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$\mathcal{N} = 4$ supersymmetric AdS_5 vacua and their moduli spaces

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Abstract: We classify the $\mathcal{N} = 4$ supersymmetric AdS_5 backgrounds that arise as solutions of five-dimensional $\mathcal{N} = 4$ gauged supergravity. We express our results in terms of the allowed embedding tensor components and identify the structure of the associated gauge groups. We show that the moduli space of these AdS vacua is of the form $\mathrm{SU}(1,m)/\big(\mathrm{U}(1)\times\mathrm{SU}(m)\big)$ and discuss our results regarding holographically dual $\mathcal{N} = 2$ SCFTs and their conformal manifolds.

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Contents

1 Introduction

Supersymmetric Minkowski compactifications of string or M-theory on Ricci-flat spaces generically result in effective field theories with a large number of perturbatively flat directions of the scalar potential. The geometry of these moduli spaces is well understood, often even beyond the classical level, and exploring the mechanisms that lead to moduli stabilization in realistic string backgrounds is an important task of string phenomenology.

Much less, by contrast, is known about the structure of moduli spaces of anti-de Sitter (AdS) vacua. While such AdS moduli might be encountered in intermediate steps of moduli stabilization scenarios, e.g. prior to de Sitter "uplifts", they play an even more fundamental role in the context of the AdS/CFT correspondence, where they correspond to exactly marginal operators of the holographically dual conformal field theory (CFT). The space of exactly marginal couplings is known as the conformal manifold, $\mathcal{C}$, of the CFT, and it comes equipped with the Zamolodchikov metric [1]. Therefore, knowledge of AdS moduli spaces can provide valuable information about $\mathcal{C}$. Within the AdS/CFT correspondence, the study of $\mathcal{C}$ started in [2–4].

A first step towards a better understanding of general AdS moduli spaces of string compactifications is the investigation of anti-de Sitter solutions of lower-dimensional supergravity theories. These moduli spaces form submanifolds of the scalar field spaces, $\mathcal{M}$, of the corresponding supergravity theories and may depend on additional data such as the gauge couplings or other deformation parameters. Uncovering the interrelations between these geometric structures defines an interesting mathematical problem in its own right that is highly sensitive to the spacetime dimension and the amount of supersymmetry present.

In [5, 6], the moduli spaces of AdS_4 vacua that preserve all the available supersymmetries of four-dimensional (4D), $\mathcal{N} = 1, 2, 4$ supergravity were investigated. For $\mathcal{N} = 1$ supergravity, it was found in [5] that the moduli space $\mathcal{C}$ is a real submanifold of the original Kähler manifold $\mathcal{M}$ with at best half the dimension. For $\mathcal{N} = 2$ supergravity, $\mathcal{C}$ is generically a product of a real submanifold of the special-Kähler geometry of the vector multiplet sector and a Kähler submanifold of the quaternion Kähler space of the hypermultiplets [5]. For $\mathcal{N} = 4$ supergravity, on the other hand, the moduli space was found to be trivial in that only isolated AdS backgrounds can exist [6]. Although 4D supergravity is expected to capture at best parts of the holographic dual of a 3D SCFT, the above results are consistent with what is known on conformal manifolds of 3D superconformal field theories [7–10]. Motivated by the results of [6] and the fact that $\mathcal{N} = 2$ SCFTs in 4D are intensely studied,[1] we investigate, in this paper, fully supersymmetric AdS_5 vacua of 5D, $\mathcal{N} = 4$ supergravity theories (i.e. AdS backgrounds that preserve all of the 16 real supercharges).[2]

5D, $\mathcal{N} = 4$ gauged supergravities were constructed in [13–16], and several specific examples of fully supersymmetric AdS_5 vacua have previously appeared in the literature. In [13], for instance, pure $\mathcal{N} = 4$ supergravity with a gauge group $SU(2) \times U(1)$ was constructed and shown to exhibit a fully supersymmetric AdS_5 background. In this case, two of the six graviphotons have to be dualized to tensor fields, which carry charge under the U(1) factor of the gauge group. In [15], the coupling of $\mathcal{N} = 4$ supergravity to vector (or dual tensor) multiplets was studied and particular AdS_5 backgrounds were found — again for the gauge group $SU(2) \times U(1)$. From the AdS/CFT perspective, the necessity of this gauge group was discussed in [17] for orbifold compactifications of type IIB string theory dual to 4D, $\mathcal{N} = 2$ superconformal quiver gauge theories [18]. The 5D candidate gauged supergravity theory of the $\mathbb{Z}_n$ orbifolds of the five-sphere was identified in [17] to be a specific $\mathcal{N} = 4$ truncation of $\mathcal{N} = 8$ supergravity with additional vector and tensor multiplets from the twisted sectors. A moduli space of the form $SU(1,m)/\big(U(1) \times SU(m)\big)$ was implicitly identified in [17] for these theories by looking at the set of holographic RG-flows induced by certain mass deformations.

Using the most general gaugings [16] in terms of the embedding tensor formalism [19–21], we determine here the general gauge group that can lead to an $\mathcal{N} = 4$ AdS_5 vacuum and identify the possible moduli spaces. The most general gauge group turns out to be of the form $G = U(1) \times H$, where H must contain an SU(2) subgroup gauged by three vector fields from the supergravity multiplet, and the U(1) must act at least on two tensor fields from the supergravity multiplet. The general moduli space of these theories is shown to be of the form[3]

$$\mathcal{C} = \frac{SU(1,m)}{U(1) \times SU(m)} \,. \tag{1.1}$$

[1] For a recent review see [11] and references therein.

[2] In [12] a similar analysis is performed for supersynmmetric AdS_7 backgrounds of seven-dimensional half-maximal supergravities where, as in $D = 4$, no supersymmetric moduli space exists. Correspondingly, it can be shown that on the dual SCFT side no supersymmetric exactly marginal operators exist [9, 10, 12].

[3] This resembles the result for two-dimensional (4,4) SCFTs that have $SO(4,m)/\big(SO(4) \times SO(m)\big)$ as conformal manifold [22, 23].

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Our analysis is intrinsically five-dimensional and at the classical level, so that within the AdS/CFT correspondence one would generally expect it to capture only part of the full story. A sufficient condition for the validity of a purely five-dimensional analysis in AdS backgrounds is when the five-dimensional fields form a consistent truncation of the ten-dimensional theory, as is for example the case for the untwisted sector of 5-sphere orbifold compactifications [17]. Working with classical supergravity means that the moduli space given in (1.1) should a priori only hold in the large-N limit (N being the number of colors in the dual SCFT). For an SU(2) gauge group, for instance, it has indeed been shown in [24, 25] that the Zamolodchikov metric has a more complicated form which agrees with the metric on $\mathcal{C}$ given in (1.1) only at leading order. Ref. [26] showed that the conformal manifold in any 4D, $\mathcal{N}=2$ SCFT is a Kähler manifold which in addition obeys the relations of tt^* geometry [27]. Finally, ref. [28] established that the corresponding Kähler potential is given by the sphere partition function of the SCFT while ref. [29] proved Kählerness of the metric using supersymmetric Ward identities. As we will show, consistency of (1.1) with the tt^* geometry of [26] imposes a constraint on the leading and subleading large-N behaviour of the three-point functions that appear in the OPE of exactly marginal operators.

The paper is organized as follows. In section 2, we recall the properties of $\mathcal{N}=4$ gauged supergravity that we need for our analysis. In section 3, we analyze $\mathcal{N}=4$ AdS_5 backgrounds and determine the constraints on the embedding tensor. We then show that an $\text{SU}(2)\times\text{U}(1)$ group is necessarily gauged by the graviphotons, and we also determine the allowed structure of the full gauge group G, thereby classifying all possible $\mathcal{N}=4$ AdS_5 vacua. In section 4, we determine the moduli space of the above AdS vacua, and in section 5 we discuss our results in terms of dual 4D, $\mathcal{N}=2$ SCFT. Finally, appendix A summarizes our Γ-matrix conventions, while in appendix B we discuss the large-N behaviour of correlation functions in the SCFT and the constraints which can be derived from the consistency with (1.1).

2 $\mathcal{N}=4$ gauged supergravity

In this section, we recall the properties of 5D, $\mathcal{N}=4$ gauged supergravity [13–16] that are relevant for our analysis. The generic spectrum of ungauged $\mathcal{N}=4$ supergravity consists of the gravity multiplet together with n vector multiplets. The gravity multiplet contains the graviton $g_{\mu\nu}$, four gravitini ψ_μ^i, $i=1,\ldots,4$, six vectors $A_\mu^{[ij]}$, A_μ^0, four spin-1/2 fermions χ^i, and one real scalar Σ. The vector fields $A_\mu^{[ij]}$ are antisymmetric in i and j and satisfy the additional condition

$$A_\mu^{[ij]}\Omega_{ij}=0\,, \tag{2.1}$$

where Ω_{ij} is the symplectic metric of USp(4), the R-symmetry group of 5D, $\mathcal{N}=4$ supersymmetry. Thus, the A_μ^{ij} transform in the **5** of USp(4), while A_μ^0 is a USp(4) singlet.

We label the vector multiplets with the index $a=1,\ldots,n$. Each vector multiplet contains a vector A_μ^a, four spin-1/2 gaugini λ^{ai}, and 5 scalars $\phi^{a[ij]}$, which are also antisymmetric in i and j and symplectic traceless analogous to (2.1). Altogether, the spectrum thus features the graviton, four gravitini, $(6+n)$ vector bosons, $(4+4n)$ spin-1/2 fermions, and $(5n+1)$ scalars.

The target space, $\mathcal{M}$, of the scalar fields is the coset

$$\mathcal{M} = \mathrm{SO}(1,1) \times \frac{\mathrm{SO}(5,n)}{\mathrm{SO}(5) \times \mathrm{SO}(n)}\,, \tag{2.2}$$

where the first factor is spanned by Σ while the second factor is spanned by the scalars $\phi^{a[ij]}$ in the vector multiplets.

