# Dimensioning shared-per-node recirculating fiber delay line buffers in an optical packet switch 

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## ARTICLE INFO

## Article history:

Received 16 July 2012
Received in revised form 3 July 2013
Accepted 20 September 2013
Available online 1 October 2013

## Keywords:

Optical packet switching
Optical burst switching
Markov modulated Poisson process
Overflow models
Fiber delay lines


#### Abstract

Optical buffering based on fiber delay lines (FDLs) has been proposed as a means for contention resolution in an optical packet switch. In this article, we propose a queuing model for feedback-type shared-per-node recirculating FDL optical buffers in asynchronous optical switching nodes. In this model, optical packets are allowed to recirculate over FDLs as long as the total number of recirculations is less than a pre-determined limit to meet signal loss requirements. Markov Modulated Poisson Process (MMPP)-based overflow traffic models and fixed-point iterations are employed to provide an approximate analysis procedure to obtain blocking probabilities as a function of various buffer parameters in the system when the packet arrival process at the optical switch is Poisson. The proposed algorithm is numerically efficient and accurate especially in a certain regime identified with relatively long and variably-sized FDLs, making it possible to dimension optical buffers in next-generation optical packet switching systems.


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## 1. Introduction

One of the candidate transport mechanisms for next-generation Internet is based on the optical packet switching paradigm that promises to provide efficient utilization of very high fiber capacities offered by WDM technologies by means of switching at finer granularities. This is in contrast with the current state-of-the-art optical circuit switching paradigm that provides point-to-point connectivity where bandwidth sharing at sub-wavelength levels is costly. The literature in this field comprises two key packet switching-based paradigms: Optical Packet Switching (OPS) and Optical Burst Switching (OBS) [1-4]. OBS does not rely on optical header processing or optical buffers as in OPS but it is possible to deploy optical buffers at OBS nodes to enhance burst blocking performance. Optical packets have fixed or variable sizes that are integer multiples of a time unit, called slot, in synchronous switching [5]. On the other hand, optical packets in asynchronous switching systems are of variable length and therefore packet arrivals need not be aligned. Using a retrial-queuing framework, we analytically study the performance of an asynchronous OPR (Optical Packet Router), referring to an asynchronous OPS or OBS node, that uses FDLs (Fiber Delay Lines) for optical buffering.

Contention is said to occur when there are multiple optical packets on the same wavelength that are simultaneously destined to the same output link. When the incoming packet's wavelength is busy at the destination link, then wavelength conversion can be used [6]. If all wavelengths are busy, then optical buffering is one of the alternative means to resolve contention [3,7]. Due to the lack of optical random access memory with current technologies, FDLs are often used for optical buffering where an optical packet finding all wavelengths occupied at the destination link is instead sent over a coil of fiber

[^0]that provides the packet with a deterministic delay with the potential of resolving contention at a later time. For different architectures proposed for FDL buffering, we refer the reader to [3,8,9]. Optical buffers can be single-stage or multi-stage, the latter having multiple blocks of delay lines cascaded together [10]. In a feed-forward architecture, an output port of a switching element at a given stage is connected by an FDL to an input port of a switching element at the next stage. In a feedback architecture, the output port of a switching element at a given stage is connected to an input port of a switching element at the same stage [10]. A packet that is sent over an FDL reserves the output in two different ways [4,11]:

- If the output link is reserved prior to entering the buffer, this architecture is called Pre-Reservation (PreRes).
- In the Post-Reservation (PostRes) scheme, the packet attempts to reserve the output link once it is about to leave the FDL.

With PreRes reservation mechanism, a packet leaving the FDL buffer is guaranteed to find an idle wavelength on the destination link whereas in the PostRes scheme, a packet may still find all wavelengths busy at its destination link at the epoch of exit from the FDL. When such a situation arises, this packet will go through multiple but limited number of FDL circulations. The advantage of PostRes buffers is the reduction in state that is maintained at the node since the switch controller only needs to keep track of whether each wavelength channel is busy or idle at a given time for PostRes. In contrast, the PreRes reservation mechanism requires the switch controller to keep track of future channel occupancy information. PreRes schemes are generally known to be more efficient in terms of performance since packets would not waste FDL resources in PreRes if they would get blocked eventually. FDL buffers can be shared for all wavelength channels on a given link in which case we have shared-per-link buffering. On the other hand, if FDLs are shared for all wavelength channels for a given node, then we have the so-called shared-per-node FDL buffering in which case we have performance benefits due to economy of sharing.

The scope of this article is on shared-per-node feedback-type recirculating FDL buffers using the PostRes reservation model due to its relative implementation simplicity. For this purpose, we consider the OPR architecture in Fig. 1 with $N$ input and output links (or fibers) where each link comprises $K$ wavelength channels. This architecture is based on the tune-and-select (TAS) architecture with shared FDL buffers and full range tuneable wavelength converters (TWCs) described in [12]. We assume a shared pool of $B$ FDLs which can all be of the same length or variable-length delay lines can be used. To accommodate both cases, we introduce an FDL spreading parameter $\alpha, 0 \leq \alpha<1$, and a delay parameter $D>0$. In our model, the $i$ th delay line introduces a delay of

$$
\begin{equation*}
D_{i}=D(1-\alpha)+(i-1) \frac{2 D \alpha}{B-1}, \quad i=1,2, \ldots, B, \text { if } B>1 . \tag{1}
\end{equation*}
$$

