Nonlinear subband decomposition structures in GF-(N) arithmetic[1]

M. Nafi Gürcan, Ömer N. Gerek, A. Enis Çetin*

Department of Electrical and Electronics Engineering, Bilkent University, Bilkent, Ankara, TR-06533, Turkey

Abstract

In this paper, perfect reconstruction filter bank structures for GF-(N) fields are developed. The new filter banks are based on the nonlinear subband decomposition and they are especially useful to process binary images such as document and fingerprint images. © 1998 Elsevier Science B.V. All rights reserved.

Zusammenfassung

In diesem Artikel werden Filterbankstrukturen zur perfekten Rekonstruktion von GF-(N) Feldern entwickelt. Die neuartigen Filterbänke basieren auf der nichtlinearen subband-Zerlegung und eignen sich besonders zur Verarbeitung digitaler binärer Bilder wie Abbildungen von Schriftstücken oder von Fingerabdrücken. © 1998 Elsevier Science B.V. All rights reserved.

Résumé

Cet article développe des structures en bancs de filtres à reconstruction parfaite pour les champs GF-(N). Les nouveaux bancs de filtres sont basés sur la décomposition en sous-bandes non-linéaire et sont particulièrement utiles pour traiter des images binaires telles que les images de documents ou d'empreintes digitales. © 1998 Elsevier Science B.V. All rights reserved.

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1. Introduction

The subband decomposition is a widely used tool in signal processing and pattern recognition. Traditionally, linear filters with perfect reconstruction (PR) property are employed for the decomposition. Recently, a variety of nonlinear 'subband' decomposition structures with perfect reconstruction

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property have been proposed and successfully used in image compression and analysis [4,5,7–9]. The key idea is to replace the linear filters in a classical subband decomposition scheme with nonlinear 'half-band' filters.

In this paper, we show that the nonlinear subband decomposition structures of [4,5,8] can be implemented in Galois Field (GF) arithmetic. In Section 2, the Galois field concepts are reviewed. In Section 3, the nonlinear subband decomposition structure of [8] is explained and it is proved that the filter bank provides perfect reconstruction in GF-(N) arithmetic as well. In Section 4 examples of binary and 256-ary nonlinear subband decompositions are presented.

2. Galois field arithmetic

A field is defined as a set with two arithmetic operations, *addition* and *multiplication* if
1. the set is closed under addition and multiplication;
2. addition and multiplication are associative and commutative; and
3. the distributive law holds.

A field with a finite number of elements, q, is called a finite field or Galois field and is denoted by GF-(q) [1]. Most signal processing operations are carried out over real and complex fields whose sizes are infinite. However, binary images such as document images, fingerprint images, etc., constitute an important class and GF-(2)-based image processing is not only computationally efficient for such images but also sufficient in many applications.

The simplest Galois field is GF-(2), with the set $\{0, 1\}$, and modulo-2 addition, and multiplication. Two binary sequences $u[n]$ and $v[n]$ can be convolved in this field as follows:

$$c[n] = \sum_l u[n - l]v[l] \quad \text{(modulo-2)}. \tag{1}$$

For example, the convolution of the sequences $u = \{0, 0, 1, 1\}$ and $v = \{1, 1, 0, 1\}$ in GF-(2) results in $c = \{0, 0, 1, 0, 1, 1, 1\}$ in GF-(2). If real arithmetic were used then the result would be $\{0, 0, 1, 2, 1, 1, 1\}$.

Nonlinear filtering operations can also be performed in GF-(2). For some morphological filtering operations such as erosion, dilation, the filter outputs are binary for binary inputs [3]. Hence in such filters the output is also in GF-(2). Another important nonlinear filter is the median filter. For binary images, median filtering can be efficiently computed simply by counting the number of ones in the region of support. If the number of ones is greater than half of the number of pixels then the median value is one, otherwise it is zero. In general, if the result of the nonlinear filter is an integer different from 0 and 1 then it can still be expressed in GF-(2) after a modulo-2 operation.

The above concepts can be generalized to GF-(q), which is based on the set $\{0, 1, \ldots, q - 1\}$ and modulo-q arithmetic, if q is a prime number. If q is not a prime number but an integer power of a prime number, p, then GF-(q) can still be constructed as an extension of GF-(p). For example, GF-($256 = 2^8$) is an extension of GF-(2).

3. Nonlinear decomposition in GF-(N) arithmetic

In Fig. 1, the block diagram of the nonlinear subband decomposition structure proposed by Hampson and Pesquet is shown [8]. In this structure H and G are nonlinear operators. The approximate signal y_a and the detail signal y_d are obtained from the input signal $x[n]$ as follows [8]:

$$y_d[n] = x_2[n] - H(x_1)[n], \tag{2}$$

$$y_a[n] = x_1[n] + G(y_d)[n], \tag{3}$$

where $x_1[n] = x[2n - 1]$, $x_2[n] = x[2n]$.

