A Survey of Signal Processing Problems and Tools in Holographic Three-Dimensional Television

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(Invited Paper)

***Abstract*—Diffraction and holography are fertile areas for application of signal theory and processing. Recent work on 3DTV displays has posed particularly challenging signal processing problems. Various procedures to compute Rayleigh–Sommerfeld, Fresnel and Fraunhofer diffraction exist in the literature. Diffraction between parallel planes and tilted planes can be efficiently computed. Discretization and quantization of diffraction fields yield interesting theoretical and practical results, and allow efficient schemes compared to commonly used Nyquist sampling. The literature on computer-generated holography provides a good resource for holographic 3DTV related issues. Fast algorithms to compute Fourier, Walsh–Hadamard, fractional Fourier, linear canonical, Fresnel, and wavelet transforms, as well as optimization-based techniques such as best orthogonal basis, matching pursuit, basis pursuit etc., are especially relevant signal processing techniques for wave propagation, diffraction, holography, and related problems. Atomic decompositions, multiresolution techniques, Gabor functions, and Wigner distributions are among the signal processing techniques which have or may be applied to problems in optics. Research aimed at solving such problems at the intersection of wave optics and signal processing promises not only to facilitate the development of 3DTV systems, but also to contribute to fundamental advances in optics and signal processing theory.**

***Index Terms*—Diffraction, discretization, fast transforms, Fresnel transform, holographic 3DTV, holography, sampling, 3DTV.**

I. Introduction

ACHIEVING true 3-D video display is the ultimate goal in research in visual technologies. Naturally, optics will play a central role in research along this direction, and signal processing techniques will be heavily used at every stage of an end-to-end 3DTV system. Holographic 3DTV is a highly desirable end product. This survey focuses on signal processing issues in diffraction and holography, with an emphasis towards issues arising at the display end of envisioned holographic 3DTV systems. We start with a brief overview of different techniques used in 3DTV displays in Section II. An analytical approach to holographic 3DTV is only possible if the underlying fundamentals of diffraction are understood; therefore, we provide a brief introduction to diffraction in Section III and point out that basic forms describing diffraction are already familiar to signal processing community. Section IV gives a short but clear definition of the key problems in holographic 3DTV display research from a signal processing point of view. In addition, we pose the proper discretization of diffraction related signals as a distinct problem in this section since discretization is the natural first step before any subsequent digital processing. Sections V–VII review some solutions to these problems that have appeared in the literature. Section V is devoted to techniques and algorithms in the computation of diffraction. Most of the techniques utilized in computer generated holography (CGH), are relevant to holographic 3DTV problems; these are briefly reviewed CGH in Section VI. Section VII highlights interesting observations regarding sampling and discretization of diffraction signals. Despite considerable work, the fundamental problems associated with holographic 3DTV are far from being solved at a satisfactory level at present. We believe that existing signal processing tools are significantly underutilized for solving these problems. Therefore, we provide an overview of what we consider the most suitable signal processing techniques which can be applied to solve the presented problems, and thus, advance the current level of holographic 3DTV technology, in Section VIII. Although an exhaustive survey in such a multidisciplinary area is almost impossible, the content of the paper and the list of references may ease the efforts of researchers who would like to understand the state-of-the-art in signal processing issues in holographic 3DTV, and to identify further research directions to address the underlying problems.

II. Some 3DTV Display Techniques

A. Holography and Holographic 3DTV Displays

In the broad sense, holography involves recording of all physical properties of the light in a 3-D environment containing objects, and its subsequent reconstruction (playback). If the reconstructed light is the same as the recorded original, any observer interacting with the reconstructed light will see the same scene as the original. Therefore, in principle, holography creates true 3-D images, with all correct color, depth, shape information and parallax relation. This broad sense definition involves all classical holographic techniques where coherent light is used to record the complex valued wavefront via interference

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[1], [2] and other true 3-D imaging techniques like ideal integral imaging [3].

Dynamic holographic display devices are necessary for holographic video. However, this does not necessarily mean that the displayed image has been captured holographically. It is envisaged that the future 3DTV systems will have decoupled input and display units where the capture unit forms an abstract 3-D representation, and then, some intermediate units convert that data to driver signals for a specific display device.

Candidate technologies for holographic display units include dynamically writable/erasable chemical films [4], or on electronically controllable arrays of pixels that can alter the phase and amplitude of light passing through (or reflected by) them, called spatial light modulators (SLMs) [5]–[22] Specific forms, like deflectable mirror array devices (DMADs) are also among potential technologies that can be adapted for 3-D display; these can also be considered as special forms of SLMs [23]–[25]. Currently, dynamic chemical film technology is not mature enough for acceptable performance. Unfortunately, the size, quality and geometries of SLMs are currently not sufficient for acceptable quality 3-D displays, either. However, it is expected that both technologies will develop in time to yield the desired quality. There are also other techniques which are based on interaction of light with acoustic signals [26]–[31] Some experimental holographic 3DTV systems usually choose to sacrifice from the ultimate true 3-D display quality, for example by eliminating vertical parallax, and thus achieve higher resolution and fidelity in other features, or reduction in computational complexity [26], [28], [29], [32], [30], [33], [31]. Ability to steer light from each point of a display device to arbitrary directions provides solutions to the 3DTV display problem; such commercial displays are available [34], [35]. Speckle noise in case of coherent illumination is another disadvantage, and there are proposed techniques to cope with this problem [36].

B. Integral Imaging

Integral photography [3] has been revitalized after the progress in active pickup devices and microlens manufacturing processes [37]. It relies on a capture device based on a microlens array to encode a true 3-D optical model of the object as a planar intensity distribution which can then be reconstructed by reversing the direction of incident optical rays. Analysis of integral imaging devices can be carried out both by ray [38] and diffractive optics [39], [40]. Improvements in related computational procedures [41] and the incorporation of novel techniques like moving lens arrays solved many problems in integral imaging [42], [43]. Solutions for viewing-zone enhancement have been tested using dynamic barrier arrays [44], microconvex-mirror arrays [41], [45] or lens switching techniques [46]. Very large-scale [47] and projection based integral imaging systems [45] with increased resolution and viewing angle are reported. Techniques have been proposed to improve the depth of viewing field based on amplitude modulated microlenses [48], a change of the optical path length [49], synthesis of real and virtual image fields [50], or on the use of microlenses with nonuniform focal lengths and aperture sizes [51]. The issues of scene occlusion [52] as well as removal of the multifacet structure [39] or suppression of color moire [53] in the reconstructed images have been successfully resolved. The maximum information capacity of integral imaging and image compression by the Karhunen-Loeve transform are discussed in [54], [55]. Holograms can be computed from captured images during integral photography [56].

