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Disorder and localization in the lowest Landau level in the presence of dilute point scatterers

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Abstract

We study the localization properties of a two-dimensional noninteracting electron gas in the presence of randomly distributed short-range scatterers in very high magnetic fields. We evaluate the participation number of the eigenstates obtained by exact diagonalization technique. At low impurity concentrations we obtain self-averaged values showing that all states, except those exactly at the Landau level, are localized with finite localization length. We conclude that in this dilute regime the localization length does not diverge. We also find that the maximum localization length increases exponentially with impurity concentration. Our calculations suggest that scaling behavior may be absent even for higher concentrations of scatterers. © 1999 Elsevier Science Ltd. All rights reserved.

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There has been a long lasting interest in understanding the localization problem in two-dimensional (2D) systems. According to scaling theory of localization [1,2], all states in a 2D system are localized if a disordered potential is present. However, in the presence of a strong perpendicular magnetic field, where the time reversal symmetry is broken, extended states appear in the center of impurity-broadened Landau bands [3]. If the scattering between Landau levels can be neglected, these extended states exist only at a single energy [4,5]. The width of the quantized plateaus of the integer quantum Hall effect (QHE) depends on the ratio of number of localized to extended states [6].

In analogy with the quantum critical phenomena and other localization transitions, it has been proposed that localization length $\xi(E)$ diverges as E approaches the critical energy E_c, which is equal to Landau level energy, so that

$$\xi(E) \propto |E - E_c|^{-\nu} \tag{1}$$

where ν is the localization critical exponent [7,8]. After the initial calculations of Aoki and Ando [9–11], several groups attempted to determine this critical exponent [12–23]. Some of the experimental results [24–28] are generally in good agreement with the calculated values. Various techniques have allowed the computation of the exponent ν, and they strongly suggest a universal value $\nu = 2.3 \pm 0.1$ for the lowest Landau level (LLL). However, in spite of a great deal of experimental evidence and numerical simulations in its favor, there is no rigorous derivation of power law divergence in the localization length. Furthermore, even if the power law divergence is true, it is not clear whether the localization critical exponent is universal, independent of impurity concentration or parameters of the disordered potential [21,29–32].

Recently, we developed a method for a particle in the LLL moving in an arbitrary potential [33], where we investigate the energy spectrum and eigenvalues for periodic distribution of point scatterers. In this study we apply the method, which is basically an exact diagonalization technique, to a potential formed by randomly distributed short-range scatterers. We concentrate on low impurity concentrations, i.e. large average impurity–impurity separations in comparison to the magnetic length, where it is difficult to perform calculations by other methods due to the presence of zero eigenvalues associated with the extended states at the band center. At low concentrations, we obtain self-averaged values where energy spectrum or localization property of eigenstates do not change with increasing system size. Contrary to the widely accepted view, localization length

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does not diverge but instead the maximum localization length grows exponentially with impurity density. Extrapolation to less pure systems suggests that localization length can become as large as the sizes of the samples used in QHE experiments which explains the observed divergence in measurements.

The Hamiltonian for a particle of mass m and charge q, moving in 2D in the presence of magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ perpendicular to the plane and potential V, is given by $H = H_0 + V$ where

$$H_0 = \frac{1}{2m}(\mathbf{p} - \frac{q}{c}\mathbf{A})^2 \tag{2}$$

Using the symmetric gauge $\mathbf{A} = \frac{1}{2}\mathbf{B} \times \mathbf{r}$ and complex coordinates $z = X + iY = \sqrt{qB/2\hbar c}(x + iy)$ where $\mathbf{r} = (x, y)$, the unperturbed Hamiltonian can be written as $H_0 = \hbar\omega(a^\dagger a + 1/2)$ where $a^\dagger = -\partial/\partial z + z^*/2$. Since $[a, a^\dagger] = 1$, the energy eigenvalues are given by $E_n = \hbar\omega(n + 1/2)$ where $\omega = qB/2mc$ (q is assumed to be positive) and $n = 0, 1, 2, \ldots$ When the magnetic field is very high the particle is confined into the LLL. This is a good approximation as long as the potential is small in comparison to Landau level splitting $\hbar\omega$. We are going to measure energies from the LLL so that $E_n = 0$.

