Transmission spectra and the effective parameters for planar metamaterials with omega shaped metallic inclusions

Zhaofeng Li *, Koray Aydin, Ekmel Ozbay

Nanotechnology Research Center, Department of Physics, and Department of Electrical and Electronics Engineering, Bilkent University, Bilkent, 06800 Ankara, Turkey

ARTICLE INFO

ABSTRACT

Planar metamaterials with omega shaped metallic inclusions were studied experimentally and theoretically. Our results show that when the incidence is perpendicular to the plane of the omega structure, the omega medium acts effectively as an electric resonator metamaterial. The stop band of the omega medium is due to the negative part of the electric resonance of the omega structure. The transmission band of the composite metamaterial (CMM) that is based on the omega medium is due to the strong positive part of the electric resonance of the omega structure. Consequently, the transmission band of the CMM does not coincide with the stop band of the omega medium. Furthermore, the transmission band of the CMM is a band with positive refractive indices. Our experimental and numerical results are in good agreement.

© 2010 Elsevier B.V. All rights reserved.

1. Introduction

Metamaterials are artificially structured materials that can possess exotic and intriguing properties that are not available in natural materials. Motivated by Pendry's proposal of the construction of a perfect lens [1], many efforts have been made to design and optimize metamaterials with an effective negative refractive index [2–5]. One widely investigated design is a metamaterial that is composed of infinite metallic wires [6] and split ring resonators (SRR) [7]. In this composite metamaterial (CMM), the bianisotropy of the SRR structure is usually thought of as an undesired property and should be avoided. Several studies have been reported concerning the elimination of the magnetoelectric coupling effect [8–11]. However, it was recently proposed that bianisotropy may be useful for the construction of a metamaterial in certain situations [12]. An omega medium (Ω medium) is a typical bianisotropic medium that is a composite electromagnetic (EM) material with a proper combination of Ω-shaped metallic inclusions in a host dielectric medium [13]. The properties of metamaterials that are composed of omega structures are indeed different from the traditional metamaterials that are composed of infinite wires and SRRs [14,15].

On the other hand, along with the operation frequency in turn increasing from the microwave region into the optical region, traditional metamaterials that are composed of infinite wires and SRRs instill a great challenge for nano-fabrications. This is because the resonance of the effective permeability requires the light to

propagate parallel to the SRR plane and the magnetic field to oscillate perpendicularly to this plane, in which such a configuration is difficult to realize in the optical domain. Accordingly, planar metamaterials are proposed, among which cut-wire pair [16] structures and fishnet structures [17] are two typical examples. For the planar metamaterials, the k vector of the incident wave is perpendicular to the plane of the resonator, in which this configuration is suitable for planar fabrications. Moreover, apart from the applications of negative index materials, planar metamaterials are also proposed to be used as functional devices, e.g., filters as well as switching or modulation devices [18].

In our previous works [14], we reported our studies on the omega media with the incidence in the plane of the omega structures, in which the properties of the omega media are different from a conventional metamaterial because of its anisotropy. In the present report, we will study the characteristics of the omega media for the incidence being perpendicular to the plane of the omega structures. In this case, the omega media and its corresponding composite metamaterial act as planar metamaterials and will not exhibit bianisotropy. Besides, since the omega media and its corresponding composite metamaterial are designed to be asymmetric along the incident direction, a retrieval method [19] that is suitable for asymmetric metamaterials is used to extract the effective parameters for the omega media and its corresponding composite metamaterial.

