INVITED PAPER

Accurate Solutions of Extremely Large Integral-Equation Problems in Computational Electromagnetics

This paper reviews accurate solutions for extremely large integral-equation problems in computational electromagnetics.

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ABSTRACT | Accurate simulations of real-life electromagnetics problems with integral equations require the solution of dense matrix equations involving millions of unknowns. Solutions of these extremely large problems cannot be achieved easily, even when using the most powerful computers with state-of-the-art technology. However, with the multilevel fast multipole algorithm (MLFMA) and parallel MLFMA, we have been able to obtain full-wave solutions of scattering problems discretized with hundreds of millions of unknowns. Some of the complicated real-life problems (such as scattering from a realistic aircraft) involve geometries that are larger than 1000 wavelengths. Accurate solutions of such problems can be used as benchmarking data for many purposes and even as reference data for high-frequency techniques. Solutions of extremely large canonical benchmark problems involving sphere and National Aeronautics and Space Administration (NASA) Almond geometries are presented, in addition to the solution of complicated objects, such as the Flamme. The parallel implementation is also extended to solve very large dielectric problems, such as dielectric lenses and photonic crystals.

KEYWORDS | Computational electromagnetics; iterative solutions; large-scale problems; multilevel fast multipole algorithm (MLFMA); parallelization; surface integral equations

I. INTRODUCTION

Real-life electromagnetics problems often involve large-scale objects discretized with millions of unknowns. These problems can be solved accurately and efficiently with the multilevel fast multipole algorithm (MLFMA) [1]–[3]. Parallelization of MLFMA on parallel computers enables the solution of even larger problems [4]. Recently, various parallel implementations have been developed to increase the problem size from millions to more than one billion [5]–[17]. Thanks to advances in both computer hardware and parallelization techniques, solutions of electromagnetics problems involving realistic objects discretized with hundreds of millions of unknowns have become not only possible but also increasingly more common [17]–[19]. This paper presents an overview of a sophisticated simulation environment based on a parallel implementation of MLFMA that is capable of such large-scale solutions on moderately powerful computers. We present numerical examples involving both canonical and complicated targets that are larger than 1000 wavelengths. We also present the solution of large-scale dielectric problems, such as transmission through dielectric lenses and photonic crystals. Details of the parallel implementation can be found in [12] and [20].

Before presenting the details of large-scale simulations, we need to answer the following question: Is it required to solve large-scale electromagnetics problems using a full-wave

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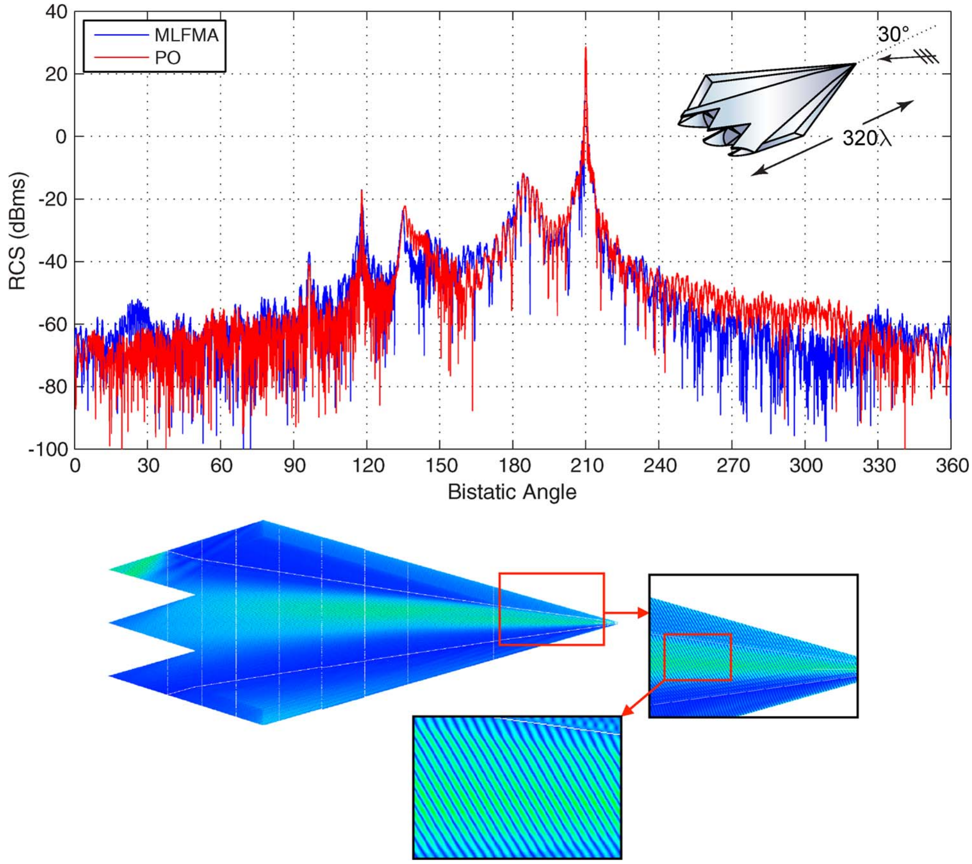

