CUMULANT-BASED PARAMETRIC MULTICHANNEL FIR SYSTEM IDENTIFICATION METHODS

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ABSTRACT

In this paper, least squares and recursive methods for simultaneous identification of four nonminimum phase linear, time-invariant FIR systems are presented. The methods utilize the second- *and* fourth-*order cumulants of outputs of the four FIR systems whose common input is an independent, identically distributed (i.i.d.) non-Gaussian process. The new methods can be extended to the general problem of simultaneous identification of three or more FIR systems with some modifications. To illustrate the effectiveness of our methods, various simulation examples are included.*

1 Introduction

Nonminimum phase system (or signal) identification is an important problem in many signal processing applications including data communication, seismic signal processing, and optical imaging [5], [6].

In this paper, we address the problem of simultaneous reconstruction of the impulse responses of four minimum or nonminimum phase FIR systems using the power spectrum and cross-trispectrum of the output sequences. We present parametric multichannel system identification methods.

Recent work [2],[8],[9] on nonminimum phase multichannel system identification includes the work by Brooks and Nikias [2] who showed that three nonminimum phase systems driven by an independent and identically distributed (i.i.d.) non-Gaussian process can be reconstructed simultaneously from their output *cross-bispectrum*. Their method is a nonparametric cepstral technique which computes the complex cepstra of the impulse response sequences of the unknown systems from the third-order cross-cumulants of output sequences. Higher order statistical identification schemes which utilize complex cepstrum have been widely used in practice. These schemes have some disadvantages when poles and zeros come close to the unit circle [10]-[12]. Our parametric methods do not suffer from this limitation. However, they require exact knowledge of systems' orders and yield consistent parameter estimation only in a class of colored Gaussian noise.

The organization of the paper is as follows. In Section 2 we define the problem and introduce the basic concepts. In Section 3 we develop a least squares type method which is based on solving a system of linear equations obtained from a relationship derived in Section 2. We prove the uniqueness of the least squares solution in Section 4 by devising a recursive method to determine the unknown impulse response parameters. We investigate the robustness of the new methods to additive noise in Section 5. In Section 6 we present simulation examples.

2 Problem Definition

In this section, we describe the multichannel system identification problem. Consider the following signal model:

$$\begin{aligned} y_i(n) &= z_i(n) + w_i(n) \\ &= \sum_{k=0}^{q_i} h_i(k)x(n-k) + w_i(n) \qquad (1) \end{aligned}$$

for $i = 1,2,3,4$, where $y_i(n)$ is the output of the i-th FIR system whose impulse response is $h_i(n)$; q_i is the order of the i-th system; $\{w_i(n)\}$ is an additive zero-mean Gaussian noise; and $z_i(n)$ is the output of the i-th system in the absence of noise. For convenience, the impulse responses, $h_i(n)$, $i = 1,2,3,4$, are numbered such that $q_1 \leq q_2 \leq q_3 \leq q_4$. The input sequence $\{x(n)\}$ is assumed to be an i.i.d. non-Gaussian process with $E\{x(n)\} = 0$, $E\{x(n)x(n+\tau_1)\} = \beta_2\delta(\tau_1)$, $E\{x(n)x(n+\tau_1)x(n+\tau_2)\} = 0$, and $c_x(\tau_1,\tau_2,\tau_3) = \beta_4\delta(\tau_1,\tau_2,\tau_3)$ where $c_x(\tau_1,\tau_2,\tau_3)$ denotes the fourth-order cumulants of the input, $x(n)$.

In most digital communication applications the system input, $\{x(n)\}$, is derived from a signal constellation which is symmetric around the origin. Therefore the third-order cumulants of $x(n)$ are identically zero. In such a case we use the fourth-order cumulants of the system outputs. The methods developed in this paper can be extended to the general problem of simultaneous reconstruction of even number of FIR systems. If the input sequence $x(n)$ is chosen to be asymmetric around the origin, odd number of systems can also be identified by using our algorithm.