2. Results of the experiment and simulation

Fig. 1a shows a unit cell of the omega medium under study. Fig. 1b is a unit cell of a composite metamaterial, which is a

---

* Corresponding author.
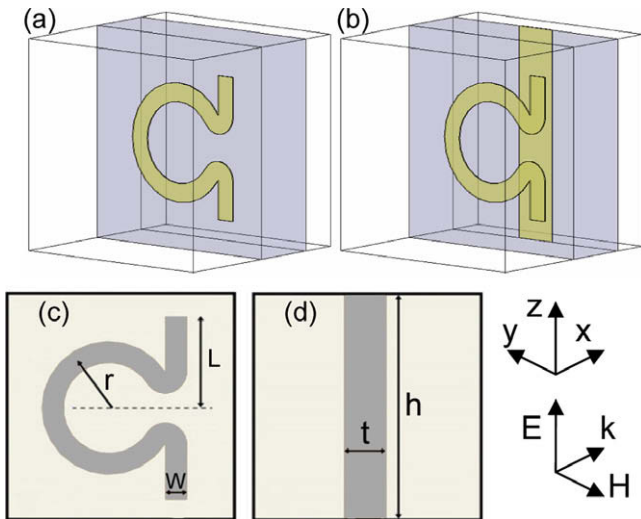 E-mail address: zhaofengli@bilkent.edu.tr (Z. Li).

Fig. 1. (a) and (b) The schematics of the unit cells for an omega medium and a CMM based on the omega medium. (c) The parameters for the omega structure. (d) The parameters for the infinite wire in the CMM medium.

combination of the omega structures and infinite wire structures. Fig. 1c shows the detailed structure of the omega structure with its parameters. The parameters in Fig. 1c are $r = 1.19$ mm, $W = 0.45$ mm, and $L = 1.8$ mm. The omega structures are made of copper on a FR4 printed circuit board (PCB). The thickness of the copper and FR4 are 30 μm and 1.6 mm, respectively. By arranging these omega structures periodically in three orthogonal directions, an omega medium can be obtained. In the experiments, we arrange Ω-resonator units periodically with 5, 40, and 30 unit cells in the x, y, and z directions, respectively. The lattice constants are $ax = ay = az = 5$ mm. While in the simulations, we use periodic boundary conditions in the y and z directions. We performed the numerical simulations by using a commercial software package (CST STUDIO microwave) that is based on the finite integration technique. In order to investigate the properties of CMM based on omega structures, a periodic arrangement of continuous thin copper wires were adopted to achieve negative permittivity at microwave frequencies. A unit cell of the continuous wire is shown in Fig. 1d. The wire is on the opposite side of the PCB. The thickness of the metal is 30 μm. The width of the thin wire is $t = 1.44$ mm, and the height is $h = 5$ mm, which is equal to the periodic constant in the y direction. In our experiments, the lattice constants and number of layers of continuous metallic wires are equal to that of periodic omega media in the x and z directions. While in the y direction, the wires are continuous and the total length of the wires is 150 mm. During the experiments, transmission measurements were performed in free space by using an HP 8510-C network analyzer. Microwave horn antennas are used as transmitters and receivers, in which the transmission through the samples was measured.

