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Overnight borrowing, interest rates and extreme value theory

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Abstract

We examine the dynamics of extreme values of overnight borrowing rates in an inter-bank money market before a financial crisis during which overnight borrowing rates rocketed up to (simple annual) 4000 percent. It is shown that the generalized Pareto distribution fits well to the extreme values of the interest rate distribution. We also provide predictions of extreme overnight borrowing rates using pre-crisis data. The examination of tails (extreme values) provides answers to such issues as to what are the extreme movements to be expected in financial markets; is there a possibility for even larger movements and, are there theoretical processes that can model the type of fat-tails in the observed data? The answers to such questions are essential for proper management of financial exposures and laying ground for regulations.

JEL classification: G0; G1; C1

Keywords: Financial crises; Risk management; Extreme value theory; Overnight rate; Federal funds rate; Turkish economy

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1. Introduction

The Turkish government began implementing a far reaching restructuring and reform program after the general elections in April 1999. The aim of the program was to reduce inflation from its 60–70 percent level per year to single digits by the end of the year 2002. The program gained further momentum after the country made a stand-by arrangement with the International Monetary Fund (IMF) in December 1999 and announced the technical aspects of the disinflation program. The main tool of the disinflation program was the adoption of a *tablita* with an exit, that is, the percent change in the value of the Turkish Lira (TRL) against a basket of foreign currencies was fixed beginning January 2000. Meanwhile, the stand-by arrangement determined a ceiling for the net domestic assets of the Central Bank. Accordingly, the Central Bank was able to create TRL liquidity only through net foreign capital inflows. The structure of the stabilization program implied that the interest rate would be market determined in line with the exchange rate depreciation and capital flows, and the volatility of interest rates would be higher than the volatility before the program. A close inspection of Fig. 1 reveals that this was the case.

It was well-known by the market participants that one of the commercial banks (Demirbank) had an extremely risky position during the year 2000. The bank (with a

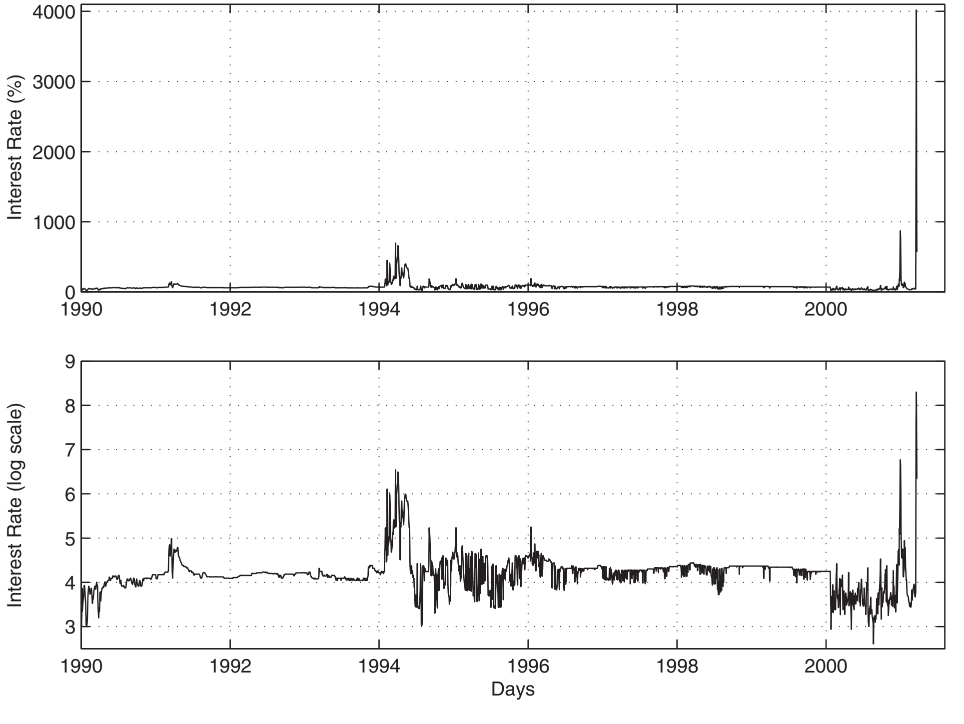

Fig. 1. Daily average overnight interest rates in Turkey (annualized simple) including the February 2001 crisis. The daily rates are weighted average of intraday rates. The sample period is from January 2, 1990 to February 23, 2001 with 2806 observations. *Source*: Central Bank of Turkey.

paid capital of USD 300 million) was funding its estimated USD 7.5 billion government securities portfolio mostly from the money market with very short-term obligations. On Monday, November 20, 2000, Demirbank was not able to borrow from the money market and the Central Bank stepped in to cover Demirbank's position. In the following days, market makers in the government securities market stopped posting prices and Demirbank was not able to liquidate its positions. As a result, overnight interest rates started to increase and there was a rapid capital outflow, starting on Wednesday, November 22.

The heavy capital outflow and continuous decrease in the Central Bank reserves caused further liquidity pressure in the market. The Central Bank started to provide liquidity by violating the rule set by the stand-by agreement. However, the injected liquidity bounced back to the Central Bank in the form of additional demand for foreign currency. Therefore, the Central Bank stopped providing liquidity on Thursday, November 30, 2000. Immediately, the overnight interest rate reached its peak at (simple annual) 873 percent on Friday, December 1, 2000. The IMF rushed in an "emergency team" to discuss an emergency loan. On Tuesday, December 5, Turkish authorities announced a USD 7.5 billion rescue package with the IMF. The following morning before the markets opened, Demirbank became the 11th Turkish bank to be taken under control of the Saving Deposits Insurance Fund.