C. Stereoscopic 3DTV Displays

Past and present implementations of most 3DTV systems rely on stereoscopy, or multiview video. In these approaches, no attempt is made to duplicate the original optical field; instead, two or more 2-D images are captured at slightly different viewing angles. The human visual system interprets the received images. 3-D perception relies on the processing of several depth cues. Older type systems require special goggles to direct different images to each eye; however, newer systems utilize autostereoscopic systems to guide different 2-D views to different angles [57]. Systems based on stereoscopic principles usually create a feeling like motion sickness especially when some associated alignments are not perfect [58]. Signal processing issues related with such display schemes are discussed in the review paper by Isgro *et al.* [59]. While the stereoscopy-based techniques are the most popular 3-D imaging techniques to date, holography-based techniques will most likely be the ultimate choice for 3DTV in the future.

III. Basics of Diffraction

Propagating optical waves in 3-D space and the associated 3-D optical field are the primary focus of diffraction and related problems. The complex valued amplitude information over a surface is sufficient to determine the field over the entire 3-D space. Computing the amplitude pattern over a plane given the amplitude pattern over another parallel plane is a classical textbook problem, and its solutions are well known [60]. The exact solution, for the scalar case, is conveniently formulated as a 2-D linear shift invariant (LSI) system whose transfer function is $\exp[jZ(k^2-k_x^2-k_y^2)^{1/2}]$, where k_x and k_y are the spatial frequencies along the two spatial axes, respectively; and Z is the distance between the planes. Wavelength of the light is λ and $k=2\pi/\lambda$. Modeling the LSI system in the Fourier domain, and then writing the inverse Fourier transform to find the desired field pattern, one gets the so called plane-wave decomposition approach to diffraction. The associated impulse response of the 2-D LSI system is the kernel of the famous Rayleigh–Sommerfeld integral which represents the 2-D convolution [61].

When the bandwidth of the 2-D input pattern is restricted to smaller $|k_x|$ and $|k_y$ around zero (paraxial approximation), we get the Fresnel diffraction where the impulse response and the associated transfer function become [60]:

$$h_Z(x,y)=\frac{1}{j\lambda Z}\exp\left(j\frac{2\pi}{\lambda}Z\right)\exp\left[j\frac{\pi}{\lambda Z}(x^2+y^2)\right] \tag{1}$$

$$H_Z(k_x,k_y)=\exp\left(j\frac{2\pi}{\lambda}Z\right)\exp\left[-j\frac{\lambda Z}{4\pi}\left(k_x^2+k_y^2\right)\right]. \tag{2}$$

The Fresnel diffraction relation between parallel planes is given as the convolution of one of the patterns by the above kernel;

this convolution is also called the Fresnel transform. Due to nature of the above kernel, this convolution can be converted to a single Fourier transform with pre- and post-multiplications by the quadratic phase function $\exp\left[[j(\pi)/(\lambda Z)](\xi^2+\eta^2)\right]$. Provided that the given pattern has a finite extent, and if the quadratic phase term which multiplies the function representing the pattern is approximately equal to one where the function is nonzero, we get the Fraunhofer diffraction which is nothing but a chirp modulated Fourier transform.

Therefore, scalar diffraction between two parallel planes involves fundamental signal processing concepts such as linear shift-invariant filtering, Fourier transformation, and modulation. More complicated problems, such as diffraction between two planes tilted with respect to each other, can also be modeled with the aid of similar signal processing concepts [62], [63].

IV. Fundamental Problems in Holographic 3DTV

Two fundamental signal processing problems in holographic 3DTV are what we will refer to as the forward and inverse problems [64].

The forward problem is the computation of the light field distribution which arises over the entire 3-D space from a given 3-D scene or object. In traditional optical holography, this light field would have been optically created and recorded by interferometric or other techniques, but in envisioned 3DTV systems there will be no direct coupling between the input and output, and it is most likely that some other abstract digital representation of the 3-D scene will be transmitted instead. Therefore, the associated field must be computed. This is a considerably more difficult problem than the classical textbook problems outlined in Section III, because the 3-D scene consists of nonplanar surfaces.

Once the desired field is computed, physical devices will be used to create it at the display end; the field generated by these devices will propagate in space and reach the viewer, creating the perception of the original 3-D scene. These devices impose many constraints on the 3-D light distributions they can generate, as a consequence of their particular characteristics and limitations. Therefore, given a physical device, such as a specific SLM, finding the driving signals to get the best approximation to the desired time-varying 3-D light field is a challenging inverse problem. A precise definition of this, so called, synthesis problem, and some proposed solutions can be found in the literature [65]–[71].

Both the forward and the inverse problems require processing of large amounts of data. Sparse signal representations and fast techniques are of crucial importance for achieving a feasible processing time.

Computation of the field depends on the foundations of diffraction theory [60], [72]–[76]. Approaches in solving diffraction problems can be investigated under four categories. From rather simple to more complicated, these categories are ray optics, wave optics, electromagnetic optics and quantum optics. Ray optics describes the propagation of light by using geometrical rules and rays [75]. In wave optics, the propagation of light is described by a scalar wave function [60]. The scalar function is a solution of the wave equation [75].

Signal processing approaches have been extensively employed in various problems related to wave optics; we present some of these important contributions in the next section. However, the present state-of-the-art does not seem to be sufficient for solving some of the problems arising in real-time holographic 3-D display. In order to facilitate further developments, we discuss several signal processing tools which, we believe, have the potential of advancing the state-of-the-art in Section VIII.

