Theory and Methodology

Tool allocation and machining conditions optimization for CNC machines

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Abstract

In the literature, there exist many variations of machining economics problem in terms of modelling approaches and solution methodologies. However most of the existing studies focus on the single machining operation which is seldom in practice. On the other hand, tool management approaches at the system level fail to relate the tooling issues to the machining conditions, and ignore the tool availability and tool wear restrictions. A new solution methodology is developed to determine the optimum machining conditions and tool allocation simultaneously to minimize the production cost of a multiple operation case where there can be alternative tools for each operation. As a result, we can both improve the solution by exploiting the interaction between these two decisions, and also prevent any infeasibility that might occur for the tool allocation problem due to tool contention among the operations for a limited number of tool types by considering the tool availability and tool life limitations.

Keywords: Manufacturing; Modelling; Tool allocation; Machining parameters

1. Introduction

In view of the high investment and operating costs of CNC machines and hence of flexible manufacturing systems (FMSs), attention should be paid to their effective utilization. However, the flexibility and the complex nature of such systems result in also more complex planning and control problems, which usually require a nonlinear mixed-integer programming (MIP) or 0–1 integer programming (IP) formulations resisting to exact solutions [12]. In the literature, the planning and control problems of FMSs are usually decomposed into a hierarchical structure corresponding to different time spans of a planning period [13]. Tool management considerations, such as tool scheduling, tool wear and tool replacement, are handled at the lowest level of the hierarchy which corresponds to the real-time operation of the system. However, the upper levels of the hierarchy, which govern the medium or long time spans of a planning horizon, consider production planning problems including the part type selection and loading, and tool allocation at the system level. Stecke [12] formulates the FMS loading problem as a nonlinear MIP and solves it through linearization techniques. Sarin and Chen [11] give a MIP formulation under the assumption that the total machining costs depend upon the tool-machine combination. The tool life is

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considered as a constraint in the formulation. Kouvelis and Lee [10] present an alternative IP formulation of the FMS loading problem by utilizing a block angular structure to avoid nonlinearity of the constraints.

Gray et al. [6] give an extensive survey on the tool management issues of automated manufacturing systems, and also emphasize that the lack of tooling considerations has resulted in poor performance of these systems. For solving the tool allocation problem at the system level, most of the published studies use 0–1 binary variables to represent tool requirements, and assume constant processing times and tool lives as a priori information by ignoring their interaction with the machining conditions selection and the tool availability restrictions. Therefore, they cannot consider the actual tool wear and the corresponding tool life limitations, hence the resulting tool replacement needs and their impact on the total cost. Furthermore, these studies determine the tool requirements for each operation independently, and fail to consider the contention among the operations for a limited number of tools. Consequently, their solution could lead to either infeasible or inferior results.

At the equipment level, there exist several studies paying attention to tooling issues like the tool selection, tool magazine loading and the minimization of tool switches due to a change in a part mix, at both the long term planning and operational level [1,3,9,14]. Unfortunately, these studies also assume constant processing times and tool lives, even though the tool wear, consequently the tool replacement frequency, is directly related with the machining conditions selection. Further, in the multiple operation case, non-machining time components, such as the tool replacements, can have a significant impact on the total cost of production because of the relatively short tool lives of many turning tools as stated by Gray et al. [6]. In the same study, they reported that tools are changed ten times more often due to tool wear than due to part mix.

The machining conditions optimization for a single operation is a well known problem, where the decision variables are the cutting speed and feed rate. Several models and solution methodologies have been developed in the literature [4,7]. However, these models only consider the contribution of machining time and tooling cost to the total cost of operation, and they usually ignore the contribution of non-machining time components to the operating cost, which could be very significant for the multiple operation case. Further, the existing studies exclude the tooling issues such as the tool availability and the tool life capacity limitations. As a result, their results can lead to infeasibility due to tool contention among the operations for a limited number of tool types. We propose a new solution methodology to make tool allocation and machining conditions selection decisions simultaneously by considering the related tooling considerations of tool wear, tool availability, and tool replacing and loading times, since they affect both the machining and the non-machining time components, hence the total cost of production.

2. Problem definition

The aim of this research is to determine the optimum machining conditions and tool allocation to manufacture a batch of parts by a CNC machine on a minimum cost basis. The following assumptions are made to define the scope of this study:

- Each machining operation has a set of candidate tools from a variety of available tool types with limited quantities on hand.
- For the machining operations, the cutting speed and the feed rate will be taken as the decision variables, and the depth of cut is assumed to be given as an input.
- For every tool, the remaining tool life prior to the tool replacement is taken into consideration as a tool waste cost.
- Since the tool changing events during an operation might adversely affect the surface finish requirements, each machining operation is assumed to be completed by a single tool type, even though alternative tools are considered for each operation.
- The tool magazine arrangement problem, in conjunction with the tool sharing and operation sequencing decisions, is preceded by the proposed tool allocation and machining conditions optimization problem.

The machining time expression for a turning operation, $t_{m_{ij}}$, is given below [8]. A list of notations used

throughout the paper is provided in Appendix A.

$$t_{m_{ij}} = (\pi D_i L_i)/(12 v_{ij} f_{ij}). \qquad (1)$$

The relationship between the tool life and the machining conditions is expressed by using the following extended form of *Taylor's tool life* equation:

$$T_{ij} = C_j/\left(v_{ij}^{\alpha_j} f_{ij}^{\beta_j} d_i^{\gamma_j}\right). \qquad (2)$$

By combining Eqs. (1) and (2), a new expression can be derived for the machining time to tool life ratio of the turning operation, which is called as the *usage rate of tool j in operation i*, and denoted by U_{ij}. Consequently, $p_{ij} = \lfloor 1/U_{ij} \rfloor$ and $n_{ij} = \lceil N_B/p_{ij} \rceil$. It is also possible to derive similar expressions for the other machining operations.

$$U_{ij} = \frac{t_{m_{ij}}}{T_{ij}} = \frac{\pi D_i L_i d_i^{\gamma_j}}{12 C_j v_{ij}^{(1-\alpha_j)} f_{ij}^{(1-\beta_j)}}. \qquad (3)$$

All time consuming events except the actual cutting operation are called non-machining time components. Even though there might be many distinct non-machining time components such as tool tuning, workpiece loading/unloading, etc., we only consider tool replacing times, t_{r_j}, and loading times, t_{l_j}, since they are the only ones that can be expressed as a function of both machining conditions and alternative operation–tool pairs.