Fig. 1. ***Solutions of a scattering problem involving the Flamme at 160 GHz. Bistatic RCS (dBms) is plotted as a function of the observation angle from 0° to 360°, where 30° and 210° correspond to the backscattering and forward-scattering directions, respectively. Imaginary part of the electric current induced on the object (calculated with MLFMA) is also depicted.***

solver, considering that much more efficient high-frequency techniques are available in the literature? The answer depends on the level of desired accuracy. For example, Fig. 1 presents the solution of a scattering problem involving a stealth airborne target called the Flamme [21]. The target is located in free space and illuminated by a plane wave propagating at 30° angle from the nose with the electric field polarized horizontally. The scaled size of the target is 0.6 m, corresponding to approximately 320λ at 160 GHz, where λ is the wavelength. Fig. 1 shows the bistatic radar-cross-section (RCS) values of the Flamme in decibel meter squared (dBms) as a function of the bistatic angle. In the plot, 30° and 210° correspond to the backscattering and forward-scattering directions, respectively. RCS values are obtained with parallel MLFMA (with 1% maximum error) in hours and with physical optics (PO) in seconds. From one point of view, PO is quite successful by providing reasonably accurate RCS values, especially in and around the forward-scattering direction. On the other hand, there are large discrepancies (as large as 10–20 dB) between the results in other directions. Furthermore, the oscillations with respect to the observation angle of the RCS values predicted by MLFMA and PO do not agree well with each other. This might be expected since PO is an approximate technique and it is based on the high-frequency behavior of electromagnetic waves. Unlike PO, the MLFMA solver takes all multiple-scattering mechanisms into account. As a result, the backscattering RCS, which is a fundamentally important quantity, may be in error by as much as 14.75 dB with PO, and this may not be acceptable for many applications. Fig. 1 also depicts the imaginary part of the electric current induced on the surface of the Flamme calculated with MLFMA. It can be observed that the current behaves as predicted by PO, e.g., there are lit and shadow regions and oscillations with a periodicity of the wavelength. Nevertheless, a detailed inspection reveals that the current and electromagnetic fields are actually more complicated, i.e., there are multiple reflections, diffractions, resonances, etc. All these phenomena can accurately be modeled only by using a full-wave solver, such as MLFMA.

It should be emphasized that full-wave solvers, including fast solvers (such as MLFMA), cannot compete (in terms of time) against the approximation techniques (such as PO) with today's computing technology. Approximation techniques are still thousands of times faster and most of them can be implemented on a desktop computer, without any need to employ high-performance computing. Nevertheless, full wave solvers are now capable of solving very large real-life problems with unprecedented levels of accuracy and details. In fact, results of full-wave simulations, including those presented in this paper, can be used as reference data to test the accuracy of various high-frequency techniques.

II. SIMULATION ENVIRONMENT

The simulation environment that is considered in this paper for full-wave solutions of large-scale electromagnetics problems in the frequency domain involves the following basic components:

- surface integral equations for efficient formulations;
- triangulations of surface geometries and low-order discretizations of unknown functions for flexible modeling and efficient numerical solutions;
- Krylov-subspace algorithms for fast iterative solutions;
- a robust implementation of MLFMA for fast matrix–vector multiplications, without sacrificing the accuracy;
- parallelization to solve extremely large electromagnetics problems.

