SUPER-TWISTING SLIDING MODE CONTROLLER FOR FREEWAY RAMP METERING

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ABSTRACT: *This paper proposes a new feedback controller for freeway ramp metering based on a second order sliding mode technique. It is called "super twisting sliding mode controller (STSMC)". STSMC is characterized by its simplicity and its robustness. Moreover, the STSMC is less sensitive to parameter variations and the model uncertainty. The chattering phenomenon is reduced compared to the first order sliding mode controller. And then, this new controller is compared with a well known feedback controller for freeway ramp metering called ALINEA*. Numerical simulations and a comparative study with the well known "ALINEA" show the relevance of the STSMC and provide a serious way for future développements.*

KEYWORDS: *Freeway ramp metering, super-twisting, ALINEA, traffic flow model*

1 INTRODUCTION

The considerable expansion of car-ownership has led to the daily appearance of congestion on urban and interurban motorways, especially during peak hours. The steadily increasing number and length of traffic jams on motorways put inconvenience to the road users, increase total travel time, economic losses and environmental pollution, and reduce traffic safety. As a result, the total social welfare decreases. This is a matter of control problems. What is urgently needed is to restore and maintain the full utilization of the motorways' capacity, instead of simply increasing capacity by expansion of infrastructure. Ramp metering has proven to be one of the most efficient means to solve this problem, as one of several dynamic traffic management (DTM) measures (Yuan; 2008).

Ramp metering or admissible control, represents the most efficient and direct way to solve congestion problems and to upgrade freeway traffic (Papageorgiou et al.; 2003) (Smaragdis et al.; 2004). This control action consists in regulating the ramp flow at the entrance of the freeway (Kotsialos and Papageorgiou; 2004). As stated in (Papageorgiou and Kotsialos; 2002), ramp metering strategies are a valuable tool for an efficient traffic management that can be classified into: *reactive strategies* that aim to maintain the traffic conditions in freeway close to pre-specified set values using real-time measurements, and *proactive strategies*, aiming at specifying optimal traffic conditions for a whole freeway network based on the demand and the model prediction over a time horizon (Hegyi; 2004) (Smaragdis et al.; 2004).

Ramp metering strategies can be implemented locally (*isolated ramp metering*) in the vicinity of each ramp to calculate the corresponding ramp metering values. It can be implemented simultaneously (*coordinated ramp metering*) when the objective is to use the available traffic measurements from larger freeway sections. It can be integrated with another control tool like variable speed limit (VSL) (Hegyi et al.; 2005).

In this paper, we focus on isolated ramp metering problem. In the literature, many isolated ramp control algorithms can be found. Feedback control strategy allows a better management of traffic situation near a ramp. Indeed, application of feedback ramp metering leads to maintain the traffic state density at the merge segment around a critical one which insures a maximum throughput. Several feedback control strategies have been proposed in the literature (Iordanova et al.; 2008) (Papageorgiou et al.; 1991) (Papageorgiou and Kotsialos; 2002). The most popular one is ALINEA. It is the first feedback control implemented in a real site (Haj-Salem et al.; 2001). Nevertheless, ALINEA as a simple integrator, is not robust and it can not react quickly when some perturbations do exist in the system. In this framework, the objective of this paper is to present an alternative to such linear control (ALINEA algorithm) by introducing a robust algorithm which is reactive and less sensitive to parameter variations and model uncertainty.

*ALINEA is "Asservissement LINéaire d'Entrée Autoroutière"

This paper is organized as follows:

- Section 2 outlines the macroscopic model (METANET) used to evaluate and compare the control algorithms.
- Section 3 explains the ramp metering algorithms in details; STSMC and ALINEA.
- In section 4 shows several numerical simulations for no-control case, ALINEA as well as STSMC.
- The conclusion, section 5, summarizes the main results and lists some promising perspectives and further researches.

2 Traffic Flow Model

METANET is a second order macroscopic modeling tool for simulating traffic flow phenomena in motorway networks of arbitrary topology and characteristics, including motorway stretches, bifurcations, ramps, and off-ramps. We refer to Messemer and Papageorgiou (1990) Kotsialos et al. (2002) Bellemans (2003) for a full description of the METANET model.. This modeling approach allows to simulate of all kinds of traffic conditions (free, dense, congested) and of capacity-reducing events (incidents) with prescribed characteristics (location, intensity, duration). Furthermore METANET allows to take into account control actions such as ramp metering, route guidance, etc (Kotsialos et al.; 2002).