In case $B=1$, we have a single FDL with delay $D$. Based on the definition above, the minimum (maximum) length delay line provides a delay of $D_{\min }=D_{1}=D(1-\alpha)\left(D_{\max }=D_{B}=D(1+\alpha)\right)$ and the average delay of the delay lines is exactly $D$. Moreover, all the other delay lines are uniformly placed in between the minimum and maximum delay line lengths. When $\alpha=0$, all delay lines are of the same length, i.e., of length $D$. On the other hand, when $\alpha \rightarrow 1$, the delay line lengths tend to range between 0 and $2 D$. The spreading parameter $\alpha$ can therefore be used to study the impact of fixed- or variable-length delay lines. The input signals in Fig. 1 are first wavelength de-multiplexed and the optical packets are converted to the desired output wavelength using the TWCs in the tune stage. Subsequently, each signal is split up to be sent to all output fibers and FDLs, but only one will be selected by switching on the corresponding semiconductor optical amplifier (SOA) in the select stage. Finally, all selected signals destined to the same fiber are combined. This architecture can be modified for the special case $\alpha=0$ for which we can use $B / W$ physical FDLs where each FDL accommodates $W>1$ parallel WDM channels as in $[12,13]$, as opposed to using $B$ separate physical FDLs. This modification can also be extended for the general $\alpha \neq 0$ case by noting recent FDL buffering technologies that promise to achieve different time delays for different wavelength channels on the same physical FDL [14].

We assume an exponentially distributed packet duration with mean set to unity which is equivalent to saying that the time unit we use is the time it takes to transmit a packet with average size. We also assume a symmetric Poisson packet arrival process with intensity $\lambda^{(0)}$ for each destination link. Thus, the total arrival rate to the switch is $N \lambda^{(0)}$. The system load is denoted by $\rho=\lambda^{(0)} / K$. The assumptions of Poisson call arrivals and exponentially distributed call-holding times have been successfully used for circuit-switched networks handling call-oriented traffic. Although it is not clear at this point what type of traffic models to be appropriate for use in next-generation OPS networks, we still employ in the current article the same tele-traffic models from the circuit switching literature for gaining insight into the operation of optical buffers.

An arriving (fresh) optical packet which finds all $K$ wavelength channels occupied at its destination link is forwarded to the FDL pool comprising $B$ delay lines. Otherwise, it is transmitted on one of the idle wavelength channels randomly. For a packet directed to the FDL buffer, if all FDLs are busy at their entrance points, then this packet is dropped. Otherwise, the packet is transmitted on one of the FDLs that is idle at its entrance point. Motivated by the PostRes model, the FDL selection process is all random. Once the packet completes its journey on the FDL, it becomes a retrial packet as opposed to a fresh packet. A retrial packet again checks if one of the $K$ wavelength channels at its destination link is idle and if so it is randomly transmitted on one of them. If none of the channels is available and the total FDL circulation count of this packet at this switch is less than $M$, then this packet is forwarded to one of the $B$ FDLs again randomly, upon availability. Otherwise, the retrial packet gets dropped. Based on the PostRes reservation model, the switch controller only needs to keep track of the binary information on whether each wavelength channel and each FDL is idle (at its entrance point) or not. Moreover, we note that void-filling-based channel scheduling is not supported with PostRes.


Fig. 1. An optical packet router with $N$ input and output fibers (links) using $B$ shared-per-node feedback-type recirculating FDLs.
Our goal is to build a stochastic model that accurately captures the behavior of shared-per-node feedback-type FDL optical buffers using the PostRes reservation scheme. Most of the existing work capitalizes on shared-per-link buffers with $K=1$ wavelength channel using the PreRes reservation model. An approximate method is proposed in [15] for this particular case which employs an iterative procedure which is simple to implement for Poisson arrivals and exponential packet lengths whereas Ref. [16] relaxes the assumptions on inter-arrival and service times of [15]. Again, for the same model, closed-form expressions for loss probabilities and expected delays are obtained in [17] for certain sub-cases whereas an exact analysis procedure for the general case of Markovian arrivals has been recently proposed in [18] using the theory of feedback fluid queues. For general $N$ and $K$, and for PreRes shared buffering, an approximate model is proposed in [10]. For $K=1$ but feedback-type shared-per-link FDLs, a queuing model is presented in [19] for the case of limited number of recirculations. A retrial-queuing model for the case $K>1$ and for feedback-type shared-per-link FDLs is proposed in [20] but with probabilistic circulations. However, to meet signal loss requirements, a limit is generally imposed on the maximum allowable number of FDL circulations [19]. In [21], a renewal model has been proposed for analyzing OPRs with shared-pernode PreRes FDL buffers. However, variable-length FDLs and recirculations are not employed in this work. This method is then used in [22] to derive the end-to-end burst blocking probability in a network of OPRs using a two-moment-based approximative scheme. On the other hand, we use Markov Modulated Poisson Process (MMPP)-based traffic models based on [23] (also used in [20]) for overflow traffic modeling that allows one to match three moments in addition to the DC value of the power spectrum of the overflow traffic. In the current article, we extend the work in [20] in three directions:

- A more efficient FDL sharing scheme, namely shared-per-node FDL buffering, as opposed to shared-per-link FDL buffering, is employed in the current study.
- In the current study, an optical packet can travel over an FDL at most $M \geq 1$ times as opposed to the less realistic probabilistic recirculation scheme of [20].
- We allow varying FDL lengths using the spreading parameter $\alpha>0$ as opposed to fixed-length FDLs proposed in [20], which will be shown later to have significant benefits in terms of blocking performance and amenability to analysis.
The motivation behind the introduction of a queuing model for FDL buffers is the need to assess if optical packet-switched networks can be operated at reasonably high levels of utilization using feedback-type recirculating FDLs. By simulations, we specify a range of $D$ and $\alpha$ values for which the system performance is relatively improved and also our proposed queuing model works acceptably well. Although the queuing model does not take into consideration the particular values of $D$ and $\alpha$, it is capable of studying the impact of other FDL parameters on blocking performance such as the FDL buffer size $B$, recirculation count $M$, number of wavelength channels $K$, and number of fibers $N$. FDLs need to be as short as possible since they increase the delays in the network and the physical size of the buffer. However, statistical multiplexing gains will be reduced with shorter FDLs since then a retrying packet will more likely see an all-occupied system again. Moreover, increased number of FDLs used in the system lead to increased hardware complexity and it is also important to minimize this number without sacrificing from performance. Recirculations are also intuitively useful since they allow multiple opportunities for reserving a channel at the destination link. However, recirculations introduce signal losses and additional delays and they
should be kept at a minimum. Simulations always provide an answer but using simulations for dimensioning purposes especially in a regime of low loss probabilities is a tedious task. The retrial-queuing model we introduce in this article can help fill this gap so as to be used for engineering and dimensioning purposes in the context of shared-per-node feedbacktype recirculating FDL buffers.