The second factor in (2.2) is conveniently parametrized by the vielbein $\mathcal{V} = (\mathcal{V}_M^m, \mathcal{V}_M^a)$, with $M = 1, \ldots, n+5$, $m = 1, \ldots, 5$. $\mathcal{V}$ is an element of $\mathrm{SO}(5,n)$ and thus obeys

$$\eta_{MN} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a\,, \tag{2.3}$$

where $\eta_{MN} = \mathrm{diag}(-1,-1,-1,-1,-1,+1,\ldots,+1)$ is the flat $\mathrm{SO}(5,n)$ metric. Alternatively, the coset can be represented by the positive definite scalar metric

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a = 2\mathcal{V}_M^m \mathcal{V}_N^m + \eta_{MN}\,, \tag{2.4}$$

which also plays the role of the gauge kinetic matrix for the $(5+n)$ vector fields combined as $A_\mu^M = (A_\mu^{[ij]}, A_\mu^a)$.

The isometry group of the scalar manifold, $\mathrm{SO}(1,1) \times \mathrm{SO}(5,n)$, extends to a global symmetry of the entire ungauged supergravity action, which is also subject to a local composite invariance under $\mathrm{Spin}(5) \times \mathrm{SO}(n)$. In order to express the boson-fermion couplings in a way that makes these symmetries manifest, one uses the group isomorphism between $\mathrm{USp}(4)$ and $\mathrm{Spin}(5)$ to express the $\mathrm{SO}(5)$ index m of the scalar vielbeine $\mathcal{V}_M^m$ in terms of $\mathrm{USp}(4)$ indices i, j via $\mathrm{SO}(5)$ gamma matrices,

$$\mathcal{V}_M^{ij} := \mathcal{V}_M^m \, (\Gamma_m)^{ij}. \tag{2.5}$$

$\mathcal{V}_M^{ij}$ is then antisymmetric and symplectic traceless in i and j and hence transforms in the $\mathbf{5}$ of $\mathrm{USp}(4)$. More details on this and our precise conventions are given in appendix A.

In the gauged versions of these theories, a subgroup of the global symmetry group $\mathrm{SO}(1,1) \times \mathrm{SO}(5,n)$ is promoted to a local gauge symmetry by introducing minimal couplings to the gauge fields and a few further terms to restore supersymmetry. This breaks part of the global symmetry group and, as a special feature of five dimensions, may require the conversion of some of the vector fields to antisymmetric tensor fields [13, 15]. This conversion concerns vector fields that would transform in nontrivial representations of the gauge group other than the adjoint representation and also occurs for $\mathcal{N} = 8$ [30–32] and $\mathcal{N} = 2$ [33] supergravity. In the case at hand, a conversion to tensor fields would in particular be necessary if the original representation[4] $(\mathbf{5}+\mathbf{n})_{-\mathbf{1}} \oplus \mathbf{1}_{\mathbf{2}}$ of the global symmetry group $\mathrm{SO}(5,n) \times \mathrm{SO}(1,1)$ decomposes w.r.t. the gauge group $G \subset \mathrm{SO}(5,n) \times \mathrm{SO}(1,1)$ as

$$(\mathbf{5}+\mathbf{n})_{-\mathbf{1}} \oplus \mathbf{1}_{\mathbf{2}} \longrightarrow \text{singlets of } G \oplus \text{non-singlets of } G \oplus \text{adj. of } G\,, \tag{2.6}$$

and would then affect the non-singlets of G.

[4]The subscripts denote the charge under $\mathrm{SO}(1,1)$.

In the so-called embedding tensor formalism [19–21], one can rewrite the theory such that the original global symmetry $\mathrm{SO}(1,1)\times\mathrm{SO}(5,n)$ remains manifest. In order to do this, one has to work with a redundant field content that contains a tensor field for each of the original vector fields. The gauge couplings are then described by three field-independent $\mathrm{SO}(1,1)\times\mathrm{SO}(5,n)$-tensors (the embedding tensors) denoted by $\xi_M, \xi_{[MN]}, f_{[MNP]}$. Their transformation under $\mathrm{SO}(5,n)$ follows from the indicated index structure, and, with respect to $\mathrm{SO}(1,1)$, ξ_M and $f_{[MNP]}$ carry charge $-1/2$, while $\xi_{[MN]}$ has charge $+1$. The entries of the embedding tensors are real numbers, and supersymmetry imposes a set of coupled consistency conditions on them known as the quadratic constraints [16][5]

$$\begin{aligned} \xi^M\xi_M = 0\,, \qquad \xi_{MN}\xi^N = 0\,, \qquad \xi^P f_{PMN} = 0\,, \\ 3f_{R[MN}f_{PQ]}{}^R = 2f_{[MNP}\xi_{Q]}\,, \qquad \xi_M{}^Q f_{QNP} = \xi_M\xi_{NP} - \xi_{[N}\xi_{P]M}\,. \end{aligned} \tag{2.7}$$

The possible solutions to these constraints parameterize the different consistent gauged $\mathcal{N}=4$ supergravity theories. In particular, they determine the gauge group and its precise embedding in the global symmetry group $\mathrm{SO}(1,1)\times\mathrm{SO}(5,n)$, the order parameters for spontaneous supersymmetry breaking, and the scalar potential.

The full bosonic Lagrangian is recorded in [16] but for the analysis in this paper, we only need the potential V and the kinetic terms of the scalar fields, which are given by

$$e^{-1}\mathcal{L} = \frac{1}{16}(D_\mu M_{MN})(D^\mu M^{MN}) - \frac{3}{2}\Sigma^{-2}(D_\mu\Sigma)(D^\mu\Sigma) - V(M,\xi,f) + \ldots\,. \tag{2.8}$$

The gauge covariant derivative reads

$$D_\mu = \nabla_\mu - A_\mu^M f_M{}^{NP}t_{NP} - A_\mu^0\xi^{NP}t_{NP} - A_\mu^M\xi^N t_{MN} - A_\mu^M\xi_M t_{\hat{0}}\,, \tag{2.9}$$

where $t_{MN}=t_{[MN]}$ are generators of $\mathrm{SO}(5,n)$, $t_{\hat{0}}$ is the generator of $\mathrm{SO}(1,1)$, and we have absorbed the gauge coupling into the embedding tensor components.

The conditions for a supersymmetric AdS-background can be concisely formulated in terms of the scalar components of the $\mathcal{N}=4$ supersymmetry transformations. For the four gravitini ψ^i_μ, the four spin-1/2 fermions in the gravitational multiplet χ^i, and the gaugini λ^i_a, they are given by [16]

$$\begin{aligned} \delta\psi_{\mu i} &= D_\mu\epsilon_i + \frac{\mathrm{i}}{\sqrt{6}}\Omega_{ij}A_1^{jk}\Gamma_\mu\epsilon_k + \ldots\,, \\ \delta\chi_i &= \sqrt{2}\Omega_{ij}A_2^{kj}\epsilon_k + \ldots\,, \\ \delta\lambda_i^a &= \sqrt{2}\Omega_{ij}A_2^{a\,kj}\epsilon_k + \ldots\,, \end{aligned} \tag{2.10}$$

where ϵ_j are the four supersymmetry parameters, and the dots indicate terms that vanish in a maximally symmetric space-time background. The fermion shift matrices in these

[5] Here and in the following, the $\mathrm{SO}(5,n)$ indices $M,N,\ldots$ are raised and lowered with η^{MN} and η_{MN} as in [16]. Consistency with $\mathcal{V}_M^A\mathcal{V}_A^N=\delta_M^N$ and $\mathcal{V}_A^M\mathcal{V}_M^B=\delta_A^B$ then requires raising and lowering the $\mathrm{SO}(5)\times\mathrm{SO}(n)$ indices $A,B,\ldots$ with η^{AB} and η_{AB}, i.e. we have $\mathcal{V}_M^a=\mathcal{V}_{Ma}$ and $\mathcal{V}_M^m=-\mathcal{V}_{Mm}$. This differs from the conventions used in [14, 15], where $M,N,\ldots$ are raised and lowered with M_{MN} and its inverse, while $A,B,\ldots$ are raised and lowered with the Kronecker delta to ensure consistency with $\mathcal{V}_B^N$ being the inverse of $\mathcal{V}_M^A$.

expressions are defined as

$$A_1^{ij} = \frac{1}{\sqrt{6}}\big(- \zeta^{(ij)} + 2\rho^{(ij)}\big)\,,$$
$$A_2^{ij} = \frac{1}{\sqrt{6}}\left(\zeta^{(ij)} + \rho^{(ij)} + \frac{3}{2}\tau^{[ij]}\right),$$
$$A_2^{a\,ij} = \frac{1}{2}\left(- \zeta^{a[ij]} + \rho^{a(ij)} - \frac{\sqrt{2}}{4}\tau^a\Omega^{ij}\right), \tag{2.11}$$

where

$$\tau^{[ij]} = \Sigma^{-1}\mathcal{V}_M^{ij}\xi^M, \qquad \tau^a = \Sigma^{-1}\mathcal{V}_M^a\xi^M,$$
$$\zeta^{(ij)} = \sqrt{2}\Sigma^2\Omega_{kl}\mathcal{V}_M^{ik}\mathcal{V}_N^{jl}\xi^{MN}, \qquad \zeta^{a[ij]} = \Sigma^2\mathcal{V}_M^a\mathcal{V}_N^{ij}\xi^{MN}, \tag{2.12}$$
$$\rho^{(ij)} = -\frac{2}{3}\Sigma^{-1}\mathcal{V}_M^{ik}\mathcal{V}_N^{jl}\mathcal{V}_{kl}^P f^{MN}{}_P\,, \qquad \rho^{a(ij)} = \sqrt{2}\Sigma^{-1}\Omega_{kl}\mathcal{V}_M^a\mathcal{V}_N^{ik}\mathcal{V}_P^{jl} f^{MNP}.$$

In terms of the shift matrices, the scalar potential is given by

$$\frac{1}{4}\Omega^{ij}V = \Omega_{kl}\big(A_2^{a\,ik}A_2^{a\,jl} + A_2^{ik}A_2^{jl} - A_1^{ik}A_1^{jl}\big)\,. \tag{2.13}$$

3 $\mathcal{N} = 4$ AdS$_5$ backgrounds

In this section, we study $\mathcal{N} = 4$ gauged supergravities that admit a fully supersymmetric AdS$_5$ background, i.e. with all sixteen supercharges left unbroken. The latter requirement demands that the supersymmetry variations (2.10) have to vanish in the AdS$_5$ background. Inspecting (2.10) and (2.13), we see that this implies

$$\langle A_2^{ij}\rangle = \langle A_2^{a\,ij}\rangle = 0\,, \tag{3.1}$$

$$\langle A_1^{ij}A_{1kj}\rangle = \frac{1}{4}|\mu|^2\,\delta_k^i\,, \tag{3.2}$$

where $\langle V\rangle = -|\mu|^2$ is the cosmological constant, which arises from the covariant derivative in the gravitino variation, and $\langle\cdot\rangle$ indicates that a quantity is evaluated in the AdS-background.

