STATUS-SEEKING AND CATCHING UP IN THE STRATEGIC RAMSEY MODEL

A Master's Thesis

by

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by

MEHMET ÖZER

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September 2008

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

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September 2008

This thesis analyzes the qualitative implications of the strategic interaction on the standard Ramsey model in terms of catching up. We have shown that the strategic interaction among agents in the economy leads the poor to be able to catch up with the rich, which is not the case for the standard Ramsey model where the initial wealth differences perpetuate. Secondly, within this framework, we incorporate the relative wealth effect and conclude that the catching up amoung agents depends on the share of two classes in the economy. If the share of two classes is same, there exist unique symetric steady state, whereas if the share of two classes are different the steady state is assymetric. Morever, the steady state level of aggregate capital stock is higher than the that of standard Ramsey model. Finally, we introduce the relative consumption effect and reach the conclusion that whatever the share of classes, the gap between the initial wealth level of two classes will disappear in the long run. In addition, the steady state level of aggregate wealth level is same with the that of standard Ramsey model.

Keywords: Strategic interaction, Status-seeking, Catching up, Ramsey model.

ÖZET

STRATEJİK RAMSEY MODELİNDE STATÜ ARAYIŞI VE YAKINSAMA

Mehmet Özer

Yüksek Lisans, Ekonomi Bölümü

Tez Yöneticisi: Yrd. Doç. Dr. Hüseyin Çağrı Sağlam

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Bu tezde stratejik etkileşimin, standart Ramsey modelin sınıflarn birbirine yakınsaması hakkındaki sonuçlarını niteliksel olarak nasıl değiştirdiğini inceledik. Ekonomideki bireyler arasındaki stratejik etkileşimin, fakirler ve zenginler arasındaki başlangıç servet farklılığını durağan dengede ortadan kaldırdığını gösterdik. Standart Ramsey modelde, başlangıç servet farklılıkları uzun dönem durağan dengede de sürmektedir. İkinci modelde, stratejik ilişki iskeletine sadık kalarak göreceli servet etkisini fayda fonksiyonuna eklemledik. Bu model altında, ekonomideki iki sınıfın birey sayılarının eşit olması durumunda durağan dengenin tek ve simetrik olduğu sonucuna vardık. Diğer taraftan, iki sınıfın birey sayılarının farklı olması, başlangıç servet farklılıklarının durağan dengede de süregelmesine neden olmaktadır. Bunun yanısıra, göreceli servet etkisinin ekonomideki durağan denge toplam sermaye miktarının standart Ramsey modeldekinden fazla olmasına neden olduğu sonucuna ayrıca ulaştık. Son olarak, yine stratejik etkileşim iskeletine sadık kalarak, modele göreceli tüketim etkisini ekledik ve vardığımız sonuç sınıfların ekonomideki oranlarından bağımsız olarak durağan dengede fakirlerin zenginlere yetişmekte olduğudur. Bu model altında, ekonomideki durağan denge toplam sermaye miktarı standart Ramsey modelindekiyle aynı olmaktadır.

Anahtar Kelimeler: Stratejik Etkileşim, Statü Arayışı, Yakınsama, Ramsey Modeli.

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CHAPTER 1

INTRODUCTION

1.1 Literature Survey

The dynamic general equilibrium model developed by Ramsey (1928) is one of the most popular frameworks for dynamic macroeconomic analysis. It can be viewed as a model of competitive capital accumulation that describes the interaction of firms and households on the markets for output, labor, and capital. The model economy consists of a representative firm and infinitely many rational households. Households are infinitely lived and own the production factors, capital and labor services. The firm hires capital and labor from the households and produces a single output on a perfectly competitive market. The output is bought by the households and is either used for consumption or saving to form future capital. Households maximize their discounted lifetime utility depending on their consumption of the output good subject to their intertemporal budget constraint. Under the assumption that there are infinitely many households, it must be noted that each household acts as a price taker on all markets (see *Sorger, 2007*).

Several extensions of the standard Ramsey model have been proposed to analyze the long-run distribution of wealth among heterogeneous agents in the economy. There are important differences in representative agent dynamic equilibrium models and models with heterogeneous agents. Becker

(1980) demonstrated that the economy's long-run stationary state capital would be concentrated in the most patient household if the households, forbidden to borrow against their future labor income, differ in their discount rates (Ramsey conjecture). On the other hand, if all households have the same time-preference rate, then the long-run distribution of wealth distribution depends on the initial distribution of wealth and is therefore history-dependent (see Kemp and Shimomura, 1992; Sorger, 2006). Consistent with these, Van Long and Shimomura (2004) showed that the initial wealth inequality will persist in the long run so that the poor individuals will never be able to catch up with the rich in such a Ramsey model economy.

In order to overcome such an important drawback of Ramsey model economy, wealth-dependent preference or time-preference rates, wealth-dependent capital returns have been proposed[1]. Lucas and Stokey (1984), for example, assume that the wealthier a household is the more impatient it becomes. Sarte (1997) assumes progressive taxation of capital income, which implies that the return to capital is decreasing with respect to the wealth of a household. Van Long and Shimomura (2004) consider relative wealth (status seeking) as an argument of the reduced form utility functions of the individuals as relative wealth yields greater social status and status matters for individual well-being[2].

In this analysis, we will concentrate on yet another mechanism that leads to catching up in such a Ramsey model economy. This mechanism rests on the observation that the price taking behavior is no longer a reasonable assumption if one considers finite number of agents or finite number of income groups (classes) in the economy. As a matter of fact, as there are only

[1] Another modification of the standard Ramsey–Cass–Koopmans model proposed to address the problem with the long-run wealth distribution relies on uncertainty. See Becker and Zilcha (1997), Aiyagari (1994) or Krusell and Smith (1998).

[2] Formally, all of these approaches imply that the wealth of a household appears in its Euler equation.

finitely many households, each one of them can take the effects of their decisions on market prices into account and realize that the strategic action is inevitable. Indeed, Sorger (2002) proposed a strategic model in which the households understood that their capital accumulation decisions directly influenced capital's rental price, although they still behaved competitively in the labor market and showed that this prevents the Ramsey conjecture from coming true. As the most patient household should realize that a reduction of its own capital supply increases the rental rate on capital and, hence, its own capital income, the resulting higher return on capital, in turn, would induce also less patient households to acquire positive capital stocks in the long run. These observations become especially important if one takes into account the fact that relative wealth yields greater social status and status matters for individual well-being.

In what follows, we will analyze the competitive and strategic forms of the Ramsey model and identify to what extent the strategic interaction among agents in the economy affects the long run wealth distribution and hence, catching up. However, since the simpler case of the competitive model provides guidance for the strategic model's possibilities, we will first present, or recall in detail, some of the results from that theory.

1.2 Ramsey Model and Catching Up

Kemp and Shimomura (1992) and Van Long and Shimomura (2004) examine a heterogeneous agent version of Ramsey model where private agents were assumed to differ only in their initial wealth. They analyze whether the initial wealth differences will perpetuate and persist in the steady state or fade away so that catching up occurs. To do so, the economy is assumed to consist of two groups of individuals; those who are initially rich and those who are initially poor. The measure of the set of initially rich and initially poor individuals are α_1 and α_2, respectively with $\alpha_1 + \alpha_2 = 1$. The initial capital stock of a poor individual is k_{10} and that of rich one is $k_2(0) > k_1(0)$. Each individual taking the paths of the rental rate, $r(t)$ and the wage rate, $w(t)$ as given in a perfectly competitive economy solves the following optimization problem where $U(c_i)$ denotes the utility obtained from consumption and k_{i0} are exogenously given initial level of wealth:

$$\forall i \in \{1,2\} \qquad \max_{c_i(t)} \int_0^\infty e^{-\rho t} U(c_i(t))dt \tag{$\mathcal{P}$}$$

subject to

$$\begin{aligned} \dot{k_i}(t) &= r(t)k_i(t) + w(t) - c_i(t), \\ k_i(0) &= k_{i0}, \end{aligned}$$

and

$$\lim_{t\to\infty} k_i(t) e^{\left(\int_0^t r(s)ds\right)} = 0,$$

The utility function is assumed to have constant elasticity of marginal utility ($\beta =$ constant) and the production function is assumed to verify all neoclassical properties. Since in a competitive equilibrium the rental rate is given by $r = f^{'}(k)$ and the wage rate is $f(k) - kf^{'}(k)$, one can easily obtain the following system of four differential equations:

$$\dot{k_i}(t) = f^{'}(k(t))k_i(t) + f(k(t)) - k(t)f^{'}(k(t)) - c_i(t), \tag{1.1}$$

$$\dot{c_i}(t) = \beta c_i(f^{'}(k(t)) - \rho), \tag{1.2}$$

where $k = \alpha_1 k_1 + \alpha_2 k_2$.