The basic equations used to compute the traffic variables (traffic density $\rho_{m,i}$ in veh/km, traffic flow $q_{m,i}$ in veh/h and mean speed $v_{m,i}$ in km/h) for every segment i of motorway link m are shown at the following, see Figure 1:

$$\rho_{m,i}(k+1) = \rho_{m,i}(k) + \frac{T}{L_m \lambda_m}[q_{m,i-1}(k) - q_{m,i}(k)] \quad (1)$$

$$q_{m,i}(k) = \rho_{m,i}(k) v_{m,i}(k) \lambda_m \quad (2)$$

$$\begin{aligned} v_{m,i}(k+1) &= v_{m,i}(k) + \frac{T}{\tau}\{V[\rho_{m,i}(k)] - v_{m,i}(k)\} \\ &+ \frac{T}{L_m} v_{m,i}(k)\left[v_{m,i-1}(k) - v_{m,i}(k)\right] \\ &- \frac{\nu T}{\tau L_m} \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\rho_{m,i}(k) + \kappa} \end{aligned} \quad (3)$$

$$V[\rho_{m,i}(k)] = v_{free,m} \exp\left[-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{cr,m}}\right)^{a_m}\right] \quad (4)$$

where $v_{free,m}$ is the free-flow speed of link m, $\rho_{cr,m}$ is the critical density per lane of link m, and a_m is a parameter of the fundamental diagram as shown in Figure 2 (see equation 4) of link m. Furthermore, τ, a time constant. ν, an anticipation constant, and κ are constant parameters.

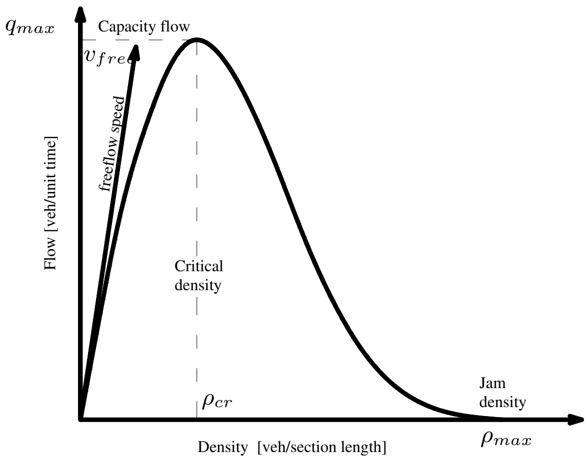

Figure 2: Payne fundamental diagram, flow-density relationship.

In order for the speed calculation to account for the speed decrease in segment i caused by merging phenomena, the following term is added to the Eq. 3:

$$-\frac{\delta T q_o v_{m,i}(k)}{L_m \lambda_m (\rho_{m,i}(k) + \kappa)} \quad (5)$$

where δ denotes a model parameter, λ_m is the number of lanes, T is the simulation time step, L_m is the length of the link m.

Origins are modeled with a simple queue model. The length of the queue equals the previous queue length plus the demand $d_o(k)$, minus the outflow $q_o(k)$.

$$w_o(k+1) = w_o(k) + T[d_o(k) - q_o(k)] \quad (6)$$

The outflow $q_o(k)$ of an origin link o depends on the traffic conditions on the mainstream and, for the metered ramp, on the ramp metering rate, where $r_o(k) \in [0, 1]$.

$$\begin{aligned} q_o(k) &= \min\left[d_o(k) + \frac{w_o(k)}{T}, Q_o r_o(k), Q_o \right. \\ &\left. \left(\frac{\rho_{max,m} - \rho_{m,i}(k)}{\rho_{max,m} - \rho_{cr,m}}\right)\right] \end{aligned} \quad (7)$$

where Q_o denotes the ramp capacity flow and $\rho_{max,m}$ the maximum density of link m.