3.1 Constraints on the gauging

We will now extract the constraints that are imposed by (3.1) and (3.2) on the embedding tensor components, i.e. on the possible gaugings that can lead to $\mathcal{N} = 4$ AdS vacua.

Let us begin with the evaluation of (3.1). Inspection of (2.11) reveals that $A_2^{a\,ij}$ decomposes into three different representations of USp(4), so that all three terms in $A_2^{a\,ij}$ have to vanish separately in the vacuum. Similarly, in A_2^{ij} the last term is antisymmetric and thus also has to vanish in the vacuum, while the first two terms in A_2^{ij} have to cancel each other. Thus, eqs. (3.1) are equivalent to

$$\langle\tau^{[ij]}\rangle = \langle\tau^a\rangle = \langle\zeta^{a[ij]}\rangle = \langle\rho^{a(ij)}\rangle = 0\,, \qquad \langle\zeta^{(ij)}\rangle + \langle\rho^{(ij)}\rangle = 0\,. \tag{3.3}$$

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Using (2.12), the vanishing of $\langle\tau^{[ij]}\rangle$ and $\langle\tau^{a}\rangle$ immediately gives[6]

$$\xi^M = 0\,. \tag{3.4}$$

In order to evaluate the rest of (3.3), it is convenient to convert the SO$(5,n)$ covariant embedding tensor components f^{MNP} and ξ^{MN} to the SO(5) $\times$ SO(n) covariant tensors $f^{ABC} := \langle\mathcal{V}_M{}^A\rangle\langle\mathcal{V}_N{}^B\rangle\langle\mathcal{V}_P{}^C\rangle f^{MNP}$ and $\xi^{AB} := \langle\mathcal{V}_M{}^A\rangle\langle\mathcal{V}_N{}^B\rangle\xi^{MN}$. The splitting $A = (m,a)$ then defines components such as f^{mnp} or ξ^{ma}, i.e.

$$f^{mnp} \equiv \langle\mathcal{V}_M{}^m\rangle\langle\mathcal{V}_N{}^n\rangle\langle\mathcal{V}_P{}^p\rangle f^{MNP}, \qquad \xi^{ma} \equiv \langle\mathcal{V}_M{}^m\rangle\langle\mathcal{V}_N{}^a\rangle\xi^{MN}, \qquad \text{etc.}\,, \tag{3.5}$$

which is the way the embedding tensor appears in the background values of the fermion shift matrices (2.12). We recall that the indices $a,b,\ldots$ and $m,n,\ldots$ are raised and lowered with, respectively, plus and minus the Kronecker delta.

Using this and the SO(5) γ-matrix notation of appendix A, the remaining three equations of (3.3) are now equivalent to

$$\xi^{am}\Gamma_m = 0\,, \qquad f^{amn}\Gamma_{mn} = 0\,, \qquad \frac{3}{\sqrt{2}}\langle\Sigma^3\rangle\,\xi^{mn}\Gamma_{mn} = -f^{mnp}\Gamma_{mnp}\,, \tag{3.6}$$

or, using (A.11)–(A.13),

$$\xi^{am} = 0\,, \qquad f^{amn} = 0\,, \qquad 3\sqrt{2}\langle\Sigma^3\rangle\,\xi_{qr} = \epsilon_{mnpqr}f^{mnp}\,. \tag{3.7}$$

It remains to analyze (3.2). Using the last equation in (3.3), it can be expressed solely in terms of $\zeta^{(ij)}$ so that it becomes a constraint on ξ^{mn}:

$$\frac{1}{4}|\mu|^2\,\mathbf{1}_4 = -3\langle\Sigma^4\rangle\,\xi^{mn}\xi^{pq}\Gamma_{mn}\Gamma_{pq} = -\frac{3}{2}\langle\Sigma^4\rangle\,\xi^{mn}\xi^{pq}\,\{\Gamma_{mn},\Gamma_{pq}\}\,. \tag{3.8}$$

With (A.17), this decomposes into the two conditions

$$\xi^{[mn}\xi^{pq]} = 0\,, \tag{3.9}$$

$$\xi^{mn}\xi_{mn} = \frac{|\mu|^2}{24\langle\Sigma^4\rangle} \neq 0\,. \tag{3.10}$$

Note that for $\xi^{mn} = 0$, no $\mathcal{N} = 4$ supersymmetric AdS$_5$ background can occur. The condition $\xi^{mn} \neq 0$ means that among the 5-plet of graviphotons of the ungauged theory some are necessarily charged under the U(1) gauge group, so that these must be converted to antisymmetric tensor fields in order to carry out the gauging, cf. our discussion around (2.6). Interestingly, also in 4D, $\mathcal{N} = 4$ gauged supergravity, an $\mathcal{N} = 4$ AdS vacuum requires a gauging with a special feature, namely magnetic gaugings [6].[7]

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[6] We also see from (2.9) that $D_\mu\Sigma$ depends only on ξ_M and thus, for $\mathcal{N} = 4$ AdS backgrounds, Σ is uncharged and $D_\mu\Sigma$ reduces to an ordinary partial derivative.

[7] In [34], the dimensional reduction of 5D, $\mathcal{N} = 2$ supergravity with charged tensor fields to 4D was found to lead to magnetic gaugings in 4D. This does not necessarily mean, however, that a dimensional reduction of the above 5D, $\mathcal{N} = 4$ AdS vacua would yield the 4D, $\mathcal{N} = 4$ vacua of [6].

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Finally, inserting $\xi^M = 0$ in (2.7), we see that the quadratic constraints considerably simplify, leaving only the Jacobi identity for the structure constants $f_{MN}{}^P$ and their orthogonality to ξ^{MN}:

$$f_{RM[N}f_{PQ]}{}^R = 0\,, \tag{3.11}$$
$$\xi^{MQ}f_{QNP} = 0\,. \tag{3.12}$$

3.2 Solving the constraints for ξ^{MN} and f^{MNP}

What is left to do is to solve the constraints (3.7), (3.9)–(3.12), which will then specify the possible gauge groups and their precise embeddings in SO$(5,n)$. These group structures become most transparent if one works with the actual representation matrices of the gauge group as they appear in the gauge covariant derivative (2.9) (subject to $\xi^M = 0$) when it acts on the scalar vielbein $\mathcal{V}_M^A$. The latter transforms in the fundamental representation of SO$(5,n)$, where the generators t_{MN} take the form

$$(t_{MN})_P{}^Q = \delta^Q_{[M}\eta_{N]P}\,, \tag{3.13}$$

so that the $\mathcal{V}_M^A$ couple to the gauge fields A_μ^0 and A_μ^M with, respectively, the representation matrices

$$(T_0)_N{}^P := -\xi^{QR}(t_{QR})_N{}^P = \xi_N{}^P, \tag{3.14}$$
$$(T_M)_N{}^P := -f_M{}^{QR}(t_{QR})_N{}^P = f_{MN}{}^P. \tag{3.15}$$

Eqs. (3.11) and (3.12) imply that these representation matrices satisfy the commutation relations

$$[T_0, T_M] = 0\,, \qquad [T_M, T_N] = -f_{MN}{}^P T_P\,, \tag{3.16}$$

i.e. T_0 generates an Abelian group factor. Let us now evaluate how (3.7), (3.9) and (3.10) further constrain T_0, T_M and their commutation relations.

We start with the equation $\xi^{ma} = 0$, which implies that the U(1) factor gauged by A^0 acts on the gravity multiplet (via the generator $(T_0)_m{}^n = \xi_m{}^n$) and on the vector multiplets (via the generator $(T_0)_a{}^b = \xi_a{}^b$) independently, i.e. this U(1) is a subgroup of SO(5) $\times$ SO(n) in SO$(5,n)$.

Next we consider the condition $f^{mna} = 0$. It implies that the T_m close among themselves and hence generate a proper subgroup of the gauge group. Moreover, $(T_m)_M{}^N = f_{mM}{}^N$ must be block diagonal so that this subgroup does not mix fields from the gravity multiplet with fields from the vector multiplets, i.e. it is a subgroup of SO(5) $\times$ SO(n). We have thus found that T_0 and T_m generate compact subgroups that do not mix gravity multiplet and vector multiplet sector, so that their action on these two sectors can be studied independently.

We begin with the action of T_0 and T_m within the gravity multiplet, which is described by the components ξ^{mn} and f_{mnp}. Note that both of these tensors must be non-zero (and proprtional to the AdS curvature), as eq. (3.10) requires $\xi^{mn} \neq 0$, which then also implies $f_{mnp} \neq 0$ by the last of eqs. (3.7). We now use that, by certain SO(5) transformations, the antisymmetric bilinear form ξ^{mn} can always be brought to canonical form where at most $\xi^{12} = -\xi^{21}$ and $\xi^{34} = -\xi^{43}$ are non-zero.[8] Without loss of generality, we can assume $\xi^{12} = -\xi^{21} \neq 0$. The primitivity condition (3.9) then implies $\xi^{34} = -\xi^{43} = 0$.

Since ξ^{12} is the only nontrivial component of ξ^{mn}, it implies, via (3.7), that the only non-vanishing structure constants f_{mnp} are f_{345} and permutations thereof, so that the total gauge group that acts within the gravity multiplet is $\mathrm{U}(1) \times \mathrm{SU}(2)$. Note that $\xi^{mn} f_{npq} = 0$ (cf. (3.12)) is then automatically satisfied. In the following, we split the index m into $\tilde{m} = 1, 2$ and $m' = 3, 4, 5$, so that $\xi^{m'n'} = 0 = f^{\tilde{m}\tilde{n}\tilde{p}}$.

We now turn to the part of the gauge group that acts nontrivially on the vector multiplet sector, i.e. to the components ξ^{ab}, f_{abm} and f_{abc}. Note that unlike ξ^{mn} and f_{mnp} none of these components necessarily needs to be non-vanishing for an $\mathcal{N} = 4$ supersymmetric AdS vacuum to exist.