The standard neoclassical properties assumed for the utility and the production functions ensure the existence of a unique positive steady state. At the steady state, the level of aggregate capial stock is characterized by

$f^{'}(k^*) = \rho$. One can easily note from (1.2) that

$$\frac{\dot{c_1}(t)}{\dot{c_2}(t)}\frac{c_1(t)}{c_2(t)} = \beta(f^{'}(k(t)) - \rho),$$

and the ratio of $c_2(t) \setminus c_1(t)$ is constant over time.

In what follows, it is shown that if $k_1(0) \neq k_2(0)$, then $k_1(t)$ and $k_2(t)$ will converge to two different stock levels, $k_1^* \neq k_2^*$.

Proposition 1 *We have, $k_1^* = k_2^*$ if and only if the initial wealth levels are identical. Moreover, there exists a continuum of steady state wealth distributions and a corresponding continuum of one-dimensional stable manifolds so that inequalities persist.*

Proof See Van Long and Shimomura (2004). □

The studies of Kemp and Shimomura (1992) and Van Long and Shimomura (2004) show that the initial wealth inequality will persist in the long run and hence, the poor individuals will never be able to catch up with the rich in such a Ramsey model economy.

In order to reach a better understanding of the notion of catching up in neoclassical growth models, it is inevitable to introduce wealth (status seeking) as an argument of the reduced form utility functions of the individuals, and consider whether the poor individuals may be willing to sacrifice consumption in the early stage of their life to build up wealth and eventually catch up with the rich as well (see Corneo and Jeanne, 2001 and Van Long and Shimomura, 2004).

1.3 Status-Seeking and Catching Up

The assumption that the households take utility only from their own consumption has been shown to be not so realistic in a dynamic perspective.

Under this assumption, as the income of an individual, and hence his consumption increase, one would expect that its welfare will increase as well. However, despite the continually rising prosperity in the developed countries, there are considerable fluctuations in the percentage of those who say they were very satisfied in terms of their welfare. Consistent with this, Ehrhardt and Veenhoven (1995) shows that the percentage of those who have attained the highest level of welfare over time is almost constant and even sometimes declining as the prosperity increase. Therefore, it is inevitable to think of a model in which the households do not take utility only from their own consumption but also from their relative position in the society.

Veblen (1922) notes that, it is not wealth but relative wealth which is important for the human being. It is argued that relative wealth yields greater social status and status matters for individual well-being. Bakshi and Chen (1996) provide empirical support to the spirit of the capitalism hypothesis (wealth accumulation not only for consumption) and show that the investors acquire wealth not just for its implied consumption, but also for its induced status. Cole *et al.* (1992), Corneo and Jeanne (1997) present that when individuals care about their social status, optimal saving behavior is affected in systematic ways and the normative properties of the equilibrium path strongly differ from the conventional models.

In order to analyze the effect of such an empirically relevant status seeking motive on catching up, Van Long and Shimomura (2004) incorporate the relative wealth of each agent in their utility function in an additively separable manner. In a perfectly competitive set up, each individual takes also the path of aggregate capital stock in the economy as given and solves,

$$\forall i \in \{1,2\} \text{ and } \forall t \in \mathbb{R}_+ \qquad \max_{c_i(t)} \int_0^\infty e^{-\rho t} \left(U(c_i(t)) + V\left(\frac{k_i(t))}{k(t)}\right)\right) dt \qquad (\mathcal{P}')$$

subject to

$$\dot{k}_i(t) = r(t)k_i(t) + w(t) - c_i(t),\ k_i(0) = k_{i0},\ \text{and}$$
$$\lim_{t\to\infty} k_i(t)e^{\left(\int_0^t r(s)ds\right)} = 0.$$

The necessary conditions of optimality leads to following system of differential equations:

$$\dot{k}_i(t) = f^{'}(k(t))k_i(t) + f(k(t)) - k(t)f^{'}(k(t)) - c_i(t), \quad (1.3)$$
$$\dot{c}_i(t) = \beta c_i \left(f^{'}(k(t)) - \rho + \frac{1}{k(t)} \frac{V'\left(\frac{k_i(t))}{k(t)}\right)}{U'(c_i(t))} \right), \forall i \in \{1,2\}. \quad (1.4)$$

Assuming a strictly concave function $V(.)$ implies, a strong incentive for the poor to accumulate as a poor individual attributes a higher value to a marginal increase in his relative wealth than a rich individual. Indeed, if the elasticity of marginal utility of relative wealth is greater than the elasticity of marginal utility of consumption, there exists a symmetric steady state ($k_1^* = k_2^* = k^*$ and $c_1^* = c_2^* = c^* = f(k^*)$) and there are no asymmetric steady states to this model implying that poor individuals will be able to catch up with the rich.

Recently, the "envy" effect namely, the "keeping up with Joneses" assumption has been put forward as a way of incorporating the status-seeking motives of the individuals in neoclassical growth models. According to this, people take utility from their relative consumption with respect to the level of consumption in their peer group or in the aggregate economy. This is based on the observation that the property acquisition and the conspicuous consumption are two conventional bases for social esteem in the sense that the households would consume conspiciously in order to increase their social status (see Veblen, 1922). Raucher (1997) analyzes whether this social status-seeking behavior, accelerate economic growth and whether the capital accumulation should be subsidized to correct for status externality. Fisher and Hof (2000) incorporates the envy effect into the Ramsey model and study the match between the decentralized and social planner solutions and propose the optimal taxation in presence of such consumption externality. Turnovsky and Penelosa (2007) have shown that in case of heterogeneous agents, this effect cause less inequality than the case of no consumption externality. De la Croix (1998) internalizes the relativity of satisfaction using habit formation and analyzes its dynamic implications in a Ramsey model economy. However, it must be noted that these papers do not take into account the strategic interactions among agents in the economy. As relativity concern directly leads agents to decide their path of consumptions and savings strategically, the analysis of the strategic Ramsey model with status-seeking agents becomes increasingly important.

CHAPTER 2

MODEL

The main assumption in all of the models discussed so far is that there are infinitely many agents in the economy and hence, the agents are price takers in all markets. However, as pointed in Sorger (2006), this common assumption of infinitely many households is obviously unjustified. In reality, we have finite number of agents but this number is so great that someone can ignore the individual's effect on the aggregate variables. As a matter of fact, social or economic similarities enforce individuals to constitute small number of groups (classes) containing the same type of individuals (in terms of their relative position in the economy). The members of each group have similar tendency in their social and economic decisions. Therefore, we are in an economic structure where there are powerful groups affecting the economy wide variables. Since members of each group are rational, they should be aware of their market power. Indeed, this awareness directly motivates individuals to act strategically. Thus, a model with relativity and catching up concern including the agents' awareness of their market power on the variables would be more consistent if one considers finite number of agents in the economy. Thus, our primary aim is to incorporate strategic behavior into such models and to analyze the qualitative implications of them in terms of long run wealth distribution, and hence catching up.

In what follows, we will analyze how the conclusions of Ramsey growth model on the long run distribution of wealth, and hence catching up would change when one takes into account the influence of the decisions of each agent on the aggregate variables as well.

2.1 Strategic Ramsey and Catching up

We propose a strategic Ramsey equilibrium model in which the households understand that their capital accumulation decisions directly influence the capital's rental price, and the wage rate. Households differ only in their initial wealth, so each supplies one unit of labor inelastically. The set of households is $H = \{1, 2, ..., n\}$, where $n \in \mathbb{N}$ denotes the number of households. Following Van Long and Shimomura (2004), we assume that there are two groups of households; those who are initially rich and those who are initially poor. The measure of the set of initially rich and poor individuals are α and $(1 - \alpha)$, respectively. Except for the assumption that households realize their market power, this is the standard infinite-horizon model with heterogeneous agents.

Let c_i denote the consumption level of type i individual and K_i his wealth. The aggregate production function, $Y = f(K) = F(K, N)$ has the usual neoclassical properties. The properties of the utility and the production functions are detailed in the following assumptions.

Notation 1 $U_i : \mathbb{R}_+ \to \mathbb{R}_+$ *is twice continuously differentiable on* $\mathbb{R}_{++}$ *with* $U_i(0) = 0$, $U_i^{'} > 0$, $U_i^{'}(0_+) = \infty$, *and* $U_i^{''} < 0$ *for each* $i \in \{1, 2\}$.

Notation 2 $f : \mathbb{R}_+ \to \mathbb{R}_+$ *is twice continuously differentiable on* $\mathbb{R}_{++}$ *with* $f^{'} > 0$, $f^{''} < 0$ *and satisfies the Inada conditions, i.e.* $\lim_{k\to 0} f'(K) = \infty$ *and* $\lim_{k\to\infty} f'(K) = 0$.