The remainder of this article is organized as follows. In Section 2, we present an overview of the MMPP and two-state MMPP construction that is crucial for the development of the article. Section 3 presents the queuing model we propose for feedback-type FDL buffers. In Section 4, we provide numerical examples for validating the accuracy of this approach as well as the use of these models for engineering and dimensioning purposes. Finally, we conclude.

## 2. Markov modulated Poisson process

The following is based on [24]. An MMPP is a point process whose intensity depends on the state of a background process that is an irreducible finite-state continuous-time Markov chain. Let us assume $m>0$ states for the background process. The MMPP is characterized by the infinitesimal generator $Q$ of the underlying Markov chain and by $\Lambda$, an $m \times m$ diagonal matrix with diagonal elements $\lambda_{1}, \lambda_{2}, \ldots, \lambda_{m}$, i.e., $\Lambda=\operatorname{diag}\left(\lambda_{1}, \lambda_{2}, \ldots, \lambda_{m}\right)$. In this case, we simply say the MMPP is characterized with the matrix pair $(Q, \Lambda)$. The operation of the MMPP is as follows. When the background process is in state $i$, the MMPP is said to be in phase $i$ and arrivals occur according to a Poisson process with rate $\lambda_{i}$. Let $\pi$ be the steady-state vector of $Q$, i.e., $\pi Q=0, \pi e=1$, where $e$ is a column vector of ones of appropriate size. The $r$ th non-central moment of the arrival rate of the MMPP characterized with the pair ( $Q, \Lambda$ ) is denoted by $\mu_{r}$ and is given in [24] as $\mu_{r}=\pi \Lambda^{r} e, r \geq 1$.

As opposed to renewal processes, successive inter-arrival times are correlated for MMPPs. This feature of MMPP as well as its amenability to analysis has attracted researchers in the field of telecommunication network traffic modeling. As one of the classical examples, a superposition of packetized voice sources with silence detection is modeled as a two-state MMPP so as to analytically study the performance of a voice multiplexer [25]. The Ref. [26] proposes an MMPP-based model that mimics the real hierarchical behavior of the packet generation process by Internet users. An MMPP model is provided for multimedia traffic in [27]. An Interrupted Poisson Process (IPP) is a two-state MMPP for which one of the two Poisson intensities is zero which amounts to interrupting the arrival process in that particular state of the background process. The most relevant application example to this article is the use of MMPPs and in particular IPPs, in modeling overflow traffic in circuit-switched networks [28,23].

In most cases, using MMPPs with large state-spaces is either computationally infeasible or impractical. Most of the existing research concentrates on two-state MMPP modeling due to its versatility [25]. In this article, we employ a technique from Heffes [29] that approximates a multi-state MMPP with one with two states. The approach of [29] matches the first three non-central moments of the instantaneous arrival rate of the MMPP and in addition an appropriately defined time constant for the process, which is defined in terms of the integral of the covariance function of the instantaneous arrival rate of the MMPP. Note that the above-mentioned integral amounts to the DC value of the associated power spectrum which is very critical as far as queuing performance is concerned [30]. Let the original multi-state MMPP be characterized with the matrix pair $\left(Q^{(m)}, \Lambda^{(m)}\right)$ with $m>2$ states. Let the two-state approximative MMPP be characterized with the pair $\left(Q^{(2)}, \Lambda^{(2)}\right)$. Also let

$$
Q^{(2)}=\left[\begin{array}{cc}
-\sigma_{1} & \sigma_{1} \\
\sigma_{2} & -\sigma_{2}
\end{array}\right], \quad \Lambda^{(2)}=\left[\begin{array}{cc}
\lambda_{1} & 0 \\
0 & \lambda_{2}
\end{array}\right]
$$

Let $\pi^{(m)}$ denote the stationary vector of the modulating Markov chain of the original MMPP $\left(Q^{(m)}, \Lambda^{(m)}\right)$ such that $\pi^{(m)} Q^{(m)}=0$ and $\pi^{(m)} e=1$. Also let $\mu_{r}$ denote the $r$ th non-central moment of the arrival rate of the MMPP $\left(Q^{(m)}, \Lambda^{(m)}\right)$ and $v$ denote its variance that is given by $v=\mu_{2}-\mu_{1}^{2}$. The time constant $\tau_{c}$ of the original MMPP is expressed as

$$
\begin{equation*}
\tau_{c}=\frac{1}{v} \int_{0}^{\infty} r(t) d t \tag{2}
\end{equation*}
$$

where $r(t)$ is the covariance function of the arrival rate. Ref. [23] provides an expression for $\tau_{c}$ that is easy to obtain:

$$
\tau_{c}=\frac{1}{v}\left[\pi^{(m)} \Lambda^{(m)}\left(e \pi^{(m)}-Q^{(m)}\right)^{-1} \Lambda^{(m)} e-\mu_{1}^{2}\right]
$$

Heffes [29] proposes to choose the parameters of the approximating two-state MMPP as follows:

$$
\sigma_{1}=\frac{1}{\tau_{c}(1+\eta)}, \quad \sigma_{2}=\frac{\eta}{\tau_{c}(1+\eta)}, \quad \lambda_{1}=\mu_{1}+\sqrt{v / \eta}, \quad \lambda_{2}=\mu_{1}-\sqrt{v \eta}
$$

where

$$
\delta=\frac{\mu_{3}-3 \mu_{1} v-\mu_{1}^{3}}{v^{3 / 2}}, \quad \eta=1+\frac{\delta}{2}\left(\delta-\sqrt{4+\delta^{2}}\right)
$$

The above choices are shown in [29] to match the first three non-central moments as well as the time constant defined in (2). When this method is used for MMPP model reduction, we will say that the pair $\left(Q^{(m)}, \Lambda^{(m)}\right)$ is reduced to the matrix
pair $\left(Q^{(2)}, \Lambda^{(2)}\right)$ using Heffes' method or mathematically, $\left(Q^{(2)}, \Lambda^{(2)}\right)=f_{H}\left(Q^{(m)}, \Lambda^{(m)}\right)$ where the function $f_{H}$ represents Heffes' method.