A general mathematical formulation of the problem is stated below, where the total cost of manufacturing for a particular batch is expressed as the sum of operating cost due to machining time and non-machining time components, the tooling cost, and tool waste cost, respectively. The objective function is a function of machining conditions selection decisions, v_{ij} and f_{ij}, and tool allocation decisions, x_{ij} and n_{ij}.

Minimize

$$C_{tm} = N_B C_0 \left(\sum_{i \in I} \sum_{j \in J} x_{ij} t_{m_{ij}} \right)$$

$$+ C_0 \left(\sum_{i \in I} \sum_{j \in J} x_{ij} \left((n_{ij} - 1) t_{r_j} + t_{l_j} \right) \right)$$

$$+ \sum_{i \in I} \sum_{j \in J} x_{ij} n_{ij} C_{t_j}$$

$$+ \sum_{i \in I} \sum_{j \in J} C_{t_j} \lfloor N_B/p_{ij} \rfloor (1 - p_{ij} U_{ij})$$

subject to
• Tool assignment constraints:

$$\sum_{j \in J} x_{ij} = 1, \quad \text{for every } i \in I,$$

$$\sum_{i \in I} \sum_{j \in J} (1 - y_{ij}) \, x_{ij} = 0;$$

• Tool availability constraint:

$$\sum_{i \in I} x_{ij} \cdot n_{ij} \le t_j, \quad \text{for every } j \in J;$$

• Tool life covering constraint:

$$x_{ij} U_{ij} \le 1, \quad \text{for every } i \in I, \, j \in J;$$

• Machine power constraint:

$$x_{ij} C_m v_{ij}^b f_{ij}^c d_i^e \le HP_{\max}, \quad \text{for every } i \in I, \, j \in J;$$

• Surface roughness constraint:

$$x_{ij} C_s v_{ij}^g f_{ij}^h d_i^L SF_{\max_i}, \quad \text{for every } i \in I, \, j \in J.$$

In this nonlinear MIP formulation, there exist three types of constraints, namely, operational, tool related and machining operation constraints. The first set of constraints represents the operational constraints which ensure that each operation is assigned to a single tool type of its candidate tools set. The tool availability and tool life covering constraints are the tool related constraints which guarantee that the solution will not exceed the available quantity on hand and the available tool life capacity for any tool type, respectively. Finally, last two set of constraints represent the usual machining operation constraints. The surface roughness presents the quality requirement on the operation and the machine power constraint provides to operate machine tool without being subject to any damage.

We will now discuss the complexity of solving the tool allocation and machining conditions optimization problem, call it $\mathscr{R}$, formulated above. Consider a special case of the general formulation in which the number of tools required by each operation, n_{ij}, are already found by solving a single-machining optimization problem for each candidate tool. So we define a new problem $\mathscr{R}'$, called a feasible tool allocation problem, by relaxing the machining operation constraints. Consequently, assigning the set of operations I into tool types of their candidate tool set, J_i, where each tool type has a limited

quantity on hand, amounts to packing the set of operations I into the minimum number of bins, where each bin has a capacity t_j. Therefore, this bin packing problem is a special case of the original problem $\mathscr{R}$. Garey and Johnson [5] showed that the bin packing problem is $\mathscr{NP}$-complete by using a transformation from the PARTITION problem. Hence, we can conclude that the tool allocation and machining conditions optimization problem is $\mathscr{NP}$-complete, since the transformation function from problem $\mathscr{R}$ to $\mathscr{R}'$ is of polynomial complexity as shown in the next section.

3. Single machining operation problem

In order to solve the tool allocation and machining conditions optimization problems simultaneously, we can devise a two-stage decision scheme by using the classical single machining operation problem (SMOP) as a key. In the SMOP, the objective function, $C_{m_{ij}}$, subject to the machining operation constraints, can be expressed as follows:

$$C_{m_{ij}} = \text{Operating Cost} + \text{Tooling Cost}$$

$$= C_0 t_{m_{ij}} + C_{t_j} U_{ij}.$$

Further, it is possible to solve this problem by combining the tool availability and tool life covering constraints in the form of tool life constraint given in below. In this new constraint, p_{ij} is a positive integer corresponding to a desired level of tool requirement, n_{ij}.

$$U_{ij} \leq 1/p_{ij}. \tag{4}$$

Now, by substituting the Eqs. (1) and (3), and rearranging the terms, the following standard mathematical formulation of geometric programming (GP) can be written for the SMOP [2] for every possible operation and tool pair:

Minimize $C_{m_{ij}} = C_1 v_{ij}^{-1} f_{ij}^{-1} + C_2 v_{ij}^{(\alpha_j-1)} f_{ij}^{(\beta_j-1)}$

subject to

$$C_t' v_{ij}^{(\alpha_j-1)} f_{ij}^{(\beta_j-1)} \leq 1,$$

$$C_m' v_{ij}^b f_{ij}^c \leq 1,$$

$$C_s' v_{ij}^g f_{ij}^h \leq 1,$$

$$v_{ij}, f_{ij} > 0,$$

where,

$$C_1 = \frac{\pi D_i L_i C_0}{12},$$

$$C_2 = \frac{\pi D_i L_i d_i^{\gamma_j} C_{t_j}}{12 C_j},$$

$$C_t' = \frac{\pi D_i L_i d_i^{\gamma_j} p_{ij}}{12 C_j},$$

$$C_m' = \frac{C_m d_i^e}{HP_{max}},$$

$$C_s' = \frac{C_s d_i^l}{SF_{max_i}}.$$

In the above formulation, the first constraint is the tool life constraint, and the others are machine power and surface roughness constraints, respectively. The associated GP-Dual problem for the above single machining optimization formulation is given below. Even the objective function for the dual problem is still a nonlinear one, the constraints of the dual formulation are well-defined linear equations.

Maximize Q^*

$$= \left(\frac{C_1}{Y_1}\right)^{Y_1} \left(\frac{C_2}{Y_2}\right)^{Y_2} (C_t')^{Y_3} (C_m')^{Y_4} (C_s')^{Y_5}$$

subject to:
- Normality condition:

$$Y_1 + Y_2 = 1;$$

- Orthogonality conditions:

$$-Y_1 + (\alpha_j - 1)Y_2 + (\alpha_j - 1)Y_3 + bY_4 + gY_5 = 0,$$

$$-Y_1 + (\beta_j - 1)Y_2 + (\beta_j - 1)Y_3 + cY_4 + hY_5 = 0;$$

- $Y_1, Y_2, Y_3, Y_4, Y_5 \geq 0.$

The dual problem can be solved by using the complementary slackness conditions between dual variables and primal constraints, which are given below, in addition to constraints of both the primal and dual problems.

$$Y_3\left(C_t' v_{ij}^{(\alpha_j-1)} f_{ij}^{(\beta_j-1)} - 1\right) = 0,$$

$$Y_4\left(C_m' v_{ij}^b f_{ij}^c - 1\right) = 0,$$

$$Y_5\left(C_s' v_{ij}^g f_{ij}^h - 1\right) = 0.$$

Each of the constraints of primal problem can be either loose or tight at the optimality. Therefore, the principle to solve this dual problem is checking every possibility for the constraints of primal problem and solving the corresponding dual. If a dual feasible solution exists then the corresponding primal solution can be evaluated in terms of its decision variables, and consequently the primal feasibility of the solution will be checked. At the optimality, the corresponding solution should be feasible in both the dual and primal problems, and the objective function value for both problems should be the same. Since we have three constraints in the primal problem, there exist eight different cases for the dual problem, but only six of them are feasible as stated below.