The represantative firm maximizes its profit where $f'(K) = F_K(K, N)$, $F_L(K, N) = [f(K) - Kf^{'}(K)] \setminus N$ and $K = \alpha N K_1 + (1 - \alpha) N K_2$. The rental return on capital is $f^{'}(K(t))$ and the wage earning for the one unit of inelastically supplied labor is $[f(K(t)) - K(t)f^{'}(K(t))] \setminus N$.

In strategic equilibrium, each household when maximizing his lifetime utility subject to the usual budget constraints will take into account the influence of his accumulation decisions on the capital's rental price and the wage rate. Then, the problem of an individual i recast as follows:

$$\forall i \in \{1,2\} \text{ and } \forall t \in \mathbb{R}_+ \qquad \underset{c_i(t)}{max} \int_0^\infty e^{-\rho t} U(c_i(t))dt \tag{2.1}$$

s.to

$$\dot{K}_i(t) = f'(K(t))k_i(t) + \frac{f(K(t)) - K(t)f'(K(t))}{N} - c_i(t), \tag{2.2}$$

$$K_i(0) = K_{i0}. \tag{2.3}$$

The Hamiltonian for the optimization problem of an individual belonging to first group is

$$H(c_1(t),\ K_1(t),\ \lambda_1(t)) = e^{-\rho t}U(c_1(t)) + \lambda_1(t)\left(f'(K(t))K_1(t) + \frac{f(K(t)) - K(t)f'(K(t))}{N} - c_1(t)\right) \tag{2.4}$$

The set of necessary conditions of optimality will then be written as folows:

$$H_{c_1}(\ c_1(t),\ K_1(t),\ \lambda_1(t)\ ) = 0 \Longrightarrow e^{-\rho t}U'(c_1(t)) = \lambda_1(t), \tag{2.5}$$

$$H_{K_1}(c_1(t), K_1(t), K(t), \lambda_1(t)) = -\dot{\lambda}_1(t)$$

which implies

$$-\frac{\dot{\lambda}_1(t)}{\lambda_1(t)} = f'(K(t)) + K_1(t)\alpha N f''(K(t)) + \frac{f'(K(t))\alpha N - \alpha N f'(K(t)) - K(t)\alpha N f''(K(t))}{N}, \tag{2.6}$$

and

$$H_{\lambda_1(t)} = \dot{K}_1(t) \Longrightarrow \dot{K}_1(t) = f^{'}(K(t))K_1(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_1(t).$$

The necessary conditions are also sufficient if the limiting transversality condition $\lim_{t \to 0} e^{-\rho t}\lambda_1(t)K_1(t) = 0$ holds. From equations (2.5) and (2.6), we get the Euler equation:

$$\frac{\dot{c_1}(t)}{c_1(t)} = \frac{1}{\theta}\left[f^{'}(K(t)) - \rho + (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t))\right] \tag{2.7}$$

under CIES form of utility funtion with an intertemporal elasticity of substitution, θ.

Similarly, one can easily write the Hamiltonian and the corresponding set of necessary conditions of optimality for the problem of a type 2 agent ($1-\alpha$ share group) and solve accordingly. We have then the following system of four differential equations:

$$\frac{\dot{c_1}(t)}{c_1(t)} = \frac{1}{\theta}\left[f^{'}(K(t)) - \rho + (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t))\right] \tag{2.8}$$

$$\frac{\dot{c_2}(t)}{c_2(t)} = \frac{1}{\theta}\left[f^{'}(K(t)) - \rho - (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t))\right] \tag{2.9}$$

$$\dot{K}_1(t) = f^{'}(K(t))K_1(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_1(t) \tag{2.10}$$

$$\dot{K}_2(t) = f^{'}(K(t))K_2(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_2(t) \tag{2.11}$$

A steady state is a quadruple $(K_1^*, K_2^*, c_1^* c_2^*)$ such that the right hand sides of the equations (2.8)-(2.11) equal to zero. A steady state is said to be symmetric if $K_1^* = K_2^*$, and $c_1^* = c_2^*$. Accordingly, a steady state is said to be asymmetric if $K_1^* \neq K_2^*$.

In the following proposition, taking into account the strategic interaction amoung agents, we show that the catching up prevails in the economy, so that even if the agents have initially different level of wealth, they will reach to the equal level of wealth at the steady state.

Proposition 2 *Under Assumptions (1) and (2), there exists a unique symmetric steady state and there are no asymmetric steady states.*

Proof From equations (2.10) and (2.11), at the steady state we have:

$$\begin{aligned} f^{'}(K^*) - \rho &= -(1-\alpha)\alpha N f^{''}(K^*)(K_1^* - K_2^*) \\ f^{'}(K^*) - \rho &= (1-\alpha)\alpha N f^{''}(K^*)(K_1^* - K_2^*) \end{aligned}$$

It is obvious that the left hand sides of these two equations are equal. Therefore, the right hand sides should also be equal. Since we have the assumption of strict concavity on production function, $f^{''}(K^*) < 0$, the condition that satisfies these two equations simultaneously is $K_1^* = K_2^*$. □

2.1.1 Steady state and the stability analysis

Linearizing the equations (2.8)-(2.11) around their unique steady state gives the following 4×4 Jacobian matrix[1]:

$$\text{J} \equiv \begin{bmatrix} 0 & 0 & \theta N f(K^*) f^{''}(K^*)\alpha(2-\alpha) & \theta N f(K^*) f^{''}(K^*)(1-\alpha)^2 \\ 0 & 0 & \theta N f(K^*) f^{''}(K^*)\alpha^2 & \theta N f(K^*) f^{''}(K^*)(1-\alpha^2) \\ -1 & 0 & f^{'}(K^*) & 0 \\ 0 & -1 & 0 & f^{'}(K^*) \end{bmatrix}$$

In order to find the characteristic roots of the Jacobian matrix, we solve $\det[J - \mu I] = 0$ and obtain the following eigenvalues:

[1] The linearization of dynamic equations is given in the Appendix.

$$
\begin{aligned}
\mu_1 &= \frac{1}{2}(B - \sqrt{B^2 - 4AC\phi}) \\
\mu_2 &= \frac{1}{2}(B + \sqrt{B^2 - 4AC\phi}) \\
\mu_3 &= \frac{1}{2}(B - \sqrt{B^2 - 8\alpha AC\phi + 8\alpha^2 AC\phi}) \\
\mu_4 &= \frac{1}{2}(B + \sqrt{B^2 - 8\alpha AC\phi + 8\alpha^2 AC\phi})
\end{aligned}
$$

where $A = f(K(t))$, $B = f^{'}(K(t))$, $C = f^{''}(K(t))$, and $\phi = \theta N$. One can easily see that μ_2 and μ_4 are positive whereas μ_1 and μ_3 are negative. Therefore, we have two positive and two negative real characteristic roots implying that the system is stable in the saddle point sense. This implies that the poor will be able to catch up with the rich in a strategic Ramsey economy.

It is clear from this analysis that introducing strategic interaction among agents changes the qualitative properties of the standard Ramsey model. In the absence of strategic interaction, poor will never be able to catch up with the rich as pointed in Van Long and Shimomura (2004). However, incorporating the strategic behavior among agents leads to the wealth level of the two classes to be the same at the stationary state. However, it must be noted that the strategic interaction among agents leads to a change in the transitional dynamics and the catching up property of the standard Ramsey model, the aggregate level of capital stock is left unchanged at the stationary state.

2.2 Strategic Ramsey Model with relative wealth

Van Long and Shimomura (2004) proved that if relative wealth appears in the reduced form utility function (because of the status concern) then the poor will catch up with the rich if the elasticity of the marginal utility of relative wealth is greater than the elasticity of marginal utility of consumption. The crucial questions here are as follows: if the agents in the economy realize their effects on the aggregate variables, will this affect the qualitative results of the Van Long and Shimomura (2004)? How will the results differ from the strategic Ramsey model of the previous section?

Since Veblen (1922), economists take relative wealth into account as a proxy of the social status. In these models, individuals do not take utility only from their consumption but also from their relative wealth. This relative wealth effect has been put into utility function in an additively separable way[2]. However, in our model the decision of an individual on consumption and accumulation of capital affects the average level of wealth and the factor incomes.

Again, we have N individuals populated in the economy and two groups of people differing only in terms of their initial capital stock. The share of the two groups in the population are α and $1-\alpha$, respectively. In addition, individuals supply their one unit of labor inelastically.

