\substack{j\in N_{FM}\\ j\neq i}} x_{ijv} \quad \forall i \in N_M, v \in V_F \tag{3.5a}$$

$$\sum_{\substack{j\in N_{MC}\\ j\neq i}} x_{jiv} = \sum_{\substack{j\in N_{MC}\\ j\neq i}} x_{ijv} \quad \forall i \in N_C, v \in V_M \tag{3.5b}$$

Recall that commercial trucks can be allocated only to Fixed-TPs and ammo trucks only to Mobile-TPs. Constraints (3.3) indicate that a vehicle can not be allocated to more than one transfer point. From another point of view, they maintain that a vehicle can start its route from one and only one transfer point. Constraints (3.4) force each vehicle to turn back to its home transfer point where it is allocated. Constraints (3.5) require that each vehicle leaves the node that it enters. Constraints (3.3)-(3.5) together also maintain that each route contains only one transfer point and they guarantee that a vehicle can not go from a node to two or more nodes at the same time.

3.1.3 Capacity constraints

We use binary decision variable y_i, where $y_i = 1$ if transfer point i is established and $y_i = 0$ otherwise.

$$\sum_{v \in V_F} \sum_{j \in N_M} f_{ijvp} \leq CD_{ip} \cdot y_i \quad \forall i \in N_F, p \in P \tag{3.6a}$$

$$\sum_{v \in V_M} \sum_{j \in N_C} f_{ijvp} \leq CD_{ip} \cdot y_i \quad \forall i \in N_M, p \in P \tag{3.6b}$$

$$\sum_{v \in V_F} \sum_{\substack{j \in N_M \\ j \neq i}} f_{ijvp} \leq \left(\sum_{l \in N_C} Q_{lp} \right) \cdot y_i \quad \forall i \in N_M, p \in P \tag{3.6c}$$

$$\begin{aligned} f_{ijvp} \leq CV_{vp} \cdot x_{ijv} \quad & \forall i \in N_F, j \in N_M, v \in V_F, p \in P \\ & \forall i, j \in N_M, i \neq j, v \in V_F, p \in P \\ & \forall i \in N_M, j \in N_C, v \in V_M, p \in P \\ & \forall i, j \in N_C, i \neq j, v \in V_M, p \in P \end{aligned} \tag{3.7}$$

$$\begin{aligned} \sum_{p \in P} f_{ijvp} \leq CT_v \cdot x_{ijv} \quad & \forall i \in N_F, j \in N_M, v \in V_F \\ & \forall i, j \in N_M, i \neq j, v \in V_F \\ & \forall i \in N_M, j \in N_C, v \in V_M \end{aligned} \tag{3.8}$$

$$\forall i, j \in N_C, i \neq j, v \in V_M$$

Constraints (3.6) ensure that transfer points can not send/receive ammo type p more than their capacity for that ammo type. They also guarantee that there is no flow from/through any closed transfer point. Constraints (3.7) require that vehicle capacities are not exceeded and maintain that unused vehicles can not carry any flow. All vehicles also have total capacities that are respected by constraints (3.8).

3.1.4 Relation constraints

We use binary decision variable w_{ijp}, where $w_{ijp} = 1$ if ammo type p travels from node i to node j and $w_{ijp} = 0$ otherwise.

$$\sum_{p \in P} f_{ijvp} \geq x_{ijv} \quad \forall i \in N_F, j \in N_M, v \in V_F \tag{3.9}$$

$$\forall i, j \in N_M, i \neq j, v \in V_F$$

$$\forall i \in N_M, j \in N_C, v \in V_M$$

$$\forall i, j \in N_C, i \neq j, v \in V_M$$

$$\left(\sum_{l \in N_C} Q_{lp} \right) \cdot w_{ijp} \geq f_{ijvp} \quad \forall i \in N_F, j \in N_M, v \in V_F, p \in P \tag{3.10}$$

$$\forall i, j \in N_M, i \neq j, v \in V_F, p \in P$$

$$\forall i \in N_M, j \in N_C, v \in V_M, p \in P$$

$$\forall i, j \in N_C, i \neq j, v \in V_M, p \in P$$

$$\sum_{v \in V_F} f_{ijvp} \geq w_{ijp} \quad \forall i \in N_F, j \in N_M, p \in P \tag{3.11a}$$

$$\forall i, j \in N_M, i \neq j, p \in P$$

$$\sum_{v \in V_M} f_{ijvp} \geq w_{ijp} \quad \forall i \in N_M, j \in N_C, p \in P \tag{3.11b}$$

$$\forall i, j \in N_C, i \neq j, p \in P$$

Constraints (3.9) require that a vehicle carries some type and amount of ammo if it is dispatched. Note that these constraints do not force vehicles to carry ammo on their way back to their home transfer points. They also guarantee that if a vehicle does not carry anything from node i to node j then it should not travel between these two nodes. Constraints (3.10) and (3.11) set the correct logical relationships between the decision variables f and w. They maintain that if ammo type p does not travel between nodes i and j then no flow of p should exist between these nodes and reversely if ammo type p does travel from i to j then there must exist some positive flow of p in between.

Note that in constraints (3.9) and (3.11), ammo flow is measured in undefined units. Hence, one needs to be careful about defining the unit of flow, because these constraints do not permit a truck to carry an ammo type less than 1 unit. If one wants to do so, then the right hand sides should be multiplied with an appropriate multiplier. For example, if our unit is 1 ton, and if we do not want to carry an ammo type less than 0.2 tons with a single truck, then our multiplier would be 0.2.

3.1.5 Time related constraints

We use nonnegative decision variable tp_{ip} to denote the arrival time of ammo type p at node i.

$$tp_{ip} \geq TE_{ip} \quad \forall i \in N_C, p \in P \tag{3.12}$$

$$tp_{ip} \leq TL_{ip} \quad \forall i \in N_C, p \in P \tag{3.13}$$

$$tp_{ip} = 0 \quad \forall i \in N_F, p \in P \tag{3.14}$$

$$tp_{ip} + TI_{ij} \cdot w_{ijp} - TM_p \cdot (1 - w_{ijp}) \leq tp_{jp} \quad \forall i \in N_F, j \in N_M, p \in P \tag{3.15}$$

$$\forall i, j \in N_M, i \neq j, p \in P$$
$$\forall i \in N_M, j \in N_C, p \in P$$
$$\forall i, j \in N_C, i \neq j, p \in P$$

Constraints (3.12) and (3.13) impose the time window requirements of combat units on the model for all ammo types. Constraints (3.14) define the initial condition by setting the arrival time of all ammo types at Fixed-TPs to time zero. Constraints (3.15) compute the arrival times of ammo types at nodes.

In fact, since constraints (3.15) refer to the latest ammo arrival, constraints (3.12) ensure that the latest ammo arrival respects the time windows of units. Note that waiting of ammo at combat units is allowed, and in the context of this dissertation, a time window indicates the time interval in which a unit can halt in battle, and receive the waiting or newly arrived supplies. Hence, ammo is allowed to reach a unit before the earliest time, and wait there until the unit actually takes it.

Recall that the decision variable w_{ijp} does not carry any information about vehicles. Hence constraints (3.12)-(3.15), which are written for w_{ijp}'s, can not prevent sub-tours of vehicles. To remedy this condition we introduce subtour elimination constraints of [25] as constraints (3.16). Note that we use nonnegative decision variable tv_{iv} to denote the arrival time of vehicle v at node i.

$$tv_{iv} + TI_{ij} \cdot x_{ijv} - TM \cdot (1 - x_{ijv}) \leq tv_{jv} \quad \forall i \in N_F, j \in N_M, v \in V_F \tag{3.16}$$
$$\forall i, j \in N_M, i \neq j, v \in V_F$$
$$\forall i \in N_M, j \in N_C, v \in V_M$$
$$\forall i, j \in N_C, i \neq j, v \in V_M$$

3.2 Objective function

In Mobile-ADS design problem two different objectives exists each of which could be applicable depending on the situation. The first objective considers the costs of transfer point establishment, vehicle acquisition, and ammo distribution. The second one considers again the costs of transfer point establishment and vehicle acquisition plus the cost of truck driving. As can be seen, two of the cost components are common to both objectives and are shown below.

$$\sum_{i \in N_{FM}} FC_i \cdot y_i \tag{3.17}$$

$$\sum_{i \in N_F} \sum_{j \in N_M} \sum_{v \in V_F} VC_v \cdot x_{ijv} + \sum_{i \in N_M} \sum_{j \in N_C} \sum_{v \in V_M} VC_v \cdot x_{ijv} \tag{3.18}$$

(3.17) is the total fixed cost of opening transfer points, and (3.18) is the total acquisition cost of used trucks. Now, we present the last component of each objective.

Depending on the mission, available forces, enemy threat, country's economy, etc. different factors may gain more importance or urgency above others on the battlefield. If we put economy and financial concerns over others, then total transportation cost of ammo becomes critical. This cost constitutes the third component of the first objective and is shown below.

$$\sum_{i \in N} \sum_{j \in N} \sum_{v \in V} \sum_{p \in P} TC_{vp} \cdot TI_{ij} \cdot f_{ijvp} \tag{3.19}$$

If enemy has the ability to detect our logistics convoys, then the more traffic we have the more our convoys are exposed to enemy fire. Moreover, we may want to concentrate some of our forces on a particular region of the combat area without enemy's notice. In such circumstances, stealth becomes a big concern, and we again do not want much traffic on the battlefield. Hence, total driving time of vehicles becomes critical, and constitutes the third component of the second objective that is shown below.

$$\sum_{i \in N} \sum_{j \in N} \sum_{v \in V} DC_v \cdot TI_{ij} \cdot x_{ijv} \tag{3.20}$$

To summarize, our objective functions are as follows.

- $z1 = (3.17) + (3.18) + (3.19)$
- $z2 = (3.17) + (3.18) + (3.20)$

It is important to note that on the same battlefield and at the same time, different objectives may gain priority for different units. For example, one brigade may move to a different direction in concealment while others keep their positions as they are. Hence, for the first brigade $z2$ and for the rest $z1$ may become the objective on the same battlefield at the same time.

3.3 Model

In light of above explanations Mobile-ADS design model is,

$$\begin{aligned}
\min \quad & z1 \text{ or } z2 \\
\text{s.t.} \quad & (3.1) - (3.16) \\
& f_{ijvp} \geq 0 \quad \forall i, j \in N, i \neq j, v \in V, p \in P \\
& tp_{ip} \geq 0 \quad \forall i \in N, p \in P \\
& tv_{iv} \geq 0 \quad \forall i \in N, v \in V \\
& x_{ijv} \in \{0,1\} \quad \forall i, j \in N, i \neq j, v \in V \\
& w_{ijp} \in \{0,1\} \quad \forall i, j \in N, i \neq j, p \in P \\
& y_i \in \{0,1\} \quad \forall i \in N_{FM}.
\end{aligned}$$

3.4 Valid inequalities

We model the Mobile-ADS design problem as a mixed integer programming model. In this section we present several valid inequalities to improve its performance in terms of solution time and quality.

3.4.1 Valid inequalities related to product flow balance

$$\sum_{v\in V_F}\sum_{i\in N_F}\sum_{j\in N_M} f_{ijvp} = \sum_{i\in N_C} Q_{ip} \quad \forall p \in P \tag{$V1a$}$$

$$\sum_{v\in V_M}\sum_{i\in N_M}\sum_{j\in N_C} f_{ijvp} = \sum_{i\in N_C} Q_{ip} \quad \forall p \in P \tag{$V1b$}$$

Valid inequalities ($V1$) require that outflow from all Fixed-TPs and from all Mobile-TPs be equal to the total demand of all combat units for each ammo type.

3.4.2 Valid inequalities related to vehicle flow balance

$$\sum_{\substack{j\in N_{FM}\\ j\neq i}} x_{ijv} \leq 1 \quad \forall i \in N_M, v \in V_F \tag{$V2a$}$$

$$\sum_{\substack{j\in N_{MC}\\ j\neq i}} x_{ijv} \leq 1 \quad \forall i \in N_C, v \in V_M \tag{$V2b$}$$

$$\sum_{v\in V_F}\sum_{i\in N_F}\sum_{j\in N_M} x_{ijv} \geq \left\lceil \frac{\sum_{p\in P}\sum_{i\in N_C} Q_{ip}}{\max_{v\in V_F}\{CT_v\}} \right\rceil \tag{$V3a$}$$

$$\sum_{v\in V_M}\sum_{i\in N_M}\sum_{j\in N_C} x_{ijv} \geq \left\lceil \frac{\sum_{p\in P}\sum_{i\in N_C} Q_{ip}}{\max_{v\in V_M}\{CT_v\}} \right\rceil \tag{$V3b$}$$

Valid inequalities ($V2$) maintain that a vehicle can not travel from a node to two or more nodes in a single planning period. Valid inequalities ($V3$) set a lower bound for the total number of vehicles that must be dispatched from transfer points to carry the total demand of all combat units.

3.4.3 Valid inequalities related to variable and logical relations

$$
\begin{aligned}
w_{ijp} \leq y_i \quad & \forall i \in N_F, j \in N_M, p \in P \\
& \forall i, j \in N_M, i \neq j, p \in P \\
& \forall i \in N_M, j \in N_C, p \in P
\end{aligned} \tag{$V4$}
$$

$$
\sum_{p \in P} \sum_{j \in N_M} w_{ijp} \geq y_i \quad \forall i \in N_F \tag{$V5a$}
$$

$$
\sum_{p \in P} \sum_{j \in N_C} w_{ijp} \geq y_i \quad \forall i \in N_M \tag{$V5b$}
$$

Valid inequalities ($V4$) ensure that ammo types can not pass through closed transfer points. Valid inequalities ($V5$) provide that at least one ammo type must pass through an open transfer point.

$$
w_{ijp} \leq \sum_{v \in V_F} x_{ijv} \quad \forall i \in N_F, j \in N_M, p \in P \tag{$V6a$}
$$

$$
\begin{aligned}
& \forall i, j \in N_M, i \neq j, p \in P \\
w_{ijp} \leq \sum_{v \in V_M} x_{ijv} \quad & \forall i \in N_M, j \in N_C, p \in P \\
& \forall i, j \in N_C, i \neq j, p \in P
\end{aligned} \tag{$V6b$}
$$

$$
\begin{aligned}
\sum_{p \in P} w_{ijp} \geq x_{ijv} \quad & \forall i \in N_F, j \in N_M, v \in V_F \\
& \forall i, j \in N_M, i \neq j, v \in V_F \\
& \forall i \in N_M, j \in N_C, v \in V_M \\
& \forall i, j \in N_C, i \neq j, v \in V_M
\end{aligned} \tag{$V7$}
$$

Valid inequalities ($V6$) state that if an ammo type travels from node i to node j then there must exist at least one vehicle travelling between these two nodes. Valid inequalities ($V7$) maintain the reverse condition by preventing any vehicle from traveling between nodes i and j if no ammo type travels between these two nodes.

$$\sum_{j \in N_M} x_{ijv} \leq y_i \quad \forall i \in N_F, v \in V_F \tag{V8a}$$

$$\sum_{\substack{j \in N_{FM} \\ j \neq i}} x_{ijv} \leq y_i \quad \forall i \in N_M, v \in V_F \tag{V8b}$$

$$\sum_{j \in N_C} x_{ijv} \leq y_i \quad \forall i \in N_M, v \in V_M \tag{V8c}$$

$$\sum_{v \in V_F} \sum_{j \in N_M} x_{ijv} \geq y_i \quad \forall i \in N_F \tag{V9a}$$

$$\sum_{v \in V_M} \sum_{j \in N_C} x_{ijv} \geq y_i \quad \forall i \in N_M \tag{V9b}$$

$$\sum_{v \in V_F} \sum_{j \in N_M} x_{ijv} \leq |V_F| \cdot y_i \quad \forall i \in N_F \tag{V10a}$$

$$\sum_{v \in V_M} \sum_{j \in N_C} x_{ijv} \leq |V_M| \cdot y_i \quad \forall i \in N_M \tag{V10b}$$

Valid inequalities ($V8$) provide that no vehicle can be dispatched from or pass through a closed transfer point. Valid inequalities ($V9$) require that an open transfer point must dispatch at least one vehicle. Valid inequalities ($V10$) guarantee that no transfer point can dispatch more vehicles than there exist in the system.

$$\sum_{i \in N_F} y_i \leq |N_F| \tag{V11a}$$

$$\sum_{i \in N_M} y_i \leq |N_M| \tag{V11b}$$

$$\sum_{i \in N_F} y_i \geq \left\lceil \frac{\sum_{p \in P} \sum_{i \in N_C} Q_{ip}}{\max_{p \in P, i \in N_F}\{CD_{ip}\}} \right\rceil \tag{$V12a$}$$

$$\sum_{i \in N_M} y_i \geq \left\lceil \frac{\sum_{p \in P} \sum_{i \in N_C} Q_{ip}}{\max_{p \in P, i \in N_M}\{CD_{ip}\}} \right\rceil \tag{$V12b$}$$

Valid inequalities ($V11$) and ($V12$) set the upper and lower bounds for the number of opened transfer points.

3.4.4 Valid inequalities related to time

$$\sum_{\substack{i \in N_{MC} \\ i \neq j}} \sum_{j \in N_C} TI_{ij} \cdot x_{ijv} \leq TM - \min_{i \in N_F, j \in N_M}\{TI_{ij}\} \quad \forall v \in V_M \tag{$V13$}$$

Consider the tours of ammo trucks an example of which can be seen in Figure 3.1 and consider deleting the returning arc of each tour from combat units to Mobile-TPs. Valid inequalities ($V13$) set the upper bound for the total traveling time of these modified routes of ammo trucks. The total traveling time of the modified route of ammo truck v is represented by $\sum_{\substack{i \in N_{MC} \\ i \neq j}} \sum_{j \in N_C} TI_{ij} \cdot x_{ijv}$. In fact this summation also defines the serving time of the last combat unit on the tour of ammo truck v. Now, let the maximum of the latest arrival times of all ammo types at combat units be 24, that is $TM = 24$. In other words, all ammo types must be delivered to combat units in 24 hours. Suppose, minimum traveling time between Fixed-TPs and Mobile-TPs is 7, that is $\min_{i \in N_F, j \in N_M}\{TI_{ij}\} = 7$. Hence, the earliest time that a Mobile-TP can dispatch an ammo truck is 7. However, all ammo types must arrive at combat units before 24. Combining these two observations, all ammo trucks have at most 14 hours to serve all combat units. Mathematically, we have $\sum_{\substack{i \in N_{MC} \\ i \neq j}} \sum_{j \in N_C} TI_{ij} \cdot x_{ijv} \leq 14$ meaning that each ammo truck should deliver the demand of the last combat unit on its tour in at most 14 hours.

Chapter 4

Static 3-Index Model Development

In Chapter 3 we present a 4-index mathematical formulation of Mobile-ADS design problem with an arc-based product flow approach. In this chapter we develop a 3-index mathematical formulation of the same problem with a node-based product flow approach.

As in the 4-index model, we still have the same directed and connected network $G = (N, A)$ with $N = N_F \bigcup N_M \bigcup N_C$ and $A = A_1 \bigcup A_2$. We also have the same vehicle set $V = V_F \bigcup V_M$ and travel time matrix T. In addition, we use the same parameters as we did in the 4-index model. Finally, our problem definition and the answers we are expecting from the 3-index model are the same.

In the 4-index model we consider vehicle and product flows on the arcs of the network. We indicate the traversal of vehicle $v \in V$ on arc $(i, j) \in A$ using the binary decision variable x_{ijv}. We also denote the flow of product $p \in P$ on arc $(i, j) \in A$ with vehicle $v \in V$ by the positive decision variable f_{ijvp}.

In the 3-index model we still use the same indicator variables for vehicle traversals on arcs. However, rather than product flow on arcs we consider product flow on the nodes of the network. To do so, we consider the product flow on arc

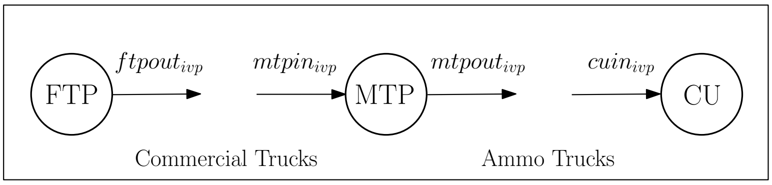


Figure 4.1: Node-based product flow decision variables

(i, j) in two parts. The first part is the outgoing flow from node $i \in N$, denoted by f_{ivp}, and the second part is the incoming flow to node $j \in N$, denoted by f_{jvp}. In other words, f_{ivp} represents the amount of product p that is sent from node i on vehicle v and f_{jvp} represents the amount of product p that is dropped to node j by vehicle v.

4.1 Constraints

4.1.1 Product flow balance constraints

We use four nonnegative product flow variables that can be seen in Figure 4.1. $ftpout_{ivp}$ ($mtpout_{ivp}$) denote the amount of ammo type p that is sent from Fixed-TP (Mobile-TP) i with commercial (ammo) truck v. $mtpin_{ivp}$ (cu_{ivp}) represent the amount of ammo type p that is dropped to Mobile-TP (combat unit) i with commercial (ammo) truck v. Note that in $ftpout_{ivp}$ and $mtpin_{ivp}$, v is a commercial truck, that is $v \in V_F$. However, in $mtpout_{ivp}$ and $cuin_{ivp}$, v is an ammo truck, that is $v \in V_M$. This partition of trucks is shown in Figure 4.1.

$$\sum_{v \in V_M} cuin_{ivp} = Q_{ip} \quad \forall i \in N_C, p \in P \tag{4.1}$$

Constraints (4.1) ensure that demand of a combat unit for each ammo type must be satisfied by ammo trucks.

$$\sum_{i \in N_F} ftpout_{ivp} = \sum_{i \in N_M} mtpin_{ivp} \quad \forall v \in V_F, p \in P \tag{4.2a}$$

$$\sum_{i \in N_M} mtpout_{ivp} = \sum_{i \in N_C} cuin_{ivp} \quad \forall v \in V_M, p \in P \tag{4.2b}$$

$$\sum_{v \in V_F} mtpin_{ivp} = \sum_{v \in V_M} mtpout_{ivp} \quad \forall i \in N_M, p \in P \tag{4.3}$$

Constraints (4.2) guarantee that each commercial (ammo) truck drops all its load, which it loads from a Fixed-TP (Mobile-TP), to Mobile-TPs (combat units). Constraints (4.3) ensure that total inflow of an ammo type to a Mobile-TP that is dropped by commercial trucks is equal to the total outflow of that ammo type from that Mobile-TP that is sent by ammo trucks.

4.1.2 Vehicle flow balance constraints

We use the same binary decision variable x_{ijv} as in the 4-index model, where $x_{ijv} = 1$ if vehicle v travels from node i to node j and $x_{ijv} = 0$ otherwise. We also use the same vehicle flow balance constraints (3.3), (3.4) and (3.5).

4.1.3 Capacity constraints

We use the same binary decision variable y_i, where $y_i = 1$ if transfer point i is established and $y_i = 0$ otherwise.

$$\sum_{v \in V_F} ftpout_{ivp} \leq CD_{ip} \cdot y_i \quad \forall i \in N_F, p \in P \tag{4.4a}$$

$$\sum_{v \in V_M} mtpout_{ivp} \leq CD_{ip} \cdot y_i \quad \forall i \in N_M, p \in P \tag{4.4b}$$

Constraints (4.4) ensure that transfer points can not send ammo type p more than their capacity for that ammo type. They also guarantee that there is no flow from/through any closed transfer point.

$$\sum_{i \in N_M} mtpin_{ivp} \leq CV_{vp} \cdot \sum_{i \in N_F} \sum_{j \in N_M} x_{ijv} \quad \forall v \in V_F, p \in P \tag{4.5a}$$

$$\sum_{i \in N_C} cuin_{ivp} \leq CV_{vp} \cdot \sum_{i \in N_M} \sum_{j \in N_C} x_{ijv} \quad \forall v \in V_M, p \in P \tag{4.5b}$$

$$\sum_{p \in P} \sum_{i \in N_F} ftpout_{ivp} \leq CT_v \cdot \sum_{i \in N_F} \sum_{j \in N_M} x_{ijv} \quad \forall v \in V_F \tag{4.6a}$$

$$\sum_{p \in P} \sum_{i \in N_M} mtpout_{ivp} \leq CT_v \cdot \sum_{i \in N_M} \sum_{j \in N_C} x_{ijv} \quad \forall v \in V_M \tag{4.6b}$$

If any positive flow is carried by truck v, the total capacity of that truck should not be exceeded but only if that truck has started its tour. Constraints (4.5) require that vehicle capacities are not exceeded and maintain that unused vehicles can not carry any flow. All vehicles also have total capacities that are respected by constraints (4.6).

4.1.4 Relation constraints

We use binary decision variable k_{ivp} that is an indicator of (1) whether ammo type p is dropped to Mobile-TP (combat unit) i with commercial (ammo) truck v or not and (2) whether ammo type p is sent from Mobile-TP i with ammo truck v or not. Mathematically, (1) $k_{ivp} = 1$ if $mtpin_{ivp} > 0$ for all $i \in N_M, v \in V_F, p \in P$ or $k_{ivp} = 0$ otherwise, (2) $k_{ivp} = 1$ if $mtpout_{ivp} > 0$ for all $i \in N_M, v \in V_M, p \in P$ or $k_{ivp} = 0$ otherwise, (3) $k_{ivp} = 1$ if $cuin_{ivp} > 0$ for all $i \in N_C, v \in V_M, p \in P$ or $k_{ivp} = 0$ otherwise.

$$ftpout_{ivp} \leq CV_{vp} \cdot \sum_{j \in N_M} x_{ijv} \quad \forall i \in N_F, v \in V_F, p \in P \tag{4.7a}$$

$$mtpin_{ivp} \leq CV_{vp} \cdot \sum_{\substack{j \in N_{FM} \\ j \neq i}} x_{jiv} \quad \forall i \in N_M, v \in V_F, p \in P \tag{4.7b}$$

$$mtpout_{ivp} \leq CV_{vp} \cdot \sum_{j \in N_C} x_{ijv} \quad \forall i \in N_M, v \in V_M, p \in P \tag{4.7c}$$

$$cuin_{ivp} \leq CV_{vp} \cdot \sum_{\substack{j \in N_{MC} \\ j \neq i}} x_{jiv} \quad \forall i \in N_C, v \in V_M, p \in P \tag{4.7d}$$

Constraints (4.7) state that if there is an outflow (inflow) of an ammo type from (to) a node by a vehicle then that vehicle must be dispatched from (enter to) that node. Conversely, they maintain that if no vehicle is dispatched from (enters to) a node then no outflow (inflow) of any ammo type can exist from (to) that node.

$$\sum_{j \in N_M} x_{ijv} \leq \sum_{p \in P} ftpout_{ivp} \quad \forall i \in N_F, v \in V_F \tag{4.8a}$$

$$\sum_{j \in N_C} x_{ijv} \leq \sum_{p \in P} mtpout_{ivp} \quad \forall i \in N_M, v \in V_M \tag{4.8b}$$

$$\sum_{\substack{j \in N_{FM} \\ j \neq i}} x_{jiv} \leq \sum_{p \in P} mtpin_{ivp} \quad \forall i \in N_M, v \in V_F \tag{4.8c}$$

$$\sum_{\substack{j \in N_{MC} \\ j \neq i}} x_{jiv} \leq \sum_{p \in P} cuin_{ivp} \quad \forall i \in N_C, v \in V_M \tag{4.8d}$$

Constraints (4.8) maintain that if there is no outflow (inflow) with a vehicle from (to) a node then that vehicle must not be dispatched from (enter to) that node. Conversely, they guarantee that if a vehicle is dispatched from (enters to) a node then some outflow (inflow) of an ammo type with that vehicle must exist from (to) that node.

$$k_{ivp} \leq mtpin_{ivp} \quad \forall i \in N_M, v \in V_F, p \in P \tag{4.9a}$$

$$k_{ivp} \leq mtpout_{ivp} \quad \forall i \in N_M, v \in V_M, p \in P \tag{4.9b}$$

$$k_{ivp} \leq cuin_{ivp} \quad \forall i \in N_C, v \in V_M, p \in P \tag{4.9c}$$

$$mtpin_{ivp} \leq CV_{vp} \cdot k_{ivp} \quad \forall i \in N_M, v \in V_F, p \in P \tag{4.10a}$$

$$mtpout_{ivp} \leq CV_{vp} \cdot k_{ivp} \quad \forall i \in N_M, v \in V_M, p \in P \tag{4.10b}$$

$$cuin_{ivp} \leq CV_{vp} \cdot k_{ivp} \quad \forall i \in N_C, v \in V_M, p \in P \tag{4.10c}$$

Constraints (4.9) and (4.10) set the correct logical relationships between the decision variables k and $mtpin$, $mtpout$, $cuin$. They maintain that if a vehicle drops (carries) an ammo type to (from) a node then there must exist some inflow (outflow) of that ammo type to (from) that node with that vehicle. Reversely, they also ensure that if a vehicle does not drop (carry) an ammo type to (from) a node then there can not exist any inflow (outflow) of that ammo type to (from) that node with that vehicle.

4.1.5 Time related constraints

We use the same time related nonnegative decision variables as in the 4-index model, namely tp_{ip} to denote the arrival time of ammo type p at node i and tt_{iv} to denote the arrival time of vehicle v at node i. We also employ the same constraints (3.12), (3.13), (3.14) and (3.16) of the 4-index model.

$$tp_{ip} - TM_p \cdot (1 - k_{ivp}) \leq tv_{iv} \quad \forall i \in N_M, v \in V_M, p \in P \tag{4.11}$$

Constraints (4.11) ensure that an ammo truck that is carrying an ammo type from a Mobile-TP can not leave that transfer point before the latest arrival time of that product to that Mobile-TP.

$$tv_{iv} - TM_p \cdot (1 - k_{ivp}) \leq tp_{ip} \quad \begin{array}{l} \forall i \in N_M, v \in V_F, p \in P \\ \forall i \in N_C, v \in V_M, p \in P \end{array} \tag{4.12}$$

Constraints (4.12) set the correct relationships between the arrival times of vehicles and products to a node. They maintain that arrival time of an ammo type to a Mobile-TP (combat unit) can not be less than the arrival time of each commercial (ammo) truck carrying that product to that transfer point (combat unit). In other words, ammo types wait for the latest arrival time of the trucks carrying them.