Theorem 1. *In the constrained SMOP, at least one of the surface roughness or machine power constraints must be tight at the optimal solution.*

Proof. There exist only two cases where both constraints can be loose at the optimality. If the tool life covering constraint is tight only, then the dual variables Y_4 and Y_5, which correspond to the machine power and surface roughness constraints, respectively, are both equal to zero. Therefore, they can be eliminated from the set of linear equations in the dual problem. We also know that the inequality of, $\alpha_j > \beta_j$, $\gamma_j > 1$, always holds for extended Taylor's tool life expression, T_{ij}, as shown by Gorczyca [8]. Since $\alpha_j \neq \beta_j$, the solution for this case is $Y_1 = 0$, $Y_2 = 1$ and $Y_3 = -1$. Therefore, this case is *infeasible* since $Y_3 < 0$. As a conclusion, the tool life covering constraint *cannot* be tight just itself. For the second case, if all the constraints are loose, then the dual variables Y_3, Y_4 and Y_5 are equal to zero. This system is infeasible since α_j and β_j cannot be equal to each other, which makes the system of equality inconsistent. Therefore, the occurrence of such a case in constrained SMOP is also impossible. The remaining cases include one of the mentioned constraints □.

The exact solution for the extended version of SMOP can be found by solving each of the aforementioned six cases at the worst case. Lets look at the two of the remaining six cases to show how we derived closed form expressions for primal and dual

variables. If only the surface roughness constraint is tight then Y_5 should be nonnegative because of the dual feasibility constraints. Furthermore, the tool life covering and the machine power constraints are loose, so the corresponding dual variables Y_3 and Y_4 are both equal to zero due to the complementary slackness conditions. Therefore, the constraints of GP-dual problem are reduced to the following system:

$$Y_1 + Y_2 = 1,$$

$$-Y_1 + (\alpha_j - 1)Y_2 + gY_5 = 0,$$

$$-Y_1 + (\beta_j - 1)Y_2 + hY_5 = 0.$$

The solution for this system can be stated explicitly as follows:

$$Y_1 = 1 - Y_2, \qquad Y_2 = \frac{g - h}{g\beta_j - h\alpha_j},$$

$$Y_5 = \frac{\alpha_j - \beta_j}{h\alpha_j - g\beta_j},$$

where $g\beta_j - h\alpha_j \neq 0$, since $g < 0$, α_j, $\beta_j > 1$ and $h > 0$.

The following conditions should be satisfied to verify dual feasibility of the solution:

$$0 \leq Y_1, Y_2 \leq 1, \qquad Y_5 \geq 0.$$

When both surface roughness and tool life covering constraints are tight, Y_3 and Y_5 should be nonnegative, whereas Y_4 is equal to zero. Therefore, the following system can be written by using the complementary slackness conditions:

$$C_t' v_{ij}^{(\alpha_j - 1)} f_{ij}^{(\beta_j - 1)} = 1,$$

$$C_s' v_{ij}^g f_{ij}^h = 1.$$

By taking logarithmic transform, above system turns to a system of linear equations with two equations and two unknowns, which is solved for v_{ij} and f_{ij}, as follows:

$$v_{ij} = \exp\left(\frac{h \ln(1/C_t') - (\beta_j - 1) \ln(1/C_s')}{h(\alpha_j - 1) - g(\beta_j - 1)} \right),$$

$$f_{ij} = \exp\left(\frac{(\alpha_j - 1) \ln(1/C_s') - g \ln(1/C_t')}{h(\alpha_j - 1) - g(\beta_j - 1)} \right),$$

where $h(\alpha_j - 1) - g(\beta_j - 1) \neq 0$.

After finding v_{ij}, f_{ij} and corresponding $C_{m_{ij}}$, dual variables Y_1 and Y_2 can be calculated as they give the weight of each term in the primal objective function:

$$Y_1 = \frac{C_1 v_{ij}^{-1} f_{ij}^{-1}}{C_{m_{ij}}}, \qquad Y_2 = 1 - Y_1.$$

If the solution is dual feasible in terms of Y_1 and Y_2, i.e. $0 \le Y_1, Y_2 \le 1$, then the following system is solved for Y_3 and Y_5:

$$(\alpha_j - 1)Y_3 + gY_5 = Y_1 - (\alpha_j - 1)Y_2,$$

$$(\beta_j - 1)Y_3 + hY_5 = Y_1 - (\beta_j - 1)Y_2.$$

The overall solution for this case is dual feasible if $Y_3, Y_5 \ge 0$. Therefore, we can find the exact solution very quickly as shown in Section 6 on a numerical example, since the explicit analytic expressions of the solution in each case are derived due to the proposed decomposition procedure. As a result, the proposed approach finds the optimum machining conditions after solving J_i problems for each operation $i \in I$ and has a polynomial time complexity of $O(IJ_i)$.

4. Proposed heuristic method

The following heuristic is proposed to reduce the initial candidate tool set to a single tool for every operation by considering the tool availability constraint, and to determine the machining conditions for every selected tool and operation pair.

Step 1. For every possible operation (i, j), such that $(i, j) \in \{(i, j) \mid y_{ij} = 1\}$, solve SMOP using the procedure defined in Section 3, and p_{ij} values are initially equal to $\lceil N_B/t_j \rceil$ to ensure the feasibility in terms of tool availability constraint. Then, update p_{ij} according to the optimum v_{ij}, f_{ij} and U_{ij}, and calculate the corresponding n_{ij}.