After the IMF's backing of the country in December 2000, the Central Bank reserves returned to its pre-crisis level. Interest rates decreased, albeit stabilizing at a higher level than the pre-crisis average. Nevertheless, there were concerns about the Treasury's ability to borrow from the domestic market at favorable terms. A scheduled domestic debt auction of the Treasury on February 20, 2001, the day before the maturing USD 7 billion domestic debt, was aimed at borrowing approximately USD 5 billion (around 10 percent of the total domestic debt). Suddenly, the day before the auction, Turkish Prime Minister Bulent Ecevit stormed out of a key meeting of top political and military leaders stating that a "dispute" had arisen between himself and the country's president. The news hit the market and the stock market plunged 18 percent in 1 day. The same day, the Central Bank sold USD 7.5 billion (approximately one-third of the total official reserves) and refused to provide TRL liquidity to the system. The daily average overnight interest rates shot up to (simple annual) 2000 percent on February 20, and 4000 percent on February 21. The government responded by dropping its exchange-rate control and the TRL/USD exchange rate went up 40 percent in 1 week.

The Turkish financial crisis in February 2001 is a case study for extreme risks and risk management practices. In recent years, the problem of extreme risks in financial markets has become topical following the crises in the Asian and Russian markets, and the unexpected big losses of investment banks such as Barings and Daiwa. The value-at-risk (VaR) measures with Gaussian-type innovations failed to cope with these turmoils mainly because the evaluation of extreme risks in the Gaussian model is directly related to the variance. However, the underlying distribution may not even have a finite variance.

We investigate the dynamics of the extreme values of overnight borrowing rates in the inter-bank money market for the TRL *before* the financial crisis of February 2001.

It is shown that the extreme value theory (EVT) is a useful tool in assessing the financial fragility. Estimated parameters imply that the Turkish interest rates have unbounded support and the interest rates may come from a distribution with infinite variance. We also provide estimates of overnight borrowing rates at 0.999 percentile (1 day in every 4 years) on pre-crisis data. The results indicate that every 4 years one can expect to see a day with overnight interest rates as high as 1000 percent (simple annual) in Turkey. In other words, the extraordinary levels observed during the February 2001 crisis were in the nature of the economy before they actually materialized.

We also show that such extraordinary levels are a statistical impossibility for the United States. Estimated parameters of the daily US Effective Federal Funds Rate distribution imply that the distribution of US interest rates has a bounded support and all moments exist. As a result, estimated interest rates even at the 0.9999th quantile (1 day in every 10,000 days, approximately 30 years) for the US are less than the historical high of 22.4 percent. We postulate that the difference between the US and Turkey stems from the fact that the two countries have a different degree of development of the financial markets and institutions.

This paper is structured as follows. In Section 2, the methodological framework is introduced. Section 3 reports the descriptive statistics and estimation results for the Turkish and the US overnight interest rates. We conclude afterwards.

2. Extreme value theory

Extreme value theory is a powerful and fairly robust framework to study the tail behavior of a distribution. Embrechts et al. (1997) is a comprehensive source of the EVT for the finance and insurance literature. Reiss and Thomas (1997) and Beirlant et al. (1996) also provide extensive coverage.[1] In the following section, we present the parametric framework for our study.

2.1. Fisher–Tippett theorem

The theorem of Fisher and Tippett (1928) is the core of the EVT. The theory deals with the convergence of maxima. Suppose that $X_1, X_2, \ldots, X_n$ is a sequence of independently and identically distributed random variables from an unknown distribution function $F(x)$. Denote the maximum of the first $m<n$ observations of X by $M_m = \max(X_1, X_2, \ldots, X_n)$. Given a sequence of $a_m>0$ and b_m such that $(M_m - b_m)/a_m$, the sequence of normalized maxima converges in distribution to the following so-called Generalized Extreme Value (GEV) distribution:

$$H_\xi(x) = \begin{cases} e^{-(1+\xi x)^{-1/\xi}} & \text{if } \xi \neq 0, \\ e^{-e^{-x}} & \text{if } \xi = 0, \end{cases} \tag{1}$$

[1]There have been a limited number of extreme value studies in finance literature. See Danielsson and de Vries (1997), Embrechts (2000) and Gençay and Selçuk (2004) and references therein for EVT applications in finance.

where X is such that $1+\xi X>0$ and ξ is the shape parameter. When $\xi>0$, the distribution is known as the Fréchet distribution and it has a fat-tail. The larger the shape parameter, the more fat-tailed the distribution. If $\xi<0$, the distribution is known as the *Weibull* distribution. Finally, if $\xi=0$, it is the *Gumbel* distribution.[2]

In general, we are not only interested in the maxima of observations, but also in the behavior of large observations which exceed a high threshold. Given a high threshold u, the distribution of excess values of X over threshold u is defined by

$$F_u(y) = \mathrm{P}\{X-u \leqslant y | X>u\} = \frac{F(y+u)-F(u)}{1-F(u)} \tag{2}$$

which represents the probability that the value of X exceeds the threshold u by at most an amount $y \geqslant 0$ *given* that X exceeds over the threshold u. A theorem by Balkema and de Haan (1974) and Pickands (1975) shows that for a sufficiently high threshold u, the distribution function of the excess can be approximated by the generalized Pareto distribution (GPD), i.e., the excess distribution $F_u(y)$ converges to the GPD below as the threshold gets large:

$$G_{\xi,\beta}(x) = \begin{cases} 1-\left(1+\xi\dfrac{x}{\beta}\right)^{-1/\xi} & \text{if } \xi \neq 0, \\ 1-\mathrm{e}^{-x/\beta} & \text{if } \xi = 0, \end{cases} \tag{3}$$

where ξ is the shape parameter. The GPD embeds a number of other distributions. When $\xi>0$, it takes the form of the ordinary Pareto distribution and $\mathrm{E}[X^k]$ is infinite for $k \geqslant 1/\xi$. For the security returns or high frequency foreign exchange returns, the estimates of ξ are usually less than 0.5, implying that the returns have finite variance. See, for instance, Longin (1996) and Dacorogna et al. (2001).

