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Radiation phase and Stokes parameters

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Abstract

The approach which has been proposed by one of us [Optics Comm. 136 (1997) 219] is developed. The quantum phase properties of radiation are determined via the conservation of the angular momentum in the interaction with a source. It is shown that the use of two dual representations of the angular momentum of the dipole transition leads to the definition of five operators similar in some sense to the Stokes operators of the radiation. The approach is compared with that by Pegg and Barnett [20]. © 1998 Elsevier Science B.V.

This paper reports some new results relating to the quantum phase of an electric dipole radiation. It builds upon an earlier investigation by one of us [1]. We begin with a brief discussion of the approach determining the quantum phase via the angular momentum.

Since the vacuum state of the electromagnetic field has a uniform phase distribution, we might imagine that the phase properties of radiation are obtained in the process of generation. Then, the phase properties of radiation are determined by the corresponding properties of a source (atom, molecule, etc.). The hypothesis made in Ref. [1] is that the conservation of the total angular momentum is responsible for the transmission of the quantum phase “information” from the source to the radiation. In the case of a source, the angular momentum \mathbf{J} corresponding to the radiative transition is well defined in terms of the SU(2) algebra, the enveloping algebra of which contains the uniquely determined Casimir operator. Therefore, the phase of \mathbf{J} is simply determined by the polar decomposition of the SU(2) algebra (see, e.g. Ref. [2]).

Unlike the case of the source, the SU(2) sub-algebra in the Weyl-Heisenberg algebra, describing the angular momentum of radiation \mathbf{M} , has no well-defined Casimir operator in the whole Hilbert space of photons. Therefore, the polar decomposition of \mathbf{M} cannot be determined in a unique way. To avoid this difficulty, in Ref. [1], we

defined the quantum phase operators of radiation as the complements of the corresponding operators of the polar decomposition of \mathbf{J} with respect to the integrals of motion, describing the conservation of the total angular momentum $\mathbf{J} + \mathbf{M}$. Following this idea, we have determined the Hermitian sine S_R and cosine C_R of the phase operators of the radiation such that $[S_R, C_R] = 0$ and $[S_R, \hat{n}] = [C_R, \hat{n}] = 0$ where $\hat{n} \equiv \sum_m a_m^\dagger a_m$ is the total photon number. In accordance with the construction, these operators S_R, C_R should correspond to the azimuthal phase of the angular momentum of radiation.

Before we begin to discuss the properties of the operators S_R, C_R we note that the above approach [1] is in logical agreement with the approaches, treating the quantum phase in terms of measuring phase properties which can be determined either via the phase distributions [3–7] or in the operational way [8,9] (see, for a review Refs. [10,11]). Naturally, any measurement follows the process of generation. Therefore the measured phase properties are obtained by radiation in the process of generation although they can be modified by interaction with the macroscopic detecting device. This measured phase should correspond to some intrinsic quantum variable responsible for the phase of radiation [12]. Considering a photon as a quantum particle, we have the energy, momentum and spin (total angular momentum) as the possible dynamical variables.

Among them, we have to choose just the angular momentum because the other two variables do not contain non-trivial angular dependence.

It should be noted that our treatment of the phase in terms of the angular momentum has a quite simple physical meaning. In fact, within the framework of quantum optics, the polarization of light is described in terms of the spin state of photons, forming a given beam. In the classical domain, the polarization of light is specified by the Stokes parameters, determining the phase difference between components with different polarization [13]. The quantum properties of this phase difference can be examined in the operational way [14]. They have also been considered with the aid of polar decomposition of the Stokes operators in a finite sub-space of the Hilbert space [15]. Below we show that the radiation phase determined in Ref. [1] is directly connected to the Stokes operators which also can be determined via the conservation of the angular momentum in the process of radiation.