We start with ξ^{ab}. If $\xi^{ab} \neq 0$, we see from (3.14) that the U(1) gauged by A^0_μ is a diagonal U(1) of a U(1) in SO(5) and a U(1) in SO(n), whereas for $\xi^{ab} = 0$ it is entirely contained in SO(5). Just as we did for ξ^{mn}, we can use suitable SO(n) transformations to bring also ξ^{ab}, and hence the U(1) generator T_0, into canonical block-diagonal form,

$$T_0 = \mathrm{diag}(\alpha\epsilon, \mathbf{0}_3, \beta_1\epsilon, \beta_2\epsilon, \ldots, \beta_p\epsilon, 0, \ldots, 0)\,, \tag{3.17}$$

where $\alpha, \beta_1, \ldots, \beta_p$ are non-vanishing real numbers, which can always be assumed positive after possible exchanges of the relevant rows and coloumns, and $\epsilon = i\sigma_2$. Here, the special case $\xi^{ab} = 0$ is meant to corresponds to $p = 0$, i.e. there would then be no ϵ-blocks with β-coefficients. In analogy with the above decomposition $m = (\tilde{m}, m')$, we then decompose the indices $a, b, \ldots$ and use $\tilde{a}, \tilde{b}, \ldots = 1, \ldots, 2p$ for the directions in which ξ^{ab} is non-trivial, and $a', b', \ldots = 2p+1, \ldots, n$ for the rest. The conditions $\xi^{\tilde{m}M} f_{MNP} = \xi^{\tilde{m}\tilde{n}} f_{\tilde{n}NP} = 0$ and $\xi^{\tilde{a}M} f_{MNP} = \xi^{\tilde{a}\tilde{b}} f_{\tilde{b}NP} = 0$ then imply that all components f_{MNP} with at least one $\tilde{a}$ or one $\tilde{m}$ index must vanish, so that modulo index permutations only $f_{m'n'p'}$, $f_{a'b'm'}$ and $f_{a'b'c'}$ can be non-zero. The $(5+n) \times (5+n)$-matrices $T_{m'}, T_{a'}$ thus may have the following general form:

$$T_{m'} = \begin{pmatrix} \mathbf{0}_2 & & & \\ & {f_{m'n'}}^{p'} & & \\ & & \mathbf{0}_{2p} & \\ & & & {f_{m'a'}}^{b'} \end{pmatrix}, \qquad T_{a'} = \begin{pmatrix} \mathbf{0}_2 & & & \\ & \mathbf{0}_3 & & {f_{a'm'}}^{c'} \\ & & \mathbf{0}_{2p} & \\ & {f_{a'b'}}^{n'} & & {f_{a'b'}}^{c'} \end{pmatrix}. \tag{3.18}$$

Using the the above pattern of possibly nontrivial structure constants, the Jacobi identity (3.11) implies that the three matrices ${f_{m'a'}}^{b'}$ form a representation of SO(3) on the vector multiplet sector,

$${f_{m'a'}}^{b'} {f_{n'b'}}^{c'} - {f_{n'a'}}^{b'} {f_{m'b'}}^{c'} = -{f_{m'n'}}^{p'} {f_{p'a'}}^{c'}\,, \tag{3.19}$$

or, equivalently, that the $T_{m'}$ as given in (3.18) satisfy the SO(3) algebra,

$$[T_{m'}, T_{n'}] = -{f_{m'n'}}^{p'} T_{p'}\,, \tag{3.20}$$

[8]An SO(5) rotation about the 1-axis can rotate the vector ξ^{1m} into the 2-direction, followed by a rotation about the 2-axis that rotates ξ^{2m} along the 1-direction. Subsequent SO(3) rotations about the 4- and 3-axis can similarly eliminate all remaining components of ξ^{3m} and ξ^{4m} up to $\xi^{34} = -\xi^{43}$.

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whereas the remaining commutators are of the form

$$[T_{m'}, T_{a'}] = -f_{m'a'}{}^{b'} T_{b'} \,, \qquad [T_{a'}, T_{b'}] = -f_{a'b'}{}^{c'} T_{c'} - f_{a'b'}{}^{m'} T_{m'} \,. \tag{3.21}$$

If $f_{m'a'b'} = 0$, the gauge group, G, obviously simplifies to $G = \mathrm{U}(1) \times \mathrm{SU}(2) \times H_c$, where $H_c \subset \mathrm{SO}(n-2p) \subset \mathrm{SO}(n)$ is a compact subgroup with structure constants $f_{a'b'}{}^{c'}$ that only acts on the vector multiplets and whose adjoint representation can be embedded into the fundamental representation of $\mathrm{SO}(n-2p)$.[9]

In the case $f_{m'a'b'} \neq 0$, the gauge group is instead given by $G = \mathrm{U}(1) \times H$, where $H \subset \mathrm{SO}(3, n-2p) \subset \mathrm{SO}(3,n) \subset \mathrm{SO}(5,n)$ must contain $\mathrm{SO}(3)$ as a subgroup and is in general non-compact with commutation relations of the form (3.20)–(3.21). The simplest nontrivial example of this kind occurs for $n = 3$ and is given by $f_{m'n'p'} = -\epsilon_{m'n'p'}$, $f_{m'a'b'} = +\epsilon_{(m'-2)a'b'}$, and $f_{a'b'c'} = 0$, i.e. the $T_{m'}$ generate $\mathrm{SO}(3)$, and the $T_{a'}$ generate three non-compact directions that transform as a triplet under the $\mathrm{SO}(3)$. Since their algebra closes again in the $T_{m'}$, the $T_{a'}$ and the $T_{m'}$ altogether generate the simple gauge group $H = \mathrm{SO}(3,1)$. By turning on $f_{a'b'c'} = \lambda \epsilon_{a'b'c'}$, the $T_{a'}$ get an admixture of a compact direction of the $\mathrm{SO}(3)$ acting on the vector multiplet sector. For $\lambda < 2$, the gauge group remains $\mathrm{SO}(3,1)$. For $\lambda > 2$, the gauge group becomes $\mathrm{SO}(3) \times \mathrm{SO}(3)$ instead. In the case of $\lambda = 2$, the gauge group becomes the non-semi-simple gauge group of Euclidean rotations and translations in three dimensions.

We should point out that in general for H to be simple, one has to make sure that the non-degenerate Cartan-Killing metric of H can be embedded into the $\mathrm{SO}(3, n-2p)$ metric $\mathrm{diag}(- - - + \ldots +)$, with the negative entries corresponding to $\mathrm{SO}(3) \subset H$. This means that a simple H must have $\mathrm{SO}(3)$ as its maximally compact subgroup. Similar to the 4D case [35], this severely restricts the possible simple gauge groups H that can lead to $\mathcal{N} = 4$ AdS vacua and leaves essentially the above $H = \mathrm{SO}(3,1)$ and $H = \mathrm{SL}(3,\mathbb{R})$ as the only possibilities. For non-simple H there are of course many more possibilities.

To summarize, the necessary gauge group structure for an $\mathcal{N} = 4$ AdS_5 vacuum is

$$G = \mathrm{U}(1) \times H_{\mathrm{nc}} \times H_{\mathrm{c}} \,, \tag{3.22}$$

where H_{nc} has the $\mathrm{SU}(2)$ as its maximally compact subgroup that is gauged by three graviphotons, and H_{c} is a compact group that is gauged only under vector multiplet gauge fields. The $\mathrm{U}(1)$ is a diagonal subgroup of a necessary $\mathrm{SO}(2) \subset \mathrm{SO}(5)$ and an optional $\mathrm{SO}(2) \subset \mathrm{SO}(n)$. In the case of H_{nc} being simple we find that it is either $\mathrm{SO}(3)$, $\mathrm{SO}(3,1)$ or $\mathrm{SL}(3,\mathbb{R})$.

We finally note that all vector fields of the ungauged theory that are acted on non-trivially by T_0 must be dualized to antisymmetric tensor fields in the gauged theory, which is in particular true for A^1_μ and A^2_μ from the gravity multiplet. This together with the gauge group $\mathrm{U}(1) \times \mathrm{SU}(2)$ in the pure supergravity sector is consistent with the fact that the $\mathcal{N} = 4$ AdS_5 superalgebra has R-symmetry group $\mathrm{U}(1) \times \mathrm{SU}(2)$ and that the gravity

[9] Any semisimple compact group H_c can be embedded in this way into an $\mathrm{SO}(N)$ for sufficiently large $N \geq \dim(H_c)$ by identifying the Cartan-Killing metric of $\mathrm{Lie}(H_c)$ with the (relevant part of the) $\mathrm{SO}(N)$ metric.

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multiplet representing this R-symmetry group has four vector fields transforming as $\mathbf{3_0} \oplus \mathbf{1_0}$ and two antisymmetric tensor fields transforming as singlets under SU(2) and a doublet under U(1) (see e.g. [17] for a related discussion).