Step 2. In the multiple operation case, a lower cost measure can be obtained while increasing the cost of SMOP, $C_{m_{ij}}$, due to a possible decrease in tool waste and tool replacement costs. Therefore, for every operation (i, j), the alternative setting having the minimum cost measure must be searched among

the possible p_{ij} and n_{ij} pairs. The following cost measure is proposed to rank a set of alternative tools for a particular operation in terms of their desirability for this operation.

$$\overline{C}_{ij} = N_B C_{m_{ij}} + C_0 \big[(n_{ij} - 1)t_{r_j} + t_{l_j} \big]$$
$$+ C_{t_j} \lfloor N_B/p_{ij} \rfloor (1 - p_{ij} U_{ij}), \qquad (5)$$

where the first term projects the cost of SMOP over the batch, while the second and third terms account for operating costs due to the non-machining time components and the tool waste cost, respectively. In this cost measure, if the tool life constraint is inactive, then an increase in the total machining cost can be justified by a decrease in the tool waste cost. Therefore, if the tool life constraint is inactive at the optimal solution of SMOP, we search among the p_{ij} values corresponding to the initial n_{ij} value found in the first step and pick the p_{ij} value that gives the minimum cost measure.

Step 3. In the multiple operation case, the solution of the SMOP may not correspond to the global minimum of proposed cost measure as stated above. Therefore the initial n_{ij} value is decreased to the next alternative n'_{ij} setting, which corresponds to a different p'_{ij} and U'_{ij} pair, and the cost measure is evaluated for the new parameters. The proposed cost measure is a convex function of the integer n_{ij} values, provided that:

$$p_{ij} U_{ij} \le p'_{ij} U'_{ij} \quad \text{for } n'_{ij} < n_{ij}.$$

The convexity of the proposed cost measure has been proven in Theorem 2 given in Appendix B. This theorem implies that if an increase in the cost measure is found then we stop and the previous solution corresponds to the global minimum of the proposed cost measure.

Step 4. Create a primal tools set, J_p, such that $J_p = \{j \mid y_{ij} = 1 \text{ and arg min}_{j \in J} \overline{C}_{ij} \text{ for every } I \in I\}$. For every $j \in J_p$, define the corresponding set of operation assignments, I_j, such that $I_j = \{i \mid y_{ij} = 1 \text{ and arg min}_{i \in I} \overline{C}_{ij}, \text{ for every } j \in J_p\}$.

Step 5. For the operations having only a single candidate tool, allocate the candidate tool j to operation i, such that $x_{ij} = 1$. If $n_{ij} = t_j$ then remove the tool j from the available tools set, J, and J_p. Otherwise, reduce the available number of tools, t_j, for further allocations. Update sets $\overline{I}$ and $\overline{J}$.

Step 6. For every $j \in J_p$, calculate the total tool requirement, $R_j = \sum_{i \in I_j} n_{ij}$. If $R_j \le t_j$, allocate tool j for $\forall i \in I_j$, and update $\bar{I}$, $\bar{J}$, and t_j. Otherwise, calculate the deficit tool amount, $\delta_j = R_j - t_j$, and the perturbation ratio, $\rho_j = \delta_j / R_j$.

Step 7. Since the tool availability constraint is violated for the deficit tools, a reduction in their tool requirements is needed, and in this case, the alternative tools should also be considered because a possible increase in the cost of SMOP due to a reduction of tool usage might justify the use of them. Starting from the most critical tool j, or equivalently with the largest perturbation ratio, for every operation $i \in I_j$ span a set of possible perturbations which is presented by an index set of $\Pi_{ij} = P_{ij} \cup S_{ij}$. In this index set, the subset $P_{ij} = \{0, \ldots, \min\{n_{ij}, \delta_j\}\}$ presents the possible perturbations $\pi \in \Pi_{ij}$ in terms of reducing the tool requirement of the best tool in operation i, where $\pi = 0$ corresponds to the no reduction case. The other indices $\pi \in S_{ij} = \{\min\{n_{ij}, \delta_j\} + 1 \ldots, \min\{n_{ij}, \delta_j\} + s(J_i) - 1\}$ represent the situations in which an alternative tool j' can replace the best tool j in operation i. For every $\pi \in \Pi_{ij}$, calculate the corresponding cost increment, as follows:

- For every perturbation $\pi \in P_{ij}$, the cost increment is $\Delta \bar{C}_i^\pi = \bar{C}_{ij}^\pi - \bar{C}_{ij}^0$, where $\bar{C}_{ij}^0$ corresponds to the initial cost measure found at Step 3.
- For every alternative tool $\pi \in S_{ij}$, the cost increment is $\Delta \bar{C}_i^\pi = \bar{C}_{ij'}^\pi - \bar{C}_{ij}^0 + \mu_j' n_{ij'}$, where $\bar{C}_{ij'}^\pi$ corresponds to the cost measure for alternative tool j' and μ_j is the opportunity cost of using an alternative deficit tool, which is equal to $\mu_j' = \max_{k \in I_j} \{\bar{C}_{kj'}^1 - \bar{C}_{kj'}^0\}$ for the deficit tools, and zero otherwise.

Step 8. Solve the following 0–1 IP to find the best perturbation combination that satisfies the related tool availability constraints with a minimum total cost increment, A_j, where z_i^π is a 0–1 binary variable which is equal to 1 if the πth perturbation is selected for operation $i \in I_j$.

Minimize $A_j = \sum_{i \in I_j} \sum_{\pi \in \Pi_{ij}} z_i^\pi \Delta \bar{C}_i^\pi$

subject to

$\sum_{\pi \in \Pi_{ij}} z_i^\pi = 1$, for every $i \in I_j$,

$$\sum_{i \in I_j} \sum_{\pi \in P_{ij}} z_i^\pi n_{ij}^\pi = t_j,$$

$$\sum_{i \in I_j} \sum_{\pi \in S_{ij}} z_i^\pi n_{ij'}^\pi \le t_{j'}, \quad \text{for every } j' \in \left(\bigcup_{i \in I_j} J_i \right)/j.$$

In the above model, the first constraint ensures that a single perturbation will be selected for each operation, and the second constraint represents that the tool usage equals to the available quantity. Third constraint identifies the set of alternative tools for each operation and guarantees that tool availability constraint for these alternative tools will be satisfied too.

Step 9. According to the solution of the above model, update sets $\bar{I}$ and $\bar{J}$, and reduce the available number of tools for every allocated tool type. Remove the tool j from the sets J and J_p. If the set J_p is nonempty then continue with the next tool having the largest perturbation ratio, go to Step 7. Otherwise stop.

5. Exact approach

We now derive a lower bound for the tool allocation and machining conditions optimization problem by relaxing the set of tool availability constraints, which can be called coupling constraints. In this resource directed decomposition procedure, we first find the optimum machining conditions for every possible operation–tool pair, and select the tool that gives the minimum cost measure as outlined below. These steps are similar to the first four steps of the proposed heuristic method described in the previous section.