Fig. 2a and b shows the results of the experiment and simulation, respectively. Our simulation results are in good agreement with that of the experiment. Fig. 2a shows the transmission spectra for the incidence with the E field polarized in the z direction. One sees a stop band around 11 GHz. This stop band is reminiscent of the stop band when the incidence is in the plane of the omega structure [14], in which the bianisotropy of the omega medium is active. This means that the incident E field in the z direction has excited the resonance of the omega structure. However, this resonance does not couple to the incident H field and, therefore, the entire metamaterial here does not show bianisotropy. Fig. 2b shows the transmission spectra for the incidence with the E field
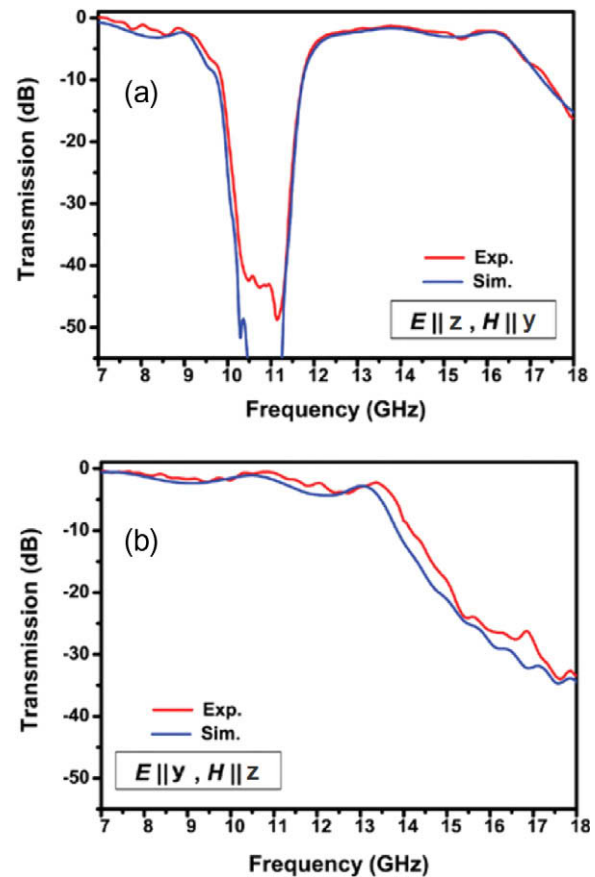

Fig. 2. The experimental and simulation results of transmission spectra for the omega medium. (a) The incidence with E field polarized in the z direction. (b) The incidence with E field polarized in the y direction.

polarized in the y direction. Different from the result of Fig. 2a, one does not see any stop band around 11 GHz. This means that the incident E field in the y direction does not excite the desired resonance of the omega structure in the frequency range under study. Consequently, in the following part of the present report, we will concentrate our investigation on the incidence with the E field polarized in the z direction.

Fig. 3a shows the experimental results of the transmission spectra for the omega medium, wire medium, and CMM. It is seen that for the CMM there is a transmission band that is below the plasmonic frequency of the wire medium. However, unlike the traditional metamaterials [2–5], this transmission band does not coincide with the stop band of the omega medium. Instead, this transmission band of CMM lies just below the stop band of the omega medium. This phenomenon is similar to the case when we were studying the bianisotropic characteristics of an omega medium [14], in which the incidence was in the plane of the omega structure. The above results imply that the transmission band of CMM should not be a band of the negative refractive index [14]. Fig. 3b shows the results of the simulation. It is clearly seen that our simulation results are in good agreement with the experimental results.