The importance of the Balkema and de Haan (1974) and Pickands (1975) results is that the distribution of excesses may be approximated by the GPD by estimating ξ and β as a function of a high threshold u. The parameters of the GPD can be estimated with various methods such as the method of probability weighted moments or the maximum likelihood method. For $\xi > -0.5$ which corresponds to heavy-tails, Hosking and Wallis (1987) presents evidence that maximum likelihood regularity conditions are fulfilled and the maximum likelihood estimates are asymptotically normally distributed. Therefore, the approximate standard errors for the estimators of β and ξ can be obtained through maximum likelihood estimation. Notice that even if the identical and independent distribution condition fails, the EVT may still be an accurate approximation of the actual distribution function of maxima (Reiss and Thomas, 1997, pp. 10 and 172). If there is dependency in the data, the estimation result should be modified with an estimated extremal index. In this case, the quantile estimate would be different than the one estimated with weak dependency or independency assumption. We refer the reader to Longin (2000), Embrechts et al. (1997, Chapter 8) and Reiss and Thomas (1997,

[2]Extensive coverage can be found in Gumbel (1958).

Chapter 6) for further discussion on dependency, extremal index and its implications in practice.

2.2. The tail estimation

Since $F_u(y)$ in Eq. (2) converges to the GPD for sufficiently large u, we have the following representation:

$$F(x) \approx [1 - F(u)]G_{\xi,\beta,u}(x-u) + F(u), \tag{4}$$

where $x = y + u$ and $y \geqslant 0$. The last term on the right-hand side can be determined by the empirical estimator $(n - N_u)/n$ where N_u is the number of exceedances of u and n is the sample size. The tail estimator, therefore, is given by

$$\widehat{F(x)} = 1 - \frac{N_u}{n}\left(1 + \hat{\xi}\,\frac{x-u}{\hat{\beta}}\right)^{-1/\hat{\xi}}. \tag{5}$$

For a given probability level q, a percentile $(\hat{x}_q)$ at the tail is estimated by inverting the tail estimator in (5),

$$\hat{x}_q = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}(1-q)\right)^{-\hat{\xi}} - 1\right). \tag{6}$$

In statistics, this is the quantile estimation and it can be utilized in VaR estimations in finance applications.

2.3. Preliminary data analysis

In statistics, a QQ-plot (quantile–quantile plot) is a convenient visual tool to examine whether a sample comes from a specific distribution. Specifically, the quantiles of a hypothesized distribution are plotted against the quantiles of an empirical distribution. If the sample comes from the hypothesized distribution, the QQ-plot is linear. In EVT and its applications, the QQ-plot is typically plotted against the Gumbel distribution, i.e., a distribution with a thin-sized tail. If the data come from a distribution in the maximum domain of attraction of the Gumbel, the points on the graph would lie along a positively sloped straight line. If there is a concave presence, this would indicate a fat-tailed distribution, whereas a convex departure is an indication of short-tailed distribution.

A second tool is the sample mean excess function (MEF) which is the sum of the excesses over the threshold u divided by the number of data points which exceed the threshold u. It is an estimate of the MEF which describes the expected overshoot of a threshold once an exceedance occurs. If the empirical MEF is a positively sloped straight line above a certain threshold u, it is an indication that the data follows the GPD with a positive shape parameter ξ. On the other hand, if the data comes from a distribution which is in the maximum domain of attraction of the Gumbel, one would observe a horizontal MEF while short-tailed data would result

Table 1
Descriptive statistics of the (simple annual) overnight interest rates before and after the February 2001 crisis

	n	Mean	Std.	Ku.	Sk.	Med.	Min.	Max.	Low	High
Pre-crisis	2801	73.0	49.4	82.62	7.5	67.7	13.6	873.1	34.0	107.7
Full sample	2806	75.7	99.6	943.2	26.4	67.7	13.6	4018.6	34.4	109.6

n: Sample size; Mean: Sample mean; Std: Standard deviation; Ku: Kurtosis; Sk: Skewness; Min: Minimum observed rate; Max: Maximum observed rate; Low: Rate corresponding to the 5th percentile; High: Rate corresponding to the 95th percentile. The full sample period is from January 2, 1990 to February 23, 2001 with 2806 observations. Pre-crisis period excludes the last five business days. *Source*: Central Bank of Turkey. Overnight interest rates are daily weighted average of inter-bank interest rates as calculated by the Central Bank of Turkey

in a negative slope. See Reiss and Thomas (1997, Chapter 2) and Embrechts et al. (1997, Chapter 6) for these and other diagnostic tools and exploratory data analysis for extremes.

3. Empirical results

The data source for the daily overnight interest rates (simple annual) is the Central Bank of Turkey. The daily rates are calculated by the Central Bank as a weighted average of intraday transactions in the inter-bank money market.[3] The descriptive statistics of daily average simple annual overnight interest rates before and after the February 2001 crisis are given in Table 1. The full sample period is January 2, 1990–February 23, 2001 with 2806 observations. It includes all available daily interest rate data from the inter-bank money market in Turkey. In this paper, all calculations and predictions are carried out with simple annual interest rates.