The upstream speed of the mainstream link is assumed to be equal to the speed of the first segment, i.e., $v_{m,0} = v_{m,1}$. The downstream density of the

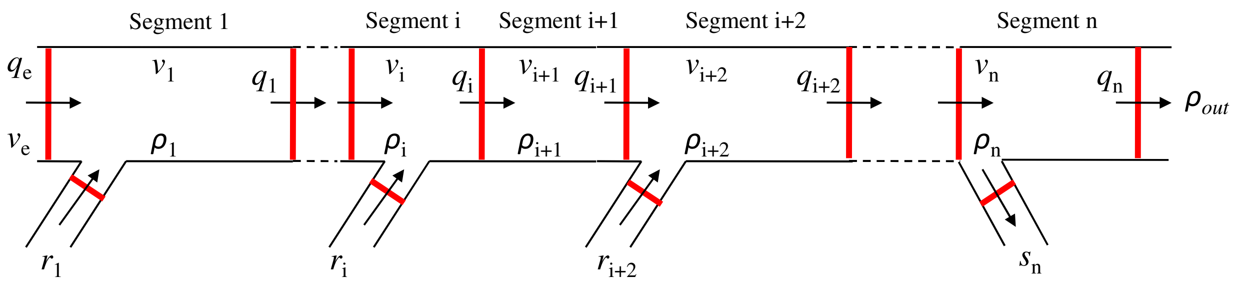

Figure 1: Example of a freeway mainstream section with n segments in the Link m.

mainstream link is assumed to be equal to the density of the last segment N in free flow, and to be equal to the critical density in congested flow.

$$\rho_{m,N+1}(k) = \left\{ \begin{array}{ll} \rho_{m,N}(k) & if\ \rho_{m,N}(k) < \rho_{cr,m} \\ \rho_{cr,m} & if\ \rho_{m,N}(k) \geq \rho_{cr,m} \end{array} \right\} \tag{8}$$

3 Real-Time Ramp Metering

3.1 STSMC

In this paper, we exploit one of the most attractive methods that can be applied to a broad class of nonlinear systems resulting in controllers that are robust to modeling errors and unknown disturbances. We investigate variable structure control (VSC) as a high-speed switched feedback control resulting in sliding mode. The gains in each feedback path switch between two values according to a rule that depends on the value of the state at each instant. The purpose of the switching control law is to derive the non-linear system state trajectory on a prespecified surface in the state space and to maintain the system state trajectory on this surface (switching surface) for subsequent time.

The sliding mode control is a well documented technique. The fundamentals of this kind of control can be found in Utkin (1993) and Utkin et al. (2009). The first order sliding mode control design procedure consists of a sliding surface ($s = 0$) design with relative degree 1 with respect to the control and a discontinuous control action that ensures a sliding mode (Rivera et al.; 2011).

The main disadvantage of the first order sliding mode is the chattering phenomenon which is characterized by small oscillations at the output of the system that can result in harmful control machines in the systems (Rivera et al.; 2011).

High order sliding mode technique can overcome the chattering phenomenon (Levant; 1993) and (Levant; 1998). There are several algorithms to realize such high order sliding mode. For instance, the sub-optimal controller, the terminal sliding mode controllers, the twisting controller and the super-twisting controller.

In this paper, we design an ramp control algorithm using the super-twisting sliding mode. It is a very simple algorithm, reducing the chattering phenomenon and robust as classique (first order) sliding mode technique. An open-loop based on the inverse dynamic control technique (trajectory planning) is designed and closed-loop control (trajectory tracking) is obtained using the super twisting sliding mode control algorithm. The objective is to ensure the convergence of the state density value of the ramp segment to the bounded desired trajectory at finite time.

The state space of the dynamic system at segment i (see Figure 1) is as follows:

$$\begin{aligned} \rho_i(k) &= \rho_i(k-1) + \frac{1}{L_i \lambda_i}[q_{i-1}(k) + q_i(k) + q_{r_i}(k)] \\ y(k) &= \rho_i(k) \end{aligned} \tag{9}$$

This system can be inversed. Therefore, it can be controlled directly in open-loop by using the inverse dynamic control technique. In this technique, the control law forces the state variables to follow the desired prespecified trajectory. To this end, some assumptions are needed (Derafa et al.; 2012):

- All the state variables (ρ and v of all segments) can be measured or estimated by sensors.
- The desired trajectory and its first and second time derivatives should be bounded.
- The variable which is controlled (traffic density of the ramp segment) should be bounded.

- The state variables (densities and mean speeds of all the segments) are limited to $0 < \rho < \rho_{max}$, $0 < v < v_{max}$.

