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A Calogero formulation for four-dimensional black-hole microstates

Olaf Lechtenfeld [a,*], Suresh Nampuri [b]

[a] Institut für Theoretische Physik and Riemann Center for Geometry and Physics, Leibniz Universität Hannover, Appelstraße 2, 30167 Hannover, Germany
[b] CAMGSD-IST, Universidade de Lisboa, Av. Rovisco Pais, 1049-001, Lisbon, Portugal

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ABSTRACT

We extract the leading-order entropy of a four-dimensional extremal black hole in $\mathcal{N}=2$ ungauged supergravity by formulating the CFT$_1$ that is holographically dual to its near-horizon AdS$_2$ geometry, in terms of a rational Calogero model with a known counting formula for the degeneracy of states in its Hilbert space.

1. Introduction

A successful statistical mechanical description of black-hole microstates constitutes one of the most precise tests of any purported theory of quantum gravity such as string theory. The most outstanding insight to be gleaned from string theory can be formulated in terms of the holographic AdS/CFT correspondence which establishes an isomorphism between the Hilbert space of quantum gravity in asymptotically AdS spaces and that of a conformal field theory living on the lower-dimensional boundary of the AdS space. Hence, non-perturbative objects in gravity such as black holes have a microstate description as thermal ensembles in the holographically dual theory. The least well understood of the well-studied AdS/CFT correspondences is the AdS$_2$/CFT$_1$ pair, where the dual conformal quantum mechanics is still an outstanding formulation problem in string theory. AdS$_2$ is of more interest than just as a two-dimensional toy model of quantum gravity: Every extremal black hole in four dimensions possesses a near-horizon geometry that can be expressed as the direct product of a black hole in AdS$_2$ and a spherical, planar or hyperbolic horizon of the four-dimensional black hole. The deep-throat geometry of the AdS isolates the constant modes in it from the asymptotic modes of fields in the black-hole background that affect the black-hole horizon and hence its entropy. In fact, the constant modes in the near-horizon geometry are fixed in terms of the quantum numbers of the black hole, and they are independent of their asymptotic values. This is the well known attractor mechanism displayed by these extremal black holes (see [1] and [2] and the references therein for a detailed explication). The holographic Bekenstein–Hawking entropy of the black hole is therefore determined purely by states in the near-horizon region. Hence, an encoding of these states in the dual conformal quantum mechanics attains significance in identifying the holographically dual conformal quantum mechanics and counting the microstates of the black hole. In this article, we look at the induced worldline superconformal quantum mechanics of an n-particle BPS system moving in the background of a black-hole in AdS$_2$. This quantum mechanics has a reformulation [3] in terms of an n-particle rational Calogero model (of type A_{n-1}),[1] and we argue that this encodes the thermal-ensemble states corresponding to the black hole in the holographically dual CFT$_1$. We justify this assertion by counting the large-charge degeneracy of states in this model to arrive at the Bekenstein–Hawking entropy of the dual black hole in AdS$_2$.

2. Calogero dynamics and extremal black holes

The near-horizon geometry of a zero-temperature BPS black-hole solution in four-dimensional ungauged supergravity is a black hole in AdS$_2 \times S^2$, whose geometry is described as

$$\mathrm{d}s^2 = -\frac{r^2-\Delta^2}{b_*^2}\mathrm{d}t^2 + \frac{b_*^2}{r^2-\Delta^2}\mathrm{d}r^2 + b_*^2\mathrm{d}\Omega_2^2\,. \tag{2.1}$$

We restrict ourselves to only bosonic backgrounds in the theory. The scalar fields ϕ^i that make up the moduli space in this background and do not correspond to flat directions of the scalar potential are driven to a critical point of this potential. They flow from the asymptotically flat space to the near-horizon geometry, and their extremum values ϕ^i_* are fixed entirely in terms of the quantum numbers of the system, independent of the asymptotic

* Corresponding author.
E-mail addresses: olaf.lechtenfeld@itp.uni-hannover.de (O. Lechtenfeld), nampuri@gmail.com (S. Nampuri).

[1] See also [4,5] for recent related work.