Another problem of fundamental nature is the discretization of signals associated with propagating optical waves. At the acquisition stage, CCD or CMOS arrays capture holographic patterns and convert them into digital signals [77]–[81]. While sampling and quantization is an extensively studied and mature field in the general sense, direct application of general results will not be efficient, interesting, nor sufficient in most diffraction related problems. Instead, systematic approaches which take the specific properties of the underlying signals into consideration and merge them with modern digital signal processing methods are highly desirable. The literature dealing with discretization and quantization issues in diffraction and holography is reviewed in Section VII. We also present an overview of signal processing tools related to sampling in Section VIII and indicate that these tools may form a sound basis for further developing efficient sampling strategies and thus ease the solution of difficult holographic 3DTV related problems.

V. Review of Techniques and Algorithms for Wave Propagation, Diffraction, and Holography

We have already presented the fundamental problems in Section IV. Here we give an overview of some of the available techniques and algorithms which facilitate, or offer, solutions to these and related problems.

Sherman gave an elegant proof of the equivalence of the Rayleigh diffraction integral and the exact scalar solution based on the planewave superposition of waves propagating in the $z > 0$ direction [61]. This is an important contribution because the fast direct calculation of the Rayleigh integral is difficult but efficient procedures based on FFT can be developed by using the planewave decomposition.

Grella examined diffraction and free-space propagation of an optical scalar field by using the Fresnel approximation [82]. The author states that Fresnel approximation can be represented as a superposition of planewaves besides the original approach based on the series expansion of the spherical wavelet exponent. The author provides a unified approach for Fresnel approximation.

Ganci gives a simplified representation of diffraction of a planewave through a tilted slit by using Fraunhofer approximation [83]. Rabal *et al.* generalized the method proposed by Ganci by examining the amplitude of diffraction patterns due to a tilted aperture [84]. They use the Fourier transform to calculate the intensity pattern from a tilted plane onto another plane perpendicular to the initial optical axis. As in [83] and [84] Leseberg and Frére were interested in the computation of the diffraction pattern between tilted planes, and they generalized the approach proposed by Rabal and Ganci [85]. Leseberg and Frére used their proposed method to obtain computer-generated holograms of larger objects [86].

Tommasi and Bianco investigated the relation of the angular spectra between rotated planes [87]. They also proposed a solution to the diffraction problem between tilted and shifted planes [88]. Implementation employs the FFT. In [89], continuous domain representation and limitations of the algorithm are highligthed. The mathematical and physical basis of the method together with several simulation results and their physical meaning are available in [89] and [63]. Another method is proposed by Matsushima *et al.* [90] to compute diffraction pattern on tilted planes, but the presented method is essentially based on the method given in [87], [88], and [62]. The significance of the procedure proposed in [90] is the comparison of several interpolation algorithms together with their effects on the computed diffraction patterns.

Mas *et al.* compare fast Fourier transform methods and fractional Fourier transform methods for calculation of diffraction patterns [91]. They state that discrete Fourier transform methods are valid only for a specific range of distances. On the other hand, fractional Fourier transform methods provide an accurate and easy implementation and give much better results in reproducing the amplitude patterns. In another paper, Mas *et al.* investigate the diffraction pattern calculation under convergent illumination [92]. They conclude that fractional Fourier transform gives a unified solution of calculation of diffraction field in all ranges of distances. Mendlovic *et al.* undertake similar investigations, comparing different numerical approaches and identifying the more advantageous one as a function of the distance of propagation [93]. Hennelly and Sheridan provide a very general and uniform framework to compare most such approaches [94]. Ozaktas *et al.* propose an algorithm based on the fractional Fourier transform that solves most of the problems associated with earlier algorithms applicable to the Fresnel regime, and is also applicable to a broader family of integrals [95].

Sypek compares the two computational approaches associated with the Fresnel diffraction [96]: one of them is the direct convolution, whereas the other one is based on a single Fourier transform with pre- and post-multiplications with chirp functions. Two modifications on the convolution based approach are proposed. The first one uses length $2N$ vectors instead of length N vectors. The second one divides the propagation distance into several segments. This reduces aliasing errors.

Veerman *et al.* propose a method that integrates the Rayleigh–Sommerfeld diffraction integral numerically [97]. They exploit the slow varying nature of the envelope of the highly oscillatory quadratic phase function in diffraction patterns. However, the method is not as fast as methods based on the planewave decomposition or Fresnel approximation. An FFT-based computation of the Rayleigh–Sommerfeld diffraction is also presented in [98].

Optical diffraction can also be represented by using wavelet transformation [99]–[101]. Sheng *et al.* have shown that optical wavelets proposed by Onural [99], [100] are the Huygens spherical wavelets under Fresnel approximation [101].

Some basis functions have been designed to deal especially with holographic signals. The wavelet-like *fresnelets*, which are reviewed in Subsection VII.A.2, have been constructed for Fresnel hologram processing [102], [103]. A Fresnel transform is applied to a standard B-spline biorthogonal wavelet basis to simulate the propagation in the hologram formation process. The obtained basis functions are well localized in the sense of the uncertainty principle for the Fresnel transform and have excellent approximation characteristics. The fresnelet transform allows for the reconstruction of complex scalar waves at several user-defined, wavelength-independent resolutions.

Cywiak *et al.* use the linearity of the Fresnel transform for fast computation [104], They first decompose the input function into Gaussian functions. Since it is easy to compute the Fresnel transform of a single Gaussian function, a final superposition of the individual results gives the desired Fresnel transform. It would be a much more elegant presentation if they first observed that the Gaussian (more generally, the Hermite polynomials times the Gaussian) functions are the eigenfunctions of the Fourier, and therefore the fractional Fourier transforms; and thus associate the easy computation of their Fresnel transform to this property.

Onural and Scott mainly concentrated on eliminating the twin-image in in-line holograms [105]. Since the twin image and the desired image overlap with each other in in-line holography, twin-image elimination is more important compared to the off-axis case. Moreover Onural [106] presented and compared the two digital Fresnel diffraction computation algorithms: one based on direct convolution with a chirp, and the other one based on a single Fourier transform with pre- and post-multiplications by a chirp. There are earlier works in the literature that discuss the application of DFT for hologram computation and the associated aliasing effects due to sampling [107].