4.2 Model

In light of above explanations Mobile-ADS design model is,

$$
\begin{aligned}
\min \quad & z2 \\
\text{s.t.} \quad & (3.4)-(3.5),(3.12)-(3.14),(3.16),(4.1)-(4.12) \\
& ftpout_{ivp} \geq 0 \quad \forall i \in N_F, v \in V_F, p \in P \\
& mtpin_{ivp} \geq 0 \quad \forall i \in N_M, v \in V_F, p \in P \\
& mtpout_{ivp} \geq 0 \quad \forall i \in N_M, v \in V_M, p \in P \\
& cuin_{ivp} \geq 0 \quad \forall i \in N_C, v \in V_M, p \in P \\
& tp_{ip} \geq 0 \quad \forall i \in N, p \in P \\
& tv_{iv} \geq 0 \quad \forall i \in N, v \in V \\
& x_{ijv} \in \{0,1\} \quad \forall i,j \in N, i \neq j, v \in V \\
& k_{ivp} \in \{0,1\} \quad \forall i \in N_M, v \in V_M, p \in P \\
& y_i \in \{0,1\} \quad \forall i \in N_{FM}.
\end{aligned}
$$

4.3 Valid inequalities

As in the 4-index model we use valid inequalities ($V2$), ($V3$), ($V8$), ($V9$), ($V10$), ($V11$), ($V12$) and ($V13$) in the 3-index model without any change. We use valid inequalities ($V1$) after a modification and develop 7 new valid inequalities.

We modify valid inequalities ($V1$) as follows.

$$\sum_{v \in V_F} \sum_{i \in N_F} ftpout_{ivp} = \sum_{i \in N_C} Q_{ip} \quad \forall p \in P \tag{V1c}$$

$$\sum_{v \in V_M} \sum_{i \in N_M} mtpout_{ivp} = \sum_{i \in N_C} Q_{ip} \quad \forall p \in P \tag{V1d}$$

Valid inequalities ($V1$) require that outflow from all Fixed-TPs and from all Mobile-TPs be equal to the total demand of all combat units for each ammo type.

New valid inequalities are as follows.

$$\sum_{i\in N_F} ftpout_{ivp} \leq CV_{vp} \cdot \sum_{i\in N_F}\sum_{j\in N_M} x_{ijv} \quad \forall v \in V_F, p \in P \tag{$V14a$}$$

$$\sum_{i\in N_M} mtpout_{ivp} \leq CV_{vp} \cdot \sum_{i\in N_M}\sum_{j\in N_C} x_{ijv} \quad \forall v \in V_M, p \in P \tag{$V14b$}$$

Valid inequalities ($V14$) state that commercial or ammo trucks can not carry an ammo type more than their capacity for that ammo type. In fact, they are different versions of valid inequalities ($1.10a$) and ($1.10c$) summed over Fixed-TPs and Mobile-TPs.

$$\sum_{i\in N_F}\sum_{v\in V_F} ftpout_{ivp} = \sum_{i\in N_M}\sum_{v\in V_F} mtpin_{ivp} \quad \forall p \in P \tag{$V15a$}$$

$$\sum_{i\in N_M}\sum_{v\in V_M} mtpout_{ivp} = \sum_{i\in N_C}\sum_{v\in V_M} cuin_{ivp} \quad \forall p \in P \tag{$V15b$}$$

Valid inequalities ($V15$) maintain the product flow balance between Fixed-TPs and Mobile-TPs as well as between Mobile-TPs and combat units. They require that total outflow of an ammo type from all Fixed-TPs with commercial trucks (from all Mobile-TPs with ammo trucks) must be equal to the total inflow of that ammo type to all Mobile-TPs with commercial trucks (to all combat units with ammo trucks). In fact, they are different versions of valid inequalities (4.2) summed over commercial and ammo trucks.

$$mtpin_{jvp} - (1 - x_{ijv}) \cdot CV_{vp} \leq ftpout_{ivp} \quad \forall i \in N_F, j \in N_M, v \in V_F, p \in P \tag{$V16a$}$$

$$cuin_{jvp} - (1 - x_{ijv}) \cdot CV_{vp} \leq mtpout_{ivp} \quad \forall i \in N_M, j \in N_C, v \in V_M, p \in P \tag{$V16b$}$$

Valid inequalities ($V16$) set the correct relationships between the product flow decision variables. Consider commercial truck v goes from Fixed-TP i to Mobile-TP j carrying ammo type p. First of all, obviously we have $x_{ijv} = 1$. Then valid inequalities ($V16a$) take the form $mtpin_{jvp} \leq ftpout_{ivp}$ which states that commercial truck v can not drop ammo type p to Mobile-TP j more than it loaded from the Fixed-TP i where it started its tour. If on the other hand, had we had $x_{ijv} = 0$ (meaning that commercial truck v does not travel between i and j) then constraints ($V16a$) would have been redundant. Constraints ($V16b$) work similarly for the decision variables $mtpout$ and $cuin$.

$$y_i \leq \sum_{v \in V_F} \sum_{p \in P} k_{ivp} \quad \forall i \in N_M \tag{$V17a$}$$

$$y_i \leq \sum_{v \in V_M} \sum_{p \in P} k_{ivp} \quad \forall i \in N_M \tag{$V17b$}$$

Valid inequalities ($V17$) state that each open Mobile-TP must receive some ammo by commercial trucks and must send some ammo with ammo trucks.

$$k_{ivp} \leq \sum_{\substack{j \in N_{FM} \\ j \neq i}} x_{jiv} \quad \forall i \in N_M, v \in V_F, p \in P \tag{$V18a$}$$

$$k_{ivp} \leq \sum_{j \in N_C} x_{ijv} \quad \forall i \in N_M, v \in V_M, p \in P \tag{$V18b$}$$

$$k_{ivp} \leq \sum_{\substack{j \in N_{MC} \\ j \neq i}} x_{jiv} \quad \forall i \in N_C, v \in V_M, p \in P \tag{$V18c$}$$

Valid inequalities ($V18$) require that if a vehicle drops (takes) an ammo type to (from) a node then that node must be on that vehicle's route.

$$\begin{aligned} k_{ivp} \leq tv_{iv} \quad & \forall i \in N_M, v \in V_F, p \in P \\ & \forall i \in N_M, v \in V_M, p \in P \\ & \forall i \in N_C, v \in V_M, p \in P \end{aligned} \tag{$V19$}$$

Valid inequalities ($V19$) ensure that if a vehicle drops (takes) an ammo type to (from) a node then the arrival time of that vehicle at that node must be a positive number, assuming that minimum traveling time between Fixed-TPs and Mobile-TPs is greater than one unit time. If there exist such a traveling time that is less than one, then we can divide the left hand side by a big enough number.

$$\sum_{v \in V_M} k_{ivp} \geq 1 \quad \forall i \in N_C, p \in N_P \tag{$V20$}$$

Valid inequalities ($V20$) maintain that the demand of each combat unit for each ammo type must be satisfied by at least one ammo truck.

$$\begin{aligned} k_{ivp} \leq y_i \quad & \forall i \in N_M, v \in V_F, p \in P \\ & \forall i \in N_M, v \in V_M, p \in P \end{aligned} \tag{$V21$}$$

Valid inequalities ($V21$) require that no vehicle can drop (take) any ammo to (from) a closed Mobile-TP.

$$\begin{aligned} tv_{iv} \leq TM_p \cdot \sum_{p \in P} k_{ivp} \quad & \forall i \in N_M, v \in V_F \\ & \forall i \in N_M, v \in V_M \\ & \forall i \in N_C, v \in V_M \end{aligned} \tag{$V22$}$$

Valid inequalities ($V22$) state that if a vehicle does not drop (take) any ammo to (from) a node then its arrival time at that node should be zero.

4.4 Model comparison

In this section we compare 4-index and 3-index models based on the total number of the decision variables and constraints.

Let n be the total number of transfer points and combat units ($n = |N|$), n_F be the total number of Fixed-TPs ($n_F = |N_F|$), n_M be the total number of

Table 4.1: Number of decision variables

4-index		3-index	
variable	number	variable	number
f_{ijvp}	n^2vp	$ftpout_{ivp}$	n_Fv_Fp
		$mtpin_{ivp}$	n_Mv_Fp
		$mtpout_{ivp}$	n_Mv_Mp
		$cuin_{ivp}$	n_Cv_Mp
tp_{ip}	np	tp_{ip}	np
tv_{iv}	nv	tv_{iv}	nv
x_{ijv}	n^2v	x_{ijv}	n^2v
y_i	n_F+n_M	y_i	n_F+n_M
w_{ijp}	n^2p	k_{ivp}	n^2p

Mobile-TPs ($n_M = |N_M|$), n_C be the total number of combat units ($n_C = |N_C|$), v be the total number of trucks ($v = |V|$), v_F be the total number of commercial trucks ($v_F = |V_F|$), v_M be the total number of ammo trucks ($v_M = |V_M|$), and p be the total number of ammo types ($p = |P|$). Number of variables for both models are presented in Table 4.1. After subtracting the common elements total number of the decision variables are;

$$\begin{aligned}
4index &= n^2\Big(vp + p\Big),\\
3index &= n_F\Big(v_Fp\Big) + n_M\Big(2vp\Big) + n_C\Big(2v_Mp\Big)\\
&< \Big(n_F + n_M + n_C\Big)\Big(2vp\Big)\\
&= n\Big(2vp\Big).
\end{aligned}$$

It is clear that for $n \geq 2$ we always have;

$$4index = n\Big(vp + p\Big) > 2vp > 3index,$$

which proves that 3-index model has always fewer variables than 4-index model.

Number of constraints for both models are presented in Figure 4.2. After subtracting the common elements total number of the constraints are;

$$\begin{aligned}
4index = {} & n_Fv_F(2n_M + 2n_Mp) + n_Mv_F(2n_M + 2n_Mp) + n_Mv_M(2n_C + 2n_Cp)\\
& + n_Cv_M(2n_C + 2n_Cp) + n_Mp(2n+1) + 2n_M^2p,\\
3index = {} & n_Fv_F(1+p) + n_Mv_F(1+3p) + n_Mv_M(1+4p) + n_Cv_M(1+3p)
\end{aligned}$$

Table 4.2: Number of constraints

4-index		3-index	
constraint	number	constraint	number
(3.1)	$p(n_M + n_C)$	(4.1)	$n_C p$
(3.2)	$p(n_M v_F + n_C v_M)$	(4.2)	vp
(3.3)	$v_F + v_M$	(4.3)	$n_M p$
(3.4)	$n_F v_F + n_M v_M$	(3.3)	$v_F + v_M$
(3.5)	$n_M v_F + n_C v_M$	(3.4)	$n_F v_F + n_M v_M$
(3.6)	$p(n_F + 2n_M)$	(3.5)	$n_M v_F + n_C v_M$
(3.7)+(3.10)	$2\Big(n_M v_F p(n_F + n_M)$	(4.4)	$p(n_F + n_M)$
	$+n_C v_M p(n_M + n_C)\Big)$	(4.5)	vp
(3.8)+(3.9)+(3.16)	$3\Big(n_M v_F(n_F + n_M)$	(4.6)	v
	$+n_C v_M(n_M + n_C)\Big)$	(4.7)+(4.9)+(4.10)	$v_F p(n_F + 3n_M)$
(3.11)	$n_M p(n_F + n_M)$		$+3v_M p(n_M + n_C)$
	$+n_C p(n_M + n_C)$	(4.8)	$v_F(n_F + n_M)$
(3.12)+(3.13)+(3.14)	$p(n_F + 2n_C)$		$v_M(n_M + n_C)$
(3.15)	$n_M p(n_F + n_M)$	(3.12)+(3.13)+(3.14)	$p(n_F + 2n_C)$
	$+n_C p(n_M + n_C)$	(3.16)	$n_M v_F(n_F + n_M)$
			$+n_C v_M(n_M + n_C)$
		(4.11)	$n_M v_M p$
		(4.12)	$p(n_M v_F + n_C v_M)$

$$+ v(1 + 2p).$$

Remember that our Mobile-ADS problem is an LRP problem in which both facility location and vehicle routing decisions exist. In our problem setting, there will be at least two potential Fixed-TPs (n_F), Mobile-TPs (n_M) and combat units (n_C). For this reason, we have at least $n_F, n_M, n_C = 2, n = 6$, and total number of the constraints are;

$$\begin{aligned}
4index &= 4v(4 + 4p) + 34p, \\
3index &= 2v_F(1 + p) + 2v_F(1 + 3p) + 2v_M(1 + 4p) + 2v_M(1 + 3p) + v(1 + 2p) \\
&< 4v(1 + 4p) + v(1 + 2p).
\end{aligned}$$

Remember that commercial (ammo) trucks distribute ammo between Fixed-TPs (Mobile-TPs) and Mobile-TPs (combat units). Hence, we will have at least one commercial and one ammo truck, which is $v_F, v_M = 1, v = 2$, and we have;

$$4index = 66p + 32 > 36p + 10 > 3index,$$

which proves that 3-index model has fewer constraints than 4-index model in realistic problem settings.

Chapter 5

Computational Experiments (Part I)

In this chapter we test our valid inequalities (V), which are derived for 4-index and 3-index formulations, on 6 moderate size test problem instances. All computations are conducted on a laptop computer with 1.83 GHz CPU, 1 GB RAM and Windows XP [69] operating system. We use GAMS/Cplex 9.1 [31] as the mixed integer programming solver and GAMS 22.0 [20] as the modeling language.

5.1 Experiments on test bed problem instances

In order to gain insight in using valid inequalities, we test all of them in 6 moderate size problem instances. In each problem instance there are; 3 potential Fixed-TP locations ($|N_F| = 3$), 8 potential Mobile-TP locations ($|N_M| = 8$), 10 combat units ($|N_C| = 10$), 5 commercial trucks ($|V_F| = 5$), 10 ammo trucks ($|V_M| = 10$) and 3 ammo types ($|P| = 3$). Table 5.1 shows the differences among the problem instances. For example, in problem instance A (PI A) at least one Fixed-TP, two Mobile-TPs must be opened and two commercial trucks, three ammo trucks must be dispatched to satisfy the total demand of the combat units in time and time window tightness is low. Table 5.2 clarifies what we mean by time window

Table 5.1: Differences of problem instances

	min number to open/dispatch				Time window
	$\lvert N_F \rvert$	$\lvert N_M \rvert$	$\lvert V_F \rvert$	$\lvert V_M \rvert$	tightness
PI A	1	2	2	3	low
PI B	1	3	3	5	low
PI C	1	2	2	3	medium
PI D	1	3	3	5	medium
PI E	1	2	2	3	high
PI F	1	3	3	5	high

Table 5.2: Time window tightness

<table>
<tr><th></th><th></th><th colspan="10">Combat Units</th></tr>
<tr><th></th><th></th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr>
<tr><td>low</td><td>TE_{ip}</td><td colspan="3">1</td><td colspan="4">6</td><td colspan="3">11</td></tr>
<tr><td></td><td>TL_{ip}</td><td colspan="3">10</td><td colspan="4">16</td><td colspan="3">24</td></tr>
<tr><td>medium</td><td>TE_{ip}</td><td colspan="3">1</td><td colspan="2">4</td><td colspan="2">10</td><td colspan="3">16</td></tr>
<tr><td></td><td>TL_{ip}</td><td colspan="3">8</td><td colspan="2">14</td><td colspan="2">20</td><td colspan="3">24</td></tr>
<tr><td>high</td><td>TE_{ip}</td><td colspan="2">2</td><td>4</td><td colspan="2">4</td><td>16</td><td colspan="2">10</td><td colspan="2">14</td></tr>
<tr><td></td><td>TL_{ip}</td><td colspan="2">8</td><td>8</td><td colspan="2">16</td><td>20</td><td colspan="2">20</td><td colspan="2">22</td></tr>
</table>

tightness. For example, low tightness means there exists three different groups of combat units with different time windows. To be exact, $TE_{ip} = 1$ and $TL_{ip} = 10$ for $i = 1, 2, 3$ and $p \in P$, $TE_{ip} = 6$ and $TL_{ip} = 16$ for $i = 4, 5, 6, 7$ and $p \in P$, $TE_{ip} = 11$ and $TL_{ip} = 24$ for $i = 8, 9, 10$ and $p \in P$.

All computational results from now on are obtained by using strong branching for selecting the branching variable and best-estimate search for selecting the next node when backtracking. For the other parameters we use CPLEX's default settings. When comparing valid inequalities, we base the comparison on the optimality gap reported by CPLEX. Computations for a problem are terminated after 3600 seconds.

5.2 4-index model

In Table 5.3 we compare the performance of single valid inequalities in each problem instance. First row presents the optimality gaps after running the original formulation for one hour and dashes indicate that no feasible solution can be found within the given time limit. Subsequent rows demonstrate the optimality gaps of the formulations after adding the corresponding valid inequalities shown in the rows. As can be seen in the table in two problem instances no feasible solution is found with the original formulation in one hour. $(V2)$ dominate the original formulation. In other words the gap of the formulation with $(V2)$ is superior to the gap of the original formulation in each problem instance. $(V13)$ perform better than other valid inequalities in terms of solution quality with an average gap of 7.64%. In addition, the results indicate that $(V4)$ are a good candidate for further examination.

Since $(V2)$, $(V4)$ and $(V13)$ are the best performing valid inequalities, we first test all pairwise combinations of other valid inequalities with them. The results that can be seen in Tables 5.4, 5.5 and 5.6 show that no pairwise combination is superior to $(V2)$, $(V4)$ and $(V13)$. Following the same methodology we test all triple combinations of other valid inequalities with some promising pairwise combinations and no improvement is obtained.

Then we propose that $(V2)$, $(V4)$ and $(V13)$ need further examination. Tables 5.7 and Table 5.8 present the performance of these valid inequalities in 30 and 15 minutes. It can be seen that in 30 minutes $(V13)$ can not find a feasible solution to two problem instances whereas $(V2)$ and $(V4)$ able to find all of them. In 15 minutes though, $(V4)$ can not find a solution to three problem instances whereas $(V2)$ can not find to only one problem instance. After such extensive testing we conclude that $(V2)$ offer the best improvements.

Table 5.3: Performance of the valid inequalities

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	16.36	-	15.76	4.35	16.80	-	4.35	-	-
($V1$)	20.64	9.86	14.87	23.22	12.19	17.46	9.86	16.37	23.22
($\boldsymbol{V2}$)	**14.44**	**8.45**	**11.16**	**0.61**	**16.16**	**14.11**	**0.61**	**10.82**	**16.16**
($V3$)	16.53	-	8.60	5.32	9.96	5.08	5.08	-	-
($\boldsymbol{V4}$)	**11.96**	**7.93**	**10.41**	**4.79**	**17.69**	**6.11**	**4.79**	**9.82**	**17.69**
($V5$)	19.85	1.39	16.84	5.09	20.34	2.10	1.39	10.94	20.34
($V6$)	32.77	-	16.02	-	23.56	-	16.02	-	-
($V7$)	15.96	-	12.11	5.22	19.08	5.13	5.13	-	-
($V8$)	15.39	5.47	21.37	8.95	18.06	4.50	4.50	12.29	21.37
($V9$)	15.37	5.32	10.03	6.42	14.10	-	5.32	-	-
($V10$)	12.97	-	11.29	4.50	37.52	-	4.50	-	-
($V11$)	16.36	-	15.76	4.35	16.83	-	4.35	-	-
($V12$)	5.69	13.47	10.29	-	14.26	9.75	5.69	-	-
($\boldsymbol{V13}$)	**12.38**	**1.78**	**8.60**	**7.86**	**10.74**	**4.45**	**1.78**	**7.64**	**12.38**

Table 5.4: Performance of the two-combination of valid inequalities (*Part 1*)

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	16.36	-	15.76	4.35	16.80	-	4.35	-	-
$(V2)$-$(V1)$	9.88	10.70	13.07	12.53	10.96	13.42	9.88	11.76	13.42
$(V2)$-$(V3)$	10.75	-	10.07	26.88	24.33	9.38	9.38	-	-
$(V2)$-$(V4)$	8.79	-	37.43	9.30	10.72	-	8.79	-	-
$(V2)$-$(V5)$	12.73	18.71	19.66	18.67	12.49	5.37	5.37	14.61	19.66
$(V2)$-$(V6)$	17.15	1.41	26.20	15.85	38.15	25.56	1.41	20.72	38.15
$(V2)$-$(V7)$	12.52	-	12.88	12.48	12.25	9.12	9.12	-	-
$(V2)$-$(V8)$	15.73	8.62	11.76	-	27.80	8.27	8.27	-	-
$(V2)$-$(V9)$	11.93	-	20.89	1.24	10.17	5.43	1.24	-	-
$(V2)$-$(V10)$	23.97	-	13.30	-	12.58	5.65	5.65	-	-
$(V2)$-$(V11)$	14.44	8.45	11.16	0.61	16.16	14.11	0.61	10.82	16.16
$(V2)$-$(V12)$	15.40	-	11.56	15.26	9.21	17.31	9.21	-	-
$(V2)$-$(V13)$	11.47	15.67	14.32	-	12.06	-	11.47	-	-

Table 5.5: Performance of the two-combination of valid inequalities (*Part 2*)

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	16.36	-	15.76	4.35	16.80	-	4.35	-	-
$(V4)$-$(V1)$	18.26	36.91	16.25	22.91	18.97	20.80	16.25	22.35	36.91
$(V4)$-$(V3)$	10.96	-	14.09	17.43	13.45	-	10.96	-	-
$(V4)$-$(V5)$	15.52	-	6.62	10.40	8.54	5.00	5.00	-	-
$(V4)$-$(V6)$	16.12	-	30.82	-	41.75	-	16.12	-	-
$(V4)$-$(V7)$	23.99	-	12.41	-	12.70	9.27	9.27	-	-
$(V4)$-$(V8)$	10.98	12.40	15.79	8.57	24.89	4.71	4.71	12.89	24.89
$(V4)$-$(V9)$	15.06	-	9.97	-	10.54	11.40	9.97	-	-
$(V4)$-$(V10)$	9.53	18.21	22.94	2.28	16.26	5.77	2.28	12.50	22.94
$(V4)$-$(V11)$	11.96	7.93	10.41	4.79	17.85	6.11	4.79	9.84	17.85
$(V4)$-$(V12)$	8.27	8.26	23.18	23.27	5.58	-	5.58	-	-
$(V4)$-$(V13)$	19.65	-	12.56	-	32.80	13.84	12.56	-	-

Table 5.6: Performance of the two-combination of valid inequalities (*Part 3*)

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	16.36	-	15.76	4.35	16.80	-	4.35	-	-
$(V13)$-$(V1)$	16.45	13.48	13.21	14.13	19.47	-	13.21	-	-
$(V13)$-$(V3)$	15.22	4.34	31.43	12.62	9.23	9.21	4.34	13.68	31.43
$(V13)$-$(V5)$	23.50	6.53	13.06	-	11.72	2.07	2.07	-	-
$(V13)$-$(V6)$	15.73	-	15.82	-	-	-	15.73	-	-
$(V13)$-$(V7)$	11.31	-	-	-	21.14	9.18	9.18	-	-
$(V13)$-$(V8)$	22.86	8.72	18.57	11.71	12.44	4.61	4.61	13.15	22.86
$(V13)$-$(V9)$	-	-	15.81	-	13.05	1.37	1.37	-	-
$(V13)$-$(V10)$	20.09	4.49	28.93	-	8.32	-	4.49	-	-
$(V13)$-$(V11)$	12.28	1.78	8.60	7.86	10.74	4.38	1.78	7.61	12.28
$(V13)$-$(V12)$	9.55	-	11.83	10.81	5.91	-	5.91	-	-

Table 5.7: Performance of the valid inequalities

	Gaps (%) after 30 minutes								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	18.91	-	15.94	11.53	17.66	-	-	-	-
($V2$)	14.58	37.72	12.17	0.62	20.83	14.12	0.62	16.67	37.72
($V4$)	12.52	7.96	11.03	4.79	19.82	9.50	4.79	10.94	19.82
($V13$)	16.49	11.33	10.25	-	16.25	-	-	-	-

Table 5.8: Performance of the valid inequalities

	Gaps (%) after 15 minutes								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	25.98	-	16.11	-	-	-	-	-	-
($V2$)	16.08	-	12.84	0.63	20.93	14.12	-	-	-
($V4$)	13.00	-	11.29	-	19.89	-	-	-	-
($V13$)	25.98	-	16.11	-	-	-	-	-	-

5.3 3-index model

In Table 5.9 we compare the performance of single valid inequalities in each problem instance. As can be seen in the table in one problem instance no feasible solution is found with the original formulation in one hour. ($V12$) perform better than other valid inequalities in terms of solution quality with an average gap of 1.56%.

Then, we test all pairwise combinations of other valid inequalities with ($V12$), the results of which can be seen in Table 5.10. These results show that pairwise combinations ($V12$-$V1$), ($V12$-$V2$) and ($V12$-$V18$) are better than the others. Thus, following the same methodology we test all triple combinations of other valid inequalities with these pairwise combinations. Finally: in Table 5.11 triple combinations ($V12$-$V1$-$V8$), ($V12$-$V1$-$V11$), ($V12$-$V1$-$V13$), ($V12$-$V1$-$V14$), ($V12$-$V1$-$V15$), ($V12$-$V1$-$V19$), ($V12$-$V1$-$V22$); in Table 5.12 ($V12$-$V2$-$V10$), ($V12$-$V2$-$V18$), ($V12$-$V2$-$V19$); in Table 5.13 ($V12$-$V18$-$V8$), ($V12$-$V18$-$V20$) give promising results that need further examination.

Tables 5.14 and Table 5.15 present the performance of these triple combinations in 30 and 15 minutes. It can be seen that in 30 minutes triple combinations ($V12$-$V1$-$V22$), ($V12$-$V2$-$V10$), ($V12$-$V2$-$V18$) and ($V12$-$V18$-$V20$) outperform others. In 15 minutes, however, ($V12$-$V18$-$V20$) clearly obtain a much better average optimality gap than that of the others. After such extensive testing we conclude that triple combination ($V12$-$V18$-$V20$) offer the best improvements.

Table 5.9: Performance of the valid inequalities

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	0.65	1.05	-	5.72	6.97	0.97	0.65	-	-
$(V1)$	1.06	0.52	7.40	0.53	0.70	1.68	0.52	1.98	7.40
$(V2)$	0.77	0.54	7.56	0.71	7.94	-	0.54	-	-
$(V3)$	7.06	0.60	13.41	0.58	-	0.70	0.58	-	-
$(V8)$	1.35	0.70	0.78	0.59	6.77	0.73	0.59	1.82	6.77
$(V9)$	0.59	1.17	7.37	0.67	1.02	1.47	0.59	2.05	7.37
$(V10)$	0.76	0.69	-	1.08	7.17	0.52	0.52	-	-
$(V11)$	0.65	1.05	-	5.72	6.97	1.07	0.65	-	-
$(V12)$	0.72	5.66	0.60	0.92	0.81	0.66	0.60	1.56	5.66
$(V13)$	1.07	0.80	6.97	0.62	7.20	0.79	0.62	2.91	7.20
$(V14)$	0.65	1.05	-	5.72	6.97	1.05	0.65	-	-
$(V15)$	-	0.57	0.71	1.18	7.51	5.62	0.57	-	-
$(V16)$	0.89	0.72	8.02	0.60	12.30	0.56	0.56	3.85	12.30
$(V17)$	-	1.15	33.95	0.56	7.16	6.16	6.16	-	-
$(V18)$	7.08	0.84	0.68	1.22	0.61	0.90	0.90	1.89	7.08
$(V19)$	0.64	0.83	6.90	5.59	1.28	1.16	1.16	2.73	6.90
$(V20)$	0.78	0.55	6.97	0.59	7.08	0.57	0.57	2.76	7.08
$(V21)$	0.74	19.82	6.68	0.56	1.08	1.20	1.20	5.01	19.82
$(V22)$	0.65	0.97	-	0.52	0.71	0.72	0.72	-	-

Table 5.10: Performance of the two-combination of valid inequalities

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
($V12$)-($V1$)	0.59	0.82	0.70	0.58	0.71	0.59	0.58	0.67	0.82
($V12$)-($V2$)	0.76	0.65	0.64	0.88	0.71	0.50	0.50	0.69	0.88
($V12$)-($V3$)	0.56	0.46	0.78	14.35	0.70	-	0.46	-	-
($V12$)-($V8$)	0.78	5.99	0.77	-	0.72	5.87	0.72	-	-
($V12$)-($V9$)	0.85	0.69	0.74	0.75	0.72	0.53	0.53	0.71	0.85
($V12$)-($V10$)	0.77	1.03	0.70	0.82	0.58	0.78	0.58	0.78	1.03
($V12$)-($V11$)	0.68	5.66	0.60	1.09	0.85	0.66	0.60	1.59	5.66
($V12$)-($V13$)	0.67	0.65	0.59	0.63	0.62	6.06	0.59	1.54	6.06
($V12$)-($V14$)	0.68	5.66	0.60	1.09	0.85	0.66	0.60	1.59	5.66
($V12$)-($V15$)	0.76	0.49	0.71	0.91	1.14	0.96	0.49	0.83	1.14
($V12$)-($V16$)	0.80	0.71	0.86	0.61	0.86	0.82	0.61	0.78	0.86
($V12$)-($V17$)	0.78	0.93	0.65	10.38	0.77	0.50	0.50	2.34	10.38
($V12$)-($V18$)	0.85	0.66	0.58	0.73	0.71	0.47	0.47	0.67	0.85
($V12$)-($V19$)	0.62	0.76	0.65	10.71	0.55	-	0.55	-	-
($V12$)-($V20$)	0.60	6.49	0.70	0.50	0.66	0.67	0.50	1.60	6.49
($V12$)-($V21$)	0.75	0.75	0.62	0.67	0.85	-	0.62	-	-
($V12$)-($V22$)	0.91	1.02	0.55	0.92	0.62	-	0.55	-	-

Table 5.11: Performance of the three-combination of valid inequalities

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
$(V12)$-$(V1)$-$(V2)$	0.65	0.84	0.72	-	0.61	18.03	-	-	-
$(V12)$-$(V1)$-$(V3)$	0.66	0.63	0.70	0.94	0.91	0.86	0.63	0.78	0.94
$(\boldsymbol{V}12)$-$(\boldsymbol{V}1)$-$\boldsymbol{V}8)$	**0.78**	**0.69**	**0.67**	**0.71**	**0.65**	**0.53**	**0.53**	**0.67**	**0.78**
$(V12)$-$(V1)$-$(V9)$	0.70	0.49	0.71	5.59	0.62	6.01	0.49	2.35	6.01
$(V12)$-$(V1)$-$(V10)$	6.94	0.66	0.60	0.94	0.63	0.64	0.60	1.74	6.94
$(\boldsymbol{V}12)$-$(\boldsymbol{V}1)$-$\boldsymbol{V}11)$	**0.62**	**0.82**	**0.70**	**0.58**	**0.75**	**0.59**	**0.58**	**0.68**	**0.82**
$(\boldsymbol{V}12)$-$(\boldsymbol{V}1)$-$\boldsymbol{V}13)$	**0.77**	**0.57**	**0.53**	**0.72**	**0.69**	**0.79**	**0.53**	**0.68**	**0.79**
$(\boldsymbol{V}12)$-$(\boldsymbol{V}1)$-$\boldsymbol{V}14)$	**0.62**	**0.82**	**0.70**	**0.58**	**0.75**	**0.59**	**0.58**	**0.68**	**0.82**
$(\boldsymbol{V}12)$-$(\boldsymbol{V}1)$-$\boldsymbol{V}15)$	**0.62**	**0.82**	**0.70**	**0.58**	**0.75**	**0.59**	**0.58**	**0.68**	**0.82**
$(V12)$-$(V1)$-$(V16)$	0.77	0.83	0.71	0.65	0.57	5.76	0.57	1.55	5.76
$(V12)$-$(V1)$-$(V17)$	0.63	0.66	0.72	0.67	0.66	1.16	0.63	0.75	1.16
$(V12)$-$(V1)$-$(V18)$	0.73	1.03	0.70	0.70	0.85	1.44	0.70	0.91	1.44
$(\boldsymbol{V}12)$-$(\boldsymbol{V}1)$-$\boldsymbol{V}19)$	**0.71**	**0.71**	**0.66**	**0.51**	**0.87**	**0.63**	**0.51**	**0.68**	**0.87**
$(V12)$-$(V1)$-$(V20)$	0.65	0.63	0.93	0.61	0.62	10.87	0.61	2.39	10.87
$(V12)$-$(V1)$-$(V21)$	0.64	0.75	0.75	14.87	0.65	10.58	0.64	4.71	14.87
$(\boldsymbol{V}12)$-$(\boldsymbol{V}1)$-$\boldsymbol{V}22)$	**0.74**	**0.84**	**0.64**	**0.54**	**0.65**	**0.68**	**0.54**	**0.68**	**0.84**