3. Retrieval of the effective parameters

In order to investigate the underlying physical mechanism of the above experimental and simulation results. We performed retrieval works for the two metamaterials that are shown in Fig. 1a and b. It should be noted that the unit cells shown in Fig. 1a and
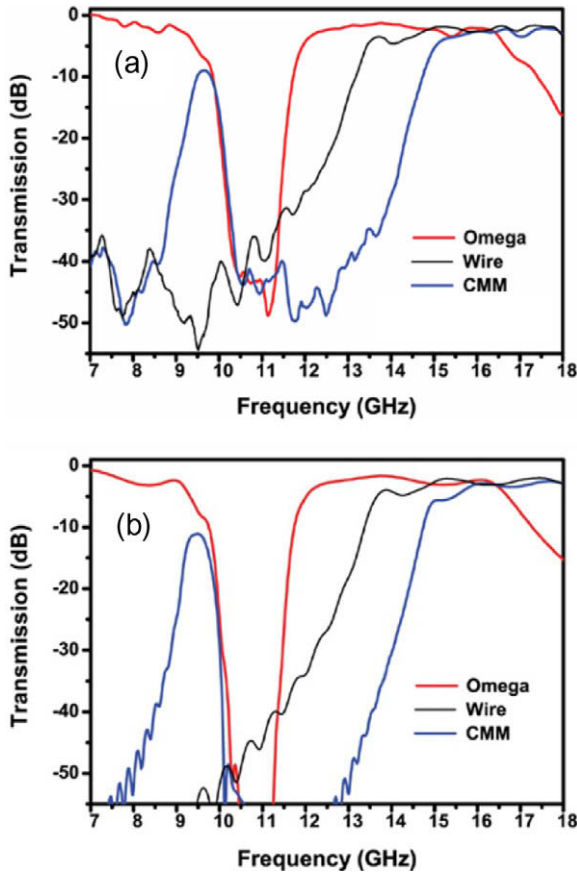
Fig. 3. The experimental (a) and simulation (b) results of transmission spectra for the omega medium, wire medium, and CMM medium. All of the results are for the incidence with the E field polarized in the z direction.
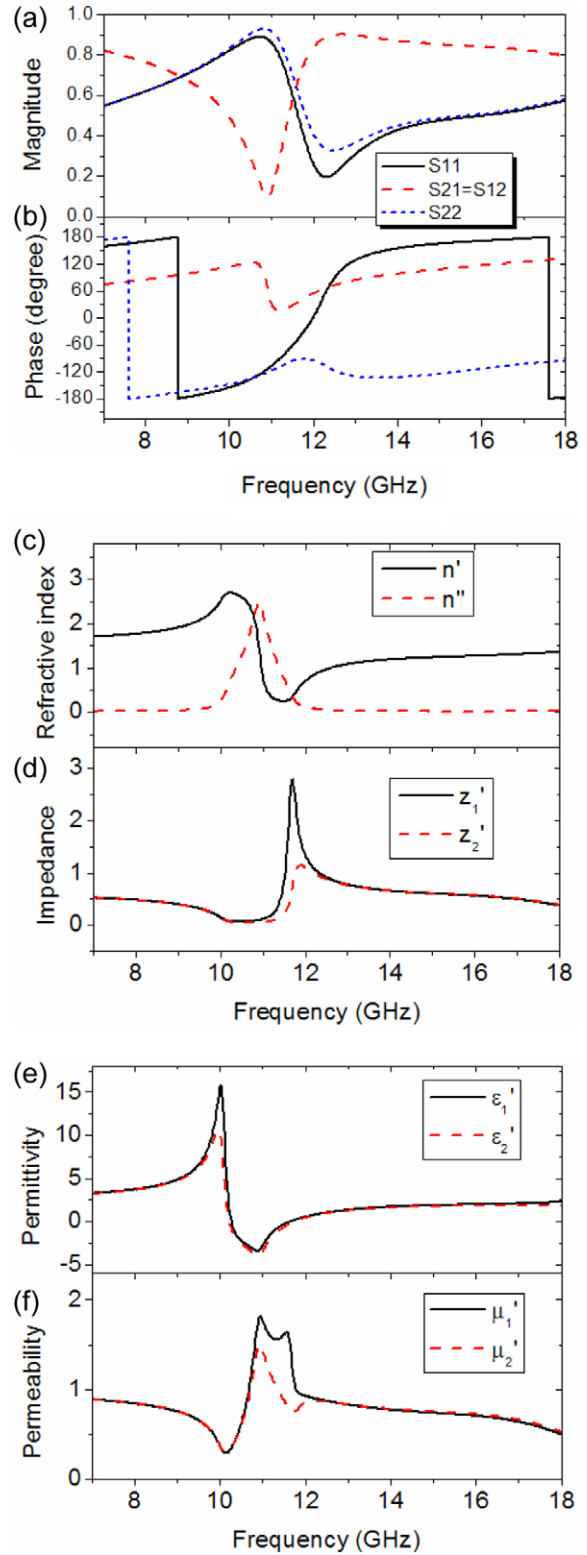

Fig. 4. The retrieval data for the omega medium. (a) and (b) The magnitude and phase data of the S parameters. (c–f) The retrieval results for the effective parameters of refractive index n, real parts of impedance z, real parts of permittivity ε, and real parts of permeability μ.