Let us consider the Jaynes-Cummings model (hereafter JCM) describing the electric dipole transition. The model Hamiltonian has the form

$$H = \sum_{m=-1}^{+1} [\omega a_m^\dagger a_m + \omega_0 R_{mm} + ig(R_{mG} a_m - a_m^\dagger R_{Gm})],$$

$$g = D \sqrt{\frac{c^4 \omega_0^2}{\hbar V \omega_3}}. \quad (1)$$

Here the atomic operators $R_{\alpha\beta} \equiv |\alpha\rangle\langle\beta|$, the states $|m\rangle \equiv |j=1; m\rangle$, $m=0, \pm 1$, correspond to the triple degenerated excited state, $|G\rangle \equiv |j'=0; 0\rangle$ describes the atomic ground state, the operators a_m^\dagger , a_m describe the electric dipole photons, ω is the radiation frequency, ω_0 is the transition frequency, g is the coupling constant depending on the effective dipole factor D and the volume of quantization V . Let us note that similar Hamiltonians have been considered in many problems of quantum optics and solid state physics (see Refs. [16,17] and references therein).

In the basis of the atomic states $|m\rangle$, the representation of the generators of the SU(2) algebra describing the angular momentum \mathbf{J} , has the form

$$J_z = R_{++} - R_{--}, \quad J_+ = \sqrt{2}(R_{+0} + R_{0-}),$$

$$J_- = \sqrt{2}(R_{0+} + R_{-0}), \quad (2)$$

with the standard commutation relations

$$[J_z, J_\pm] = \pm J_\pm, \quad [J_+, J_-] = 2J_z, \quad (3)$$

and the Casimir operator

$$(\mathbf{J})^2 = 2 \sum_m R_{mm} \equiv 2 \times \mathbf{1},$$

where $\mathbf{1}$ is the unit operator. Then, the polar decomposition of (2) is provided by the exponential of the phase operator [2]

$$E = R_{+0} + R_{0-} + e^{i\psi} R_{-+}, \quad EE^\dagger = \mathbf{1},$$

$$E^3 = e^{i\psi} \mathbf{1}, \quad (4)$$

and the radial operator $J_r = \sqrt{2}(\mathbf{1} - R_{--})$ such that $J_+ = J_r E$. Here ψ is an arbitrary real parameter. Clearly E is similar to the Coxeter operator [18]. Then, the Hermitian sine and cosine of the atomic phase operators are

$$S = \frac{E - E^\dagger}{2i}, \quad C = \frac{E + E^\dagger}{2}. \quad (5)$$

One can see that $[S, C] = 0$ and $S^2 + C^2 = \mathbf{1}$. Using the transformation

$$|\varphi_m\rangle = \sum_{n=-1}^{+1} \frac{e^{-2imn\pi}}{\sqrt{3}} |m\rangle, \quad \varphi_m = \frac{2m\pi - \psi}{3},$$

one can introduce a new basis $|\varphi_m\rangle$, $m=0, \pm 1$, such that $E|\varphi_m\rangle = e^{i\varphi_m} |\varphi_m\rangle$. Then, the Hermitian atomic phase operator ϕ clearly is

$$\phi = \sum_{m=-1}^{+1} \varphi_m |\varphi_m\rangle\langle\varphi_m|$$

$$= -\frac{\psi}{3} \mathbf{1} - \frac{2i\pi}{3\sqrt{3}} (e^{-i\psi/3} E - e^{i\psi/3} E^\dagger). \quad (6)$$

It describes the azimuthal phase of the angular momentum \mathbf{J} . One can see that in Eqs. (5) $S = \sin\phi$ and $C = \cos\phi$. The representation of the SU(2) algebra in the basis $|\varphi_m\rangle$ is of the form

$$\Phi_z = \sum_{m=-1}^{+1} m |\varphi_m\rangle\langle\varphi_m| = -2 \left(S \cos \frac{\psi}{3} + C \sin \frac{\psi}{3} \right),$$

$$\Phi_+ = \sum_{m=-1}^{+1} \sqrt{2 - m(m+1)} |\varphi_{m+1}\rangle\langle\varphi_m|,$$

$$\Phi_- = (\Phi_+)^+, \quad (7)$$

where the operators Φ obey the commutation relations (3). Clearly the representation (7) is dual to (2) [2]. It follows from (7) that $\Phi_z = -2 \sin(\phi + \psi/3)$. The polar decomposition in the dual representation of SU(2) is determined by the corresponding unitary exponential operator

$$\epsilon = |\varphi_+\rangle\langle\varphi_0| + |\varphi_0\rangle\langle\varphi_-\rangle + e^{i\chi} |\varphi_-\rangle\langle\varphi_+|,$$

$$\epsilon\epsilon^\dagger = \mathbf{1}, \quad \epsilon^3 = e^{i\chi} \mathbf{1} \quad (8)$$

and the radial operator

$$\Phi_r = \sum_m \sqrt{2 - m(m+1)} |\varphi_{m+1}\rangle\langle\varphi_{m+1}|.$$

