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Brief Paper

Dwell-time computation for stability of switched systems with time delays

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Abstract: The aim of this study is to find an improved dwell time that guarantees the stability of switched systems with heterogeneous constant time-delays. Piecewise Lyapunov–Krasovkii functionals are used for each candidate system to investigate the stability of the switched time-delayed system. Under the assumption that each candidate system is stable for small delay values, a sufficient condition for dwell-time that guarantees the asymptotic stability is derived. Numerical examples are given to compare the results with the previously obtained dwell-time bounds.

1 Introduction

A stability condition is derived in this paper for switched time-delayed systems. The general form of a switched system can be expressed as

$$\dot{x}(t) = f_{q(t)}(x(t)) \quad t \geq t_0 \tag{1}$$

where $q(t) : \mathbb{R} \to \mathcal{F}$ is the 'switching signal', $\mathcal{F} := \{1, 2, \ldots, \ell\}$ for some positive integer ℓ, $x(t) \in \mathbb{R}^n$ is the state and $f_i : \mathbb{R}^n \to \mathbb{R}^n$ is a differentiable function for every $i \in \mathcal{F}$. For notational convenience, we say that each f_i represents the dynamical behaviour of a candidate system. There are several works on this topic where the candidate systems are considered as linear [1], linear parameter varying [2], non-linear [3] or both non-linear and uncertain [4]. See the survey [5] for a review of recent results and further references.

The analysis of the switched systems differs from the analysis of the time-varying systems. For the switched systems, analysis is performed for a set of switching signal, whereas for the time-varying systems, analysis is performed for a specific switching signal [6]. Many control problems involving complex systems such as non-linear systems, uncertain systems and parameter-varying systems, can be cast within the framework of switched systems, [2, 6–10]. The main challenge in a switched control system is the stability analysis. Note that by a judicious switching between two or more unstable candidate systems the overall system can be made stable, [11]. Conversely, it is also possible to obtain an unstable response by a particular switching between two stable candidate plants. We refer to [7, 12] for a general review of the switched systems.

There is a vast literature about the stability of delay-free switched systems. In [9], necessary and sufficient conditions for the quadratic stability is obtained using Filippov solutions to discontinuous differential equations and Lyapunov functionals. 'Dwell time' [8], is the minimum value of the time intervals between consecutive time instances in which switching occurs. It is shown that a sufficiently large dwell time can guarantee the stability of the system provided that the candidate plants are stable [13]. 'Average dwell time' as an alternative to the dwell time is introduced in [1]. Using the average dwell-time concept, [14] develops sufficient conditions for exponential stability and weighted L_2 gain for the switched systems; see also [15, 16]. LaSalle's invariance principle is covered in the framework of the switched systems in [6]. In [17], results of [1] is applied to linear parameter-varying systems. In [18, 19], Lie algebra is used for finding quadratic CLFs. These CLFs are used in the stability analysis of switched linear and non-linear systems [18]. Existence of the CLF for the switched system implies stability of the switched system. Reverse is shown to be true for both linear [20] and non-linear [21] switched systems. Stability of the switched non-linear systems are covered in [3]. State-feedback control design is explained for continuous uncertain switched systems in [22]. The switched filter design, for dynamic output stabilisation of continuous switched systems using Lyapunov–Metzler inequalities, is covered in [23].

In contrast to the variety in the works on delay-free switched systems, there are relatively few works on time-delayed switched systems [24–29]. Switched systems with time delays on detecting the switching signal are covered in [30]. In the present work, time-delayed linear switched

systems are considered to be in the form

$$\dot{x}(t) = A_{q(t)}x(t) + \bar{A}_{q(t)}x(t - \tau(t)_{q(t)}), \quad t \geq t_0 \tag{2}$$

In (2), the system switches between infinite-dimensional systems. Owing to general difficulty of infinite-dimensional systems, stability analysis of the switched time-delayed systems are relatively more difficult [31]. Time-delayed systems are widely encountered in chemical processes, aerodynamics and communication networks [32–34]. Time delays in these systems are usually uncertain and time varying [35–37]. Robust H_∞ controllers can be designed for time-delayed systems, which guarantees the robustness within uncertainty bounds [33]. The large collection of conditions for stability analysis of time-delayed linear systems can be grouped into two categories: delay-dependent conditions and delay-independent conditions [38]. Lyapunov–Razumikhin and Lyapunov–Krasovskii methods are two main approaches in obtaining delay-dependent and delay-independent stability conditions for the time-delayed linear systems [38–42]. There are various sufficient conditions in terms of linear matrix inequalities (LMIs) and Ricatti-type inequalities for the stability of time-delayed systems [32, 36, 38, 41, 43, 44]. Many of these sufficient conditions are shown to be equivalent [38, 45]. For the switched time-delayed systems, the stability analysis and controller design issues are also discussed in some recent studies [24–26, 28, 46–50]. Additionally, see [51–54] for the discrete-time versions the related problems associated with switched systems. In particular, the stability conditions of [24, 51] are trajectory-dependent. In this paper, trajectory-independent stability is aimed. For the finite-dimensional linear systems, asymptotic stability of the system implies the exponential stability while for the infinite-dimensional systems, this is not the case [31, 55]. The papers [1, 6, 8, 20, 54] deal with finite-dimensional systems. In [28], piecewise Lyapunov–Razumikhin functions are used to find a dwell time for the stability. The approach we are proposing allows reducing the conservatism in [28] by using piecewise Lyapunov–Krasovskii functionals.