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starting values. Hence, the near-horizon geometry acts as an attractor in the moduli space. The common radius of the AdS_2 and S^2 spaces is the modulus $|Z|$ of the central charge Z of the supersymmetry algebra and, by the BPS condition, equal to the mass $M(\phi^i)$ of the black hole. Both are computed at a point in the asymptotic moduli space coinciding with the attractor point. The three U-duality invariants characterizing the black hole can hence be summarized as

$$M(\phi_*) = |Z(\phi_*)| = b_* \, . \tag{2.2}$$

As our model system, we consider a bound state of D0 and D4 branes wrapped on a $CY_3 \times T^2$ to produce a four-dimensional dyonic black solution. In the M-theory picture, this can be viewed as a collection of particle momenta on the M-theory circle S^1_M with intersecting M5 branes wrapping a (4-cycle in CY_3) $\times\, S^1_M$. As the near-horizon geometry decouples from the asymptotically flat space, the states contributing to the black-hole entropy must be localized in this region. Hence, probing the Hilbert space of these states will yield a count of the black-hole microstates from a statistical mechanics perspective. As mentioned in the previous section, the Hilbert space of quantum gravity in the near-horizon AdS_2 geometry can be formulated in terms of states in the holographically dual CFT_1 which implies that the black-hole degeneracy must be reproducible in terms of the counting formula for states in this conformal quantum mechanics. We therefore need a proposal for identifying the microstates of the AdS_2 black hole in a conformal quantum mechanical theory. One such proposal for conducting such an analysis is motivated by the observation that this system belongs to the special class of BPS black holes which can be lifted up to five dimensions to yield a near-horizon geometry of a BTZ black hole in $AdS_3 \times S^2$. The holographic correspondence with the two-dimensional BCFT is well understood in this case, and the black hole can be thought of as a chiral-ensemble excitation in the CFT, with the central charge defined by the D4 branes and with the CFT excitation number of the black hole being equal to its mass. Hence, in the 'black hole in AdS_2' scenario, we are motivated to consider the black hole as an excitation about AdS_2, described in terms of degrees of freedom that can be encoded in terms of a superconformal quantum mechanics. This suggests that the black hole is naturally represented as a halo of n BPS particles moving in the AdS_2 background. These particles are governed by a superconformal quantum mechanics, with a target space that is the symmetric product of AdS_2 and S^2. This is a putative formulation of the holographically dual CFT. We proceed to delineate this connection below.

3. AdS_2–Calogero correspondence

3.1. Rational Calogero from AdS_2

Gravity in two dimensions is a conformal quantum field theory living on a strip. States in this theory are in a one-to-one correspondence to those defined in the BCFT, which in this case is also the holographically dual field theory. This field theory is in fact some superconformal quantum mechanics and must encode all the bulk states. A single particle moving in the AdS background is described by a superconformal quantum mechanical worldline theory. For a scalar particle, in the large-radius limit of an AdS geometry, parametrized in the Poincaré patch via

$$ds^2 = -\frac{R^4}{q^4} dt^2 + R^2 \frac{dq^2}{q^2} \, , \tag{3.1}$$

this is the rational 2-body Calogero model, with the Hamiltonian[2]

$$H = \frac{p^2}{2} + \frac{\lambda^2}{q^2} \, , \tag{3.2}$$

where λ is proportional to the angular momentum Casimir of the particle in four dimensions. The energy must be evaluated with respect to the AdS global time coordinate, where the Killing vector is smooth everywhere, and the Hamiltonian for this coordinate is given by

$$H = \frac{p^2}{2} + \frac{\lambda^2}{q^2} + \omega^2 \frac{q^2}{2} \, , \tag{3.3}$$

with an undetermined non-zero force constant ω. The addition of the last term arises by passing from the Poincaré time t to the global time τ, which are related as

$$\partial_\tau = \partial_t + \omega^2 K \, , \tag{3.4}$$

where K is the special conformal transformation generator of the $SO(2,1)$ isometry group of AdS_2, given by $K = \frac{1}{2} q^2$ in the large-R limit. The ground-state wave function in this case reads

$$\psi(q) = q^\alpha \, e^{-\omega^2 q^2/4} \qquad \text{where} \qquad \alpha = \tfrac{1}{2}\big(1 + \sqrt{1+4\lambda^2}\big) \, . \tag{3.5}$$