Table 5.12: Performance of the three-combination of valid inequalities

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
($V12$)-($V2$)-($V3$)	0.60	0.82	0.78	5.94	0.68	0.54	0.54	1.56	5.94
($V12$)-($V2$)-($V8$)	0.62	0.63	0.90	5.87	0.71	0.84	0.62	1.60	5.87
($V12$)-($V2$)-($V9$)	0.63	0.79	0.62	10.71	0.68	1.19	0.62	2.44	10.71
($\boldsymbol{V12}$)-($\boldsymbol{V2}$)-($\boldsymbol{V10}$)	**0.67**	**0.62**	**0.69**	**0.50**	**0.75**	**0.70**	**0.50**	**0.66**	**0.75**
($V12$)-($V2$)-($V11$)	0.76	0.68	0.73	0.98	0.56	0.90	0.56	0.77	0.98
($V12$)-($V2$)-($V13$)	0.59	0.50	0.72	10.26	6.71	5.75	0.50	4.09	10.26
($V12$)-($V2$)-($V14$)	0.74	0.67	0.83	0.98	0.71	0.87	0.67	0.80	0.98
($V12$)-($V2$)-($V15$)	0.66	0.69	0.67	0.62	0.77	10.34	0.62	2.29	10.34
($V12$)-($V2$)-($V16$)	0.61	0.81	0.80	0.77	0.66	-	-	-	-
($V12$)-($V2$)-($V17$)	0.61	0.51	1.00	0.98	0.72	5.82	0.51	1.61	5.82
($\boldsymbol{V12}$)-($\boldsymbol{V2}$)-($\boldsymbol{V18}$)	**0.74**	**0.56**	**0.70**	**0.62**	**0.72**	**0.67**	**0.56**	**0.67**	**0.74**
($\boldsymbol{V12}$)-($\boldsymbol{V2}$)-($\boldsymbol{V19}$)	**0.62**	**0.53**	**0.51**	**0.85**	**0.50**	**0.99**	**0.50**	**0.67**	**0.99**
($V12$)-($V2$)-($V20$)	0.72	5.86	0.90	0.89	0.72	14.58	0.72	3.95	14.58
($V12$)-($V2$)-($V21$)	0.80	0.88	-	0.70	0.74	10.26	-	-	-
($V12$)-($V2$)-($V22$)	0.71	0.65	0.67	0.54	0.76	1.05	0.54	0.73	1.05

Table 5.13: Performance of the three-combination of valid inequalities

	Gaps (%) after 1 hour								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
($V12$)-($V18$)-($V3$)	0.75	1.21	0.70	5.82	0.69	-	-	1.83	5.82
($V12$)-($V18$)-($V8$)	**0.86**	**0.71**	**0.71**	**0.67**	**0.59**	**0.51**	**0.51**	**0.68**	**0.86**
($V12$)-($V18$)-($V9$)	0.66	0.69	0.70	1.12	0.76	6.25	0.66	1.70	6.25
($V12$)-($V18$)-($V10$)	0.75	0.57	0.63	0.80	0.65	0.92	0.57	0.72	0.92
($V12$)-($V18$)-($V11$)	0.85	1.08	0.58	0.73	0.71	0.47	0.47	0.74	1.08
($V12$)-($V18$)-($V13$)	0.64	0.88	0.54	-	0.62	0.53	-	-	-
($V12$)-($V18$)-($V14$)	0.85	1.08	0.67	0.73	0.71	0.47	0.47	0.75	1.08
($V12$)-($V18$)-($V15$)	0.77	0.61	0.78	10.48	0.70	11.12	0.61	4.08	11.12
($V12$)-($V18$)-($V16$)	0.73	5.86	1.09	1.06	0.66	0.74	0.66	1.69	5.86
($V12$)-($V18$)-($V17$)	0.76	0.95	0.78	0.96	0.76	0.86	0.76	0.85	0.96
($V12$)-($V18$)-($V19$)	0.71	1.07	0.62	-	0.66	0.69	-	-	-
($V12$)-($V18$)-($V20$)	**0.58**	**0.64**	**0.72**	**0.51**	**0.86**	**0.69**	**0.51**	**0.67**	**0.86**
($V12$)-($V18$)-($V21$)	0.73	6.79	0.74	0.52	0.70	1.19	0.52	1.78	6.79
($V12$)-($V18$)-($V22$)	0.68	0.75	0.72	5.95	0.64	0.57	0.57	1.55	5.95

Table 5.14: Performance of the valid inequalities

	Gaps (%) after 30 minutes								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	0.80	15.06	-	5.91	7.28	6.40	-	-	-
($V12$)	0.77	-	0.68	5.92	7.69	0.66	-	-	-
($V12$)-($V1$)	0.68	1.50	12.68	0.60	0.76	10.27	0.60	4.42	12.68
($V12$)-($V2$)	0.76	-	0.83	-	0.72	6.07	-	-	-
($V12$)-($V18$)	0.98	15.26	0.74	-	0.76	0.54	-	-	-
($V12$)-($V1$)-($V8$)	0.93	0.74	0.67	0.72	0.65	5.78	0.65	1.58	5.78
($V12$)-($V1$)-($V11$)	0.68	1.50	12.68	0.60	0.76	10.27	0.60	4.42	12.68
($V12$)-($V1$)-($V13$)	0.98	0.88	0.64	0.84	0.97	-	-	-	-
($V12$)-($V1$)-($V14$)	0.68	1.50	12.68	0.60	0.76	10.27	0.60	4.42	12.68
($V12$)-($V1$)-($V15$)	0.68	1.50	12.68	0.60	0.76	10.27	0.60	4.42	12.68
($V12$)-($V1$)-($V19$)	0.80	0.91	0.84	0.66	0.90	6.44	0.66	1.76	6.44
($V12$)-($V1$)-($V22$)	**0.98**	**1.16**	**0.66**	**0.56**	**0.70**	**1.16**	**0.56**	**0.87**	**1.16**
($V12$)-($V2$)-($V10$)	**1.08**	**0.69**	**0.75**	**0.56**	**0.86**	**1.10**	**0.56**	**0.84**	**1.10**
($V12$)-($V2$)-($V18$)	**0.77**	**0.57**	**0.74**	**0.75**	**0.76**	**0.68**	**0.57**	**0.71**	**0.77**
($V12$)-($V2$)-($V19$)	0.65	1.38	0.68	15.23	0.81	1.06	0.65	3.30	15.23
($V12$)-($V18$)-($V8$)	0.86	27.68	0.71	0.70	-	0.66	-	-	-
($V12$)-($V18$)-($V20$)	**0.62**	**0.70**	**0.83**	**0.51**	**0.90**	**0.70**	**0.51**	**0.71**	**0.90**

Table 5.15: Performance of the valid inequalities

	Gaps (%) after 15 minutes								
	PI A	PI B	PI C	PI D	PI E	PI F	Min	Average	Max
Original	17.70	15.06	-	5.93	7.43	6.41	-	-	-
($V12$)	0.92	-	0.91	5.93	-	-	-	-	-
($V12$)-($V1$)	0.83	5.77	17.62	5.57	0.82	18.28	0.82	8.15	18.28
($V12$)-($V2$)	7.15	-	6.84	-	0.74	-	-	-	-
($V12$)-($V18$)	1.27	15.26	0.74	-	-	-	-	-	-
($V12$)-($V1$)-($V8$)	0.93	5.83	0.93	1.27	0.91	5.89	0.91	2.63	5.89
($V12$)-($V1$)-($V11$)	0.83	5.77	17.62	5.57	0.82	18.28	0.82	8.15	18.28
($V12$)-($V1$)-($V13$)	0.98	5.69	0.64	0.92	1.10	-	-	-	-
($V12$)-($V1$)-($V14$)	0.83	5.77	17.62	5.57	0.82	18.28	0.82	8.15	18.28
($V12$)-($V1$)-($V15$)	0.83	5.77	-	5.57	0.82	18.28	-	-	-
($V12$)-($V1$)-($V19$)	1.32	11.17	0.94	15.13	7.22	11.37	0.94	7.86	15.13
($V12$)-($V1$)-($V22$)	7.02	1.21	0.66	0.56	0.70	6.12	0.56	2.71	7.02
($V12$)-($V2$)-($V10$)	1.56	6.00	1.05	6.76	1.05	6.09	1.05	3.75	6.76
($V12$)-($V2$)-($V18$)	0.95	0.63	0.74	0.75	0.76	-	-	-	-
($V12$)-($V2$)-($V19$)	0.65	6.41	0.72	-	0.85	1.52	-	-	-
($V12$)-($V18$)-($V8$)	17.89	-	1.00	0.70	-	11.21	-	-	-
($V12$)-($V18$)-($V20$)	**0.77**	**0.70**	**0.85**	**0.58**	**0.96**	**0.79**	**0.58**	**0.78**	**0.96**

5.4 Findings

We conclude that in the problem instances we investigated ($V2$) help reduce the solution time of 4-index model. Likewise, we find that ($V12$), ($V18$) and ($V20$) reduce the solution time of 3-index model. In the following computations we use these valid inequalities with the corresponding models.

Chapter 6

Computational Experiments (Part II)

This chapter presents large-scale applications of the 4-index and 3-index models, strengthen by adding the valid inequalities that help lessen the solution time, to some realistic battle scenarios. To evaluate the performance of the models we generate two different sets of problem instances. The first set consists of 6 small size instances, and the second set contains 12 large size instances. Table 6.1 displays the characteristics of the problem instances.

The required number of trucks shows the minimum number of trucks required to supply the combat units complying all the constraints. In small size instances (PI G - PI L) number of required commercial trucks ranges from 3 to 5, ammo trucks ranges from 6 to 10, and number of ammo types ranges from 3 to 5. There are from 8 to 12 commercial trucks, from 16 to 24 ammo trucks, from 6 to 10 required commercial and from 12 to 20 ammo trucks, and from 2 to 5 ammo types in medium and large size instances.

In all problem instances; opening cost of Fixed-TP is 100 and of mobile-TP is 50, acquisition cost of a commercial truck is 50 and of an ammo truck is 25, transportation cost of a unit of an ammo type per unit time is 1 for commercial trucks and 0.5 for ammo trucks.

Table 6.1: Characteristics of problem instances

	♯ Fixed TPs	♯ Mobile TPs	♯ Combat units	♯ Com. trucks	♯ Ammo trucks	♯ required Com. trucks	♯ required Ammo trucks	♯ Ammo types
PI G	2	8	16	6	12	3	6	3
PI H	2	8	16	6	12	5	10	3
PI I	2	8	16	6	12	3	6	4
PI J	2	8	16	6	12	5	10	4
PI K	2	8	16	6	12	3	6	5
PI L	2	8	16	6	12	5	10	5
PI1	3	8	20	8	16	6	12	2
PI2	3	8	20	12	24	6	12	2
PI3	3	8	20	12	24	10	20	2
PI4	3	8	20	8	16	6	12	3
PI5	3	8	20	12	24	6	12	3
PI6	3	8	20	12	24	10	20	3
PI7	3	8	20	8	16	6	12	4
PI8	3	8	20	12	24	6	12	4
PI9	3	8	20	12	24	10	20	4
PI10	3	8	20	8	16	6	12	5
PI11	3	8	20	12	24	6	12	5
PI12	3	8	20	12	24	10	20	5

Each Fixed-TP has a capacity of 100 for each ammo type, whereas the capacity is 50 for Mobile-TPs. There are three different kinds of commercial trucks with a capacity of 13, 20, and 32 units and three different kinds of ammo trucks with capacities 7, 10 and 16. Finally, the demand of combat units ranges from 1 to 4 units for each ammo type.

6.1 Small size instances

We consider a strategic scenario in which a country's land forces are attacking enemy forces. Generally, land forces of a country consist of several armies, corps, brigades, and battalions. Figure 6.1 provides information on the organization of land forces.

The number of soldiers in an army can vary significantly between countries, commonly from 100.000 up to 200.000 or more. A corps typically includes from 20,000 to 50,000 soldiers. Under the current doctrine of most countries' land

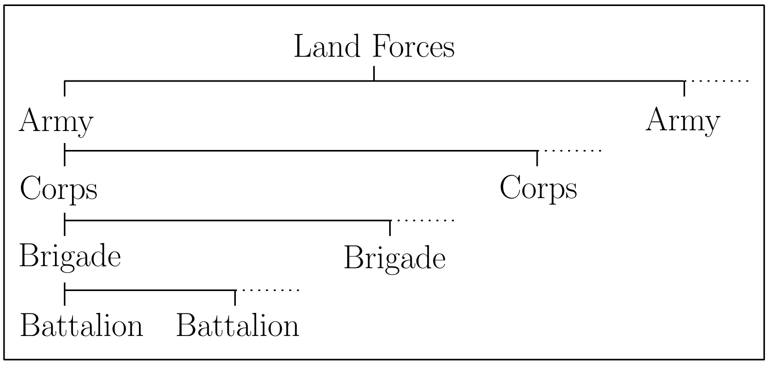


Figure 6.1: Organization of a representative land force

forces, armies are mostly concerned with both administrative and institutional missions. Usually, corps is the highest level of command that is concerned with operations on the battlefield.

Hence, in our base scenario we consider a corps that is in offense position to defeat enemy forces. The corps has 4 brigades. A brigade may have 4 or 5 battalions depending on the mission, enemy threat, etc. We assume that each brigade has 4 battalions. Therefore, there are 16 battalions in total and ammo must be pushed down to them. In other words these battalions are the so called combat units in the formulation ($|N_C| = 16$). Logistics planners determine 2 potential locations for Fixed-TPs ($|N_F| = 2$) and 8 potential locations for Mobile-TPs ($|N_M| = 8$) in the corps' control area. The layout of the corps on the battlefield can be seen in Figure 6.2, which shows the true potential locations of Fixed-TPs and Mobile-TPs and known locations of battalions. FTP/MTP/CU denotes Fixed-TP/Mobile-TP/combat unit (battalion), respectively. Dotted circles represent potential locations for Fixed-TPs and Mobile-TPs. In addition, all distances are taken from actual highway maps. Note that, the first brigade consists of battalions 1 to 4, i.e the first brigade=$\{CU_i : 1 \leq i \leq 4\}$. Likewise, the second brigade=$\{CU_i : 5 \leq i \leq 8\}$, the third brigade=$\{CU_i : 9 \leq i \leq 12\}$ and the fourth brigade=$\{CU_i : 13 \leq i \leq 16\}$.

The corps' transportation unit has 6 commercial ($|V_F| = 6$) and 12 ammo trucks ($|V_M| = 12$).

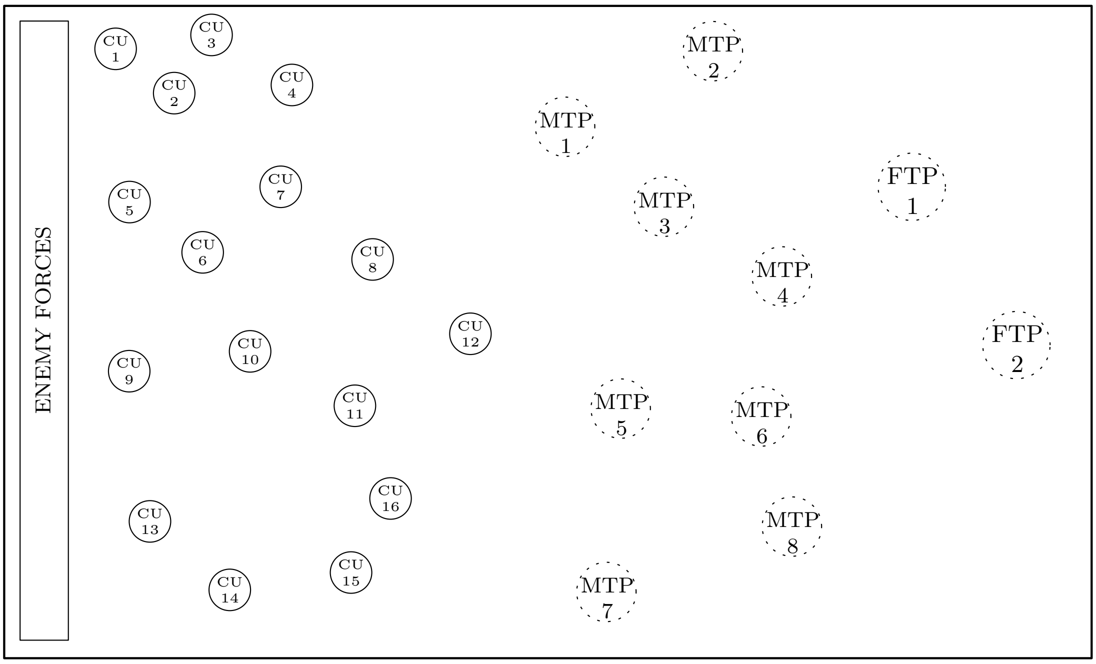
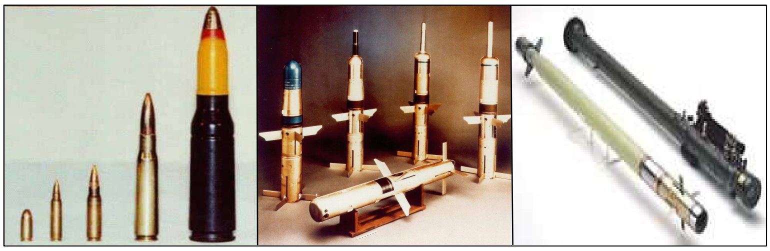


Figure 6.2: The corps' layout plan on the battlefield



High Density Ammo Medium Density Ammo Low Density Ammo

Figure 6.3: Ammunition groups

To reduce complexity, logistics planners group ammo into three groups according to their daily usage amounts as can be seen in Figure 6.3. High density group consists of mostly used ammo types such as infantry rifle bullets, etc. Medium density group consists of less frequently used ammo types such as anti-tank missiles, etc. Low density group consists of least used ammo types such as anti-aircraft missiles, etc. In other words three different ammo types ($|P| = 3$) must be distributed. The demand of each battalion is one ton for ammo type 1 (low density), two tons for ammo type 2 (medium density) and three tons for ammo type 3 (high density).

In order to be supplied, a maneuvering battalion must halt and take the security precautions. However all brigades of the corps can not halt at the same time when they are engaged with the enemy. Hence corps logistics and tactical planners decide to supply brigades in turns and determine the beginning and ending of the supply time window for each brigade. In other words battalions of a brigade have the same supply time window and brigades have nonoverlaping time windows.

Note that the two formulations (4-index and 3-index models) that we provide in the previous chapters do not consider any specific military command and control structure or any hierarchy among military units. These formulations are given in their most general form like a distributer or a manufacturer is supplying its customers with some industrial products. In doing so, our aim is to show that our model can be used in almost any (military or not) distribution system and to help readers compare Mobile-ADS design model with the existing network design or LRP models.

However, we need to state some military requirements to convert these models into a more realistic military form. To start with, for solving a real life Mobile-ADS design problem we need to consider a corps in battle. In other words, we need to design a Mobile-ADS for a corps in real life instances. The military requirements that we face in reality on the battlefield are as follows.

MILITARY REQUIREMENT 1 Due to the shortage of manpower/equipment resources and enemy threat, a corps can not establish an unlimited number of Fixed-TPs in its control area. It is preferable to open one Fixed-TP per corps.

MILITARY REQUIREMENT 2 Every brigade uses a separate wireless communication channel to communicate with its battalions on the battlefield. Hence, it is always easier for a brigade to communicate with its own battalion than to communicate with a battalion of another brigade. Besides, a battalion always reports to its own brigade about its location and demands. Thus, it is usually preferable to supply a combat unit from a Mobile-TP of its own brigade.

MILITARY REQUIREMENT 3 For the same reason stated in military requirement 1 and because of military requirement 2 it is preferable to open one Mobile-TP per brigade.

We design 6 different PIs of the base scenario by considering several combinations of two problem parameters. These problem parameters are explained in detail below.

- Number of product types (3, 4, 5): To evaluate the performance of the models in different complexity levels we consider 3 different ammo type numbers. That is, number of ammo types can be 3, 4 and 5.
- Truck usage percentage (50%, 83%): There are 6 commercial and 12 ammo trucks. We consider two different truck usage levels. In the first level, which is 50%, at least 3 commercial and 6 ammo trucks must be used. In the second level, which is 83%, at least 5 commercial and 10 ammo trucks must be used.

As we explain in Chapter 3, in a real combat environment we may need to consider two different objectives ($z1$ and $z2$). Hence, we test the 4-index model in each problem instance with both of these objective functions and test the 3-index model with the second objective function.

Table 6.2 presents the gaps and the objective function values. Note that we use ($V2$), strong branching and best-estimate search, solve LP relaxations at each node by primal simplex with devex pricing, only generate implied bounds, cover cuts and clique cuts, implement aggressive scaling, and perform presolve at nodes.

As can be seen in the table, 4-index model can attain less than 8% gaps in one hour and can not find a solution only in one instance with the first objective. With the second objective, 3-index can find a solution for four instances, whereas 4-index can find only for three. In addition, while 4-index can only reach gaps from 14% to 23%, 3-index is able to attain better gaps, at most 10%.

Table 6.2: Gaps (%) and costs in 1 hour (small instances)

	4-index, first obj.		4-index, second obj.		3-index, second obj.	
	gap	cost	gap	cost	gap	cost
PI G	7.50	923.18	14.63	712.41	0.85	662.33
PI H	6.70	1106.62	-	-	9.74	873.24
PI I	7.24	922.98	20.29	763.00	8.69	663.45
PI J	6.69	1106,16	-	-	-	-
PI K	-	-	23.06	790.32	-	-
PI L	0.72	1112,83	-	-	10.00	875.12

6.2 Large size instances

In the large size instance base scenario we again consider a corps. The corps has 4 brigades. We assume that each brigade has 5 battalions. Therefore, there are 20 battalions in total and ammo must be pushed down to them. In other words these battalions are the so called combat units in the formulation ($|N_C| = 20$). Logistics planners determine 3 potential locations for Fixed-TPs ($|N_F| = 3$) and 8 potential locations for Mobile-TPs ($|N_M| = 8$) in the corps' control area. The layout of the corps on the battlefield can be seen in Figure 6.4, which shows the true potential locations of Fixed-TPs and Mobile-TPs and known locations of battalions. FTP/MTP/CU denotes Fixed-TP/Mobile-TP/combat unit (battalion), respectively. Dotted circles represent potential locations for Fixed-TPs and Mobile-TPs. In addition, all distances are taken from actual highway maps. The first brigade consists of battalions 1 to 5, i.e the first brigade=$\{CU_i : 1 \leq i \leq 5\}$. Likewise, the second brigade=$\{CU_i : 6 \leq i \leq 10\}$, the third brigade=$\{CU_i : 11 \leq i \leq 15\}$ and the fourth brigade=$\{CU_i : 16 \leq i \leq 20\}$.

The corps' transportation unit has 8 commercial ($|V_F| = 8$) and 16 ammo trucks ($|V_M| = 16$).

Remember from Chapter 5 that we have three candidate valid inequalities for 4-index model, namely ($V2$), ($V4$) and ($V13$). Hence, we first test their performances on the base scenario. Table 6.3 provides the solutions of the strengthen 4-index model, with valid inequalities, as well as the original model. As can be seen in the table, the original model and the model with ($V13$) can not find a

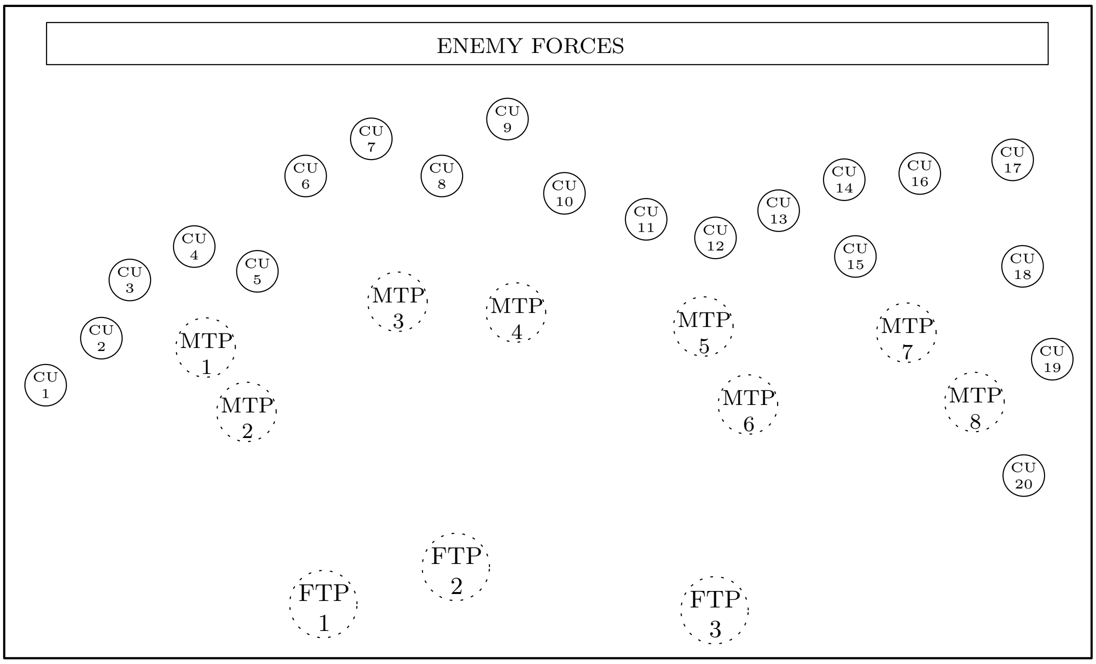


Figure 6.4: The corps' layout plan on the battlefield

feasible solution in 24 hours. However, the model with $(V2)$ solves the military scenario up to 2% in 24 hours and, in fact, this model finds the first solution in 17 minutes with a gap of 15.59%.

Then, we use strong branching and best-estimate search as we do in the valid inequality runs in Chapter 5. We solve the LP relaxations at each node by primal simplex with devex pricing. We only generate implied bounds, cover cuts and clique cuts and do not allow the generation of other cuts. We implement aggressive scaling and perform presolve at nodes. The last row pointed with $\star$ shows the performance of the model under the specified parameter settings. The results indicate that significant improvements are reached by the strengthen model with $(V2)$ under the specified parameter settings in terms of solution quality. It finds the first solution in two minutes with a gap of 12.04% and obtains a gap of 0.89% after 24 hours. In addition, the solution of this model is only 4.98% away from the optimal solution after one hour and 1.90% away after two hours.

Figure 6.5 shows the graphical representation of the solution obtained by the

Table 6.3: Results of the base scenario

	Utilization		Gaps (%)				
	N_{FM}	V	5 h	10 h	15 h	20 h	24 h
Original	-	-	-	-	-	-	-
$(V2)$	45%	75%	2.22	2.22	2.18	2.17	2.00
$(V4)$	45%	75%	-	12.99	12.82	12.05	8.65
$(V13)$	-	-	-	-	-	-	-
$(V2)^{\star}$	45%	75%	1.15	0.93	0.93	0.93	0.89

model with $(V2)$ under the specified parameter settings. As can be seen in the figure; 1 Fixed-TP point, 4 Mobile-TPs (1 for each brigade) are opened and 6 commercial trucks are dispatched from the Fixed-TP, and 12 ammo trucks are used by Mobile-TPs (each one dispatches 3 ammo trucks). Solid lines represent the routes of commercial trucks and a double arrowed line denotes a return trip. For example, commercial truck 1 is dispatched from Fixed-TP 2, drops its load at Mobile-TP 2 and comes back to Fixed-TP 2. However, commercial truck 3 starts its tour from Fixed-TP 2, serves Mobile-TP 3 and 2 in turn and returns to its home transfer point. A route (tour) of any ammo truck is represented by one of the three arrowed line types, namely dotted ($\cdots$), dashed (- - -) or dash dotted ($- \cdot -$) lines. Each line type is used only once for each Mobile-TP. In other words, each Mobile-TP has only one dotted, one dashed and one dash dotted line, meaning that each Mobile-TP dispatches three ammo trucks. For example, dashed line that emanates from Mobile-TP 2 represents the tour of ammo truck 14. It starts its tour from Mobile-TP 2, visits battalion 2 and battalion 1 in turn and returns to Mobile-TP 2 at the end. Likewise, the tour of ammo truck 4 is Mobile-TP 7 – Battalion 16 – Battalion 17 – Mobile-TP 7.

A cautious examination of the solution reveals that opening a Mobile-TP closer to Fixed-TPs is almost always (there is only one exception) more cost-effective than opening one closer to combat units. The model prefers Mobile-TP 2 over 1, Mobile-TP 6 over 5 and Mobile-TP 7 over 8. The only exception is that the model chooses Mobile-TP 3 rather than 4, but their distances from Fixed-TP 2 are almost the same. The explanation for selecting the closer Mobile-TP is in fact straightforward. Transportation cost of commercial trucks is higher than that of ammo trucks. Hence, less distance from a Fixed-TP means shorter commercial truck routes which in turn leads to less total transportation cost. As a matter of fact, opening a Mobile-TP (which can satisfy all demands of attached units in time) that is farther from the front line combat units is more advantageous tactically, too. Such a transfer point will be less vulnerable to enemy fire and unexpected counter attacks. In addition, it will have more time to change its location in case of such contingencies.

To help the reader in understanding the solution, we demonstrate the details of the solution for the first brigade in Figure 6.6. $CTruck_i$ and $ATruck_i$ denotes commercial and ammo truck i, respectively. p_i represents the amount of ammo type i. As an example, commercial truck 3 starts its route from Fixed-TP 2 and carries 20 tons of ammo 2 to Mobile-TP 3. It drops 10 tons of its load there and continues to Mobile-TP 2 with 10 tons of ammo 2. Finally, it goes back to its home transfer point after serving Mobile-TP 2 with 10 tons of ammo 2. Commercial truck 1 is dispatched from Fixed-TP 2 with a load of 5 tons of ammo 1 and 15 tons of ammo 3. It drops all ammo to Mobile-TP 2 and returns to Fixed-TP 2. Demands of battalion 2 are supplied by two different ammo trucks (which we called multi-sourcing in the literature classification). To be exact, one ton of ammo 1 and ammo 2 are provided by ammo truck 9 and one ton of ammo 2 and three tons of ammo 3 are provided by ammo truck 14.