The remaining sections of the paper are organised as follows. The problem definition and preliminary remarks are presented in Section 2. The main result is given in Section 3, where a dwell time is derived for guaranteeing stability. Two examples are presented in Section 4. Concluding remarks are made in Section 5. A brief version of this paper (results given without the proofs) has been presented at the IFAC World Congress 2011 [56].

2 Problem definition

We use $\mathbb{R}^+$, $\mathbb{R}_0^+$ and $\mathbb{Z}_0^+$ to denote the set of positive real numbers, non-negative real numbers and non-negative integers, respectively. The set of all continuous and bounded functions with domain $[a, b] \subset \mathbb{R}_0^+$ and range $\mathbb{R}^n$ is denoted by $C([a, b], \mathbb{R}^n)$. Let $||.||$ be the Euclidean norm of a vector in $\mathbb{R}^n$. Let $|f|_{|t-\tau,t|}$ be the ∞ norm of $f \in \mathbb{C}[a, b]$, defined as

$$|f|_{|t-\tau,t|} := \sup_{t-\tau \leq \theta \leq t} ||f(\theta)||$$

With the notations above, consider the following switched time-delayed system

$$\Sigma_t = \begin{cases} \dot{x}(t) = A_{q(t)}x(t) + \bar{A}_{q(t)}x(t - \tau_{q(t)}), & t \geq 0 \\ x_0(\theta) = \phi(\theta), & \forall\theta \in [-\tau_{\max}, 0] \end{cases} \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state, $q(t) : \mathbb{R}_0^+ \to \mathcal{F}$ the piecewise switching and $\mathcal{F} := \{1, 2, \ldots, \ell\}$. In other words, for all $t \in [t_j, t_{j+1})$, we have $q(t) = k_j \in \mathcal{F}$, where $j \in \mathbb{Z}_0^+$ is the jth switching time instant and $t_j \in \mathbb{R}^+$. From these definitions, it follows that the trajectory of Σ_t in an arbitrary switching interval $[t_j, t_{j+1})$ obeys

$$\Sigma_{k_j} = \begin{cases} \dot{x}(t) = A_{k_j}x(t) + \bar{A}_{k_j}x(t - \tau_{k_j}), & t \in [t_j, t_{j+1}) \\ x_{t_j}(\theta) = \phi_j(\theta), & \forall\theta \in [-\tau_{k_j}, 0] \end{cases} \tag{4}$$

where the initial condition $\phi_j(\theta)$ is defined as

$$\phi_j(\theta) = \begin{cases} x(t_j + \theta), & -\tau_{k_j} \leq \theta < 0 \\ \lim_{h\to 0^-} x(t_j + h), & \theta = 0 \end{cases} \tag{5}$$

Let the triplet $\Sigma_i = (A_i, \bar{A}_i, \tau_i) \in \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times n} \times \mathbb{R}^+$ be the ith candidate system of (3). For every time instant t, $\Sigma_t \in \mathcal{A} = \{\Sigma_i \ : \ i \in \mathcal{F}\}$, where $\mathcal{A}$ is the set of all candidate systems. In equation (3), $\tau_{\max} = \max_{i\in F} \tau_i$ is the maximal time delay of the candidate systems in $\mathcal{A}$.

The switched time-delayed system Σ_t is stable [6] if there exists a strictly increasing continuous function $\bar{\alpha} : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ with $\bar{\alpha}(0) = 0$, such that

$$||x(t)|| \leq \bar{\alpha}(|x|_{[t_0-\tau_{\max},t_0]}), \quad \forall t \geq t_0 \geq 0 \tag{6}$$

along the trajectory of (3). The system is asymptotically stable if Σ_t is stable and $\lim_{t\to\infty} x(t) = 0$.

Lemma 2.1 (see [39]): A given candidate system Σ_i can be transformed into the following system denoted by Υ_i

$$\dot{y}(t) = (A_i + \bar{A}_i)y(t) - \int_{-2\tau_i}^{-\tau_i} \bar{A}_i^2 y(t + \theta)\,\mathrm{d}\theta - \int_{-\tau_i}^{0} \bar{A}_i A_i y(t + \theta)\,\mathrm{d}\theta \tag{7}$$

with the initial condition

$$\psi_i(\theta) = \begin{cases} \phi(\theta) & -\tau_i \leq \theta < 0 \\ \phi(-\tau_i) & -2\tau_i \leq \theta < -\tau_i \end{cases} \tag{8}$$

Note that asymptotic stability of the system Υ_i implies asymptotic stability of the system Σ_i.