Let us define $c_{1234}(\tau_1,\tau_2,\tau_3)$ as the fourth-order cross-cumulant sequence of the processes $\{y_i(n)\}_{i=1}^4$, i.e.,

$$\begin{aligned} c_{1234}(\tau_1,\tau_2,\tau_3) = E\{y_1(n+\tau_1)y_2(n+\tau_2)y_3(n+\tau_3)y_4(n)\} \\ -E\{y_1(n+\tau_1-\tau_2)y_2(n)\}\cdot E\{y_3(n+\tau_3)y_4(n)\} \end{aligned}$$

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$$-E\{y_1(n+\tau_1-\tau_3)y_3(n)\}\cdot E\{y_2(n+\tau_2)y_4(n)\}$$
$$-E\{y_1(n+\tau_1)y_4(n)\}\cdot E\{y_2(n+\tau_2-\tau_3)y_3(n)\}.$$

By using the fact that the fourth-order cumulants of zero mean Gaussian noise processes are identically zero, $c_{1234}(\tau_1,\tau_2,\tau_3)$ can be related to the unknown impulse responses $\{h_i(n)\}_{i=1}^4$ as shown below:

$$c_{1234}(\tau_1,\tau_2,\tau_3)=\beta_4\sum_{k=0}^{q_4}h_1(k+\tau_1)h_2(k+\tau_2)\cdot h_3(k+\tau_3)h_4(k). \quad (2)$$

The cross-trispectrum, $C_{1234}(\omega_1,\omega_2,\omega_3)$, of the output processes, $\{y_i(n)\}_{i=1}^4$, is defined as the three-dimensional Fourier transform of the cross-cumulant sequence, $c_{1234}(\tau_1,\tau_2,\tau_3)$. From (2), it follows that

$$C_{1234}(\omega_1,\omega_2,\omega_3)=\beta_4H_1(\omega_1)H_2(\omega_2)H_3(\omega_3)\cdot H_4(-\omega_1-\omega_2-\omega_3) \quad (3)$$

where $H_i(\omega)$ is the Fourier transform of the system impulse response $h_i(n)$.

We also need the second-order cumulant sequence, $s(\tau)=E[z_4(n)z_4(n+\tau)]$, of the noise free output sequence, $z_4(n)$. The power spectrum, $S(\omega)$, of $z_4(n)$ is

$$S(\omega)=\beta_2H_4(\omega)H_4(-\omega). \quad (4)$$

2.1 A Fundamental Relationship

In this subsection, we derive a relationship between the second- and fourth-order cumulants. This relationship is the basis of our multichannel system identification method.

By multiplying both sides of Equation (3) by $H_4(\omega_1+\omega_2+\omega_3)$ and using (4) we get

$$H_4(\omega_1+\omega_2+\omega_3)C_{1234}(\omega_1,\omega_2,\omega_3)=\epsilon H_1(\omega)\cdot H_2(\omega)H_3(\omega)S(\omega_1+\omega_2+\omega_3) \quad (5)$$

where $\epsilon=\beta_4/\beta_2$. By taking the inverse Fourier Transform of both sides of (5) we obtain the following relationship;

$$\sum_{i=0}^{q_4}h_4(i)c_{1234}(\tau_1-i,\tau_2-i,\tau_3-i)=\epsilon\sum_{i=0}^{q_1}h_1(i)\cdot h_2(\tau_2-\tau_1+i)h_3(\tau_3-\tau_1+i)s(\tau_1-i) \quad (6)$$

which relates the impulse responses, $\{h_i(n)\}_{i=1}^4$, to the second order cumulants, $s(n)$, of the sequence $z_4(n)$ and the fourth-order cross-cumulants, $c_{1234}(\tau_1,\tau_2,\tau_3)$, of the output sequences, $\{y_i(n)\}_{i=1}^4$. This relationship is the four-channel version of an equation used in some parametric system identification techniques [1],[7]. Equation (6) is very important because it allows us to estimate the impulse responses, $\{h_i(n)\}_{i=1}^4$, by solving an overdetermined system of linear equations.

3 Least Squares (LS) Solution

In this section, we develop a least squares method for reconstructing the impulse response sequences, $\{h_i(n)\}_{i=1}^4$, from the second-order cumulants and the fourth-order cross-cumulants by using Equation (6). First, we assume without loss of generality that $\{h_i(n)\}_{i=1}^4$'s are scaled such that $h_i(0)=1$, $i=1,2,3,4$. Then, Equation (6) can be arranged as follows:

$$c_{1234}(\tau_1,\tau_2,\tau_3)=\epsilon\sum_{i=0}^{q_1}h_1(i)h_2(\tau_2-\tau_1+i)\cdot h_3(\tau_3-\tau_1+i)s(\tau_1-i)-\sum_{i=1}^{q_4}h_4(i)c_{1234}(\tau_1-i,\tau_2-i,\tau_3-i). \quad (7)$$