4 $\mathcal{N} = 4$ moduli space

In the previous section, we determined the general form of the gauge groups that can lead to $\mathcal{N} = 4$ supersymmetric AdS vacua. The purpose of this section is to determine the $\mathcal{N} = 4$ moduli spaces of these vacua, i.e. the manifold of scalar field deformations that preserve all four supersymmetries of a given $\mathcal{N} = 4$ AdS background. To this end, we use the same method as in [5, 6] and vary the supersymmetry conditions (3.1)–(3.2) so as to find all possible directions in the scalar field space $\mathcal{M}$ that are left undetermined when (3.1)–(3.2) are preserved. More concretely, we look for continuous solutions of

$$\delta A_1^{ij} = \delta A_2^{ij} = \delta A_{2a}^{ij} = 0\,, \tag{4.1}$$

in the vicinity of a fully supersymmetric AdS_5 background.[10] To start with, we parameterize the variations of the vielbein $\mathcal{V}$ by defining the $5n$ scalar field fluctuations $\delta\phi^{ma}$ around an AdS_5 background value $\langle \mathcal{V} \rangle$ by

$$\mathcal{V} = \langle \mathcal{V} \rangle \, \exp[2\,\delta\phi^{ma}(t_{ma})]\,, \tag{4.2}$$

where t_{ma} are the $(5+n)\times(5+n)$ matrices given in (3.13) corresponding to the coset $\mathrm{SO}(5,n)/\big(\mathrm{SO}(5)\times \mathrm{SO}(n)\big)$. This implies

$$\delta\mathcal{V}_M^m = \langle \mathcal{V}_M^a \rangle\, \delta\phi^{ma}, \qquad \delta\mathcal{V}_M^a = \langle \mathcal{V}_M^m \rangle\, \delta\phi^{ma}, \tag{4.3}$$

which are also consistent with (2.3). For the inverse vielbein, consistency with the relation $\mathcal{V}_A{}^M \mathcal{V}_M{}^B = \delta_A{}^B$ gives

$$\delta\mathcal{V}_m^M = -\langle \mathcal{V}_a^M \rangle\, \delta\phi^{ma}, \qquad \delta\mathcal{V}_a^M = -\langle \mathcal{V}_m^M \rangle\, \delta\phi^{ma}. \tag{4.4}$$

To linear order in $\delta\phi$, the metric M_{MN} defined in (2.4) is then given by

$$M_{MN} = \langle M_{MN} \rangle + 4\langle \mathcal{V}_{(M}^m \rangle \langle \mathcal{V}_{N)}^a \rangle \delta\phi^{ma} + \mathcal{O}(\delta\phi^2)\,. \tag{4.5}$$

Applying the above variations to the three equations (3.7) gives, respectively, the following conditions on $\delta\phi^{ma}$ and $\delta\Sigma$:

$$\xi^{nm}\delta\phi^{na} + \xi^{ab}\delta\phi^{mb} = 0\,, \tag{4.6}$$

$$f^{pmn}\delta\phi^{pa} + f^{abn}\delta\phi^{mb} + f^{amb}\delta\phi^{nb} = 0\,, \tag{4.7}$$

$$\delta\Sigma = 0\,, \tag{4.8}$$

[10] Note that the scalar potential is quadratic in $A_1^{ij}, A_2^{ij}, A_{2a}^{ij}$ so that the solutions of (4.1) are automatically flat directions of the scalar potential.

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where, for the last equation, we used the identities $\delta\xi^{mn} = 0$ and $\delta f^{mnp} = 0$. These are simple consequences of (4.3) and $\xi^{ma} = 0 = f^{mna}$, which, together with (4.8), also imply that (3.9) and (3.10) are automatically preserved.

Thus (4.8) fixes Σ, while (4.6) and (4.7) are the only nontrivial conditions on the other moduli. We will now show that these conditions mean that the moduli space is isomorphic to the coset space $\mathrm{SU}(1,m)/\big(\mathrm{U}(1)\times\mathrm{SU}(m)\big)$ for some $m \leq p$ where p denotes the index range for which $\xi^{\tilde{a}\tilde{b}}$ is nontrivial (cf. the discussion in the previous section below (3.17)).

To see this, we first examine (4.6). As only $\xi^{\tilde{m}\tilde{n}}$ and $\xi^{\tilde{a}\tilde{b}}$ can be non-vanishing, eq. (4.6) is trivial for $(m,a) = (m',a')$ and yields three nontrivial equations for the other index combinations:

$$\delta\phi^{\tilde{n}a'} = 0\,, \qquad \delta\phi^{m'\tilde{b}} = 0\,, \tag{4.9}$$

$$\xi^{\tilde{n}\tilde{m}}\delta\phi^{\tilde{n}\tilde{a}} + \xi^{\tilde{a}\tilde{b}}\delta\phi^{\tilde{m}\tilde{b}} = 0\,. \tag{4.10}$$

Thus, only $\delta\phi^{m'a'}$ and $\delta\phi^{\tilde{m}\tilde{a}}$ can be nontrivial, with the latter being constrained by (4.10).

Eq. (4.7), finally, only constrains the components $\delta\phi^{m'a'}$ to satisfy

$$f^{p'm'n'}\delta\phi^{p'a'} + f^{a'b'n'}\delta\phi^{m'b'} + f^{a'm'b'}\delta\phi^{n'b'} = 0\,. \tag{4.11}$$

This constraint was already discussed in detail in [6], where it was shown that its solution is given by

$$\delta\phi^{m'a'} = f^{a'b'm'}\lambda^{b'}, \tag{4.12}$$

where $\lambda^{b'}$ is an arbitrary (infinitesimal) real vector.

Eq. (4.12) implies that $\delta\phi^{m'a'}$ can only be nontrivial for $f^{a'b'm'} \neq 0$, i.e. for non-compact gauge groups. Moreover, if we consider $(X_{a'}{}^{b'm'}) := f_{a'}{}^{b'm'}$ as a $(q\times 3q)$ matrix (where $a', b' \ldots = 1,\ldots,q$), we see that the number of independent $\delta\phi^{m'a'}$ is equal to $\mathrm{rk}(X) \leq q$, which is also the number of independent non-compact gauge group generators. As the non-compact gauge symmetries have to be spontaneously broken in a given vacuum, the $\delta\phi^{m'a'}$ are the natural candidates for the Goldstone bosons eaten by the corresponding non-compact gauge fields. The physical moduli space would then only consists of the scalars $\delta\phi^{\tilde{m}\tilde{a}}$ subject to the constraint (4.10). We now confirm explicitly that the $\delta\phi^{m'a'}$ are indeed the Goldstone bosons eaten by the massive vectors and then give the geometric interpretation of the constraint (4.10) to identify the physical moduli space.

In order to identify $\delta\phi^{m'a'}$ with Goldstone bosons, we consider the gauge covariant derivative of the scalar field matrix M_{MN} (cf. (4.5)) and introduce $D_\mu M_{AB} := \langle\mathcal{V}_A^M\rangle\langle\mathcal{V}_B^N\rangle D_\mu M_{MN}$. Using (2.9) and keeping only the linear terms in $\delta\phi$ and A_μ^M, we obtain

$$D_\mu M_{AB} = \langle\mathcal{V}_A^M\rangle\langle\mathcal{V}_B^N\rangle\big(4\langle\mathcal{V}_{(M}^m\rangle\langle\mathcal{V}_{N)}^a\rangle\partial_\mu\delta\phi^{ma} + 2A_\mu^P f_{P(M}{}^Q\langle M_{N)Q}\rangle + 2A_\mu^0\xi_{(M}{}^Q\langle M_{N)Q}\rangle + \ldots\big)\,. \tag{4.13}$$

Introducing $A_\mu^C := \langle\mathcal{V}_M^C\rangle A_\mu^M$ and using $\langle\mathcal{V}_A^M\mathcal{V}_B^N M_{MN}\rangle = \delta_{AB}$, this can be written as

$$D_\mu M_{AB} = 4\delta_{(A}^m\delta_{B)}^a\partial_\mu\delta\phi^{ma} + 2A_\mu^C f_{C(A}{}^D\delta_{B)D} + 2A_\mu^0\xi_{(A}{}^D\delta_{B)D} + \ldots\,, \tag{4.14}$$

which for $(A,B) = (m',a')$ becomes, using (4.12),

$$2f^{m'a'b'}\partial_\mu\lambda^{b'} - 2A_\mu^{b'}f^{m'a'b'} + \ldots\,. \tag{4.15}$$

From this expression, we read off that under a local gauge transformation $\delta A_\mu^{b'} = \partial_\mu \Lambda^{b'} + \ldots$ with $\Lambda^{b'} = \lambda^{b'}$, the nontrivial flat directions $\delta\phi^{m'a'}$ are absorbed by the vector fields $A_\mu^{b'}$. Moreover, we see that the kinetic term $D_\mu M_{MN} D^\mu M^{MN} = D_\mu M_{AB} D^\mu M^{AB}$ in the action results in mass terms of the form $\hat{M}^2_{a'b'} \sim f_{a'}{}^{c'm'} f_{b'}{}^{c'm'} = (XX^T)_{a'b'}$. This precisely gives mass to the rk(X) non-compact gauge bosons, which thus eat all independent $\delta\phi^{m'a'}$, as claimed above. One also notes that in the $\mathcal{N}=4$ supersymmetric AdS-backgrounds all four graviphotons $A^{m'}$, A^0 remain massless and thus, as expected, the SU(2) × U(1) part of the gauge symmetry is always unbroken.

We now return to the only true moduli, the $\delta\phi^{\tilde{m}\tilde{a}}$ that are subject to the constraint (4.10). For convenience we will assume the form (3.17) for T_0. In the following we show that, for $\beta_i = \alpha$ $(i = 1, \ldots, p)$, this constraint describes the canonical embedding of

$$\frac{\mathrm{SU}(1,p)}{\mathrm{U}(1)\times\mathrm{SU}(p)} \subset \frac{\mathrm{SO}(2,2p)}{\mathrm{SO}(2)\times\mathrm{SO}(2p)} \subset \frac{\mathrm{SO}(5,n)}{\mathrm{SO}(5)\times\mathrm{SO}(n)}\,, \tag{4.16}$$

and hence that the $\mathcal{N}=4$ moduli space is isomorphic to $\mathrm{SU}(1,p)/\big(\mathrm{U}(1)\times\mathrm{SU}(p)\big)$. If not all β_i are equal to α, the moduli space becomes $\mathrm{SU}(1,m)/\big(\mathrm{U}(1)\times\mathrm{SU}(m)\big)$ for some $m < p$.