b are asymmetric along the x direction (the wave propagation direction) and, therefore, it is not suitable for retrieving the effective parameters by using the conventional retrieval method [20]. Instead, we used an improved retrieval method that is capable of extracting the effective parameters for asymmetric metamaterials [19]. In the retrieval procedure, we employed a single layer of omega medium (or CMM) along the x axis. Hence, the simulation setup coincides with a slab of omega medium (or CMM) that consists of single period layer. The effective parameters were then derived from the transmission and reflection coefficients of this single layer of omega medium (or CMM). In the retrieval results, we will obtain one refractive index n, two impedances (z_1 and z_2 for $+x$ and $-x$ directions, respectively), two permittivity (ε_1 and ε_2), and two permeability (μ_1 and μ_2).

Fig. 4a and b shows the magnitude and phase information of the scattering parameters (S parameters) for the omega medium shown in Fig. 1a. It can be seen that S21 is equal to S12, but S11 is not equal to S22. The results of the S parameters confirm the fact that the omega medium under study is an asymmetric medium, because S11 is equal to S22 for a conventionally symmetric metamaterial. Fig. 4c shows the retrieved results of the effective refractive index n, where $(.)'$ denotes the real part operator, and $(.)''$ denotes the imaginary part operator, respectively. It can be seen that the imaginary part of the refractive index has large positive values in the frequency range from 10 to 11.8 GHz, which exactly corresponds to the stop band of the omega medium (cf. Fig. 3). Fig. 4d shows the retrieved results for the real parts of the impedances. Due to the asymmetry of the unit cell, here we obtained two impedances. z_1 is the impedance for wave propagation in the $+x$ direction, and z_2 is the impedance for wave propagating in the

$-x$ direction. Except for the frequencies in the stop band, z_1' is equal to z_2'. This implies that in the frequencies away from the resonance, the omega medium is much like a homogeneous material [19]. However, for the frequencies close to the resonance, the
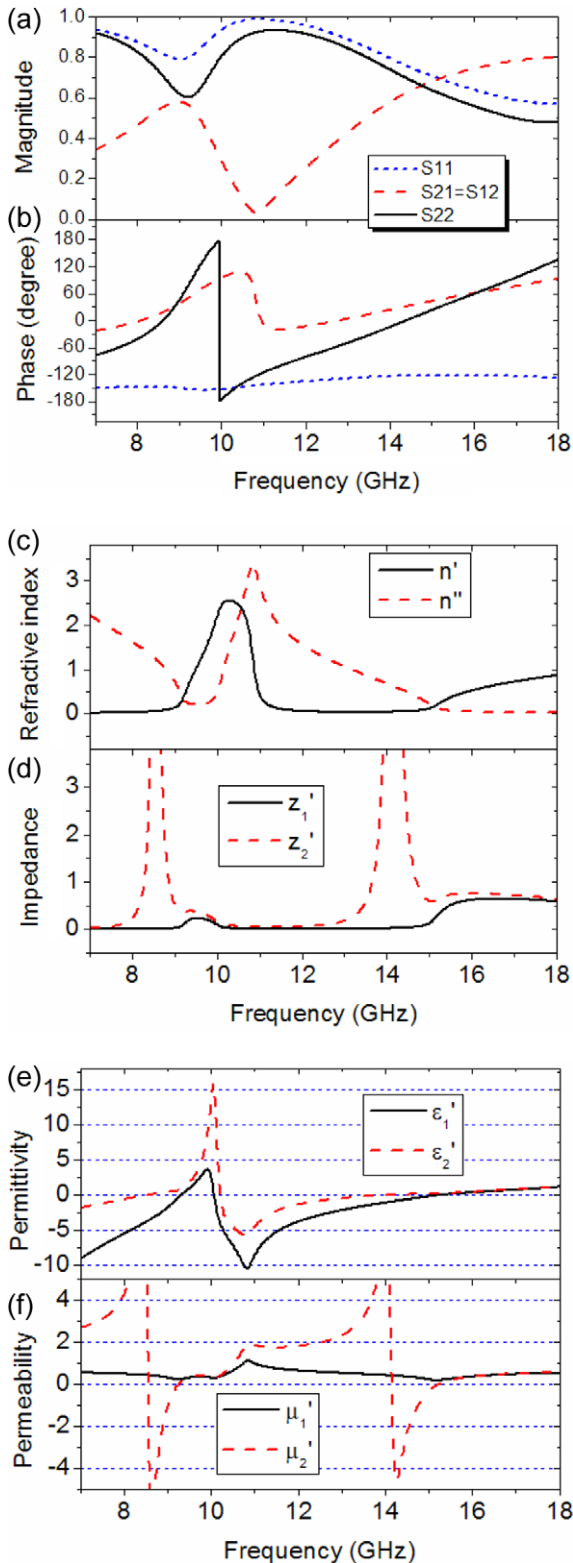
Fig. 5. The retrieval data for the CMM. (a) and (b) The magnitude and phase data of the S parameters. (c–f) The retrieval results for the effective parameters of refractive index n, real parts of impedance z, real parts of permittivity ε, and real parts of permeability μ.