The superposition of independent MMPPs is also an MMPP. Consider the superposition of $n$ not-necessarily identical two-state MMPPs. The state-space, in lexicographic order, can be described by Kronecker calculus [31]. Given $A=\left\{A_{i j}\right\}$, a $p \times p$ matrix, and a $q \times q$ matrix $B$, the Kronecker product of the matrices $A$ and $B$ is denoted by $A \otimes B$ and is given as a matrix with block elements $\left\{A_{i j} B\right\}$. Therefore, the size of the square matrix $A \otimes B$ is $p q$. The Kronecker sum of the matrices $A$ and $B$ is denoted by $A \oplus B$ and is given by $A \otimes I_{q}+I_{p} \otimes B$, where $I_{k}$ denotes an identity matrix with size $k$. With this notation, the superposition of $n$ independent two-state MMPPs characterized with $\left(Q_{i}, \Lambda_{i}\right), 1 \leq i \leq n$, can be represented by the superposition MMPP $(Q, \Lambda)$ [23] where

$$
\begin{aligned}
Q & =Q_{1} \oplus Q_{2} \oplus \cdots \oplus Q_{n} \\
& =Q_{1} \otimes I_{2} \otimes \cdots \otimes I_{2}+I_{2} \otimes Q_{2} \otimes I_{2} \otimes \cdots \otimes I_{2}+\cdots+I_{2} \otimes I_{2} \otimes \cdots \otimes I_{2} \otimes Q_{n} \\
\Lambda & =\Lambda_{1} \oplus \Lambda_{1} \oplus \cdots \oplus \Lambda_{n}
\end{aligned}
$$

The superposition can be handled only with $n+1$ states instead of $2^{n}$ states when the individual two-state MMPPs are identically distributed. In this case, we have

$$
Q_{i}=\left[\begin{array}{cc}
-\sigma_{1} & \sigma_{1} \\
\sigma_{2} & -\sigma_{2}
\end{array}\right], \quad \Lambda_{i}=\left[\begin{array}{cc}
\lambda_{1} & 0 \\
0 & \lambda_{2}
\end{array}\right], \quad 1 \leq i \leq n
$$

and the superposition MMPP can be represented by $(Q, \Lambda)$ where

$$
Q=\left[\begin{array}{cccc}
-n \sigma_{1} & n \sigma_{1} & & \\
\sigma_{2} & -\left(\sigma_{2}+(n-1) \sigma_{1}\right) & \ddots & \\
& \ddots & \ddots & \sigma_{1} \\
& & n \sigma_{2} & -n \sigma_{2}
\end{array}\right], \quad \Lambda=\left[\begin{array}{llll}
n \lambda_{1} & & & \\
& (n-1) \lambda_{1}+\lambda_{2} & & \\
& & \ddots & \\
& & & n \lambda_{2}
\end{array}\right]
$$

since in this case, the state $i, 0 \leq i \leq n$, keeps track of the number of individual MMPPs which are in their first state and we do not have to keep track of the states of individual MMPPs.

## 3. Analytical model

Since there is symmetry among the output links, we concentrate on one single tagged output link that comprises $K$ wavelength channels. This tagged link is associated with the so-called fiber process $\{X(t): t \geq 0,0 \leq X(t) \leq K\}$ which keeps track of the number of busy wavelength channels on the tagged link at time $t$. Recall that the fresh traffic destined to the tagged link is Poisson with intensity $\lambda^{(0)}$. Let the retrial process $\left\{Y_{i}(t): t \geq 0,1 \leq i \leq M\right\}$ denote the arrival process to the tagged link stemming from optical packets that have already traversed FDLs $i$ times. Note that $\left\{Y_{i}(t)\right\}$ is a count process that counts up each time a packet (destined for the tagged link) leaves the FDL buffer and this amounts to the $i$ th circulation of the packet over the FDLs. Equivalently, the retrial process $\left\{Y_{i}(t)\right\}$ has overflown from the tagged link $i$ times and the packet has found an idle FDL at the epoch of overflows. Provided that FDL delays are sufficiently long, i.e., $D \gg 1$, and the spreading parameter $\alpha$ is relatively large, i.e., $\alpha \gg 0$, we conjecture that the retrial process $\left\{Y_{i}(t)\right\}$ can well be approximated with a Poisson process with intensity $\lambda^{(i)}, 1 \leq i \leq M$. In this operating regime, i.e., $D \gg 1, \alpha \gg 0$, the processes $\left\{\left(Y_{1}(t), \ldots, Y_{M}(t), X(t)\right)\right\}$ are also approximated as independent processes. We note that the analytical model we propose is based on these two approximations which are valid in the above-mentioned operating regime and therefore does not take into consideration the particular choices of the parameters $D$ and $\alpha$. We will later show the validity of these approximations by simulations in Section 4.

