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Short Communication

## Information and preference reversals in lotteries

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### ABSTRACT

Several approaches have been proposed for evaluating information in expected utility theory. Among the most popular approaches are the expected utility increase, the selling price and the buying price. While the expected utility increase and the selling price always agree in ranking information alternatives, Hazen and Sounderpandian [11] have demonstrated that the buying price may not always agree with the other two. That is, in some cases, where the expected utility increase would value information *A* more highly than information *B*, the buying price may reverse these preferences. In this paper, we discuss the conditions under which all these approaches agree in a generic decision environment where the decision maker may choose to acquire arbitrary information bundles.

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### 1. Introduction

Information gathering is an essential element of decision making under uncertainty. Expected utility theory states that a decision maker (DM) is never worse off with additional information in non-strategic decision environments (see, [5]). In fact, empirical studies suggest that DMs may also seek information that is non-instrumental to their decisions (see, [2,19]). However, even when DMs are assumed to behave optimally as prescribed by the expected utility theory, their preferences toward information do not necessarily exhibit monotonicity with respect to the critical attributes in the decision environment such as the initial wealth, risk aversion and action flexibility (see, [10,12]). Lack of monotonicity between the value of information and risk aversion is also confirmed by the experiments presented in [16].

Other experimental studies offer conflicting insights into how DMs evaluate information in real life settings. For example, when DMs engage in strategic interactions, they tend to overvalue information (see, [9]). A similar behavior is observed for gathering information in organizations (see, [8]). In non-strategic settings, however, experiments in [6,14,15] showed that DMs undervalue available information, sometimes even to the point of discounting ambiguous information (see, [20]). Perhaps, as argued in [7], this kind of behavior is observed because ‘the valuation of information relies heavily on consequential reasoning’; a cognitive activity in which DMs do not always perform well (see, [17,18]). Delquíe [7] illustrates through a series of experiments that information is valued less than an equivalent option in a simple two-stage decision environment.

Despite its shortcomings in fully characterizing the real life human behavior in certain decision settings, expected utility theory predicts preference reversals (see, [11]). Such reversals occur when a DM is willing to pay less to acquire an information alternative that is otherwise preferred if information is free. There is a vast body of experimental evidence on lottery preference reversals going back to [13]. In this paper, we investigate the conditions under which two approaches to evaluate information agree in ranking information alternatives: buying price approach and expected utility increase approach. Following [1], we define information (or in our vocabulary, information *bundles*) as algebras of events on the outcome space. Acquisition of an information bundle enables a DM to learn whether a collection of events have occurred. We consider a problem in which a DM with initial wealth level  $w$  has the option of playing one of the multiple lotteries or ignoring this option to remain at wealth level  $w$ . The DM holds a prior belief on the lottery’s probability law and obtains information to revise his beliefs. Such situations arise commonly when an engineering designer must choose among alternate designs in an uncertain environment, and where information may be obtained regarding, for example, the market size or product performance.

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**2. Problem definition**

We begin with a DM who has a continuous and monotonically increasing utility function  $u : \mathbb{R} \rightarrow \mathbb{R}$ . The DM could select among lotteries  $\Pi_j : \Omega \rightarrow \mathbb{R}, j \in \{1, \dots, m\}$  where  $\Omega$  denotes the state space. Each lottery is a random variable mapping the states to monetary outcomes. A lottery  $\Pi_j$  is distinguished by the cumulative distribution function  $F_j$  (and an associated density  $f_j$ ) over the monetary outcomes, which may be either positive or negative. The DM may select one of the lotteries or may choose not to play at all.

Before committing a decision, information bundles are available to the DM. Information bundles are generated by a collection of disjoint events  $\{A_1, \dots, A_k\}$  that satisfy  $\cup_{j=1}^k A_j = \Omega$ . The information bundle  $\mathcal{I}$  generated by this collection includes  $A_1, \dots, A_k$  and their complements as well as the finite unions and intersections of the events in  $\{A_1, \dots, A_k\}$ . The *expected utility increase*<sup>2</sup>  $V(w, \mathcal{I}, u)$  of information bundle  $\mathcal{I}$  with utility function  $u$  and initial wealth  $w$  is defined as

$$V(w, \mathcal{I}, u) := \sum_i P(A_i) \cdot \max_{j \in \{1, \dots, m\}} \{u(w), \mathbb{E}[u(w + \Pi_j)|A_i]\} - \max_{j \in \{1, \dots, m\}} \{u(w), \mathbb{E}[u(w + \Pi_j)]\}. \tag{1}$$