Hence, the particle has no support at the center of AdS, and its wave function is localized farther out. The limiting value of the wave function at the boundary acts as a local insertion on the BCFT and, hence, defines the operator in the BCFT corresponding to some state in the bulk. As a consequence, a state corresponding to an excitation in AdS_2 can be mapped to a superparticle moving in the bulk and such states can be organized in terms of the asymptotic symmetry group of AdS_2. Thus, we can regard the black hole as an ensemble of n BPS particles in AdS, which define a superconformal quantum mechanics with a target space given by n symmetrized copies of $AdS_2 \times S^2$. In the fully symmetric sector, the $SU(2)$ R-charge of the superconformal quantum mechanics will be simply the common R-charge of the n particles multiplied by n. It follows that the angular momentum matrix of this system is a multiple of the identity matrix.

Quantizing the spectrum of this system will generate the Hilbert space that counts the entropy of the BPS black hole. To this end, we observe that, in our chosen model of the dyonic black hole as a supersymmetric D0–D4 bound-state ensemble, the microstate counting is essentially a field-theory computation of the Witten index for n particles. Their momenta are equal to the $D0$-brane quantum numbers in the two-dimensional worldvolume theory of intersecting M5 branes on $CY_3 \times S^1_M$, at a point in the moduli space corresponding to $V_{CY_3} \ll R_{S^1_M}$. This theory is simply two-dimensional $SU(n)$ super Yang–Mills on a cylinder,[3] which has been shown in [7] to be equivalent to an n-particle rational Calogero model governed by the Hamiltonian

$$H = \sum_i \frac{p_i^2}{2} + \sum_{i<j} \frac{\lambda^2}{(q_i - q_j)^2} \, . \tag{3.6}$$

As in the single-particle case, the spectrum of the system is computed with respect to the global time τ, and the corresponding Hamiltonian can be related to the Schwarzschild-time Hamiltonian by adding the superconformal generator K. This introduces a confining harmonic well to the rational Calogero model,

[2] See [3] and [6] for a detailed exposition.

[3] See [3] and the references therein for details.

$$H=\sum_i \frac{p_i^2}{2}+\sum_{i<j}\frac{\lambda^2}{(q_i-q_j)^2}+\omega^2\sum_i \frac{q_i^2}{2}\,. \tag{3.7}$$

In the Higgs limit where the spacing between the positions of all particles vanish and all the particles are driven to the origin of the coordinate system, the analysis of the ground states is similar to that of the single-particle system, and so the discussion for the single-particle case goes through for the multi-particle system. Hence, this model offers a putative formulation of the CFT_1 required to count the large-charge leading-order black-hole entropy. We now proceed to show how this model encodes the vacuum states of the holographically dual quantum mechanics and how the AdS_2 geometry emerges in the bulk by analyzing the flow of the ground state in the space of its coupling constants. We test this model by deriving the degeneracy formula for this system.

For the purposes of the ensuing discussion and noting that for the $\mathcal{N}=2$ D0–D4–D4–D4 system, the SU(2) R-symmetry is identified with the SU(2) group of rotations in four dimensions, we see that all microstates of this static spherically symmetric system have the same fermion number, and hence the Witten index is the same as the full partition function. Therefore, in order to derive the Bekenstein–Hawking entropy for theses systems, we concentrate on the bosonic part of the Calogero model. The BPS particles are then simply assumed to be extremal particles with a unit ratio of mass to charge and with a mutual Coulomb interaction.

3.2. AdS_2 from Calogero

How may an asymptotically AdS_2 bulk background arise from a rational Calogero model? It is necessary to check that the ground state of this model in some limit (corresponding to approaching the boundary, i.e. $q\to 0$) must move in the space of coupling constants of the deformed model, such that this state feels the vacuum geometry of the bulk gravity theory, namely AdS_2. The metric it should see is nothing but the Fischer information metric for the ground-state wave function of the deformed Calogero Hamiltonian (3.3), given by

$$\psi(q,\alpha,\omega)=A\,q^{\alpha}\,e^{-\omega^2q^2/2} \quad \text{where}\quad A^2=(2\alpha{+}1)(\tfrac{\omega}{2})^{2\alpha+1}\,. \tag{3.8}$$