The economic problem of an individual $i \in \{1,2\}$ is the maximization of the lifetime utility subject to the law of motion of the respective capital stock. For all $i \in \{1,2\}$,

$$\max_{c_i(t)} \int_0^{\infty} e^{-\rho t} \left(U(c_i(t)) + V\left(\frac{K_i(t))}{\alpha K_1(t) + (1-\alpha)K_2(t)} \right) \right) dt \qquad (2.12)$$

subject to

$$\dot{K}_i(t) = f^{'}(K(t))K_i(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_i(t), \forall t, \qquad (2.13)$$

$$K_i(0) = K_{i0} \qquad (2.14)$$

[2]Corneo and Jeanne (2001) formalizes the relative wealth as $v(a_t - A_t)$ where A_t denotes average wealth at time t and $a_t$denotes the individual's wealth. This is precisely the way in which Akerlof (1997) incorporates social status into his model. However, for comparison purposes we use the model structure of Van Long and Shimomura (2004).

where

$$V\left(\frac{K_i(t))}{\alpha K_1(t)+(1-\alpha)K_2(t)}\right)=V(s_i).$$

Notation 3 $\forall i\in\{1,2\}$, $\mathbf{V}_i:\mathbb{R}_+\rightarrow\mathbb{R}_+$ *is twice continuously differentiable on* $\mathbb{R}_{++}$ *with* $V_i(0)=0$, $V_i^{'}>0$, $V_i^{'}(0_+)=\infty$ *and* $V_i^{''}<0$.

The strict concavity of $V(s_i)$ means that a poor person gets more pleasure from a marginal increase in his relative wealth than a rich person. It should be noted that this creates a strong incentive for the poor to accumulate.[3]

The Hamiltonian for the optimization problem of an individual belonging to the first group is :

$$\begin{aligned}H(c_1(t),\ K_1(t),\lambda_1(t))&=e^{-\rho t}\{U(c_1(t))+V(\frac{K_1(t))}{\alpha K_1(t)+(1-\alpha)K_2(t)})\}+\\ &\lambda_1(t)\{f^{'}(K(t))K_1(t)+\frac{f(K(t))-K(t)f^{'}(K(t))}{N}-c_1(t)\}\quad(2.15)\end{aligned}$$

Hence, we get the Euler equation:

$$\begin{aligned}\dot{c_1}(t)=\frac{1}{\theta}c_1(t)\{f^{'}(K(t))-\rho+(1-\alpha)\alpha Nf^{''}(K(t))(K_1(t)\\ -K_2(t)-c_1(t)\frac{V'(s_1(t))}{U''(c_1(t))}\frac{(1-\alpha)K_2(t)}{(\alpha K_1(t)+(1-\alpha)K_2(t))^2}\}\end{aligned}$$

and the law of motion of the capital stock:

$$\dot{K_1}(t)=f^{'}(K(t))K_1(t)+\frac{f(K(t))-K(t)f^{'}(K(t))}{N}-c_1(t).$$

Similarly, we can write the problem of an individual belonging to the second group ($1-\alpha$, share group) and solve accordingly. Since the problem

[3]One may notice that individual's relative wealth is defined as the individual's wealth over the average level of wealth in the economy.

and the steps are similar, the exposition of the problem is omitted to avoid repetition. After solving the two problems, we have obtained following the dynamic equations for the Ramsey model in case of strategic interaction with the presence of relative wealth in the reduced form utility function:

$$
\begin{aligned}
\dot{c_1}(t) &= \frac{1}{\theta}c_1(t)\{f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t) - \\
&\quad c_1(t)\frac{V'(s_1(t))}{U''(c_1(t))}\frac{(1-\alpha)K_2(t)}{(\alpha K_1(t) + (1-\alpha)K_2(t))^2})\} &(2.16)\\
\dot{c_2}(t) &= \frac{1}{\theta}c_2(t)\{f'(K(t)) - \rho - (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t) - \\
&\quad c_2(t)\frac{V'(s_2(t))}{U''(c_2(t))}\frac{\alpha K_1(t)}{(\alpha K_1(t) + (1-\alpha)K_2(t))^2})\} &(2.17)\\
\dot{K_1}(t) &= f'(K(t))K_1(t) + \frac{f(K(t)) - K(t)f'(K(t))}{N} - c_1(t) &(2.18)\\
\dot{K_2}(t) &= f'(K(t)K_2(t) + \frac{f(K(t)) - K(t)f'(K(t))}{N} - c_2(t) &(2.19)
\end{aligned}
$$

The effect of relative wealth can be isolated in the terms containing $V'(.)$ in the Euler equations In order to make the steady state analysis possible, we use two different cases in which we put a restriction on the share of classes, α and $1-\alpha$.We will show that the qualitative properties of the steady state changes depending on the value of α.

Proposition 3 *If the share of two classes are different, then there exists at least one economy such that steady state is asymmetric(no catching up) and if the share of two classes are same (i.e $\alpha = 1 \setminus 2$) then the unique steady state is symetric(poor will be able to catch up with the rich).*

Proof **Case 1:** $\alpha \neq 1 \setminus 2$.

Proof of this statement is by contradiction. Assume $\alpha \neq 1 \setminus 2$ and let the steady state be $c_1^* = c_2^* = c^*$ and $k_1^* = k_2^* = k^*$. Then, from the dynamic equations we have

$$-\rho-(1-\alpha)\alpha N f^{''}(k^*)(K_1^*-k_2^*)-\frac{V^{'}(1)}{U''(c^*)}\frac{\alpha K_1(t)}{(\alpha K_1(t)+(1-\alpha)K_2(t))^2}=$$
$$-\rho-(1-\alpha)\alpha N f^{''}(K^*)(K_1^*-K_2^*)-\frac{V^{'}(1)}{U''(c^*)}\frac{\alpha K_1(t)}{(\alpha K_1(t)+(1-\alpha)K_2(t))^2}.$$

Then, we have,

$$\frac{V^{'}(1)}{U''(c^*)}\frac{(1-\alpha)K^*}{(K^*)^2}=\frac{V^{'}(1)}{U''(c^*)}\frac{\alpha K^*}{(K^*)^2},$$

which implies $\alpha=(1-\alpha)$ so that $\alpha=1\backslash 2$; a contradiction. Thus, if $\alpha\neq 1\backslash 2$ the steady state is asymmetric.

Case 2: $\alpha = 1 \setminus 2$

In this part, we take the following functional forms for the utility and the production functions

$$\begin{aligned} f(K(t)) &= (\alpha N K_1(t)+(1-\alpha)N K_2(t))^{\beta}, \\ U(c_i(t)) &= \ln c_i(t), \\ V(z_i(t)) &= \ln z_i(t). \end{aligned}$$

where $\beta = 0.3$ and the population size $N = 100$. From the steady state conditions, we have

$$f^{'}(K^*)K_1^*+\frac{f(K^*)-K^*f^{'}(K^*)}{N}=c_1^*,$$

and

$$f^{'}(K^*)K_2^*+\frac{f(K^*)-K^*f^{'}(K^*)}{N}=c_2^*.$$

Substituting the production function, we obtain that

$$\begin{aligned} c_1^* &= (\frac{1}{2})^\beta N^\beta (K_1^*+K_2^*)^\beta \frac{K_1^*+K_2^*(1-N)}{N(K_1^*+K_2^*)}, \\ c_2^* &= (\frac{1}{2})^\beta N^\beta (K_1^*+K_2^*)^\beta \frac{K_2^*+K_1^*(1-N)}{N(K_1^*+K_2^*)}. \end{aligned}$$

At the steady state, the two dynamic equations, (2.18) and (2.19) are equal to zero, implying that

$$c_2^* \frac{V'(s_2(t))}{U''(c_2(t))} \frac{K_1^*}{2(K_1^*+K_2^*)} - c_1^* \frac{V'(s_1(t))}{U''(c_1(t))} \frac{K_1^*}{2(K_1^*+K_2^*)} = \frac{1}{2} f''(K^*)(K_1^*-K_2^*).$$

Substituting utility and production functions and $\alpha = 1 \setminus 2$ into the above equations, we come up with,

$$\begin{aligned} (\frac{1}{2})^\beta N^{2\beta}(K_1^*+K_2^*)^{2\beta}((\frac{K_2^*+(N-1)K_1^*}{N(K_1^*+K_2^*)})^2 K_1^{*2} - (\frac{K_1^*+(N-1)K_2^*}{N(K_1^*+K_2^*)})^2 K_2^{*2}) = \\ (\frac{1}{2})^{\beta+3} N^\beta \beta(\beta-1)(K_1^*+K_2^*)^{\beta-2}(K_1^*-K_2^*)K_1^*K_2^*. \end{aligned}$$

We know that at steady state we have $K_1^* = \eta K_2^*$ for $\eta \in (0,\infty)$. A further investigation leads that η which satisfies the above equation is either -1 or $+1$. Since the amount of capital stock for each individual at the steady state cannot be negative, the only possible case is $\eta = 1$.Results are robust for the values of N and β. Thus, at the steady state the capital stocks of the two classes are equal so that catching up occurs. □

These two cases imply that if the share of two heterogeneous groups and their initial level of capital stocks are different, then in the long run (i.e. at the steady state) the capital stock for the two groups will be different as well. In other words, there is no catching up among the agents. However, if they have equal shares in the society, whatever the initial level of wealth they have, at steady state, the level of capital stock will be same for all agents. The intuition behind this conclusion is that α and $1-\alpha$ shares shows the relative degree of effectiveness on the aggregate variables where the agents are strategic status seekers and consumers. If the weight of each class is same, classes converge towards each other in terms of their long run wealth.