Step 1. For every possible operation (i, j), such that $(i, j) \in \{(i, j) | y_{ij} = 1\}$, solve SMOP with $p_{ij} = \lceil N_B / t_j \rceil$ and update p_{ij} and n_{ij} accordingly.

Step 2. Evaluate the proposed cost measure defined in Eq. (5).

Step 3. For every operation, find the global minimum of the proposed cost measure for each candidate tool.

Step 4. Determine sets J_p and I_j.

Step 5. Lower bound is equal to, LB = $\sum_{j \in J_p} \sum_{i \in I_j} \bar{C}_{ij}$.

After deciding on a lower bound on the minimum cost value, the following enumerative approach is proposed to solve the general formulation described in Section 2 optimally.

Step 1. For every possible operation (i, j), solve SMOP to determine n_{ij}.

Step 2. Resolve SMOP for the requirement level, $k \in \{1, 2, \ldots, n_{ij}\}$, of every operation (i, j) to find p_{ij}^k, U_{ij}^k, and the corresponding $C_{m_{ij}}^k$.

Step 3. Evaluate the following cost measure for every operation–tool pair (i, j) at the tool requirement level k.

$$\overline{C}_{ij}^k = N_B C_{m_{ij}}^k + C_0 \left[(k-1)t_{r_j} + t_{l_j} \right] + C_{t_j} \lfloor N_B / p_{ij}^k \rfloor \left(1 - p_{ij}^k U_{ij}^k \right).$$

Step 4. Solve the following IP to find the best allocation for every operation that satisfies the tool availability constraints:

$$\text{Minimize} \sum_{i \in I} \sum_{j \in J_i} \sum_{k=1}^{n_{ij}} \overline{C}_{ij}^k x_{ij}^k$$

subject to:

$$\sum_{j \in J_i} \sum_{k=1}^{n_{ij}} x_{ij}^k = 1 \quad \forall i \in I,$$

$$\sum_{i \in I} \sum_{k=1}^{n_{ij}} k x_{ij}^k \leq t_j \quad \forall j \in J,$$

where x_{ij}^k is a 0–1 binary decision variable which is equal to 1 if the machining of volume i is assigned to tool j at the tool requirement level of k tools. In this formulation, the first constraint ensures that a single allocation will be selected for each operation. The second constraint guarantees that total number

of tool allocations will not exceed the tool availability constraints.

6. A numerical example

In this section, an example part is studied which has twelve prespecified machinable volumes with the geometrical data and the required surface qualities given in Table 1. The geometric description of the part is also illustrated in Fig. 1, in which each machinable volume, V_i, can be machined by a set of candidate tools denoted by an operation–tool pair (i, j). There are six different tool types available. Their technological parameters and the other input data are presented in Tables 2 and 3, respectively.

The possible operation–tool assignments are given by the following 0–1 matrix Y:

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}^T.$$

In the first two steps of the algorithm, the best machining conditions for every possible operation-tool pair is determined for different n_{ij} values. In Table 4, this procedure is illustrated for the Volume-11 and Tool-6 pair, i.e. operation $(11, 6)$, as an example.

In the multiple operation case, the optimal solution of the SMOP may not correspond to the minimum of proposed cost measure as illustrated in Fig. 2 for the operation $(12, 1)$, and also in Table 4 for the operation $(11, 6)$. We found a better solution by decreasing the number of tool requirements, which slightly increased the cost of SMOP but decreased

Table 1
Machinable volume data

	D_i	L_i	d_i	$SF_{\max_i}$		D_i	L_i	d_i	$SF_{\max_i}$
V_1	4	3	0.2	300	V_7	2.6	2	0.05	50
V_2	4	9	0.2	400	V_8	2.6	3	0.25	400
V_3	3.6	3	0.05	75	V_9	2.6	4	0.25	300
V_4	3.6	9	0.25	400	V_{10}	2.1	3	0.25	300
V_5	3.1	2	0.25	300	V_{11}	2.1	4	0.05	40
V_6	3.1	7	0.25	400	V_{12}	1.6	3	0.05	30
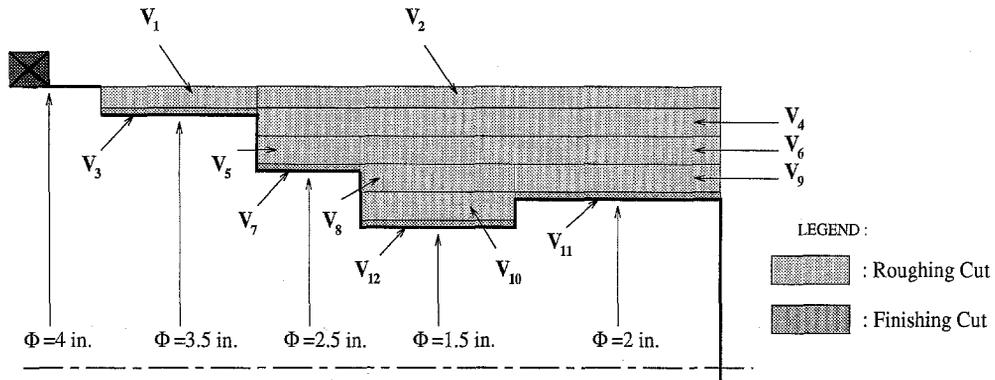
Fig. 1. Machinable volume presentation.

Table 2
Technological exponents and coefficients of the available tools

	α	β	γ	C_j	b	c	e	C_m	g	h	l	C_s
T_1	4.0	1.40	1.16	40960000	0.91	0.78	0.75	2.394	-1.52	1.004	0.25	204620000
T_2	4.3	1.60	1.20	37015056	0.96	0.70	0.71	1.637	-1.60	1.005	0.30	259500000
T_3	3.7	1.28	1.05	11001020	0.80	0.75	0.70	2.415	-1.63	1.052	0.30	205740000
T_4	4.1	1.26	1.05	48724925	0.80	0.77	0.69	2.545	-1.69	1.005	0.40	204500000
T_5	3.7	1.30	1.05	13767340	0.83	0.75	0.73	2.321	-1.63	1.015	0.30	203500000
T_6	4.2	1.65	1.20	56158018	0.90	0.78	0.65	1.706	-1.54	1.104	0.32	211825000

the overall cost measure for the multiple operation case. Furthermore, we can easily conjecture that the proposed cost measure, $\overline{C}_{ij}$, is more effective than the SMOP approaches, which do not consider the non-machining time components and the tool waste cost.