The *buying price*  $B(w, \mathcal{I}, u)$  of information bundle  $\mathcal{I}$  satisfies the equation

$$\max_{j \in \{1, \dots, m\}} \{u(w), \mathbb{E}[u(w + \Pi_j)]\} = \sum_i P(A_i) \cdot \max_{j \in \{1, \dots, m\}} \{\mathbb{E}[u(w + \Pi_j - B(w, \mathcal{I}, u))|A_i], u(w - B(w, \mathcal{I}, u))\}. \tag{2}$$

A DM is said to exhibit a *preference reversal* on information bundles  $\mathcal{I}_1$  and  $\mathcal{I}_2$  if he ranks them differently using the expected utility increase and the buying price approaches; that is  $V(w, \mathcal{I}_1, u) > V(w, \mathcal{I}_2, u)$ , but  $B(w, \mathcal{I}_1, u) < B(w, \mathcal{I}_2, u)$ , or vice versa.

After the acquisition of  $\mathcal{I}$ , the DM may update his initial decision. Let  $\mathbb{P}$  be the power set of  $\Omega$ . We define an optimal decision function  $d_u : \mathbb{R} \times \mathbb{P} \rightarrow \mathbb{R}$  as follows,

$$d_u(w, A) = \begin{cases} +j, & \mathbb{E}[u(w + \Pi_j)|A] \geq u(w) \text{ and } \mathbb{E}[u(w + \Pi_j)|A] > \mathbb{E}[u(w + \Pi_k)|A] \quad \forall k \in \{1, \dots, m\} \setminus \{j\}, \\ -1, & o.w. \end{cases}$$

The optimal decision function informs us whether the DM chooses to play a lottery  $\Pi_j$  at a wealth level  $w$  given he knows that  $A$  occurs. For each  $\mathcal{I}$ , we will group outcomes of the lottery in  $m + 1$  sets. We place an outcome  $\pi \in \mathbb{R}$  into one of these sets depending on the optimal action on the set among  $A_1, \dots, A_k$  that contains  $\pi$ . We define  $\Gamma_j(u, w, \mathcal{I}) = \{\pi \in \mathbb{R} : d_u(w, A) = +j \text{ for } A \in \{A_1, \dots, A_k\} \text{ and } \pi \in A\}$ . For example, all outcomes in  $A_j \in \{A_1, \dots, A_k\}$  is in  $\Gamma_j(u, w, \mathcal{I})$  if the optimal decision given  $A_j$  is to play lottery  $\Pi_j$ . Note that,  $\cup_{j \in \{-1, 1, \dots, m\}} \Gamma_j(u, w, \mathcal{I}) = \mathbb{R}$  where  $\Gamma_{-1}(u, w, \mathcal{I})$  denotes the set of outcomes on which no lottery is played. In what follows, we consider a single DM with a utility function  $u$  and thus suppress the variable  $u$  in  $V(w, \mathcal{I}, u), B(w, \mathcal{I}, u), \Gamma_j(u, w, \mathcal{I})$  and  $d_u(w, A)$ .

**3. Comparison of arbitrary information bundles**

For a strictly increasing and continuous utility function  $u$ , our first result establishes conditions under which both approaches agree in ranking of generic information bundles.

**Proposition 1.** *Let  $u$  be a strictly increasing, continuous and strictly concave utility function exhibiting decreasing degree of risk aversion. Let  $\mathcal{I}_1$  and  $\mathcal{I}_2$  be two arbitrary information bundles. Then if  $V(x, \mathcal{I}_1) > V(x, \mathcal{I}_2)$  for  $x \in [0, w]$ , then  $B(w, \mathcal{I}_1) > B(w, \mathcal{I}_2)$ .*

**Proof.** The equations for the buying price are,

$$\begin{aligned} \max_{j \in \{1, \dots, m\}} \{\mathbb{E}[u(w + \Pi_j)], u(w)\} &= \sum_{j \in \{1, \dots, n\}} P(\Gamma_j(w - B(w, \mathcal{I}_z), \mathcal{I}_z)) \cdot \mathbb{E}[u(w + \Pi_j - B(w, \mathcal{I}_z)) | \Gamma_j(w - B(w, \mathcal{I}_z), \mathcal{I}_z)] \\ &+ P(\Gamma_{-1}(w - B(w, \mathcal{I}_z), \mathcal{I}_z)) \cdot u(w - B(w, \mathcal{I}_z)), \quad z = 1, 2. \end{aligned}$$