Since we are focusing on catching up, we will now analyze the level of capital stock and the consumption at this unique symmetric steady state ($\alpha = 1 \setminus 2$) and perform the stability analysis.

2.2.1 Steady state and the stability analysis

The level of capital stock and the consumption at this unique symetric steady state can easily be obtained as follows:

$$f^{'}(K^*) = \rho + c^* \frac{V'(1)}{U''(c_1(t))} \frac{1}{2K^*} \tag{2.20}$$

$$c^* = \frac{f(K^*)}{N} \tag{2.21}$$

After linearizing the dynamic equations around the steady state, we derive the following 4×4 Jacobian matrix[4]:

$$\begin{bmatrix} \frac{c^{*2}}{K^*} & 0 & c^*(\frac{3}{8}Nf^{''}(K^*) - \frac{3c^{*2}}{4K^*}) & c^*(\frac{1}{4}Nf^{''}(K^*) + \frac{c^{*2}}{4K^*}) \\ 0 & \frac{c^{*2}}{K^*} & c^*(\frac{1}{4}Nf^{''}(K^*) + \frac{c^{*2}}{4K^*}) & c^*(\frac{3}{8}Nf^{''}(K^*) - \frac{3c^{*2}}{4K^*}) \\ -1 & 0 & f^{'}(K^*) & 0 \\ 0 & -1 & 0 & f^{'}(K^*) \end{bmatrix}$$

This Jacobian matrix have two positive and two negative real characteristic roots (-0.066, -0.019, 0.1028, 0.1507) implying that the system is stable in the saddle path sense. These results are robust to the parameter values provided that the utility function is in the form of *CIES* and the production function is strictly concave and increasing.

In contrast with Van Long and Shimomura (2004), the relationship between the elasticity of the marginal utility of relative wealth and the elasticity

[4]The linearization of system is in Appendix.

of marginal utility of consumption is not important for the catching up. The crucial element that affects the catching up turns out to be the share of the two classes in the economy. However, our result concerning the steady state level of aggregate capital stock is consistent with that of Van Long and Shimomura (2004). Introducing status concern a la relative wealth and incorporating strategic interaction cause an increase in the steady state level of aggregate capital stock of the economy.

In standard Ramsey model with and without the strategic interaction incorporated, the marginal productivity of capital at the steady state is equal to the constant time preference rate. However, the second term in equation (2.20) implies that the marginal productivity of capital at the steady state is less than the constant time preference rate. Since the production function is strictly increasing and strictly concave, the steady state capital stock in the economy will be higher provided that the share of groups is the same, if individuals are status seekers and act strategically. This result confirms the empirical evidence provided by Bakshi and Chen (1996) as well.

2.3 Strategic Ramsey model with envy effect:

The aim of this section is to investigate the influence of the status seeking behavior a la relative consumption in a standard version of the Ramsey model with heterogeneous agents acting strategically. To do so, we use an additively separable utility function in terms of the agent's own and relative consumption. The other assumptions of the model are the same as in the previous two sections. Accordingly, the individual $i \in \{1, 2\}$, solves the following problem:

$$\max_{c_i(t)} \int_0^\infty e^{-\rho t} \left(U\left(c_i(t)\right) + V\left(\frac{c_i(t))}{\alpha c_1(t) + (1-\alpha) c_2(t)}\right)\right) dt \qquad (2.22)$$

subject to

$$\dot{K}_i(t) = f^{'}(K(t))K_i(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_i(t) \quad (2.23)$$

$$K_i(0) = K_{i0}. \quad (2.24)$$

where the aggregate capital stock is:

$$K(t) = \alpha N K_1(t) + (1-\alpha) N K_2(t),$$

and the average level of consumption is:

$$\bar{c} = \alpha c_1(t) + (1-\alpha) c_2(t)$$

The specifications of function V are stated in the following assumption.

Notation 4 $\forall i \in \{1,2\}$, $V_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ *is twice continuously differentiable on* $\mathbb{R}_{++}$ *with* $V_i(0) = 0$, $V_i^{'} > 0$, $V_i^{'}(0_+) = \infty$ *and* $V_i^{''} < 0$.

We set up the Hamiltonian for the problem of an individual belonging to the first class (α, share group) :

$$H(c_1(t),\ K_1(t),\ \lambda_1(t)) = e^{-\rho t}\left(U(c_1(t)) + V(\frac{c_1(t))}{\alpha c_1(t) + (1-\alpha)c_2(t)})\right) + \lambda_1(t)\left(f^{'}(K(t))K_1(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_1(t)\right). \quad (2.25)$$

The necessary conditions of optimality are as follows:

$$H_{c_1}(\ c_1(t),\ K_1(t),\ \lambda_1(t)\ ) = 0 \Rightarrow$$

$$e^{-\rho t}\left(U^{'}(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V^{'}(z_1(t))\right) = \lambda_1(t),$$

where we denote

$$z_1(t) = \frac{c_1(t))}{\alpha c_1(t) + (1-\alpha)c_2(t)}. \tag{2.26}$$

$$H_{k_1}(c_1(t),\ K_1(t),\ \lambda_1(t)) = -\dot{\lambda_1}(t) \Rightarrow$$

$$-\frac{\dot{\lambda_1}(t)}{\lambda_1(t)} = f^{'}(K(t)) + K_1(t)\alpha N f^{''}(K(t)+$$

$$\frac{f^{'}(K(t))\alpha N - \alpha N f^{'}(K(t)) - K(t)\alpha N f^{''}(K(t))}{N} \tag{2.27}$$

$$\lambda_1(t)\{f^{'}(K(t)) + \alpha N f^{''}(K(t))(K_1(t) - K_2(t))\} = -\dot{\lambda_1}(t) \tag{2.28}$$

$$H_{\lambda_1}(c_1(t),\ K_1(t),\ \lambda_1(t)) = \ \dot{K_1}(t) \Rightarrow$$

$$\dot{K_1}(t) = f^{'}(K(t))K_1(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_1(t) \tag{2.29}$$

$$-\dot{\lambda_1}(t) = \rho e^{-\rho t}\left(U^{'}(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V^{'}(z_1(t))\right) -$$

$$\{\dot{c_1}(t)U^{''}(c_1(t)) + \frac{(1-\alpha)(\dot{c_2}(t)(\bar{c})^2 - 2(\alpha\dot{c_1}(t) + (1-\alpha)\dot{c_2}(t))(\bar{c})c_1(t)}{(\bar{c})^4}V^{'}(z_1(t)) +$$

$$\frac{(1-\alpha)c_2(t)(\dot{c_1}(t)(\bar{c}) - (\alpha\dot{c_1}(t) + (1-\alpha)\dot{c_2}(t))c_1(t))}{(\bar{c})^4}V^{''}(z_1(t))\}$$

Hence, we get the Euler equation:

$$\dot{c_1}(t) = -\left(\frac{U'(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V'(z_1(t))}{G}\right)$$
$$\left(f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t))\right) +$$
$$\frac{V'(z_1(t))2(1-\alpha)^2\frac{\dot{c_2}(t)}{(\bar{c})^2} + \frac{(1-\alpha)^2V''(z_2(t))c_2(t)\dot{c_2}(t)}{(\bar{c})^4}}{U'(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V'(z_1(t))}$$

where

$$G = U''(c_1(t)) - \frac{\partial V(z(t))}{\partial z(t)}\frac{2(1-\alpha)\alpha c_2(t)}{\left(\bar{c}\right)^3} + V''(z_1(t))\frac{c_2(t)^2(1-\alpha)^2}{(\bar{c})^4}$$