We design 12 different PIs of the base scenario by considering several combinations of two problem parameters. These problem parameters are explained in detail below.

- Number of product types (2, 3, 4, 5): To evaluate the performance of the models in different complexity levels we consider 4 different ammo type numbers. That is, number of ammo types can be 2, 3, 4 and 5.

- Truck usage percentage (75%, 50%, 83%): We consider three different truck usage levels. In the first level, which is 75%, there exist 8 commercial and 16 ammo trucks out of which at least 6 commercial and 12 ammo trucks must be used to supply the total demand of combat units. In the second level, which is 50%, there exist 12 commercial and 24 ammo trucks out of

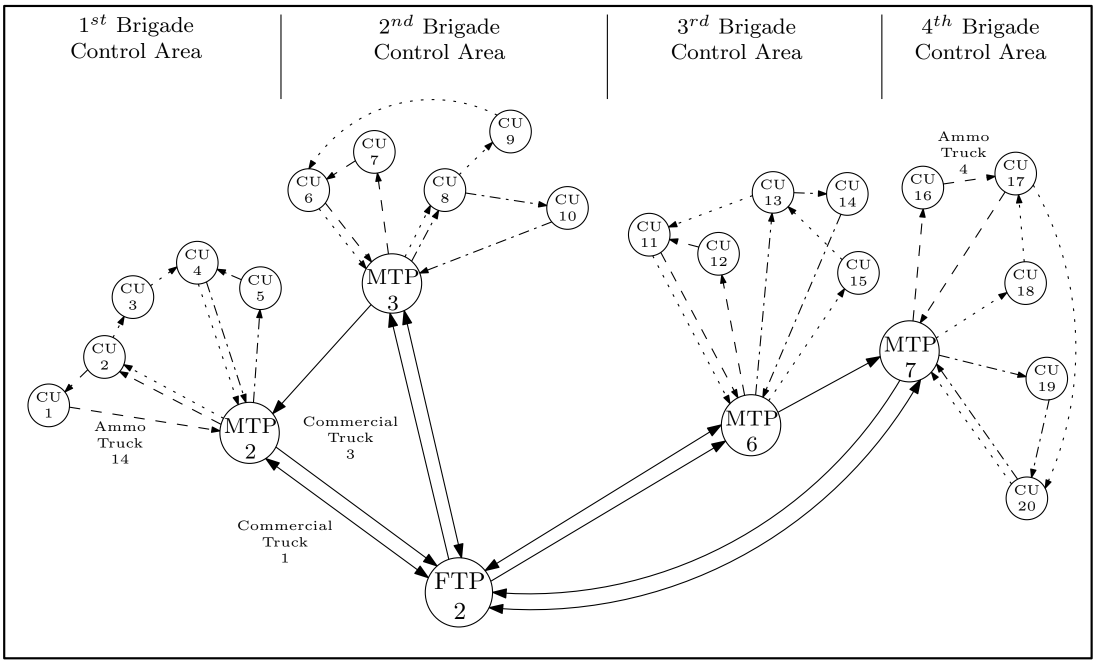


Figure 6.5: Layout of the base scenario solution

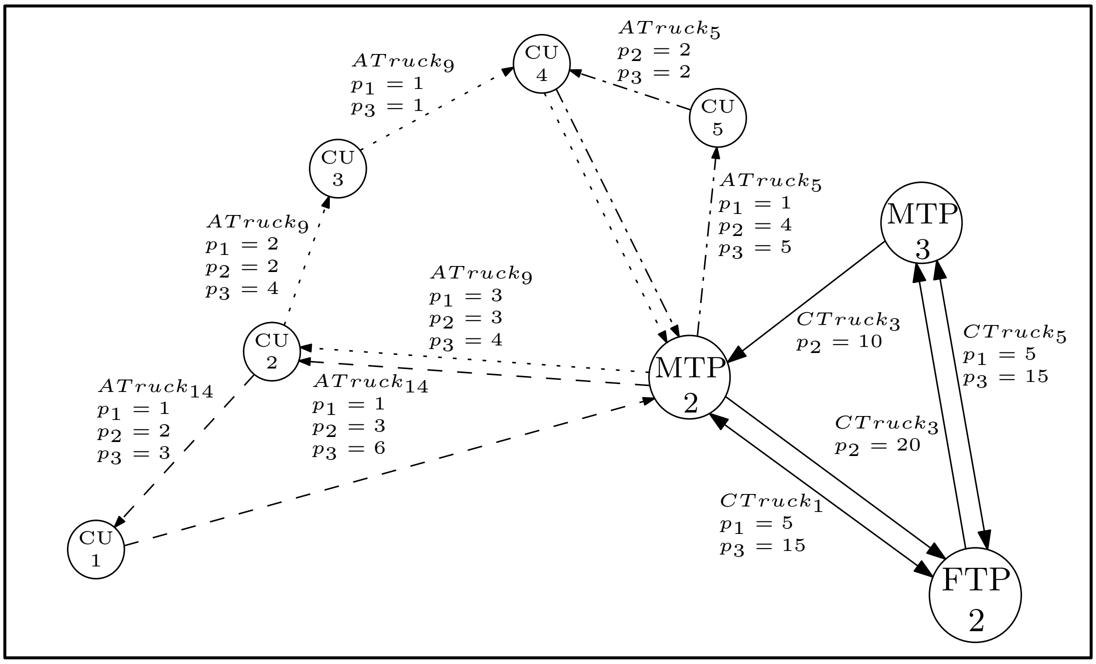


Figure 6.6: Detailed solution of the first brigade

which at least 6 commercial and 12 ammo trucks must be used. Finally, in the third level, which is 83%, there exist 12 commercial and 24 ammo trucks out of which at least 10 commercial and 20 ammo trucks must be used.

As we explain in Chapter 3, in a real combat environment we may need to consider two different objectives ($z1$ and $z2$). Hence, we test the 4-index model in each problem instance with both of these objective functions and test the 3-index model with the second objective function.

6.3 4-index model

Tables 6.4 and 6.5 presents the gaps and the objective function values with the first objective function. Abbreviations m and h stand for minutes and hours. Note that we use ($V2$), strong branching and best-estimate search, solve LP relaxations at each node by primal simplex with devex pricing, only generate implied bounds, cover cuts and clique cuts, implement aggressive scaling, and perform presolve at nodes.

As can be derived from the table, after 24 hours the average gaps with the first objective are 2.85%, 2.84%, 4.82% and 6.55% for $|P| = 2$, 3, 4 and 5, respectively. In addition, they are 2.04%, 3.39% and 7.37% for 75%, 50% and 83% truck usage levels, respectively. It is clear that the problem gets harder to solve as the number of ammo types and the number of available and used trucks increase.

Tables 6.6 and 6.7 exhibit the gaps and the objective function values with the second objective function. As can be derived from Table 6.6, after 24 hours the average gaps with the second objective are 3.25%, 3.31%, 3.40%and 4.23% for $|P| = 2$, 3, 4 and 5, respectively. In addition, they are 0.49%, 1.29% and 8.87% for 75%, 50% and 83% truck usage levels, respectively. Similar to the first objective, the problem gets harder to solve as the number of ammo types and the number of available and used trucks increase.

Table 6.4: Gaps of the problem instances with 4-index model and with the first objective

	Obj.	$\|P\|$	Truck	Gaps (%) after								
			Usage	2 m	8 m	30 m	1 h	2 h	4 h	5 h	10 h	24 h
PI1	$z1$	2	75%	-	-	12.41	10.57	2.77	2.02	2.02	1.93	0.86
PI2			50%	16.10	15.95	10.15	8.49	7.45	5.26	3.09	2.01	0.88
PI3			83%	14.79	14.15	10.15	7.97	6.82	6.82	6.82	6.82	6.82
PI4		3	75%	12.04	10.75	10.16	4.98	1.90	1.34	1.15	0.93	0.89
PI5			50%	15.72	15.34	12.86	10.51	6.79	4.07	3.00	1.97	0.91
PI6			83%	-	-	10.42	8.71	6.71	6.71	6.71	6.71	6.71
PI7		4	75%	-	13.22	8.93	8.59	7.27	4.20	3.65	2.98	2.45
PI8			50%	-	-	-	-	14.26	6.78	5.73	3.24	2.74
PI9			83%	-	-	-	-	-	10.58	10.36	10.07	9.26
PI10		5	75%	-	-	-	11.39	10.29	5.28	4.72	4.24	3.94
PI11			50%	-	-	20.87	18.28	15.67	13.28	12.97	10.87	9.02
PI12			83%	-	20.83	16.12	14.03	10.67	6.83	6.69	6.69	6.69

Table 6.5: Costs of the problem instances with 4-index model and with the first objective

	Obj.	$\|P\|$	Truck	Costs after								
			Usage	2 m	8 m	30 m	1 h	2 h	4 h	5 h	10 h	24 h
PI1	$z1$	2	75%	-	-	1413.58	1403.60	1326.48	1317.18	1317.18	1316.04	1301.84
PI2			50%	1428.37	1428.37	1358.43	1353.68	1352.97	1352.04	1331.29	1316.79	1301.84
PI3			83%	1743.97	1743.97	1688.32	1688.32	1688.32	1688.32	1688.32	1688.32	1688.26
PI4		3	75%	1362.83	1362.83	1360.15	1309.48	1307.59	1307.57	1305.24	1302.38	1301.84
PI5			50%	1414.09	1413.06	1387.89	1361.47	1327.28	1305.67	1304.00	1303.40	1301.84
PI6			83%	-	-	1694.12	1686.44	1686.17	1686.17	1686.17	1686.17	1686.17
PI7		4	75%	-	1377.94	1332.15	1331.28	1328.81	1308.90	1308.82	1308.42	1304.80
PI8			50%	-	-	-	-	1421.22	1323.00	1323.00	1318.37	1314.80
PI9			83%	-	-	-	-	-	1689.19	1687.73	1687.73	1686.17
PI10		5	75%	-	-	-	1374.57	1372.09	1361.34	1353.32	1346.79	1342.75
PI11			50%	-	-	1509.48	1465.69	1455.91	1428.02	1428.02	1412.02	1386.06
PI12			83%	-	1867.45	1779.13	1750.05	1686.23	1686.17	1686.17	1686.17	1686.17

Table 6.6: Gaps of the problem instances with 4-index model and with the second objective

	Obj.	$\lvert P\rvert$	Truck Usage	Gaps (%) after								
				2 m	8 m	30 m	1 h	2 h	4 h	5 h	10 h	24 h
PI1	$z2$	2	75%	-	-	-	-	-	-	-	0.41	0.35
PI2			50%	20.59	20.56	18.70	1.14	1.02	0.87	0.70	0.66	0.64
PI3			83%	12.56	12.56	9.17	9.07	9.03	8.97	8.96	8.75	8.75
PI4		3	75%	-	-	-	-	-	-	0.48	0.44	0.44
PI5			50%	23.99	23.99	20.73	12.76	3.37	3.31	3.29	0.54	0.52
PI6			83%	-	-	-	-	9.31	9.16	9.16	8.98	8.96
PI7		4	75%	-	-	-	-	-	3.72	1.07	1.02	0.58
PI8			50%	-	-	18.92	13.59	6.12	6.04	6.00	5.99	0.75
PI9			83%	-	-	11.03	9.06	9.04	9.00	8.98	8.87	8.87
PI10		5	75%	-	-	-	-	-	-	-	-	0.57
PI11			50%	-	-	-	-	20.71	13.47	10.85	8.60	3.25
PI12			83%	-	-	-	-	-	9.20	9.16	9.11	8.88

Table 6.7: Costs of the problem instances with 4-index model and with the second objective

	Obj.	$\|P\|$	Truck Usage	Costs after 2 m	8 m	30 m	1 h	2 h	4 h	5 h	10 h	24 h
PI1	$z2$	2	75%	-	-	-	-	-	-	-	922.47	921.98
PI2			50%	1251.84	1251.84	1125.83	926.25	925.76	924.99	923.65	923.56	923.46
PI3			83%	1386.14	1386.14	1334.84	1334.84	1334.84	1334.84	1334.84	1331.72	1331.72
PI4		3	75%	-	-	-	-	-	-	922.40	922.07	922.03
PI5			50%	1203.66	1203.66	1155.41	1050.08	948.38	948.27	948.27	922.03	921.89
PI6			83%	-	-	-	-	1336.27	1334.59	1334.59	1332.33	1332.01
PI7		4	75%	-	-	-	-	-	950.81	925.53	925.53	922.02
PI8			50%	-	-	1129.05	1104.18	975.24	974.46	974.06	974.06	922.53
PI9			83%	-	-	1362.61	1333.14	1332.85	1332.76	1332.76	1331.72	1331.72
PI10		5	75%	-	-	-	-	-	-	-	-	922.05
PI11			50%	-	-	-	-	1154.98	1058.61	1027.65	1002.74	947.32
PI12			83%	-	-	-	-	-	1335.86	1335.86	1335.44	1332.01

6.4 3-index model

Tables 6.8 and 6.9 exhibit the gaps and the objective function values with the second objective function. Note that we use ($V18$), ($V20$), strong branching and best-estimate search.

As can be derived from the table, after 24 hours the average gaps with the second objective are 3.64%, 3.72%, 5.57% and 9.02% for $|P| = 2$, 3, 4 and 5, respectively. In addition, they are 2.86%, 4.24% and 9.35% for 75%, 50% and 83% truck usage levels, respectively. It is clear that the problem gets harder to solve as the number of ammo types and the number of available and used trucks increase.

Table 6.8: Gaps of the problem instances with 3-index model and with the second objective

	Obj.	$\|P\|$	Truck Usage	Gaps (%) after								
				2 m	8 m	30 m	1 h	2 h	4 h	5 h	10 h	24 h
PI1	$z2$	2	75%	-	-	-	-	-	1.13	1.12	1.10	0.81
PI2			50%	-	-	1.47	0.97	0.94	0.91	0.91	0.91	0.89
PI3			83%	-	-	-	11.60	9.84	9.52	9.51	9.23	9.21
PI4		3	75%	-	-	-	-	-	-	-	6.32	0.90
PI5			50%	-	-	2.05	1.04	0.99	0.97	0.97	0.95	0.90
PI6			83%	-	-	-	-	-	11.17	9.42	9.38	9.35
PI7		4	75%	-	-	-	-	-	-	-	0.96	0.90
PI8			50%	-	-	-	-	-	6.78	6.78	6.78	6.60
PI9			83%	-	-	-	-	-	-	-	-	9.20
PI10		5	75%	-	-	-	-	-	11.55	11.51	8.96	8.83
PI11			50%	-	-	-	-	15.83	15.77	15.75	15.72	8.58
PI12			83%	-	-	-	-	-	12.14	12.07	11.91	9.64

Table 6.9: Costs of the problem instances with 3-index model and with the second objective

	Obj.	$\|P\|$	Truck	Costs after								
			Usage	2 m	8 m	30 m	1 h	2 h	4 h	5 h	10 h	24 h
PI1	$z2$	2	75%	-	-	-	-	-	924.29	924.24	924.19	921.60
PI2			50%	-	-	926.92	922.25	922.02	921.77	921.77	921.77	921.60
PI3			83%	-	-	-	1368.03	1341.33	1336.64	1336.55	1332.39	1332.00
PI4		3	75%	-	-	-	-	-	-	-	975.15	921.84
PI5			50%	-	-	932.28	922.78	922.43	922.18	922.18	922.09	921.60
PI6			83%	-	-	-	-	-	1361.21	1335.00	1334.41	1333.90
PI7		4	75%	-	-	-	-	-	-	-	922.07	922.35
PI8			50%	-	-	-	-	-	979.70	979.70	979.70	978.16
PI9			83%	-	-	-	-	-	-	-	-	1332.00
PI10		5	75%	-	-	-	-	-	1031.70	1031.25	1002.37	998.76
PI11			50%	-	-	-	-	1083.98	1083.37	1083.02	1082.65	998.17
PI12			83%	-	-	-	-	-	1376.36	1375.26	1372.76	1338.36

Table 6.10: Summary of the problem instances

		number of ammo types			
		2	3	4	5
truck	75%	PI1	PI4	PI7	PI10
usage	50%	PI2	PI5	PI8	PI11
levels	83%	PI3	PI6	PI9	PI12

6.5 Findings with valid inequalities

Recall that 12 large size instance are introduced by considering several combinations of two problem parameters, namely number of ammo types ($|P| = 2, 3, 4, 5$) and truck usage percentage ($75\%, 50\%, 83\%$). Figure 6.10 reminds these instances.

To summarize the results of the computational experiments graphically, we arrange 12 problem instances into 3 groups according to truck usage percentages. In detail: the first group consists of PI1, PI4, PI7 and PI10 whose truck usage percentage is 75%; the second group consists of PI2, PI5, PI8 and PI11 whose truck usage percentage is 50%; the third group consists of PI3, PI6, PI9 and PI12 with a 83% truck usage percentage. In the 75% group 18 trucks are dispatched out of 24 trucks, 50% group 18 trucks are dispatched out of 36 trucks, and 83% group 30 trucks are dispatched out of 36 trucks. In each group we investigate the effect of the ammo type number on the performance of the models. In detail, in each group the first/second/third/fourth problem instance has 2/3/4/5 ammo types. For example, in the 50% truck usage group PI2/PI5/PI8/PI11 has 2/3/4/5 ammo types.

First of all we present the branch and bound details in Table 6.11. Node shows the number of nodes and iteration shows the number of iterations that are used by the MIP solver in solving of our problem instances. It can be seen that as the problem gets harder to solve, the number of nodes and iterations decrease, since solving linear sub-problems at each node takes more time.

For each instance, model and objective function, we present the CPU time and the cost of the linear programming bound in Tables 6.12, 6.13 and 6.14. With the first objective function, the results show that there is not much difference between

Table 6.11: The number of nodes and iterations

	4-index, first obj.		4-index, second obj.		3-index, second obj.	
	iteration	node	iteration	node	iteration	node
PI1	92,464,288	193,120	75,757,047	241,601	16,865,584	237,201
PI2	82,535,826	152,678	91,279,736	156,016	16,789,905	137,573
PI3	89,415,817	166,601	57,261,215	167,710	7,636,301	207,130
PI4	96,944,058	145,680	88,736,917	159,301	19,308,708	160,325
PI5	93,782,520	128,101	93,488,993	101,401	10,308,708	109,524
PI6	68,194,211	134,294	69,523,100	119,631	9,721,280	129,768
PI7	128,189,366	119,684	87,283,387	102,501	18,784,518	98,547
PI8	104,855,439	84,270	69,771,864	88,601	129,750,852	77,053
PI9	80,879,446	80,630	60,356,325	87,717	14,580,055	79,642
PI10	117,715,796	60,452	96,405,506	59,301	20,150,938	86,912
PI11	97,524,207	50,162	80,531,638	55,201	14,881,716	43,803
PI12	61,760,104	75,801	48,907,082	73,101	16,190,386	46,371

the original and enhanced (with $(V2)$) 4-index model in terms of the computation time for solving the linear programming relaxations. Run time ranges between 1 to 15 seconds, which consider reasonable. As for the second objective, including all the valid inequalities into the models certainly deteriorates the computation time. For 4-index model not a big difference exists between the original model and the enhanced model. However, running the 3-index model without any valid inequality clearly gives better run times. In addition, the results reveals that 3-index model solves the linear relaxations faster than 4-index model. Briefly, for all the instances the linear programming lower bounds can be computed very quickly.

As for the solution quality, generally including all valid inequalities into the both models helps to attain a better lower bound in the hard instances. Furthermore, in each instances 4-index provides a better lower bound that 3-index model.

6.5.1 4-index model

We present the objective values of the 4-index model with the first objective function in Figure 6.7. As can be seen in Figures 6.7(a), 6.7(b) and 6.7(c) for

Table 6.12: Linear programming bound of 4-index model with the first objective

	Original model		with $(V2)$		with all valid inq.s	
	run time	cost	run time	cost	run time	cost
PI1	0.78	1187.55	0.60	1187.55	0.94	1187.55
PI2	1.24	1187.55	1.41	1187.55	1.52	1187.55
PI3	1.30	1477.66	1.68	1477.66	1.39	1537.55
PI4	1.50	1187.55	1.17	1187.55	1.32	1187.55
PI5	2.86	1187.55	2.89	1187.55	2.41	1187.55
PI6	3.14	1477.66	3.72	1477.66	2.36	1537.55
PI7	2.86	1187.55	3.27	1187.55	1.77	1187.55
PI8	5.34	1187.55	5.93	1187.55	2.96	1187.55
PI9	6.81	1477.66	6.22	1477.66	3.36	1537.55
PI10	4.00	1187.55	4.96	1187.55	1.92	1187.55
PI11	7.74	1187.55	9.00	1187.55	4.94	1187.55
PI12	9.32	1477.66	11.17	1477.66	15.12	1537.55

Table 6.13: Linear programming bound of 4-index model with the second objective

	Original model		with $(V2)$		with all valid inq.s	
	run time	cost	run time	cost	run time	cost
PI1	2.73	914.46	2.35	914.46	4.48	914.46
PI2	5.00	914.46	4.96	914.46	8.62	914.46
PI3	5.33	1211.72	6.12	1211.72	10.04	1272.60
PI4	5.85	914.46	5.52	914.46	12.57	914.46
PI5	13.04	914.46	12.46	914.46	24.79	914.46
PI6	12.48	1211.72	12.57	1211.72	27.94	1272.60
PI7	10.86	914.46	14.37	914.46	29.34	914.46
PI8	22.67	914.46	26.66	914.46	140.11	914.46
PI9	22.96	1211.72	26.14	1211.72	66.32	1272.60
PI10	22.28	914.46	24.16	914.46	292.31	914.46
PI11	38.88	914.46	49.18	914.46	747.81	914.46
PI12	49.13	1211.72	46.15	1211.72	303.62	1272.60

Table 6.14: Linear programming bound of 3-index model with the second objective

	Original model		with ($V18$)-($V20$)		with all valid inq.s	
	run time	cost	run time	cost	run time	cost
PI1	0.58	911.45	1.07	912.44	2.20	912.51
PI2	0.95	911.45	2.09	912.44	6.31	912.48
PI3	0.96	1207.35	1.96	1208.05	4.18	1269.09
PI4	0.67	910.99	1.52	912.16	8.16	912.21
PI5	1.78	910.99	4.01	912.16	49.14	912.18
PI6	1.79	1206.66	4.09	1207.60	18.13	1268.59
PI7	1.27	910.53	2.76	911.88	11.04	911.92
PI8	2.57	910.53	4.93	911.88	18.90	911.90
PI9	3.21	1205.97	8.63	1207.17	29.99	1268.12
PI10	1.50	910.25	4.65	911.71	21.17	911.76
PI11	4.45	910.25	15.13	911.71	49.21	911.74
PI12	3.88	1205.56	9.42	1206.91	62.36	1267.87

the 75% and 50% groups the objective values improve fast in the first 4-6 hours and for the 83% group it improves in the first 2 hours. After these time intervals no significant improvements can be obtained and this observation is valid for all ammo type numbers in each group.

In addition, for the 75% and 50% groups ammo type number affects the performance of the model more than it does for the 83% group. For the 83% level in 2 hours the model can attain a similar objective for 5-ammo type instance (PI12) as in the other instances, whereas for the 75% and 50% levels the objective value of the 5-ammo type instances (PI10 and PI11) stays higher than those of the other instances.

The objective values of the 4-index model with the second objective function are presented in Figure 6.8. Figure 6.8(a) shows that for the 75% level except 5-ammo type instance (PI10) the objective values do not improve much after the first 10 hours. It can be seen in Figure 6.8(b) that for the 50%, again except 5-ammo type instance (PI11), objective values mostly improve in the first 2 hours. In both levels the objective function values of the 5-ammo type level take more than 20 hours to get close to the level of the objective values of the other instances that have less number of ammo types. In 83% level, as can be seen in Figure

6.8(c), all instances attain low objective function values in the first 4 hours.

Furthermore, similar to the results of the first objective, ammo type number affects the performance of the model more in the 75% and 50% levels than it does in the 83% level.

6.5.2 3-index model

Objective values of the 3-index model with the second objective function is presented in Figure 6.9. Figures 6.9(a), 6.9(b) and 6.9(c) exhibit that, different than the 4-index model, for the 3-index model in all truck usage levels 5-ammo type instances (PI10, PI11 and PI12) attain an objective value higher than those of the other instances. In addition, it takes longer for the 5-ammo type instance to lower the objective values, i.e. 10 hours for the 75% level, and more than 20 hours for the 50% and 83% levels.

A comparative analysis between the 4-index and 3-index models reveals that in the situations similar to our problem setting it is more advantageous to use the 4-index model with the valid inequalities if there are more than 4 ammo types.

6.6 Findings without valid inequalities

What we notice after extensive computational experiments is that performance of the valid inequalities may slightly differ from one scenario to another. In other words, while in the scenarios we covered in Chapter 5 and 6, ($V2$) helps reduce the solution time for 4-index model, there may exist other scenarios where some other valid inequality or inequalities outperform it or no valid inequality can help at all. Hence we also want to compare 4-index and 3-index models without the effects of the valid inequalities.

In addition, recall that in all the computations with 4-index model we use strong branching and best-estimate search, solve LP relaxations at each node by primal simplex with devex pricing, only generate implied bounds, cover cuts and clique cuts, implement aggressive scaling, and perform presolve at nodes. However, these specifications may also be case dependent. Hence, in the following computations we use the default parameter settings of the GAMS/Cplex except for strong branching and best-estimate search.

Briefly, we evaluate the 4-index and 3-index models in their original forms without the effects of the valid inequalities and of the specific GAMS/Cplex parameter settings.

We compare the models according to two different criteria of which the first is the general statistics (number of equations, nonzero elements and variables) of the models generated by the formulations and the second is the optimality gaps reached in a certain time period. As for the first comparison, we present three statistics obtained from the Model Statistics section of GAMS 22.0 in Figure 6.10.

For the sake of simplicity we do not present the results of the instances that have two ammo types (PI1 - PI3) for their results are similar. We also show the results of the 75% and 50% truck usage levels (PI5 - PI6, PI8- PI9 and PI11 - PI12) together since they have the same statistics.

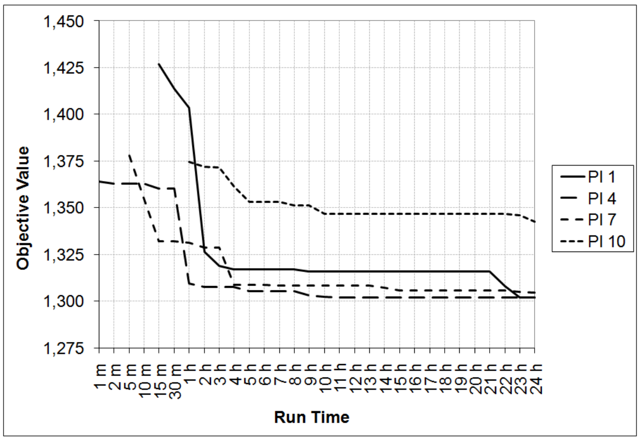
Figure 6.11 compares the 4-index and 3-index models. It can be seen that 4-index model has; 260% more single equations, 300%-360% more non-zero elements and 310%-450% more single variables than 3-index model. We expect that having a lower number of equations, nonzero elements and variables will lead to better computational results for 3-index model.

Table 6.15 exhibits the computational results of both models. We use strong branching for selecting the branching variable, best-estimate search for selecting the next node when backtracking and GAMS/Cplex's default settings for the other parameters. No valid inequalities are used for both models. Computations for a problem are terminated after 14 hours. Note that these computations are conducted on a computer with 2.4 GHz CPU, 4 GB RAM and Windows XP operating system.

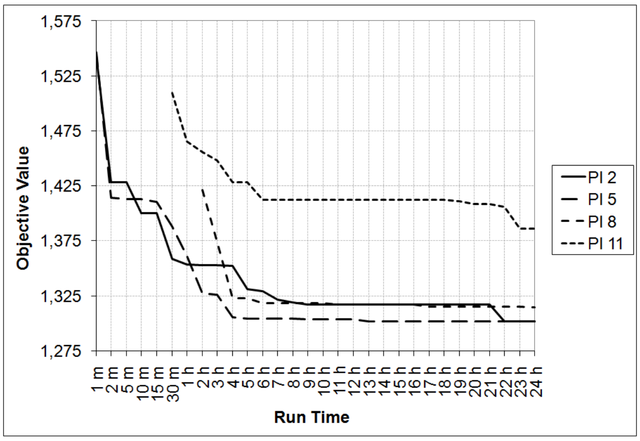
Table 6.15: Computational results of the 4-index and 3-index models

		Gaps (%) after						
		5 m	15 m	1 h	2 h	4 h	11 h	14 h
PI4	4-index	-	-	-	-	-	9.11	6.71
PI5		-	-	-	-	-	8.74	8.74
Other PIs		-	-	-	-	-	-	-
PI4	3-index	-	4.58	1.42	1.40	1.37	1.35	1.35
PI5		-	-	6.95	1.44	1.44	1.43	1.43
PI6		-	-	-	-	-	-	-
PI7		-	-	-	7.27	4.32	1.42	1.42
PI8		12.00	11.89	6.88	1.50	1.44	1.42	1.40
PI9		-	-	-	-	-	-	-
PI10		-	-	-	-	-	-	15.64
PI11		-	-	-	-	15.71	11.22	11.22
PI12		-	-	-	-	-	-	-

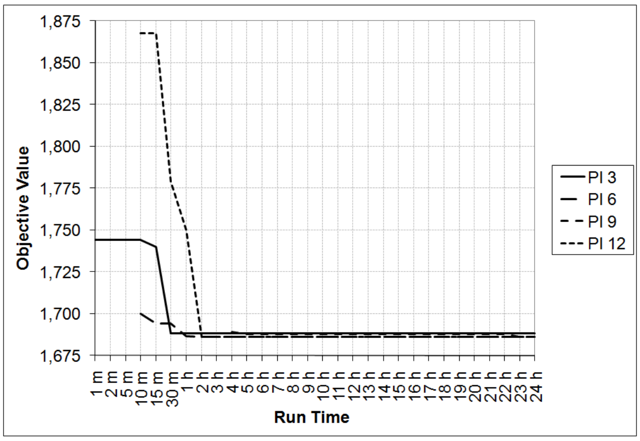
It can be seen that 4-index model finds a solution in only two problem instances, whereas 3-index model finds a solution in six instances. 3-index model can not find a solution only to 5-ammo type instances (PI6, PI9 and PI12). In addition, even in these two instances (PI4 and PI5) 3-index model finds an initial solution in less than an hour (4-index finds an initial in 11 hours) and reaches a better gap (6.71% and 8.74% for 4-index, whereas 1.35% and 1.43% for 3-index) in 14 hours. It is clear from the results that 3-index model performs better than 4-index model in a fixed amount of time without using valid inequalities and specific GAMS/Cplex parameter settings.