Esmer *et al.*, presented algorithms based on pseudo matrix inversion, projections onto convex sets and conjugate gradient methods, together with performance comparisons for computing the diffraction pattern over a reference plane due to distributed discrete data in 3-D space [108].

Mapping from a 3-D problem into its 2-D counterpart, and other issues associated with resolution and accuracy, involves issues related to degrees of freedom in optics [69], [109], [110], [68]. A special case of optical field generation is presented as an optimization problem in [111]. Some associated algorithms based on optimization techniques are proposed [68], [67]. Wave field synthesis methods found applications in synthesis of some important beams and unconventional waves [112], [113], [70], [114]–[118]. Specific solutions of the wave equation for different purposes may be adopted to solve the 3DTV display related problems [112], [119]–[121], [70], [122], [111], [123]–[130], [113]. Interesting solutions provided for some other related cases, that might be applicable also to the holographic 3DTV problems, can be found in the literature [131]–[141].

Efficient and effective computation of holograms using modern computer graphics procedures and hardware are also reported [142]. Furthermore, 3-D objects are extracted from holograms digitally and displayed on conventional 2-D displays using computer graphics methods [143].

Compression of holographic signals require special techniques for improved compression performance due to the specific form and nature of such signals [144]. It is also shown that the 3-D objects can be reconstructed only from the phase information of the optical field calculated from the phase-shifting digital holograms [145]. Compression of holographic signals by constructing the hologram by pre-computed, indexed, stored small-size fringe patterns is demonstrated to yield real-time operation for horizontal parallax only (HPO) holograms [32], [30], [33].

A vast literature, which may offer solutions to problems related to those posed in Section IV, exists in the area of CGH The next section is devoted to a brief review of CGH techniques.

VI. Computer Generated Holography

CGH have about a forty years of history [146]–[149]. Instead of optical recording, the hologram associated with the wavefront representing the object is generated by employing different computational techniques and numerical approaches by mathematically simulating the optical wave propagation. An ideal CGH should achieve complex light modulation at a high diffraction efficiency and precise reconstruction of the target image. The CGHs outperform conventional refractive and diffractive components as a consequence of their ability to create any desired wavefront and thus to modify the input wavefront with much better flexibility [150]. For this reason CGHs find a wide range of application as display elements, optical interconnectors, aberration compensators in optical testing, spatial filters for optical signal processing and computing, beam manipulators and array generators etc. CGHs can be considered as thin optical elements with a complex amplitude transmittance; however, in many cases, they are phase only elements [12]. There are different classification of CGHs depending on the complex amplitude representation on the recording media (binary, phase,amplitude and combined phase-amplitude media), and the encoding method [151]. The algorithm to form a CGH is chosen according to the desired image characteristics and the associated computational complexity. Analytical approaches such as phase-detour method, kinoform method, double or multiple phase methods, explicit spatial carrier methods, 2-D simplex representation, representation by orthogonal and bi-orthogonal components, coding by "symmetrization," etc., can be used for computing digital holograms [151]. There are cell-oriented and point-oriented methods. In cell-oriented CGHs the hologram plane is divided into small resolution elements. The number of resolution cells needed depends on the complexity of the wavefront that is to be produced [149], [152], [153]. Iterative approaches such as iterative Fourier transform algorithm [154], direct binary search [155], simu1ated annealing [156] have been proposed and used. These methods are computationally demanding.

However, CGHs which are intended for dynamic displays need faster algorithms. It is difficult to realize SLMs which can provide the desired complex phase [157]. SLMs with only binary modulation are particularly desirable for display of CGHs. Computer generated binary reflection holograms may be displayed using micromirror devices (DMD) [23]. The SLM properties are crucial for the quality of the optical reconstruction of digital holograms. A comparison of the the optical reconstruction of phase and amplitude holograms by different modulators in terms of diffraction efficiency and recovery quality is presented in [158]. CGHs offer the possibility of displaying high quality 3-D images of 3-D objects with appropriate depth cues based on various algorithms [159]–[162]. A Fourier transform based algorithm for fast calculation of diffractive structures, which permits image reconstruction on cylindrically and spherically curved surfaces, is developed in [163]. Another popular approach is to calculate the CGH as a superposition of analytic distributions by decomposing the object surface into a certain number of discrete independent point sources, line segments or higher-order image elements. The modeled underlying physical phenomenon is the interference between the light waves coming from the analytically defined "holoprimitives" constructing the object and the reference wave to form the resulting complex amplitude distribution on the hologram plane [85], [164]. Hardware [165] and look-up table based computations are proposed [166], [167]. Representation of image elements at different locations by scaling and translation of similar elemental diffractive structures permits fast updating of the CGH by the so called incremental computing [168]. Real color fractional Fourier transform holography is proposed in [169]. Many other techniques for CGHs can be found in the literature [170]–[173], [167], [174]–[177].

VII. Discretization and Quantization Issues in Diffraction and Holography

The discretization of diffraction related signals by taking their specific characteristics into consideration is an interesting and fruitful area. Unfortunately, general approaches in sampling are not efficient nor adequate for such signals as also described in Section IV. Here we present available work in the literature in this area.