Similarly, we can write the individual problem for the second group and solve accordingly. We have obtained the following system of dynamic equations for the Ramsey model in case of strategic interaction with the presence of relative consumption in the reduced form utility function:

$$\dot{c_1}(t) = -\left(\frac{U'(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V'(z_1(t))}{G}\right)$$
$$\left(f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t))\right) +$$
$$\frac{V'(z_1(t))2(1-\alpha)^2\frac{\dot{c_2}(t)}{(\bar{c})^2} + \frac{(1-\alpha)^2V''(z_1(t))c_2(t)\dot{c_2}(t)}{(\bar{c})^4}}{U'(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V'(z_1(t)),} \tag{2.31}$$

$$\dot{c_2}(t) = -\left(\frac{U^{'}(c_2(t)) + \frac{\alpha c_1(t)}{(\bar{c})^2}V^{'}(z_2(t))}{U''(c_2(t)) - V'(z_2(t))\frac{2(1-\alpha)\alpha c_1(t)}{(\bar{c})^3} + V''(z_2(t))\frac{c_1(t)^2(1-\alpha)^2}{(\bar{c})^4}} \right)$$

$$\Big(f^{'}(K(t)) - \rho - (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t))\Big) +$$

$$\frac{V^{'}(z_2(t))2(1-\alpha)^2\frac{\dot{c_1}(t)}{(\bar{c})^2} + \frac{(1-\alpha)^2 V^{''}(z_2(t))c_1(t)\dot{c_1}(t)}{(\bar{c})^4}}{U^{'}(c_2(t)) + \frac{\alpha c_2(t)}{(\bar{c})^2}V^{'}(z_2(t)),} \qquad (2.32)$$

$$\begin{aligned} \dot{K_1}(t) &= f^{'}(K(t))K_1(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_1(t), \\ \dot{K_2}(t) &= f^{'}(K(t))K_2(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_2(t). \qquad (2.33) \end{aligned}$$

2.3.1 Catching-up and stability analysis

Proposition 4 *If each agents realize their effect on the interest rate and wage earning and if relative consumption effect appear in the reduced form utility function, then even if their initial capital stocks are different, in the long run (at steady state) there will be catching up amoung agents.*

Proof As we can easily see from the equations (2.33) and (2.33), at steady state we have, like in the Strategic Ramsey model, following conditions:

$f^{'}(K^*) - \rho - (1-\alpha)\alpha N f^{''}(K^*)(K_1^* - K_2^*) = f^{'}(K^*) - \rho + (1-\alpha)\alpha N f^{''}(K^*)(K_1^* - K_2^*)$

Since by assumption $\alpha \in (0,1)$ and $f^{''}(K(t)) < 0$ (strict concavity of production function), we have at equilibrium $K_1^* = K_2^*$. □

Unlike the status seeking a la relative wealth in which the catching up depends on the share of two classes, whatever the share of two classes in the economy and whatever their initial capital stock is , poor will be able to catch up with the rich at the steady state. The other important result of this section is that when compared with the strategic Ramsey model, the envy effect just cause to change transitional dynamics of the economy, the steady state capital stock and consumption levels are same for the both models. As we can see from equations (2.31)-(2.33), the steady state values of consumption and aggregate capital are:

$$
\begin{aligned}
f^{'}(K^{*}) &= \rho && (2.34)\\
c^{*} &= \frac{f(K^{*})}{N} && (2.35)
\end{aligned}
$$

It must be noted from equations (2.34) and (2.35)that in the model with relative consumption appearing in reduced form utility function and the presence of strategic interaction among agents leads to a change in the transitional dynamics and the catching up property of the standard Ramsey model, the aggregate level of capital stock is left unchanged at the stationary state.

To look at the stability of system, we linearize the equations (2.31)-(2.33) around the unique symmetric steady state, where $c_1^* = c_2^* = f(K(t)) \setminus N$ and $K_1^* = K_2^* = K^*$ so that 4×4 Jacobian matrix will be the following:[5]

$$
\begin{vmatrix} 0 & 0 & L & M \\ 0 & 0 & P & Q \\ -1 & 0 & f^{'}(k^{*}) & 0 \\ 0 & -1 & 0 & f^{'}(k^{*}) \end{vmatrix} \text{ where}
$$

$$
\begin{aligned}
L &= \frac{2-\alpha}{2-\alpha^2}\frac{(c^*(1+2\alpha-\alpha^2)-\alpha(1-\alpha^2))}{(1+2\alpha-\alpha^2)}Nf^{''}(K^*)\left(\frac{\alpha(2-\alpha)+\alpha^2(1-\alpha^2)}{(1+2\alpha-\alpha^2)}\right)\\
M &= \frac{2-\alpha}{2-\alpha^2}\frac{(c^*(1+2\alpha-\alpha^2)-\alpha(1-\alpha^2))}{(1+2\alpha-\alpha^2)}Nf^{''}(K^*)\left((1-\alpha)^2+\frac{(1-\alpha^2)^2}{(1+2\alpha-\alpha^2)}\right)\\
P &= \frac{1+\alpha}{2-\alpha^2}\frac{(c^*(1+2\alpha-\alpha^2)-\alpha(1-\alpha^2))}{(1+2\alpha-\alpha^2)}Nf^{''}(K^*)\left(\alpha^2+\frac{\alpha^2(2-\alpha)^2}{2-\alpha^2}\right)\\
Q &= \frac{1+\alpha}{2-\alpha^2}\frac{(c^*(1+2\alpha-\alpha^2)-\alpha(1-\alpha^2))}{(1+2\alpha-\alpha^2)}Nf^{''}(K^*)\left(\alpha^2+\frac{\alpha^2(2-\alpha)(1-\alpha)^2}{2-\alpha^2}\right)
\end{aligned}
$$

Without loss of generalization we assume the following form of utility and

[5] The linearization of dynamic equations are in Appendix.

production function:

$$
\begin{aligned}
U(c_i(t)) &= \ln c_i(t), \\
V(z_i(t)) &= \ln z_i(t), \\
f(K(t)) &= K(t)^{\beta},
\end{aligned}
$$

where $N = 100$, $\beta = 0.3, \rho = 0.05$. We have four eigenvalues, two of which are real and in opposite sign and two of which are complex and positive impliying that the system is stable in the saddle point sense. The results are robust for any values of the parameters β, ρ, and N.

CHAPTER 3

CONCLUSION

The main assumption in all one sector growth models is that there are infinitely many agents in the economy and hence, the agents are price takers in all markets. However, this assumption is unjistified for two reasons. In reality, we have finite number of agents but this number is so great that someone can ignore the individual's effect on the aggregate variables. As a matter of fact, social or economic similarities enforce individuals to constitute small number of groups (classes) containing the same type of individuals (in terms of their relative position in the economy). The members of each group have similar tendency in their social and economic decisions. Therefore, we are in an economic structure where there are powerful groups affecting the economy wide variables. Since members of each group are rational, they should be aware of their market power. Indeed, this awareness directly motivates individuals to act strategically. Thus, a model with relativity and catching up concern including the agents' awareness of their market power on the variables would be more consistent if one considers finite number of agents in the economy. Secondly, empirical evidences show that agents are status-seekers (in terms of relative wealth or relative consumption). Hence, individuals are affected by the other agents' decisions. Indeed, agents are strategic status-seekers.

In this thesis, we have analyzed the qualitative implications of the strategic interaction on the standard Ramsey model in terms of catching up among heteregenous agents. We have shown that the strategic interaction among agents in the economy leads the poor to be able to catch up with the rich, which is not the case for the standard Ramsey model where the initial wealth differences perpetuate. Secondly, within this framework, we incorporate the relative wealth effect and conclude that the catching up amoung agents depends on the share of two classes in the economy. If the share of two classes is same, there exist unique symetric steady state, whereas if the share of two classes are different the steady state is assymetric. Morever, the steady state level of aggregate capital stock is higher than that of standard Ramsey model. Finally, we introduce the relative consumption effect and reach the conclusion that whatever the share of classes, the gap between the initial wealth level of two classes will disappear in the long run. In addition, the steady state level of aggregate wealth level is same with the that of standard Ramsey model.