By concatenating (7) for $(\tau_1,\tau_2,\tau_3)\in S$ where S is a region which is described below, we obtain the following overdetermined system of linear equations:

$$\mathbf{d}=\mathbf{M}\mathbf{r} \quad (8)$$

where $\mathbf{r}=[h_4(1)\ldots h_4(q_4)\ \epsilon\ \epsilon h_1(1)\ldots\epsilon h_1(q_1)\ \epsilon h_2(1)\ldots\epsilon h_2(q_2)\epsilon h_3(1)\ldots\epsilon h_3(q_3)\epsilon h_1(1)h_2(1)\ldots\epsilon h_1(q_1)h_2(q_2)\ \epsilon h_1(1)h_3(1)\ldots\epsilon h_1(q_1)h_3(q_3)\ \epsilon h_2(1)h_3(1)\ldots\epsilon h_2(q_2)h_3(q_3)\epsilon h_1(1)h_2(1)h_3(1)\ldots\epsilon h_1(q_1)h_2(q_2)h_3(q_3)]^T$ is a $(q_4(q_1+1)(q_2+1)(q_3+1))$ column vector, $\mathbf{d}=[c_{1234}(\tau_1,\tau_2,\tau_3):(\tau_1,\tau_2,\tau_3)\in S]^T$ is a $N(q_1,q_2,q_3,q_4)$ column vector, and $\mathbf{M}$ is a matrix of size $N(q_1,q_2,q_3,q_4)\times(q_4(q_1+1)(q_2+1)(q_3+1))$ whose entries are determined according to (7). $N(q_1,q_2,q_3,q_4)$ is the number of points in the region S which is determined as follows. It follows from (2) that $c_{1234}(\tau_1,\tau_2,\tau_3)$ is nonzero for $-q_4\le\tau_1\le q_1$, $-q_4\le\tau_2\le q_2$, and $-q_4\le\tau_3\le q_3$. Hence, left hand side of (6), $\sum_{i=0}^{q_4}h_4(i)c_{1234}(\tau_1-i,\tau_2-i,\tau_3-i)$, is nonzero for $-q_4\le\tau_1\le q_1+q_4$, $-q_4\le\tau_2\le q_2+q_4$, and $-q_4\le\tau_3\le q_3+q_4$. In addition, we should maintain that $h_2(\tau_2-\tau_1+i)h_3(\tau_3-\tau_1+i)$ term at the right hand side of (6) is nonzero; yielding $0\le\tau_2-\tau_1+i\le q_2$, $0\le\tau_3-\tau_1+i\le q_3$ for $i=0,1,2,\ldots,q_1$. This leads to $-q_1\le\tau_2-\tau_1\le q_2$ and $-q_1\le\tau_3-\tau_1\le q_3$. Thus, the region S is defined by the following set,

$$S=\{(\tau_1,\tau_2,\tau_3)\ :-q_4\le\tau_1\le q_1+q_4,\ -q_4\le\tau_2\le q_2+q_4,-q_4\le\tau_3\le q_3+q_4,\ -q_1\le\tau_2-\tau_1\le q_2,-q_1\le\tau_3-\tau_1\le q_3\}. \quad (9)$$

By counting the number of points in this region, we obtain the size of the column vector $\mathbf{d}$, $N(q_1,q_2,q_3,q_4)$, as

$$N(q_1,q_2,q_3,q_4)=q_1(q_1+1)(2q_1+1)/3+(q_2+q_3+2)q_1(q_1+1)+2(q_1+1)(q_2+1)(q_3+1)+(2q_4-q_1-1)(q_1+q_2+1)(q_1+q_3+1). \quad (10)$$