We prove by contradiction. Assume  $B(w, \mathcal{I}_2) > B(w, \mathcal{I}_1)$ . Using the above equations,

$$\begin{aligned} &\sum_{j \in \{1, \dots, m\}} \int_{\Gamma_j(w - B(w, \mathcal{I}_1), \mathcal{I}_1)} u(w + \pi - B(w, \mathcal{I}_1)) \cdot f_j(\pi) d\pi + P(\Gamma_{-1}(w - B(w, \mathcal{I}_1), \mathcal{I}_1)) \cdot u(w - B(w, \mathcal{I}_1)) \\ &< \sum_{j \in \{1, \dots, m\}} \int_{\Gamma_j(w - B(w, \mathcal{I}_2), \mathcal{I}_2)} u(w + \pi - B(w, \mathcal{I}_1)) \cdot f_j(\pi) d\pi + P(\Gamma_{-1}(w - B(w, \mathcal{I}_2), \mathcal{I}_2)) \cdot u(w - B(w, \mathcal{I}_1)). \end{aligned} \tag{3}$$

We know that  $w - B(w, \mathcal{I}_2) < w - B(w, \mathcal{I}_1)$ , which in turn implies  $\Gamma_{-1}(w - B(w, \mathcal{I}_2), \mathcal{I}_2) \supset \Gamma_{-1}(w - B(w, \mathcal{I}_1), \mathcal{I}_2)$  as  $u$  exhibits decreasing degree of risk aversion. Let  $\Gamma_d = \Gamma_{-1}(w - B(w, \mathcal{I}_1), \mathcal{I}_2)^c - \Gamma_{-1}(w - B(w, \mathcal{I}_2), \mathcal{I}_2)^c$ . Note that on  $\Gamma_d$ , the decision is to play some lottery at the wealth level of  $w - B(w, \mathcal{I}_1)$ . This implies,

$$\begin{aligned} &\sum_{j \in \{1, \dots, m\}} \int_{\Gamma_j(w - B(w, \mathcal{I}_2), \mathcal{I}_2)} u(w + \pi - B(w, \mathcal{I}_1)) \cdot f_j(\pi) d\pi + [P(\Gamma_d) + P(\Gamma_{-1}(w - B(w, \mathcal{I}_1), \mathcal{I}_2))] \cdot u(w - B(w, \mathcal{I}_1)) \\ &\leq \sum_{j \in \{1, \dots, m\}} \int_{\Gamma_j(w - B(w, \mathcal{I}_1), \mathcal{I}_2)} u(w + \pi - B(w, \mathcal{I}_1)) \cdot f_j(\pi) d\pi + P(\Gamma_{-1}(w - B(w, \mathcal{I}_1), \mathcal{I}_2)) \cdot u(w - B(w, \mathcal{I}_1)). \end{aligned} \tag{4}$$

<sup>2</sup> In what follows, we compare between expected utility increase and buying price, since it has been previously shown that expected utility increase and selling price are equivalent for ranking information.

The left hand side of (4) and the right hand side of (3) are exactly the same. Hence, both (3) and (4) can be combined to yield,

$$\sum_{j \in \{1, \dots, m\}} \int_{\Gamma_j(w-B(w, \mathcal{I}_1), \mathcal{I}_1)} u(w + \pi - B(w, \mathcal{I}_1)) \cdot f_j(\pi) d\pi + P(\Gamma_{-1}(w - B(w, \mathcal{I}_1), \mathcal{I}_1)) \cdot u(w - B(w, \mathcal{I}_1)) < \sum_{j \in \{1, \dots, m\}} \int_{\Gamma_j(w-B(w, \mathcal{I}_1), \mathcal{I}_2)} u(w + \pi - B(w, \mathcal{I}_1)) \cdot f_j(\pi) d\pi + P(\Gamma_{-1}(w - B(w, \mathcal{I}_1), \mathcal{I}_2)) \cdot u(w - B(w, \mathcal{I}_1)). \tag{5}$$

A contradiction follows because (5) states that  $V(w - B(w, \mathcal{I}_1), \mathcal{I}_2) > V(w - B(w, \mathcal{I}_1), \mathcal{I}_1)$  and  $w - B(w, \mathcal{I}_1) \in [0, w]$ . Hence, the proposition is proved. □

Proposition 1 holds for a fairly large class of utility functions. The caveat is that the DM at a wealth level  $w$  should not switch his ranking of two information bundles in the range  $[0, w]$ . As first noted in [13], lottery preference reversals occur as the amount paid for information acquisition may change the risk preferences. The buying price equations evaluate the preferences of the DM at a lower wealth level. Consequently, if the DM changes his optimal action after paying for information acquisition, Proposition 1 states that a preference reversal may occur.