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APPENDIX

The linearization of the dynamic equations of capital stock: Since the evolution of capital stocks are same for all three models, first of all we linearize these differential equations.

$$\dot{K_1}(t) = f^{'}(K(t))K_1(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_1(t)$$

$$\begin{aligned} \frac{\partial(\dot{K_1}(t))}{\partial c_1} &= -1 \\ \frac{\partial(\dot{K_1}(t))}{\partial c_2} &= 0 \end{aligned}$$

$$\frac{\partial(\dot{K_1}(t))}{\partial K_1} = f^{'}(K(t)) + \alpha N K_1(t) f^{''}(K(t)) + \frac{\alpha N f^{'}(K(t)) - \alpha N f^{'}(K(t)) - K(t)\alpha N f^{''}(K(t))}{N},$$

which can be recast as

$$\frac{\partial(\dot{K_1}(t))}{\partial K_1} = f^{'}(K(t)) + (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t)).$$

$$\frac{\partial(\dot{K_1}(t))}{\partial K_2} = (1-\alpha)K_1(t)Nf^{''}(K(t)) + \frac{(1-\alpha)Nf^{'}(K(t)) - (1-\alpha)Nf^{'}(K(t)) - K(t)(1-\alpha)Nf^{''}(K(t))}{N},$$

that simplifies to

$$\frac{\partial(\dot{K_1}(t))}{\partial K_2} = (1-\alpha)^2 N f^{''}(K(t))(K_1(t) - K_2(t)).$$

Since at steady state $K_1^* = K_2^* = K^*$, $c_1^* = c_2^* = c^*$, we have

$$\frac{\partial(\dot{K_1}(t))}{\partial K_1} \quad | \quad_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = f^{'}(K(t)),$$
$$\frac{\partial(\dot{K_1}(t))}{\partial K_2} \quad | \quad_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = 0.$$

From the evolution of the second individual's capital stock:

$$\dot{K_2}(t) = f^{'}(K(t))K_2(t) + \frac{f(K(t)) - K(t)f^{'}(K(t))}{N} - c_1(t),$$

it is clear that

$$\frac{\partial(\dot{K_2}(t))}{\partial c_1} = 0,$$
$$\frac{\partial(\dot{K_2}(t))}{\partial c_2} = -1.$$

$$\frac{\partial(\dot{K_2}(t))}{\partial K_1} = \alpha K_2(t) N f^{''}(K(t)) + \frac{\alpha N f^{'}(K(t)) - \alpha N f^{'}(K(t)) - K(t)\alpha N f^{''}(K(t))}{N}$$

$$\frac{\partial(\dot{K_2}(t))}{\partial K_1} = -\alpha^2 N f^{''}(K(t))(K_1(t) - K_2(t))$$

$$\frac{\partial(\dot{K_2}(t))}{\partial K_2} = [f^{'}(K(t)) + (1-\alpha)NK_1(t)f^{''}(K(t)) + (1-\alpha)f^{'}(K(t)) - \frac{(1-\alpha)Nf^{'}(K(t)) + K(t)(1-\alpha)Nf^{''}(K(t))}{N}]$$
$$\frac{\partial(\dot{K_2}(t))}{\partial K_2} = f^{'}(K(t)) - (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t))$$

Since at steady state $K_1^* = K_2^* = K^*$, $c_1^* = c_2^* = c^*$, we obtain

$$\frac{\partial(\dot{K_2}(t))}{\partial K_2} \quad | \quad_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = 0$$
$$\frac{\partial(\dot{K_2}(t))}{\partial K_2} \quad | \quad_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = f^{'}(K(t))$$

Linearization of the Euler equations of the model with strategic interaction:

: Taking the partial derivatives of equations (2.8) and (2.9) with respect to c_1, c_2, K_1, K_2, and evaluating at the symetric steady state, we have

$$\frac{\partial(\dot{c_1}(t))}{\partial c_1} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = 0,$$
$$\frac{\partial(\dot{c_1}(t))}{\partial c_2} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = 0,$$
$$\frac{\partial(\dot{c_1}(t))}{\partial K_1} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = \theta N f(K^*) f^{''}(K^*)\alpha(2-\alpha),$$
$$\frac{\partial(\dot{c_1}(t))}{\partial K_2} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = \theta N f(K^*) f^{''}(K^*)(1-\alpha)^2,$$

and

$$\frac{\partial(\dot{c_2}(t))}{\partial c_1} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = 0,$$
$$\frac{\partial(\dot{c_2}(t))}{\partial c_2} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = 0,$$
$$\frac{\partial(\dot{c_2}(t))}{\partial K_1} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = \theta N f(K^*) f^{''}(K^*)\alpha^2,$$
$$\frac{\partial(\dot{c_2}(t))}{\partial K_2} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = \theta N f(K^*) f^{''}(K^*)(1-\alpha^2).$$

Linearization of Euler equations of the model with relative wealth and strategic interaction:

Since the capital accumulation equations are same with the previous model we need only to linearize the dynamic equation for consumption with recpect to $c_1(t)$, $c_2(t)$, $K_1(t)$, $K_2(t)$ around the steady state values K_1^*, K_2^*, c_1^*, c_2^*.

$$\dot{c_1}(t) = \frac{1}{\theta}c_1(t)\{f^{'}(K(t)) - \rho + (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t)) - c_1(t)\frac{V'(s_1(t))}{U'(c_1(t))}\frac{(1-\alpha)K_2(t)}{(\alpha K_1(t) + (1-\alpha)K_2(t))^2})\}$$

One can easily obtain that

$$\frac{\partial(\dot{c_1}(t))}{\partial c_1} = \frac{1}{\theta}c_1(t)\{\frac{V'(s_1(t))}{U'(c_1(t))}\frac{(1-\alpha)K_2(t)}{(\alpha K_1(t) + (1-\alpha)K_2(t))^2} - c_1(t)\frac{U''(s_1(t))}{U'(c_1(t))^2}V'(s_1(t))\frac{(1-\alpha)K_2(t)}{(\alpha K_1(t) + (1-\alpha)K_2(t))^2}\},$$

$$\frac{\partial(\dot{c_1}(t))}{\partial c_2} = 0,$$

and

$$\frac{\partial(\dot{c_1}(t))}{\partial K_2} = \frac{1}{\theta} c_1(t)\{(1-\alpha)Nf^{''}(K(t))+$$
$$(1-\alpha)^2\alpha N^2 f^{'''}(K(t))(K_1(t)-K_2(t))+$$
$$\frac{(1-\alpha)c_1(t)}{U'(c_1(t))}[\frac{V'(s_1(t))}{(\alpha K_1(t)+(1-\alpha)K_2(t))^3}(\alpha K_1(t)-(1-\alpha)K_2(t))+$$
$$\frac{(1-\alpha)K_1(t)K_2(t)V''(s_1(t))}{(\alpha K_1(t)+(1-\alpha)K_2(t))^4}]\}.$$

Similarly for $\dot{c_2}(t)$,

$$\frac{\partial(\dot{c_2}(t))}{\partial c_1} = 0,$$

$$\frac{\partial(\dot{c_2}(t))}{\partial c_2} = \frac{1}{\theta} c_1(t)\{\frac{V'(s_2(t))}{U'(c_2(t))}\frac{\alpha K_1(t)}{(\alpha K_1(t)+(1-\alpha)K_2(t))^2}-$$
$$c_2(t)\frac{U''(s_2(t))}{U'(c_2(t))^2}V'(s_2(t))\frac{\alpha K_1(t)}{(\alpha K_1(t)+(1-\alpha)K_2(t))^2}\},$$

$$\frac{\partial(\dot{c_2}(t))}{\partial K_1} = \frac{1}{\theta} c_2(t)\{\alpha^2 Nf^{''}(K(t))-(1-\alpha)\alpha^2 N^2 f^{'''}(K(t))(K_1(t)-K_2(t))+$$
$$\frac{\alpha c_2(t)}{U'(c_2(t))}\{\frac{V'(s_2(t))}{(\alpha K_1(t)+(1-\alpha)K_2(t))^3}(1-\alpha)K_2(t)-\alpha K_1(t)$$
$$+\frac{-\alpha K_1(t)K_2(t)V''(s_2(t))}{(\alpha K_1(t)+(1-\alpha)K_2(t))^4}\}\},$$

$$\frac{\partial(\dot{c_2}(t))}{\partial K_2} = \frac{1}{\theta} c_2(t)\{\alpha(1-\alpha^2)Nf^{''}(K(t))-$$
$$\alpha(1-\alpha)^2 N^2 f^{'''}(K(t))(K_1(t)-K_2(t))+$$
$$\frac{\alpha c_2(t)K_1(t)}{U'(c_2(t))}\{-\frac{V'(s_2(t))}{(\alpha K_1(t)+(1-\alpha)K_2(t))^3}2(1-\alpha)K(t)+$$
$$\frac{\alpha K_1(t)V''(s_2(t))}{(\alpha K_1(t)+(1-\alpha)K_2(t))^4}\}\}.$$

Without loss of generalization we assume the following form of utility and production function:$U(c_i(t)) = \ln c_i(t)$, $V(z_i(t)) = \ln z_i(t)$ and $f(K(t)) = K(t)^\beta$ where $K(t) = \alpha N K_1(t) + (1-\alpha)NK_2(t)$,and parameters values are $n = 100$, $\alpha = 0.4, \beta = 0.3, \rho = 0.05$. Then we obtained the first two rows of the Jacobian matrix as follow:

$$\frac{\partial(\dot{c_1}(t))}{\partial c_1} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = \frac{c^{*2}}{k^*},$$
$$\frac{\partial(\dot{c_1}(t))}{\partial c_2} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = 0,$$
$$\frac{\partial(\dot{c_1}(t))}{\partial K_1} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = c^*(\frac{3}{8}Nf^{''}(k^*) - \frac{3c^{*2}}{4K^*}),$$
$$\frac{\partial(\dot{c_1}(t))}{\partial K_2} \mid_{\{c_1^*, c_2^*, K_1^*, K_2^*\}} = c^*(\frac{1}{4}Nf^{''}(K^*) + \frac{c^{*2}}{4K^*}).$$