According to these assumptions, one can inverse the system (9). In addition, it could be controlled in open-loop using this algorithm:

$$q_{r_i}(k) = U_{eq}(k) = (\rho_i(k) - \rho_i(k-1))L_i\lambda_i - q_{i-1}(k) + q_i(k) \quad (10)$$

To ensure the steady-state and to reduce the influence of parameter variations and disturbances, the traffic flow has to be operated in closed loop. The feedback control is designed by adding a super twisting sliding mode algorithm to the open-loop algorithm shown in Eq. (10). The feedback control is of important use to achieve desired dynamic behaviour and to compensate the external disturbances (see Figure 3):

$$U = \overbrace{(\rho_i(k) - \rho_i(k-1))L_i\lambda_i - q_{i-1}(k) + q_i(k)}^{Open\ loop\ terms} \underbrace{-k_1|S|^{\frac{1}{2}}\,sign(S) - k_2\int_0^k sign(S)dk}_{Feedback\ terms} \quad (11)$$

where U is the closed-loop control variable, k_1 and k_2 are two positive parameters that must be selected to satisfy the desired performances of the closed loop system and to ensure the finite time convergence at zero.

The sliding mode controller is designed based on the Lyapunov theorem that guarantee an asymptotic stability:
If there exist a scalar funciton $V(S)$ with continuous first partial derivatives such that

$$\left.\begin{array}{l} V(S)\ is\ positive\ definite. \\ \dot{V}(S)\ is\ negative\ definite \end{array}\right\} V\dot{V} < 0$$

Then the equilibrium point is asymptotically stable

3.2 ALINEA

ALINEA is a traffic responsive strategy based on classical automatic control methods. It is the first algorithm in highway traffic domain based on a feedback philosophy (Papageorgiou et al.; 1991). The optimal slip road volume is selected for each control interval, based on keeping the occupancy level downstream of the ramp at a preset 'critical' value, typically set just below that which would cause traffic flow to break down.

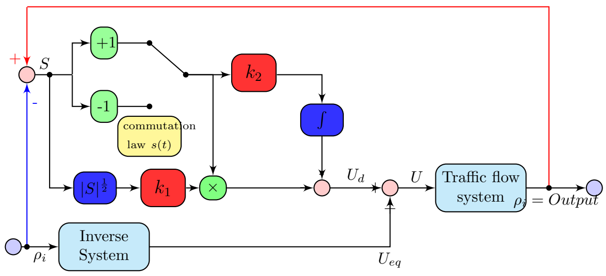

Figure 3: Closed-loop control structure

The ALINEA strategy calculates at each period $k = 1, 2, ...$ (e.g., every minute) (Papageorgiou et al.; 1997):

$$q_r(k) = q_r(k-1) + K_R[\hat{O} - O_{out}(k)] \quad (12)$$

where $K_R > 0$ is a regulator parameter, $\hat{O}$ is the critical occupancy. In field experiments, it was found that ALINEA is not very sensitive to the choice of the regulator parameter K_R. A value of $K_R = 70\ veh/h$ was found to yield excellent results at many different sites. The value of $q_r(k-1)$ should be set equal to the measured actual ramp volume in the last period (i.e., not equal to the calculated ramp volume in the last period) (Papageorgiou et al.; 1997) (Papageorgiou and Hadj-Salem; 1995).

The difference between ALINEA and the open loop algorithms is that ALINEA reacts smoothly even to slight differences $\hat{O} - O_{out}(k)$, and thus it may prevent congestion in an elegant way and stabilize traffic flow at a high throughput level (Papageorgiou et al.; 1997).

ALINEA requires only one mainstream detector station for $O_{out}(k)$ downstream of the ramp entrance. The measurement location should be such that a congestion, originating from excessive ramp volumes, is visible in the measurement, see Figure 4.

4 Numerical simulations and results

For the numerical simulations, we consider the freeway section depicted in Figure 5. The freeway section is discretized into 6 equal segments, each with 2 lanes λ and length $L = 1\ km$, with one ramp at the second segment. All the model parameters, shown in Table 1, for all the segments are considered equals for the two algorithms. The main origin and ramp traffic demands are chosen in order to obtain a simulation with high density where the traffic control can improve the behavior of the system, see Figure 6. The simulation time steps are 900.