In the limit of $q\to 0$, the wave function can be approximated to leading order by

$$\psi(q,\alpha,\omega)=A\,q^{\alpha}\qquad \text{for}\quad 0\le q\le \tfrac{2}{\omega}\,, \tag{3.9}$$

while all higher-order deformations of the Hamiltonian can be neglected. This yields a two-parameter space graded by α and ω. The Fischer information metric for a space parametrized by n variables $\Theta=(\theta_i)$, with $i=1,\dots,n$, is given as

$$g_{\theta_i\theta_j}=\int_0^{\frac{2}{\omega}}\mathrm{d}q\,q^{2\alpha}\,\partial_{\theta_i}\log|\psi(q,\Theta)|^2\,\partial_{\theta_j}\log|\psi(q,\Theta)|^2\,|\psi(q,\Theta)|^2, \tag{3.10}$$

where $|\psi(q,\Theta)|^2$ is the probability density on the wave-function space. The Fischer metric on the two-dimensional space under consideration is computed explicitly to be

$$\mathrm{d}s^2=\tilde{\alpha}^{-2}\big(\mathrm{d}\tilde{\alpha}^2+\omega^{-2}\mathrm{d}\omega^2\big)\qquad \text{where}\quad \tilde{\alpha}=\frac{1}{2\alpha{+}1}\,. \tag{3.11}$$

Hence, to summarize, if one considers a deformation of the conformal Calogero Hamiltonian by a harmonic oscillator term which initiates a flow in the space of coupling constants, then in the limit of $q\to 0$, to observe the change in the ground state, we need to consider only the quadratic deformation so as to obtain a two-dimensional space of coupling constants. The latter is found to be essentially Euclidean AdS_2.[4] This explicitly goes to show that the flow of the ground state in the space of relevant coupling constants, near the boundary of AdS_2, falls into a representation of the $\mathrm{SL}(2,\mathbb{R})$ symmetry group that annihilates the vacuum of the dual CFT_1. Hence, we now have a dynamical model which is a putative candidate for counting the degrees of freedom of the holographically dual CFT_1. We now run our first check of this counting by computing the degeneracy of states in the spectrum of this Hamiltonian, dual to the ground state of a BPS particle moving in the background of a black hole in AdS_2.

4. Degeneracy from the Calogero Hamiltonian

The presence of the harmonic oscillator discretizes the n-body spectrum in (3.7) so that it acquires energy eigenvalues [8,10],

$$E_n(m)=\omega\big(f(\lambda)+\tfrac{n}{2}+m\big)\qquad \text{with}\quad m\in\mathbb{Z}_{\ge 0}\,. \tag{4.1}$$

In the above, $f(\lambda)$ is a linear function of λ. Here, the quantum number m is actually partitioned into positive-integer parts of maximum size n,

$$m=m_1+m_2+m_3+\dots\qquad \text{with}\quad m_r\in\{1,2,\dots,n\}\,, \tag{4.2}$$

which determines the multiplicity of $E_n(m)$ to be the number $p_n(m)$ of correspondingly restricted partitions of m. Its generating function reads [11]

$$\sum_m p_n(m)\,q^m=\prod_{1\le k\le n}\frac{1}{(1-q^k)}\qquad \text{with}\quad q=\mathrm{e}^{-\beta}\,. \tag{4.3}$$

Here, β is the periodicity of the Euclidean time circle and (up to numerical factors) equal to the inverse of the black-hole temperature. We work in the large-n limit, which implies $p_n(m)\to p(m)$ and simplifies the generating function to

$$\sum_m p(m)\,\mathrm{e}^{-\beta m}=\prod_{k\in\mathbb{N}}\frac{1}{(1-q^k)}=\frac{1}{\eta(\beta)}\,\mathrm{e}^{\frac{\pi\beta}{24}}\,. \tag{4.4}$$

The asymptotic growth of $p(m)$ can be obtained by a saddle-point approximation of the Laplace transform of the degeneracy formula, in the low β limit, and by using the transformation property of the Dedekind η function under Poisson resummation to give

$$p(m)\;\approx\;\mathrm{e}^{2\pi\sqrt{\frac{m}{6}}}\,, \tag{4.5}$$

where the approximation sign indicates a suppression of all quadratic corrections to the saddle point and of other subleading terms. As the system we are studying exhibits no classical mass gap,[5] we need to pick the largest possible Euclidean time periodicity to define the Euclidean temperature, and hence we take the Euclidean periodicity to be $\frac{2\pi n}{\omega}$. This is equivalent to rescaling ω in the spectrum by a factor of n and demanding that we count only eigenvalues with m being integral multiples of n. Therefore, in the above expression, m should actually be replaced by mn.