Furthermore, we define the overflow process $\left\{Z_{j}(t): t \geq 0,1 \leq j \leq M\right\}$ that represents the overflow process of the tagged link corresponding to packets that have overflown from the tagged link $j$ times and are in the process of searching for an idle FDL. Obviously, when $B \rightarrow \infty$, for $1 \leq j \leq M,\left\{Z_{j}(t)\right\}$ and $\left\{Y_{j}(t)\right\}$ correspond to the same count process since in this case an overflown packet will always get to find an idle FDL. We do not know $\lambda^{(i)}, 1 \leq i \leq M$, yet but we will attempt to find them using an iterative procedure as described below.

Let us assume the quantities $\lambda^{(i)}, 1 \leq i \leq M$, are now available. Then the process $\{X(t)\}$ is a birth-death process with constant birth rate $\lambda=\sum_{i=0}^{M} \lambda^{(i)}$ and death rate $k$ when $X(t)=k, 1 \leq k \leq K$. Let $P$ denote the generator of this process. Let $x$ denote the stationary vector of $P$ such that $x P=0, x e=1$, and partition $x=\left(x_{0}, x_{1}, \ldots, x_{K}\right)$. As shown in the classical circuit switching literature [23], the overflow process $\left\{Z_{j}(t)\right\}$ is not Poisson since the inter-event times associated with the process $\left\{Z_{j}(t)\right\}$ are highly correlated due to the way overflows occur. Therefore, there is a need for more elaborate modeling of the overflow traffic taking into consideration such correlation effects. Actually, $\left\{Z_{j}(t)\right\}$ can exactly be modeled through a $(K+1)$-state MMPP characterized with the matrix pair $\left(P, \Lambda_{j}\right), 1 \leq j \leq M$, where $\Lambda_{j}$ is an all-zeros matrix except for its single south-east corner entry set to $\lambda^{(j-1)}$. To cope with the state-space explosion problem, we use Heffes' method described in the previous section to reduce the $(K+1)$-state MMPP to a two-state MMPP. With this method in place, we suggest that the process $\left\{Z_{j}(t)\right\}, 0 \leq j \leq M$, is to be approximated by a two-state MMPP characterized with the pair $\left(P_{j}, C_{j}\right)$
defined through $\left(P_{j}, C_{j}\right)=f_{H}\left(P, \Lambda_{j}\right)$ for $1 \leq j \leq M$. Let us then write

$$
P_{j}=\left[\begin{array}{cc}
-\kappa_{1}^{(j)} & \kappa_{1}^{(j)}  \tag{3}\\
\kappa_{2}^{(j)} & -\kappa_{2}^{(j)}
\end{array}\right], \quad C_{j}=\left[\begin{array}{cc}
c_{1}^{(j)} & 0 \\
0 & c_{2}^{(j)}
\end{array}\right], \quad 1 \leq j \leq M .
$$

In this particular scenario, due to the way $\Lambda_{j}$ is structured, the two-state MMPP $\left(P_{j}, C_{j}\right)$ is actually an IPP, i.e., $c_{1}^{(j)}=0$, as shown in [23] but we keep the notation more general for the sake of convenience.

Note that $\left\{Z_{j}(t)\right\}$ is the contribution of overflow traffic due to the single tagged fiber only. Let $\left\{\bar{Z}_{j}(t) ; 1 \leq j \leq M\right\}$ be the overflow process for the entire OPR corresponding to packets that have overflown from any one of the $N$ fiber links $j$ times and which are in the process of finding an idle FDL. The overflow process $\left\{\bar{Z}_{j}(t)\right\}$ is then called the $j$ th parcel using the terminology of [23]. Moreover, this process can approximately be represented by a two-state MMPP characterized with the matrix pair $\left(Q_{j}, R_{j}\right)$ [23]. Since $\left\{\bar{Z}_{j}(t)\right\}$ is obtained through the superposition of $N$ individual overflow processes, this two-state MMPP model can be obtained by using Heffes' model reduction method [29] similar to the approach in [23]: $\left(Q_{j}, R_{j}\right)=$

Let $\eta_{j}$ denote the mean arrival rate for the $j$ th parcel which is the mean rate of the MMPP $\left(Q_{j}, R_{j}\right)$. The superposition of $M$ independent MMPPs parameterized by $\left(Q_{j}, R_{j}\right), 1 \leq j \leq M$, can be represented by the MMPP $(Q, R)$ with

$$
\begin{equation*}
Q=Q_{1} \oplus Q_{2} \oplus \cdots \oplus Q_{M}, \quad R=R_{1} \oplus R_{2} \oplus \cdots \oplus R_{M} \tag{5}
\end{equation*}
$$

The MMPP $(Q, R)$ is offered to $B$ FDLs of varying lengths. An accepted packet into the buffer occupies the entrance point of the FDL for a duration which is equal to the packet transmission time. This observation leads to an MMPP/M/B/B queuing system on the state-space $\left\{\left(l, l^{\prime}\right), 1 \leq l \leq B, 1 \leq l^{\prime} \leq 2^{M}\right\}[23]$ where $l$ corresponds to the number of FDLs that are occupied at their entrance points and $l^{\prime}$ represents the state of the incoming MMPP with infinitesimal generator $Q$. As in [23], the infinitesimal generator of the MMPP/M/B/B system is given by the following matrix:

$$
V=\left[\begin{array}{cccccc}
Q-R & R & & & &  \tag{6}\\
I & Q-R-I & R & & & \\
& 2 I & Q-R-2 I & R & & \\
& & \ddots & \ddots & \ddots & \\
& & & & B I & Q-B I
\end{array}\right]
$$

where $I$ denotes an identity matrix of size $2^{M}$. The performance measures of the system can be calculated by studying the stationary vector $\pi=\left(\pi_{0}, \pi_{1}, \ldots, \pi_{B}\right)$ of $V$ which satisfies

$$
\begin{equation*}
\pi V=0, \quad \pi e=1 \tag{7}
\end{equation*}
$$

It is clear that $V$ is of size $(B+1) 2^{M}$, but one can utilize the block-tridiagonal structure of $V$ to obtain $\pi$ with a computational complexity that is linear in $B$; see for example the block-tridiagonal LU factorization algorithm [32]. When a packet belonging to parcel $j$ finds all the $B$ FDLs occupied, then blocking occurs due to the lack of an idle FDL. In this case, the blocking probability for parcel $j$, denoted by $\gamma_{j}$, is given by

$$
\begin{equation*}
\gamma_{j}=\frac{\pi_{B} V_{j} e}{\eta_{j}} \tag{8}
\end{equation*}
$$

where

$$
\begin{equation*}
V_{j}=0 \oplus 0 \oplus \cdots \oplus \underbrace{R_{j}}_{j \text { th position }} \oplus 0 \oplus \cdots \oplus 0 \tag{9}
\end{equation*}
$$