(a) 75% truck usage

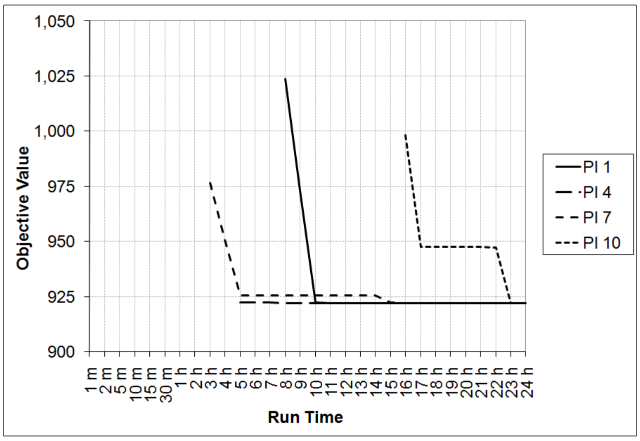


(b) 50% truck usage

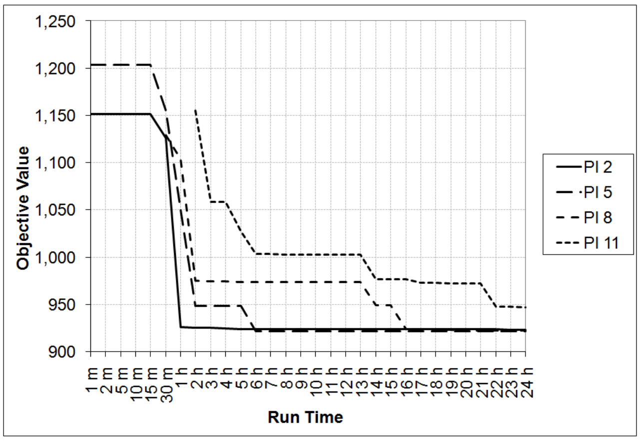


(c) 83% truck usage

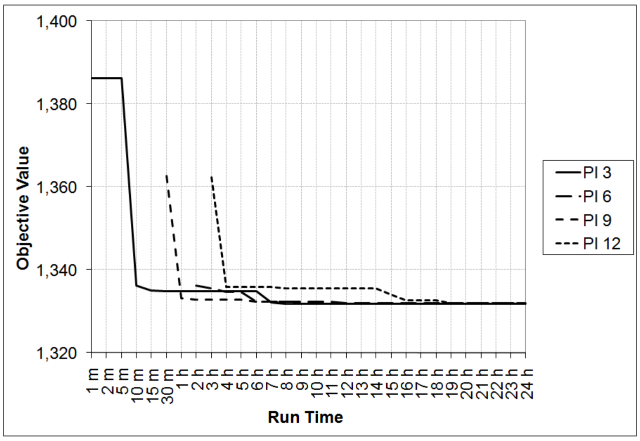
Figure 6.7: 4-index model run times with the first objective



(a) 75% truck usage

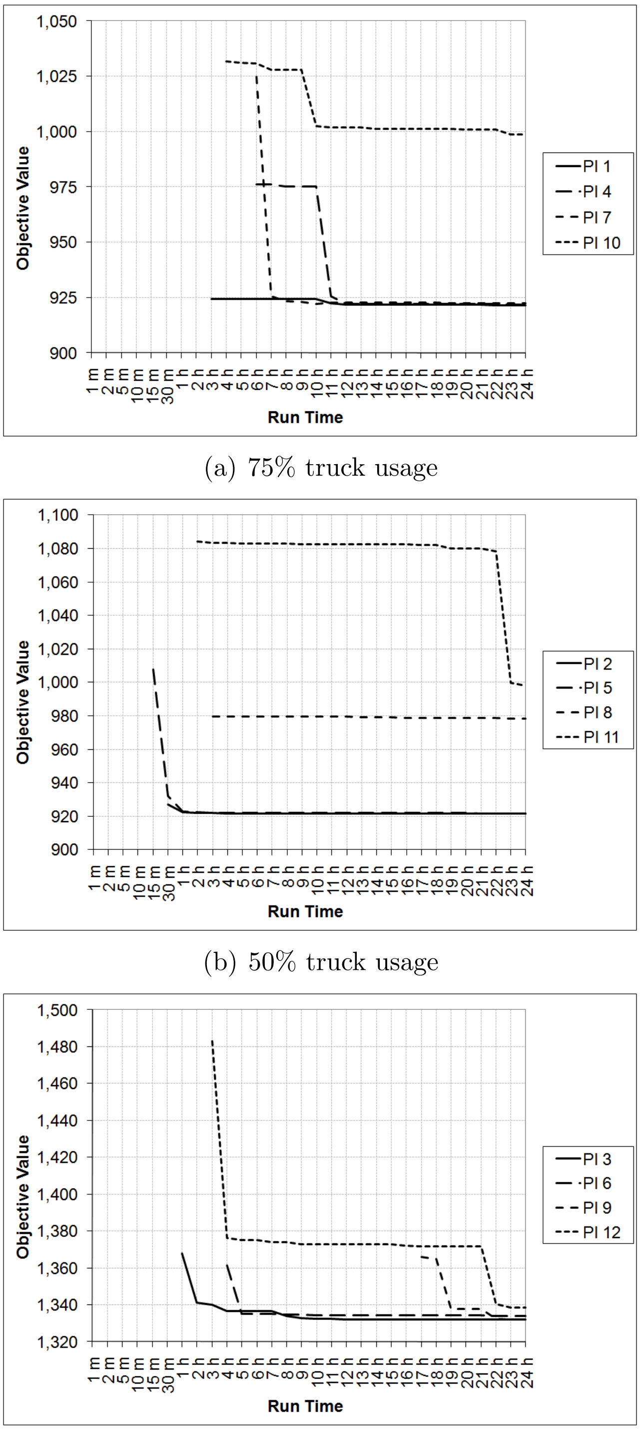


(b) 50% truck usage



(c) 83% truck usage

Figure 6.8: 4-index model run times with the second objective

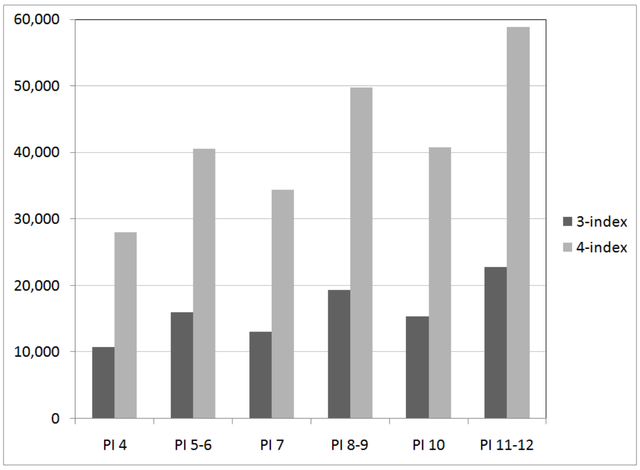


(a) 75% truck usage

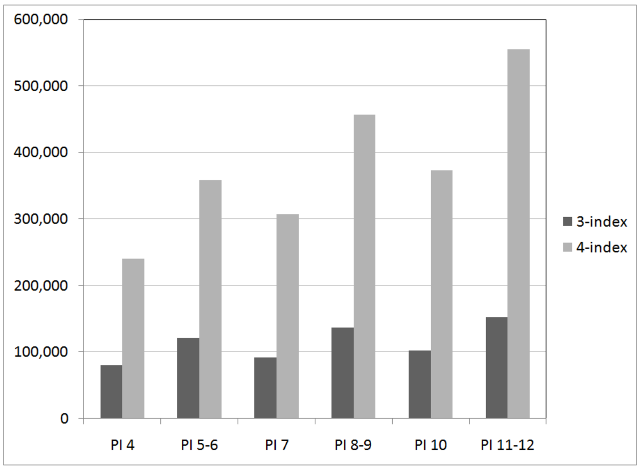
(b) 50% truck usage

(c) 83% truck usage

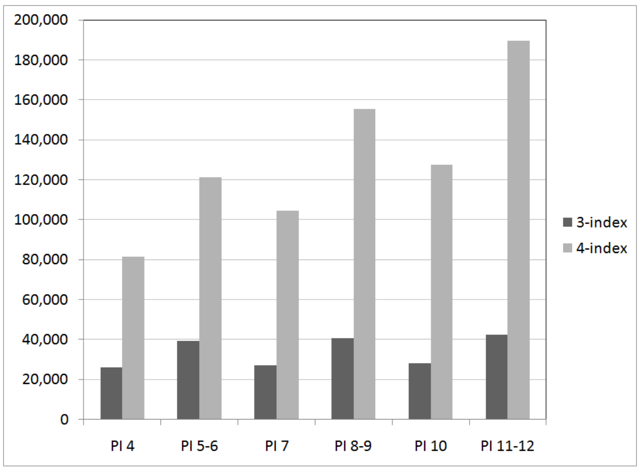
Figure 6.9: 3-index model run times with the second objective



(a) Number of single equations



(b) Number of nonzero elements



(c) Number of single variables

Figure 6.10: Comparison of the general statistics of 4-index and 3-index models

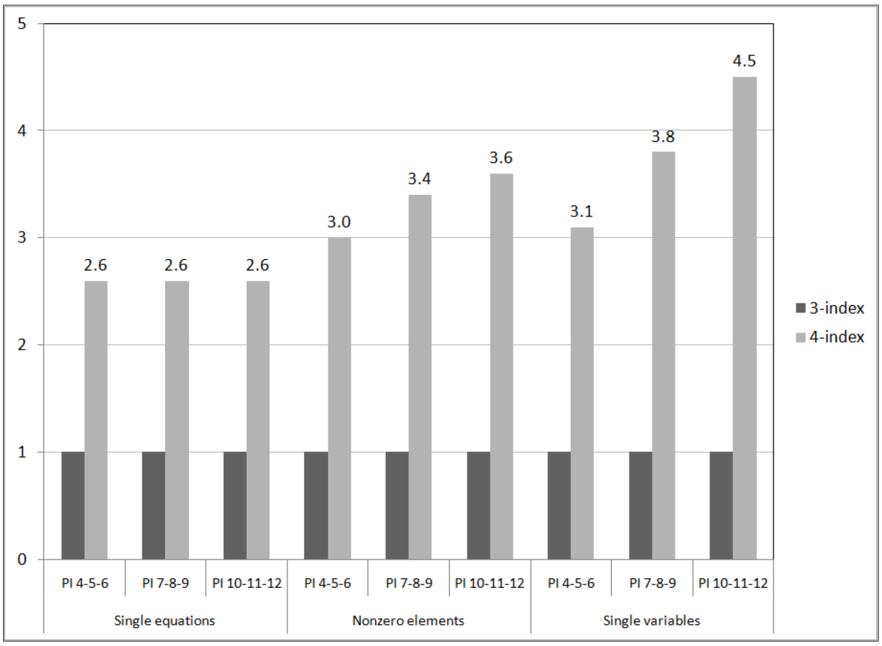


Figure 6.11: Comparison summary of the 4-index and 3-index models

Chapter 7

Heuristic Solution Methodology

In this chapter we present two heuristic methods of which the first is "VRP first-LRP second" and the second is "LRP first-VRP second" method to solve the Mobile-ADS design problem more efficiently. Broadly speaking, it can be said that the first method falls under the route first, location-allocation second and the second method falls under the location-allocation first, route second categories of Min et al. [59] with some differences. For example, in the second method we also do routing at the location-allocation phase.

Recently, Nagy and Salhi [64] classify LRP heuristics into four groups, namely; sequential, clustering-based, iterative and hierarchical methods. In general, all methods decompose an LRP into its major components, that is location, allocation and routing. Then, they solve these parts either repeatedly, iteratively or simultaneously.

In detail, sequential methods usually first solve a location problem to decide which depots to open and to allocate customers to open depots. Then, given the locations of the open depots a vehicle routing problem (VRP) is solved.

Clustering-based methods first group the customers into some clusters such that each cluster contains one potential depot or vehicle. Then, for each cluster a VRP is solved either after or before locating a depot.

Iterative methods usually construct two or more subproblems each one including one or two of the major components. Then, these subproblems are solved in a loop sequentially. Note that in the solution process a subproblem provides some input to the next subproblem in an iterative manner.

Hierarchical methods treat the location subproblem as the main problem and the routing subproblem as the subordinate problem that is embedded into the main problem. A hierarchical method then solves the location problem while in each step of the location problem it solves a routing problem which in turn provides information to the location problem.

Our solution approach in this study is a clustering-based heuristic according to this categorization. In general terms, we first partition all combat units into some clusters such that each cluster contains at least one potential Mobile-TP site. Then, in the "VRP first-LRP second" heuristic we solve a VRP for each cluster and using the solutions of VRPs we solve an LRP for the rest of the problem. Whereas, in the "LRP first-VRP second" heuristic, we solve an LRP and using the solution of this LRP we solve a VRP for each cluster. The details of these methods are as follows.

7.1 VRP first-LRP second heuristic

"VRP first-LRP second" heuristic consists of three phases. Phase 1 is the clustering part that partitions the combat units into clusters. Phase 2 is the VRP part that finds the routes of ammo trucks distributing ammo from Mobile-TPs to units. Phase 3 is the LRP part that decides the locations of the transfer points to open and the routes of commercial trucks distributing ammo from Fixed-TPs to Mobile-TPs. The flowchart for this method can be seen in Figure 7.1.

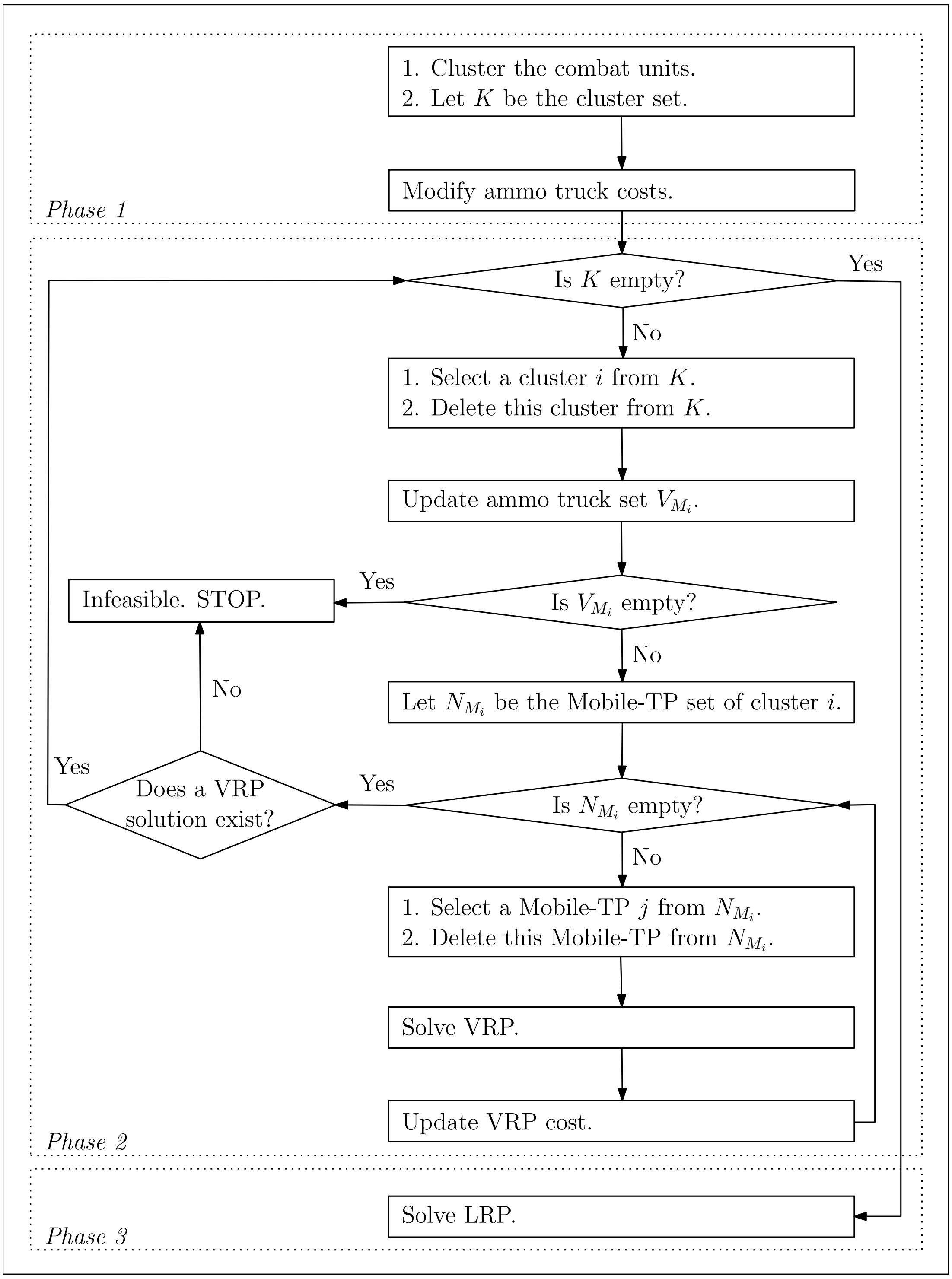


Figure 7.1: Flowchart for the VRP first-LRP second heuristic method

7.1.1 Phase 1. Clustering

In this phase we group all combat units into clusters such that for each cluster there exists at least one potential Mobile-TP, which can serve the total demand of clustered units within the specified time windows.

7.1.1.1 Step 1. Form the clusters

Because of the military requirements that are provided in section 6.1, it is clear that we already have these clusters. The second and the third requirements state that each brigade opens a single Mobile-TP and the units of that brigade can be served by only that transfer point. Hence, each brigade forms a cluster with at least one potential Mobile-TP. Since each unit belongs to a single brigade there exist as many distinct clusters as there are brigades. Let K be the cluster set and proceed to Step 2.

7.1.1.2 Step 2. Modify ammo truck costs

Each cluster contains at least one potential Mobile-TP meaning that it may contain two or more. Note that in the next phase a VRP will be solved for each potential Mobile-TP within each cluster. Consider that we have two clusters each having two potential Mobile-TPs. We first solve a VRP for the first Mobile-TP of the first cluster. This Mobile-TP can use any ammo truck in its solution as long as its feasible. Then, we solve another VRP for the second Mobile-TP of the same cluster. Since, only one of them will be opened in the end the second Mobile-TP can also use any ammo truck, too. In other words it can use the same ammo trucks that the first Mobile-TP used for the same cluster.

Next, we solve the VRP of the first Mobile-TP of the second cluster. This time, since we do not know yet which transfer point of the first cluster will be opened in the final solution, this transfer point can not use the ammo trucks that are used by either the first or the second transfer points of the first cluster. In addition, the second Mobile-TP of the second cluster can use the trucks that are used by the first transfer point but it can not use (like the first transfer point) the trucks that are used by the transfer points of the first cluster.

If not taken care of, this situation may cause false infeasibility. To solve this problem we must increase the probability of same truck usage of the Mobile-TPs of the same cluster. To do so, we modify the acquisition costs of ammo trucks (VC_v for all $v \in V_M$) in a decreasing/increasing fashion slightly such that every truck has a different cost. With the modified costs if using trucks a and b are less costly then it is more probable that transfer points of the same cluster will use them. Proceed to Step 3.

7.1.2 Phase 2. Vehicle routing problem (VRP)

In this phase we solve a VRP for each potential Mobile-TP complying with all of the original constraints.

7.1.2.1 Step 3. Select a cluster

If set K is empty this means that all clusters have been processed already and we are ready to proceed to the next phase, hence go to Step 10. Otherwise, select a cluster , i, delete it from K and proceed to Step 4.

7.1.2.2 Step 4. Update ammo truck set (V_M)

In this step we try to increase computational efficiency. To do so we make an additional (not unrealistic though) assumption that all combat units require less than truck loads (LTL). If this is the case, then a Mobile-TP dispatches at most one truck for each unit of its cluster. Hence, the number of trucks that are used by any Mobile-TP can be bounded above by the number of units of its cluster. In other words, we can modify the ammo truck set for each cluster such that it includes exactly the same number of trucks, which are not used by any Mobile-TP of the previously solved cluster, as the number of units in the cluster. To do so we employ the following procedure. Note that $|V_{M_i}|$ is the cardinality of this set and $|\text{cluster } i|$ is the number of units in this cluster. Let V_{M_i} be the modified ammo truck set that will be used by the potential Mobile-TPs of cluster i.

let $V_{M_i} = \emptyset$

for $(v \in V_M)$ do

while $(|V_{M_i}| \leq |\text{cluster } i|)$ do

if $v \notin \cap_{k \in K: i \neq k} V_{M_k}$

let $v \in V_{M_i}$

end while

end for

Proceed to Step 5.

7.1.2.3 Step 5. Check infeasibility

If set V_{M_i} is empty this means that there is no unused ammo truck left for that cluster to dispatch and the problem is infeasible, hence STOP. Otherwise, proceed to Step 6.

7.1.2.4 Step 6. Select a potential Mobile-TP

Let N_{M_i} be the set of all potential Mobile-TPs of cluster i. If set N_{M_i} is empty this means that a VRP for all potential Mobile-TPs of that cluster is solved and nothing is remained to be processed, hence proceed to Step 7. Otherwise, select a Mobile-TP, j^*, delete it from N_{M_i} and go to Step 8.

7.1.2.5 Step 7. Check infeasibility

If no VRP has a feasible solution this means that the demands of the units of this cluster can not be satisfied in the given problem setting according to the specified constraints. In other words the problem is infeasible, hence STOP. Otherwise, if at least one feasible solution exists for the VRP of a Mobile-TP then this means that we processed all potential Mobile-TPs of this cluster and no transfer point is remained to be solved a VRP for. Then we are ready to process a new cluster, hence go to Step 3.

7.1.2.6 Step 8. Solve VRP

In this step we are going to solve a VRP for cluster i and Mobile-TP j^* using vehicle set V_{M_i}. Let N_{C_i} be the set of all combat units of cluster i and $N^*_{C_i} = N_{C_i} \bigcup j^*$. In other words, $N^*_{C_i}$ includes all combat units of cluster i and Mobile-TP j^*. The VRP to be solved in this step is presented below.

4-index VRP

If we want to use the 4-index model, then we are going to solve the following model in this step.

$$\min \quad o1 \text{ or } o2$$

s.t.

$$\sum_{v \in V_{M_i}} \left(\sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} f_{hgvp} - \sum_{\substack{r \in N_{C_i} \\ r \neq g}} f_{grvp} \right) = Q_{gp} \quad \forall g \in N_{C_i}, p \in P \tag{7.1}$$

$$\sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} f_{hgvp} \geq \sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} f_{ghvp} \quad \forall g \in N_{C_i}, v \in V_{M_i}, p \in P \tag{7.2}$$

$$\sum_{g \in N_{C_i}} x_{j^*gv} \leq 1 \quad \forall v \in V_{M_i} \tag{7.3}$$

$$\sum_{g \in N_{C_i}} x_{j^*gv} = \sum_{g \in N_{C_i}} x_{gj^*v} \quad \forall v \in V_{M_i} \tag{7.4}$$

$$\sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} x_{ghv} = \sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} x_{hgv} \quad \forall g \in N_{C_i}, v \in V_{M_i} \tag{7.5}$$

$$\sum_{v \in V_{M_i}} \sum_{g \in N_{C_i}} f_{j^*gvp} \leq CD_{j^*p} \quad \forall p \in P \tag{7.6}$$

$$f_{j^*gvp} \leq CV_{vp} \cdot x_{j^*gv} \quad \forall g \in N_{C_i}, v \in V_{M_i}, p \in P \tag{7.7}$$

$$f_{ghvp} \leq CV_{vp} \cdot x_{ghv} \quad \forall g, h \in N_{C_i}, g \neq h, v \in V_{M_i}, p \in P \tag{7.8}$$

$$\sum_{p \in P} f_{j^*gvp} \leq CT_v \cdot x_{j^*gv} \quad \forall g \in N_{C_i}, v \in V_{M_i} \tag{7.9}$$

$$\sum_{p \in P} f_{ghvp} \leq CT_v \cdot x_{ghv} \quad \forall g, h \in N_{C_i}, g \neq h, v \in V_{M_i} \tag{7.10}$$

$$\sum_{p \in P} f_{j^*gvp} \geq x_{j^*gv} \quad \forall g \in N_{C_i}, v \in V_{M_i} \tag{7.11}$$

$$\sum_{p \in P} f_{ghvp} \geq x_{ghv} \quad \forall g, h \in N_{C_i}, g \neq h, v \in V_{M_i} \tag{7.12}$$

$$\sum_{v \in V_{M_i}} f_{j^*gvp} \geq w_{j^*gp} \quad \forall g \in N_{C_i}, p \in P \tag{7.13}$$

$$\sum_{v \in V_{M_i}} f_{ghvp} \geq w_{ghp} \quad \forall g, h \in N_{C_i}, g \neq h, p \in P \tag{7.14}$$

$$\left(\sum_{h \in N_{C_i}} Q_{hp} \right) \cdot w_{j^*gp} \geq f_{j^*gvp} \quad \forall g \in N_{C_i}, v \in V_{M_i}, p \in P \tag{7.15}$$

$$\left(\sum_{h \in N_{C_i}} Q_{hp} \right) \cdot w_{grp} \geq f_{grvp} \quad \forall g, r \in N_{C_i}, g \neq r, v \in V_{M_i}, p \in P \tag{7.16}$$

$$tp_{gp} \geq TE_{gp} \quad \forall g \in N_{C_i}, p \in P \tag{7.17}$$

$$tp_{gp} \leq TL_{gp} \quad \forall g \in N_{C_i}, p \in P \tag{7.18}$$

$$tp_{j^*p} = \min_{g \in N_F} \{TI_{gj^*}\} \quad \forall p \in P \tag{7.19}$$

$$tp_{j^*p} + TI_{j^*g} \cdot w_{j^*gp} - TM_p \cdot (1 - w_{j^*gp}) \leq tp_{gp} \quad \forall g \in N_{C_i}, p \in P \tag{7.20}$$

$$tp_{gp} + TI_{gh} \cdot w_{ghp} - TM_p \cdot (1 - w_{ghp}) \leq tp_{hp} \quad \forall g, h \in N_{C_i}, g \neq h, p \in P \tag{7.21}$$

$$tv_{j^*v} + TI_{j^*g} \cdot x_{j^*gv} - TM \cdot (1 - x_{j^*gv}) \leq tv_{gv} \quad \forall g \in N_{C_i}, v \in V_{M_i} \tag{7.22}$$

$$tv_{gv} + TI_{gh} \cdot x_{ghv} - TM \cdot (1 - x_{ghv}) \leq tv_{hv} \quad \forall g, h \in N_{C_i}, g \neq h, v \in V_{M_i} \tag{7.23}$$

$$\sum_{g \in N_{C_i}} x_{j^*gv} \leq 1 \quad \forall v \in V_{M_i} \tag{7.24}$$

$$\sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} x_{ghv} \leq 1 \quad \forall g \in N_{C_i}, v \in V_{M_i} \tag{7.25}$$

$$f_{ghvp} \geq 0 \quad \forall g, h \in N^*_{C_i}, g \neq h, v \in V_{M_i}, p \in P \tag{7.26}$$

$$tp_{gp} \geq 0 \quad \forall g \in N^*_{C_i}, p \in P \tag{7.27}$$

$$tv_{gv} \geq 0 \quad \forall g \in N^*_{C_i}, v \in V_{M_i} \tag{7.28}$$

$$x_{ghv} \in \{0,1\} \quad \forall g, h \in N^*_{C_i}, g \neq h, v \in V_{M_i} \tag{7.29}$$

$$w_{ghp} \in \{0,1\} \quad \forall g, h \in N^*_{C_i}, g \neq h, p \in P \tag{7.30}$$

where

$$o1 = \sum_{g \in N_{C_i}} \sum_{v \in V_{M_i}} VC_v \cdot x_{j^*gv} \tag{7.31}$$

$$+ \sum_{g \in N^*_{C_i}} \sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} \sum_{v \in V_{M_i}} \sum_{p \in P} TC_{vp} \cdot TI_{gh} \cdot f_{ghvp} \tag{7.32}$$

$$o2 = \sum_{g \in N_{C_i}} \sum_{v \in V_{M_i}} VC_v \cdot x_{j^*gv} \tag{7.33}$$

$$+ \sum_{g \in N^*_{C_i}} \sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} \sum_{v \in V_{M_i}} DC_{vp} \cdot TI_{gh} \cdot x_{ghv} \tag{7.34}$$

3-index VRP

If we want to use the 3-index model, then we are going to solve the following model in this step.

$$\min \quad o2$$

s.t.

$$\sum_{v \in V_{M_i}} cuin_{gvp} = Q_{gp} \quad \forall g \in N_{C_i}, p \in P \tag{7.35}$$

$$mtpout_{j^*vp} = \sum_{g \in N_{C_i}} cuin_{gvp} \quad \forall v \in V_{M_i}, p \in P \tag{7.36}$$

$$\sum_{g \in N_{C_i}} x_{j^*gv} \leq 1 \quad \forall v \in V_{M_i} \tag{7.37}$$

$$\sum_{g \in N_{C_i}} x_{j^*gv} = \sum_{g \in N_{C_i}} x_{gj^*v} \quad \forall v \in V_{M_i} \tag{7.38}$$

$$\sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} x_{ghv} = \sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} x_{hgv} \quad \forall g \in N_{C_i}, v \in V_{M_i} \tag{7.39}$$

$$\sum_{v \in V_{M_i}} mtpout_{j^*vp} \leq CD_{j^*p} \quad \forall p \in P \tag{7.40}$$

$$\sum_{g \in N_{C_i}} cuin_{gvp} \leq CV_{vp} \cdot \sum_{g \in N_{C_i}} x_{j^*gv} \quad \forall v \in V_{M_i}, p \in P \tag{7.41}$$

$$mtpout_{j^*vp} \leq CV_{vp} \cdot \sum_{g \in N_{C_i}} x_{j^*gv} \quad \forall v \in V_{M_i}, p \in P \tag{7.42}$$

$$cuin_{gvp} \leq CV_{vp} \cdot \sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} x_{hgv} \quad \forall g \in N_{C_i}, v \in V_{M_i}, p \in P \tag{7.43}$$

$$\sum_{p \in P} mtpout_{j^*vp} \leq CT_v \cdot \sum_{g \in N_{C_i}} x_{j^*gv} \quad \forall v \in V_{M_i} \tag{7.44}$$

$$\sum_{g \in N_{C_i}} x_{j^*gv} \leq \sum_{p \in P} mtpout_{j^*vp} \quad \forall v \in V_{M_i} \tag{7.45}$$

$$\sum_{\substack{h \in N^*_{C_i} \\ h \neq g}} x_{hgv} \leq \sum_{p \in P} cuin_{gvp} \quad \forall g \in N_{C_i}, v \in V_{M_i} \tag{7.46}$$

$$k_{j^*vp} \leq mtpout_{j^*vp} \quad \forall v \in V_{M_i}, p \in P \tag{7.47}$$

$$k_{gvp} \leq cuin_{gvp} \quad \forall g \in N_{C_i}, v \in V_{M_i}, p \in P \tag{7.48}$$

$$mtpout_{j^*vp} \leq CV_{vp} \cdot k_{j^*vp} \quad \forall v \in V_{M_i}, p \in P \tag{7.49}$$

$$cuin_{gvp} \leq CV_{vp} \cdot k_{gvp} \quad \forall g \in N_{C_i}, v \in V_{M_i}, p \in P \tag{7.50}$$

$$tp_{gp} \geq TE_{gp} \quad \forall g \in N_{C_i}, p \in P \tag{7.51}$$

$$tp_{gp} \leq TL_{gp} \quad \forall g \in N_{C_i}, p \in P \tag{7.52}$$

$$tp_{j^*p} = \min_{g \in N_F} \{TI_{gj^*}\} \quad \forall p \in P \tag{7.53}$$

$$tv_{j^*v} + TI_{j^*g} \cdot x_{j^*gv} - TM \cdot (1 - x_{j^*gv}) \leq tv_{gv} \quad \forall g \in N_{C_i}, v \in V_{M_i} \tag{7.54}$$

$$tv_{hv} + TI_{hg} \cdot x_{hgv} - TM \cdot (1 - x_{hgv}) \leq tv_{gv} \quad \forall g, h \in N_{C_i}, g \neq h, v \in V_{M_i} \tag{7.55}$$

$$tp_{j^*p} - TM_p \cdot (1 - k_{j^*vp}) \leq tv_{j^*v} \quad \forall v \in V_{M_i}, p \in P \tag{7.56}$$

$$tv_{gv} - TM_p \cdot (1 - k_{gvp}) \leq tp_{gp} \quad \forall g \in N_{C_i}, v \in V_{M_i}, p \in P \tag{7.57}$$

$$mtpout_{j^*vp} \geq 0 \quad \forall v \in V_{M_i}, p \in P \tag{7.58}$$

$$cuin_{gvp} \geq 0 \quad \forall g \in N_{C_i}, v \in V_{M_i}, p \in P \tag{7.59}$$

$$tp_{gp} \geq 0 \quad \forall g \in N^*_{C_i}, p \in P \tag{7.60}$$

$$tv_{gv} \geq 0 \quad \forall g \in N^*_{C_i}, v \in V_{M_i} \tag{7.61}$$

$$x_{ghv} \in \{0,1\} \quad \forall g, h \in N^*_{C_i}, g \neq h, v \in V_{M_i} \tag{7.62}$$

$$k_{gvp} \in \{0,1\} \quad \forall g \in N^*_{C_i}, v \in V_{M_i}, p \in P \tag{7.63}$$

Note that in realistic problem instances a brigade has 4 or 5 combat units meaning that each cluster contains at most 5 units, that is $|N_{C_i}| \leq 6$ for all $i \in K$. Hence, the VRP in this step contains a single depot and at most 5 customers. We assume that such a problem size can be solved in a reasonable amount of time with either 4-index or 3-index model.