A. Sampling in Optics, Diffraction and Holography

1) Sampling of Optical Signals With Finite Extent in Different Domains: A signal can be space- or band-limited but never both. For optical signals, the so called (α)-*Fresnel limited* functions turned to be more convenient and efficient than the band-limited functions in terms of sampling and recoverability [178]. (α)-Fresnel limited functions are defined to have finite extent of ξ_0 in their Fresnel transform domain associated with the parameter α. Such functions are not band-limited, however, they can be reconstructed from their samples taken at a rate $T = 1/(2|\alpha|\xi_0)$. The proof of this result is given by Gori [178]. Another theorem proven in [178] indicates that the Fresnel transform of a *space-limited* function (a function $f(x)$ vanishing for $|x| > x_0$) can be fully recovered from its α-Fresnel domain samples. The same result was also proven later independently by Onural [179] who also stated the prefect reconstruction conditions for both band- and space-limited cases. In particular, it is shown that full recovery of space-limited signals from their below Nyquist rate sampled Fresnel diffraction patterns is possible. It is also shown in [180] and [181] that it is possible to reconstruct objects from hologram samples obtained below the Nyquist rate; real-life applications by considering finite number of samples and finite (nonimpulsive) area of the capturing charge coupled devices (CCD) array elements are discussed. Furthermore, the effect of sampling in noisy conditions is also analyzed. The possibility of full recovery from undersampled holographic signal is observed also in [182]. The authors considered the case of large numerical apertures, where the nonconstructive superposition of planar-wave components of the propagating diffraction field at the locations of the replicas essentially washes out the unwanted replicas of the original, and thus naturally accomplishes the full reconstruction. Full mathematical proof of this phenomena is recently given by Onural [183].

The effects of the shape of the sensing elements and the overall array size to the CCD captured optical data and subsequent digital reconstruction of off-axis holography are examined in [184]. A frequency domain analysis of the overall transfer function is carried out for both the planar and the spherical reference beam cases.

In a work by Stern and Javidi [185] it is shown that neither band-, nor space-limited functions can be fully recovered from their samples if the replicas of their Wigner distributions due to sampling do not overlap.

Several nonuniform sampling schemes have been suggested based on the observation that the bandwidth of the object remains unchanged as a consequence of the all-pass nature of the linear system that represents the diffraction [186], [187], [188]. Another approach observes that the information of interest in a hologram is carried in the complex envelope of the fringe pattern and not in the carrier ([189]). Based on this, Khare and George have suggested sampling the recorded hologram about twice the Nyquist rate for the object (or baseband) signal. This may be regarded as a generalization in the shift-invariant space spirit of [190]. Connection with the work in [102], where the modulation is replaced by the Fresnel transform, can be noted as well.

2) Wavelet-Inspired Discretization of Optical Signals: Wavelets have inspired several interesting approaches in the area of optical signal sampling and reconstruction. In [99], the diffraction integral is viewed as a continuous wavelet transform. The light field at different distances is regarded as the result of an inner product of the light distribution at some initial plane and scaled-shifted chirp functions. In contrast to conventional wavelet analysis, these scaling functions however, are not limited in neither the spatial nor the frequency domain. The transform has been named *scaling chirp transform* and shown to be valid and reversible in [100]. A number of inversion formulas are provided with a discussion on their redundancy and ways to possibly exploit this redundancy. For fixed scale, the scaled and shifted chirp functions form a complete orthogonal set, while they form a redundant frame over different scales. This also suggests a way to sample the light field throughout the space by using scaled chirp expansions.

Some related wavelet-like functions, called *chirplets* have been suggested in [191] and [192], and used for instantaneous frequency measurements [193]. A chirplet is a compact support signal with increasing (decreasing) frequency [191], [192]. It is band and time localized version of the scaling chirp function mentioned above. Chirplets are rather attractive for representation of holograms since they have minimal energy spread for the Fresnel transform in a similar sense as Gabor functions [102]. In [194], [195], and [196], methods for finding a sparse chirplet signal representation are suggested.

An interesting strategy to construct bases suitable for processing digital holograms is presented in [102]. Based on the observation that digital holography tends to spread out sharp details such as object edges over the entire imaging plane, standard wavelets have been ruled out as directly applicable to holograms. Instead, a Fresnel transform is applied to a wavelet basis to simulate the propagation in the hologram formation process and thus to build an adapted *fresnelet* basis. In contrast to classical wavelets, where multiresolution spaces are generated through dilation of one single function, in the fresnelets case there is one generating function for each scale. B-spline biorthogonal wavelets have been used to construct the fresnelet dictionaries due to their excellent approximation characteristics and analytical expression in spatial domain. Subsequently, their Fresnel transform associated wavelets are derived explicitly [102]. Thus, this new diffracted basis can be used to analyze the light field distribution at some distance and once a decomposition is obtained, the field can be calculated immediately in the original (initial) plane.

Digital reconstructions of diffraction patterns or holograms require algorithmic digital implementations of the underlying continuous mathematical models which represent diffraction. Common implementations of the Fresnel case are either based on convolution, or on a single Fourier transformation [106], [197]. Inevitably, either the kernel which represent the wave-propagation (diffraction), or its analytically known Fourier transform (the transfer function) of (2) should be discretized when the convolution is implemented digitally. This problem is in the focus of the paper [198] where some well known properties of the continuous Fresnel kernel, together with rather overlooked ones are presented. Furthermore, efficient computation of the exact Fresnel transform of some periodic input (object) functions at some specific discrete distances is given, too. Another observation is the perfectly discrete and periodic nature of the continuous Fresnel transform of periodic and discrete input functions for certain distances.

B. Quantization

From a theoretical point of view, the diffraction is an operation which disperses the information content of simple object patterns over the entire space; therefore, it is quite immune to noise or loss of information: reconstructions from partial holograms could be pretty much satisfactory, with some bearable quality degradation. Therefore, it is expected that grossly digitized holograms would still yield reasonable reconstructions. Indeed, this fact was utilized for the computer-generation of holographic masks, going all the way to binary holograms. It might be interesting to look at oversampled, but coarse digitized cases.