Since $\left\{Y_{j}(t)\right\}$ is assumed to be Poisson and is obtained from $\left\{Z_{j}(t)\right\}$ by choosing those packets that get to find an idle FDL, we have

$$
\begin{equation*}
\lambda^{(j)}=\frac{\eta_{j}\left(1-\gamma_{j}\right)}{N}, \quad 1 \leq j \leq M \tag{10}
\end{equation*}
$$

Once we know the arrival intensities $\lambda^{(i)}, 1 \leq i \leq M$, we can obtain the pair ( $Q, R$ ) that characterizes the MMPP traffic offered to the $B$ FDL buffers. On the other hand, given the pair ( $Q, R$ ), one can obtain $\lambda^{(i)}, 1 \leq i \leq M$, as in (10). Therefore, the overall problem can be solved by means of a fixed-point procedure described in Table 1.

Table 1
Algorithm to find $P_{B}$ given $\lambda^{(0)}, K, N, B$, and $M$.

```
Set }\mp@subsup{\lambda}{}{(i)}=0,1\leqi\leqM
Form the generator P for the fiber process which is of birth-and-death type. Also form the matrices }\mp@subsup{\Lambda}{j}{},1\leqj\leqM\mathrm{ .
For 1\leqj\leqM, using Heffes' method, reduce the (K+1)-state MMPP (P, \Lambda
For 1\leqj\leqM, obtain the MMPP ( }\mp@subsup{Q}{j}{},\mp@subsup{R}{j}{})\mathrm{ (parcel }j\mathrm{ ) as in (4).
Calculate }\mp@subsup{\overline{\eta}}{j}{}\mathrm{ which is the mean arrival rate of the MMPP ( }\mp@subsup{Q}{j}{},\mp@subsup{R}{j}{})\mathrm{ that is written as }\mp@subsup{\eta}{j}{}=\mp@subsup{y}{j}{}\mp@subsup{R}{j}{}e\mathrm{ where }\mp@subsup{y}{j}{}\mathrm{ is the stationary vector of Q Q,i.e.,
yj Q = 0, yje = 1.
Obtain the MMPP (Q,R) based on (5).
7 Form the generator V for the buffer process as in (6) and calculate its stationary vector }\pi\mathrm{ satisfying }\piV=0,\pie=1\mathrm{ . Also partition
\pi=(\mp@subsup{\pi}{0}{},\mp@subsup{\pi}{1}{},\ldots,\mp@subsup{\pi}{B}{}).
Calculate the parcel-j blocking probability }\mp@subsup{\gamma}{j}{}\mathrm{ as in (8).
9 For 1\leqj\leqM, write }\mp@subsup{\lambda}{}{(j)}\mathrm{ as in (10).
10 If the successive values for }\mp@subsup{\lambda}{}{(j)},1\leqj\leqM\mathrm{ are sufficiently close, convergence is reached. Upon convergence, write the loss probability }\mp@subsup{P}{B}{}\mathrm{ as in
(11) and exit. Otherwise, go to Step 2.
```

Upon convergence of the fixed-point procedure, we can find the overall blocking probability $P_{B}$ based on the expression below:

$$
\begin{equation*}
P_{B}=\frac{\chi_{K} \lambda^{(M)} N+\sum_{j=1}^{M} \pi_{B} V_{j} e}{N \lambda^{(0)}} \tag{11}
\end{equation*}
$$

The term $x_{K} \lambda^{(M)}$ in the numerator above amounts to the rate of packets that are blocked due to the lack of an idle wavelength channel at the tagged link once the recirculation limit is reached. This term is multiplied with the factor $N$ since there are $N$ such links. On the other hand, the second term in the numerator represents the rate of packets that are blocked due to the lack of an idle FDL at one of the retrial attempts. The denominator gives the rate of fresh packets into the OPR and the ratio gives the packet blocking probability. Similarly, one can find the distribution of the number of FDL recirculations. For this purpose, let $H$ denote the number of retrials required for a successful packet. It is easy to show that

$$
\begin{equation*}
P(H=h)=\frac{\lambda^{(h)}\left(1-x_{K}\right)}{\lambda^{(0)}\left(1-P_{B}\right)}, \quad 0 \leq h \leq M \tag{12}
\end{equation*}
$$

## 4. Numerical results

For the fixed-point procedure, let $\lambda_{k}$ denote the vector of retrial rates $\lambda^{(i)}, 1 \leq i \leq M$, at the end of the $k$ th iteration. We stop the iterations when $\left|\lambda_{k}-\lambda_{k-1}\right|_{2} /\left|\lambda_{k}\right|_{2}<\varepsilon$ for some tolerance parameter $\varepsilon$ which is set to 0.00001 for the numerical examples of this article. We ran the fixed-point procedure for all combinations involving $N \in\{4,8,16\}, K \in\{8,16,32\}, B \in$ $\{8,16,32\}, M \in\{1,2,3\}$ and $\rho \in\{0.5,0.7,0.9\}$. The iterations converged rapidly in all cases; the minimum (maximum) number of iterations required was 4 (27) for this experiment. The number of required iterations appear to increase with increased $B$ and $\rho$ but with decreased $N$ values but convergence was acceptably fast in all cases we tested.