Here χ is an additional real parameter. Since we are primarily interested in the qualitative results, we may as-

sume that $\chi=0$. It enables us to fairly simplify the analysis with no loss of generality. Then

$$\epsilon = e^{2i\pi/3}R_{++} + R_{00} + e^{-2i\pi/3}R_{--}.$$

Thus, in addition to (5) and (6), one can introduce the dual sine, cosine and the phase operators as follows,

$$\begin{aligned} S_\phi &= \frac{\epsilon - \epsilon^+}{2i} = \frac{\sqrt{3}}{2}(R_{++} - R_{--}), \\ C_\phi &= \frac{\epsilon + \epsilon^+}{2} = R_{00} - \frac{1}{2}(R_{++} + R_{--}), \\ \phi_\phi &= \frac{2\pi}{3}(R_{++} - R_{--}) = \frac{4\pi}{3\sqrt{3}}S_\phi. \end{aligned} \quad (9)$$

Thus, the quantum phase properties of the atomic angular momentum \mathbf{J} are completely determined by the set of nine Hermitian operators $\mathbf{1}$, J_z , S , C , ϕ , Φ_z , S_ϕ , C_ϕ , ϕ_ϕ . Among them, only five are independent at any real ψ . Therefore, below we assume $\psi = 0$ for simplicity and turn our attention to the operators $\mathbf{1}$, S , C , S_ϕ , C_ϕ . According to the basis idea of the approach [1], let us determine the following Hermitian operators for the radiation field,

$$\begin{aligned} S_0 &= \sum_{m=-1}^{+1} a_m^+ a_m \equiv \hat{n}, \\ S_1 &= \frac{1}{2}(a_+^+ a_0 + a_0^+ a_- + a_-^+ a_+ + \text{h.c.}), \\ S_2 &= \frac{1}{2i}(a_+^+ a_0 + a_0^+ a_- + a_-^+ a_+ - \text{h.c.}), \\ S_3 &= \frac{\sqrt{3}}{2} \sum_{m=-1}^{+1} m a_m^+ a_m, \\ S_4 &= a_0^+ a_0 - \frac{1}{2}(a_+^+ a_+ + a_-^+ a_-), \end{aligned} \quad (10)$$

such that the combinations $S_0 + \mathbf{1}$, $S_1 + C$, $S_2 + S$, $S_3 + S_\phi$, and $S_4 + C_\phi$ are the integrals of motion for the model Hamiltonian (1). It can be seen that

$$[S_0, S_1] = [S_0, S_2] = [S_0, S_3] = [S_0, S_4] = [S_1, S_2] = 0$$

although

$$[S_{1,2}, S_{3,4}] \neq 0.$$

Thus, S_0 , S_1 , S_2 can be measured at once as well as S_0 , S_3 , S_4 while $S_{1,2}$ and S_3 , S_4 cannot be measured at once.

To clarify the notations and physical meaning of the operators (10), let us consider the radiation generated by the transitions $|j=1; m=\pm 1\rangle \leftrightarrow |j'=0; m'=0\rangle$ while the mode with $m=0$ is chosen to be in the vacuum state. Then, the radiation field consists of two circularly polarized modes with opposite helicities. It is clear that the expectation values of the operators (10) formally coincide (up to constant factors) in this case with the Stokes parameters s_i determined in the circularly polarized basis (see,

for the notations Ref. [13]). Therefore, one can choose to interpret the operators (10) as the generalized Stokes operators of the electric dipole radiation.