A recent paper [199] discusses the quantization effects in phase-shifting holography. It provided both numerical simulations and experimental quality assessment and concludes that, for both uniform (specular) and random (diffuse) objects a 4-bit quantization is sufficient to recognize the reconstructed objects and the difference between 6 and 8 bits is not perceivable. Above 4 bits, the effect of quantization on the reconstructed image quality seems to be independent of the object phase distribution. In [200] it has been observed, that the quality of the reconstructed images from recorded holograms is more influenced by the phase information than the magnitude information. The paper assumes, with relevant arguments, that the magnitude has a Raleigh distribution, whereas the phase is uniformly distributed over the $[-\pi, \pi]$ interval. Then, a solution for minimum-mean-squared-error quantizer in polar form is formulated and numerically solved for some quantization levels. The allocation of bits between phase and magnitude is discussed. It is observed that even though the phase and magnitude are statistically independent, the optimum magnitude quantization scheme depends on the number of phase quantization levels. The effects of phase quantization in Fourier holography is discussed in [201]. Binary and three-level hologram recordings are considered. It is concluded that phase quantization results in ghost images located at different depths; it is further concluded that these ghost images are less disturbing particularly for high-contrast images, due to their different depths. Nonuniform quantization through companding of complex numbers by employing nonuniform grid patterns over the complex plane is shown to be efficient for digital holograms with a reconstruction quality comparable to that obtained by quantization by the k-means algorithm [202]. Quantization issues associated with holographic signals are discussed in [144]. It is shown that degradation in reconstructed image quality is minimal for 10 bits or more, and the distortion becomes severe below 5 bits; numerical error plots together with reconstructed images are presented.

VIII. Signal Processing Tools for Diffraction and Holography Related Problems

So far, we presented the fundamental signal processing problems in holographic 3DTV (Section IV), and gave an overview of related work in the literature (Sections V, VI and VII). Here in this section we turn our attention to signal processing tools and techniques which we believe have the potential to significantly advance the state-of-the art in holographic 3DTV related issues.

A. Sampling From Shannon's Theorem to Frame Theory

The most well known and by far the most influential paper in sampling is published in 1949 by Shannon [203]–[205] The theorem formalized by Shannon simply states that a band-limited function can be fully recovered from its equispaced samples taken at a rate which is at least twice the highest frequency component of the function. The reconstruction (interpolation) formula is based on shifted sinc functions [203] and is known as cardinal series expansion, a term introduced by E. T. Whittaker [206] and used by Shannon through the work by J. M. Whittaker [207], [208]. The roots of this result have been traced back to Cauchy in 1841 [209], [210]. Kotel'nikov [211] formulated the same theorem independently in 1933. Japanese authors (e.g., in [212]) pay credit for this result to Someya [213]. For the huge amount of work done on uniform sampling after Shannon we refer to milestone reviews, tutorials and books and the references therein [214]–[218].

Naturally, the theory and applications of sampling and reconstruction have been significantly developed to handle various other constraints than the band-limited case since Shannon's work. Recent works have addressed the sampling through the more general shift-invariant space framework [219]. Other basis functions than the shifted sinc functions have been favored for sampling and reconstruction of real-life signals [190], and their approximation properties are studied [220]–[222]. Optimized designs lead to functions minimizing approximation error kernels [223]–[225]. An important subset of the shift-invariant spaces is the subset of wavelet spaces possessing additional multiresolution property [226], [227]. Sampling theorems [228] and sampling techniques for wavelet spaces have been studied extensively [229]–[231].

For the case of nonuniform sampling, Benedetto and Fereira [209, Sec. 1], have emphasized the results by Paley and Wiener [232] and Kadec [233]. These results have also inspired the study of nonharmonic Fourier series which then evolved into the theory of frames (see [234] and the references therein). Frames are a generalization of bases and in the most general case they provide the harmonics for signal reconstruction formulas. In Bendetto's work [234], most of the proofs of nonuniform sampling theorems have been stated from that frame theory point of view.

In an attempt to unify uniform and nonuniform sampling within the shift-invariant space framework, Aldroubi and Gröchenig have surveyed some 119 sources "...bringing together wavelet theory, frame theory, reproducing kernel Hilbert spaces, approximation theory, amalgam spaces, and sampling" ([235, p. 591]). Interested readers can find precise mathematical proofs together with practical iterative frame algorithms for signal reconstruction in that survey. The formulation of the sampling problem from a shift-invariant space perspective might turn to be quite important for the problems in diffraction and holography. The Fresnel approximation, extensively used for description of diffraction processes, is in fact a convolution integral which preserves the shift-invariance. Therefore, the nonbandlimited sampling and reconstruction schemes proved to be efficient for digital images can be appropriately modified and extended to handle holography problems. One example is the so-called Fresnel-splines [102].

A straightforward extension of the classical sampling and interpolation is presented in [236], where the so called quasi-Fourier transform is introduced by replacing the exponent x in the Fourier basis functions $e^{j\omega x}$ by a function $\phi(x)$. Thus a new band-limited function, which is recoverable from its periodic samples, is generated.

B. Transformation Theory and Space-Frequency Analysis

As elementary as it is, *planewave decomposition* remains a key tool for understanding optical diffraction. Plane wave decomposition is directly related to Fourier decomposition, with planewaves propagating in different directions corresponding to different spatial frequencies. Therefore, the Fourier transform has been the most natural tool for space-frequency analysis of optical signals. Algorithms for fast implementation of its discrete version, the *discrete Fourier transform* (DFT), the so-called FFT algorithms are extensively studied. The famous Cooley–Tukey algorithm is just one from this family. Among others are prime-factor (Good–Thomas) FFT algorithm [237], [238], Bruun's FFT algorithm [239], Rader's FFT algorithm

[240], and Bluestein's FFT algorithm [241]. See also a tutorial review on FFT algorithms [242]. A rather new approach to the efficient implementation of Fourier transforms is computing it via the Walsh–Hadamard transform (WHT) [243].The approach is based on the Good's theorem [244], which suggests factorizing a Kronecker product structured transform matrix into a product of several sparse matrices. Since the WHT matrix has exactly such a recursive Kronecker product structure the WHT coefficients can be computed very efficiently, and then converted into FT coefficients by a special conversion matrix [243].

A number of newer transforms have been found applicable or at least promising for analysis of signals modeling optical diffraction. They can be unified under the notion of *atomic decompositions*. More specifically, these are signal representations in terms of basis sets with particular features especially suited to an application, allowing the capture of the signal characteristics by only a few significant coordinates. A selection of references to the most important atomic decompositions are given below.