**4. Information about a single event**

For information bundles generated by a single arbitrary event, we can relax the condition in Proposition 1 when we restrict ourselves to evaluation of a single lottery  $\Pi$  with a cumulative distribution function  $F$  (and density  $f$ ). We state that for one-switch utility functions, it suffices to check the expected utility increase ranking at two wealth levels. For the general case, imposing a one-switch rule on utility functions does not yield the result without assuming  $V(x, \mathcal{I}_1) > V(x, \mathcal{I}_2)$  for all  $x \in (0, w)$ . The family of one-switch utility functions admits four different functional forms: quadratic, sumex, linear plus exponential and linear times exponential (see, [4]). Among these, linear plus exponential is the only utility function in which the DM’s preferences toward a riskier lottery increase consistently as the DM gets wealthier (see, [3]).

Let  $A$  and  $B$  be two arbitrary events. Information bundles generated by these events are  $\mathcal{I}_A$  and  $\mathcal{I}_B$ , respectively. In this section, the optimal decision function  $d_u(w, A) = +1$  when the DM chooses to play  $\Pi$  given  $A$  occurs, and  $d_u(w, A) = -1$  if  $\Pi$  is not played when  $A$  occurs.

**Proposition 2.** *Let  $u$  be a strictly increasing and continuous utility function obeying the one-switch rule. Let  $A$  and  $B$  be two arbitrary events whose outcomes can be resolved before making a decision. If  $V(w, \mathcal{I}_B) > V(w, \mathcal{I}_A)$  and  $V(\bar{w}, \mathcal{I}_B) > V(\bar{w}, \mathcal{I}_A)$  for some  $\bar{w} \leq 0$ , then  $B(w, \mathcal{I}_B) > B(w, \mathcal{I}_A)$ .*

**Proof.** The buying price equation for  $\mathcal{I}_A$  is,

$$\max \{ \mathbb{E}[u(w + \Pi)], u(w) \} = P(A) \cdot \max \{ \mathbb{E}[u(w + \Pi - B(w, \mathcal{I}_A)) | A], u(w - B(w, \mathcal{I}_A)) \} + (1 - P(A)) \cdot \max \{ \mathbb{E}[u(w + \Pi - B(w, \mathcal{I}_A)) | A^c], u(w - B(w, \mathcal{I}_A)) \}.$$

The equation for  $\mathcal{I}_B$  is similar and can be obtained by substituting  $B$  for  $A$  in the above equation. We consider four cases.

Case 1:  $d(w, A) = d(w, B) = -1$ . The buying price equations are combined to yield,

$$P(B) \cdot u(w - B(w, \mathcal{I}_B)) + \int_{B^c} u(w + \pi - B(w, \mathcal{I}_B)) \cdot f(\pi) d\pi = P(A) \cdot u(w - B(w, \mathcal{I}_A)) + \int_{A^c} u(w + \pi - B(w, \mathcal{I}_A)) \cdot f(\pi) d\pi. \tag{6}$$

If we assume that  $B(w, \mathcal{I}_A) > B(w, \mathcal{I}_B)$ , and substitute  $B(w, \mathcal{I}_A)$  with  $B(w, \mathcal{I}_B)$  in Eq. (6), we obtain after a little rearrangement,

$$P(B - A) \cdot u(w - B(w, \mathcal{I}_B)) + \int_{A-B} u(w + \pi - B(w, \mathcal{I}_B)) \cdot f(\pi) d\pi < P(A - B) \cdot u(w - B(w, \mathcal{I}_B)) + \int_{B-A} u(w + \pi - B(w, \mathcal{I}_B)) \cdot f(\pi) d\pi. \tag{7}$$