To argue this assumption, let us stress that the general picture of the electric dipole radiation both classical and quantum should take into account all three types of polarization in the near zone as well as in the far zone (see, e.g. Ref. [19], chapter 16). In this case, the standard polarization tensor consists of nine components. Only five among them are independent because the natural parameters are the intensities of three components and three phase differences $\delta_{mm'}$, $m \neq m'$ such that $\sum_m \delta_{m,m'} = 0$. It is not hard to see that the operators S_0 , S_3 , S_4 in (10) determine three photon numbers, corresponding to the components with different polarizations. At the same time, the operators S_1 , S_2 determine the phase difference between the components. Thus, the above interpretation of the operators (10) as the generalized Stokes operators is valid.

It should be emphasized that the set of the operators (10) is determined here via the integrals of motion corresponding to the conservation of the angular momentum in the process of radiation according to what has been proposed in Ref. [1]. At the same time, the operators S_1 , S_2 in (10) can be interpreted, in accordance with their construction, as the cosine and sine of the azimuthal phase of the angular momentum (spin) of the electric dipole radiation [1] which we will call below the radiation phase.

Of course, the above consideration within the framework of JCM has led to the result for a single-photon case. In order to generalize it to the multi-photon case of common interest, it is necessary to examine the set of atoms, interacting with the electric dipole radiation. The Dicke model could be used for this aim. At the same time, the analogy with the Stokes operators permits us to find some interesting results immediately, using Eqs. (10). As in classical optics [13,19], to give the operators S_1 , S_2 the meaning of the cosine and sine respectively, one can multiply them by a normalization factor, depending on the intensity and providing the natural limits for the averages. Following this way, we introduce the radiation cosine and sine as follows,

$$C_R = K S_1, \quad S_R = K S_2. \quad (11)$$

It follows from the definitions (10) that the Hermitian operators (11) commute with each other and with the total photon number \hat{n} . In the JCM (1), the constant K is clearly equal to 1 due to the integrals of motion. In a more general case of multi-photon radiation, a convenient form of K is afforded by requiring that

$$\langle C_R^2 + S_R^2 \rangle = 1. \quad (12)$$

It is clear that the definition of the radiation cosine and sine (11) is quite similar to that done within the operational approach [9] for a simple homodine detection

scheme. However, unlike the operational approach, there is no necessity of introducing two different constants here. Actually, one can consider the “exponential of the phase operator” $E_R = C_R + iS_R$ which is supposed to be a unitary one *in average* $\langle E_R E_R^\dagger \rangle = 1$. This natural requirement is equivalent to (12) if we use one and the same constant K for both C_R and S_R in Eq. (11). Then it follows from (12) and (10) that

$$K = \left[\langle \hat{n}_+(1 + \hat{n}_0) + \hat{n}_0(1 + \hat{n}_-) + \hat{n}_-(1 + \hat{n}_+) \rangle + \langle a_+^{\dagger 2} a_0 a_- + a_0^{\dagger 2} a_+ a_- + a_-^{\dagger 2} a_+ a_0 + \text{h.c.} \rangle \right]^{-1/2}. \quad (13)$$

It can be seen that if any one of the modes obey the condition $\langle a_m \rangle = 0$, the second average in (13) vanishes. In the simplest case of only two circularly polarized modes when $\langle a_0 \rangle = 0$, we get

$$K = [I_+ + I_- + I_+ I_-]^{-1/2}, \quad (14)$$

where $I_m \equiv \langle \hat{n}_m \rangle$. As a justification test for the choice of the normalization constant K (13), let us consider all three modes in the number state. Employing Eqs. (10)–(14) then gives $\langle C_R \rangle = \langle S_R \rangle = 0$ and $V(C_R) = V(S_R) = 1/2$ where $V(\cdot)$ denotes the variance. Thus, the definition (11), (12) is consistent with the standard idea of a uniform phase distribution in the number state [10,11]. The same result is clearly valid in the case of the vacuum state. Moreover, if any two modes are in the number or vacuum state, the radiation phase described by the operators (11) has uniform distribution. Therefore, consideration of the radiation phase in a single-mode case has no meaning.