7.1.2.7 Step 9. Update VRP cost

A careful examination reveals two differences between $z1$ and $o1$ and $z2$ and $o2$. The first difference is the fixed opening costs of Mobile-TPs. $z1$ includes these costs but $o1$ does not, since there is only one transfer point in each VRP of Step 8 and this transfer point is already considered open. The second difference is the vehicle acquisition costs. Remember that ammo truck costs are modified in Step 2 and VRP in Step 8 considers these modified costs rather than the actual costs. Hence to get the real total cost and to compare it with the cost of the original formulation we need to modify the VRP costs as follows.

Firstly, add the fixed cost of Mobile-TP j^* (FC_{j^*}) to the VRP cost. Secondly, add the difference between the actual and modified cost of each used ammo truck to the VRP cost. Let ε_v represent the difference between the actual and modified cost of ammo truck v. Then, we need to update the VRP costs as follows.

$$o1^* = o1 + FC_{j^*} + \sum_{g \in N_{C_i}} \sum_{v \in V_{M_i}} \varepsilon_v \cdot x_{j^*gv} \tag{7.64}$$

$$o2^* = o2 + FC_{j^*} + \sum_{g \in N_{C_i}} \sum_{v \in V_{M_i}} \varepsilon_v \cdot x_{j^*gv} \tag{7.65}$$

These modified costs are the actual costs and will be used in the LRP that will be explained in Phase 3. Since we are done with Mobile-TP j^* go to Step 6.

7.1.3 Phase 3. Location routing problem (LRP)

In the first two phases we determine the routes and schedules of ammo trucks that distribute ammo from each Mobile-TP to the units of the cluster where its home Mobile-TP belongs to. In other words, until now we are interested in the distribution from Mobile-TPs to units. In this phase we will be interested in the distribution network design and the distribution from Fixed-TPs to Mobile-TPs using commercial trucks. By distribution network design we mean that we are going to decide which Mobile-TPs and Fixed-TPs to open. Since, both location and routing decisions exist in this phase we will solve an LRP model including all Fixed-TPs and Mobile-TPs and complying with all of the original constraints.

7.1.3.1 Step 10. Solve LRP

In this step no combat unit exists and we solve a two layer LRP in which Fixed-TPs lie on the first and Mobile-TPs lie on the second layer. We need to decide (1) which Fixed-TP and Mobile-TP (one for each cluster) to open and (2) routes of the commercial trucks among open transfer points.

Let $o1^*_j$ and $o2^*_j$ represent the $o1^*$ and $o2^*$ of Mobile-TP j. For example, $o1^*_j$ is the fixed cost of opening Mobile-TP j, the acquisition cost of the ammo trucks that are used by Mobile-TP j and the distribution cost of ammo to units from Mobile-TP j. Hence, by using $o1^*$ and $o2^*$ in this step we incorporate the VRP into the LRP as a simple cost parameter that will be used in the objective function.

4-index LRP

If we want to use the 4-index model, then we are going to solve the following model in this step.

$$\min \quad s1 \quad \text{or} \quad s2$$

s.t.

$$(3.2a), (3.3a), (3.4a), (3.5a), (3.6a), (3.11a), (3.14), (V8b)$$

$$(3.7) \quad \forall i \in N_F, j \in N_M, v \in V_F, p \in P; \forall i, j \in N_M, i \neq j, v \in V_F, p \in P$$

$$(3.8) - (3.9) \quad \forall i \in N_F, j \in N_M, v \in V_F; \forall i, j \in N_M, i \neq j, v \in V_F$$

$$(3.10) \quad \forall i \in N_F, j \in N_M, v \in V_F, p \in P; \forall i, j \in N_M, i \neq j, v \in V_F, p \in P$$

$$(3.15) \quad \forall i \in N_F, j \in N_M, p \in P; \forall i, j \in N_M, i \neq j, p \in P$$

$$(3.16) \quad \forall i \in N_F, j \in N_M, v \in V_F; \forall i, j \in N_M, i \neq j, v \in V_F$$

$$\sum_{v \in V_F} \left(\sum_{\substack{j \in N_{FM} \\ j \neq i}} f_{jivp} - \sum_{\substack{j \in N_M \\ j \neq i}} f_{ijvp} \right) = \sum_{j \in N_{C_i}} Q_{jp} \cdot y_i \quad \forall i \in N_M, p \in P \tag{7.66}$$

$$tp_{ip} \leq \min_{g \in N_{C_i}} \{TL_{gp} - TI_{ig}\} \quad \forall i \in N_M, p \in P \tag{7.67}$$

$$\sum_{j \in N_{M_i}} y_j = 1 \quad \forall i \in K \tag{7.68}$$

$$f_{ijvp} \geq 0 \quad \forall i, j \in N_{FM}, i \neq j, v \in V_F, p \in P \tag{7.69}$$

$$tp_{ip} \geq 0 \quad \forall i \in N_{FM}, p \in P \tag{7.70}$$

$$tv_{iv} \geq 0 \quad \forall i \in N_{FM}, v \in V_F \tag{7.71}$$

$$x_{ijv} \in \{0, 1\} \quad \forall i, j \in N_{FM}, i \neq j, v \in V_F \tag{7.72}$$

$$w_{ijp} \in \{0, 1\} \quad \forall i, j \in N_{FM}, i \neq j, p \in P \tag{7.73}$$

$$y_i \in \{0, 1\} \quad \forall i \in N_{FM} \tag{7.74}$$

where

$$s1 = \sum_{i \in N_F} FC_i \cdot y_i \tag{7.75}$$

$$+ \sum_{i \in N_F} \sum_{j \in N_M} \sum_{v \in V_F} VC_v \cdot x_{ijv} \tag{7.76}$$

$$+ \sum_{i \in N_{FM}} \sum_{j \in N_{FM}} \sum_{v \in V_F} \sum_{p \in P} TC_{vp} \cdot TI_{ij} \cdot f_{ijvp} \tag{7.77}$$

$$+ \sum_{i \in N_M} o1_i^* \cdot y_i \tag{7.78}$$

$$s2 = \sum_{i \in N_F} FC_i \cdot y_i \tag{7.79}$$

$$+ \sum_{i \in N_F} \sum_{j \in N_M} \sum_{v \in V_F} VC_v \cdot x_{ijv} \tag{7.80}$$

$$+ \sum_{i \in N_{FM}} \sum_{j \in N_{FM}} \sum_{v \in V_F} DC_v \cdot TI_{ij} \cdot x_{ijv} \tag{7.81}$$

$$+ \sum_{i \in N_M} o2_i^* \cdot y_i \tag{7.82}$$

$$\tag{7.83}$$

3-index LRP

If we want to use the 3-index model, then we are going to solve the following model in this step.

$$\min \quad s2$$

s.t.

$(3.3a), (3.4a), (3.5a), (4.2a), (4.4a), (4.5a), (4.6a), (4.7a), (4.7b), (3.14)$

$(4.8a), (4.8c), (4.9a), (4.10a), (7.67), (7.68), (V8b)$

$(3.16) \quad \forall i \in N_F, j \in N_M, v \in V_F; \forall i, j \in N_M, i \neq j, v \in V_F$

$(4.12) \quad \forall i \in N_M, v \in V_F, p \in P$

$$\sum_{v \in V_F} mtpin_{ivp} = \sum_{j \in N_{C_i}} Q_{jp} \cdot y_i \quad \forall i \in N_M, p \in P \tag{7.84}$$

$$ftpout_{ivp} \geq 0 \quad \forall i \in N_F, v \in V_F, p \in P \tag{7.85}$$

$$mtpin_{ivp} \geq 0 \quad \forall i \in N_M, v \in V_F, p \in P \tag{7.86}$$

$$tp_{ip} \geq 0 \quad \forall i \in N_{FM}, p \in P \tag{7.87}$$

$$tv_{iv} \geq 0 \quad \forall i \in N_{FM}, v \in V_F \tag{7.88}$$

$$x_{ijv} \in \{0, 1\} \quad \forall i, j \in N_{FM}, i \neq j, v \in V_F \tag{7.89}$$

$$k_{ivp} \in \{0, 1\} \quad \forall i \in N_{FM}, v \in V_F, p \in P \tag{7.90}$$

7.2 LRP first-VRP second heuristic

"LRP first-VRP second" is a three phase heuristic method. Phase 1 is the clustering part that partitions the combat units into clusters. Phase 2 is the location and routing part that decides the locations of the transfer points to open and the routes of commercial trucks distributing ammo from Fixed-TPs to Mobile-TPs. Phase 3 is the routing part that finds the routes of ammo trucks distributing ammo from Mobile-TPs to units. The flowchart for this method can be seen in Figure 7.2.

7.2.1 Phase 1. Clustering

In this phase we group all combat units into clusters such that each cluster includes at least one potential Mobile-TP and this Mobile-TP can serve the total demand of units within the specified time windows.

7.2.1.1 Step 1. Form the clusters

This step is the same as Step 1 of the first heuristic method.

7.2.2 Phase 2. Location routing problem (LRP)

In this phase, as in the Phase 3 of the first heuristic, we will be interested in the distribution network design and the distribution from Fixed-TPs to Mobile-TPs using commercial trucks. In other words, we are going to decide which Mobile-TPs and Fixed-TPs to open and how ammo will be distributed from open Fixed-TPs to open Mobile-TPs using commercial trucks.

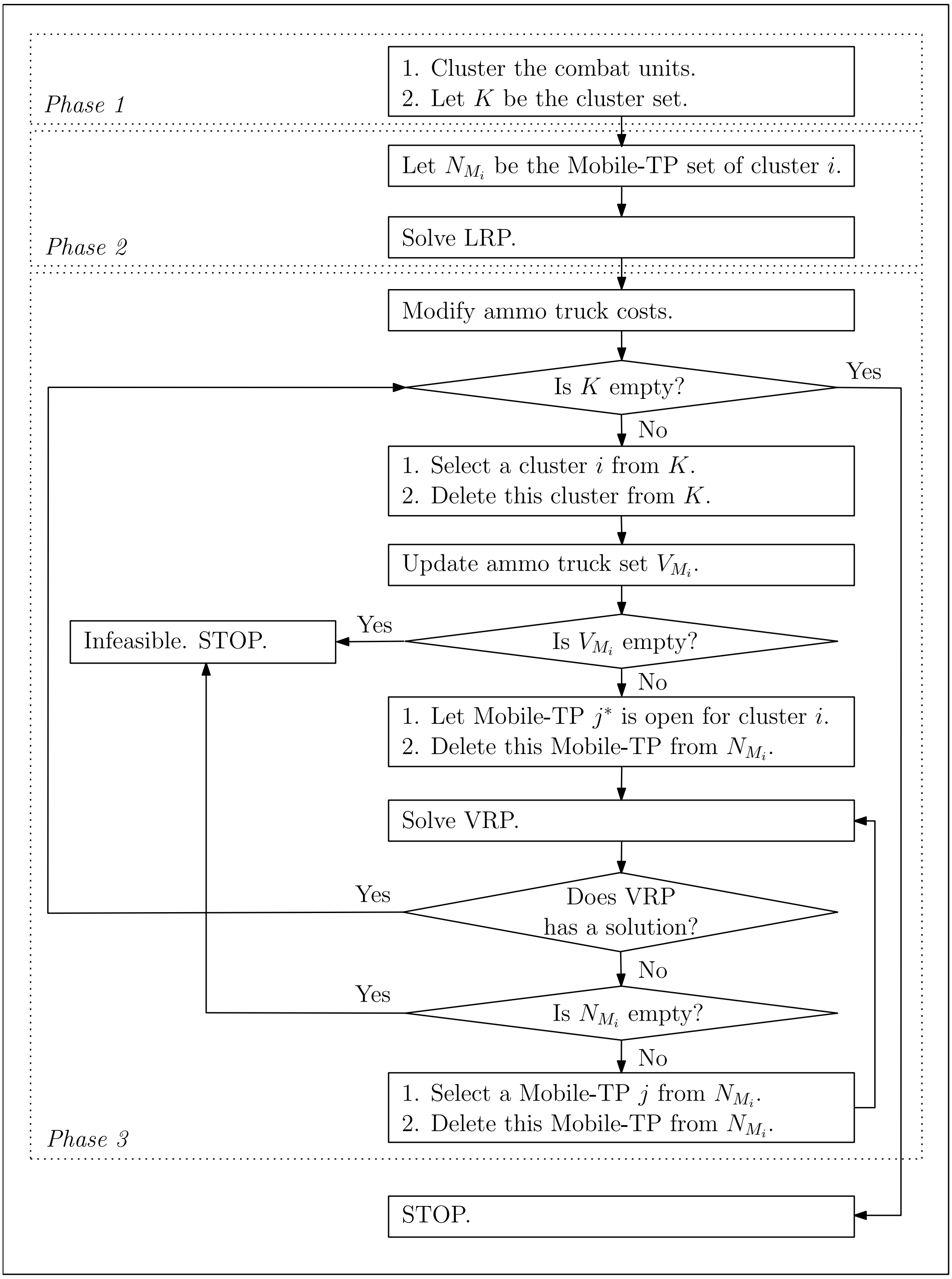


Figure 7.2: Flowchart for the LRP first-VRP second heuristic method

7.2.2.1 Step 2. Solve LRP

In this step, we solve an LRP model similar (except the objective functions) to that of the first heuristic to decide (1) which Fixed-TP and Mobile-TP (one for each cluster) to open and (2) routes of the commercial trucks among open transfer points. Let N_{M_i} be the set of all potential Mobile-TPs of cluster i. The LRP model to be solved in this step is presented below.

4-index LRP

If we want to use the 4-index model, then we are going to solve the following model.

$$\begin{aligned} \min \quad & w1 \quad \text{or} \quad w2 \\ \text{s.t.} \quad & \text{the same constraint set of 4-index LRP of the first heuristic.} \end{aligned}$$

where

$$w1 = \sum_{i \in N_{FM}} FC_i \cdot y_i \tag{7.91}$$

$$+ \sum_{i \in N_F} \sum_{j \in N_M} \sum_{v \in V_F} VC_v \cdot x_{ijv} \tag{7.92}$$

$$+ \sum_{i \in N_{FM}} \sum_{j \in N_{FM}} \sum_{v \in V_F} \sum_{p \in P} TC_{vp} \cdot TI_{ij} \cdot f_{ijvp} \tag{7.93}$$

$$w2 = \sum_{i \in N_{FM}} FC_i \cdot y_i \tag{7.94}$$

$$+ \sum_{i \in N_F} \sum_{j \in N_M} \sum_{v \in V_F} VC_v \cdot x_{ijv} \tag{7.95}$$

$$+ \sum_{i \in N_{FM}} \sum_{j \in N_{FM}} \sum_{v \in V_F} DC_v \cdot TI_{ij} \cdot x_{ijv} \tag{7.96}$$

3-index LRP

If we want to use the 3-index model, then we are going to solve the following model in this step.

$$\min \quad w2$$

s.t. the same constraint set of 3-index LRP of the first heuristic.

Note that the only difference between the LRP models of the first and the second heuristics are in the objective functions. In the second method we do not solve any VRP for any Mobile-TP before solving LRP. Hence, we do not have any knowledge about the distribution system beyond Mobile-TPs. The only thing we know about the combat units is their demand. Thus, in the LRP model we open one Mobile-TP for each cluster and we try to send each open Mobile-TP an amount equal to the total demand of combat units that belong to the same cluster. That is why we include the fixed opening costs of Mobile-TPs into the objective function. In other words in the LRP model we consider fixed opening costs of transfer points, commercial truck acquisition costs and distribution among transfer points.

7.2.3 Phase 3. Vehicle routing problem (VRP)

In this phase we solve a VRP for each open Mobile-TP complying with all of the original constraints.

7.2.3.1 Step 3. Modify ammo truck costs

This step is the same as Step 2 of the first heuristic method.

7.2.3.2 Step 4. Select a cluster

If set K is empty this means that all clusters have been processed already and we are done, hence STOP. Otherwise, select a cluster , i, delete it from K and proceed to Step 5.

7.2.3.3 Step 5. Update ammo truck set (V_M)

This step is the same as Step 4 of the first heuristic method.

7.2.3.4 Step 6. Check infeasibility

If set V_{M_i} is empty this means that there is no unused ammo truck left for that cluster to dispatch and the problem is infeasible, hence STOP. Otherwise, proceed to Step 7.

7.2.3.5 Step 7. Select the open Mobile-TP

There exists exactly one open Mobile-TP for cluster i in the LRP solution and let Mobile-TP j^* be this one. Select this Mobile-TP, j^*, delete it from N_{M_i} and proceed to Step 8.

7.2.3.6 Step 8. Solve VRP

In this step we are going to solve exactly the same VRP model of the first heuristic model for cluster i and Mobile-TP j^* using vehicle set V_{M_i} to determine the routes of ammo trucks distributing ammo to combat units. The VRP to be solved in this step is presented below.

4-index VRP

If we want to use the 4-index model, then we are going to solve the following model in this step.

$$\begin{aligned} \min \quad & o1 \;\text{ or }\; o2 \\ \text{s.t.} \quad & \text{the same constraint set of 4-index VRP of the first heuristic.} \end{aligned}$$

where constraint (7.19) is changed as shown below and $\widehat{y}_g$ is the value of the decision variable y_i for all $i \in N_F$ in the LRP solution.

$$tp_{j^*p} = \max_{g \in N_F} \{TI_{gj^*} \cdot \widehat{y}_g\} \quad \forall p \in P \tag{7.97}$$

3-index VRP

If we want to use the 3-index model, then we are going to solve the following model in this step.

$$\begin{aligned} \min \quad & o2 \\ \text{s.t.} \quad & \text{the same constraint set of 3-index VRP of the first heuristic.} \end{aligned}$$

where constraint (7.53) is replaced with (7.97).

7.2.3.7 Step 9. Check VRP solution

If VRP does not have a feasible solution this means that the demands of the units of this cluster can not be satisfied in the given problem setting according to the specified constraints from Mobile-TP j^*. Then we need to check a new Mobile-TP, hence proceed to Step 10. Otherwise, if VRP has a feasible solution with Mobile-TP j^* then this means that we are ready to process a new cluster, hence go to Step 4.

7.2.3.8 Step 10. Check infeasibility

If no Mobile-TP is left to be solved a VRP for, this means that the demands of the units of this cluster can not be satisfied in the given problem setting according to the specified constraints from any Mobile-TP of this cluster. In other words the problem is infeasible, hence STOP. Otherwise, select a Mobile-TP j from N_{M_i} and delete it from N_{M_i}. Go to Step 8.

Chapter 8

Computational Experiments (Part III)

This chapter compares the performances of the two heuristic approaches that are introduced in Chapter 7 by using the same 12 problem instances.

Remember that we introduce 12 problem instances in Section 6.2 that are developed according to their ammo type numbers ($|P| = 2, 3, 4, 5$) and truck usage percentages ($75\%, 50\%, 83\%$). A summary of the instances can be found in Figure 6.10. In the 75% group 18 trucks are dispatched out of 24 trucks, in the 50% group 18 trucks are dispatched out of 36 trucks, and in the 83% group 30 trucks are dispatched out of 36 trucks.

8.1 4-index model

8.1.1 VRP first-LRP second heuristic

Table 8.1 demonstrates the run times of heuristic 1 for each problem instance with the first objective. To calculate the run times of heuristic 1 we introduce the following rule. Remember that in the first heuristic, that is "VRP first-LRP second" heuristic, we solve a VRP for each MTP and using the solutions of these VRPs we solve a single LRP for the rest of the problem. Remember also that in each problem instance we have 8 MTPs. Hence, in heuristic 1 we solve eight VRPs and one LRP for each problem instance. We are interested in the time needed for heuristic 1 to reach the objective value of the original model after 24 hours. To be exact, for example we run heuristic 1 with the first objective for problem instance 1 until it reaches a cost of 1301.84. Note that Table 6.5 shows corresponding costs of problem instances

To calculate the run time we also introduce the following procedure. We allow each VRP to run for at most 60 seconds. In other words, each VRP is terminated after 60 seconds if it can not find the optimum until that time. For example, in problem instance 1, VRP for MTP 1 finds the optimum in 6.7 seconds, but VRP for MTP 3 is terminated after 60 seconds at a feasible (may be sub-optimal) solution, before it finds the optimum.

After running all 8 VRPs in this manner, we run the LRP until it reaches to the objective value that is obtained by the original model in 24 hours. The LRP is terminated after 3600 seconds if it can not reach to it. For example, in problem instance 1, LRP for FTPs reaches a cost of 1301.84 in 2 seconds. The total run time for heuristic 1 for each problem instance is the sum of the run times of 8 VRPs plus the run time of the single LRP. For example, total run time of heuristic 1 for problem instance 1 is 195.7 seconds.

We can summarize the running procedure of heuristic 1 as follows:

- Run 8 VRPs either to completion or at most 60 seconds,
- Run a single LRP until it reaches to the objective value that is obtained by the original model in 24 hours or at most 3600 seconds,
- Total run time is the sum of the run times of all 8 VRPs and that of the single LRP.

Table 8.2 demonstrates the run times of heuristic 1 for each problem instance with the second objective.

Table 8.1: Run times (seconds) of heuristic 1 with the first objective

	Problem instances											
	PI1	PI2	PI3	PI4	PI5	PI6	PI7	PI8	PI9	PI10	PI11	PI12
MTP 1	6.7	10.3	60.0	16.1	9.8	60.0	27.3	19.0	60.0	60.0	60.0	60.0
MTP 2	6.5	5.5	60.0	6.3	4.5	60.0	21.4	9.9	60.0	60.0	60.0	60.0
MTP 3	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 4	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 5	11.0	14.6	60.0	13.4	16.0	60.0	30.1	28.0	60.0	60.0	60.0	60.0
MTP 6	13.5	14.8	60.0	14.5	13.5	60.0	24.2	39.5	60.0	49.2	60.0	60.0
MTP 7	18.7	18.3	60.0	28.1	27.8	60.0	37.2	40.3	60.0	60.0	60.0	60.0
MTP 8	17.3	29.0	60.0	35.8	22.0	60.0	43.3	31.4	60.0	60.0	60.0	60.0
FTP	2.0	155.7	1886.4	54.0	513.7	3299.8	61.1	55.6	201.7	48.5	50.4	651.7
TOTAL	**195.7**	**368.3**	**2366.4**	**288.2**	**727.3**	**3779.8**	**364.6**	**343.7**	**681.7**	**517.7**	**530.4**	**1131.7**

Table 8.2: Run times (seconds) of the problem instances with heuristic 1 and with the second objective

	Problem instances											
	PI1	PI2	PI3	PI4	PI5	PI6	PI7	PI8	PI9	PI10	PI11	PI12
MTP 1	27.6	33.0	60.0	49.5	53.4	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 2	25.9	29.5	60.0	42.1	60.0	60.0	52.6	52.7	60.0	60.0	60.0	60.0
MTP 3	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 4	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 5	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 6	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 7	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 8	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
FTP	155.4	33.1	251.8	41.1	1038.4	874.9	103.3	142.0	213.7	180.9	19.6	302.6
TOTAL	**568.4**	**455.7**	**731.8**	**492.7**	**1511.8**	**1354.9**	**575.9**	**614.7**	**693.7**	**660.9**	**499.6**	**782.6**

8.1.2 LRP first-VRP second heuristic

Run times of heuristic 2 for each problem instance can be seen in Table 8.3. To calculate the run times of heuristic 2 we introduce the following rule. Remember that in this heuristic we solve an LRP first and using the solution of this LRP we solve a VRP for each open MTP. Note that in each problem instance we have only 4 open MTPs, one for each brigade. Hence, in heuristic 2 we solve 4 VRPs and one LRP in each instance.

To calculate the run time we introduce the following procedure. We allow the LRP to run for at most 3600 seconds. The LRP is terminated after 3600 seconds if it can not find the optimum until that time. For example, in problem instance 1, the LRP finds the optimum in 74.8 seconds, but LRP for problem instance 6 is terminated after 3600 seconds without even finding a feasible solution.

After running the LRP in this manner, we run one VRP for each open MTP in the solution of the LRP. We let each VRP to run at most 60 seconds. For example, in problem instance 1, no VRP is solved for MTP 1 because it is not opened in the solution of the LRP. VRP for MTP 2 finds the optimum in 1.7 seconds. In addition, VRP for MTP 3 is terminated after 60 seconds at a feasible (may be sub-optimal) solution, before it finds the optimum. Note that no VRP is initialized for problem instance 6 since the LRP can not find a feasible solution. The total run time for heuristic 2 for each problem instance is the sum of the run times of four VRPs plus the run time of the single LRP.

We can summarize the running procedure of heuristic 2 as follows:

- Run a single LRP either to completion or at most 3600 seconds,
- Run 4 VRPs either to completion or at most 60 seconds,
- Total run time is the sum of the run times of all VRPs and of LRP.

Run times of heuristic 2 with the second objective for each problem instance can be seen in Table 8.4.

Table 8.3: Run times (seconds) of heuristic 2 with the first objective

	Problem instances											
	PI1	PI2	PI3	PI4	PI5	PI6	PI7	PI8	PI9	PI10	PI11	PI12
MTP 1	-	-	-	-	-	-	-	-	-	-	-	-
MTP 2	1.7	2.9	60.0	6.2	6.8	-	7.4	20.4	-	60.0	60.0	-
MTP 3	60.0	60.0	60.0	60.0	60.0	-	60.0	60.0	-	60.0	60.0	-
MTP 4	-	-	-	-	-	-	-	-	-	-	-	-
MTP 5	-	-	-	-	-	-	-	-	-	-	-	-
MTP 6	9.3	10.1	60.0	14.5	13.5	-	24.1	39.8	-	56.8	60.0	-
MTP 7	12.5	12.5	60.0	27.9	27.8	-	37.2	40.4	-	60.0	60.0	-
MTP 8	-	-	-	-	-	-	-	-	-	-	-	-
FTP	74.8	3600.0	3600.0	48.6	3020.6	-	68.6	214.5	-	230.3	911.8	-
TOTAL	**158.3**	**3685.5**	**3840.0**	**157.1**	**3128.7**	**-**	**197.3**	**375.1**	**-**	**467.1**	**1151.8**	**-**

Table 8.4: Run times (seconds) of the problem instances with heuristic 2 and with the second objective

	Problem instances											
	PI1	PI2	PI3	PI4	PI5	PI6	PI7	PI8	PI9	PI10	PI11	PI12
MTP 1	-	-	-	-	-	-	-	-	-	-	-	-
MTP 2	27.3	28.5	60.0	34.3	56.8	60.0	60.0	47.9	60.0	60.0	60.0	60.0
MTP 3	60.0	60.0	60.0	60.0	60.0	-	60.0	60.0	60.0	60.0	60.0	60.0
MTP 4	-	-	-	-	-	60.0	-	-	-	-	-	-
MTP 5	-	-	-	-	-	-	-	-	-	-	-	-
MTP 6	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 7	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 8	-	-	-	-	-	-	-	-	-	-	-	-
FTP	1692.8	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0
TOTAL	**1900.1**	**3808.5**	**3840.0**	**3814.3**	**3836.8**	**3840.0**	**3840.0**	**3827.9**	**3840.0**	**3840.0**	**3840.0**	**3840.0**

8.2 3-index model

8.2.1 VRP first-LRP second heuristic

Table 8.5 demonstrates the run times of heuristic 1 for each problem instance with the second objective. We calculate the run times as explained above.

8.2.2 LRP first-VRP second heuristic

Run times of heuristic 2 with the second objective for each problem instance can be seen in Table 8.6. Run times are calculated as explained above.