In Step 4, the following sets are formed by using the best machining operation conditions for every possible pair: $I_3 = \{1, 2, 4, 5, 6, 8, 9, 10\}$, $I_5 = \{3\}$, $I_6 = \{7, 11, 12\}$ and $J_p = \{3, 5, 6\}$. Therefore, a lower bound on the minimum cost value is equal to 119.84. Since there is no operation having a single candidate tool, we skip Step 5. In Step 6, we determine the

Table 3
Tooling information

	T_1	T_2	T_3	T_4	T_5	T_6
t_{r_j}	0.75	0.75	0.75	0.75	1	0.75
t_{l_j}	1	1	1	1	1.5	0.75
t_j	2	3	20	10	4	2
C_{t_j}	0.50	0.70	0.70	0.70	0.75	0.75

tools of the set J_p for which the tool availability constraint is violated, as follows:

$$R_3 = n_{1,3} + n_{2,3} + n_{4,3} + n_{5,3} + n_{6,3} + n_{8,3}$$
$$+ n_{9,3} + n_{10,3}$$
$$= 3 + 6 + 6 + 2 + 4 + 2 + 3 + 2 = 28 > t_3 = 20,$$

$$R_5 = n_{3,5} = 2 < t_5 = 4,$$

$$R_6 = n_{7,6} + n_{11,6} + n_{12,6} = 1 + 2 + 1 = 4 > t_6 = 2.$$

For the Tool-5, there exists an excess amount of 2 tools, so this tool and its corresponding volume are appended in the following reservation sets and the available quantity on hand is updated: $\bar{I} = \{3\}$, $\bar{J} = \{5\}$ and $t_5 = 2$. For the others tools, the deficit ratios are as follows:

$$\rho_3 = \frac{28 - 20}{28} \approx 0.2857, \qquad \rho_6 = \frac{4 - 2}{4} = 0.5.$$

From the above values, Tool-6 is found as the most scarce resource. Therefore, we first allocate Tool-6, then continue with the Tool-3. For this purpose, all possible perturbations of the Tool-6 for its

Table 4
Finding the minimum cost measure for operation (11, 6)

n_{ij}	p_{ij}	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	$\overline{C}_{ij}$
3	12	659.02	0.01655	0.2015	2.5721	0.0784	0.1595	6.00
2	15	633.60	0.01567	0.2214	3.3217	0.0667	0.1607	5.57
1	30	535.20	0.01238	0.3318	9.9528	0.0333	0.1909	6.10

$N_B = 30$ parts, $C_0 = \$0.5/\text{min}$, and $HP_{max} = 5$ hp.

operation assignments are generated as explained in Step 7. An example perturbation set is given for the operation (11, 6) of Tool-6 in Table 5, where the cases $\pi = 2$ and $\pi = 3$ correspond the use of alterna-tive tools, Tool-1 and Tool-2, respectively, instead of the primary tool of Tool-6.

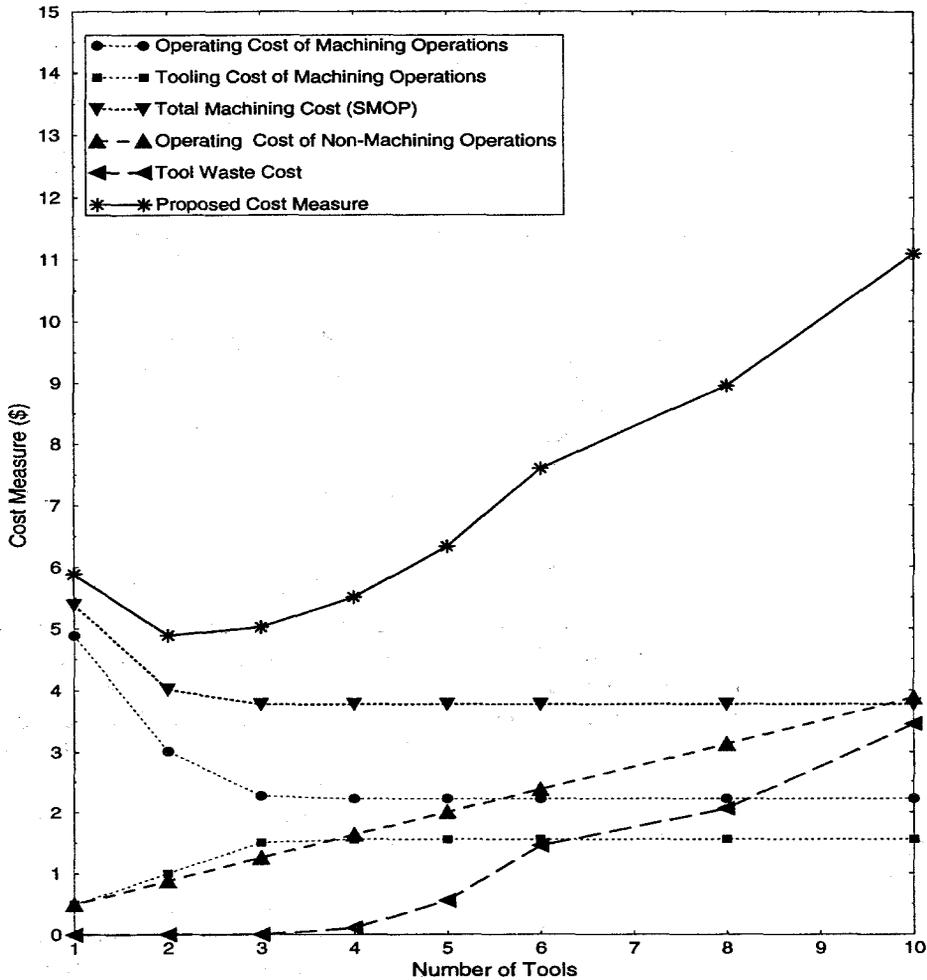For the allocation of Tool-6, the following 0–1 IP is solved to find the best combination of the possible

Fig. 2. The detailed analysis of cost measure for operation (12, 1).

Table 5
Perturbations of operation (11, 6)

π	T#	p_{ij}	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	n_{ij}^{π}	$\overline{C}_{ij}^{\pi}$	$\Delta\overline{C}_i^{\pi}$
0	6	15	633.60	0.01567	0.2214	3.3217	0.0667	0.1607	2	5.57	0
1	6	30	535.20	0.01238	0.3318	9.9528	0.0333	0.1909	1	6.10	0.53
π	T#	p_{if}	v_{if}	f_{if}	$t_{m_{if}}$	T_{if}	U_{if}	$C_{m_{if}}$	n_{if}^{π}	$\overline{C}_{if}^{\pi}$	$\Delta\overline{C}_i^{\pi}$
2	1	15	651.89	0.00799	0.4222	6.3335	0.0667	0.2445	2	8.21	2.64
3	2	10	538.40	0.00908	0.4495	4.4947	0.1000	0.2947	3	10.09	4.52

perturbations for the operations 7, 11 and 12 as discussed in Step 8.