The conditions for the expected utility increase approach imply,

$$P(B - A) \cdot u(w) + \int_{A-B} u(w + \pi) \cdot f(\pi) d\pi > P(A - B) \cdot u(w) + \int_{B-A} u(w + \pi) \cdot f(\pi) d\pi, \tag{8}$$

$$P(B - A) \cdot u(\bar{w}) + \int_{A-B} u(\bar{w} + \pi) \cdot f(\pi) d\pi > P(A - B) \cdot u(\bar{w}) + \int_{B-A} u(\bar{w} + \pi) \cdot f(\pi) d\pi.$$

Consider lotteries  $\mathcal{F}$  and  $\mathcal{G}$  such that  $\mathcal{F} = \{0, \Pi(\omega) \in B - A; \pi, \Pi(\omega) = \pi \in A - B; x, o.w.\}$  for some  $x \in \mathbb{R}$  (i.e.,  $\mathcal{F}$  offers 0 when  $\Pi(\omega) \in B - A$ , offers  $\pi$  when  $\Pi(\omega) = \pi \in A - B$ , and offers some value  $x$  otherwise) and  $\mathcal{G} = \{0, \Pi(\omega) \in A - B; \pi, \Pi(\omega) = \pi \in B - A; x, o.w.\}$  for the same  $x \in \mathbb{R}$ . Since  $u$  obeys the one-switch rule, and since Eqs. (7) and (8) evaluate and rank lotteries  $\mathcal{F}$  and  $\mathcal{G}$  for utility function  $u$  at different wealth levels, we arrive at a contradiction. Note that, since  $u$  is strictly increasing and one-switch,  $B(w, \mathcal{I}_A) = B(w, \mathcal{I}_B)$  is not possible either. Then,  $B(w, \mathcal{I}_B) > B(w, \mathcal{I}_A)$ .

Case 2:  $d(w, A) = d(w, B) = +1$ . Since  $B(w, \mathcal{I}_A) = B(w, \mathcal{I}_{A^c})$  and  $B(w, \mathcal{I}_B) = B(w, \mathcal{I}_{B^c})$ , the proof of this case is identical to Case 1.

Case 3:  $d(w, A) = +1, d(w, B) = -1$ . Again, if we assume  $B(w, \mathcal{I}_A) > B(w, \mathcal{I}_B)$  and incorporate this into the buying price equation,

$$P(A^c \cap B^c) \cdot u(w - B(w, \mathcal{I}_B)) + \int_{A \cap B} u(w + \pi - B(w, \mathcal{I}_B)) \cdot f(\pi) d\pi > P(A \cap B) \cdot u(w - B(w, \mathcal{I}_B)) + \int_{A^c \cap B^c} u(w + \pi - B(w, \mathcal{I}_B)) \cdot f(\pi) d\pi. \tag{9}$$

Using the expected utility increase conditions,

$$P(A \cap B) \cdot u(w) + \int_{A^c \cap B^c} u(w + \pi) \cdot f(\pi) d\pi > P(A^c \cap B^c) \cdot u(w) + \int_{A \cap B} u(w + \pi) \cdot f(\pi) d\pi, \tag{10}$$

$$P(A \cap B) \cdot u(\bar{w}) + \int_{A^c \cap B^c} u(\bar{w} + \pi) \cdot f(\pi) d\pi > P(A^c \cap B^c) \cdot u(\bar{w}) + \int_{A \cap B} u(\bar{w} + \pi) \cdot f(\pi) d\pi.$$

Define lotteries  $\mathcal{H}$  and  $\mathcal{E}$  as follows:  $\mathcal{H} = \{0, \Pi(\omega) \in A^c \cap B^c; \pi, \Pi(\omega) = \pi \in A \cap B; y, o.w.\}$  for some  $y \in \mathbb{R}$  and  $\mathcal{E} = \{0, \Pi(\omega) \in A \cap B; \pi, \Pi(\omega) = \pi \in A^c \cap B^c; y, o.w.\}$  for the same  $y \in \mathbb{R}$ .

Similarly, evaluation of lotteries  $\mathcal{H}$  and  $\mathcal{E}$  and the assumption that  $u$  obeys the one-switch rule yields the contradiction. As in Case 1,  $B(w, \mathcal{I}_A) = B(w, \mathcal{I}_B)$  does not conform with the one-switch behavior. Hence the conclusion follows in this case.