In Table 1, the sample means of 73.0 and 75.7 percent correspond to compound annual interest rates of approximately 107 and 113 percent, respectively. Although it is high by the standards of a developed market, it reflects the high inflation levels and associated risk in the economy. The annual average percent increase in consumer prices (inflation) during the sample period was 75.4 percent, implying an average annual real interest rate of 18–21 percent. Both kurtosis and skewness estimates show that the interest rates are far from being normally distributed. The estimated kurtosis 82.62 before the crisis shows that the interest rate distribution has a fat-tail. The estimated skewness of 7.5 before the crisis points out that the distribution is skewed. After the crisis, both skewness (26.4) and kurtosis (943.2) estimates indicate that fat-tailness and skewness of the distribution have substantially increased.

[3]The observed maximum *intraday* interest rate is (simple annual) 6200 percent. This means borrowing TRL at an interest rate of 17.2 percent for 1 day ($17.2 \times 360 \approx 6200$). We use *weighted* average of intraday rates as provided by the Central Bank.

3.1. Tail estimation of excess interest rates

It is necessary to determine a threshold interest rate to estimate the parameters of the GPD. As presented in Section 2.4, the QQ-plot and the MEF are two empirical tools for this task. In a QQ-plot, the quantiles of the empirical distribution function on the x-axis are plotted against the quantiles of the Gumbel distribution function on the y-axis. The points should lie approximately along a straight line if the data is from the assumed Gumbel distribution. Since the Gumbel distribution has a medium-sized tail, a concave relationship between the quantiles of the empirical and the Gumbel distributions indicate a heavy-tailed distribution for the time series under study. The top panel of Fig. 2 indicates that the sample points start deviating from linear behavior at around 80 percent and form a concave pattern.

The sample MEF is another diagnostic tool used to determine a threshold. If the points of the MEF exhibit an upward trend, this indicates heavy-tailed behavior. Short-tailed data exhibit negatively sloped behavior whereas an exponential distribution has a flat MEF. The bottom panel of Fig. 2 demonstrates that the sample MEF is approximately linear and positively sloped after the 80 percent interest rate threshold. The examination of both panels in Fig. 2 indicate that the approximate threshold value corresponds to 80 percent.

For a given threshold level, a tail estimation involves the estimation of the parameters of the GPD. The maximum likelihood estimation is a convenient way to obtain standard errors of the parameter estimates.[4] For the threshold $u = 80$, the threshold exceedances are 389 data points which constitute the upper 13.9 percent tail of the original sample of 2801 sample points. Notice that an 80 percent overnight interest rate (simple annual) is equivalent to an annualized compound interest rate of 122 percent. During the sample period, the average annual percent increase in TRL/USD exchange rate is 75 percent. Therefore, the threshold implies a 23 percent annualized dollar interest rate.

The maximum likelihood estimates of ξ and β at threshold $u = 80$ are (standard errors are in parenthesis) 0.73 (0.086) and 22 (2.03), respectively. The estimated parameters are statistically significant at the 1 percent level. As discussed in Section 2.1, when $\xi > 0$, the distribution is known as the Fréchet distribution and it has a fat-tail with tail index $1/\xi$. The larger the shape parameter, the more fat-tailed the distribution. The value of $\hat{\xi} = 0.73$ indicates that the overnight interest rate series may come from a distribution with infinite variance.

The estimated GPD model with a threshold of 80 percent interest rate ($u = 80$) before the February 2001 crisis is presented in the top panel of Fig. 3. The estimated model is plotted in a solid curve while the interest rates above the threshold are shown in circles (on logarithmic scale). The estimated model successfully captures

[4]EVIM (Extreme Value Analysis in Matlab, Gençay et al. (2001)) is a comprehensive toolbox for EVT analysis. This toolbox is available at http://www.bilkent.edu.tr/~faruk. EVIS (Extreme Values in S-Plus) is another free suit of S-Plus functions for EVT analysis. The package may be obtained from http://www.math.ethz.ch/~mcneil. We used both packages in our calculations and estimations.