This section provides an overview of these components, also considering the recent developments in the literature.

A. Surface Integral Equations

Electromagnetics problems involving metallic and dielectric objects can be formulated both rigorously and efficiently with surface integral equations [22]. For metallic objects, the electric-field integral equation (EFIE), the magnetic-field integral equation (MFIE), and the combined-field integral equation (CFIE) [23] are the most popular formulations. EFIE is applicable to open and closed surfaces, but it usually leads to ill-conditioned matrix equations and suffers from the well-known low-frequency breakdown problem. Recent studies mainly focused on improving the conditioning of EFIE by using advanced discretizations based on decompositions [24], [25] and dual functions [26]. For closed objects, CFIE is preferred since it is free of the internal resonances [23] and less affected by the low-frequency breakdown problem. However, this formulation is contaminated with the inaccuracies due to the low-order discretizations of the identity operator [27], [28]. If the MFIE contribution is large in CFIE, compared to the EFIE contribution, accuracy of CFIE may be significantly hampered since MFIE contains an identity operator. Accuracy of CFIE can be improved by using the linear–linear functions [29].

Recently, novel integral-equation formulations for dielectric objects have been developed. Among them, the combined tangential formulation (CTF) [30], which is a rescaled version of the Poggio–Miller–Chang–Harrington–Wu–Tsai (PMCHWT) formulation [22], provides very accurate solutions with low-order discretizations. For better conditioned matrix equations, however, the electric and magnetic current combined-field integral equation (JMCFIE) [31]–[35] is preferred. This is a mixed formulation based on a linear combination of CTF and the combined normal formulation (CNF) [30]. Similar to CFIE for conducting objects, a Galerkin discretization of JMCFIE results in well-tested identity operators, which lead to very efficient iterative solutions [33], but may reduce the accuracy of the results. Therefore, the combination parameter in JMCFIE is extremely important for the tradeoff between the accuracy and the efficiency [32], [36].

B. Discretization

Probably, the most popular functions for low-order discretizations of the surface integral equations in computational electromagnetics are the Rao–Wilton–Glisson (RWG) functions [37]. Despite their simplicity, the RWG functions provide the necessary flexibility to model the current distributions on arbitrarily complicated surfaces. They are also shown to possess the ability to properly model the charge distributions [38]. In order to use the RWG functions, surfaces are discretized with triangles that are small in terms of the wavelength. Dense matrix equations are obtained by expanding the currents and by testing the boundary conditions using a set of RWG functions. Electromagnetic interactions between the discretization elements (i.e., basis and testing functions) can be calculated via numerical integrations on triangular domains. Using the RWG functions, numerical integrations can be performed very accurately and efficiently [39]. The RWG functions are also very suitable for the acceleration methods based on the factorization and diagonalization, such as MLFMA. Unsurprisingly, some of the largest integral-equation problems solved in the literature are discretized with the RWG functions [5]–[17].

C. Iterative Algorithms

Many Krylov-subspace algorithms are available in the literature to perform iterative solutions of dense matrix equations derived from integral equations. Among them, the generalized minimal residual (GMRES) algorithm [40] has become popular due to its stability and efficiency for ill-conditioned matrix equations. Flexible variants of this algorithm have also been used to construct inner–outer schemes [41]. For well-conditioned matrix equations, however, alternative methods, such as the biconjugate-gradient-stabilized (BiCGStab) algorithm [42], can be more efficient than GMRES by providing fast convergence without increasing the memory requirement. BiCGStab is particularly useful for the matrix equations derived from CFIE and JMCFIE [5], [33]. Solutions of these formulations can be further accelerated with simple block-diagonal preconditioners [1], [33] that are suitable for low-complexity algorithms, such as MLFMA.