[4] As $\alpha>\frac{1}{2}$, this is not a complete Poincaré patch, since $0\le\tilde{\alpha}<\frac{1}{2}$. However, in what follows, we will simply refer to it as the Poincaré patch and leave the inherent subtleties in this metric for future study.

[5] This is completely consistent with looking at the most symmetric sector of the theory.

Now, let us consider the physically relevant values of this model for the black-hole statistical mechanics. Essentially, we are counting a Witten index on the full Hilbert space of the system, and so we should be looking at the ground-state degeneracy. The full conformal quantum gravity has a net central charge of zero, which is the sum of the conformal anomalies due to diffeomorphisms, ghosts and matter. As the matter content in the black-hole background does not differ from that of 'empty' AdS, the matter contribution to the stress tensor is the same in both cases, and hence the only matter contribution to the stress tensor can come from modes which are fully annihilated by the complete $SO(2,1)$ isometry of the AdS_2 vacuum. This fixes the excitation quantum number to

$$m = \frac{c}{24}\,. \tag{4.6}$$

Another argument for the above relation can be put forward as follows. The ground-state degeneracy we are counting is in the black-hole background, while the Calogero spectrum has been evaluated in the Poincaré patch of AdS_2. A conformal transformation can be used to map the ground state of the black-hole background to that of the Poincaré patch. Under this transformation the stress tensor picks up an inhomogeneous term coming from the Schwartzian derivative, which raises the ground-state energy by an amount of $\frac{c}{24}$ in the Poincaré patch [12]. Here, c is the ground-state Casimir energy or central charge of the holographically dual CFT. From a dual CFT perspective, this implies that all such black holes must have a Casimir energy equal to $\frac{c}{24}$, implying again that $m = \frac{c}{24}$.

The number n of particles in the Calogero model is equal to the number of degrees of freedom of the CFT_1 and thus equal to c. Consequently, $mn = \frac{c^2}{24}$, and the leading-order contribution to the black-hole entropy is found to be

$$S = 2\pi\sqrt{\frac{c^2}{6\times 24}} = 2\pi\,\frac{c}{12}\,, \tag{4.7}$$

which matches the standard Bekenstein–Hawking black-hole entropy of the four-dimensional black hole reduced on the two-sphere in the near-horizon geometry [13]. Note that the relevant degrees of freedom that go into the computation of the black-hole entropy can be interpreted as the degrees of freedom of the AdS vacuum that the black-hole observer does not see, resulting in an entanglement entropy. Hence, one can extract leading-order information about the microstate description of bulk states in AdS by using general properties of an equivalent formulation of the BCFT in terms of a known superconformal Calogero model.

5. Discussion and conclusions

The formulation of a holographic dual to quantum gravity in AdS_2 has been the least well understood of the frequently analyzed gauge–gravity correspondences. Concurrently, extremal black holes in four dimensions with a near-horizon geometry have a density of states that is related to the square root of the energy, reflecting an underlying degeneracy of microstates that is captured by a CFT_2 as opposed to a CFT_1. This article builds upon a proposed formulation in [3] of the microstates of a black hole in AdS_2 in terms of the worldline quantum mechanics of conformal Calogero particles in AdS_2.

The AdS_2 factor in the near-horizon geometry of all extremal black objects produces a universal expression for black-hole entropy in terms of the two-dimensional central charge. The specific details of the model, including supersymmetry and rotations, go into the definition of the two-dimensional central charge. A parallel situation exists in the case of black holes which can be lifted, at an appropriate point of their asymptotic moduli space, to five dimensions to get AdS_3 near-horizon geometries. The corresponding expression for the central charge in terms of the geometrical length scale of AdS_3 is obtained by a KK uplift along a third compact direction of the AdS_2. The extra information involving, say, a $U(1)$ charge corresponding to translation along this compact direction is encoded in the definition of the three-dimensional Newton constant. Similarly, though the actual computation of the dependence of the entropy on the charge quantum numbers for any given system involves a model-dependent computation of the central charge, the relation between the Bekenstein–Hawking entropy and the two-dimensional central charge is universal and is derived here. There is, by now, a rich body of literature explaining macroscopic counting in AdS_2 with Sen's entropy function and, for BPS black holes, a quantum entropy function construction. For details on the supergravity backgrounds referred to here and on the corresponding macroscopic analysis see, for instance, [9].