As an additional example of some considerable interest, we now investigate the field in the state $\prod_m |\alpha_m\rangle$ provided by the coherent states of three possible components of the electric dipole radiation. This case permits us to examine the classical limit of strong coherent fields. It is a straightforward matter to arrive at the conclusion that $V(C_R)$, $V(S_R) \rightarrow 0$ when the intensities of all three modes tend to infinity. Thus, the radiation phase determined by Eqs. (11), (12) shows the right behavior in the classical limit.

Since in the far zone the intensity of the linearly polarized component of the electric dipole radiation is quite small it is reasonable to choose $\alpha_0 = 0$ which enables us to fairly simplify the analysis. Carrying out averaging, we get

$$\begin{aligned} s_0 &\equiv \langle S_0 \rangle = I_+ + I_-, \\ s_1 &\equiv \langle S_1 \rangle = \sqrt{I_+ I_-} \cos \delta_{+-}, \\ s_2 &\equiv \langle S_2 \rangle = \sqrt{I_+ I_-} \sin \delta_{+-}, \\ s_3 &\equiv \langle S_3 \rangle = \frac{\sqrt{3}}{2} (I_+ - I_-), \\ s_4 &\equiv \langle S_4 \rangle = -\frac{1}{2} (I_+ + I_-) = -\frac{1}{2} s_0, \end{aligned} \quad (15)$$

where $\delta_{+-} \equiv \arg \alpha_+ - \arg \alpha_-$ and $I_m \equiv |\alpha_m|^2$. In this special case $s_4 \sim s_0$. Thus, averaging (10) with respect to the state $|\alpha_+\rangle |0_0\rangle |\alpha_-\rangle$ determines the standard set of four Stokes parameters determined in the circularly polarized basis [13]. In this case, the parameters s_1, s_2 determine the cosine and sine of the phase difference between two components with opposite helicities. At first sight, this correspondence leads to the choice of the constant in (11) as $K = (I_+ I_-)^{-1/2}$ instead of (13). Actually, this choice leads to unphysical behavior of the averages $\langle C_R \rangle$, $\langle S_R \rangle$ and variances $V(C_R)$, $V(S_R)$ at $I_+, I_- \rightarrow 0$. Employing the conditions (11)–(14) then gives

$$\langle C_R \rangle = \frac{\cos \delta_{+-}}{\sqrt{1 + I_+^{-1} + I_-^{-1}}}, \quad \langle S_R \rangle = \frac{\sin \delta_{+-}}{\sqrt{1 + I_+^{-1} + I_-^{-1}}}. \quad (16)$$

Then, in the classical limit of high intensities we get $\langle C_R \rangle \rightarrow \cos \delta_{+-}$, $\langle S_R \rangle \rightarrow \sin \delta_{+-}$ and $V(C_R)$, $V(S_R) \rightarrow 0$. Consider now the variance $V(C_R)$ as a function of I_+ at fixed I_- . Since

$$V(C_R) = \frac{I_+ + I_- + \sqrt{I_+ I_-} \cos \delta_{+-}}{2(I_+ + I_- + \sqrt{I_+ I_-})} \quad (17)$$

we get $V(C_R) \rightarrow 1/2$ at $I_+ \rightarrow 0$. Under the condition $1 \geq \cos \delta_{+-} > \sqrt{I_+ I_-}$ which can be realized in the strong quantum case of low intensities, the value of the variance can exceed 1/2 at some I_+ . The maximum of (17) is achieved at

$$I_+ = I_- \left[\frac{\sqrt{I_-^2 + (1 + I_-) \cos \delta_{+-}} - I_-}{(1 + I_-) \cos \delta_{+-}} \right]^2.$$

At the same point, the variance $V(S_R)$ has a minimum. The qualitative explanation of this effect is based on the consideration of the probability to have a given phase difference. At $I_+ = 0$, there is a uniform probability distribution in the system. Creation of very few photons of the mode $m = +1$ leads to the formation of some domains with almost equal probabilities having phase difference δ_{+-} and $\delta_{+-} + \pi$. So, it looks like a “phase bunching”. Further increase of I_+ leads to formation of a more or less sharp probability distribution which cannot reach the δ -function because the variance (17) achieves the saturation point

$$\lim_{I_+ \rightarrow \infty} = \frac{1}{2(1 + I_-)}.$$

The saturation as well as the “phase banching” cannot occur at $I_+ = I_- \equiv I$. In this case, we get the formal limit

$$\lim_{I \rightarrow 0} = \frac{2 + \cos \delta_{+-}}{4}.$$

To avoid the illusory contradiction with the above result for the vacuum state, one has to average this expression over $\delta_{+-} \in [0, 2\pi]$.