D. Multilevel Fast Multipole Algorithm

Matrix–vector multiplications required for iterative solutions can be performed efficiently and accurately with MLFMA [1]–[3]. Using the factorization and diagonalization of electromagnetic interactions [43], MLFMA performs the multiplications of $N \times N$ matrices with arbitrary vectors in $\mathcal{O}(N \log N)$ time and using $\mathcal{O}(N \log N)$ memory. Each matrix–vector multiplication is formulated as a sequence of aggregation, translation, and disaggregation stages, which are realized on a multilevel tree structure that is constructed by the recursive clustering of the object. MLFMA implementations have various controllable error sources, in addition to the discretization, numerical integration, and iterative-residual errors. Specifically, two important accuracy parameters are the number of harmonics for the factorization/diagonalization and the size of stencils to interpolate/anterpolate [44] electromagnetic fields. The number of harmonics can be determined by the excess bandwidth formulas [45], [46], whereas the number of interpolation points can be determined heuristically [47]. The latter is relatively easy, considering that the same stencil can be applied at all levels of the tree structure. Global (instead of local) interpolation [48] is also an option to suppress the error if $\mathcal{O}(N \log^2 N)$ complexity is acceptable.

The conventional MLFMA has a low-frequency breakdown that inhibits its applicability to nonuniform and/or dense discretizations [49]. A straightforward solution to this problem is to discard the diagonalization (that is responsible for the breakdown) and employ multipole expansions directly for the subwavelength interactions [49]–[52].

E. Parallelization

Solutions of large-scale problems require the parallelization of MLFMA on parallel computers [4]–[20]. The idea is to distribute the computational tasks into several processes so that the solution can be performed quickly. In addition, parallelization enables the solution of larger problems by increasing the total memory available for solutions. Unfortunately, the parallelization of MLFMA is a formidably challenging mission due to complicated structure of this algorithm. Different parallelization schemes have appeared in the literature, especially for distributed-memory architectures, to increase the parallelization efficiency [4]–[17]. A successful technique is the hierarchical partitioning strategy [6], [10], [12], [20], which facilitates highly efficient parallelizations on hundreds of processors. The hierarchical strategy is based on optimizing the (subdomain and field) partitioning at each level of MLFMA such that the overall tree structure is distributed among processors in the best possible manner. Using the hierarchical strategy, the load balancing is improved and communications are accelerated. In this work, we use the hierarchical strategy for the solution of extremely large electromagnetics problems discretized with hundreds of millions of unknowns on moderate-size computers.

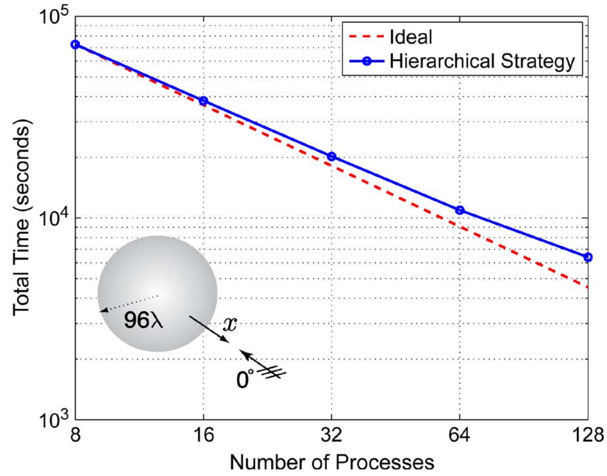

Fig. 2. *Solutions of a scattering problem involving a conducting sphere of radius of 96λ discretized with 33 791 232 unknowns. The total computing time is plotted as a function of the number of processes from 8 to 128.*

In order to demonstrate the improved parallelization efficiency with the hierarchical strategy, Fig. 2 presents solutions of a scattering problem involving a conducting sphere of radius 96λ discretized with 33 791 232 unknowns. The problem is solved with a maximum of 1% error in the scattered fields. The solution is parallelized into 8, 16, 32, 64, and 128 processes on a cluster of Intel Xeon Nehalem-EX L7555 processors with 1.87-GHz clock rate. Fig. 2 depicts the total computing time, including problem input/output, setup, and 39 BiCGStab iterations (for 0.001 residual error). The ideal case assuming 100% parallelization efficiency is also shown for comparisons. We observe that the computing time is significantly reduced from 72 591 s to only 6400 s when the number of processes is increased from 8 to 128.