Lemma 2.2 (see [39]): Suppose for a given triplet $\Sigma_i \in \mathcal{A}$, $i \in \mathcal{F}$, there exist real symmetric matrices $P_i > 0$, S_{1i} and S_{2i} that solves the LMI

$$\begin{bmatrix} M & -\tau_i P_i \bar{A}_i A_i & -\tau_i P_i \bar{A}_i^2 \\ -\tau_i A_i^T \bar{A}_i^T P_i & -\tau_i S_{1i} & 0 \\ -\tau_i (\bar{A}_i^2)^T P_i & 0 & -\tau_i S_{2i} \end{bmatrix} < 0 \tag{9}$$

where

$$M = P_i(A_i + \bar{A}_i) + (A_i + \bar{A}_i)^T P_i + \tau_i S_{1i} + \tau_i S_{2i} \tag{10}$$

then Υ_i is asymptotically stable. This guarantees the asymptotic stability of Σ_i for all delays in the interval $[0, \tau_i]$.

It is easy to check that (9) implies $S_{1i} > 0$, $S_{2i} > 0$ and $A_i + \bar{A}_i$ is Hurwitz stable. If all candidate systems of (3), $\Sigma_i \in \mathcal{A}$ are asymptotically stable satisfying (9), then the set

$\mathcal{A}$ is denoted as $\tilde{\mathcal{A}}$. It is assumed that $\mathcal{A} = \tilde{\mathcal{A}}$ for the rest of the discussion. In this paper, sufficient condition that guarantees the asymptotic stability of the switched system (3) will be constructed using piecewise Lyapunov–Krasovskii functionals. One method in the stability analysis of switched systems is to find common Lyapunov function (CLF). In [57], CLFs are found for switched time-delay systems assuming that each candidate system has the same time delay τ, each candidate is assumed to be delay-independently stable, A matrix is symmetric and $\tilde{A}$ matrix is in the form δI. Even without these assumptions, method of finding CLFs are very conservative due to the fact that it is usually difficult to find a CLF for all the candidate systems, especially for time-delay systems whose stability criteria are only sufficient in most cases. A recent work found asymptotic stability conditions using piecewise Lyapunov–Razumikhin functions [28]. In our work, by using piecewise Lyapunov–Krasovkii functionals, we will try to reduce the conservatism in [28].

Although there are less conservative conditions than (9) for the stability of time-delayed linear systems (see e.g. [38, 41]), for the purpose of this paper the condition (9) is more useful. Typically, less conservative results are obtained by additional terms in the Krasovskii functional. However, this complicates the analysis in finding an bound such as (20) obtained below. For example, inclusion of the derivative of the state in the Lyapunov–Krasovskii functional as in [41], makes it difficult to bound the Lyapunov–Krasovskii functional by a function that depends 'only' on the state. The inequalities (20) and (24) that are obtained from the particular Lyapunov–Krasovskii functional chosen here play crucial roles in our analysis.

3 Main results

For a given $\tau_D > 0$, the switching signal set based on the dwell time τ_D is denoted as $S[\tau_D]$ where for any switching signal $q(t) \in S[\tau_D]$, the distance between any consecutive discontinuities of $q(t)$, that is, $t_{j+1} - t_j$ for $j \in \mathbb{Z}_0^+$, is greater than τ_D [1, 8, 28]. Dwell-time-based switching is independent of the trajectory of the solutions [6]. Before presenting the main result of the paper, we need to recall some lemmas and prove some propositions, which will be useful in the proof of our main result.

Lemma 3.1 (see [39]): Suppose $u, v, w : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ are continuous, non-decreasing functions, $u(0) = v(0) = 0$, $w(s) > 0$ for $s > 0$. If there exists a continuous functional V, such that

$$u(||x(t)||) \le V(t, x_t) \le v(|x|_{[t-\tau,t]}), \quad \forall t \ge t_0 \tag{11}$$

$$\dot{V}(t, x_t) \le -w(||x(t)||), \quad \forall t \ge t_0 \tag{12}$$

then the solution $x = 0$ of the switched time-delay system (3) is uniformly asymptotically stable.

For functions defined in Lemma 3.1, we say that (V, u, v, w) is a stability quadruple for the switched time-delay system (3). Construct the following piecewise Lyapunov–Krasovskii functional for the transformed system Υ_i of the candidate system

$$V_i(t, x_t) = x^T(t)P_i x(t) + \int_{-\tau_i}^{0}\int_{t+\theta}^{t} x^T(\xi)S_{1i}x(\xi)\,\mathrm{d}\xi\,\mathrm{d}\theta + \int_{-2\tau_i}^{-\tau_i}\int_{t+\theta}^{t} x^T(\xi)S_{2i}x(\xi)\,\mathrm{d}\xi\,\mathrm{d}\theta \tag{13}$$

where $P_i > 0$, $S_{1i} > 0$ and $S_{2i} > 0$ are real symmetric matrices and $\theta \in [-2\tau, 0]$. This functional can be bounded by

$$u_i(||x(t)||) \le V_i(t, x_t) \le v_i(|x|_{[t-2\tau_i,t]}), \quad \forall t \ge t_0, \ \forall x \in \mathbb{R}^n$$

where

$$u_i(s) = \sigma_{\min}[P_i]s^2 \tag{14}$$

and

$$v_i(s) = \left(\sigma_{\max}[P_i] + \frac{\tau_i^2}{2}\sigma_{\max}[S_{1i}] + \frac{3\tau_i^2}{2}\sigma_{\max}[S_{2i}]\right)s^2 \tag{15}$$

Here $\sigma_{\min}[.]$ and $\sigma_{\max}[.]$ denote the minimum and maximum singular values, respectively.