omega medium shows asymmetric and inhomogeneous features. Fig. 4e and f shows the retrieval results for the effective parameters of permittivity (ε') and permeability (μ'). From these two results, one can clearly see that the stop band of the omega medium is

due to the negative value of the permittivity (ε') since the permeability (μ') is always positive in the entire frequency range. Obviously, based on the above retrieval results, one can conclude that when the k vector of the incident wave is perpendicular to the plane of the omega structure, the omega medium acts effectively as an electric resonator.

Now let us check the retrieval results for the CMM. Fig. 5a and b shows the magnitude and phase information of the S parameters for the CMM. It can be seen that S21 is equal to S12, but S11 is still not equal to S22 because the unit cell of the CMM is asymmetric along the wave propagation direction. Fig. 4c shows the retrieved results of effective refractive index n. It is seen that in the frequency range from 9.2 to 10 GHz, the real part of the refractive index n' is positive, at the same time the imaginary part of the refractive index n'' is nearly zero. This frequency range exactly corresponds to the transmission band of the CMM. Fig. 4d shows the retrieval results of the real parts for the effective impedances z_1' and z_2'. Corresponding to the transmission band of CMM (from 9.2 to 10 GHz), z_1' and z_2' have positive but different values. Fig. 4e and f shows the retrieved effective parameters of real parts of the permittivity (ε') and the permeability (μ'). From the data of permittivity (ε'), one sees that corresponding to the frequency range of the transmission band of CMM, ε_1' and ε_2' have positive but different values. The difference between ε_1' and ε_2' comes from the asymmetry of the CMM unit cell. Meanwhile, μ_1' and μ_2' have positive values in the same frequency range. Nonetheless, the results of μ_2' show some abnormal values in other two frequency ranges, i.e., the frequency range from 8.6 to 9.2 GHz, and the frequency range from 14.15 to 15.2 GHz. In these two frequency ranges, the value of μ_2' is negative. This phenomenon is counterintuitive because no element in the CMM is resonant with the H field of the incidence. On the other hand, the value of ε_1' is positive in the same two frequency ranges. This means that these two frequency ranges are within stop bands for an EM wave propagating in the $-x$ direction. Moreover, in these two frequency ranges, the two impedances z_1' and z_2' have quite different values. This means that the CMM is not suitable to be considered as a homogeneous material within there two frequency ranges [20]. Consequently, the retrieved effective parameters of permittivity and permeability have very weak physical meanings for the two frequency ranges. Now, one can conclude that the transmission band of the CMM is a band of positive refraction. Although this transmission band is below the plasmonic frequency of the infinite wire medium, the electric resonance of the omega structure is so strong that the combined medium still has a positive value of permittivity around the resonance frequency range.