III. NUMERICAL EXAMPLES

In this section, we present examples to the solution of large-scale electromagnetics problems with parallel MLFMA. All objects are located in free space, and the ensuing electromagnetics problems are formulated with CFIE or JMCFIE with a combination parameter of 0.5, i.e., CFIE $= 0.5 \times$ EFIE $+ 0.5 \times$ MFIE and JMCFIE $= 0.5 \times$ CTF $+ 0.5 \times$ CNF. All solutions are performed with a target of 1% maximum error in the scattered fields. In order to reach this target, all components of the error sources are kept well below 1%. For example, numerical integrations of matrix elements (near-field and far-field interactions) are calculated with maximum 1% error, and, for each problem, iterations are carried out until the residual error is reduced to below 0.001 or 0.005.

Similarly, other sources of error due to MLFMA and CFIE are also monitored and kept under 1%.

A. Benchmark Problems

Fig. 3 presents the solution of a scattering problem involving a conducting sphere of radius 340λ discretized with 540 659 712 unknowns. For the solution, MLFMA is parallelized into 64 processes on a cluster of Intel Xeon Nehalem-EX L7555 processors with 1.87-GHz clock rate. The total computing time (including input/output, setup, and 65 BiCGStab iterations for 0.001 residual error) is 60.5 h. The normalized bistatic RCS (i.e., RCS/πa^2, where a is the radius in meters) is plotted as a function of the observation angle from 0° to 180°, where 0° and 180° correspond to the backscattering and forward-scattering directions, respectively. RCS values around the backscattering and forward-scattering directions are zoomed in. We

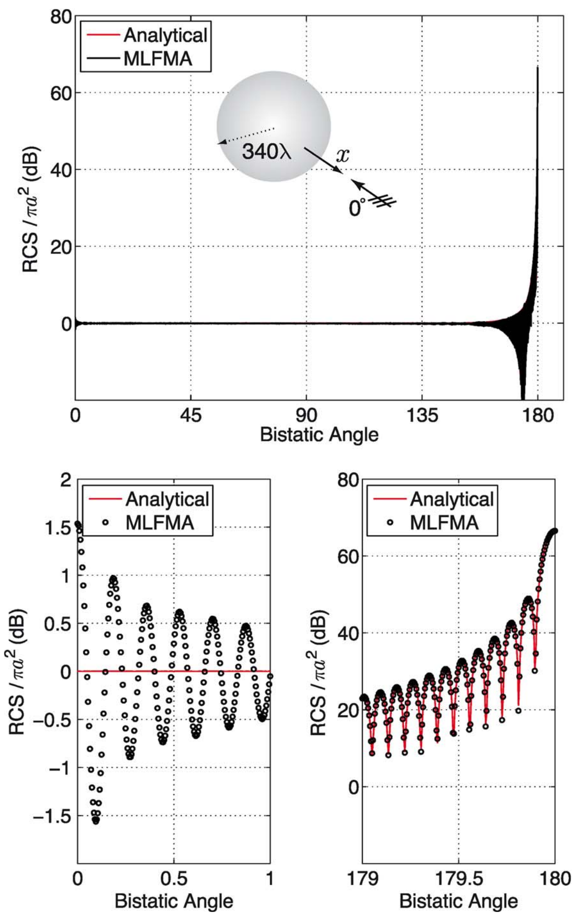

Fig. 3. ***Normalized bistatic RCS (RCS/πa^2) values of a sphere with a radius of 340λ discretized with 540 659 712 unknowns from 0° to 180°, where 0° and 180° correspond to the backscattering and forward-scattering directions, respectively. RCS values are zoomed in around the backscattering and forward-scattering directions in separate plots. Computational values provided by the parallel MLFMA implementation with 1% maximum error agree well with an analytical Mie-series solution.***

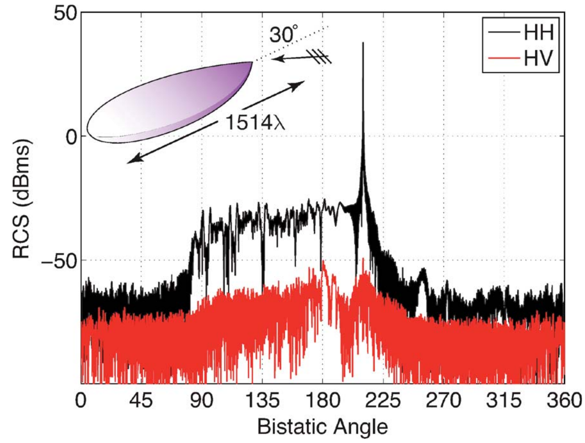

Fig. 4. ***Co-polar (HH) and cross-polar (HV) bistatic RCS values (dBms) of the NASA Almond at 1.8 THz. The target is illuminated by a plane wave propagating at 30° angle from its nose with the electric field polarized horizontally. For numerical solutions, the target is discretized with 552 310 272 unknowns.***