The least squares solution of the overdetermined system of linear equations given by (8) is

$$\mathbf{r}=\left(\mathbf{M^TM}\right)^{-1}\mathbf{M^Td}. \quad (11)$$

$h_4(1), h_4(2), \ldots, h_4(q_4)$ can then be determined as the first q_4 elements of the vector $\mathbf{r}$. Other impulse response coefficients $\{h_i(n)\}_{i=1}^{3}$ can be directly obtained by dividing the corresponding element of $\mathbf{r}$ by $r(q_4+1)$, which is ϵ. However, directly obtained results could be inaccurate due to measurement noise and estimation errors. In that case, we identify $\{h_i(n)\}_{i=1}^{3}$ by using a method [1] which is based on the singular value decomposition (SVD). This method exploits all the available information provided by the vector $\mathbf{r}$. We form three matrices $\mathbf{R}[\mathbf{h_1}, \mathbf{h_2}], \mathbf{R}[\mathbf{h_1}, \mathbf{h_3}], \mathbf{R}[\mathbf{h_2}, \mathbf{h_3}]$ from the elements of the vector $\mathbf{r}$; such that, the matrix, $\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}]$, is of rank one and can be written in the following form:

$$\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}] = \epsilon \begin{bmatrix} 1 \\ h_i(1) \\ h_i(2) \\ \vdots \\ h_i(q_i) \end{bmatrix} [1 \;\, h_j(1) \;\, h_j(2) \;\, \cdots h_j(q_j)]$$

where $i, j = 1, 2, 3$ and $i \neq j$. The unknown impulse response sequences $h_i(n)$ and $h_j(n)$ can be identified from $\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}]$ using the singular value decomposition, i.e.,

$$\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}] = \mathbf{Z}\mathbf{V}\mathbf{U}^{\mathbf{T}} \tag{12}$$

where $\mathbf{V}$ is a diagonal matrix, the diagonal elements of which are the singular values of $\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}]$. The columns of the orthogonal matrix $\mathbf{Z}$, $\mathbf{z_1}, \mathbf{z_2}, \ldots, \mathbf{z_{q_i+1}}$, are the left singular vectors of $\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}]$, and the columns of the orthogonal matrix $\mathbf{U}$, $\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_{q_j+1}}$, are the right singular vectors of $\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}]$. Since $\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}]$ is of rank one, it has only one nonzero singular value whose corresponding singular vectors determine the impulse responses $h_i(n)$ and $h_j(n)$. From the properties of the SVD, it can be shown that [4]

$$h_i(n) = k_1 z_1(n) \qquad 0 \leq n \leq q_i \tag{13}$$

and

$$h_j(n) = k_2 u_1(n) \qquad 0 \leq n \leq q_j \tag{14}$$

where k_1 and k_2 are constants chosen to scale the singular vectors, $\mathbf{z_1}$ and $\mathbf{u_1}$, so that $h_i(0) = h_j(0) = 1$.

We should mention that theoretically only one singular value of $\mathbf{R}[\mathbf{h_i}, \mathbf{h_j}]$ is nonzero. In practice, due to noise and estimation errors, there may be many nonzero singular values, but only a single dominant one. In such a case we keep the dominant singular value and its corresponding singular vectors.

4 Uniqueness of the LS Solution and the Recursive Method

The least squares method described in the previous section yields a unique (least squares) solution if the matrix $\mathbf{M}$ has full rank. In order to show that the matrix $\mathbf{M}$ is of full rank we first show that elements of the unknown vector, $\mathbf{r}$, can be uniquely determined from (6) using a recursive algorithm. By setting $\tau_1 = \tau_2 = \tau_3 = -q_4$ in (6) and by using the fact that $h_i(0) = 1$, $i = 1, 2, 3, 4$, we obtain

$$\epsilon = \frac{c_{1234}(-q_4, -q_4, -q_4)}{s(-q_4)}. \tag{15}$$

Similarly, by setting $\tau_1 = -q_4$ only, we obtain

$$\epsilon h_2(\tau_2 + q_4) h_3(\tau_3 + q_4) = \frac{c_{1234}(-q_4, \tau_2, \tau_3)}{s(-q_4)} \tag{16}$$

for $\tau_2 = -q_4, \ldots, q_2 - q_4, \tau_3 = -q_4, \ldots, q_3 - q_4$, and

$$h_2(\tau_2 + q_4) h_3(\tau_3 + q_4) = \frac{c_{1234}(-q_4, \tau_2, \tau_3)}{c_{1234}(-q_4, -q_4, -q_4)}. \tag{17}$$