The corresponding synthesis equations are

$$x_1'[n] = y_a[n] - G(y_d)[n], \tag{4}$$

$$x_2'[n] = y_d[n] + H(x_1)[n]. \tag{5}$$

In GF-(N) arithmetic, Eqs. (4) and (5) take the following form:

$$x_1'[n] = y_a[n] \ominus G(y_d)[n] \tag{6}$$

$$= x_1[n] \oplus G(y_d)[n] \ominus G(y_d)[n] \tag{7}$$
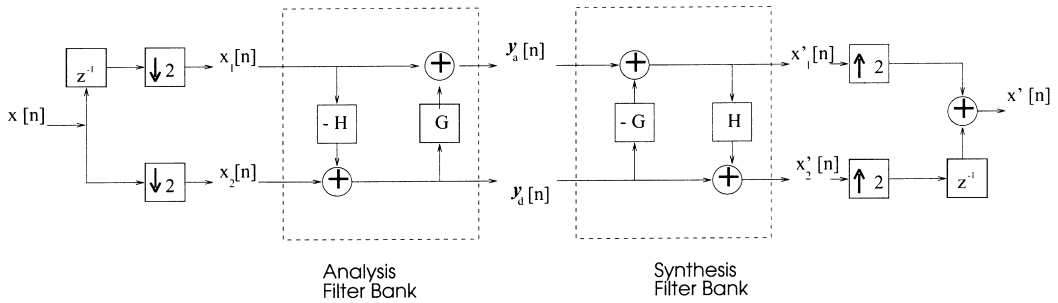
$$= x_1[n], \tag{8}$$

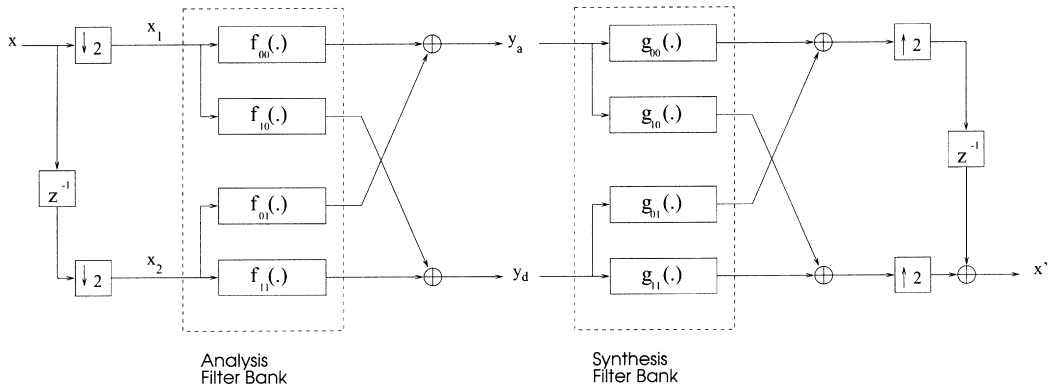Fig. 1. *N*-ary subband decomposition structure.

Fig. 2. The nonlinear perfect reconstruction filter bank structure in [2].

where $\ominus, \oplus$ represent the GF-(N) subtraction and addition operations, respectively (they do not represent morphological erosion and dilation operations). Similarly,

$$x_2'[n] = y_d[n] \oplus H(x_1)[n] \tag{9}$$

$$= x_2[n] \ominus H(x_1)[n] \oplus H(x_1)[n] \tag{10}$$

$$= x_2[n]. \tag{11}$$

The Perfect Reconstruction (PR) property is valid as long as the outputs of the nonlinear filters G and H are in the GF-(N) field. Therefore, Eqs. (8) and (11) prove that the PR property of this nonlinear decomposition structure is preserved, if GF-(N) arithmetic is used. We call the resulting structure

nonlinear GF-(N) subband decomposition filter bank.

Some of the nonlinear subband decomposition structures are related to each other by appropriately choosing the nonlinear filters. For example, the structure depicted in Fig. 2 [5] is a more general version of the structures developed in [8,9]. The analysis stage of Fig. 1 can be obtained by cascading two analysis banks of the structure of [2] and by choosing the filters as $f_{00} = 1$, $f_{01} = 0, f_{10} = -H$ and $f_{11} = 0$ in the first analysis filter bank and $f_{00} = 0, f_{01} = G, f_{10} = 0$ and $f_{11} = 1$ in the second analysis filter bank. Similarly, the analysis filter bank of [9] can be composed of as a cascade connection of the filter banks $f_{00} = 1, f_{01} = \alpha(\cdot), f_{10} = 0$ and $f_{11} = 0$ with $f_{00} = 0$, $f_{01} = 0$, $f_{10} = \beta(\cdot)$ and $f_{11} = 1$. It can
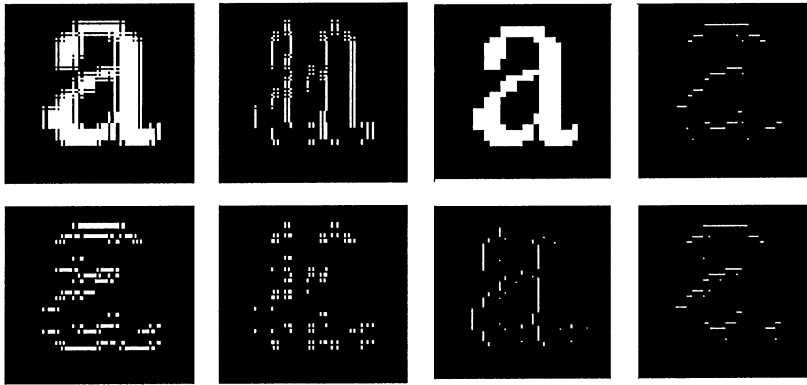
Fig. 3. Binary subband decomposition of the letter 'a': (a) binary wavelet transform, (b) binary nonlinear subband decomposition.