Proposition 3.2: For each time-delay system Υ_i with Lyapunov–Krasovskii functional (13), assume that (11) and (12) are satisfied for some $w_i(s)$ with u and v defined as in (14) and (15) respectively, then we have the following result

$$|x|_{[t_m-\tau_i,t_m]} \le \sqrt{\frac{\sigma_{\max}[P_i] + \frac{\tau_i^2}{2}\sigma_{\max}[S_{1i}] + \frac{3\tau_i^2}{2}\sigma_{\max}[S_{2i}]}{\sigma_{\min}[P_i]}}\,|x|_{[t_n-2\tau_i,t_n]}, \quad \forall t_m \ge t_n + \tau_i \tag{16}$$

Proof: Υ_i is stable and V_i is an admissible functional satisfying (11), $V_i(t_m, x_t) \le V_i(t_n, x_t)$ for all $t_m \ge t_n$. Thus

$$u_i(||x(t_m)||) \le V_i(t_m, x_t) \le V_i(t_n, x_t) \le v_i(|x|_{[t_n-2\tau_i,t_n]})$$

$$\sigma_{\min}[P_i]||x(t_m)|| \le u_i(||x(t_m)||) \le v_i(|x|_{|t_n-2\tau_i,t_n|}) \le \left(\sigma_{\max}[P_i] + \frac{\tau_i^2}{2}\sigma_{\max}[S_{1i}] + \frac{3\tau_i^2}{2}\sigma_{\max}[S_{2i}]\right)|x|_{[t_n-2\tau_i,t_n]}$$

Since $P_i > 0$

$$||x(t_m)|| \le \sqrt{\frac{\sigma_{\max}[P_i] + \frac{\tau_i^2}{2}\sigma_{\max}[S_{1i}] + \frac{3\tau_i^2}{2}\sigma_{\max}[S_{2i}]}{\sigma_{\min}[P_i]}}\,|x|_{[t_n-2\tau_i,t_n]}, \quad \forall t_m \ge t_n$$

$$||x(t_m - \tau_i)|| \le \sqrt{\frac{\sigma_{\max}[P_i] + \frac{\tau_i^2}{2}\sigma_{\max}[S_{1i}] + \frac{3\tau_i^2}{2}\sigma_{\max}[S_{2i}]}{\sigma_{\min}[P_i]}}\,|x|_{[t_n-2\tau_i,t_n]} \tag{17}$$

for all $t_m \ge t_n + \tau_i$. Since $t_m > t_n + \tau_i$ is arbitrary, this equation is also valid for all $t \in [t_m - \tau_i, t_m]$. □

Assume that Lemma 3.1 is satisfied for system (3) and $\lim_{s\to\infty} u(s) \to \infty$. Then if $|\phi|_{[t_0-\tau,t_0]} \le \delta_1$ and $\delta_1 > 0$, Lemma 3.1. implies that there exists $\delta_2 > \delta_1 > 0$, such that $u(\delta_2) = v(\delta_1)$ and $||x(t)|| < \delta_2$ for all $t > t_0$. For such a δ_2, consider the following:

Proposition 3.3: For system (3) satisfying Lemma 3.1 with $\lim_{s\to\infty} u(s) \to \infty$, for an arbitrary η, $0 < \eta < \delta_2$, $|\phi|_{[t_0-\tau,t_0]} \le \delta_1 < \delta_2$ implies

$$||x(t)|| \le \eta, \quad \forall t > t_0 + T(\eta) \tag{18}$$

where $T(\eta) = [v(\delta_1)]/\gamma$, v is defined as in the Lemma 3.1 and $\gamma = \inf_{\eta \le s \le \delta_2} w(s)$

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Proof: Let $T_* > 0$ and let $||x(t_1)|| > \eta$ for a time instant $t_1 > t_0 + T_*$. Let $\gamma = \inf_{\eta \le s \le \delta_2} w(s)$. Since the system is stable and V is a Lyapunov–Krasovskii functional, from Lemma 3.1, we have the following

$$\dot{V}(t, x_t) \le -w(||x(t)||) < -\gamma \quad \forall t \ge t_0$$

This implies $V(t, x_t) \le V(t_0, \phi) - (t - t_0)\gamma \le v(\delta_1) - (t - t_0)\gamma$. Let $T_* > [v(\delta_1)]/\gamma$. Then for every $t > t_0 + T_*$, we have $V(t, x_t) \le 0$. However, we assume that there is a time instant $t_1 > t_0 + T_*$ such that $||x(t_1)|| > \eta$. This implies that

$$V(t, x_t) \ge u(||x(t_1)||) \ge u(\eta) > 0$$

This is a contradiction. Therefore time instant t_1 cannot exists and this implies

$$||x(t)|| \le \eta \quad \forall t > t_0 + \frac{v(\delta_1)}{\gamma}$$

which concludes the proof. □

Assumption 3.4: For every transformed candidate system Υ_i defined in Lemma 2.1, the corresponding candidate system Σ_i satisfies the stability condition of Lemma 2.2, that is, $\mathcal{A} = \tilde{\mathcal{A}}$.