Case 4:  $d(w, A) = -1, d(w, B) = +1$ . Since  $B(w, \mathcal{I}_A) = B(w, \mathcal{I}_{A^c})$  and  $B(w, \mathcal{I}_B) = B(w, \mathcal{I}_{B^c})$ , this case is identical to Case 3. Hence the proposition follows.  $\square$

### 5. Illustrative example

In this example, we illustrate the effect of DM's utility function on information value. We consider a simple lottery rather than a continuous lottery for analytical convenience,

Outcome	$\omega_1$	$\omega_2$	$\omega_3$	$\omega_4$	$\omega_5$	$\omega_6$	$\omega_7$	$\omega_8$
Probability	0.25	0.10	0.055	0.045	0.25	0.05	0.10	0.15
Payoff, \$	30	-20	80	-40	-70	90	10	-5

Assume that the DM has an initial wealth level  $w$  of 130. First, we assume his preferences are represented by the one-switch utility function  $u(x) = x - be^{-cx}$  where  $b = 20$  and  $c = 0.001$ . We consider two events:  $A = \{\omega_1, \omega_2, \omega_4, \omega_5\}$  and  $B = \{\omega_3, \omega_4, \omega_5\}$ . If we check the expected utility conditions of Proposition 2, we observe that

$$V(130, \mathcal{I}_B) > V(130, \mathcal{I}_A) \quad \text{and} \quad V(0, \mathcal{I}_B) > V(0, \mathcal{I}_A).$$

In this case  $B(130, \mathcal{I}_B) \approx 10.25$  and  $B(130, \mathcal{I}_A) \approx \$9.14$ , so  $B(130, \mathcal{I}_B) > B(130, \mathcal{I}_A)$  as well.

If, on the other hand, the DM's preferences are represented by the  $n$ -switch utility function  $u(x) = ax^3 + bx^2 + cx$  where  $a = 10^{-7}$ ,  $b = 2 \times 10^{-7}$ , and  $c = 3 \times 10^{-7}$ , one can check that,

$$V(130, \mathcal{I}_B) > V(130, \mathcal{I}_A) \quad \text{and} \quad V(-53, \mathcal{I}_B) > V(-53, \mathcal{I}_A),$$

but  $B(130, \mathcal{I}_B) \approx \$2.95$  and  $B(130, \mathcal{I}_A) \approx 2.98$ , so

$$B(130, \mathcal{I}_B) < B(130, \mathcal{I}_A).$$

As such, Proposition 2 does not hold for  $n$ -switch utility functions, and thus one-switch condition is necessary. This example also illustrates one aspect of reversals that was encountered in many counterexamples that we generated. When preference reversals occur, we observed that the decisions with respect to both approaches were close (albeit in different directions). Hence, reversals generally occur when small changes in wealth level shift preferences with respect to two information bundles. This causes a small difference in the buying prices of two information bundles as well (i.e., approximately 3¢ in the above example). We expect under expected utility theory that most information preference reversals should be a result of such slight changes in evaluation of information bundles.

### 6. Conclusions

In this paper, we present sufficient conditions for agreement between the expected utility increase and the buying price approaches in ranking information bundles. For an unrestricted class of information bundles, we show that both approaches agree if the ranking using the expected utility approach does not switch on a bounded range of initial wealth levels. When we restrict our attention to one-switch utility functions, information bundles generated by single arbitrary events and the choice of a single lottery, we show that it suffices to impose conditions on ranking using the expected utility increase approach at only two wealth levels to obtain an agreement. We illustrate the role of the one-switch condition in Proposition 2 with an example. In particular, we show that the result in Proposition 2 does not necessarily hold for  $n$ -switch utility functions and hence one-switch is a necessary condition for the result to hold.

The implications of our results are twofold. First, expected utility theory predicts that risk averse DMs should not reverse their preferences toward risky lotteries as long as their lottery preferences remain robust to negative changes in wealth level. Preferences at higher wealth levels do not have any bearing on whether the DM will reverse his preferences at some initial wealth level  $w$ . Second, if DMs' preferences are well-behaved in that their ranking of two risky lotteries change only once as a function of wealth level, then it is sufficient to check the DM's preferences at two separate wealth levels. It would be interesting to see how these behavior predictions made by our results match the real life behavior through future experimental studies. Furthermore, experiments could determine whether, in practice, information preference reversals occur when slight changes in DMs' wealth level cause a shift in preferences.

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