Table 8.5: Run times (seconds) of the problem instances with heuristic 1 and with the second objective

	Problem instances											
	PI1	PI2	PI3	PI4	PI5	PI6	PI7	PI8	PI9	PI10	PI11	PI12
MTP 1	7.1	6.4	1.0	19.9	13.4	0.9	40.0	37.9	2.2	60.0	60.0	60.0
MTP 2	5.1	5.5	2.9	14.9	13.1	60.0	53.8	37.7	7.8	60.0	60.0	60.0
MTP 3	37.8	53.2	7.9	60.0	60.0	16.5	60.0	60.0	60.0	60.0	60.0	60.0
MTP 4	48.5	53.5	5.4	60.0	60.0	16.0	60.0	60.0	60.0	60.0	60.0	57.1
MTP 5	22.4	24.6	8.3	55.6	36.8	21.4	60.0	60.0	27.1	60.0	60.0	60.0
MTP 6	56.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 7	16.9	17.3	0.7	38.3	51.1	0.5	60.0	60.0	60.0	60.0	60.0	60.0
MTP 8	27.4	24.8	20.8	44.7	46.5	29.3	60.0	60.0	60.0	60.0	60.0	60.0
FTP	558.4	619.8	942.5	114.6	912.0	155.3	28.1	84.1	500.9	3.9	35.0	19.6
TOTAL	**779.6**	**865.1**	**1049.5**	**468.0**	**1252.9**	**359.9**	**481.9**	**519.7**	**838.1**	**483.9**	**515.0**	**496.7**

Table 8.6: Run times (seconds) of the problem instances with heuristic 2 and with the second objective

	Problem instances											
	PI1	PI2	PI3	PI4	PI5	PI6	PI7	PI8	PI9	PI10	PI11	PI12
MTP 1	-	-	-	-	-	-	-	-	-	-	-	-
MTP 2	4.5	4.0	4.5	14.2	16.9	9.2	36.1	31.4	15.9	60.0	60.0	28.8
MTP 3	37.9	53.3	-	60.0	60.0	-	60.0	60.0	60.0	60.0	60.0	-
MTP 4	-	-	5.4	-	-	16.0	-	-	-	-	-	56.7
MTP 5	-	-	-	-	-	-	-	-	-	-	-	-
MTP 6	60.0	59.5	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0	60.0
MTP 7	8.6	8.7	0.7	40.2	34.8	0.7	60.0	60.0	1.6	60.0	60.0	61.5
MTP 8	-	-	-	-	-	-	-	-	-	-	-	-
FTP	3073.2	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0	3600.0
TOTAL	**3184.2**	**3725.5**	**3670.6**	**3774.4**	**3771.7**	**3685.9**	**3816.1**	**3811.4**	**3737.5**	**3840.0**	**3840.0**	**3747.0**

Table 8.7: Summary of 4-index model with the first objective

	Original model		Heuristic 1		Heuristic 2	
	run time	cost	run time	cost	run time	cost
PI1	80879	1301.84	196	1301.84	158	1301.84
PI2	78086	1301.84	368	1301.84	3686	1301.84
PI3	78344	1688.26	2366	1688.26	3840	1686.17
PI4	79236	1301.84	288	1301.84	157	1301.84
PI5	78035	1301.84	727	1301.84	3129	1301.84
PI6	7005	1686.17	3780	1686.17	-	-
PI7	83157	1304.80	365	1301.84	197	1301.84
PI8	85911	1314.80	346	1305.83	375	1301.85
PI9	82383	1686.17	682	1686.17	-	-
PI10	86198	1342.75	518	1304.79	467	1304.62
PI11	84541	1386.06	530	1378.83	1152	1306.13
PI12	13605	1686.17	1132	1686.17	-	-
AVERAGE	**69782**	**1441.88**	**941**	**1437.12**	**-**	**-**

8.3 Findings

8.3.1 4-index model

Table 8.7 compares both the run times (seconds) and the first objective function values of the original model with those of the two heuristics. On the average, the original model reaches an objective value of 1441.88 in approximately 19 hours (69782 seconds), whereas heuristic 1 reaches an objective value of 1437.12 in approximately 16 minutes (941 seconds). From another point of view, heuristic 1 provides a better solution with 0.3% less cost within 98.7% less time. Heuristic 2 can not find a solution in three problem instances within the allowed run time.

Table 8.8 displays the cost structure of the 4-index model with the first objective. As can be seen cost of the VRP phase generally constitutes 45% percent of the overall costs with the first heuristic, whereas it is 34% with the second heuristic.

Table 8.9 compares both the run times (seconds) and the objective function values of the original model with those of the two heuristics. On the average,

Table 8.8: 4-index model cost structure with the first objective

	Heuristic 1			Heuristic 2		
	VRP	LRP	VRP/TOTAL	VRP	LRP	VRP/TOTAL
PI1	587.04	714.80	0.45	437.04	864.80	0.34
PI2	587.04	714.80	0.45	437.04	864.80	0.34
PI3	776.47	911.79	0.46	626.47	1059.70	0.37
PI4	587.04	714.80	0.45	437.04	864.80	0.34
PI5	587.04	714.80	0.45	437.04	864.80	0.34
PI6	776.47	909.70	0.46	-	-	-
PI7	587.04	714.80	0.45	437.04	864.80	0.34
PI8	587.05	718.78	0.45	437.05	864.80	0.34
PI9	776.47	909.70	0.46	-	-	-
PI10	589.99	714.81	0.45	439.82	864.80	0.34
PI11	592.63	786.20	0.43	441.33	864.80	0.34
PI12	776.47	909.70	0.46	-	-	-

the original model reaches an objective value of 1060.90 in approximately 18 hours (65354 seconds), whereas heuristic 1 reaches an objective value of 1058.43 in approximately 12 minutes (745 seconds), and heuristic 2 reaches an objective value of 1058.42 in approximately 1 hour (3672 seconds). From another point of view, heuristic 1 provides a better solution with 0.2% less cost within 98.9% less time, and heuristic 2 provides a better solution with 0.2% less cost within 94.4% less time.

The cost structure of the 4-index model with the second objective is shown in Table 8.10. Likewise with the first objective, the cost of the VRP phase is bigger in the first heuristic than that of the second heuristic. In detail, generally 55% of the overall cost is VRP cost in the first heuristic, whereas it is around 40% in the second heuristic.

8.3.2 3-index model

Table 8.11 compares both the run times (seconds) and second objective function values of the original model with those of the two heuristics. On the average, the original model reaches an objective value of 1076.70 in approximately 21 hours (75483 seconds), whereas heuristic 1 reaches an objective value of 1068.77

Table 8.9: Summary of 4-index model with the second objective

	Original model		Heuristic 1		Heuristic 2	
	run time	cost	run time	cost	run time	cost
PI1	79286	921.98	569	921.44	1900	921.64
PI2	81395	923.46	456	921.98	3809	921.93
PI3	30461	1331.72	732	1331.67	3840	1331.72
PI4	52175	922.03	493	921.60	3814	921.64
PI5	81250	921.89	1512	921.60	3837	921.64
PI6	41800	1332.01	1355	1331.94	3840	1332.01
PI7	77941	922.02	576	921.94	3840	921.64
PI8	83123	922.53	615	921.99	3827	921.77
PI9	30340	1331.72	694	1331.67	3840	1331.72
PI10	80375	922.05	661	921.85	3840	921.85
PI11	82937	947.32	500	921.75	3840	921.75
PI12	63165	1332.01	783	1331.67	3840	1331.72
AVERAGE	**65354**	**1060.90**	**745**	**1058.43**	**3672**	**1058.42**

Table 8.10: 4-index model cost structure with the second objective

	Heuristic 1			Heuristic 2		
	VRP	LRP	VRP/TOTAL	VRP	LRP	VRP/TOTAL
PI1	507.60	413.84	0.55	358.28	563.36	0.39
PI2	507.56	414.42	0.55	358.28	563.65	0.39
PI3	709.02	622.65	0.53	560.20	771.52	0.42
PI4	507.56	414.04	0.55	358.28	563.36	0.39
PI5	507.56	414.04	0.55	358.28	563.36	0.39
PI6	708.33	623.61	0.53	560.49	771.52	0.42
PI7	507.56	414.38	0.55	358.28	563.36	0.39
PI8	507.69	414.30	0.55	358.42	563.35	0.39
PI9	709.02	622.65	0.53	560.20	771.52	0.42
PI10	508.49	413.36	0.55	358.49	563.36	0.39
PI11	507.67	414.08	0.55	358.39	563.36	0.39
PI12	709.02	622.65	0.53	560.20	771.52	0.42

Table 8.11: Summary of 3-index model with the second objective

	Original model		Heuristic 1		Heuristic 2	
	run time	cost	run time	cost	run time	cost
PI1	78724	921.60	780	921.60	3184	921.63
PI2	39880	921.60	865	921.60	3726	921.64
PI3	44365	1332.00	1050	1331.72	3671	1332.01
PI4	80718	921.84	468	921.84	3774	921.64
PI5	79641	921.60	1253	921.60	3772	921.64
PI6	78905	1333.90	360	1332.01	3686	1332.01
PI7	85829	922.35	482	922.19	3816	921.65
PI8	80781	978.16	520	975.24	3811	921.64
PI9	83322	1332.00	838	1331.99	3738	1331.72
PI10	81992	998.76	484	978.30	3840	998.10
PI11	85638	998.17	515	931.90	3840	923.65
PI12	85999	1338.36	497	1335.27	3747	1332.03
AVERAGE	**75483**	**1076.70**	**676**	**1068.77**	**3717**	**1064.95**

in approximately 11 minutes (676 seconds), and heuristic 2 reaches an objective value of 1064.95 in approximately 1 hour (3717 seconds). From another point of view, heuristic 1 provides a better solution with 0.7% less cost within 99.1% less time, and heuristic 2 provides a better solution with 1.1% less cost within 95.1% less time.

Table 8.12 presents the cost structure of the 3-index model with the second objective. The results are similar to those of the 4-index model with the second objective. VRP phase generally constitutes 55% percent of the overall cost, whereas it is around 40% with the second heuristic.

8.3.3 Comparison of the heuristics

With the 4-index formulation and the first objective, on the average heuristic 1 (VRP first-LRP second) reaches a better objective function value than that of the original formulation within 98.7% less time (941 seconds). There exist three instances that heuristic 2 (LRP first-VRP second) can not find a solution within the allowed time.

Table 8.12: 3-index model cost structure with the second objective

	Heuristic 1			Heuristic 2		
	VRP	LRP	VRP/TOTAL	VRP	LRP	VRP/TOTAL
PI1	507.56	414.07	0.55	358.28	563.35	0.39
PI2	507.56	414.08	0.55	358.28	563.36	0.39
PI3	709.02	622.99	0.53	560.49	771.52	0.42
PI4	507.56	414.08	0.55	358.28	563.36	0.39
PI5	507.56	414.08	0.55	358.28	563.36	0.39
PI6	710.49	621.52	0.53	560.49	771.52	0.42
PI7	507.57	414.08	0.55	358.29	263.36	0.58
PI8	507.56	414.08	0.55	358.28	563.36	0.39
PI9	709.07	622.65	0.53	560.20	771.52	0.42
PI10	510.62	487.48	0.51	434.74	563.36	0.44
PI11	509.70	413.95	0.55	360.29	563.36	0.39
PI12	708.83	623.20	0.53	560.50	771.53	0.42

With the 4-index formulation and the second objective, on the average both heuristics are better than the original formulation. However, heuristic 1 attains a similar objective function value (1058.43) to that of the heuristic 2 in 745 seconds, whereas the run time of heuristic 2 is 3672 seconds.

With the 3-index formulation and the second objective, on the average both heuristics perform better than the original formulation. Nevertheless, in 676 seconds heuristic 1 reaches a slightly higher objective value (1068.77) that is attained by heuristic 2 in 3717 seconds.

These computational results assert that the "VRP first-LRP second" heuristic outperforms the "LRP first-VRP second" heuristic.

8.3.4 Comparison of the models

With the heuristic 1, on the average, 4-index model reaches a better objective function value (1058.43 versus 1068.77) in a longer amount of time (745 versus 676 seconds). With the heuristic 2, on the average, 4-index model outperforms 3-index model with a better objective function value (1058.42 versus 1064.95) that is attained in less time (3672 versus 3717 seconds).

Chapter 9

Dynamic Model Development

Remember that in Chapter 3 we take a snapshot of the battlefield, which contains Fixed-TPs, Mobile-TPs and combat units, at a particular point in time and develop a static 4-index mathematical formulation of Mobile-ADS design problem for a fixed period. In this chapter we extend this formulation over time, assuming that known locations of combat units, as well as the set of potential Mobile-TP locations change in every consecutive planning period. Note that for the sake of brevity we only provide the dynamic version of the 4-index formulation, however it is straightforward to obtain the dynamic version of the 3-index model by following similar modifications that are explained below.

9.1 Model development

We assume that the planning horizon (combat duration) T is partitioned into consecutive 24-hour time periods, represented by $t \in T$. In other words, there are $|T|$ time periods, i.e. $t \in \{1, 2, ..., |T|\}$. We also assume that potential location set for Fixed-TPs (N_F) do not change over time. N_M^t is the potential location set of Mobile-TPs and N_C^t is the known locations of combat units in period $t \in T$. An example of the movement of Mobile-TPs and combat units between the first and the second periods is shown in Figure 9.1.

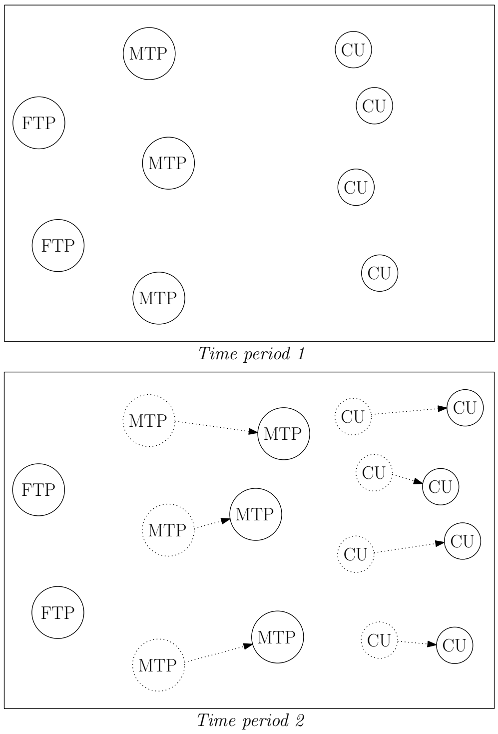


Figure 9.1: Movement of Mobile-TPs and combat units between time periods

We use the following set relations in the dynamic formulation. $N_M^t \subseteq N_M^{t+1}$ for all $t \in \{1, 2, ..., |T| - 1\}$. $N^t = N_F \bigcup N_M^t \bigcup N_C^t$, $N_{FM}^t = N_F \bigcup N_M^t$, $N_{MC}^t = N_M^t \bigcup N_C^t$ for all $t \in T$ and $N_{FM} = N_F \bigcup N_M^{|T|}$. These set relationships are depicted in Figure 9.2.

We permit the opening of new Mobile-TPs at the beginning of any time period and the closing of existing ones at the end of any time period. We also let an existing Mobile-TP to re-open once it is closed or a new Mobile-TP to re-close once it is open. We assume that if a Mobile-TP is to be moved to another potential location then its transportation will take a relatively short time compared to 24-hour planning period; hence, we suppose it changes its location instantaneously.

We know that the available number of trucks or the capacity of Mobile-TPs may vary between the time periods due to breakdowns, enemy fire, etc. However, we admit time-independent fleet size and transfer point capacity for the sake of simplicity. Nevertheless, these parameters could easily be made time-dependent.

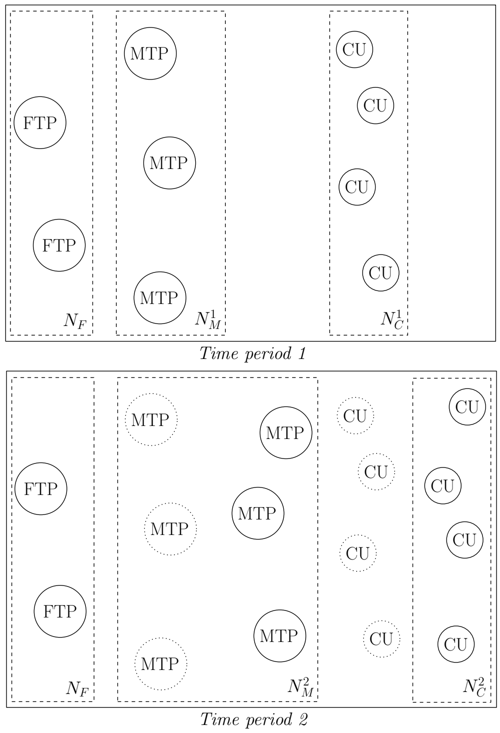


Figure 9.2: Fixed-TP, Mobile-TP and combat unit sets in the dynamic case

Note that in the dynamic formulation, we use the same decision variables as in the static model with an extra index $t \in T$, and they all refer to the related time period t. We also have two new binary decision variables: yy_{ij}^t is 1 if Mobile-TP of a brigade is opened at potential location i at the beginning of time period t and relocated at a different potential location j (such that i and j are potential locations of the same brigade) at the beginning of time period $t+1$, 0 otherwise; and xx_v is 1 if vehicle v is dispatched from a transfer point in at least one time period, 0 otherwise. The dynamic Mobile-ADS design problem can be formulated as follows:

$$\begin{aligned}
\min \quad & \sum_{\substack{i \in N_{FM} \\ t=1}} FC_i \cdot y_i^t + \sum_{t \in T} \sum_{i \in N_M^t} \sum_{j \in N_M^t} \sum_{v \in V_M} DC_v^t \cdot TI_{ij}^t \cdot yy_{ij}^t \\
& + \sum_{v \in V} VC_v \cdot xx_v + \sum_{t \in T} \sum_{i \in N^t} \sum_{j \in N^t} \sum_{v \in V} \sum_{p \in P} TC_{vp}^t \cdot TI_{ij}^t \cdot f_{ijvp}^t \\
& + \sum_{t \in T} \sum_{i \in N_C^t} \sum_{j \in N_M^t} \sum_{v \in V_M} DC_v^t \cdot TI_{ij}^t \cdot x_{ijv}^t
\end{aligned}$$

subject to

$$\sum_{v \in V_F} \left(\sum_{\substack{j \in N_{FM}^t \\ j \neq i}} f_{jivp}^t - \sum_{\substack{j \in N_M^t \\ j \neq i}} f_{ijvp}^t \right) = \sum_{v \in V_M} \sum_{j \in N_C^t} f_{ijvp}^t \quad \forall i \in N_M^t, p \in P, t \in T \tag{D-1a}$$

$$\sum_{v \in V_M} \left(\sum_{\substack{j \in N_{MC}^t \\ j \neq i}} f_{jivp}^t - \sum_{\substack{j \in N_C^t \\ j \neq i}} f_{ijvp}^t \right) = Q_{ip}^t \quad \forall i \in N_C^t, p \in P, t \in T \tag{D-1b}$$

$$\sum_{\substack{j \in N_{FM}^t \\ j \neq i}} f_{jivp}^t \geq \sum_{\substack{j \in N_M^t \\ j \neq i}} f_{ijvp}^t \quad \forall i \in N_M^t, v \in V_F, p \in P, t \in T \tag{D-2a}$$

$$\sum_{\substack{j \in N_{MC}^t \\ j \neq i}} f_{jivp}^t \geq \sum_{\substack{j \in N_C^t \\ j \neq i}} f_{ijvp}^t \quad \forall i \in N_C^t, v \in V_M, p \in P, t \in T \tag{D-2b}$$

$$\sum_{i \in N_F} \sum_{j \in N_M^t} x_{ijv}^t \leq 1 \quad \forall v \in V_F, t \in T \tag{D-3a}$$

$$\sum_{i \in N_M^t} \sum_{j \in N_C^t} x_{ijv}^t \leq 1 \quad \forall v \in V_M, t \in T \tag{D-3b}$$

$$\sum_{j \in N_M^t} x_{jiv}^t = \sum_{j \in N_M^t} x_{ijv}^t \quad \forall i \in N_F, v \in V_F, t \in T \tag{D-4a}$$

$$\sum_{j \in N_C^t} x_{jiv}^t = \sum_{j \in N_C^{t+1}} x_{ijv}^{t+1} \quad \forall i \in N_M^{t+1}, v \in V_M, t \in \{1, 2, ..., |T| - 1\} \tag{D-4b-1}$$

$$\sum_{j \in N_C^t} x_{jiv}^t = \sum_{j \in N_C^t} x_{ijv}^t \quad \forall i \in N_M^t, v \in V_M, t = |T| \tag{D-4b-2}$$

$$\sum_{\substack{j \in N_{FM}^t \\ j \neq i}} x_{jiv}^t = \sum_{\substack{j \in N_{FM}^t \\ j \neq i}} x_{ijv}^t \quad \forall i \in N_M^t, v \in V_F, t \in T \tag{D-5a}$$

$$\sum_{\substack{j \in N_{MC}^t \\ j \neq i}} x_{jiv}^t = \sum_{\substack{j \in N_{MC}^t \\ j \neq i}} x_{ijv}^t \quad \forall i \in N_C^t, v \in V_M, t \in T \tag{D-5b}$$

$$\sum_{v \in V_F} \sum_{j \in N_M^t} f_{ijvp}^t \leq CD_{ip} \cdot y_i^t \quad \forall i \in N_F, p \in P, t \in T \tag{D-6a}$$

$$\sum_{v \in V_M} \sum_{j \in N_C^t} f_{ijvp}^t \leq CD_{ip} \cdot y_i^t \quad \forall i \in N_M^t, p \in P, t \in T \tag{D-6b}$$

$$\sum_{v \in V_F} \sum_{j \in N_M^t} f_{ijvp}^t \leq \left(\sum_{l \in N_C^t} Q_{lp}^t \right) \cdot y_i^t \quad \forall i \in N_M^t, i \neq j, p \in P, t \in T \tag{D-6c}$$

$$\begin{aligned} f_{ijvp}^t \leq CV_{vp} \cdot x_{ijv}^t \quad & \forall i \in N_F, j \in N_M^t, v \in V_F, p \in P, t \in T; \\ & \forall i, j \in N_M^t, i \neq j, v \in V_F, p \in P, t \in T \end{aligned} \tag{D-7}$$

$$\forall i \in N_M^t, j \in N_C^t, v \in V_M, p \in P, t \in T;$$
$$\forall i, j \in N_C^t, i \neq j, v \in V_M, p \in P, t \in T$$

$$\sum_{p \in P} f_{ijvp}^t \leq CT_v \cdot x_{ijv}^t \quad \forall i \in N_F, j \in N_M^t, v \in V_F, t \in T; \tag{D-8}$$
$$\forall i, j \in N_M^t, i \neq j, v \in V_F, t \in T$$
$$\forall i \in N_M^t, j \in N_C^t, v \in V_M, t \in T;$$
$$\forall i, j \in N_C^t, i \neq j, v \in V_M, t \in T$$

$$\sum_{p \in P} f_{ijvp}^t \geq x_{ijv}^t \quad \forall i \in N_F, j \in N_M^t, v \in V_F, t \in T; \tag{D-9}$$
$$\forall i, j \in N_M^t, i \neq j, v \in V_F, t \in T$$
$$\forall i \in N_M^t, j \in N_C^t, v \in V_M, t \in T;$$
$$\forall i, j \in N_C^t, i \neq j, v \in V_M, t \in T$$

$$\left(\sum_{l \in N_C^t} Q_{lp}^t \right) \cdot w_{ijp}^t \geq f_{ijvp}^t \quad \forall i \in N_F, j \in N_M^t, v \in V_F, p \in P, t \in T; \tag{D-10}$$
$$\forall i, j \in N_M^t, i \neq j, v \in V_F, p \in P, t \in T$$
$$\forall i \in N_M^t, j \in N_C^t, v \in V_M, p \in P, t \in T;$$
$$\forall i, j \in N_C^t, i \neq j, v \in V_M, p \in P, t \in T$$

$$\sum_{v \in V_F} f_{ijvp}^t \geq w_{ijp}^t \quad \forall i \in N_F, j \in N_M^t, p \in P, t \in T; \tag{D-11a}$$
$$\forall i, j \in N_M^t, i \neq j, p \in P, t \in T$$

$$\sum_{v \in V_M} f_{ijvp}^t \geq w_{ijp}^t \quad \forall i \in N_M^t, j \in N_C^t, p \in P, t \in T; \tag{D-11b}$$
$$\forall i, j \in N_C^t, i \neq j, p \in P, t \in T$$

$$TE_{ip}^t \leq tp_{ip}^t \leq TL_{ip}^t \quad \forall i \in N_C^t, p \in P, t \in T \tag{D-12}$$

$$tp_{ip}^t = 0 \quad \forall i \in N_F, p \in P, t \in T \tag{D-13}$$

$$tp_{ip}^t + TI_{ij}^t \cdot w_{ijp}^t - TM_p^t \cdot (1 - w_{ijp}^t) \leq tp_{jp}^t \quad \forall i \in N_F, j \in N_M^t, p \in P, t \in T \tag{D-14}$$
$$\forall i, j \in N_M^t, i \neq j, p \in P, t \in T;$$
$$\forall i \in N_M^t, j \in N_C^t, p \in P, t \in T;$$
$$\forall i, j \in N_C^t, i \neq j, p \in P, t \in T$$

$$tv_{iv}^t + TI_{ij}^t \cdot x_{ijv}^t - TM^t \cdot (1 - x_{ijv}^t) \leq tv_{jv}^t \quad \forall i \in N_F, j \in N_M^t, v \in V_F, t \in T \tag{D-15}$$

$$\forall i, j \in N_M^t, i \neq j, v \in V_F, t \in T;$$

$$\forall i \in N_M^t, j \in N_C^t, v \in V_M, t \in T;$$

$$\forall i, j \in N_C^t, i \neq j, v \in V_M, t \in T$$

$$2 \cdot yy_{ij}^t \leq y_i^t + y_j^{t+1} \quad \forall i \in N_M^t, j \in N_M^{t+1}, i \neq j, t \in \{1, 2, ..., |T| - 1\} \tag{D-16}$$

$$1 + yy_{ij}^t \geq y_i^t + y_j^{t+1} \quad \forall i \in N_M^t, j \in N_M^{t+1}, i \neq j, t \in \{1, 2, ..., |T| - 1\} \tag{D-17}$$

$$+xx_v \geq x_{ijv}^t \quad \forall i \in N_F, j \in N_M^t, v \in V_F, t \in T \tag{D-18a}$$

$$xx_v \geq x_{ijv}^t \quad \forall i \in N_M^t, j \in N_C^t, v \in V_M, t \in T \tag{D-18b}$$

$$f, tp, tv \text{ nonnegative}; y, x, w, yy, xx \text{ binary.} \tag{D-19}$$

The objective function contains five different cost components. The first component calculates the fixed cost of opening a new transfer point in period 1. Note that if we buy the equipment for a transfer point once, we can use it several times in different time periods. Opening a new transfer point actually means a transfer point is changing its location. Hence, in reality the cost of opening a new transfer point is its cost of repositioning and this constitutes the second component. The third and fourth components are vehicle acquisition and ammunition distribution costs. The fifth component calculates the driving cost of empty trucks returning to their home transfer points.

Note that constraints have the same equation numbers, with a D in front that represents dynamic version, as their duplicates have in the static formulation. For example, constraints (D-1a) are the dynamic version of constraints (1a). The only constraints that are different from their static duplicates are constraints (D-4b-1) and (D-4b-2). To be exact, their static version is constraints (4b) which force each ammo truck to turn back to its home Mobile-TP from where it is dispatched. Nevertheless, in the dynamic model Mobile-TPs can change their location in every consecutive time period.

Now, suppose a Mobile-TP is located at node i in time period t and dispatches ammo truck v. Then, that transfer point moves to node j in period $t+1$, meaning that any truck that will serve combat units in period $t+1$ will be dispatched from

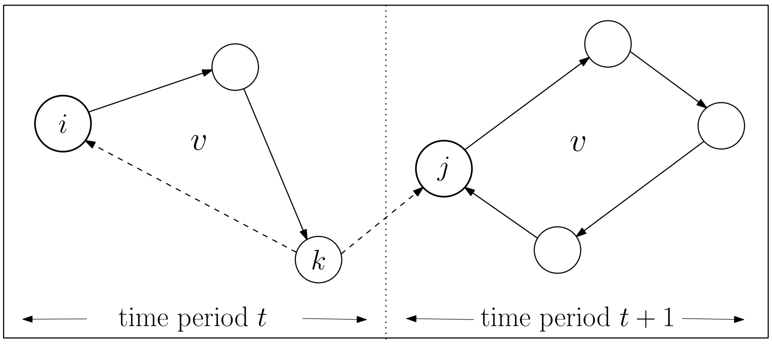


Figure 9.3: Returning arcs of ammo trucks to transfer points

node j. Consider that unit k is the last unit on the route of truck v in period t. As depicted in Figure 9.3, we must allow the truck to return to the transfer point, which may be located at a different node, of the next period (dashed line incoming to node j), as well as allowing it to return to the original transfer point (dashed line incoming to node i). Hence, we introduce constraints (D-4b-1) and (D-4b-2) in place of (4b) in the dynamic model.

We also have four new constraints that are (D-16), (D-17), (D-18a) and (D-18b). Constraints (D-16) and (D-17) provide that if a transfer point changes its location in a time period, the cost of repositioning is properly added to the overall cost. Constraints (D-18a) and (D-18b) let us count the cost of a truck only once, even if it is used many times.

9.2 Sample scenario

We present a sample multi-period scenario and its solution in Figures 9.4, 9.5 and 9.6. We again consider a corps including 20 combat units as can be seen in Figure 9.4, and also consider two consecutive planning periods, i.e. t^1 and t^2. CUi^1 (resp. CUi^2) represents the known location of combat unit i in the first (resp. second) period. Potential Mobile-TP locations for the first brigade in the first period is MTP1 and MTP2, whereas it is MTP1, MTP2, MTP9 and MTP10 in the second period. The solution for the first period is shown in Figure 9.5. Solid (resp. dashed and dotted) lines represent the routes of commercial (resp.

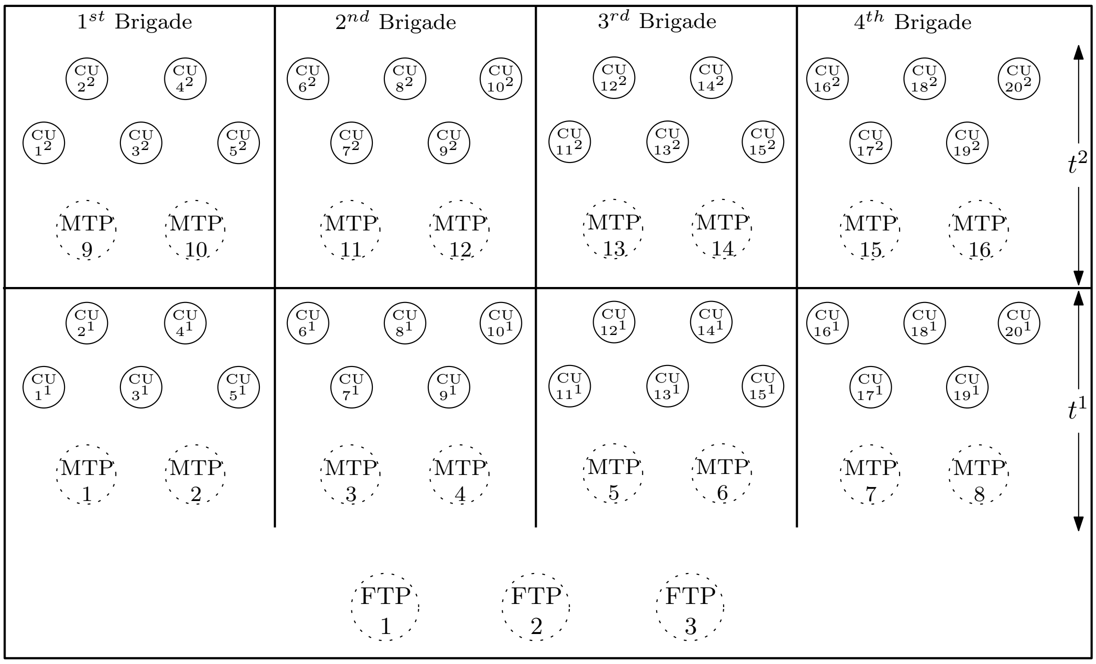


Figure 9.4: The corps' layout plan on the battlefield for two periods

ammo) trucks.