Consider an arbitrary switching interval $[t_j, t_{j+1})$ of the switching signal $q(t) \in S[\tau_D]$ with $\tau_D > \tau_{\max}$ where $q(t) = k_j$, $k_j \in \mathcal{F}$ for all $t \in [t_j, t_{j+1})$ and $t_j \in \mathbb{Z}^+ \cup 0$ is the jth switching time instant. The state variable $x_j(t)$ obeys (4) in this interval. Define $x_j(t_{j+1}) = \lim_{h \to 0^-} x(t_{j+1+h}) = x_{j+1}(t_{j+1})$ based on the fact that $x(t)$ is continuous for $t \ge 0$. With this definition $x_j(t)$ is defined on the compact set $[t_j, t_{j+1}]$. The initial condition of Σ_{k_j} is $\phi_j(t) = x(t) = x_{j-1}(t)$ where $t \in [t_j - \tau_{k_j}, t_j]$ for $j \in \mathbb{Z}^+$. Initial condition of the transformed system Υ_i is $\phi_i(t)$ as defined before. Let the Lyapunov–Krasovskii functional be

$$V_{k_j}(t, x_t) = x_j^T(t) P_{k_j} x_j(t) + \int_{-\tau_{k_j}}^{0} \int_{t+\theta}^{t} x_j^T(\xi) S_{1k_j} x_j(\xi)\, \mathrm{d}\xi\, \mathrm{d}\theta + \int_{-2\tau_{k_j}}^{-\tau_{k_j}} \int_{t+\theta}^{t} x_j^T(\xi) S_{2k_j} x_j(\xi)\, \mathrm{d}\xi\, \mathrm{d}\theta \tag{19}$$

Then for every $x_j \in \mathbb{R}^n, t \in [t_j, t_{j+1}]$, we have

$$\kappa_{k_j} ||x_j(t)||^2 \le V_{k_j}(t, x_t) \le \left(\bar{\kappa}_{k_j} + \frac{\tau_{k_j}^2}{2} \bar{\chi}_{1k_j} + \frac{3\tau_{k_j}^2}{2} \bar{\chi}_{2k_j} \right) |x_j|_{[t-2\tau_{k_j}, t]} \tag{20}$$

where $\kappa_i = \sigma_{\min}[P_i]$, $\bar{\kappa}_i = \sigma_{\max}[P_i]$, $\chi_{1i} = \sigma_{\max}[S_{1i}]$ and $\chi_{2i} = \sigma_{\max}[S_{2i}]$.

Proposition 3.5: Let

$$W_{k_j} = -(P_{k_j}(A_{k_j} + \bar{A}_{k_j}) + (A_{k_j} + \bar{A}_{k_j})^T P_{k_j}) - \tau_{k_j}(R_{1k_j} + R_{2k_j}) \tag{21}$$

where $R_{1k_j} = R_{1k_j}^T$ is the solution of the LMI

$$\begin{bmatrix} S_{1k_j} - R_{1k_j} & -\tau_{k_j} P_{k_j} \bar{A}_{k_j} A_{k_j} \\ -\tau_{k_j} A_{k_j}^T \bar{A}_{k_j}^T P_{k_j} & -\tau_{k_j} S_{1i} \end{bmatrix} < 0 \tag{22}$$

and $R_{2k_j} = R_{2k_j}^T$ is the solution of the LMI

$$\begin{bmatrix} S_{2k_j} - R_{2k_j} & -\tau_{k_j} P_{k_j} \bar{A}_{k_j}^2 \\ -\tau_{k_j} (\bar{A}_{k_j}^T)^2 P_{k_j} & -\tau_{k_j} S_{2i} \end{bmatrix} < 0 \tag{23}$$

then the upper bound on the derivative of the Lyapunov Krasovskii functional (19) can be set as

$$\dot{V}_{k_j}(t, x_t) \le -x_j^T(t) W_{k_j} x_j(t) \tag{24}$$

Proof: Take the derivative of the Lyapunov Krasovskii functional with respect to time along the trajectory.