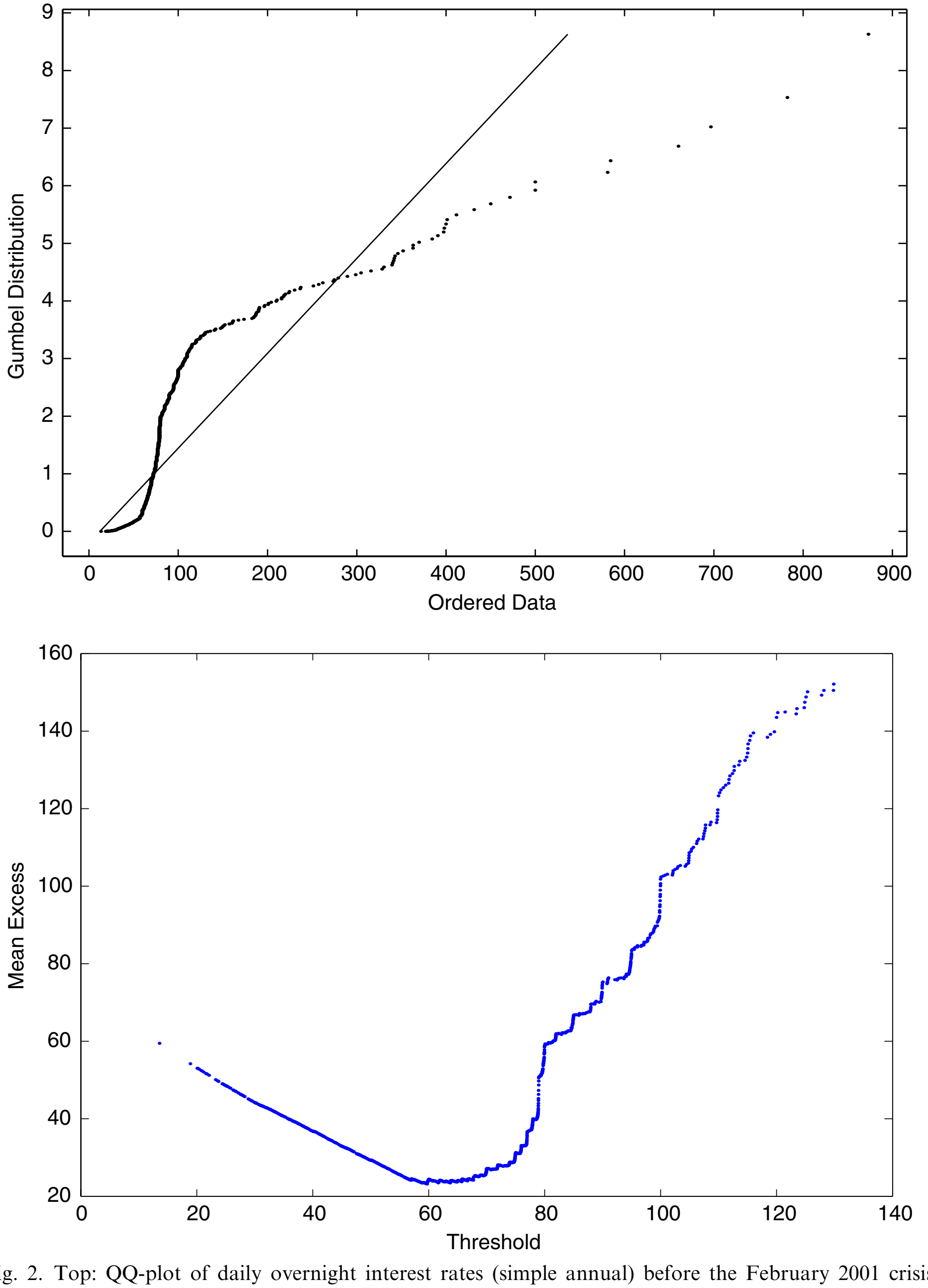

Fig. 2. Top: QQ-plot of daily overnight interest rates (simple annual) before the February 2001 crisis against the Gumble distribution. The quantiles of the empirical distribution function on the x-axis are plotted against the quantiles of the Gumbel distribution function on the y-axis. The points should lie along the straight line if the data is from the assumed Gumbel distribution. A concave presence indicates a fat-tailed distribution. *Source*: Central Bank of Turkey. Bottom: Sample mean excesses of daily overnight interest rates (simple annual) before the February 2001 crisis over increasing thresholds. A straight line with positive slope above a given threshold u is a sign of the GPD in tail. Notice that the plot is approximately linear and positively sloped after 80 percent, indicating the Pareto behavior in the tail.

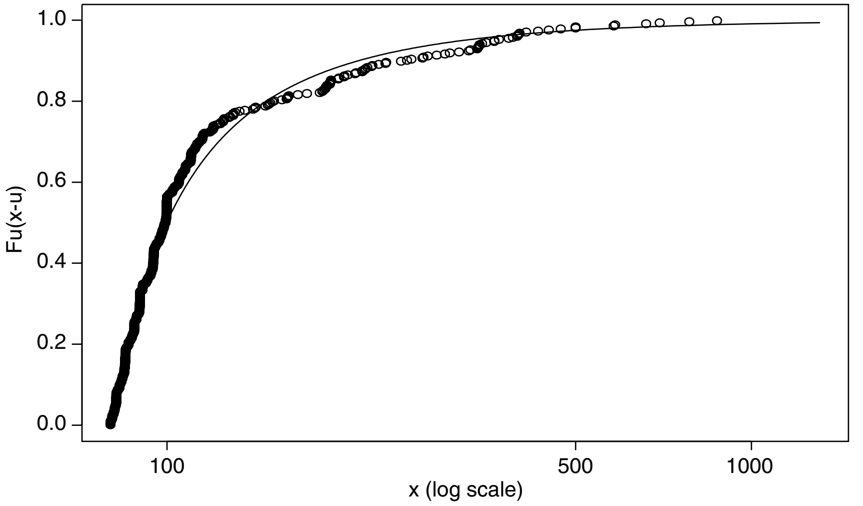

Fig. 3. Top: The estimated GPD model for the excess interest rates before the February 2001 crisis. The estimated model is plotted as a curve while the actual daily overnight interest rates (simple annual) above the threshold are shown in circles (in logarithmic scale). The threshold is 80 percent interest rate, $u = 80$. The estimated parameters are $\xi = 0.73$ and $\hat{\beta} = 22$. The number of exceedances is 389. The sample period is from January 2, 1990 to February 16, 2001 which ends 1 week before the February 2001 crisis. Bottom: Estimates of shape parameter, ξ, at different thresholds (upper x-axis) or alternatively with different number of exceedances (lower x-axis) before the February 2001 crisis. The parameter estimates are from 100 GPD models with thresholds ranging between 80 percent and 100 percent interest rates (simple annual). The sample period is from January 2, 1990 to February 16, 2001. The sample period ends at the week before the February 2001 crisis.

the underlying extreme values, and the tail behavior of excess interest rates is successfully approximated by a Fréchet-type distribution.

To examine the robustness of the results to the choice of the threshold value, the maximum likelihood estimation of the shape parameter, ξ, is carried out for GPD models with a range of thresholds between 80 and 100 percent. The estimates of ξ, as well as their asymptotic confidence intervals, are reported in the bottom panel of Fig. 3. On the lower x-axis the number of data points exceeding the threshold is plotted and on the upper x-axis the threshold is located. The estimates of the shape parameter, ξ, are plotted on the y-axis. The results indicate that the shape parameter is fairly stable within the range of 80–90 percent interest rate. Of course, there are fewer observations at higher thresholds and less statistical precision. This is reflected in the wider confidence intervals.