To see this, we recall the canonical embedding of the Lie algebra $\mathfrak{su}(1,p)$ into the Lie algebra $\mathfrak{so}(2,2p)$. Obviously, $\delta\phi^{\tilde{m}\tilde{a}}$ parameterizes the coset space $\mathrm{SO}(2,2p)/\big(\mathrm{SO}(2)\times\mathrm{SO}(2p)\big)$. Decomposing the $(2\times 2p)$ matrix $\delta\phi^{\tilde{m}\tilde{a}}$ into (2×2) blocks A_i, $i = 1,\ldots,p$,

$$(\delta\phi^{\tilde{m}\tilde{a}}) = \begin{pmatrix} A_1 & \cdots & A_p \end{pmatrix}, \tag{4.17}$$

the condition (4.10) becomes

$$\alpha\epsilon A_i - \beta_i A_i \epsilon = 0 \quad \text{(no sum)}\,. \tag{4.18}$$

If $\alpha = \beta_i$, this implies $A_i = x_i \mathbf{1}_2 + y_i\epsilon$ for some real numbers x_i, y_i, whereas $\alpha \neq \beta_i$ implies $A_i = 0$. Assuming $\alpha = \beta_i$ for all $i = 1,\ldots,p$, the $\mathfrak{so}(2,2p)$ matrix parameterized by the $\delta\phi^{\tilde{m}\tilde{a}}$,

$$\begin{pmatrix} \mathbf{0}_2 & A_1 & \cdots & A_p \\ A_1^T & \mathbf{0}_2 & \cdots & \mathbf{0}_2 \\ \vdots & \vdots & & \vdots \\ A_p^T & \mathbf{0}_2 & \cdots & \mathbf{0}_2 \end{pmatrix} \tag{4.19}$$

is thus equivalent to the non-compact part of a general $\mathfrak{su}(1,p)$ matrix,

$$\begin{pmatrix} 0 & x_1+iy_1 & \cdots & x_p+iy_p \\ x_1-iy_1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ x_p-iy_p & 0 & \cdots & 0 \end{pmatrix} \tag{4.20}$$

upon the canonical embedding $x+iy \to x\mathbf{1}_2 + y\epsilon$ of $\mathbb{C}$ into Mat$(2,\mathbb{R})$. Now the starting point of our considerations was an arbitrary $\mathcal{N}=4$ vacuum point. This means that condition (4.10) holds not only at the point of consideration, but also in a neighborhood in the space of $\mathcal{N}=4$ vacua. Therefore the moduli space is homogeneous and is given by exponentiating the modes fulfilling (4.10). For $\beta_i = \alpha\ \forall\, i$, the scalars $\delta\phi^{\tilde{m}\tilde{a}}$ thus parameterize $\mathrm{SU}(1,p)/\big(\mathrm{U}(1)\times\mathrm{SU}(p)\big)$.

If some of the β_i are not equal to α, the corresponding x_i and y_i vanish and the moduli space is $\mathrm{SU}(1,m)/\big(\mathrm{U}(1)\times\mathrm{SU}(m)\big)$ for $m<p$, where m counts the number of β_i that are equal to α. This reduced moduli space is consistent with the fact that the coefficients α and β_i determine the charges and the masses of the tensor fields. Only for a particular mass of the tensor fields will there be a massless scalar in the corresponding tensor multiplet, which just corresponds to the case $\beta_i=\alpha$ for the relevant index i.

To summarize, the moduli space of an $\mathcal{N}=4$ supersymmetric AdS_5 vacuum is always of the form

$$\mathcal{C} = \frac{\mathrm{SU}(1,m)}{\mathrm{U}(1)\times\mathrm{SU}(m)} \tag{4.21}$$

for some m with $2m\leq n$, where n denotes the original number of vector multiplets in the ungauged theory. In addition, m counts the number of tensor fields in tensor multiplets that are charged with respect to the U(1) gauge group factor with the same charge as the two tensor fields from the gravity multiplet.

The above type of moduli space was also found in [17] in a particular subset of 5D, $\mathcal{N}=4$ gauged supergravity theories that arise in type IIB compactifications on orbifolds of S^5. Our results show that *all* $\mathcal{N}=4$ AdS vacua of 5D gauged supergravity have this moduli space. Note that for $m=1$ this gives the familiar moduli space of $\mathcal{N}=4$ super Yang-Mills theory with the metric $g\sim(\tau-\bar\tau)^{-2}$, which also occurs in the untwisted sector of the half-maximally supersymmetric 5-sphere orbifolds discussed in [17].

The coset space $\mathbb{C}H^m := \mathrm{SU}(1,m)/\big(\mathrm{U}(1)\times\mathrm{SU}(m)\big)$ is sometimes called the complex (or Hermitian) hyperpolic space and has several geometric properties that are also important for the rest of this paper. We first note that $\mathbb{C}H^m$ is the non-compact Riemannian symmetric space dual[11] to the complex projective space $\mathbb{C}P^m = \mathrm{SU}(1+m)/\big(\mathrm{U}(1)\times\mathrm{SU}(m)\big)$ and that it is a Hermitian symmetric space of complex dimension m with isometry group $\mathrm{SU}(1,m)$. Like all Hermitian symmetric spaces, $\mathbb{C}H^m$ is a Kähler manifold, and a form of the Kähler potential that makes the $\mathrm{SU}(m)$ isometry subgroup manifest is

$$K = -M^3\ln(1-z^i\bar z^i)\,, \tag{4.22}$$

where z^i $(i=1,\dots,m)$ are dimensionless local complex coordinates on the manifold. For future use we also included the dependence on the five-dimensional Planck mass M which up to this point was chosen to be unity.[12] Note that for dimensionless scalar fields the metric and K have mass dimension three (in 5D) and indeed from (4.22) one finds

$$g_{i\bar j} = M^3\left(\frac{\delta^{ij}}{(1-z^k\bar z^k)}+\frac{\bar z^i z^j}{(1-z^k\bar z^k)^2}\right). \tag{4.23}$$

[11]The dual of a symmetric space G/H with Cartan decomposition $\mathrm{Lie}(G)=\mathrm{Lie}(H)\oplus\mathfrak{k}$ is the symmetric space G'/H with Cartan decomposition $\mathrm{Lie}(G')=\mathrm{Lie}(H)\oplus i\mathfrak{k}$ (cf. [36]). If G/H is compact and has positive sectional curvature, then G'/H is non-compact and has negative sectional curvature, and vice versa.

[12]For $m=1$ there exists a coordinate transformation which puts K into the form $K=-M^3\ln(\tau-\bar\tau)$.

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$\mathbb{C}H^m$ is also a special-Kähler manifold with holomorphic prepotential (see e.g. [37] for further details on the special-Kähler geometry in various symplectic frames)[13]

$$F(X) = \frac{i}{2} X^I \eta_{IJ} X^J , \tag{4.24}$$

where $(X^I) = (X^0, X^i)$, $I = 0, 1, \ldots, m$ are homogeneous special coordinates related to the z^i via $X^i/X^0 = z^i$, and $\eta_{IJ} = \mathrm{diag}(+1, -1, \ldots, -1)$. In general, the Riemann curvature tensor of special-Kähler manifolds obeys [38]

$$R^l{}_{j\bar{m}k} = -M^6 g^{l\bar{l}} C_{\bar{l}\bar{m}\bar{k}} g^{\bar{k}n} C_{nkj} + M^{-3} (g_{\bar{m}j} \delta^l_k + g_{\bar{m}k} \delta^l_j) \,, \tag{4.25}$$

where $C_{ijk} = e^{K/M^3} F_{ijk}$, with F_{ijk} being the third derivatives of the prepotential F.[14] Since for the case at hand F is quadratic, we have $C_{ijk} = 0$ and thus the Riemann tensor of $\mathbb{C}H^m$ obeys

$$R^l{}_{j\bar{m}k} = M^{-3} (g_{\bar{m}j} \delta^l_k + g_{\bar{m}k} \delta^l_j) \,. \tag{4.26}$$

This property of $\mathcal{C}$ is closely related to the tt^*-geometry of the dual SCFT, as we discuss in appendix B.

5 Holography and the $\mathcal{N} = 2$ SCFT conformal manifold

So far our analysis has been entirely within $5D$, $\mathcal{N} = 4$ gauged supergravity. As we mentioned in the introduction, one of the motivations to study supersymmetric AdS-backgrounds comes from the relation to holographically dual superconformal field theory (SCFT) within the AdS/CFT correspondence. For the case at hand, this would be a $4D$, $\mathcal{N} = 2$ SCFT with eight ordinary and eight superconformal supercharges. The holographic dictionary between higher-dimensional type IIB backgrounds of the form $\mathrm{AdS}_D \times Y_{10-D}$, where Y_{10-D} is an appropriate compact manifold, has been discussed in [40, 41] and reviewed, for example, in [42]. Here we only focussed on the AdS_D factor and did not consider any relation to solutions of higher-dimensional supergravities or string theories. It has not yet been firmly established which aspects are captured by our lower-dimensional analysis. However, for consistent truncations it is expected that the lower-dimensional supergravity does give reliable predictions for the dual SCFT in the large-N limit. General consistent truncations to five-dimensional $\mathcal{N} = 2$ and $\mathcal{N} = 4$ gauged supergravities have been performed for instance in [43–48], but most of these truncations focus on gauged supergravities where the AdS_5 vacuum is only $\mathcal{N} = 2$ supersymmetric. It would be interesting to find consistent truncations to five-dimensional supergravity for models with $\mathcal{N} = 4$ vacua, as for instance the examples of [17], and to understand whether localized sources in the higher-dimensional theory can be included in such an analysis.

[13] $\mathbb{C}H^m$ is a special-Kähler manifold of the "local" type, i.e. one that could arise in the vector multiplet sector of 4D, $\mathcal{N} = 2$ supergravity, but not in rigid 4D, $\mathcal{N} = 2$ supersymmetry. Such a distinction could not be given for the AdS_4 moduli spaces studied in [5].

[14] Here we follow the conventions of [39]. Note that C_{ijk} and $R^l{}_{j\bar{m}k}$ are dimensionless, so that with $g_{i\bar{j}} \sim M^3$ both sides of (4.25) are in fact proportional to M^3.

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If a suitable consistent truncation to 5D, $\mathcal{N} = 4$ supergravity exists, one might still wonder whether there could be moduli among the modes one has truncated out, in particular among the infinite tower of Kaluza-Klein modes. While the high masses of generic KK modes would usually prevent them from being moduli, in AdS spacetimes there could be a scalar in a KK-multiplet that has mass zero even though the other members of the multiplet have smaller and/or larger masses, as happens e.g. in the KK decomposition of type IIB supergravity on the five-sphere [49, 50]. An exactly marginal operator, however, also has to be a singlet of the R-symmetry group of the SCFT, so that any AdS-modulus candidate among the truncated modes would have to be neutral under the $SU(2) \times U(1)$ part of the 5D gauge group. If this group is realized geometrically in the compactification space, a modulus in a KK multiplet would have to be inert under this geometric symmetry, which is typically not the case.

Keeping such issues in mind, let us now become a bit more specific and discuss possible interpretations of our result. In section 3, we found that the AdS-backgrounds necessarily have an unbroken $U(1) \times SU(2)$ symmetry gauged by the graviphotons, which indeed corresponds to the $U(1) \times SU(2)$ R-symmetry of the dual $\mathcal{N} = 2$ SCFT. The unbroken gauge factor $H_c \subset SO(n)$ has to be related to an unbroken flavour symmetry of the SCFT. We also found that non-compact symmetries can be gauged, but they are always spontaneously broken in the vacuum.