$$\begin{aligned} \dot{V}_{k_j}(t, x_t) &= x_j^T(t) D_{1k_j} x_j^T(t) \\ &+ \int_{-\tau_{k_j}}^{0} \begin{bmatrix} x_j^T(t) & x_j^T(t+\theta) \end{bmatrix} D_{2kj} \begin{bmatrix} x_j(t) \\ x_j(t+\theta) \end{bmatrix} \mathrm{d}\theta \\ &+ \int_{-2\tau_{k_j}}^{-\tau_{k_j}} \begin{bmatrix} x_j^T(t) & x_j^T(t+\theta) \end{bmatrix} D_{3kj} \begin{bmatrix} x_j(t) \\ x_j(t+\theta) \end{bmatrix} \mathrm{d}\theta \end{aligned} \tag{25}$$

where

$$D_{1k_j} = P_{k_j}(A_{k_j} + \bar{A}_{k_j}) + (A_{k_j} + \bar{A}_{k_j})^T P_{k_j}$$

$$D_{2k_j} = \begin{pmatrix} S_{1k_j} & -\tau_{k_j} P_{k_j} \bar{A}_{k_j} A_{k_j} \\ -\tau_{k_j} A_{k_j}^T \bar{A}_{k_j}^T P_{k_j} & -\tau_{k_j} S_{1i} \end{pmatrix}$$

$$D_{3k_j} = \begin{pmatrix} S_{2k_j} & -\tau_{k_j} P_{k_j} \bar{A}_{k_j}^2 \\ -\tau_{k_j} (\bar{A}_{k_j}^T)^2 P_{k_j} & -\tau_{k_j} S_{2i} \end{pmatrix}$$

Add and subtract the term

$$\int_{-\tau_{k_j}}^{0} x_j^T(t) R_{1k_j} x_j(t)\, \mathrm{d}\theta + \int_{-2\tau_{k_j}}^{-\tau_{k_j}} x_j^T(t) R_{2k_j} x_j(t)\, \mathrm{d}\theta$$

to the right-hand side of equation (25) where R_{1k_j} and R_{2k_j} are the solutions of the LMIs (22) and (23), respectively. We obtain

$$\begin{aligned} \dot{V}_{k_j}(t, x_t) &= x_j^T(t) \tilde{D}_{1k_j} x_j^T(t) \\ &+ \int_{-\tau_{k_j}}^{0} \begin{bmatrix} x_j^T(t) & x_j^T(t+\theta) \end{bmatrix} \tilde{D}_{2kj} \begin{bmatrix} x_j(t) \\ x_j(t+\theta) \end{bmatrix} \mathrm{d}\theta \\ &+ \int_{-2\tau_{k_j}}^{-\tau_{k_j}} \begin{bmatrix} x_j^T(t) & x_j^T(t+\theta) \end{bmatrix} \tilde{D}_{3kj} \begin{bmatrix} x_j(t) \\ x_j(t+\theta) \end{bmatrix} \mathrm{d}\theta \end{aligned} \tag{26}$$

where

$$\tilde{D}_{1k_j} = P_{k_j}(A_{k_j} + \bar{A}_{k_j}) + (A_{k_j} + \bar{A}_{k_j})^T P_{k_j} + \tau_{k_j}(R_{1k_j} + R_{2k_j})$$

$$\tilde{D}_{2k_j} = \begin{pmatrix} S_{1k_j} - R_{1k_j} & -\tau_{k_j} P_{k_j} \bar{A}_{k_j} A_{k_j} \\ -\tau_{k_j} A_{k_j}^T \bar{A}_{k_j}^T P_{k_j} & -\tau_{k_j} S_{1i} \end{pmatrix}$$

$$\tilde{D}_{3k_j} = \begin{pmatrix} S_{2k_j} - R_{2k_j} & -\tau_{k_j} P_{k_j} \bar{A}_{k_j}^2 \\ -\tau_{k_j} (\bar{A}_{k_j}^T)^2 P_{k_j} & -\tau_{k_j} S_{2i} \end{pmatrix}$$

Since $\tilde{D}_{2k_j}$ and $\tilde{D}_{3k_j}$ are negative definite

$$\begin{aligned} \dot{V}_{k_j}(t, x_t) &= x_j^T(t) \tilde{D}_{1k_j} x_j^T(t) \\ &+ \int_{-\tau_{k_j}}^{0} \begin{bmatrix} x_j^T(t) & x_j^T(t+\theta) \end{bmatrix} \tilde{D}_{2kj} \begin{bmatrix} x_j(t) \\ x_j(t+\theta) \end{bmatrix} \mathrm{d}\theta \end{aligned}$$

$$+\int_{-2\tau_{k_j}}^{-\tau_{k_j}} \begin{bmatrix} x_j^T(t) & x_j^T(t+\theta) \end{bmatrix} \tilde{D}_{3kj} \begin{bmatrix} x_j(t) \\ x_j(t+\theta) \end{bmatrix} \mathrm{d}\theta$$

$$\leq x_j^T(t)\tilde{D}_{1k_j}x_j^T(t) = -x_j^T(t)W_{k_j}x_j(t) \qquad \square$$