Now, let us consider the potential

$$V(z, z^*) = V_0 \sum_i \delta(z - z_i)\delta(z^* - z_i^*) \tag{3}$$

where z_i denotes the position of the ith impurity in complex coordinates defined above. According to our method [33], to find the nonzero eigenvalues, the matrix to be diagonalized is

$$\langle i|\tilde{V}|j\rangle = \frac{V_0}{\pi}\exp(z_i z_j^* - |z_i|^2/2 - |z_j|^2/2) \tag{4}$$

Once $\tilde{V}$ is diagonalized, the eigenfunctions $\psi(z, z^*)$ of V can be constructed from $\tilde{\psi}_i$

$$\psi(z, z^*) = \sqrt{\frac{V_0}{\pi^2 E}} \sum_i \exp(z z_i^* - |z|^2/2 - |z_i|^2/2)\tilde{\psi}_i \tag{5}$$

The above expression is the exact solution for the original infinite system as long as $\tilde{\psi}_i$ values are known. This can be achieved for a finite number of impurities. Therefore, the solutions we obtain are exact results for infinite system with finitely many impurities which are localized in a certain region with a definite density. Our calculations verify the suggestion of Azbel and Halperin [34] that at sufficiently low impurity concentrations approximate eigenstates of the Hamiltonian can be constructed by considering those eigenstates for the individual scatterers.

We distinguish between the extended and the localized states via participation number P, which is the inverse of the mean fourth power of the amplitude [35]. Therefore, given a wave function ψ, P is defined as

$$P = \frac{\left[\int |\psi(\mathbf{r})|^2 d\mathbf{r}\right]^2}{\int |\psi(\mathbf{r})|^4 d\mathbf{r}}. \tag{6}$$

The participation number is a convenient quantity for distinguishing between localized and extended states since it takes a nonvanishing value for the former and becomes infinite for the latter. If a state is localized within a $d-$dimensional volume of average diameter D, P behaves as D^d irrespectively of the system size. For a plane wave it depends on the system size as L as L^d. In general, extended states lead to an effective dimensionality d^*, smaller than the real dimensionality d, which means that the states are not space-filling.

For $\tilde{\psi}$, participation number reduces to $\tilde{P} = 1/\sum_i |\tilde{\psi}_i|^4$ provided that $\tilde{\psi}$ is normalized. For a state localized on single impurity $\tilde{P} = 1$, while for uniform distribution over N_i impurities $\tilde{P} = N_i$. Therefore, we can interpret $\tilde{P}$ as a measure of number of scatterers on which $\tilde{\psi}$ takes nonzero value. We note that, as can be seen from Eq. (5), ψ and $\tilde{\psi}$ have the same localization behavior, i.e. they are both extended or localized. Although we can evaluate corresponding P, we prefer to use $\tilde{P}$ to distinguish between localized and extended states. In this way we get rid of four-fold sums to be performed for evaluation of P in terms of $\tilde{\psi}_i$. We note that for $f < 1$, where f is the number of impurities per flux quantum, there are $N_i(1/f - 1)$ extended states at the center of the Landau band [36]. However, the behavior given by Eq. (1) has been proposed for states with $E \neq E_c$ and our method filters the states with $E = E_c$. We use $\tilde{P}$ to decide whether a state is extended or not as follows. Let us consider N_i impurities distributed in a square so that their concentration is f. If $\tilde{P} > \sqrt{N_i}$, then $\tilde{\psi}$ is nonvanishing on approximately $\sqrt{N_i}$ sites which means that it may extend from one side of the square to its opposite. Therefore, we assume that $\tilde{\psi}$ is an extended state. On the other hand, if $\tilde{P} < \sqrt{N_i}$, then $\tilde{\psi}$ has no chance to be extended. In this way we obtain the number of extended states N_e, for a given system composed of N_i impurities. Although $\tilde{P} > \sqrt{N_i}$, corresponding state can still be localized. Therefore, N_e is only an upper bound for the number of extended states. Our definition of extendedness becomes exact if the sites at which $\tilde{\psi}$ is nonvanishing form straight lines in at least one direction.

Given f, we diagonalize the matrix $\langle i|\tilde{V}|j\rangle$ for increasing number of impurities N_i and by evaluating the participation numbers we find N_e. Repeating the calculation for various number of impurity distributions we obtain the average number of extended states $\langle N_e\rangle$. As shown in Fig. 1, we observe that first $\langle N_e\rangle$ increases with N_i, then decreases and vanishes at the end for $N_i > N_i^*$ after which there are no more extended states. We define N_i^* as the number of impurities for which $\langle N_e\rangle = 1$. If we increase N_i further, we do not get any extended states. This result is independent of the arbitrariness of our method of distinguishing between