We first validate the analytical method proposed in the previous section by simulations in Figs. 2-6. For this purpose, we first fix $N=1, K=16, B=16, \rho=0.9$ and we plot the blocking probability $P_{B}$ obtained using simulations as a function of the FDL spreading parameter $\alpha$ for various values of $D$ and for two values of the recirculation limit parameter $M$ in Fig. 2. Fig. 3 addresses the same scenario but with load $\rho$ reduced to 0.75 while all other parameters are fixed. We further increase the number of fibers $N$ with $\rho$ fixed at 0.75 to 8 in a new simulation scenario whose results are depicted similarly in Fig. 4. The results obtained via the analytical model are also depicted in Figs. 2-4. It is clear that the blocking probability $P_{B}$ decreases as $D$ increases in all scenarios for fixed $\alpha$. This stems from the observation that for low values of $D$, a retrying optical packet will see a system occupancy positively correlated with the one the same packet had attempted to join but failed $D$ time units back. Similar observations were made in [11,20]. Using FDLs of varying lengths is generally peculiar to PreRes schemes but it is clear from Figs. 2-4 that such different length FDLs also reduce blocking probabilities in PostRes schemes. The packet blocking probability $P_{B}$ first decreases with increasing FDL spreading parameter $\alpha$ but beyond a certain value of $\alpha$, it tends to slightly increase as $\alpha \rightarrow 1$ for all the three scenarios we tested. This latter behavior is more evident for relatively low values of $D$ and $M$. To explain this phenomenon, as $\alpha \rightarrow 1$, some of the delay lines will induce very small delays compared to the average packet length and consequently retrying packets using such small FDLs will likely get blocked. However, with a proper choice of the FDL spreading parameter $\alpha$, for example $\alpha=0.8$, such behavior can be avoided. Throughout the rest of the article, we set $\alpha=0.8$.

In the second simulation example, we fix $N=8, K=16$, and study the accuracy of the analytical model in terms of $B, M$, and $\rho$. For this purpose, we plot the blocking probability $P_{B}$ for various values of $D$ in Fig. 5 obtained by simulations (recall $\alpha=0.8$ used throughout all simulations) as a function of the FDL size $B$ for four different scenarios corresponding to $\rho=0.7,0.9$ and $M=1,5$. This plot also presents the results of the analytical model. We observe that for a given scenario, there is a certain value of $D$, say $D_{\max }$, beyond which there will be no significant performance improvement in terms of

(a) $\mathrm{M}=1$
FDL spreading parameter $\alpha$

(b) $M=3$
FDL spreading parameter $\alpha$
$\square-\mathrm{D}=0.5-\boxminus-\mathrm{D}=1 \cdot-*-\mathrm{D}=2-\Delta-\mathrm{D}=5 \cdots \cdots \mathrm{D}=10-$ analysis

Fig. 2. The packet blocking probability $P_{B}$ as a function of the FDL spreading parameter $\alpha$ for the scenario when $N=1, K=16, B=16, \rho=0.9$ and for various values of $D$ and for two values of recirculation limit parameter (a) $M=1$, (b) $M=3$.

(a) $M=1$
FDL spreading parameter $\alpha$

(b) $M=3$
FDL spreading parameter $\alpha$

|  |  |  |  |  |
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Fig. 3. The packet blocking probability $P_{B}$ as a function of the FDL spreading parameter $\alpha$ for the scenario when $N=1, K=16, B=16, \rho=0.75$, and for various values of $D$ and for two values of recirculation limit parameter (a) $M=1$, (b) $M=3$.
blocking probability. However, $D_{\max }$ turns out to depend on the presented scenario. Based on the results provided in Fig. 5, when the average delay length $D$ is chosen so that it is larger than $D_{\max }$, then the analytical model captures very well the blocking performance with respect to the FDL size $B$.

It is also crucial to choose an absolute value for $D$ to use in the OPR. For this purpose, we design the following simulation experiment. Fixing $M=5, K=16$, and for given $N$ and $B$, for two values of desired blocking probability $P_{B}=0.01,0.001$, we find $\lambda^{(0)}$ that meets the blocking probability requirement using the analytical model. We then simulate the corresponding


Fig. 4. The packet blocking probability $P_{B}$ as a function of the FDL spreading parameter $\alpha$ for the scenario when $N=8, K=16, B=16, \rho=0.75$, and for various values of $D$ and for two values of recirculation limit parameter (a) $M=1$, (b) $M=3$.

OPR offered with Poisson traffic with intensity $\lambda^{(0)}$ for various values of $D$. The results of this simulation experiment are presented in Fig. 6. We observe that irrespective of the choice of the switch size $N, D_{\max }$ turns out to depend on $B$; for low values of $B, D_{\max }$ is only a few packet lengths whereas $D_{\max }$ increases beyond a few packet lengths with increased $B$. However, setting $D$ to a very large value increases the delays despite the blocking performance improvement. Consequently, setting $D$ to around ten packet lengths appears to be an acceptable trade-off between delay and blocking for a wide range of scenarios including the ones given in Figs. 5 and 6. Moreover, with this choice of $D$, the analytical procedure we propose in this article appears to capture very well the blocking performance. However, the model can produce an optimistic estimate for blocking probabilities if shorter delay lines are used if the delay requirement is stringent.