observe that the computational values agree very well with the analytical Mie-series solution. Considering all observation angles (0° to 180°), the relative error (as calculated in [18]) is found to be 0.84%. Assessing the accuracy of solutions is of the utmost importance. For this purpose, an interactive web-based benchmarking tool is prepared and made available at www.cem.bilkent.edu.tr/benchmark [54].

Fig. 4 presents the solution of a scattering problem involving another canonical object, i.e., the National Aeronautics and Space Administration (NASA) Almond [53], at 1.8 THz. The size of the target is approximately 1514λ at this frequency. As depicted in the inset of the figure, the NASA Almond is illuminated by a plane wave propagating at 30° angle from the nose with the electric field polarized horizontally. The problem is discretized with 552 310 272 unknowns and solved by using MLFMA on a cluster of Intel Xeon Nehalem-EX L7555 processors with 1.87-GHz clock rate. The total computing time (including input/output, setup, and 70 BiCGStab iterations for 0.001 residual error) is 56.6 h, when the solution is parallelized into 64 processes. Fig. 4 depicts the co-polar and cross-polar RCS values (dBms) with respect to the observation angle from 0° to 360°, where 210° corresponds to the forward-scattering direction. Typical properties of the RCS of the NASA Almond are observed: low backscattered RCS, no specular reflection, and relatively large values in the 90°–210° range.

B. Flamme

As an example of the application of MLFMA to complicated 3-D objects, Fig. 5 presents the solution of a scattering problem involving the stealth airborne target Flamme [21]. Similar to the NASA Almond, the target is illuminated by a plane wave propagating at 30° angle from

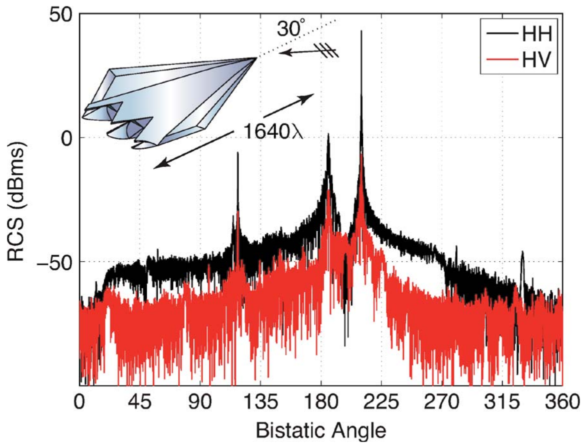

Fig. 5. *Co-polar (HH) and cross-polar (HV) bistatic RCS values (dBms) of the Flamme at 820 GHz. The target is illuminated by a plane wave propagating at 30° angle from its nose with the electric field polarized horizontally. For numerical solutions, the target is discretized with 538 967 040 unknowns.*

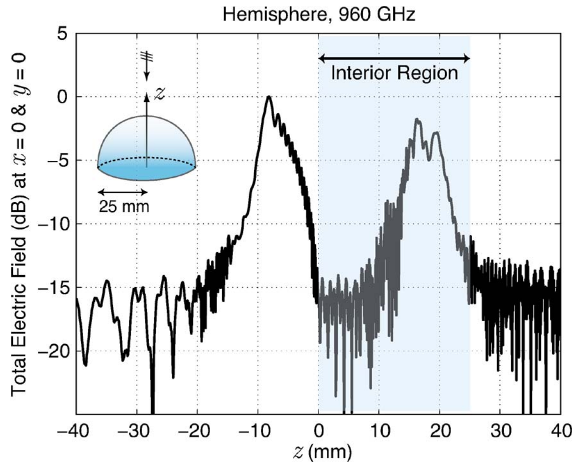

Fig. 6. *Solution of an electromagnetics problem involving a dielectric hemisphere lens with a radius of 25 mm at 960 GHz. The lens has a relative permittivity of 4.8 and the problem is discretized with 39 389 184 unknowns. The total electric field on the axis of rotation of the lens is plotted from z = −40 to 40 mm.*