One may wonder about the isospectrality of the Calogero model to a set of decoupled harmonic oscillators and conclude that the latter could be argued to capture dynamics in AdS_2. However, isospectrality is only about the energy eigenvalues. The wave functions of the Calogero model differ from those of decoupled harmonic oscillators, and the momentum operator is modified to the Dunkl operator. So, although the degeneracy of the black hole could be derived as a quaint numerology from a collection of harmonic oscillators, the Calogero model arises naturally by studying the Hamiltonian governing geodesics in the near-horizon geometry of the black hole. One is neglecting the particle motion in the angular directions, which is related to the mass gap of the angular excitations in a near-horizon geometry, allowing us to focus on an s-wave-like mode. The spectrum of this model is then used to compute the degeneracy after a motivated identification of the quantum numbers with relevant quantities of the black-hole background. Some of these arguments have been outlined before by [12], but the notion of computing the entropy by purely using the modular properties of the function in the large-n limit, without actually employing a two-dimensional Cardy formula, is genuinely a more rigorous approach.

The paper looks at the bosonic part of the super-Hamiltonian of a superparticle moving about in an AdS_2 background. This is consistent with the construction of black solutions in a purely bosonic subsector of superstring theory by suppressing the fermionic fields. Further, for static supersymmetric black holes in four dimensions, the identification of the R-charge with the angular momentum quantum number renders the Witten index calculation to be simply a count of ground states with the same fermion number, and hence, an effective partition function for the ground states. For a generic non-supersymmetric AdS_2 background, we will of course have to compute the real partition function for the theory, which is a more involved problem and not being considered here. A detailed microscopic calculation including subleading corrections will require precise identification of the fermionic constituents and also an analysis of the role played by various aspects of AdS_2 dynamics such as fragmentation.

In sum, the purpose of the paper is to model the entropy of states in AdS_2 as coming from the conformal quantum mechanics on the boundary. The wave function of the particles moving in AdS_2 have no support at its origin, and their support on the boundary can be thought of as operators on the BCFT. Hence, we model microstates of the black hole as constituents in the Hilbert space of light electric branes such as D0 branes moving about in a heavy magnetic background like D4 branes which backreact to produce an AdS_2 near-horizon geometry. Motivated by this, the boundary theory that is holographically dual to a black hole in AdS_2 is conjectured to be precisely the theory of the n-particle system moving in AdS_2. We give evidence for this conjecture by first computing ab initio the leading-order Bekenstein–Hawking component of the black-hole entropy and then verifying that the metric on the space of ground-state wave functionals of the theory is precisely AdS_2. The degeneracy of states in this model accurately reproduces the Bekenstein–Hawking entropy without taking recourse to viewing the underlying CFT as the chiral half of a two-dimensional CFT or implementing the Cardy formula. The accuracy of the computation indicates that this formulation offers a putative way to understand quantum gravity in AdS_2 and opens avenues for new checks on the gauge–gravity correspondence in two dimensions.

If the Calogero model is to be dual to string theory in AdS_2, then the metric on the space of coupling constants, as generated by the flow of generic states in this space, must be the emergent bulk metric of the geometry in which the motion of a BPS particle is governed by the worldline Hamiltonian that includes those couplings. The background so derived is dual to the states whose flow is under consideration. We have already demonstrated this for the vacuum state as a necessary condition for this theory to be a holographic candidate for AdS_2. Investigating the Fisher information metric on the full Hilbert space of the Calogero model, by which bulk geometry emerges from superconformal quantum mechanics, might yield further insights into gauge–gravity duality in two as well as in higher dimensions.

Finally, to extend this holographic dictionary, one needs to map phenomena in AdS_2 such as AdS fragmentation in terms of Calogero models, understand subleading corrections in terms of the holographically dual theory. We leave this for ongoing and future work.

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