3.2. Point predictions at the tails

It is possible to estimate a percentile value at the tail from the estimated parameters of the GPD. In Fig. 4, the tail estimates are reported with the pre-February 2001 crisis data. Since EVT is used to calculate unconditional probabilities, we do not predict the timing of the crisis but report that certain extreme interest rate levels are expected to happen every so often.

On a log–log plot as in Fig. 4, GPD becomes linear with negative $(1/\xi)$ slope and it is easier to interpret results visually. In this figure, the left y-axis indicates the tail probabilities, $1 - F(x)$. The vertical dotted line starting from the x-axis is the estimated overnight interest rate which corresponds to the 1 percent tail probability on the y-axis. This value is obtained from the intersection of the 1 percent tail probability with the estimated tail and traced down to the x-axis. Therefore, the 99 percent quantile (1 day in every 100 days) interest rate corresponds to 257 percent.

The dotted curve is the confidence interval calculated by the profile likelihood method.[5] The 95 percent confidence level corresponds to the intersection of the horizontal dotted line starting from the right y-axis. The range of the 95 percent confidence interval is 221 percent for the lower end and 313 percent for the higher end.

Fig. 4 has a straightforward message. When the interest rate corresponding to the 99 percent quantile (1 day in every 100 days) is evaluated at the 95 percent confidence level, the maximum simple annual interest rate would go as high as 313 percent. This is a substantially high overnight interest rate but it is not in the order of ten thousands or millions percent compound interest rate as it was observed during the February crisis. Let us now go further in the tail to evaluate further extreme possibilities and investigate the 99.9 percent quantile estimate (1 day in every 4 years). This is presented in Fig. 5 where the interest rate estimates at the 99.9 percent quantile (1 day in every 4 years) are plotted as a function of the threshold (upper x-axis) or alternatively as a function of exceedances (lower x-axis) with the

[5] The usual Wald standard errors are computed from the inversion of the Hessian of the log-likelihood function. Confidence limits from the inversion of the likelihood ratio statistic are called profile likelihood confidence limits.

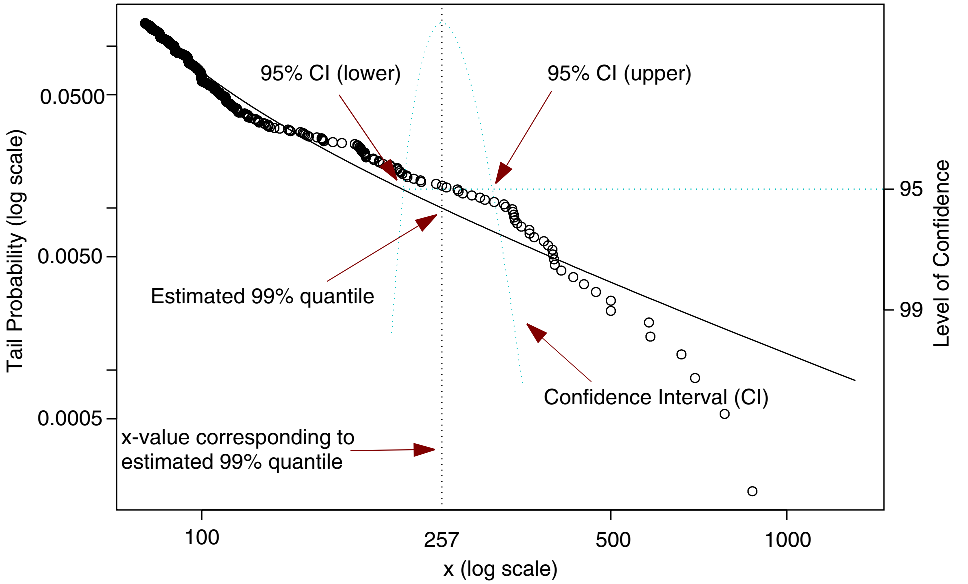

Fig. 4. Tail estimate for the excess interest rates before the February 2001 crisis. The left y-axis indicates the tail probabilities $1 - F(x)$. The vertical dotted line from x-axis is the estimated interest rate (at 257 percent) and intersects with the 1 percent tail probability on the left y-axis. Therefore, it is the estimated interest rate at the 99 percent quantile. The dotted curve is the confidence interval around this estimate, calculated by the profile likelihood method. It intersects with the 95 percent horizontal confidence line from the right y-axis and the corresponding 95 percent confidence interval is $(221, 313)$ in percent. Notice that the confidence interval becomes larger when the confidence level is increased (a down movement in the level of confidence on the right y-axis). The sample period is from January 2, 1990 to February 16, 2001 which ends 1 week before the February 2001 crisis.

pre-February crisis data. The estimated interest rate settles above 1000 percent (simple annual) for most thresholds although variations are higher at higher thresholds. The upper confidence interval indicates that before the crisis, every 4 years one could expect to see a day with an overnight interest rate as high as 4800 percent.

The results show that overnight interest rates observed during the financial crisis were in the nature of the economy before they materialized. The policy makers and the financial authorities in Turkey, along with the Turkish commercial banks, simply ignored the possibility that such extreme tail events might occur.