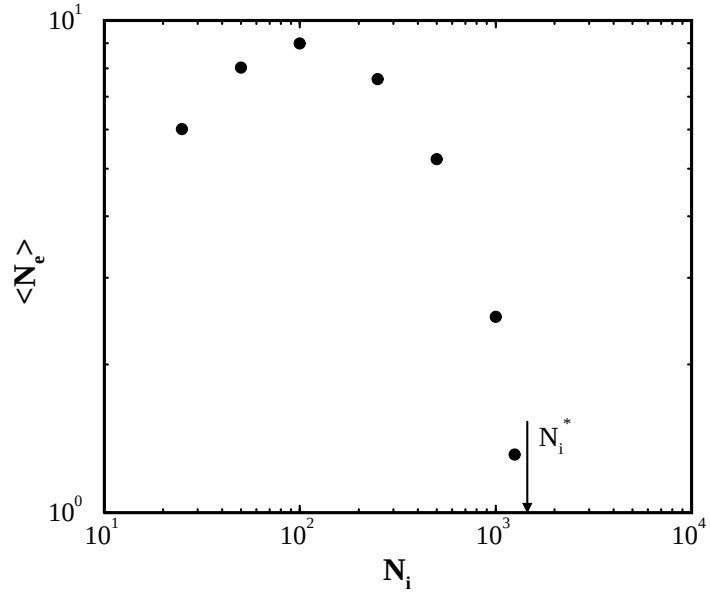

Fig. 1. The mean number of extended states N_e vs. number of impurities N_i for $f = 2/3$. Although initially $\langle N_e \rangle$ increases with N_i, after $N_i = N_i^* = 1598$ no extended states appear.

extended and localized states, since contour plots of wave functions show clearly that all states are localized for $N_i > N_i^*$.

At this stage, it is not possible to say whether there is no scaling behavior for larger concentrations but Fig. 2 gives some idea about the system sizes to be used if the same result holds in this denser region. In Fig. 2, we plot N_i^* as a function of f. We observe that $N_i^* \propto \exp(cf)$ which implies that the maximum localization length $\xi_{max}/l \propto \exp(cn_i l^2/2)$ where c is a constant. This result suggests that for large enough sizes it may be possible to observe the same behavior even for higher concentrations. For example, for

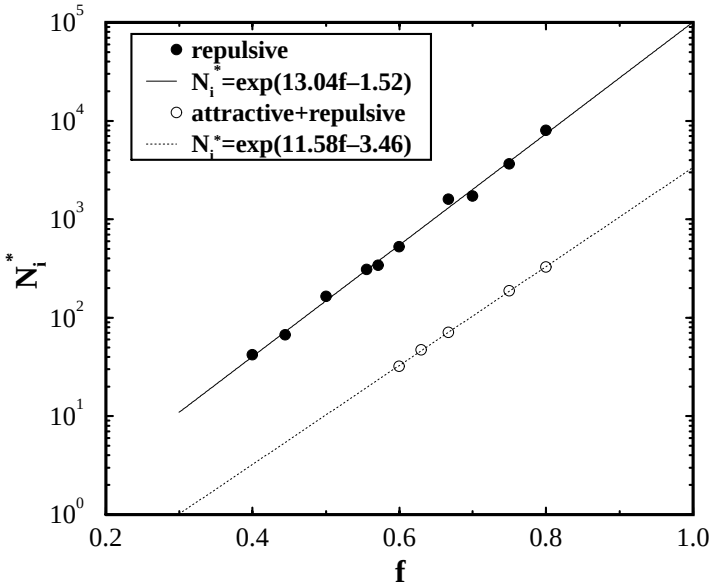

Fig. 2. Variation of N_i^* with f. (●) denotes the for pure repulsive case while (○) stands for the case with equal number of attractive and repulsive scattering centers. The solid and dotted lines are the best fits to the data points.

$f = 1.2$, we predict that $N_i^* \sim 10^6$. We also performed calculations for the case with equal number of attractive and repulsive point scatterers. We observe that there is no qualitative change in our results as can be seen from the figure. Beyond a certain size all states are localized which can be interpreted as the absence of any scaling behavior. Increase of localization length with increasing impurity concentration may look contradictory. However, if we visualize the system as percolation of wave functions localized at impurity sites, it is easy to understand why we get less localized states with increasing f.

Ando's suggestion [36] that the dilute point scatterer case is anomalous, is verified by our method. In fact, the method is based on the fact that the point scatterers preserve a finite fraction of states in a strong magnetic field at the LLL. The physical origin of the singular δ-function appearing in the density of states is that the states with the wave function which vanishes at positions of scatterers are not affected by them. The difficulty in numerical studies in the dilute limit is the presence of $N(1-f)$ zero eigenvalues, N being the total number of states. Our method decouples Nf states having nonzero eigenvalues.

In conclusion, at low impurity concentrations we obtain self-averaged values showing that all states, except those exactly at the Landau level, are localized with finite localization length. We conclude that at low impurity concentrations the localization length does not diverge. Our results suggest that the same behavior may be observed for higher concentrations. Even if this is not the case, the transition between the two regimes, scaling and no scaling behaviors, is a very interesting problem to investigate.

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