Figures 7, 8, and 9 show, respectively, the time evo-

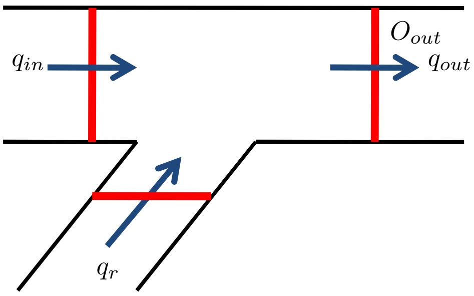

Figure 4: Local ramp-metering variables (ALINEA)

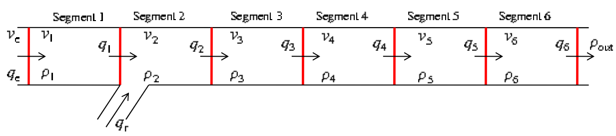

Figure 5: Simulated cross section

lution of the traffic densities, mean speed and traffic flow in all segments in no-control case. The total travel time TTT of the whole section is calculated, in the case of no control $TTT = 1715.8\ veh - h$. From latter figures, one can notice that the traffic situation is strongly congested at segment 1 and 2 because of the ramp located at the segment 2. Figure 10 shows the queue length at the main origin as well as at the ramp in the case where the ramp flow is not controlled.

In the case of the new algorithm STSMC, one can observe an improvement in traffic situation especially in the segments 1 and 2. Figures 11, 12 and 13 show, respectively, the time evolution of traffic density, traffic mean speed and traffic flow for all segments of the simulated section, respectively. For the case of STSMC, the queue lengths at origins are shown in Figure 14. In the same case the $TTT = 1552\ veh - h$ which is less that TTT found in no control case. Therefore, one can notice that the control algorithm improves the traffic situation through the freeway section. Apart from that the queue length is produced at the upstream of the ramp, nowhere congestion is produced. The STSMC control value is shown in Figure 15.

At the end, to evaluate the performance of the STSMC, we simulated the freeway section in the case of another control algorithm: ALINEA. It is a very well-know control algorithm by traffic community. It is the first feedback control implemented and tested in real life. ALINEA is exploited in many countries,

Table 1: Model Parameters

Parameter	Value
κ	40 $veh/lane/km$
ν	60 km^2/h
ρ_{max}	180 $veh/lane/km$
δ	0.0122
T	10 s
τ	18 s
ρ_{cr}	33.5 $veh/lane/km$
a	1.867
C_m	4200 veh/h
C_{ramp}	2000 veh/h

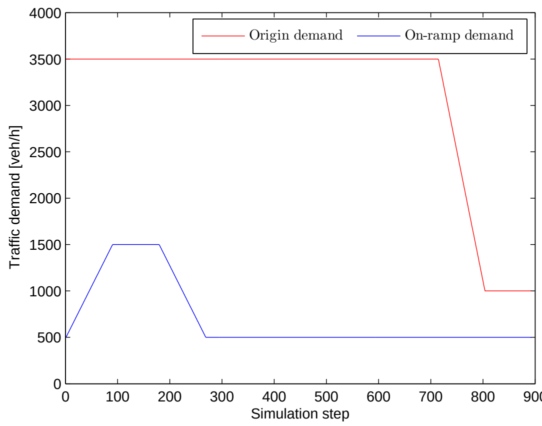

Figure 6: Traffic demands

for example, in United states, United Kingdom, France, Netherlands, and in other Europeans and developed countries in the world. ALINEA is efficient and better to those algorithms which do exist up to now. For all theses reason, the STSMC is compared to ALINEA.

Figures 16, 17, et 18 show the traffic density, traffic mean speed and traffic flow for all segments of the simulated section, respectively. For the case of ALINEA, the queue lengths at origins are shown in Figure 19. In the same case the $TTT = 1552.1\ veh{-}h$ which is comparable with the TTT found by STSMC, In ALINEA also the queue length is produced at the upstream of the ramp but nowhere else congestion is produced. The ALINEA control value is shown in Figure 20.

5 Conclusion

In this paper, a new control algorithm based on variable structure control technique was proposed. The main advantages of such kind of approach are its robustness against parametric variations and model