In section 4, we derived the coset space $SU(1,m)/\big(U(1) \times SU(m)\big)$ as the moduli space of the AdS-backgrounds. In the dual SCFT, this corresponds to the conformal manifold, i.e. the space of exactly marginal couplings φ^i [3]. They deform a given SCFT, S^*, as

$$S[\varphi] = S^* + \sum_i \int \varphi^i O_i \,, \tag{5.1}$$

where the O_i denote the exactly marginal operators of S^*.[15] This deformation space is endowed with a natural metric, the Zamolodchikov metric given by

$$g_{ij}(\varphi) = x^{2\Delta} \langle O_i(x) O_j(0) \rangle_{S[\varphi]} \,. \tag{5.2}$$

The holographic dictionary states that in the large N-limit this metric should agree with the metric on the moduli space of AdS-backgrounds. In section 4, we derived such moduli spaces in 5D supergravity, and thus it is of interest to do a more detailed comparison.

First of all there is the question to what extent the Zamolodchikov metric is already constrained by supersymmetry. Mimicking an argument first employed by N. Seiberg in [22], one can promote φ^i to a background supermultiplet. This in turn constrains the metric of this multiplet to obey the properties imposed by the supersymmetry of the given SCFT. For example in an $D = 4$, $\mathcal{N} = 1$ SCFT this argument constrains $g_{ij}(\varphi)$ to be a Kähler metric, which has indeed been shown by other means in [51].

In 4D, $\mathcal{N} = 2$ SCFT, the marginal operators O_i reside in conformal chiral multiplets with Weyl weight $w = 2$, while the deformation parameters φ^i are members of chiral

[15] The notation S^* is somewhat symbolic as we include the possibility of non-Lagrangian theories. Furthermore, the marginal operators O_i we are interested in preserve all supercharges and thus have scaling dimension $\Delta = 2$, are R-symmetry singlets and form the highest components of their $\mathcal{N} = 2$ superfields.

multiplets with $w = 0$. Unfortunately, the geometry of Weyl multiplets with arbitrary Weyl weight is not known.[16] In [26] it was shown that the metric on $\mathcal{C}$ is Kähler and additionally obeys the tt^*-geometry [27]. Moreover, the Kähler potential gives the sphere partition function, as has been shown by using localization techniques in [28] and supersymmetric Ward identities in [29]. The moduli space $\mathcal{C} = \mathbb{C}H^m$ we obtained in section 4 is both Kähler and obeys the tt^*-geometry, as discussed in appendix B. In fact, it is the specific special-Kähler manifold with a quadratic prepotential. Of course, in our approach we only capture the large-N limit of the exact Zamolodchikov metric and therefore we are led to conjecture that our result arises only in that limit. In appendix B, we discuss in more detail the large-N limit in view of [26] and argue for a specific subleading behaviour of the Zamolodchikov metric as well as the (single and double trace) operators of dimension four in cases where our analysis applies. Our result also suggests that the sphere partition function of suitable $D = 4$, $\mathcal{N} = 2$ SCFTs should simplify in the large-N limit to agree with the exponential of the Kähler potential of (4.21).

6 Conclusion

In this paper, we identified all five-dimensional, $\mathcal{N} = 4$ gauged supergravity theories that allow for $\mathcal{N} = 4$ AdS_5 vacua and determined the moduli spaces of these solutions. The requirement of a fully supersymmetric AdS vacuum constrains the gauge group of the supergravity theory to be of the general form $\mathrm{U}(1) \times H$, where H must contain an $\mathrm{SU}(2)$ subgroup gauged by three graviphotons, and the $\mathrm{U}(1)$ factor is gauged by another graviphoton and must (at least) act nontrivially on two tensor fields in the gravity multiplet. The moduli space of the resulting vacua was found to be the special-Kähler manifold $\mathrm{SU}(1,m)/\big(\mathrm{U}(1) \times \mathrm{SU}(m)\big)$, where m counts the number of tensor fields from tensor multiplets with the same $\mathrm{U}(1)$ charge as the two tensor fields from the gravity multiplet.

We discussed this result in the context of the AdS/CFT correspondence, where the holographic dual of the AdS moduli space is given by the conformal manifold of dual 4D, $\mathcal{N} = 2$ SCFTs. In cases where the truncation to five dimensions captures all essential features of the ten-dimensional theory this determines the large-N behavior of the conformal manifold and via the result of [28] also the large-N behavior of the sphere partition function of the SCFT. Comparison with the tt^*-like geometry found in [26] indicates that our result might constrain the large-N behavior of three-point functions that appear in the OPE of exactly marginal operators.

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[16]For higher-derivative couplings of the Weyl multiplet see, for example, [52, 53].

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A SO(5) vs. USp(4) bases

The R-symmetry group of the $\mathcal{N}=4$ Poincaré superalgebra in five space-time dimensions is given by $\mathrm{USp}(4) \equiv \mathrm{U}(4) \cap \mathrm{Sp}(4, \mathbb{C})$. We denote the corresponding symplectic form by Ω^{ij}, $i, j = 1, \ldots, 4$, so that USp(4) is generated by Hermitian(4×4)-matrices $\mathrm{U}_i{}^j$ that satisfy $U^T \Omega + \Omega U = 0$. The fermions of $\mathcal{N}=4$ supergravity transform in the fundamental representation of USp(4). In order to describe their couplings to the scalar fields $(\mathcal{V}_M^m, \mathcal{V}_M^a)$ of the coset space $\mathrm{SO}(5,n)/\mathrm{SO}(5) \times \mathrm{SO}(n)$, one converts the SO(5) index $m = 1, \ldots, 5$ to USp(4) indices i, j using the group isomorphism $\mathrm{USp}(4) \cong \mathrm{Spin}(5)$ that follows from properties of the SO(5) Clifford algebra. In the following, we briefly review some useful identities related to this isomorphism and match it to the supergravity conventions used in this paper (for further details see e.g. [54, 55]).

The Clifford algebra in five Euclidean dimensions is represented by (4×4) gamma matrices Γ_m, $m, n, \ldots = 1, \ldots, 5$ satisfying

$$\{\Gamma_m, \Gamma_n\} = 2\delta_{mn}\mathbf{1} \quad \Longleftrightarrow \quad \Gamma_{m\,i}{}^j \Gamma_{n\,j}{}^k + (m \leftrightarrow n) = 2\delta_{mn}\delta_i^k \,. \tag{A.1}$$

As in any odd dimension, there are actually two equivalence classes of irreducible representations of (A.1). They differ in how one defines the the fifth gamma matrix in terms of the first four, which leaves a sign ambiguity: $\Gamma_5 = \pm\Gamma_1\Gamma_2\Gamma_3\Gamma_4$. Apart from eqs. (A.13)–(A.16), all equations in this appendix are insensitive to this sign choice.

5D rotational invariance requires the Γ_m to be traceless, and for a Euclidean Clifford algebra, they may also always be chosen to be Hermitian, as we will assume from now on:

$$\Gamma_m = \Gamma_m^\dagger \,. \tag{A.2}$$

For any representation of this type, there exists then a "charge conjugation matrix" C with the following properties:

$$\Gamma_m^T = \Gamma_m^* = C\Gamma_m C^{-1} \,, \qquad C = -C^T, \qquad C^* = -C^{-1}. \tag{A.3}$$

These relations imply, in particular, that $(C\Gamma_m)$ and $(C\Gamma_{mnpq})$ are antisymmetric, whereas $(C\Gamma_{mn})$ and $(C\Gamma_{mnp})$ are symmetric matrices. Due to its antisymmetry and invertibility, we can identify C with a symplectic form Ω as follows:

$$\Omega^{ij} := C^{ij}, \qquad \Omega_{ij} := C_{ji} = -C_{ij} \,. \tag{A.4}$$

Here, C^{ij} denote the entries of C, whereas C_{ij} are meant to be the components of the inverse matrix C^{-1} so that $C^{ij}C_{jk} = \delta_k^i$, and hence $\Omega^{ij}\Omega_{kj} = \delta_k^i$. Ω can then be used to raise and lower USp(4) indices $i, j, \ldots$ according to the convention [14]

$$V^i = \Omega^{ij} V_j \,, \qquad V_i = V^j \Omega_{ji} \,. \tag{A.5}$$

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We can then define

$$\Gamma_m^{ij} := \Omega^{ik}\Gamma_{m\,k}{}^{j} = (C\Gamma_m)^{ij}, \qquad \Gamma_{m\,ij} := \Gamma_{m\,i}{}^{k}\Omega_{kj} = (\Gamma_m C^{-1\,T})_{ij}\,. \tag{A.6}$$

Γ_m^{ij} has the properties

$$\Gamma_m^{ij} = -\Gamma_m^{ji}\,, \qquad \Gamma_m^{ij}\Omega_{ij} = 0\,, \qquad (\Gamma_m^{ij})^* = \Omega_{il}\,\Omega_{jk}\Gamma_m^{lk}\,, \tag{A.7}$$

where the first identity is just the antisymmetry of $(C\Gamma_m)$, the second is the tracelessness of Γ_m, and the third equation a consequence of the reality properties (A.3). Completely analogous identities are inherited by the coset representatives

$$\mathcal{V}_M^{ij} := \mathcal{V}_M^m \Gamma_m^{ij}\,. \tag{A.8}$$

Using the above properties, it is easy to see that the SO(5) generators

$$M_{mn} := \frac{i}{4}\,[\Gamma_m, \Gamma_n] \tag{A.9}$$

are Hermitian (4×4)-matrices that also satisfy

$$(M_{mn})^T \cdot \Omega + \Omega \cdot M_{mn} = 0\,, \tag{A.10}$$

i.e. that they can be viewed as generators of USp(4) in the fundamental representation.