Linearization of Euler equations of the model with relative consumption and strategic interaction:

From the equations (2.31) and (2.32), we have the following Euler equations:

$$\dot{c_1}(t) = -\left(\frac{U^{'}(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V^{'}(z_1(t))}{U''(c_1(t)) - V'(z_1(t))\frac{2(1-\alpha)\alpha c_2(t)}{(\bar{c})^3} + V''(z_1(t))\frac{c_2(t)^2(1-\alpha)^2}{(\bar{c})^4}}\right)$$
$$\{f^{'}(K(t)) - \rho + (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t))\} +$$
$$\frac{V^{'}(z_1(t))2(1-\alpha)^2\frac{\dot{c_2}(t)}{(\bar{c})^2} + \frac{(1-\alpha)^2 V^{''}(z_1(t))c_2(t)\dot{c_2}(t)}{(\bar{c})^4}}{U^{'}(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V^{'}(z_1(t))},$$

$$\dot{c_2}(t) = -\left(\frac{U^{'}(c_2(t)) + \frac{\alpha c_1(t)}{(\bar{c})^2}V^{'}(z_2(t))}{U''(c_2(t)) - V'(z_2(t))\frac{2(1-\alpha)\alpha c_1(t)}{(\bar{c})^3} + V''(z_2(t))\frac{c_1(t)^2(1-\alpha)^2}{(\bar{c})^4}}\right)$$
$$\{f^{'}(K(t)) - \rho + (1-\alpha)\alpha N f^{''}(K(t))(K_1(t) - K_2(t))\} +$$
$$+\frac{V^{'}(z_2(t))2(1-\alpha)^2\frac{\dot{c_1}(t)}{(\bar{c})^2} + \frac{(1-\alpha)^2 V^{''}(z_2(t))c_1(t)\dot{c_1}(t)}{(\bar{c})^4}}{U^{'}(c_2(t)) + \frac{\alpha c_2(t)}{(\bar{c})^2}V^{'}(z_2(t))}.$$

For the sake of expositional simplicity, let us adopt the following notation:

$$
\begin{aligned}
X &= -\left(\frac{U'(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V'(z_1(t))}{U''(c_1(t)) - V'(z_1(t))\frac{2(1-\alpha)\alpha c_2(t)}{(\bar{c})^3} + V''(z_1(t))\frac{c_2(t)^2(1-\alpha)^2}{(\bar{c})^4}}\right), \\
Y &= \frac{V'(z_1(t))2(1-\alpha)^2\frac{\dot{c_2}(t)}{(\bar{c})^2} + \frac{(1-\alpha)^2V''(z_1(t))c_2(t)\dot{c_2}(t)}{(\bar{c})^4}}{U'(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V'(z_1(t))}, \\
Z &= \frac{V'(z_1(t))2(1-\alpha)^2\frac{\dot{c_2}(t)}{(\bar{c})^2} + \frac{(1-\alpha)^2V''(z_1(t))c_2(t)\dot{c_2}(t)}{(\bar{c})^4}}{U'(c_1(t)) + \frac{(1-\alpha)c_2(t)}{(\bar{c})^2}V'(z_1(t))}, \\
T &= \frac{V'(z_2(t))2(1-\alpha)^2\frac{\dot{c_1}(t)}{(\bar{c})^2} + \frac{(1-\alpha)^2V''(z_2(t))c_1(t)\dot{c_1}(t)}{(\bar{c})^4}}{U'(c_2(t)) + \frac{\alpha c_2(t)}{(\bar{c})^2}V'(z_2(t))}.
\end{aligned}
$$

Accordingly, the equations (2.31) and (**??**) can be recast as follows,

$$
\begin{aligned}
\dot{c_1}(t) &= X\left(f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t)) + \dot{c_2}(t)Y\right), \\
\dot{c_2}(t) &= Z\left(f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t)) + \dot{c_1}(t)T\right).
\end{aligned}
$$

By substituting the $\dot{c_2}(t)$ from equation (**??**) into equation (**??**), we obtain

$$
\begin{aligned}
\dot{c_1}(t) = \frac{X}{1-TZY}[\{f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t)) + \\
ZY\{f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t))\}].
\end{aligned}
$$

We will now take the derivatives of the equations above with respect to $\{c_1(t),\ c_2(t), K_1(t),\ K_2(t)\}$ and evaluate these derivatives at the steady state values.

$$
\begin{aligned}
\frac{\partial(\dot{c_1}(t))}{\partial c_1} = \frac{\partial(\frac{X}{1-TZY})}{\partial c_1(t)}[\{f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t)) + \\
ZY\{f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t))\}] + \\
\frac{X}{1-TZY}[\{f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t)) + \\
\frac{\partial Z}{\partial c_1(t)}Y\{f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t))\}] + \\
Z\frac{\partial Y}{\partial c_1(t)}\{f'(K(t)) - \rho + (1-\alpha)\alpha N f''(K(t))(K_1(t) - K_2(t))\}.
\end{aligned}
$$

Then, we have

$$\frac{\partial(\dot{c_1}(t))}{\partial c_1} \mid_{\{c_1^*,c_2^*,K_1^*,K_2^*\}} = 0,$$
$$\frac{\partial(\dot{c_1}(t))}{\partial c_2} \mid_{\{c_1^*,c_2^*,K_1^*,K_2^*\}} = 0.$$

Now taking derivative of equation (2.31) with respect to $K_1(t)$ and $K_2(t)$, we obtain:

$$\frac{\partial(\dot{c_1}(t))}{\partial K_1} = \frac{X}{1-TZY}\left(\alpha(2-\alpha)Nf^{''}(K(t)) + ZY\{\alpha^2 Nf^{''}(K(t))\}\right),$$
$$\frac{\partial(\dot{c_1}(t))}{\partial K_2} = \frac{X}{1-TZY}\left((1-\alpha)^2 Nf^{''}(K(t)) + ZY\{(1-\alpha^2)Nf^{''}(K(t))\}\right).$$

For the Euler equation of the second individual, we apply the same procedures and find the following entiries of the corresponding Jacobian matrix:

$$\frac{\partial(\dot{c_2}(t))}{\partial c_1} \mid_{\{c_1^*,c_2^*,K_1^*,K_2^*\}} = 0$$
$$\frac{\partial(\dot{c_2}(t))}{\partial c_2} \mid_{\{c_1^*,c_2^*,K_1^*,K_2^*\}} = 0$$

$$\frac{\partial(\dot{c_2}(t))}{\partial K_1} \mid_{\{c_1^*,c_2^*,K_1^*,K_2^*\}} = \frac{Z}{1-XTY}[\alpha^2 Nf^{''}(K(t)) + TX\{\alpha(2-\alpha)Nf^{''}(K(t))\}]$$

$$\frac{\partial(\dot{c_2}(t))}{\partial K_2} \mid_{\{c_1^*,c_2^*,K_1^*,K_2^*\}} = \frac{Z}{1-XTY}[(1-\alpha^2)Nf^{''}(K(t)) + TX\{(1-\alpha)^2 Nf^{''}(K(t))\}]$$

Without loss of generalization we assume the following form of utility and production function:$U(c_i(t)) = \ln c_i(t)$, $V(z_i(t)) = \ln z_i(t)$ and $f(K(t)) = K(t)^\beta$ where $K(t) = \alpha N K_1(t) + (1-\alpha)NK_2(t)$, $N = 100$, $\alpha = 0.4, \beta = 0.3$, and $\rho = 0.05$. Denoting $\chi = \frac{(c^*(1+2\alpha-\alpha^2)-\alpha(1-\alpha^2))}{(1+2\alpha-\alpha^2)}$, we have:

$$L = \frac{2-\alpha}{2-\alpha^2}\chi Nf^{''}(K^*)\left(\frac{\alpha(2-\alpha)+\alpha^2(1-\alpha^2)}{(1+2\alpha-\alpha^2)}\right),$$
$$M = \frac{2-\alpha}{2-\alpha^2}\chi Nf^{''}(K^*)\left((1-\alpha)^2 + \frac{(1-\alpha^2)^2}{(1+2\alpha-\alpha^2)}\right),$$
$$P = \frac{1+\alpha}{2-\alpha^2}\chi Nf^{''}(K^*)\left(\alpha^2 + \frac{\alpha^2(2-\alpha)^2}{2-\alpha^2}\right),$$
$$Q = \frac{1+\alpha}{2-\alpha^2}\chi Nf^{''}(K^*)\left(\alpha^2 + \frac{\alpha^2(2-\alpha)(1-\alpha)^2}{2-\alpha^2}\right).$$