the nose with the electric field polarized horizontally. At 820 GHz, the size of the target corresponds to approximately 1640λ and it is discretized with 538 967 040 unknowns. For the solution, MLFMA is parallelized into 64 processes on a cluster of Intel Xeon Nehalem-EX L7555 processors with 1.87-GHz clock rate. The total computing time (including input/output, setup, and 75 BiCGStab iterations for 0.001 residual error) is 58.2 h. Fig. 5 depicts the co-polar and cross-polar RCS (dBms) values with respect to the observation angle from 0° to 360°. The backscattered RCS (at 30°) is very low since the target exhibits stealth properties. On the other hand, there are specular reflections and the RCS of the Flamme makes peaks at several observation angles, in contrast with the RCS of the NASA Almond.

C. Dielectric Problems

The hierarchical parallelization of MLFMA can be extended to dielectric problems [35]. Fig. 6 presents the solution of an electromagnetics problem involving a dielectric hemisphere lens with a radius of 25 mm. The lens has a relative permittivity of 4.8 and it is illuminated by a plane wave propagating toward its convex surface at 960 GHz. The problem is discretized with 39 389 184 unknowns and the solution is parallelized into 64 processes on a cluster of Intel Xeon Nehalem X5560 processors with 2.80-GHz clock rate. The total computing time (including input/output, setup, and 68 BiCGStab iterations for 0.005 residual error) is 21.6 h. Fig. 6 depicts the total electric field on the axis of rotation of the lens from $z = -40$ to 40 mm. Focusing due to the lens is observed in the transmission region at around $z = -7$ mm, where the total electric field is maximum.

Finally, Fig. 7 presents the solution of a transmission problem involving a simple photonic crystal. Five dielectric slabs of dimensions $2 \times 2 \times 0.41$ cm^3 are placed at 0.5-cm intervals and illuminated by a plane wave at 960 GHz. The relative permittivity of the structure is 1.6. The problem is discretized with 39 628 800 unknowns and the solution is parallelized into 64 processes on a cluster of Intel Xeon Nehalem X5560 processors with 2.80-GHz clock rate. The total computing time (including input/output, setup, and 54 BiCGStab iterations for 0.005 residual error) is 10.1 h. Fig. 7 depicts the electric field on the axis of symmetry from $z = -5$ to 5 cm to demonstrate the complicated electromagnetics response of the structure.

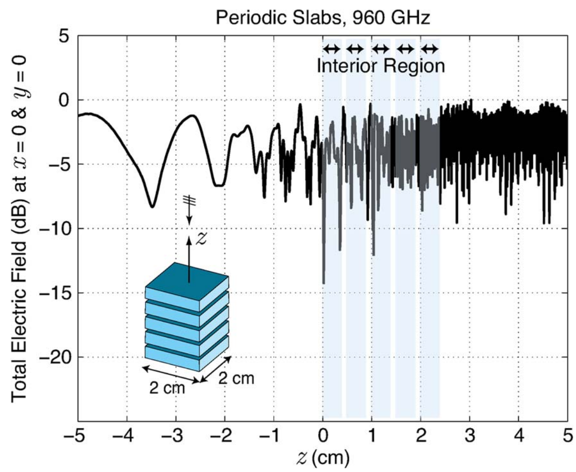

Fig. 7. *Solution of an electromagnetics problem involving five 2 × 2 × 0.41-cm³ dielectric slabs at 960 GHz. The structure has a relative permittivity of 1.6 and the problem is discretized with 39 628 800 unknowns. The total electric field on the axis of symmetry is plotted from z = −5 to 5 cm.*

IV. CONCLUSION

Fast and accurate solutions of extremely large integral-equation problems are presented. By parallelizing MLFMA with the hierarchical partitioning strategy, it is possible to solve realistic electromagnetics problems involving 3-D complicated objects discretized with hundreds of millions of unknowns. Solutions that are obtained by a full-wave solver can be used for a variety of benchmarking purposes, including as reference data for approximate high-frequency techniques. Solutions of a set of extremely large problems are available at www.cem.bilkent.edu.tr/benchmark [54]. ■

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