The best choice of W_{k_j} is obtained from the following optimisation problem. Maximise l over all $l \in \mathbb{R}^+$ and symmetric matrices $P_{k_j}, R_{1k_j}, R_{2k_j}, S_{1k_j}, S_{2k_j}$ subject to LMIs (22) and (23) and additional constraints

$$\begin{bmatrix} M & -\tau_{k_j}P_{k_j}\bar{A}_{k_j}A_{k_j} & -\tau_{k_j}P_{k_j}\bar{A}_{k_j}^2 \\ -\tau_{k_j}A_{k_j}^T\bar{A}_{k_j}{}^TP_{k_j} & -\tau_{k_j}S_{1k_j} & 0 \\ -\tau_{k_j}(\bar{A}_{k_j}^T)^2P_{k_j} & 0 & -\tau_{k_j}S_{2k_j} \end{bmatrix} < 0$$

$$P_{k_j}(A_{k_j}+\bar{A}_{k_j}) + (A_{k_j}+\bar{A}_{k_j})^TP_{k_j} + \tau_{k_j}(R_{1k_j}+R_{2k_j}) + lI \leq 0$$

where $\mathbb{R}^+$ is the set of positive real numbers, I is the identity matrix of appropriate dimension and $M = P_{k_j}(A_{k_j}+\bar{A}_{k_j}) + (A_{k_j}+\bar{A}_{k_j})^TP_{k_j} + \tau_{k_j}S_{1k_j} + \tau_iS_{2k_j}$. The matrices P_{k_j}, R_{1k_j}, R_{2k_j}, S_{1k_j} and S_{2k_j} are obtained from the solution of this optimisation problem. From these matrices, we can determine $\sigma_{\min}[P_i]$, $\sigma_{\max}[P_i]$, $\sigma_{\max}[S_{1i}]$, $\sigma_{\max}[S_{2i}]$ and

$$W_{k_j}^* = -P_{k_j}(A_{k_j}+\bar{A}_{k_j}) - (A_{k_j}+\bar{A}_{k_j})^TP_{k_j} - \tau_{k_j}(R_{1k_j}+R_{2k_j})$$

Select $w(s)$ in Lemma 3.1 as $w(s) = \varpi_{k_j}s^2$ where $\varpi_{k_j} = \sigma_{\min}[W_{k_j}^*] > 0$ is the minimum eigenvalue of the $W_{k_j}^*$. With this selection, (12) is satisfied.

Assume $|\phi_j(t)|_{[t_j-\tau_j,t_j]} \leq \delta_j$. For an arbitrary α with $0 < \alpha < 1$, let $\eta = \alpha\delta_j$ in Proposition 3.3. With this selection of η and $\delta_j = \delta_1$, we have $0 < \eta = \alpha\delta_j < \delta_1 < \delta_2$. Using the Proposition 3.3, we have

$$||x_j(t)|| \leq \alpha\delta_j \quad \forall t \geq t_j + T_j \tag{27}$$

where

$$T_j = \frac{v(\delta_j)}{\gamma} = \frac{\left(\bar{\kappa}_j + \frac{\tau_j^2}{2}\bar{\chi}_{1j} + \frac{3\tau_j^2}{2}\bar{\chi}_{2j}\right)}{\alpha^2\varpi_j} \tag{28}$$

Equation (27) implies

$$|x|_{[t_j+T_j,t_{j+1}]} \leq \alpha\delta_j \tag{29}$$

Let

$$\lambda = \max_{i\in\mathcal{F}} \frac{\sigma_{\max}[P_i] + \frac{\tau_i^2}{2}\sigma_{\max}[S_{1i}] + \frac{3\tau_i^2}{2}\sigma_{\max}[S_{2i}]}{\sigma_{\min}[P_i]}$$

$$\mu = \max_{i\in\mathcal{F}} \frac{\bar{\kappa}_i}{\varpi_i}, \quad \rho_1 = \max_{i\in\mathcal{F}} \frac{\tau_{\max}^2\bar{\chi}_{1i}}{2\varpi_i}, \quad \rho_2 = \max_{i\in\mathcal{F}} \frac{3\tau_{\max}^2\bar{\chi}_{2i}}{2\varpi_i} \tag{30}$$

Define

$$T^* = \frac{\mu+\rho_1+\rho_2}{\alpha^2}$$

Note that

$$T^* > T_j = \frac{v(\delta_j)}{\gamma} = \frac{\left(\bar{\kappa}_j + \frac{\tau_j^2}{2}\bar{\chi}_{1j} + \frac{3\tau_j^2}{2}\bar{\chi}_{2j}\right)}{\alpha^2\varpi_j}, \quad j = 0,1,2,\ldots$$

Let the dwell time to be $\tau_{\mathrm{D}} = T^* + 2\tau_{\max}$. Recall that $t_{j+1} - t_j > \tau_{\mathrm{D}}$. Thus $t_{j+1} - t_j > T^* + 2\tau_{\max} > T^* + 2\tau_{j+1} > T_j + 2\tau_{j+1}$. Also note that $|\psi_{j+1}(t)| = |x_j(t)|$ where $t \in [t_{j+1} - 2\tau_{j+1}, t_{j+1}]$. Thus, we have

$$|\psi_{j+1}|_{[t_{j+1}-2\tau_{j+1},t_{j+1}]} = |x_j|_{[t_{j+1}-2\tau_{j+1},t_{j+1}]} \leq |x_j|_{[t_j+T_j,t_{j+1}]} \leq \alpha\delta_j := \delta_{j+1} \tag{31}$$

and δ_0 is defined as $\delta_0 := |\psi|_{[-2\tau_{\max},0]} = |\phi|_{[-\tau_{\max},0]} \geq |\phi|_{[-\tau_{k_0},0]}$. Therefore we obtain a convergent sequence δ_i where $\delta_i = \alpha^i\delta_0$ with $i = 0,1,2,\ldots$.