Minimize

$$A_6 = 0.25z_7^1 + (0.26 + 1.30)\, z_7^2 + 1.09\, z_7^3 + 2.56\, z_7^4$$
$$+ 0.53\, z_{11}^1 + 2.64\, z_{11}^2 + 4.52\, z_{11}^3 + 1.35\, z_{12}^1$$
$$+ 2.94\, z_{12}^2$$

subject to:

$$z_7^0 + z_7^1 + z_7^2 + z_7^3 + z_7^4 = 1,$$
$$z_{11}^0 + z_{11}^1 + z_{11}^2 + z_{11}^3 = 1,$$
$$z_{12}^0 + z_{12}^1 + z_{12}^2 = 1,$$
$$z_7^0 + 2\, z_{11}^0 + z_{11}^1 + z_{12}^0 = t_6 = 2,$$
$$2\, z_7^3 + 2\, z_{11}^2 + 2\, z_{12}^2 \le t_1 = 2,$$
$$2\, z_7^4 + 3\, z_{11}^3 + 2\, z_{12}^2 \le t_2 = 3,$$
$$z_7^2 \le t_3 = 20$$
$$z_7^1 \le t_5 = 2.$$

The solution to the above problem is as follows: $z_7^1 = z_{11}^1 = z_{12}^0 = 1$ and $A_6 = 0.78$. This solution suggests to use Tool-5 for the manufacturing of Volume-7 instead of Tool-6, a reduction of a single Tool-6 in the processing of the Volume-11, and it leaves the original solution for the Volume-12 without any reduction in the usage of Tool-6. For the Tool-3, the same IP model has been solved with the new parameters. The possible perturbations were generated after allocating Tool-6 and updating the related sets and tool availabilities. The resulting final tool allocations with the corresponding machining conditions are tabulated in Table 6. The final tool allocation is also represented by the following sets: $\bar{I}_3 = \{1, 2, 4, 5, 6, 9\}$, $\bar{I}_4 = \{8\}$, $\bar{I}_5 = \{3, 7, 10\}$, $\bar{I}_6 = \{11, 12\}$ and $\bar{J} = \{3, 4, 5, 6\}$. As a summary, the initial solution of SMOP was inferior to the proposed cost measure for the multiple operation case as indicated in both Table 4 and Fig. 2, and it was also infeasible due to tool availability constraint resulting from the tool contention among the operations for a

Table 6
Final tool allocation and the machining conditions

V#	T#	p_{ij}	v_{ij}	f_{ij}	$t_{m_{ij}}$	T_{ij}	U_{ij}	$C_{m_{ij}}$	n_{ij}	$\overline{C}_{ij}$
1	3	15	266.13	0.02565	0.4599	6.8990	0.0667	0.2766	2	9.17
2	3	5	256.73	0.03189	1.1506	5.9650	0.1929	0.7103	6	23.83
3	5	15	528.39	0.02624	0.2038	3.0575	0.0667	0.1519	2	5.81
4	3	6	236.50	0.02635	1.3604	8.1623	0.1667	0.7969	5	25.91
5	3	30	245.79	0.02128	0.3102	9.3053	0.0333	0.1784	1	5.85
6	3	8	242.92	0.02747	0.8510	7.0095	0.1214	0.5105	4	17.00
7	5	30	555.22	0.01905	0.1286	3.8584	0.0333	0.0893	1	3.43
8	4	15	214.75	0.03025	0.3142	4.7125	0.0667	0.2038	2	6.99
9	3	15	259.98	0.02321	0.4509	6.7640	0.0667	0.2721	2	9.04
10	5	30	270.56	0.02181	0.2793	8.5375	0.0327	0.1642	1	5.69
11	6	30	535.20	0.01238	0.3318	9.9528	0.0333	0.1909	1	6.10
12	6	30	639.16	0.01222	0.1608	4.8244	0.0333	0.1054	1	3.54

Table 7
Results of the computational experiments

No. of operations	Algorithm	Total cost (in $)	Deviation
9	Lower bound	117.34	2.29%
	Exact approach	120.09	–
	Heuristic	125.82	4.77%
12	Lower bound	119.84	1.82%
	Exact approach	122.06	–
	Heuristic	122.36	0.25%
15	Lower bound	120.38	2.03%
	Exact approach	122.88	–
	Heuristic	124.24	1.11%

Table 8
Comparison of computation time requirements

Number of operations	CPU times (in seconds)		
	SMOP	Lower bound	Exact approach
9	0.04	0.02	1.13
12	0.04	0.02	0.68
15	0.05	0.02	0.90

limited number of tools.

In order to measure the effectiveness of the proposed heuristic method for various size of problems, we have generated two additional examples with 9 and 15 machinable volumes. In the first case, volumes 1–2, 5–6 and 8–9 are combined into three machinable volumes. For the second case, volumes 2, 4 and 6 are further divided into two resulting in 15 volumes. Obviously, each change will create a different solution space for the original problem. We then calculate the total cost values of proposed heuristic method, lower bound and optimal solution found by using LINDO along with the percent deviations from the optimal solution as summarized in Table 7. Furthermore, the required computation times for optimally solving SMOP for every operation–tool pair, lower bound calculations and the exact approach are reported in Table 8 for the same example problems.

7. Conclusions

In this paper, a new solution methodology for the multiple operation case has been developed to solve the tool allocation and machining conditions selection problems simultaneously to find the minimum production cost. For this purpose, the classical SMOP formulation has been extended by adding a new tool life constraint, which enabled us to include tooling issues like tool wear and tool availability. Furthermore, a new cost measure was proposed that has been particularly devised to identify possible trade-offs among these conflicting decisions of tooling and machining conditions selection, and to link operational level decisions to the system level. By using this cost measure, the heuristic method has enabled to consider alternative candidate tools of every operation and to exploit the interaction between the number of tools required with the machining, tool replacing and loading times, and tool waste cost in conjunction with the optimum machining conditions. Consequently, the proposed method can prevent any infeasibility that may occur for the tool allocation problem at the system level due to tool contention and tool life restrictions through a feedback mechanism. In this respect, this study can be considered as a part of the fully automated process planning system.