3.3. A comparison with the US overnight interest rates

Although the Turkish daily overnight rate is an excellent case study with high volatility and a thick-tailed distribution, this data set has not been studied widely in the literature and is not well-known. Hence, we have repeated the extreme value analysis with the daily US Effective Federal Funds Rate as a comparison. The Federal Funds Rate (FFR) is the interest rate that banks with excess reserves at a

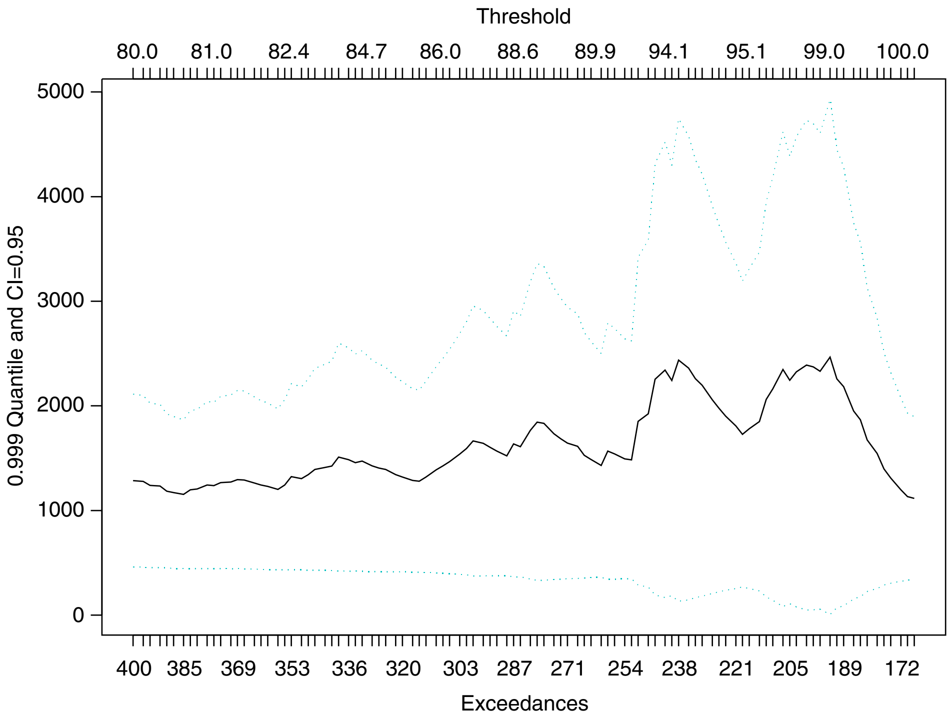

Fig. 5. Interest rate estimates of quantile 0.999 as a function of the threshold (upper x-axis) or alternatively as a function of exceedances (lower x-axis) before the February crisis. These estimates are from 100 GPD models with thresholds ranging between 80 percent and 100 percent interest rates (simple annual). The estimated interest rate at 0.999 quantile settles above 1000 percent at most thresholds although variations are higher at the higher thresholds. The Wald confidence intervals indicate that interest rates as high as 4000 percent is a possibility. The sample period is from January 2, 1990 to February 16, 2001 which ends 1 week before the February 2001 crisis.

Federal Reserve District Bank charge other banks that need overnight loans. The sample period is from July 1, 1954 to December 31, 2000. The sample size is 16,986 daily observations. The data source is the Federal Reserve Board of Governors.[6]

In Fig. 6 (bottom, left), the quantiles of the empirical distribution function of the daily US Effective Federal Funds Rate on the x-axis are plotted against the quantiles of the Gumbel distribution function on the y-axis. The convex relationship between the quantiles of the empirical and the Gumbel distributions indicate a thin-tailed distribution for the US interest rates. The GPD estimations with different numbers of exceedances in this data set indicate that the estimated shape parameter $\hat{\xi}$ is

[6]We do not intend to model the FFR dynamics in the US: It is far beyond the scope of this study. There is ongoing discussion as to whether interest rates are stationary or not. Conventional economic and finance theory often assumes that interest rates are stationary and utilize a model which is a mean-reverting process. However, some empirical studies provide evidence that interest rates are nonstationary processes.

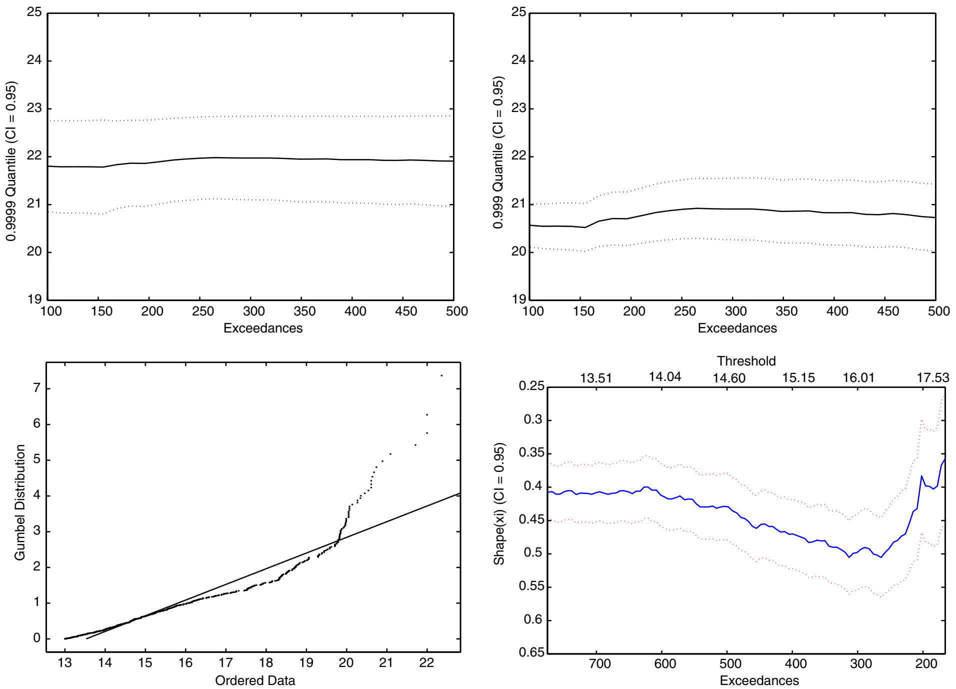

Fig. 6. Top: Effective Federal Funds Rate (daily) estimates of 0.9999th (top, left) and 0.999 (top, right) quantiles as a function of exceedances. The estimated interest rate even at 0.9999 quantile (1 day in every 30 years) is less than the historical high of 22.36 percent (July 22, 1981). Bottom left: QQ-plot of Effective Federal Funds Rate (daily, simple annual) against the Gumbel distribution. The points should lie along the straight line if the data are from a Gumbel distribution. A convex presence indicates a thin-tailed distribution. Bottom right: Estimates of shape parameter, ξ, at different thresholds (upper x-axis) or alternatively with different number of exceedances (lower x-axis) for the US. The parameter estimates are from 100 GPD models with thresholds ranging between 13 percent and 18 percent interest rates (simple annual). The sample period is from July 1, 1954 to December 31, 2000. *Data source*: Federal Reserve Board of Governors, H.15 Release.