Proposition 3.2 implies

$$|x|_{[t,t+\tau_i]} \leq \sqrt{\frac{\sigma_{\max}[P_i] + \frac{\tau_i^2}{2}\sigma_{\max}[S_{1i}] + \frac{3\tau_i^2}{2}\sigma_{\max}[S_{2i}]}{\sigma_{\min}[P_i]}}|x|_{[t_n-2\tau_i,t_n]}, \quad \forall t \geq t_j$$

Thus

$$\sup_{t\in[t_j,t_{j+1}]} ||x_j(t)|| \leq \sup_{t\in[t_j,t_{j+1}]} |x_j(t)|_{[t,t+\tau_{k_j}]} \leq \sqrt{\lambda}|x_j|_{[t_j-2\tau_{k_j},t_j]} \leq \sqrt{\lambda}\delta_j = \alpha^j\sqrt{\lambda}\delta_0 \tag{32}$$

which implies the asymptotic stability of the transformed switched time-delay system Υ_t with the switching signal $q(t) \in S[\tau_{\mathrm{D}}]$. Asymptotic stability of the transformed switched time-delay system implies the asymptotic stability of the switched time-delay system Σ_i. Thus, we can state our final result as follows.

Theorem 3.6: Under Assumption 3.4, the system Σ_t, defined in (3), is asymptotically stable for any switching rule $q(t) \in S[\tau_{\mathrm{D}}]$, where $\tau_{\mathrm{D}} = T^* + 2\tau_{\max}$ with

$$T^* = \frac{\mu+\rho_1+\rho_2}{\alpha^2} \quad \text{for any } \alpha \in (0,1)$$

and μ, ρ_1 ρ_2 are as defined in (30); here α determines the decay rate as shown in (32).

4 Numerical examples

In this section, several illustrative examples are used to demonstrate the results in Section 3 and compare the main result of this paper with [28, 29].

Example 4.1: The system given below is taken from [28] for comparison purposes. Let Σ_1 be

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad \bar{A}_1 = \begin{bmatrix} -1 & 0 \\ -0.5 & -1 \end{bmatrix}, \quad \tau_1 = 0.3\,\mathrm{s} \tag{33}$$

Let Σ_2 be

$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}, \quad \tau_2 = 0.6\,\mathrm{s} \tag{34}$$

In the paper [28], dwell time for this system is found to be $\tau_{\mathrm{D}} = 6.52$ s. Using Theorem 3.6, a dwell time is found as $\tau_{\mathrm{D}} = 1.2 + [2.15/\alpha^2]$ seconds for a fixed α. Note that the system is stable for all $\alpha \in (0,1)$. For $\alpha > 0.48$ our dwell-time result is smaller than 6.52 s. Let us take $\alpha = 0.99$. This implies $\tau_{\mathrm{D}} = 3.4$ s. Hence, the bound obtained in [28] can be improved.

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Example 4.2: Consider the numerical example in [29]. In this example, two candidate systems are stabilised by a state feedback. The stabilised individual systems have the following A, $\bar{A}$ matrices and time delays

$$A_1 = \begin{bmatrix} -1.799 & -0.814 \\ 0.2 & -0.714 \end{bmatrix}, \quad \bar{A}_1 = \begin{bmatrix} -1 & 0 \\ -0.45 & -1 \end{bmatrix}$$
$$\tau_1 = 0.155\,\text{s} \tag{35}$$
$$A_2 = \begin{bmatrix} -1.853 & -0.093 \\ -0.853 & -1.1593 \end{bmatrix}, \quad \bar{A}_2 = \begin{bmatrix} -1 & 0 \\ 0.05 & -1 \end{bmatrix}$$
$$\tau_2 = 0.2\,\text{s} \tag{36}$$

In [29], a dwell time for the stabilised uncertain switched system is obtained as $\tau_D = 0.83$ s. For the same closed loop system with no uncertainty, our method obtains the dwell time for the switched system as $\tau_D = 0.4 + [0.31/\alpha^2\,\text{s}]$. Let us take $\alpha = 0.99$. This implies $\tau_D = 0.72$ s.

5 Concluding remarks

We performed the stability analysis for the switched system by using some appropriate model transformations of the candidate systems. Piecewise Lyapunov–Krasovkii functionals are used for the derivation of a dwell time. Thus, the earlier results obtained by using piecewise Lyapunov–Razumikhin functions in [28, 29] are now improved and simplified. Two illustrative examples are given for comparisons with the previous results.

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