easily be shown that the filter structure in Fig. 2 also provides PR in GF-(N) arithmetic, for the three types of filter structures (Type I: Hierarchical DPCM-like, Type II: DPCM-like, and Type III: channel swapping) discussed in [5].

GF-(N) arithmetic was recently used in wavelet theory [2,11]. In [2], a framework for the wavelet transform of *finite cyclic groups* was developed. In [10], this idea was specifically used for the binary decomposition of binary images, and the resulting transformation was called the binary wavelet transform (BWT). Our structure differs from the other GF-(N) arithmetic decompositions in its use of nonlinear operations instead of linear operations.

The computational complexity of the binary nonlinear subband decomposition is significantly less than the BWT in which image-size matrix multiplications are required. Due to this, a typical BWT operation requires $2N^3$ logical operations (XOR) where N is the size of the square image. On the other hand, computational complexity of our binary decomposition algorithm is $N^2(M + 2)$ where M is the size of the region of support for the order-statistics filter. Since typical image size, N, is much larger than the filter size, M, our structure significantly decreases the computational complexity. Furthermore, the ringing effects which appear at sharp signal variation points are eliminated with the use of nonlinear operations. An extra computational saving is obtained if median (or any

order-statistics)-type filter is used as explained in Section 2.

4. Binary and N-ary nonlinear subband decomposition examples

Consider the images in Fig. 3(a,b). In this figure the binary wavelet transform (BWT) and the binary nonlinear subband decomposition (BNSD) of the letter 'a' are shown, respectively. The ringing artifacts of the BWT do not appear in the BNSD subimages and slightly higher document image compression results are obtained using the BNSD in [6]. In Fig. 3(b) the nonlinear binary filter H is chosen as the median filter and $G = 0$. As pointed out in Section 2, median filtering is performed by simply counting the number of ones or zeros in the region of support for binary images. If the number of ones is greater than or equal to the half of the number of pixels in the support region then the median value is one, otherwise it is zero.

Fig. 4 shows the result of the one-level subband decomposition of 512×512 'Lena' image which has 256 gray levels. The decomposition is obtained by choosing H as the median filter and $G = 0$, which corresponds to the nonlinear decomposition structure of Egger et al. [4] in GF-(256) arithmetic. Since the GF-(256) arithmetic is used in the decomposition, all the subimages in Fig. 4 are 256-gray-level images. If the median filter has a support region
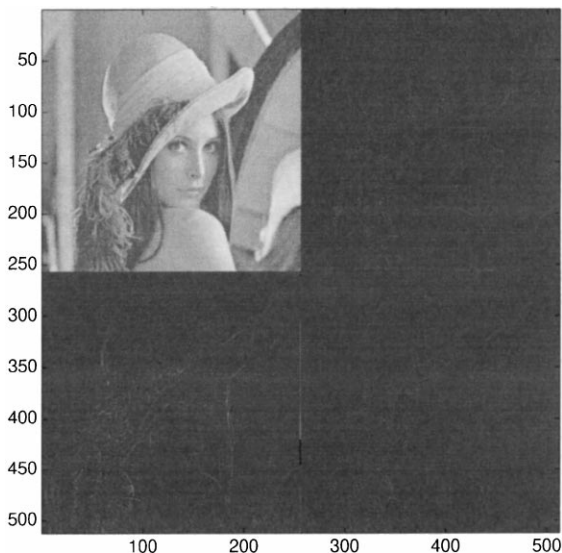
Fig. 4. 256-ary nonlinear subband decomposition of the Lena image.

with odd number of pixels then the output of the median filter will also be in GF-(256) field. Otherwise, the output of the median filter is modified as

$$x[n] = \left\lfloor \frac{x_{(L/2)} + x_{((L+2)/2)}}{2} \right\rfloor, \tag{12}$$

where $x_{(L/2)}$ is the $(L/2)$th element of rank ordered L pixels in the support region. In this way the output of the median filter remains in GF-(256).

5. Conclusions

In this paper, it is shown that the recently proposed nonlinear subband decomposition structures can be implemented in GF-(N) arithmetic. The resulting filter bank structures are especially useful to process binary images such as document and fingerprint images. Future work will include the development of lossless medical image compression algorithms in GF-(N) arithmetic.

Acknowledgements

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