Wavelets have been perhaps the most inspirational constructions due to their ability to represent transient signals by offering a trade-off between space and frequency (scale) resolution. As basis functions, they separate the space of square-integrable functions into a set of nested subspaces. We refer the reader to the book by Mallat [226] and to book review by Benedetto [245] for basic information regarding wavelets. To improve the time-frequency (space-scale) resolution, wavelets have been extended also to overcomplete schemes such as *wavelet packets* [246], [226], [247] and bases with improved directional and translational-invariant properties, such as *Gabor wavelets* [248], [249] and *Dual tree-complex wavelets* [250]. Other bases, such as *ridgelets*, *curvelets*, *beamlets*, *brushlets* have been designed for effective representation of ridges, curves, lines or oriented textures, respectively [251]–[259].

The *chirplets*, which are also already commented in the light of holographic signal sampling (cf. Section VII-A.2), can be useful in digital holography for space-frequency analysis since they are known to be good instantaneous frequency estimators [193].

In general, atoms are organized in overcomplete *dictionaries* and the task is to obtain a sparse or super-resolving representation with preferably $O(N)$ or $O(N \log N)$ number of computations [260]. Several methods have been proposed for obtaining optimal signal representations from overcomplete dictionaries, such as *frame decomposition* [261], *matching pursuits*, [262], *basis pursuits*, [263], [260], *best orthogonal basis search* [246], [247]. We briefly review them here because of their potential importance for processing of holographic signals, where, due to the high amount of data, sparse and adapted signal decompositions are highly appreciated.

The *frame decomposition* has been acknowledged as a sampling approach (see Section VIII-A) and can be stated within the classical least squares problem. In this setting, a set of linear equations relate the linear expansion coefficients with the output signal samples through a transform (frame operator) matrix, that is

$$\mathbf{Ac} = \mathbf{x}. \tag{3}$$

Here, $\mathbf{c}$ is the M-dimensional vector of input unknown coefficients, $\mathbf{A}$ is the known $N \times M$ system matrix, and $\mathbf{x}$ is the N-dimensional vector of given data (e.g., desired wavefield). Usually, $M < N$. In this case, only an approximate solution is found minimizing the l^p norm of the error vector $\|\mathbf{e}\|_p = \|\mathbf{x} - \mathbf{Ac}\|_p$. In the case of $p = 2$, this is the *least squares* (LS) solution which is found when the error vector $\mathbf{e}$ is orthogonal to all the columns of $\mathbf{A}$ and explicitly given by taking the *pseudoinverse* $\mathbf{A}^\dagger$ of the matrix $\mathbf{A}$, as $\mathbf{c} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{x} = \mathbf{A}^\dagger\mathbf{x}$, [264].

Numerical solutions of (3) involve *LU* factorization for the case of square transform matrices [264] or *QR* factorization for the case of full-rank LS problems with $N > M$, implemented via Gramm-Schmidt orthogonalization, Householder transformations or Givens rotations [265]. Considered as a particular case of optimization problem with linear equality constraints, the inverse problem (3) can be solved by employing *linear programming* (LP). It is especially appropriate for undetermined $(N < M)$ cases or for achieving partial orthogonality of the error vector in LS problems. From a geometrical point of view, two major LP approaches have been developed based on the Dantzig's simplex method [266] and on the interior point method [267], [268]. Modifications, addressing the computational efficiency by combining the benefits of these two have been suggested [354], [269]–[272].

Best Orthogonal Basis (BOB) relies on organizing the basis elements (e.g., wavelet packets) into tree structures having parent-children subspace relations and fast searching the best set of orthogonal subspaces that form a complete representation of the signal [246], [247].

Matching Pursuits, suggested by Mallat and Zhang [262] find an approximate and sparse signal decomposition employing a recursive and adaptive algorithm that builds up a signal representation one element at a time, picking the most contributive element at each step. Starting from an initial residual $R^{(0)} = s$, the element chosen at the ith step is the one which minimizes $\|R^{(i)}\|$. It is the same as the one which maximizes $|\alpha_{\gamma_i} = |\langle R^{(i-1)}, \phi_{\gamma_i}\rangle|$, since $R^{(i)} = R^{(i-1)} - \alpha_{\gamma_i}\phi_{\gamma_i}$. The MP approach is quite powerful for extracting structure from signals which consist of components with widely varying space-frequency localizations [262]. Particularly interesting works that can relate digital holography and MP algorithm are [196] and [195]. There are proposed methods for fast MP algorithm with the dictionary of Gaussian chirp functions that find m decomposition atoms in $O(mN)$ operations for a signal of length N.

Basis Pursuits (BP), suggested by Chen and Donoho, aim at finding the coefficients $\mathbf{c}$ in (3) yielding minimal l^1 norm. In this setting, the matrix $\mathbf{A}$ is an $M \times N$ matrix with the dictionary elements $\{\boldsymbol{\alpha}_\gamma\}_{\gamma\in\Gamma}$ collected as columns. It is a convex, nonquadratic optimization problem which involves considerably more effort and sophistication compared to the case of l^2 norm minimization. Solution of the BP method has been sought by employing the simplex and interior points linear programming techniques [263].

FOCUSS, suggested by Rao and Kreutz-Delgado [273]–[275] finds an optimal basis selection by minimizing diversity measures proposed by Wickerhauser and Donoho [246], [276]. The method uses a factored representation for the gradient and involves successive relaxation of the Lagrangian necessary condition. This yields algorithms that are intimately related to the affine scaling transformation (AST) based methods commonly employed by the interior point approach to nonlinear optimization [277]. In [273] and [274], the authors give comprehensive analysis of the convergence of these methods, showing that the rate of convergence can be controlled in some range. The Gaussian entropy minimization algorithm is shown to be equivalent to a well-behaved $p = 0$ norm-like optimization algorithm. Computer experiments demonstrate that the p-norm-like and the Gaussian entropy algorithms perform well, converging to sparse solutions.

K-SVD, suggested by Aharon, Elad, and Bruckstein [278] is an algorithm for adapting dictionaries to a collection of signals rather than a single signal. Given a set of training signals, they seek the dictionary that leads to the sparsest possible representation. The method can be viewed also as a generalization of the K-Means clustering process. K-SVD is an iterative method that alternates between sparse coding of the examples based on the current dictionary, and a process of updating the dictionary atoms to better fit the data.

