Fast Algorithm for Scattering from Planar Arrays of Conducting Patches

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***Abstract*— A direct (noniterative) algorithm for the solution of the electromagnetic scattering from three-dimensional planar arrays of conducting patches is developed. For an N-unknown problem, the computational complexity of this new solution technique is shown to be $O(N^2 \log^2 N)$, which is considerably lower than the $O(N^3)$ computational complexity of the conventional direct solution techniques. The advantages of the reduction in the computational complexity is pronounced in the solution of large electromagnetics problems, such as scattering from large and finite arrays of patches, synthesis and analysis of finite-sized frequency selective surfaces (FSS's), and radiation and scattering from large phased-array antennas, to name a few.**

***Index Terms*—Algorithms, antenna arrays, frequency selective surface, scattering.**

I. Introduction

SOLUTIONS of electromagnetic scattering problems involving three-dimensional (3-D) planar (Fig. 1) and quasi-planar geometries in homogeneous and layered media are of great interest due to the existence of a multitude of useful applications, such as frequency selective surfaces (FSS's), printed circuit boards (PCB's), microstrip structures, monolithic microwave integrated circuits (MMIC's), phased-array antennas, and rough surfaces.

Although several different techniques [1]–[3] existed for the solution of these problems, the need to solve larger problems with limited computational resources recently sparked the successful development of numerous new fast solvers [4]–[13]. However, no method can be expected to solve all classes of problems. For instance, the iterative solvers [4]–[9], which are perfectly suited for large problems, perform poorly for near-resonant structures. Some techniques are limited to two-dimensional (2-D) geometries [10], [11], whereas some others are limited to homogeneous-medium problems [12], [13]. Development of a new noniterative method and its application to 3-D planar geometries in homogeneous media will be outlined in this letter. The method can be extended to the cases of quasi-planar structures and/or layered-media problems. This method can also be used as a block-diagonal preconditioner and to obtain an initial guess in the framework of an iterative technique.

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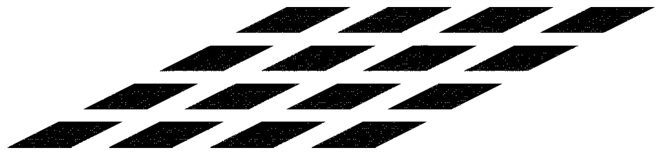

Fig. 1. Geometry of a planar array of conducting patches.

II. Fast Direct Algorithm Based on the Steepest Descent Path (FDA/SDP)

Assuming that the planar geometry of Fig. 1 is placed on the x–y plane, tangential components of the electric field on the same plane are given by

$$\begin{bmatrix} E_x(\boldsymbol{\rho}) \\ E_y(\boldsymbol{\rho}) \end{bmatrix} = i\omega\mu \int d\boldsymbol{\rho}' \left[\bar{\mathbf{I}} + \frac{\nabla_s' \nabla_s'}{k^2} \right] \cdot g(\boldsymbol{\rho}, \boldsymbol{\rho}') \begin{bmatrix} J_x(\boldsymbol{\rho}') \\ J_y(\boldsymbol{\rho}') \end{bmatrix} \tag{1}$$

where $\boldsymbol{\rho} = \hat{x}x + \hat{y}y$ and $\boldsymbol{\rho}'$ are arbitrary position vectors on the x–y plane, $d\boldsymbol{\rho}' = dx'\, dy'$, and $\nabla_s' = \hat{x}\partial_{x'} + \hat{y}\partial_{y'}$. In the above, $g(\boldsymbol{\rho}, \boldsymbol{\rho}')$ is the 3-D scalar Green's function confined to the in-plane interactions and can be expressed in terms of the 2-D Green's function using the identity [14]

$$g(\boldsymbol{\rho}, \boldsymbol{\rho}') = \frac{e^{ik|\boldsymbol{\rho}-\boldsymbol{\rho}'|}}{4\pi|\boldsymbol{\rho}-\boldsymbol{\rho}'|} \tag{2}$$

$$= \frac{i}{8\pi} \int_{-\infty}^{\infty} dk_\rho \, \frac{k_\rho}{k_z} \, H_0^{(1)}(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}'|) \tag{3}$$

$$= \frac{k}{2\pi} \int_0^{\infty} ds \, \frac{1+is^2}{\sqrt{s^2-i2}} \, H_0^{(1)}[k|\boldsymbol{\rho}-\boldsymbol{\rho}'|(1+is^2)]. \tag{4}$$

Equation (4) is obtained by deforming the path of integration in (3) to the steepest descent path (SDP), where the integrand is rapidly decaying. The SDP integral in (4) can be numerically evaluated by sampling the integrand at a set of appropriately chosen points s_m with associated weights w_m

$$\frac{e^{ik|\boldsymbol{\rho}-\boldsymbol{\rho}'|}}{4\pi|\boldsymbol{\rho}-\boldsymbol{\rho}'|} = \frac{k}{2\pi} \sum_m w_m \frac{1+is_m^2}{\sqrt{s_m^2-i2}} H_0^{(1)} \cdot [k|\boldsymbol{\rho}-\boldsymbol{\rho}'|(1+is_m^2)]. \tag{5}$$