After comparing the retrieval results of the omega medium and CMM, one finds that the stop band of the omega medium is due to the negative part of the electric resonance of the omega structure, while the transmission band of the CMM is due to the positive part of the resonance. Consequently, the transmission band of the CMM does not coincide with the stop band of the omega medium.

4. Conclusions

Planar metamaterial with omega structure inclusions are studied experimentally and numerically. When the incidence is perpendicular to the plane of the omega structure, the omega medium acts effectively as an electric resonator metamaterial. The stop band of the omega medium is due to the negative part of the electric resonance of the omega structure. For the CMM based on the omega structure, a transmission band can be observed, which is located beside the stop band of the omega medium. Our retrieval results show that this transmission band is a band with positive refractive indices. The transmission band of

CMM is due to the strong positive part of the electric resonance of the omega structure. Consequently, the transmission band of the CMM does not coincide with the stop band of the omega medium naturally.

Acknowledgments

References

[1] J.B. Pendry, Phys. Rev. Lett. 85 (2000) 3966.
[2] R.A. Shelby, D.R. Smith, S. Schultz, Science 292 (2001) 77.
[3] A.A. Houck, J.B. Brock, I.L. Chuang, Phys. Rev. Lett. 90 (2003) 137401.
[4] K. Aydin, K. Guven, C.M. Soukoulis, E. Ozbay, Appl. Phys. Lett. 86 (2005) 124102.
[5] K. Aydin, E. Ozbay, J. Opt. Soc. Am. B 23 (2006) 415.
[6] J.B. Pendry, A.J. Holden, W.J. Stewart, I. Youngs, J. Phys. Condens. Matter. 10 (1998) 4785.
[7] J.B. Pendry, A.J. Holden, D.J. Robbins, W.J. Stewart, IEEE Trans. Microw. Theory Technol. 47 (1999) 2057.
[8] C.R. Simovski, S. He, Phys. Lett. A 311 (2003) 254.
[9] J. Huangfu, L. Ran, H. Chen, X. Zhang, K. Chen, T.M. Grzegorcyzk, J.A. Kong, Appl. Phys. Lett. 84 (2004) 1537.
[10] R. Marques, F. Medina, R. Rafii-El-Idrissi, Phys. Rev. B 65 (2002) 144440.
[11] R. Marques, F. Mesa, J. Martel, F. Medina, IEEE Trans. Anten. Propag. 51 (2003) 2572.
[12] S.A. Tretyakov, C.R. Simovski, M. Hudlicka, Phys. Rev. B 75 (2007) 153104.
[13] M.M.I. Saadoun, N. Engheta, Microw. Opt. Technol. Lett. 5 (1992) 184.
[14] K. Aydin, Z. Li, M. Hudlicka, S.A. Tretyakov, E. Ozbay, New J. Phys. 9 (2007) 326.
[15] K. Aydin, Z. Li, S. Bilge, E. Ozbay, Photon. Nanostr. 6 (2008) 116.
[16] G. Dolling, C. Enkrich, M. Wegener, J.F. Zhou, C.M. Soukoulis, S. Linden, Opt. Lett. 30 (2005) 3198.
[17] M. Kafesaki, I. Tsiapa, N. Katsarakis, T. Koschny, C.M. Soukoulis, E.N. Economou, Phys. Rev. B 75 (2007) 235114.
[18] H. Chen, J.F. O'Hara, A.J. Taylor, R.D. Averitt, C. Highstrete, M. Lee, W.J. Padilla, Opt. Express 15 (2007) 1084.
[19] D.R. Smith, D.C. Vier, Th. Koschny, C.M. Soukoulis, Phys. Rev. B 71 (2005) 036617.
[20] D.R. Smith, S. Schultz, P. Markos, C.M. Soukoulis, Phys. Rev. B 65 (2002) 195104.