Let us compare our results with those obtained within the Pegg-Barnett approach (PBA) [20] which has received a lot of attention during the last ten years (see, for a recent review Refs. [10,21]) and has led to many important results. We use here the form of PBA considered recently in Ref. [22]. Then, the phase distribution over the phases of two circularly polarized modes is determined as follows,

$$P_\psi = |\langle \phi_+, \phi_- | \psi \rangle|^2,$$

where $|\phi_+, \phi_- \rangle$ is the Susskind-Glogower phase state and $|\psi \rangle$ is the state of the radiation field. To establish the connection with the results already obtained in this paper, suppose that $|\psi \rangle = |\alpha_+ \rangle |0_0 \rangle |\alpha_- \rangle$. Since our formalism is focused on the phase difference rather than individual phase we need to use the distribution function for the relative phase $\phi = \phi_+ - \phi_-$. Referring the procedure suggested in Refs. [22,23] to cast the range of ϕ into 2π range from 4π range, we take

$$P_{2\pi} = \sum_{n=0}^{\infty} |\langle \phi^{(n)} | \psi \rangle|^2,$$

$$|\phi^{(n)} \rangle = \frac{1}{\sqrt{2\pi}} \sum_{n_+=0}^n e^{i\phi n_+} |n_+, 0, n - n_+ \rangle.$$

Using this distribution function, one can calculate the mean value of any function $F(\phi)$ of the relative phase as follows,

$$\langle F(\phi) \rangle = \int_{-\pi}^{\pi} d\phi P_{2\pi}(\phi) F(\phi).$$

Then

$$\begin{aligned} \langle \cos \hat{\phi}_{\text{PB}} \rangle &= e^{-(I_+ + I_-)} \sum_{n_{\pm}=0}^{\infty} \frac{I_+^{n_+} I_-^{n_-}}{n_+! n_-!} \\ &\times \frac{\text{Re}(\alpha_+ \alpha_-^*)}{\sqrt{(n_+ + 1)(n_- + 1)}}, \\ \langle \cos^2 \hat{\phi}_{\text{PB}} \rangle &= \frac{1}{2} + \frac{e^{-(I_+ + I_-)}}{2} \sum_{n_{\pm}=0}^{\infty} \frac{I_+^{n_+} I_-^{n_-}}{n_+! n_-!} \\ &\times \frac{\text{Re}(\alpha_+ \alpha_-^*)}{\sqrt{(n_+ + 2)(n_+ + 1)(n_- + 2)(n_- + 1)}}. \end{aligned} \quad (18)$$

To clarify the difference between the two approaches let us represent our results (16), (17) as follows

$$\begin{aligned} \langle C_R \rangle &= \frac{\text{Re}(\alpha_+ \alpha_-^*) e^{-(I_+ + I_-)}}{\sqrt{I_+ + I_- + I_+ I_-}} \sum_{n_{\pm}=0}^{\infty} \frac{I_+^{n_+} I_-^{n_-}}{n_+! n_-!}, \\ \langle C_R^2 \rangle &= \frac{1}{2} + \frac{\langle C_R \rangle}{2\sqrt{I_+ + I_- + I_+ I_-}} \\ &+ \frac{[\text{Re}(\alpha_+ \alpha_-^*)]^2}{2(I_+ + I_- + I_+ I_-)} \sum_{n_{\pm}=0}^{\infty} \frac{I_+^{n_+} I_-^{n_-}}{n_+! n_-!}. \end{aligned} \quad (19)$$