We close with some useful identities:

$$\Gamma_m^{ij}\Gamma_{n\,ij} = 4\delta_{mn}\,, \tag{A.11}$$

$$\mathrm{tr}(\Gamma_{mn}\Gamma_{pq}) = 4(\delta_{mq}\delta_{np} - \delta_{mp}\delta_{nq})\,, \tag{A.12}$$

$$\Gamma_m = \pm\frac{1}{24}\epsilon_{mnpqr}\Gamma^{npqr}, \tag{A.13}$$

$$\Gamma_{mn} = \mp\frac{1}{6}\epsilon_{mnpqr}\Gamma^{pqr}, \tag{A.14}$$

$$\Gamma_{mnp} = \mp\frac{1}{2}\epsilon_{mnpqr}\Gamma^{qr}, \tag{A.15}$$

$$\Gamma_{mnpq} = \pm\epsilon_{mnpqr}\Gamma^{r}, \tag{A.16}$$

$$\{\Gamma_{mn}, \Gamma_{pq}\} = 2\Gamma_{mnpq} + 2\delta_{np}\delta_{mq} - 2\delta_{nq}\delta_{mp}\,, \tag{A.17}$$

where we use $\epsilon_{12345} = 1$, and the signs refer to the sign choice $\Gamma_5 = \pm\Gamma_1\Gamma_2\Gamma_3\Gamma_4$.[17]

B Large N counting

In [26], the Riemann tensor of the metric on the conformal manifold of a 4D, $\mathcal{N} = 2$ SCFT was found to satisfy the relation

$$R^l_{i\bar{j}k} = -C^M_{ik}\,g_{M\bar{N}}\,C^{*\bar{N}}_{\bar{j}\bar{q}}\,g^{\bar{q}l} + g_{k\bar{j}}\delta^l_i + g_{i\bar{j}}\delta^l_k\,. \tag{B.1}$$

[17] In the main body of this work we will use the plus sign.

Here, C_{ij}^M are the chiral ring coefficients between chiral primaries O_i, O_j of conformal dimension $\Delta = 2$ and O_M of conformal dimension $\Delta = 4$, whereas $g_{i\bar{j}}$ and $g_{M\bar{N}}$ denote the Zamolodchikov metrics for these operators. The chiral ring coefficients can be expressed in terms of 3-point correlator coefficients $C_{ij\bar{M}}$ as

$$C_{ij\bar{M}} = C_{ij}^N g_{N\bar{M}} \, . \tag{B.2}$$

Note that all quantities in (B.1) are dimensionless and no powers of any mass scale as in (4.25) appear.

Our 5D supergravity analysis, on the other hand, led to AdS-moduli spaces of the form $\mathrm{SU}(m,1)/\big(\mathrm{SU}(m) \times \mathrm{U}(1)\big)$, which obeys (4.26). Since (B.1) resembles (4.25), it is worthwhile to establish a closer connection. Note that the two formulas differ in that the OPE coefficients C_{ijM} do not coincide with the C_{ijk} of special geometry. Therefore a comparison is not straightforward. As the supergravity approximation in AdS/CFT is generally only valid for large N (N being the number of colors), it is useful to understand the large-N behaviour of the various terms in (B.1).

In [56], extremal 2- and 3-point correlators of single trace chiral primary operators in 4D, $\mathcal{N} = 4$ super Yang-Mills theories were computed in the weak coupling limit and at strong 't Hooft coupling $\lambda = N g_{\mathrm{YM}}^2 \gg 1$ using the dual supergravity side of the AdS/CFT correspondence. The results were found to agree. We recall here the N dependence of the correlators in the weak coupling analysis.

We normalize the Yang-Mills action as $S = -\int \frac{1}{2g_{\mathrm{YM}}^2} \mathbf{Tr} F^2 + \ldots = -\int \frac{1}{4g_{\mathrm{YM}}^2} F^a F^a + \ldots$, where $F = F^a T^a$ with the $\mathrm{U}(N)$ generators T^a ($a = 1, \ldots, N^2$), which we assume to be in the fundamental representation of $\mathrm{U}(N)$, i.e. they are $(N \times N)$ matrices with $\mathbf{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$.

The scalar fields $\phi^\alpha = \phi_a^\alpha T^a$ ($\alpha = 1, \ldots, 6$) have scaling dimension $\Delta = 1$, transform in the fundamental representation of the R-symmetry group SO(6) and have the propagators

$$\langle \phi_a^\alpha(x) \phi_b^\beta(y) \rangle = \frac{g_{\mathrm{YM}}^2 \delta_{ab} \delta^{\alpha\beta}}{(2\pi)^2 |x-y|^2} \, . \tag{B.3}$$

As we are interested in massless supergravity scalar fields (the AdS moduli), we need to focus on marginal operators in the dual SCFT. They have scaling dimension $\Delta = 2$ for the lowest component scalar field (i.e. $\Delta = 4$ for the highest component of the superfield) and can be composed from two fundamental scalar fields ϕ_a^α as a single trace operator[18]

$$\mathcal{O}^{\alpha\beta} := \mathbf{Tr}(\phi^\alpha \phi^\beta) \, . \tag{B.4}$$

Using Wick's theorem, the free 2-point function of two such single trace operators $\mathcal{O}$ is of the form

$$g(x,y) = \langle \mathcal{O}^{\alpha\beta}(x) \mathcal{O}^{\gamma\delta}(y) \rangle = \frac{N^2 g_{\mathrm{YM}}^4 (\delta^{\alpha\gamma}\delta^{\beta\delta} + \text{cyclic})}{(2\pi)^4 |x-y|^4} \, . \tag{B.5}$$

[18]Here and in the following, the SO(6) indices $\alpha, \beta, \ldots$ should always be thought of as being in a completely symmetric and traceless combination, which, however, we do not make explicit as it does not affect the large N scaling. Likewise, we are really interested in $\mathrm{SU}(N)$ instead of $\mathrm{U}(N)$ generators only.

More generally, we have [56]

$$g(x,y) = \langle \mathcal{O}^{\alpha_1 \ldots \alpha_k} \mathcal{O}^{\beta_1 \ldots \beta_k} \rangle = \frac{N^k g_{\rm YM}^{2k} (\delta^{\alpha_1 \beta_1} \ldots \delta^{\alpha_k \beta_k} + \text{cyclic})}{(2\pi)^{2k} |x-y|^{2k}} \,, \tag{B.6}$$

for the single trace operators $\mathcal{O}^{\alpha_1 \ldots \alpha_k} = \mathbf{Tr}(\phi^{\alpha_1} \ldots \phi^{\alpha_k})$. We need the case $k = 4$ for the $\Delta = 4$ single trace operators, for which we read off the scaling $N^4 g_{\rm YM}^8$.

Next, let us consider the 2-point function of the $\Delta = 4$ double trace operators defined as $\mathcal{O}^{\alpha\beta,\gamma\delta}(x) := \mathbf{Tr}\big(\phi^\alpha(x)\phi^\beta(x)\big)\mathbf{Tr}\big(\phi^\gamma(x)\phi^\delta(x)\big)$. It scales like $N^4 g_{\rm YM}^8$, because Wick's theorem gives rise to terms such as $\delta^{ab}\delta^{cd}\delta^{ef}\delta^{gh}\delta^{ae}\delta^{bf}\delta^{cg}\delta^{dh} \sim N^2 N^2$. Note that among the dimension 4 operators that can be formed from the scalars ϕ^α, there are only the single trace operators $\mathcal{O}^{\alpha\beta\gamma\delta}$ and the double trace operators $\mathcal{O}^{\alpha\beta,\gamma\delta}$, when one restricts oneself to the traceless SU(N) generators.

The 3-point functions we need to consider are thus of the form

$$\begin{gathered} \langle \mathcal{O}^{\alpha\beta} \mathcal{O}^{\gamma\delta} \mathcal{O}^{\epsilon\eta\kappa\lambda} \rangle \\ \langle \mathcal{O}^{\alpha\beta} \mathcal{O}^{\gamma\delta} \mathcal{O}^{\epsilon\eta,\kappa\lambda} \rangle \,. \end{gathered} \tag{B.7}$$

The first 3-point function scales as $\lambda^4/N \sim N^3$ [56]. The second 3-point function can be directly determined with Wick's theorem and gives a contribution that scales as $N^4 g_{\rm YM}^8$ (because it leads to $\delta_{ab}\delta^{ab}\delta_{cd}\delta^{cd} \sim N^2 N^2$), as well as one that scales as $N^2 g_{\rm YM}^8$ (coming from a contraction that collapses to $\delta_{ab}\delta^{ab} \sim N^2$). If the above scalings are also valid at strong 't Hooft coupling and also in general $\mathcal{N} = 2$ SCFTs, one would have the following scalings:

$$\begin{aligned} & g_{i\bar{j}} \sim \lambda^2 \sim N^2, \\ & g_{I\bar{J}} \sim \lambda^4 \sim N^4, \\ & C_{ijI} \sim \lambda^4/N \sim N^3, \quad (I \sim \text{single trace } \Delta = 4)\,, \\ & C_{ijI} \sim \lambda^4 \left(1 + \frac{1}{N^2}\right) \sim N^4 + N^2, \quad (I \sim \text{double trace } \Delta = 4)\,. \end{aligned} \tag{B.8}$$

Note that we inferred from (4.22) that on the supergravity side $g_{i\bar{j}} \sim M^3$, which, using the AdS/CFT dictionary, indeed implies $g_{i\bar{j}} \sim N^2$ on the dual side.

Putting everything together, the right hand side of (B.1) then scales as

$$\begin{aligned} & g_{k\bar{j}}\delta_i^l + g_{i\bar{j}}\delta_k^l \sim N^2 + N^0 + \ldots\,, \\ & \text{single trace} \sim N^3 N^3 N^{-4} N^{-2} \sim N^0 + \ldots\,, \\ & \text{double trace} \sim (N^4 + N^2)(N^4 + N^2) N^{-4} N^{-2} + N^2 \sim N^2 + N^0 + \ldots\,. \end{aligned} \tag{B.9}$$

Note that the left-hand side of (B.1) is independent of N as it is the (scale-invariant) Riemann tensor. This means that at leading order (N^2) the terms on the right-hand side universally have to cancel each other.[19] This predicts, on the one hand, a certain leading behavior for the OPE coefficients C_{ijI} for double trace operators. Moreover, it predicts a very specific subleading contributions (N^0) of the metric $g_{i\bar{j}}$ and the double trace OPE coefficients, as well as a specific leading behaviour of the OPE coefficients C_{ijI} for single trace operators. Only if they conspire in the right way, they can be consistent with the supergravity result (4.21). It would be interesting to check this in explicit SCFTs.

[19] We thank K. Papadodimas for extensive discussions on this point.

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