Thus, the 3-D Green's function for this problem can be expressed as a sum of several 2-D Green's functions. This representation of the Green's function essentially converts the

3-D problem to a "quasi-2-D" problem. At this point, the problem can be solved using any 2-D solver that has less than $O(N^3)$ computational complexity such as the recursive T-matrix algorithm (RTMA) [12], the recursive aggregate-T-matrix algorithm (RATMA) [13], or the nested equivalence principle algorithm (NEPAL) [15]. In this work, the quasi-2-D problem will be solved using RATMA, which is a direct (noniterative) solution technique. Thus, the resulting method will be called the fast direct algorithm based on the SDP or the FDA/SDP.

The FDA/SDP takes advantage of the fact that the induced currents (i.e., basis and testing functions) on planar and quasi-planar geometries interact with each other within a very limited solid angle. Thus, all the degrees of freedom that are required to solve a "truly 3-D" geometry are not required for a planar or a quasi-planar geometry, and this situation can be exploited to develop efficient solution algorithms.

A. Numerical Integration

Let $D_{\min}$ and $D_{\max}$ denote the smallest and the largest distances, respectively, separating the patches in the quasi-2-D problem such that the two may be different by several orders of magnitude. In order to solve the quasi-2-D problem using RATMA, the integrals

$$\frac{e^{ikD_{\min}}}{4\pi D_{\min}} + \cdots + \frac{e^{ikD_{\max}}}{4\pi D_{\max}} = \frac{k}{2\pi}\left\{\int_0^\infty ds\,\frac{1+is^2}{\sqrt{s^2-i2}}H_0^{(1)}[kD_{\min}(1+is^2)] + \cdots + \int_0^\infty ds\,\frac{1+is^2}{\sqrt{s^2-i2}}\,H_0^{(1)}[kD_{\max}(1+is^2)]\right\} \tag{6}$$

have to be evaluated by sampling their integrands at the same set of points s_m to obtain the same accuracy for all of the integrals. Although the decay rates of the integrands can be very different, an integration rule can be developed such that all of the above integrals can be computed using the same set of sampling points [16]. Devising an integration rule that employs logarithmically spaced sampling points, the number of sampling points N_S can be bounded by $O[\log\,(D_{\max}/D_{\min})]$. Noting that $D_{\max}/D_{\min} \propto \sqrt{N}$ for the 2-D clustered planar arrays of patches, we have $N_S \propto \log\sqrt{N} \propto \log N$. Further details of the numerical integration will be given elsewhere [17].

B. Computational Complexity

It was shown earlier that the RATMA has $O(NP^2)$ computational complexity and $O(P^2)$ memory requirement [13], where P is the number of harmonics used in the translation formulas. In the FDA/SDP, $P = N_P N_S$, where $N_P \propto \sqrt{N}$ is the number of 2-D harmonics required for the quasi-2-D problem, and $N_S \propto \log N$ is the number of sampling points to compute the SDP integrals for the dense quasi-2-D problems considered in this work. Thus, the FDA/SDP has $O(N^2 \log^2 N)$ computational complexity and $O(N \log^2 N)$ memory requirement.

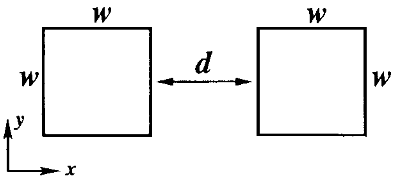

Fig. 2. A generic two-patch geometry with the separation between the patches (d) varying as a parameter.

The FDA/SDP can also be used to obtain an accurate solution for a portion of a larger geometry and this partial solution can be employed in block-diagonal preconditioning or as an initial guess in the framework of a faster iterative technique. For instance, in a single-level implementation of the fast multipole method (FMM) [4], [5], [8] where N unknowns are partitioned into $\sqrt{N}$ clusters each containing $\sqrt{N}$ unknowns, the FDA/SDP will require $O(N^{3/2} \log^2 N)$ operations for the block-diagonal preconditioning or to obtain an initial guess. In a multilevel implementation of the FMM [6], [7], the efficiency of the FDA/SDP as a block-diagonal preconditioner will allow using larger block sizes, which will result in better preconditioning and consequently reduce the number of iterations.