We can recover $h_2(n)$ and $h_3(n)$ by setting $\tau_3 = -q_4$ and $\tau_2 = -q_4$ in the above equation, i.e.,

$$h_2(\tau_2 + q_4) = \frac{c_{1234}(-q_4, \tau_2, -q_4)}{c_{1234}(-q_4, -q_4, -q_4)} \tag{18}$$

for $\tau_2 = -q_4, \ldots, q_2 - q_4$, and

$$h_3(\tau_3 + q_4) = \frac{c_{1234}(-q_4, -q_4, \tau_3)}{c_{1234}(-q_4, -q_4, -q_4)} \tag{19}$$

for $\tau_3 = -q_4, \ldots, q_3 - q_4$. Setting $\tau_3 = -q_4$ in (6) yields

$$\epsilon h_1(\tau_1 + q_4) h_2(\tau_2 + q_4) = \frac{c_{1234}(\tau_1, \tau_2, -q_4)}{s(-q_4)} \tag{20}$$

for $\tau_1 = -q_4, \ldots, q_1 - q_4$, $\tau_2 = -q_4, \ldots, q_2 - q_4$, and

$$h_1(\tau_1 + q_4) h_2(\tau_2 + q_4) = \frac{c_{1234}(\tau_1, \tau_2, -q_4)}{c_{1234}(-q_4, -q_4, -q_4)}. \tag{21}$$

We can recover $h_1(n)$ by setting $\tau_2 = -q_4$ in the above equation, as

$$h_1(\tau_1 + q_4) = \frac{c_{1234}(\tau_1, -q_4, -q_4)}{c_{1234}(-q_4, -q_4, -q_4)} \tag{22}$$

for $\tau_1 = -q_4, \ldots, q_1 - q_4$. Similarly, we set $\tau_2 = -q_4$ in (6) and we obtain

$$\epsilon h_1(\tau_1 + q_4) h_3(\tau_3 + q_4) = \frac{c_{1234}(\tau_1, -q_4, \tau_3)}{s(-q_4)} \tag{23}$$

for $\tau_1 = -q_4, \ldots, q_1 - q_4$, $\tau_3 = -q_4, \ldots, q_3 - q_4$. At this point, we compute $h_4(n)$, $1 \leq n \leq q_4$, as follows. We start with the assumption that $h_4(0) = 1$. For $n = 1$ to $\lfloor q_4/2 \rfloor$, we set $\tau_1 = -q_4 + n$, $\tau_2 = q_2 - q_4 + n$, and $\tau_3 = q_3 - q_4 + n$ in (6) and we recursively obtain

$$h_4(n) = (c_{1234}(-q_4, q_2 - q_4, q_3 - q_4))^{-1} \cdot [\epsilon h_2(q_2) \cdot h_3(q_3) s(-q_4 + n) - \sum_{i=0}^{n-1} h_4(i) \cdot c_{1234}(\mathrm{n} - \mathrm{q}_4 - \mathrm{i}, \mathrm{q}_2 - \mathrm{q}_4 + \mathrm{n} - \mathrm{i}, \mathrm{q}_3 - \mathrm{q}_4 + \mathrm{n} - \mathrm{i})] \tag{24}$$

By setting $\tau_1 = q_1 + q_4$, $\tau_2 = q_4$, $\tau_3 = q_4$ in (6),

$$h_4(q_4) = \frac{\epsilon h_1(q_1) s(q_4)}{c_{1234}(q_1, 0, 0)}. \quad (25)$$

Then, for $n = 1$ to $\lfloor q_4/2 \rfloor$, we set $\tau_1 = q_1 + q_4 - n$, $\tau_2 = q_4 - n$, $\tau_3 = q_4 - n$ in (6) and we recursively obtain

$$h_4(q_4 - n) = \frac{1}{c_{1234}(q_1, 0, 0)} [\epsilon h_1(q_1) s(q_4 - n)$$

$$- \sum_{i=0}^{n-1} h_4(q_4 - i) c_{1234}(q_1 - n + i, -n + i, -n + i)]. \quad (26)$$

We note that $\lfloor q_4/2 \rfloor = q_4/2$ if q_4 is even, and $\lfloor q_4/2 \rfloor = (q_4 - 1)/2$ if q_4 is odd.