remarkably stable around -0.40 (see Fig. 6). This implies that the maximum daily US Effective Federal Funds Rate distribution has a bounded support and all moments exist. As a result, the extraordinary overnight rates observed in Turkey are a statistical impossibility for the United states. Indeed, Fig. 6 plots estimated interest rates at the 0.9999th quantile (1 day in every 10,000 days, approximately 30 years), and at the 0.999th quantile (1 day in every 1000 days, approximately 3 years) as a function of the number of exceedances. The estimated interest rate is very stable around 20.5 percent at the 0.1 percent tail and at around 22 percent at the 0.01 percent tail regardless of the number of exceedances. Moving in the tail of the distribution from the 0.1 percent to the 0.01 percent region increases the estimated interest rate by only around 150 bases points (1.5 percentage point). Notice that it is not the observed maximum that would determine the possible extreme values at certain quantiles. It is the shape of the extreme value distribution at which maximum interest rates converge. Clearly, the maximum interest rates in Turkey converge to a distribution which may not even exhibit a finite variance while the maximum interest rates in the U.S. has bounded support.[7]

We postulate that the difference between the U.S. and Turkey stems from the fact that the two countries have a different degree of development of the financial markets and institutions. First, financial markets in Turkey are very shallow and the financial structure is very weak. Total assets of all commercial banks in Turkey (around USD 150 billion) are comparable to the assets of a medium size international bank. Second, the Turkish economy has a long history of high inflation with accommodative monetary policies conducted by the Central Bank which has been under the strong influence of the government. The FED, on the other hand, may be considered as a truly independent monetary authority. It is reasonable to expect that the distributional characteristics of interest rates in Turkey are changing in view of the fact that Turkey now has an independent central bank and the financial markets are developing very fast.

4. Lessons from the Turkish crises and conclusions

Financial crises in emerging markets in general and the Turkish crises in November 2000 and February 2001 in particular, provide several lessons for investors both in developing and developed countries. Since a significant portion of total savings in developed economies are invested in emerging markets by hedge funds, mutual funds and other institutions in the form of portfolio investment, the costs of financial crises are not confined to the residents of emerging market countries. Therefore, a careful investigation of the market dynamics and the causes of crises in these economies would benefit investors at large by increasing the investor awareness.

Fundamental macroeconomic indicators such as growth rate, current account balance, real exchange rate, budget deficit, export–import ratio and debt–income ratio are the main sources for assessing the current and future status of an economy. Therefore, they play a significant role in the decision making process of the IMF, credit rating agencies and multinational fund managers. One of the lessons from the Turkish crisis is that even if there is no deterioration in fundamental indicators, the balance sheet issues in the financial sector may create an environment in which even a small shock can lead to a total collapse of the system. In particular, a balance sheet mismatch situation (funding long-term illiquid assets with short-term obligations) combined with slack supervision and regulation is an invitation for a liquidity and currency crisis.

Among others, Eichengreen (2001) investigates both the Argentinian and Turkish crises in detail. He points to the vulnerability of the banking sector as the main

[7]Given estimated values $\hat{\xi} = 0.40$ and $\hat{\beta} = 3.75$ with a threshold interest rate 13.5 percent, the estimated endpoint of the maximum interest rate distribution in the US is 22.67 percent.

source of crisis in Turkey. Although it was expected that the lira was going to appreciate during the program, the government could not commit to an upfront devaluation as the Turkish banks and the private sector had large unhedged foreign exchange exposures. It was hoped by the program designers that the banking sector would strengthen before the economy would move into a floating exchange rate regime. As Eichengreen (2001) points out, this strategy created a moral hazard in the system and an incentive to strengthen both balance sheets and supervision diminished.[8] As a result, short foreign exchange positions of the banks doubled during the initial phase of the program. The direct consequence of this was that the fear of destabilizing the economy forced the authorities to resist any correction in the exchange rate, even if it meant extraordinary increases in overnight interest rates. Given the government's strong commitment to exchange rates, the relevant question from the investors' point of view was ''what extraordinary interest rates may be observed under extreme situations?''. Our estimation results from the pre-crisis data indicate that every 4 years one could expect to see a day with overnight interest rates over 1000 percent (simple annual). In other words, the extraordinary levels observed during the crisis were in the nature of the economy before they actually materialized. The developments during and after the crisis indicate that both policy designers and the financial sector failed to take this characteristic of the economy into account.

Another important implication of our results for risk management is that the degree of development of the financial markets and institutions in different countries implies different levels of risk and associated risk premium. Therefore, risk managers should abstain from applying the same risk model and the same value-at-risk methodology to different economies. A statistical impossibility for a developed economy might easily be ''1 day in every 100 days'' in an emerging market.

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[8]According to Eichengreen (2001), the moral hazard was created ''because exchange risk was socialized, that is to say, its strategy committed the government to preventing the exchange rate from moving and therefore to compensating the banks for their losses if the policy failed''.

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