III. Results

In order to demonstrate the accuracy of the FDA/SDP, radar cross sections (RCS's) of a number of two-patch geometries are computed and compared to the corresponding solutions obtained with the method of moments (MoM). The generic two-patch geometry is shown in Fig. 2, where $w = \lambda$ and d is varied as a parameter. Fig. 3(a)–(c) shows the RCS's of the two-patch geometries of Fig. 2 when the separation between the patches (d) is equal to $\lambda/2$, λ, and 2λ, respectively. In each case, the excitation is a plane wave with a y-polarized electric field and incident on the patches at $\theta = 45°$ and $\phi = 0°$. Both the FDA/SDP results (solid curves) and the MoM results (dots) are plotted in Fig. 3(a)–(c) to exhibit the good agreement between the two, which testifies to the accuracy of the FDA/SDP.

Separating the solution and filling times, we have compared the solution times of the FDA/SDP, the MoM, and the RATMA as shown in Fig. 4. FDA/SDP solution times are seen to increase with a smaller slope than those of the MoM and the RATMA, confirming the reduced computational complexity of the FDA/SDP. The solution times presented in Fig. 4 are obtained by solving the scattering problems of increasingly larger planar arrays of patches, as shown in Fig. 1. The RATMA was predicted and demonstrated to have $O(N^3)$ complexity for such 2-D clustering of 3-D scatterers [13]. On the other hand, the FDA/SDP employs the RATMA to solve 2-D problems to take advantage of the RATMA's reduced computational complexity for 2-D clustering of 2-D scatterers [13].

IV. Conclusions

Development of the FDA/SDP, a new fast noniterative solution technique, and its application to planar arrays of

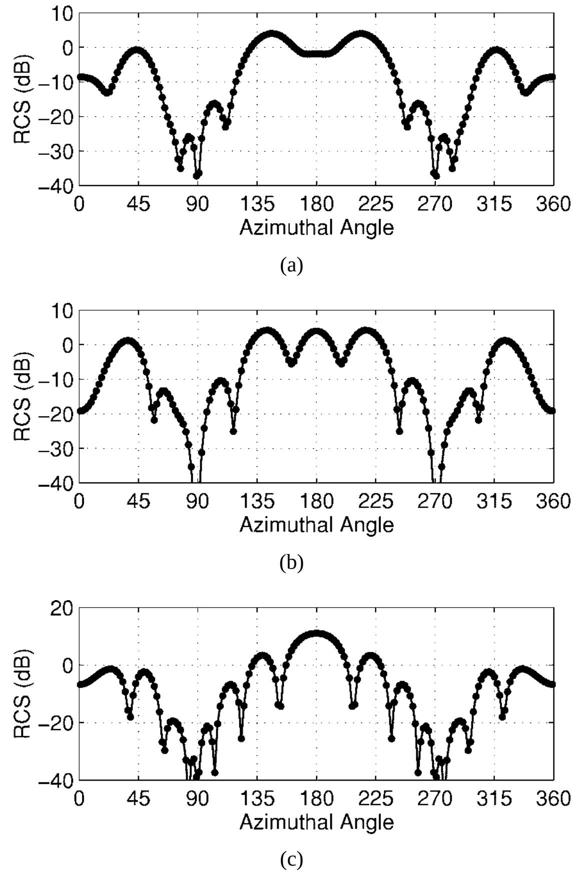

Fig. 3. RCS of the two $\lambda \times \lambda$ conducting patches (as shown in Fig. 2) on the x–y plane for various patch separations: (a) $d = \lambda/2$, (b) $d = \lambda$, and (c) $d = 2\lambda$. Plane waves with y-polarized electric fields are incident on the patches at $\theta = 45^\circ$ and $\phi = 0^\circ$.

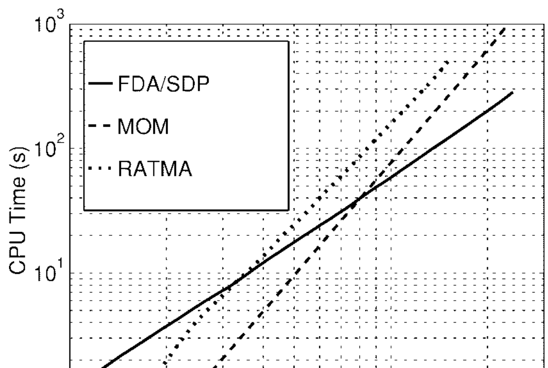

Fig. 4. Comparison of the CPU times required for various noniterative solution algorithms.

conducting patches have been presented in this letter. The FDA/SDP achieves its efficiency by essentially converting a 3-D planar geometry to a quasi-2-D geometry and then employing a fast 2-D solver to efficiently solve this resulting quasi-2-D problem.

The FDA/SDP can be extended from planar to quasiplanar structures, where the geometry is not strictly planar, however, the size of the geometry in one dimension is much smaller than the other two dimensions. Furthermore, extension from homogeneous-media problems to layered-media problems is also straightforward since the spectral-domain representation of the Green's function in (3) exists for layered media. These extensions and other issues will be reported elsewhere [17].

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