Finally, we are ready to recover the unknown parameters $\{\epsilon h_1(i) h_2(\tau_2 - \tau_1 + i) h_3(\tau_3 - \tau_1 + i)\}$. For $n = 0$ to $\lfloor q_1/2 \rfloor$, we set $\tau_1 = -q_4 + n$ in (6) and recursively compute

$$\epsilon h_1(n) h_2(\tau_2 + q_4) h_3(\tau_3 + q_4) = \frac{1}{s(-q_4)} (\sum_{i=0}^{n} h_4(i)$$

$$\cdot c_{1234}(\tau_1 - i, \tau_2 - i, \tau_3 - i) - \sum_{i=0}^{n-1} \epsilon h_1(i)$$

$$\cdot h_2(\tau_2 - \tau_1 + i) h_3(\tau_3 - \tau_1 + i) s(\tau_1 - i)) \quad (27)$$

for $\tau_2 = -q_4, ..., q_2 - q_4$ and $\tau_3 = -q_4, ..., q_3 - q_4$. The above recursive formula requires the knowledge of $\{h_4(n)\}$ and $\{\epsilon h_2(i) h_3(j)\}$ to compute $\{\epsilon h_1(i) h_2(\tau_2 - \tau_1 + i) h_3(\tau_3 - \tau_1 + i)\}$. Now, we set $\tau_1 = \tau_2 = \tau_3 = q_1 + q_4$ in (6);

$$\epsilon h_1(q_1) h_2(q_2) h_3(q_3) = \frac{h_4(q_4) c_{1234}(q_1, q_2, q_3)}{s(q_4)}. \quad (28)$$

Then, we start from $\epsilon h_1(q_1) h_2(q_2) h_3(q_3)$ by setting $\tau_1 = q_1 + q_4 - n$ in (6) for $n = 0$ to $\lfloor q_1/2 \rfloor$, and we recursively compute

$$\epsilon h_1(q_1 - n) h_2(\tau_2 - q_4) h_3(\tau_3 - q_4) = \tfrac{1}{s(q_4)} (\textstyle\sum_{i=q_4-n}^{q_4} h_4(i)$$

$$\cdot c_{1234}(\tau_1 - i, \tau_2 - i, \tau_3 - i) - \textstyle\sum_{i=q_1-n+1}^{q_1} \epsilon h_1(i)$$

$$h_2(\tau_2 - \tau_1 + i) h_3(\tau_3 - \tau_1 + i) s(\tau_1 - i)), \quad (29)$$

for $\tau_2 = q_4, ..., q_2 + q_4$ and $\tau_3 = q_4, ..., q_3 + q_4$. The recursive algorithm described above uses Equation (6) only for certain values of pτ_1, τ_2, τ_3 to uniquely determine the unknown vector $\mathbf{r}$. Therefore, it is equivalent to choosing linearly independent rows of the matrix $\mathbf{M}$ and solving the system of linear equations formed by these independent rows. It follows then that there are $q_4 + (1 + q_1)(1 + q_2)(1 + q_3)$ linearly independent rows of $\mathbf{M}$ where this number is the number of unknowns in the system of linear equations given by (8). Hence the number of linearly independent rows equals to the number of columns, and the rank of the matrix $\mathbf{M}$ is $q_4 + (1 + q_1)(1 + q_2)(1 + q_3)$. Since $\mathbf{M}$ has full column rank, there is a unique least squares solution.

5 Robustness to Additive Gaussian Noise

In practical applications, the received signals, $\{y_i(n)\}_{i=1}^4$, are usually the noise corrupted version of the system outputs, $\{z_i(n)\}_{i=1}^4$. In this section, we consider the case where the noise terms $\{w_i(n)\}_{i=1}^4$ are Gaussian noise processes, independent of each other and $\{z_i(n)\}_{i=1}^4$.

For zero mean Gaussian processes, cumulants of order greater than two are identically zero. Hence the fourth-order cumulants of $\{y_i(n)\}_{i=1}^4$ are not affected by additive Gaussian noise. However, the second-order cumulants are affected by the presence of Gaussian noise. The methods described in previous sections use the second-order cumulant sequence $s(\tau)$ of the noiseless case system output $z_4(n)$, instead of the second-order cumulant sequence $c_{y_4}(\tau)$ of $y_4(n)$. They are related to each other as follows:

$$c_{y_4}(\tau) = s(\tau) + c_{w_4}(\tau) \quad (30)$$

where $c_{w_4}(\tau)$ is the second-order cumulant sequence of $w_4(n)$. In practice we can only estimate $c_{y_4}(\tau)$, not $s(\tau)$. It follows from (23)-(29) that the recursive method described in Section 4 uses samples of $s(\tau)$ for which $q_4 - \lfloor q_4/2 \rfloor \le |\tau| \le q_4$. If the second-order cumulants of the additive noise, $c_{w_4}(\tau)$, are nonzero for lags in the range $|\tau| \le q$ where $q = q_4 - \lfloor q_4/2 \rfloor - 1$ the recursive method will not be affected by the presence of noise as $c_{y_4}(\tau) = s(\tau)$ for $q < |\tau| \le q_4$. Consequently, uniqueness and consistency of the LS solution will remain unaffected if the rows of the matrix $\mathbf{M}$ which contain the samples of $c_{w_4}(\tau)$ are removed. Both the least squares and recursive solutions are robust to additive white Gaussian noise because $c_{w_4}(\tau)$ is nonzero only for $\tau = 0$.

6 Simulation Examples

Consider the following set of systems

$$y_1(n) = x(n) - 0.6x(n-1) + w_1(n)$$

$$y_2(n) = x(n) + 0.75x(n-1) + w_2(n) \quad (31)$$

$$y_3(n) = x(n) + 0.5x(n-1) - 1.25x(n-2) + w_3(n)$$

$$y_4(n) = x(n) - 0.375x(n-1) + 0.8x(n-2) + w_4(n)$$

where the input signal, $x(n)$, is a zero mean, i.i.d., sequence with $\beta_2 = 5$, $\beta_3 = 0$ and $\beta_4 = -34$; The noise terms, $\{w_i(n)\}_{i=1}^4$, are zero mean, white Gaussian processes with variance 1, and they are uncorrelated with each other.

In our simulation examples the data records (N=2048), $\{y_i(n)\}_{i=1}^4$, $(n = 0, 1, ..., 2047)$, were generated by the above set of systems. The impulse response coefficients of the unknown systems were estimated by using the LS method for 100 output realizations for the noise-free case where noise processes, $\{w_i(n)\}_{i=1}^4$, are eliminated in the signal model, as well as the noisy case. The mean value and the standard deviation for each impulse response coefficient were computed over 100 realizations. For the noisy case, rows of the coefficient matrix $\mathbf{M}$ which contain the value, $c_{y_4}(0)$, were removed. Experimental results are presented in Table 1. It is observed that the mean values are not significantly different for the noise-free and noisy cases. However, standard deviations are slightly larger for the noisy case.

Complex-cepstra based multichannel system identification methods produce poor results when system zeros are close to the unit circle [2]. Our parametric methods do not suffer from this limitation. For example, in (31) $h_3(n)$ and $h_4(n)$ have zeros at -1.3956, 0.8956 and $0.1875 \pm i0.8746$, respectively. Although the last three zeros are close to the unit circle, our LS method produced good estimates of them.

The new methods require exact knowledge of systems' orders. In [1] an efficient system order determination scheme was developed for single channel system identification. This scheme is based on the single channel version of our fundemental Equation (6). A reliable multichannel system order estimation scheme can be developed as in [1].

A consistent behaviour of the new methods have been observed in all the simulation examples tried.

7 Conclusion

In this paper new methods for simultaneous identification of four minimum or nonminimum phase LTI FIR systems driven by an i.i.d. non-Gaussian process are presented. Our methods, a *Least Squares* (LS) method and a *recursive* method, are parametric and utilize the second- and fourth-order cumulants of the system outputs in an appropriate domain of support. The recursive method is developed to prove the uniqueness of the least squares solution. The new methods can be extended to the more general problem of simultaneous identification of three or more systems by using second-order cumulants and system output cumulants of order being equal to the number of systems to be identified.

We experimentally observed that the LS method yields consistent parameter estimation in a class of colored Gaussian noise including the white Gaussian noise.

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Table 1: Reconstructed Impulse Response Coefficients for the Noise-Free and Noisy Cases.

		Noise-free		*Noisy*	
	True	Mean	St.Dev.	Mean	St.Dev.
$h_1(1)$	-0.6	-0.6121	0.0422	-0.6096	0.0614
$h_2(1)$	0.75	0.7307	0.0380	0.7319	0.0586
$h_3(1)$	0.5	0.4866	0.0366	0.4863	0.0552
$h_3(2)$	-1.25	-1.2340	0.0421	-1.2391	0.0641
$h_4(1)$	-0.375	-0.3931	0.0358	-0.3970	0.0456
$h_4(2)$	0.8	0.7863	0.0173	0.7851	0.0281