The *fractional Fourier transform (FRT)* is a generalization of the ordinary Fourier transform with a fractional order parameter such that the zeroth order transform is the identity operation, the first order transform is the ordinary Fourier transform, and the fractional transform interpolates between them in an index-additive manner [279]–[286]. It has found a large number of applications in signal processing (for instance [287]–[290]) and optics (for instance [291]–[297]). The relationship of the FRT to optical propagation and diffraction rests on the result relating free-space propagation in the Fresnel approximation (namely the Fresnel integral or the Fresnel transform [298]) to the FRT [295], [299], [296]. Extensions of this result relate arbitrary linear canonical transforms to the fractional Fourier transform; for instance [297]. Optical systems consisting of arbitrary concatenations of lenses and section of free space can be modeled as linear canonical transforms, and thus propagation through such systems, including free-space propagation, can be viewed as an act of continual fractional transformation. The wave field evolves through fractional Fourier transforms of increasing order as it propagates through free space or the multilens system. While these results are directly relevant to holography, relatively few works have explicitly applied the FRT to holographic problems [300]. Sampling and periodicity issues related to the fractional Fourier transformation have been discussed in [301]–[306]. The discrete fractional Fourier transform has been defined in [307] and [308]; other works in this area, including some alternative definitions include [309]–[317]. The applications of other fractional transformations in optics is reviewed in [318].

Linear canonical transforms (LCT) are a class of integral transforms which include the fractional Fourier and Fresnel transforms and other important operations as special cases [319]. They are also known as quadratic-phase systems or integrals, generalized Huygens integrals, generalized Fresnel transforms, ABCD integral transforms, or similar names. Fast numerical algorithms for LCTs exist [320], [94].

Integrals involving highly oscillatory exponential terms play an important role in optics [97]. Under certain approximations these take the form of quadratic-phase integrals which are equivalent or related to linear canonical transforms. During numerical evaluation of these integrals, naive application of the Nyquist-Shannon approach may require very large sampling rates due to the highly oscillatory nature of the kernels. It has been shown that by careful consideration of sampling issues, the number of samples need not be allowed to be larger than the space-bandwidth product of the signals. A fast $N \log N$ algorithm for computing the samples of the continuous FRT of a function from the samples of that function is presented in [321] and [322], where N is the space-bandwidth product of the signal. Related issues are discussed in [323]–[329]. This algorithm has been extended, with the same properties, to arbitrary LCTs [95]. This approach employs the smallest possible number of samples implied by the space-bandwidth product of the output signal. Recalling that linear canonical transforms can model systems consisting of arbitrary concatenations of lenses and sections of free space, this algorithm can be used to compute the input-output relation for such systems with an efficiency and accuracy comparable to the use of the FFT in computing the Fourier transform Comparisons of different approaches to calculating Fresnel integrals may be found in [93], [91], [320] which shed light onto the limitations of certain earlier methods.

For a review of the literature on *space-frequency representations* we refer the reader to [330]–[334]. Some of these have received greater attention in optics, such as the *windowed/short-time Fourier transform*, which is closely related to Gabor expansions [335], [336], and the *Wigner distribution* and *ambiguity function*. Reviews of the applications of the Wigner distribution in optics are given in [337] and [338]. A Special Issue [339] is devoted to the Wigner distribution and phase space in optics. Related topics are sometimes referred to as operator optics [340]–[345].

The relationship between the Wigner distribution and linear canonical transforms and fractional Fourier transforms is of key importance. Fractional Fourier transformation corresponds to rotation of the Wigner distribution [284], [287], [289], [346]. The Wigner distribution and linear canonical transforms were established as a standard tool in optics primarily by Bastiaans [347], [348], based on a number of earlier works [349]–[351].

Recently, the diffraction problem is revisited and formulated using the projection-slice theorem as a tool using impulse functions defined over curves and impulses [352], [353].

IX. CONCLUSION

While signals and systems concepts have been applied to optical problems for decades, the degree of sophistication attained seems short of that in mainstream signal processing and insufficient to handle certain problems arising in 3DTV, possibly as a result of the interdisciplinary nature of the problems. While the accumulated knowledge in certain areas, such as in optical signal recovery, has reached a highly sophisticated state, in others it falls short. Most strikingly, the issue of sampling and quantization (and more generally finite representation) of optical fields, that lies at the heart of computational techniques, seems to be handled mostly in an ad hoc manner, despite the fact that the tools necessary to put these issues on firm ground are part of standard information theory and signal analysis topics.

While there is a bulk of articles on the topic, the recent achievements in frame and wavelet theory are still to meet the challenging problems in optics, diffraction and holography. In a very general setting, the sampled representation of the 3-D scene will be in the form of nonuniformly distributed samples over the 3-D space. Such a setting requires rather novel approaches based on frame theory instruments for dealing with irregularly distributed samples. Another promising topic for further research is the use of application-tuned bases for achieving sparse signal expansions (so-called atomic decompositions). Fresnelets, chirplets, and scaling chirps, while being defined and reasoned mathematically, are still to be implemented and tested in real holography and diffraction applications.

The problem of computing the optical field emanating from a field specified over an arbitrary surface profile does not seem to have been solved satisfactorily from a theoretical or numerical perspective. Although past work on 3-D light synthesis exhibits considerable sophistication, the various alternative approaches to synthesize desired dynamic optical fields using different SLM technologies have not been sufficiently explored. While such tools as atomic decompositions, space-frequency representations, fractional Fourier transforms, and the like, are understood to be relevant for optical problems, they have not been fully exploited for purposes of diffraction problems. Especially in the context of holography and integral imaging, there seems to be a great deal of confusion regarding the required information capacity to record 3-D images.

Improvements and developments in the above mentioned areas will not only pave the way to the realization of efficient 3DTV systems, but also constitute advances in optics and signal processing theory.

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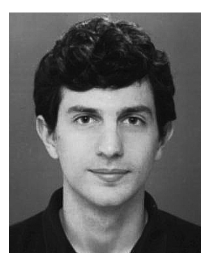

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