The success of the analytical model in the regime of relatively long FDLs and relatively large spreading parameter is now explained. Recall that the processes $X(t), Y_{1}(t), \ldots, Y_{M}(t)$ were assumed to be independent processes in the analytical model. Note that the retrial process $Y_{i}(t)$ is generally not independent from the fiber process $X(t)$. As stated before, this stems from the observation that a retrying optical packet will see a system correlated with the one the same packet had overflown $D$ time units back when $D$ is small. However, as shown in [20], as $D \rightarrow \infty$, the process $Y_{i}(t)$ becomes independent from $X(t)$ and approximations that are based on this independence turn out to produce acceptable results. In practice, it was shown that $D$ should be at least a few mean packet lengths for this approximation to hold for the shared-per-link case fed with Poisson traffic [20]. With shorter FDLs, the retrial processes and the fiber process cannot be approximated as independent processes and therefore the effectiveness of using FDLs for blocking probability reduction is relatively limited. For the spreading parameter effect on performance, let us assume $\alpha \rightarrow 0$ first. In this case, it is obvious that $Y_{i}(t)$ and $Y_{j}(t), i \neq j$, are not independent. To see this, let us assume severe congestion at the tagged fiber at around time $t_{0}$. There will be retrial traffic that will retry all together at $t_{0}+D$. Since the instantaneous rate of this retrial process will be high, the system will continue to be congested at time $t_{0}+D$ leading to an increase of instantaneous rate of retrial traffic at $t_{0}+2 D$, and so on. This shows that $X\left(t_{0}\right)$ and $Y_{i}\left(t_{0}+i D\right), 1 \leq i \leq M$, are not independent. In order to break this dependence, we introduce in this article the spreading parameter $\alpha$ and random FDL selection policy. Using Figs. 2-4, we show the impact of the choice of $\alpha$ on the independence assumption. Therefore, use of long average FDL delays $D$ and relatively large spreading parameter $\alpha$ (for example $\alpha=0.8$ ) is not only effective in improving blocking performance but also in this regime, an analytical model can be built on the basis of the independence assumption of the processes $X(t), Y_{1}(t), \ldots, Y_{M}(t)$.

For the remainder of this article, we provide results obtained only using the analytical model assuming that the average FDL size parameter $D$ is set to a value exceeding ten packet lengths as motivated before. We now define the achievable throughput $T$ as the maximum load the OPR can support under a blocking probability constraint $P_{B}$. We fix $N=8$ and plot $T$ associated with the blocking constraint $P_{B}<0.001$ as a function of the FDL size $B$ for various values of $M$ in Fig. 7 for two different choices of $K$ : (a) $K=8$, (b) $K=16$. The results are obtained only through the proposed analytical model. Note that the $B=0$ case does not employ FDLs and the corresponding throughput is obtained using the Erlang-B formula for $K$ servers. We observe that there is a certain value of $B$, say $B_{\max }$, beyond which the throughput $T$ cannot further be improved. The quantity $B_{\max }$ however increases with increasing $M$ and also with increasing $K$. When small FDL sizes are used $(B \rightarrow 0)$,


Fig. 5. The packet blocking probability $P_{B}$ as a function of the FDL delay parameter $B$ for various values of $D$ for the scenario when $\alpha=0.8, N=8, K=16$ : (a) $\rho=0.7, M=1$; (b) $\rho=0.7, M=5$; (c) $\rho=0.9, M=1$; (d) $\rho=0.9, M=5$.


Fig. 6. The packet blocking probability $P_{B}$ as a function of the FDL delay parameter $D$ for various values of $N$ and $B$ when $M=5, K=16$ and for two values of the desired blocking probability $P_{B}=0.01,0.001$.
there is limited gain in using multiple circulations $M>1$. However, one can benefit from multiple circulations for larger FDL sizes. Consider the basic case of using one circulation only and using $B_{\max }$ FDLs. In this case, the throughput is increased by $88.5 \%$ and $52.6 \%$, associated with the cases $K=8$ and $K=16$, respectively, compared with the bufferless scenario. Even with such basic schemes, the throughput can substantially be improved.

The final numerical experiment we present is on the provisioning of the FDL size $B$. Again, for a given blocking probability $P_{B}=0.001$, and for given $N, M, K$, and $B$, we iteratively calculate $T$ but also find the value of $B$, namely $B_{\text {max }}$, beyond which


Fig. 7. The achievable throughput $T$ under a desired blocking probability $P_{B}=0.001$ as a function of the FDL size $B$ for various values of $M$ for $N=8$ and for two different values of $K$ : (a) $K=8$, (b) $K=16$.
$T$ does not change significantly. In this example, $B_{\max }$ is found so that the difference in $T$ values corresponding to FDL sizes $B_{\max }$ and $B_{\max }+1$ is less than 0.001 . Once the targeted FDL size $B_{\max }$ is found, the maximum throughput achievable by using an FDL size of $B_{\max }$ is called $T_{\max }$. The FDL size requirement $B_{\max }$ and the corresponding maximum achievable throughput $T_{\max }$ are plotted in Fig. 8. Our observations are as follows.

- The FDL size requirement $B_{\max }$ changes almost linearly with $K$ and $M$.
- The FDL size requirement $B_{\max }$ increases with increased switch size $N$. However, the maximum achievable throughput $T_{\max }$ obtained using $B_{\max }$ FDLs does not change much with increased switch size $N$.
- Most of the throughput gains stem from one or two circulations with incremental changes with further number of recirculations.


## 5. Conclusion

We propose an MMPP-based queuing model along with fixed-point iterations to accurately evaluate the performance of feedback-type shared-per-node recirculating FDL buffers. Simulation results show that the proposed model allows us to accurately estimate the packet blocking performance in a certain regime of long FDL delay $D$ along with the employment of a relatively large spreading parameter $\alpha$. Benefits of using variable-length FDLs are justified in a feedback architecture using the PostRes reservation scheme, which to the best of our knowledge is novel. Moreover, the retrial-queuing model proposed in this article can effectively be used in dimensioning feedback-type shared-per-node recirculating FDL optical buffers.

## Acknowledgment

The work of N. Akar is supported in part by the Science and Research Council of Turkey (Tübitak) under project no: EEEAG-111E106.


Fig. 8. The FDL size requirement $B_{\max }$ and the corresponding maximum achievable throughput $T_{\max }$ as a function of $K$ for three values of $M$ and for two values of $N$.

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