Appendix A. Nomenclature

$\alpha_j, \beta_j, \gamma_j$: Speed, feed, depth of cut exponents for tool j
C_j : Taylor's tool life constant for tool j
C_m, b, c, e : Specific coefficient and exponents of the machine power constraint
C_0 : Operating cost of the CNC machine ($/min)
C_s, g, h, l : Specific coefficient and exponents of the surface roughness constraint
C_{t_j} : Cost of the tool j ($/per tool)
D_i : Diameter of the generated surface (in.)
d_i : Depth of cut for operation i (in.)
f_{ij} : Feed rate for operation i using tool j (ipr)
HP_{max} : Maximum available machine power for all operations (hp)
I : Set of all operations
$\bar{I}$: Set of the allocated operations
J : Set of the available tools
$\bar{J}$: Set of the allocated tools

J_i : Set of the candidate tools for the operation i

L_i : Length of the generated surface (in.)

n_{ij} : Number of tool type j required for completion of operation i

N_B : Batch size

p_{ij} : Number of times that an operation i can be performed by a tool type j

$s(\cdot)$: Cardinality of a set

$\text{SF}_{\max_i}$: Maximum allowable surface roughness for the volume i, (μin.)

t_j : Number of available tools on hand for tool type j

t_{l_j} : Tool magazine loading time for a single tool j, (min.)

t_{r_j} : Tool replacing time for tool j, (min.)

v_{ij} : Cutting speed for operation i using tool j, (fpm)

x_{ij} : 0–1 binary decision variable which is equal to 1 if tool j is assigned to operation i

y_{ij} : 0–1 binary indicator which is equal to 1 if tool j is a candidate tool for operation i

Appendix B

Theorem 2. *The following cost measure is a convex function of the integer n_{ij} values:*

$$\overline{C}_{ij} = N_B C_{m_{ij}} + C_0\left[(n_{ij} - 1)t_{r_j} + t_{l_j}\right]$$
$$+ C_{t_j}\lfloor N_B/p_{ij}\rfloor\left(1 - p_{ij}U_{ij}\right)$$

provided that:

$$p_{ij}U_{ij} \leq p'_{ij}U'_{ij} \quad \text{for } n'_{ij} < n_{ij}.$$

Proof. To prove this theorem, the following properties of the convex functions will be devised:

Property 1. *A linear function is convex.*

Property 2. *The sum of convex functions is also convex.*

The proposed cost measure has three components, namely, SMOP, operating cost due to non-machining events, and tool waste cost. The SMOP component is a convex function since its Hessian matrix is positive definite over the possible values of v_{ij} and f_{ij}, hence the integer n_{ij} values (Bazaraa et al. [2]). The non-machining time component is a linear function of the integer n_{ij} values, so it is a convex function due to Property 1. The third component of the measure is the tool waste cost:

$$C_{\text{TW}} = C_{t_j}\lfloor N_B/p_{ij}\rfloor\left(1 - p_{ij}U_{ij}\right).$$

Let us consider two consecutive integer tool requirements such that:

$$n'_{ij} < n_{ij} \quad \text{and} \quad n_{ij} - n'_{ij} \geq 1.$$

We can write the following statement in general:

$$\lfloor N_B/p_{ij}\rfloor = \begin{cases} n_{ij} & \text{if } N_B/p_{ij} \in \mathbb{Z}^+, \\ n_{ij} - 1 & \text{otherwise.} \end{cases}$$

Now, consider the worst case for these two consecutive tool requirements, such that:

$$\lfloor N_B/p'_{ij}\rfloor = n'_{ij} \quad \text{and} \quad \lfloor N_B/p_{ij}\rfloor = n_{ij} - 1.$$

That is,

$$n_{ij} - n'_{ij} \geq 1 \Rightarrow \lfloor N_B/p_{ij}\rfloor \geq \lfloor N_B/p'_{ij}\rfloor.$$

Therefore the tool waste cost component is a non-decreasing function, i.e. a convex function, if the following condition is satisfied:

$$p_{ij}U_{ij} \leq p'_{ij}U'_{ij} \quad \text{for } n'_{ij} < n_{ij}.$$

Consequently, the proposed cost measure is also a convex function over the integer values of n_{ij} due to Property 2.

References

[1] Bard, J.F., and Feo, T.A., "The cutting path and tool selection problem in computer aided process planning", *Journal of Manufacturing Systems* 8/1 (1989) 17–26.

[2] Bazaraa, M.S., Sherali, H.D., and Shetty, C.M., *Nonlinear Programming Theory and Algorithms* (2nd Ed.), Wiley, New York, 1993.

[3] Crama, Y., Kolen, A.W.J., Oerlemans, A.G., and Spieksma, F.C.R., "Minimizing the number of tool switches on a flexible machine", *The International Journal of Flexible Manufacturing Systems* 6/1 (1994) 33–54.

[4] Ermer, D.S., "Optimization of the constrained machining economics problem by geometric programming", *Journal of Engineering for Industry* 93/4 (1971) 1067–1072.

[5] Garey, M.R. and Johnson, D.S., *Computers and Intractability: A Guide to the Theory of NP-Completeness*, Freeman, New York, 1979.

[6] Gray, A.E., Seidman, A., and Stecke, K.E., "A synthesis of decision models for tool management in automated manufacturing", *Management Science* 39/5 (1993) 549–567.

[7] Gopalakrishnan, B., and Al-Khayyal, F., "Machine parameter selection for turning with constraints: An analytical approach based on geometric programming", *International Journal of Production Research* 29/9 (1991) 1897–1908.

[8] Gorczyca, F.E., *Application of Metal Cutting Theory*, Industrial Press, 1987.

[9] Kouvelis, P., "An optimal tool selection procedure for the initial design phase of a flexible manufacturing system", *European Journal of Operational Research* 55/2 (1991) 201–210.

[10] Kouvelis, P., and Lee, H.L., "Block angular structures and the loading problem in a flexible manufacturing system", *Operations Research* 39/4 (1991) 666–676.

[11] Sarin, S.C., and Chen, C.S., "The machine loading and tool allocation problem in a flexible manufacturing system", *International Journal of Production Research* 25/7 (1987) 1081–1094.

[12] Stecke, K.E., "Formulation and solution of nonlinear integer production planning problems for flexible manufacturing systems", *Management Science* 29/3 (1983) 273–288.

[13] Suri, R., and Whitney, C.K., "Decision support requirements in flexible manufacturing", *Journal of Manufacturing Systems* 3/1 (1984) 61–69.

[14] Tang, C.S., and Denardo, E.V., "Models arising from a flexible manufacturing machine, Part I: Minimization of the number of tool switches", *Operations Research* 36/5 (1988) 767–777.