As can be seen in the figure, for the third brigade, a Mobile-TP is established at location MTP6 in period 1, and it is moved to location MTP13 in period 2. Hence, at the end of their routes, all ammo trucks return to location MTP13 rather than MTP6. However, a Mobile-TP serves the second brigade at the same location, namely MTP3, in both periods. Thus, all ammo trucks return to the same location, MTP3, from where they were dispatched. Figure 9.6 shows the solution for period 2, and all ammo trucks return to their home transfer points, since this is the last planning period.

Note that the dynamic model finds an initial solution to this two-period scenario in 8 hours with a gap of 23%, and the gap is still 23% after 24 hours.

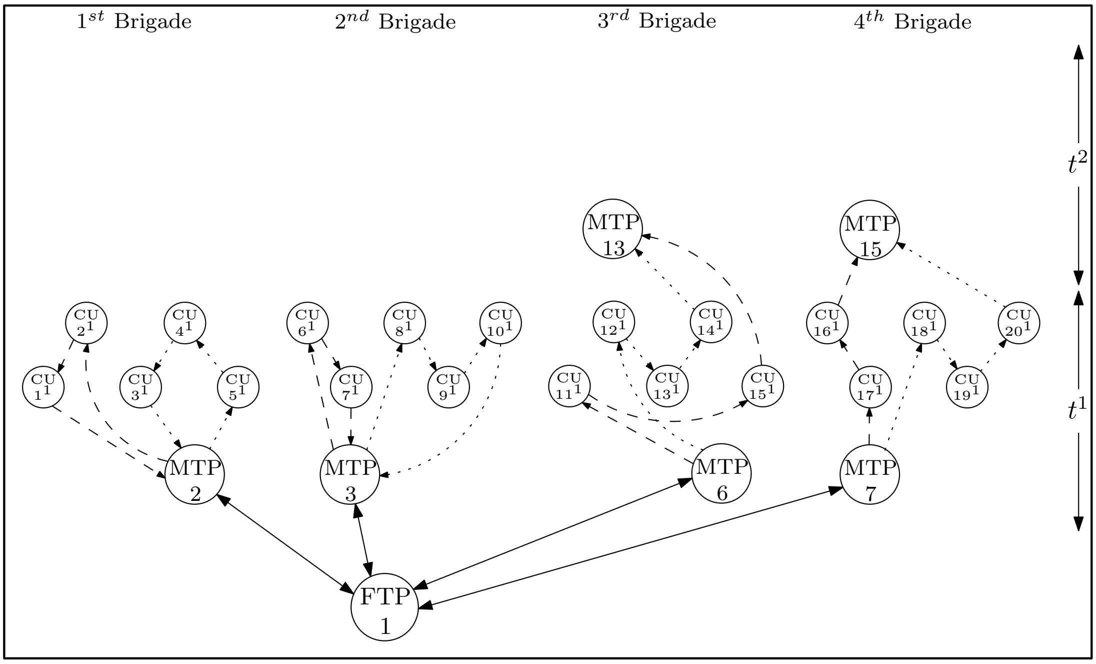


Figure 9.5: Dynamic model solution of the multi-period scenario for the first period

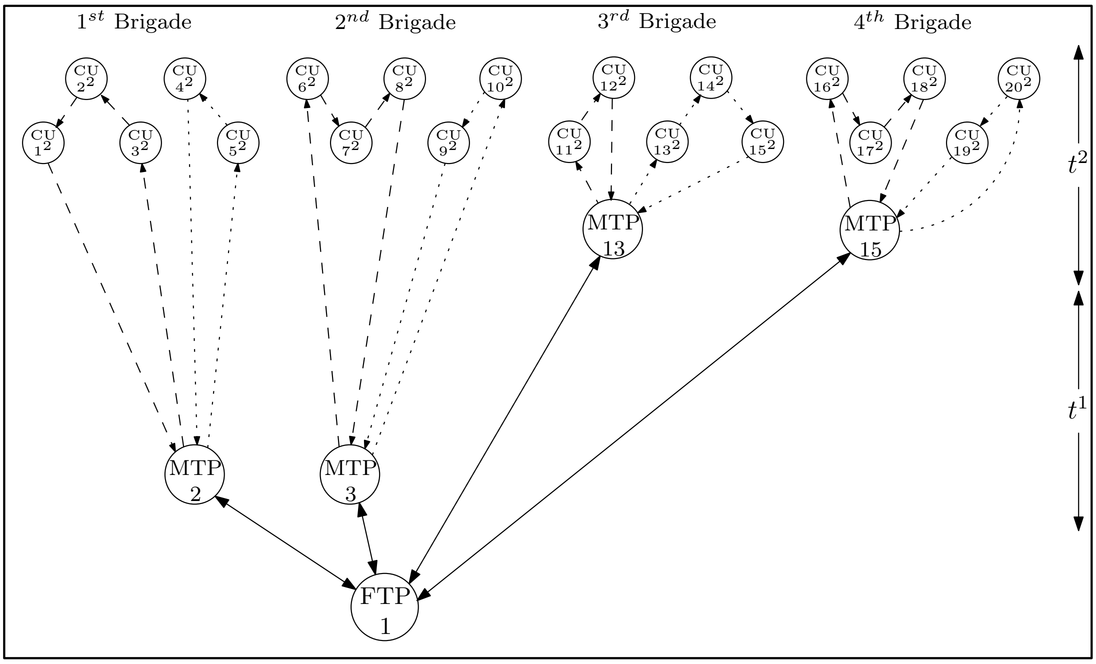


Figure 9.6: Dynamic model solution of the multi-period scenario for the second period

Chapter 10

Static Model in Real Life

In this chapter we first show how the static model can assist in a multi-period combat operation and then discuss how the model can help the logistics planners when faced with unplanned combat situations.

10.1 Multi-period Planning with the Static Model

In what follows, we provide a framework to guide in the successive use of the static model for multi-period strategic decision making. Recall that one of the main differences of the two models lies in the costing structure of the ammo trucks mainly due to the fact that in the dynamic model this cost refers to the travel cost to the new (possible) Mobile-TP location whereas in the static model this cost is neglected. In order to compare the two models more fairly, we now include the Mobile-TP return cost within the objective function of the static model. As another modification, we adjust the fixed costs of Mobile-TPs in the static model in compliance with the logic we used for the dynamic model. Note that the first component of the objective function of the dynamic model corresponds to the fixed cost of opening a new transfer point in the first period. Since we incur this cost only once in the first period, in the consecutive periods, we take the fixed costs of all opened Mobile-TPs as zero and add their repositioning costs (whenever applicable). Moreover, the potential Mobile-TP location set in each period should also contain the selected Mobile-TP locations of the previous period as output by the static model.

Consider the layout of the corps in Figure 9.4. The static model can be used to solve this problem exactly in the same way explained in Chapter 3, with the above suggested adjustment. Figure 10.1 and 10.2 present the static model solution for the first and the second periods. In a comparison with the dynamic solution we can see both similarities and differences. Briefly, both the dynamic and the static models keep MTP3 at the same place for both periods and move MTP7 to MTP15 in the second period. However, the static model continues to use MTP6, but moves MTP2 to MTP10 in the second period, whereas the dynamic model continues to use MTP2 and relocates MTP6 to MTP13.

As for the truck routes, again there are similarities and differences between the solutions of the models. For example, ammo truck routes are similar for the 2^{nd} brigade in period 1 and for the 3^{rd} and 4^{th} brigades in period 2. However, the most obvious difference is in commercial truck routes. The dynamic model uses 4 commercial trucks whereas the static model dispatches 3 commercial trucks. At the same time, it can also clearly be seen from the figures that the dynamic model has less traffic than the static model.

Which model (static or dynamic) is more advantageous depends on the combat environment and enemy capabilities. For instance, if the enemy has the ability to detect our logistics convoys then the dynamic model would be more advantageous, because it has less traffic.

Note that the static model finds the initial solution for the first period in 1 minute with a gap of 20%, and for the second period in 20 minutes with a gap of 15%, whereas the dynamic model finds an initial solution for both periods in 8 hours with a gap of 23%.

In addition, it is very straightforward to adjust the heuristic methods, which

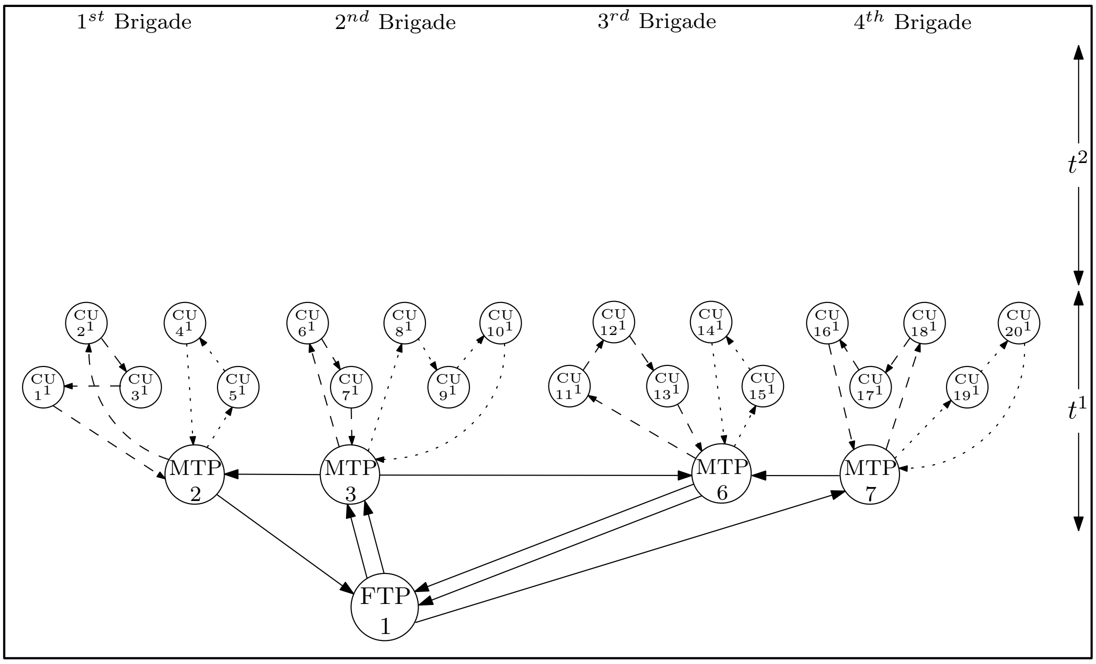


Figure 10.1: Static model solution of the multi-period scenario for the first period

are introduced in Chapter 7, to solve the static models in each period according to the framework explained above. Hence, our heuristics can easily be employed to solve the successive static models for each period to further shorten the solution time.

In fact, the framework of successive use of the static model for the solution of multi-period scenarios can be considered as a heuristic methodology in itself for the dynamic model.

10.2 Facing Unplanned Contingencies with the Static Model

Consider that logistics planners begin to execute the first day of the dynamic distribution plan that is shown in 9.5. However, at the end of the first day or just before the distribution plan for the second day is put into action, they receive the following Logistics Related Update from the Corps Headquarters: For the first

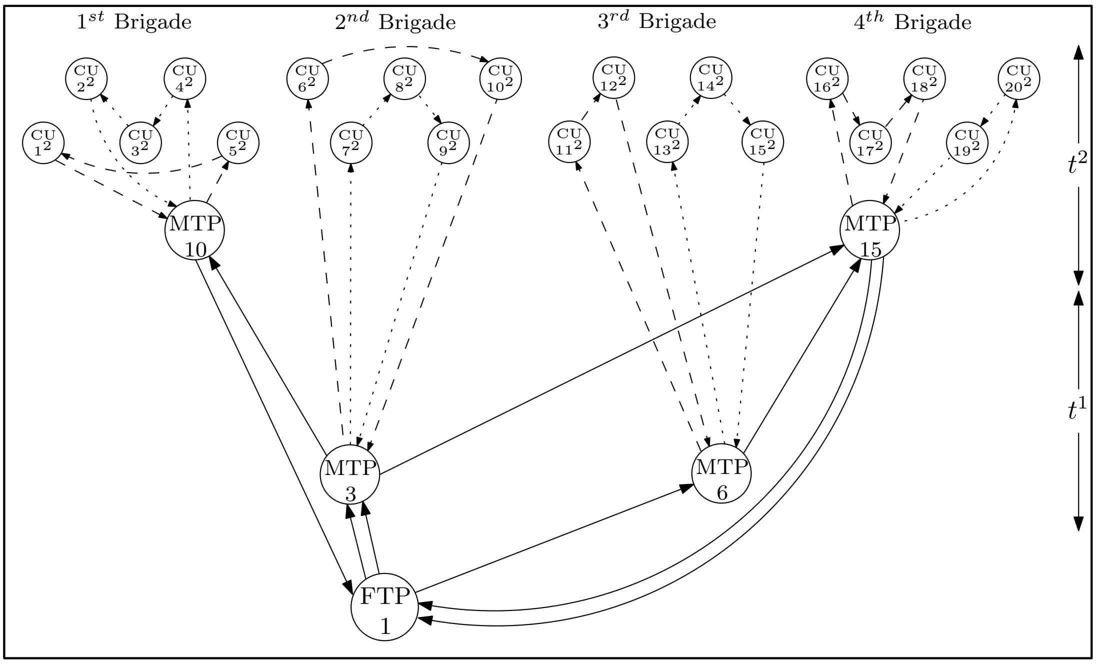


Figure 10.2: Static model solution of the multi-period scenario for the second period

brigade, due to unexpectedly high enemy presence, Battalion 4 triples its ammo requirements for each type. The bridge between MTP2 and Battalion 5 has been destroyed and other battalions of the brigade are too far away that Battalion 5 must be supplied by the second brigade (MTP3). For the second brigade, due to unexpectedly high enemy presence, Battalion 8 doubles its ammo requirements for each type. For the third brigade, everything is as planned and no change is required. For the fourth brigade, due to heavy enemy artillery fire, MTP15 is no longer suitable for any further logistics operations. All ammo trucks have been moved to MTP16 and distribution must be made from this site from now on. Transportation between Battalion 16 and 17 is blocked because of an enemy mine field.

Similar updates can be encountered at any (the first, the last or a middle) day of a given distribution plan. In such circumstances, logistics planners must prepare the new distribution plan according to the update. However, the first and utmost important challenge is not to prepare the whole distribution plan. The real issue is to save the present day as soon and as good as possible, and

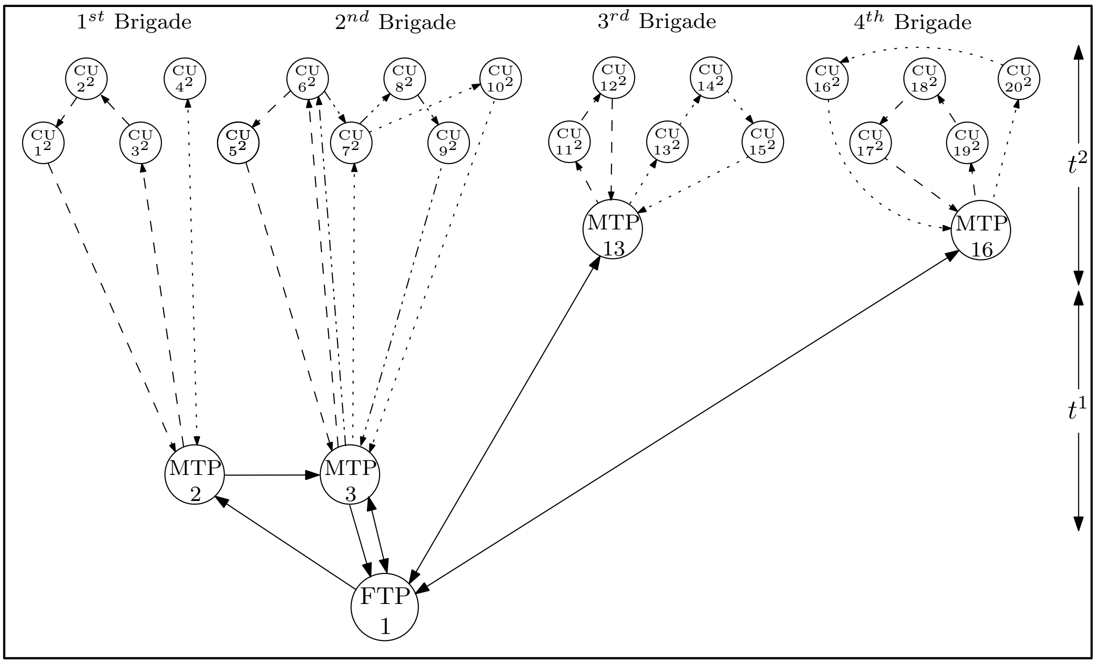


Figure 10.3: New distribution plan for the second period

then to prepare the distribution plan for the rest of the battle duration. For the present day, we have serious time pressure to answer the needs of the combat units confronting the enemy. But for the next days we will have enough time to plan. Hence, the best way to proceed is to run the static model for the present day and then run the dynamic model for the rest of the combat duration.

Figure 10.3 shows the new distribution plan for the second period that is obtained by the static model. This solution is obtained in 7 minutes, which we believe is short enough to answer the needs of the combat units in time.

Chapter 11

Summary and Conclusion

What was the main reason of the defeat in Operation Barbarossa (the code-name of German invasion of Russia with approximately three million men in 1941 during World War II) after seven months of continuous combat and very close to Moscow? In the first two weeks Russian defense forces could not stop German forces and they reached deep into the Russian territory with a shocking speed. After a short time, German supply lines were about 1,600 km long and sustaining the battle became almost impossible. Combat units could not exploit the tactical advantages because of the lack of ammunition supply. They used more than 600,000 horses due to the shortage of trucks and at last it came to a point where the transportation system could not supply the demands of the combat units for the three most important supply classes which are clothing, food and ammunition. At the end German dream turned into a nightmare after a counter-attack of Russian Army just about 30 km reach of Moscow. It could be said that logistics was not the only reason but it was the primary factor that took part in German failure.

Above example is not the only failure in which logistics is the primary contributing factor. On the contrary such a failure list would be so long including the failed Damascus siege of King Louis VII and Emperor Conrad III in 1148 during the second Crusade, the surrender of John Burgoyne at Saratoga in 1777 during the American Revolutionary War, the retreat of Napoleon from Moscow in 1812 during the Napoleonic Wars, and so on. These real events show that logistics has always remained one of the most important actors on the battlefield throughout centuries and it will remain as such in the following centuries.

"Logistics can be a force multiplier; however, if not controlled, it can be the Achilles' heel of an operation." Following this statement of [55] and realizing that ammunition is one of the most important supply classes, to provide an effective and flexible distribution system on the battlefield we propose Mobile Ammunition Distribution System (Mobile-ADS) which can support fast moving land forces in rapidly changing combat environments.

In Chapter 2, we show that Mobile-ADS design problem is in fact a Location Routing Problem (LRP). Then, to find a suitable model for the problem in this study we search the existing LRP literature. In doing so, we first create a classification scheme consisting of 17 problem characteristics. We then classify 78 previous articles and state the aspects which have received little or no attention so far. In summary; only one study gives an explicit mathematical formulation including hard time windows, three of the three-layer studies locate facilities at two different layers, all studies distribute single product except for four and only one study allows demand points to be supplied by more than one vehicle and/or depot. However, we show that the Mobile-ADS design problem includes all of these four aspects in it. Hence, the problem in this study is relatively new and the literature does not contain any model that can handle it.

In Chapter 3, we develop a static 4-index (considering arc-based product flow) mixed integer programming formulation that integrates all above mentioned aspects. We then derive several problem specific valid inequalities to lessen the solution time.

In Chapter 4, we introduce a static 3-index (considering node-based product flow) mixed integer programming formulation. To the author's knowledge, such a programming approach has not been used in LRP formulations before. We also present some valid inequalities for this model. Finally, we show that 3-index formulation has fewer decision variables and constraints then 4-index formulation.

In Chapter 5, we test the valid inequalities on 6 moderate size hypothetical problem instances. By doing so, we determine the valid inequalities that help reduce the solution time of each formulation in the problem instances we examined.

In Chapter 6, we evaluate the performances of both formulations on 6 small size and 12 large size realistic problem instances with and without the valid inequalities. We first introduce the base scenario and test some valid inequalities. Then, we present the objective values and solution times of the models in each problem instance. Briefly, we conclude that with the valid inequalities 4-index formulation performs better than 3-index formulation in the 12 problem instances. However, in general 3-index formulation outperforms the other without the valid inequalities.

In Chapter 7, we provide two heuristic solution methods of which the first is a "VRP first-LRP second" and the second is a "LRP first-VRP second" approach. In the first method, we solve a VRP for each Mobile-TP and then solve a single LRP depending on the solutions of these VRPs. In the second method, we first solve a single LRP and then solve a VRP for each opened Mobile-TP in the solution of the LRP.

In Chapter 8, we compare the performances of the two heuristics with those of the original formulation in each problem instance. We show that, in most of the instances, heuristics provide a better solution than the original model in at least 94% less time. To be more specific, VRP first-LRP second (LRP first-VRP second) attains better solutions in about 11-16 minutes (1 hour) than the original model obtains in about 18-21 hours.

In Chapter 9, we extend the static 4-index formulation over time and present a dynamic formulation to cover entire battle duration. We assume that entire planning horizon is partitioned into consecutive 24-hour time periods. We also assume that the locations of combat units and potential locations of Mobile-TPs are known in every 24-hour planning period. We solve a sample two-period scenario and present the solution.

In Chapter 10, we show how the static model could assist in a multi-day combat operation. To do so we provide a framework to guide in the successive use of the static model for multi-period strategic decision making. Finally, we discuss how the static model can help the logistics planners when faced with unplanned combat situations where spontaneous problems must be solved in a short amount of time.

As stated in [1], large scale complex applications, dynamic multiple period models, time windows, stochastic parameters, and applications of metaheuristics are either very few or not exist in the LRP literature. In one of the recent surveys, [59], authors report the key dimensions of LRPs that have not been fully incorporated as follows; stochastic parameters, time windows, dynamic multiple period models, multiple objectives, both delivery and pickups, benchmarks for solution efficiency, inventory and large scale complex applications. In the most recent review of LRP literature, [64], authors suggest the following areas for further research; using of approximation formulae, dynamic multiple period models, stochastic parameters, continuous solution space, inventory, multiple objectives, competitive location, combining several heuristics together or combining heuristics with exact methods and large scale complex applications.

Common research directions suggested by the authors of the reviews of the LRP literature can be highlighted as follows. Complex and diverse real world applications of LRP to untouched actual problems, other than well studied depot-customer setting, are needed to extend the spectrum of LRPs. Incorporation of dynamic nature as multiple periods into the LRP models are expected to enhance the realism of the models. Consideration of the time windows in LRPs is a necessity especially in todays logistics environment where the value of time becomes an important factor. Combining inventory decisions into LRP models is advised to examine the interaction among location, routing and inventory.

We believe that our dissertation enriches the LRP literature by addressing some of the key dimensions of the location-routing, that have been rarely included in the previous LRP models and suggested for further research. To begin with, we provide a broader perspective of the LRP literature encompassing most of the past classifications both in terms of the number of surveyed problem characteristics and the number of reviewed articles. We hope this wider coverage leads to a better understanding of where the LRP literature stands now and to where it should proceed, revealing the gaps that require further exploration by the researchers.

We develop a decision support tool for an actual complex real world military problem that extends the spectrum of the LRP literature. In [64], the last review of LRP literature, [60] (solves a single-period military equipment location problem by a decomposition heuristic) and [7] (solves a multi-period plant-depot-facility-customer distribution problem that includes some inventory aspects by CPLEX) are given as the examples of the most complex LRP models. However, both studies have no time windows, distribute single products, allow the demands to be met by only one vehicle or depot. In this dissertation, we first provide a complex single-period static model (for short term tactical decisions) that considers hard time windows, distributes multiple products among three layers, and supplies demand points with multiple vehicles or depots if needed. We, then, extend the static model over time (for long term strategic decisions) to the case with multi-periods within the planning horizon incorporating the dynamic nature of the problem. We assume that even our static model, let alone the dynamic version, surpasses most of the previous studies in terms of complexity and fills some of the gaps, which we determine above, in the LRP literature. Furthermore, we also demonstrate a framework, which can be utilized as a heuristic solution methodology for the dynamic model, in which the static model can be used successively such that each period's solution provides an input for the next period.

Due to its complexity LRP models are generally hard to solve. To overcome this disadvantage, researchers usually resort to heuristics or urge the use of them instead of investigating different mathematical formulations. However, the 3-index model we develop in this dissertation is a promising attempt to improve the computational efficiency of the LRP models purely through modeling. The novelty of our 3-index mathematical formulation stems from the incorporation of node-based product-flow approach that is frequently used in vehicle routing literature. The proposed formulation enables us to decrease the number of decision variables and constraints in the model, which in turn leads to better computational performances. We anticipate that applying new formulations, which are tested and proved to be useful in other areas, to LRP models has the potential to improve the efficiency, and thus may pave the way for successful applications of LRP models to significant real world problems. Hence, one other contribution of this dissertation lies in its mathematical formulation leading to an enrichment of the LRP literature.

We analyze the Mobile-ADS design problem as an LRP in great detail, however we conclude that there are several features of this problem that deserve further consideration. Including inventory decision into our formulations would increase the reality of our models, since usually some amount of ammunition inventory is kept on the battlefield. However, we need to state here that one of our main objectives in developing our models is to keep these ammo inventory levels at minimum at the Fixed-TPs and preferably no inventory at Mobile-TPs. Another possible extension would be the incorporation of Geographical Information Systems, which are becoming more and more popular every day, into our models as proposed by [1]. Frequent changes in the combat environments usually render long term logistics support plans obsolete or induces repeated updates to them. These circumstances generally include combat units, engaged with the enemy, waiting desperately for the ammunition supply–a very undesirable situation on the battlefield. Such unplanned contingencies require rapid short term decisions that can be made fast enough only by feeding the new data into the models in a short amount of time. Geographical Information Systems, with their user-friendly interfaces, can enable the easy and fast entry of large amounts data into the models leading especially to faster short term decisions and can enhance the applicability of our models as real time decision support systems on the battlefield.

Although Mobile-ADS design models developed in this dissertation may seem to be derived for the specific ammunition distribution military logistics problem, we strongly believe that they can be applied to a wide variety of distribution systems after some straightforward modifications. Since they contain most of the real world aspects, we hope that they will help model complex distribution systems more realistically.

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Appendix A

Model specifications

Tables A.1 and A.2 list the sets and parameters that are used throughout this dissertation by 4-index and 3-index formulations. Table A.3 gives the decision variables of the 4-index formulation, whereas Table A.4 presents those of the 3-index formulation.

Table A.1: Sets

N :	set of all nodes such that $N = N_F \bigcup N_M \bigcup N_C$ and N_F, and N_F, N_M, N_C are mutually exclusive.
N_F :	set of potential Fixed-TP nodes such that $N_F \subset N$.
N_M :	set of potential Mobile-TP nodes such that $N_M \subset N$.
N_C :	set of combat unit nodes such that $N_C \subset N$.
Note that $N_{FM} = N_F \bigcup N_M$ and $N_{MC} = N_M \bigcup N_C$.	
V :	set of all vehicles such that $V = V_F \bigcup V_M$ and V_F, V_M are mutually exclusive.
V_F :	set of commercial trucks (all stationed at Fixed-TPs) such that $V_F \subset V$.
V_M :	set of ammo trucks (all stationed at Mobile-TPs) such that $V_M \subset V$.
P :	set of ammo types.

Table A.2: Parameters

Q_{ip} :	demand of combat unit i for ammo type p.
CD_{ip} :	nonnegative capacity of transfer point i for ammo type p.
CV_{vp} :	nonnegative capacity of vehicle v for ammo type p.
CT_v :	nonnegative total capacity of vehicle v.
TI_{ij} :	travel time between nodes i and j, which includes the service time at node i.
TE_{ip} :	earliest time that combat unit i can receive supplies of ammo type p.
TL_{ip} :	latest time that combat unit i can receive supplies of ammo type p.
TM_p :	maximum latest arrival time of ammo type p among units, that is $TM_p = \max_{i \in N_C}\{TL_{ip}\}$.
TM :	maximum of the latest arrival times of all ammo types, that is $TM = \max_{p \in P}\{TM_p\}$.
TC_{vp} :	cost of transporting one unit of ammo type p on vehicle v per hour.
VC_v :	cost of acquiring vehicle v.
DC_v :	cost of driving vehicle v per hour.
FC_i :	fixed cost of opening transfer point i.

Table A.3: Decision variables of the 4-index formulation

f_{ijvp} :	nonnegative amount of flow of ammo type p carried from node i to j by vehicle v.
tp_{ip} :	nonnegative arrival time of ammo type p at node i.
tv_{iv} :	nonnegative arrival time of vehicle v at node i.
y_i :	1, if transfer point i is opened; 0 otherwise.
x_{ijv} :	1, if vehicle v travels from node i to j; 0 otherwise.
w_{ijp} :	1, if ammo type p travels from node i to j; 0 otherwise.

Table A.4: Decision variables of the 3-index formulation

$ftpout_{ivp}$:	nonnegative amount of flow of ammo type p sent from Fixed-TP i by commercial truck v.
$mtpout_{ivp}$:	nonnegative amount of flow of ammo type p sent from Mobile-TP i by ammo truck v.
$mtpin_{ivp}$:	nonnegative amount of flow of ammo type p dropped to Mobile-TP i by commercial truck v.
$cuin_{ivp}$:	nonnegative amount of flow of ammo type p dropped to combat unit i by ammo truck v.
tp_{ip} :	nonnegative arrival time of ammo type p at node i.
tv_{iv} :	nonnegative arrival time of vehicle v at node i.
y_i :	1, if transfer point i is opened; 0 otherwise.
x_{ijv} :	1, if vehicle v travels from node i to j; 0 otherwise.
k_{ivp} :	1, if ammo type p is dropped to Mobile-TP (combat unit) i by commercial (ammo) truck v, or if ammo type p is sent from Mobile-TP i by ammo truck v; 0 otherwise.