One can see that each term in the sums in Eqs. (18) has different normalization while in Eqs. (19), all the terms have one and the same normalization factor related to our choice of the constant K (12). In addition, the expression for $\langle C_R^2 \rangle$ contains an extra term proportional to $\langle C_R \rangle$. This term comes from the vacuum fluctuations related to the mode $m = 0$. This causes a striking difference when one of the modes $m = \pm 1$ is in the quantum domain. Exactly, the existence of the “phase banching” is stipulated just by this term (see Fig. 1). At the same time, both approaches show the saturation of the variance when one of the intensities tends to infinity while the second is kept constant. We should note as well the same qualitative difference from the results obtained by the polar decomposition of the standard Stokes operators in a finite-dimensional Hilbert space in the Ref. [22].

Let us briefly discuss the results. The approach based on the definition of the radiation phase via the conservation of the angular momentum in the process of radiation [1] leads to the definition of five operators, forming the set of the generalized Stokes operators in the case of electric dipole radiation. According to the construction, two of them can be interpreted as the Hermitian cosine and sine of the azimuthal phase operators of the angular momentum of radiation (at the corresponding normalization). These operators manifest a quite reasonable behavior in the clas-

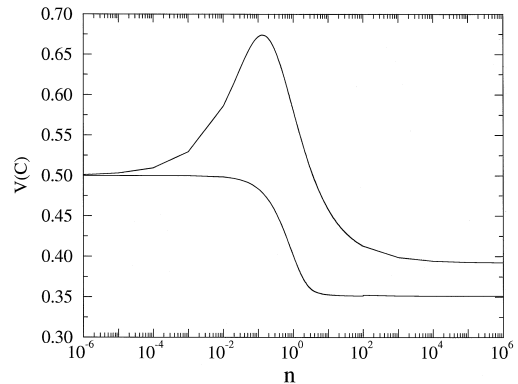


Fig. 1. Variance of cosine. Lower curve is for the Pegg-Barnett cosine, the upper curve is the C_R operator. Both curves are drawn for $\delta = 0$ and $I_+ = 0.275$.

sical limit as well as in the quantum domain. In the simplest case of only two circularly polarized modes, the radiation phase formally coincides with the phase difference between these two modes, although, in the general case of all three modes, it depends on the phase differences between all pairs of modes. It should be stressed that the contribution of the linearly polarized component $m = 0$ is important even if this component is in the vacuum state because it influences the vacuum fluctuations. This influence can lead to qualitative effects such as “phase bunching”. The role of the third component in the quantum fluctuations is a distinctive feature of our approach in comparison with the approaches based on the cutoff of the Hilbert space [20,22]. In reality, the cutoff of the Fock basis leads to the definition of the unit operator which can be considered as some approximation of the Casimir operator of the SU(2) sub-algebra, describing the angular momentum of radiation, but only in a particular sub-space of the Hilbert space. Existence of the unit operator makes it possible to perform polar decomposition and determine the corresponding quantum phase properties. At the same time, the cutoff procedure reduces the algebraic properties which are responsible for the quantum fluctuations, first of all. The limit taken after the calculation of all expectation values cannot completely restore these properties which are especially important in the quantum domain.

In connection with the measurement of the cosine and sine (11) or the Stokes operators S_1 , S_2 in (10), we should note that the standard operational eight-port scheme [8,9] can be used for this aim. Actually, since the total photon number and S_1 , S_2 commute, the different inputs should consist of a mixture of the linearly polarized component with different circularly polarized components such that each output includes all three components. It is a straightforward matter to check that the standard operational relations [9] determine in this case just the operators S_1 , S_2 and S_0 modified by the parameters of the beamsplitters.

Let us also note that the phase distribution for the radiation phase under consideration can be found using a sub-space of the Hilbert space which is provided by the eigenfunctions of the operator $(M)^2$ with given eigenvalues.

Let us stress at the conclusion that there are different phases related to different schemes of detection [9] (the geometrical phase, etc.). The above considered radiation phase is related to the angular momentum, has a simple

physical meaning in terms of the polarization properties of radiation and can be measured in the eight-port homodine detection.

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