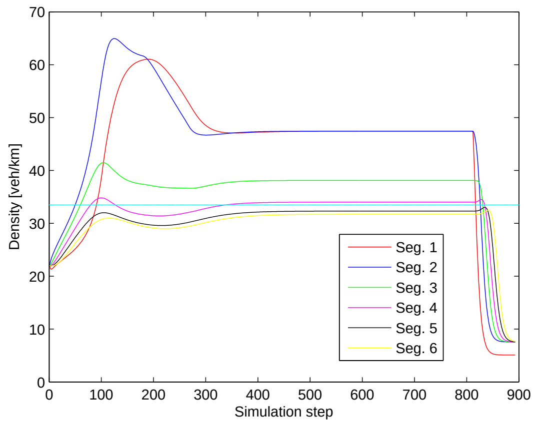

Figure 7: Traffic density - no control case

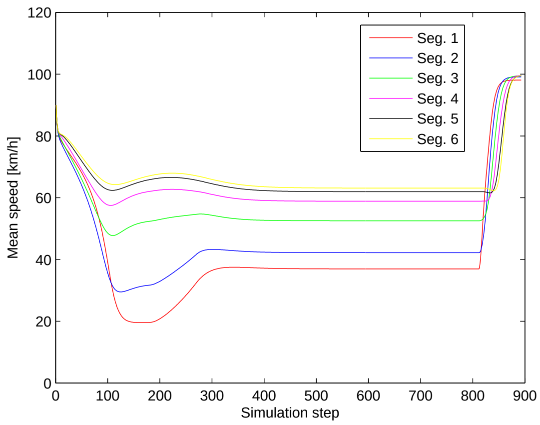

Figure 8: Traffic mean speed - no control case

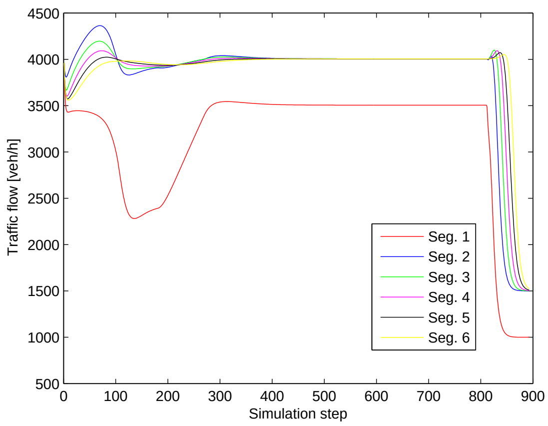

Figure 9: Traffic flow - no control case

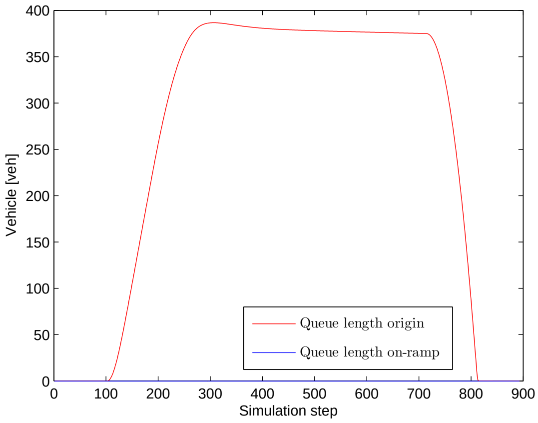

Figure 10: Queue length at the origins: no control case

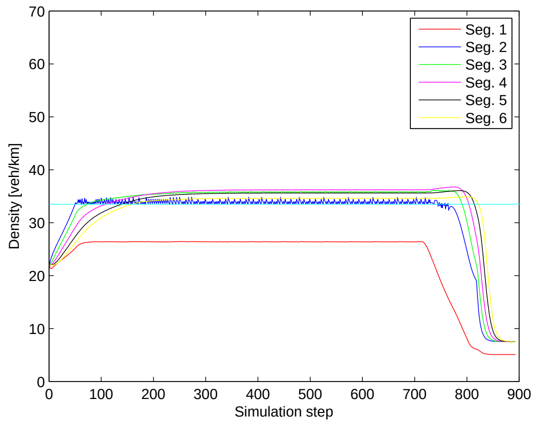

Figure 11: Traffic density, the section is controlled by STSMC algorithm

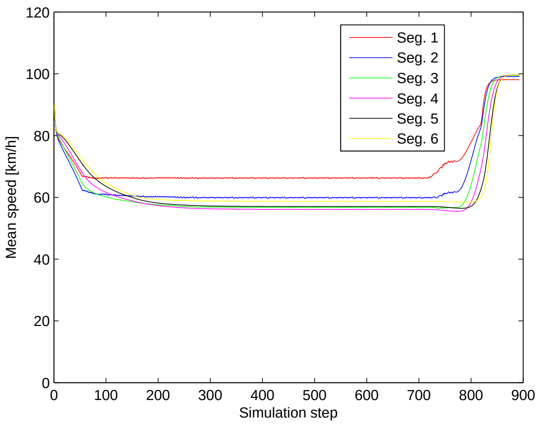

Figure 12: Traffic mean speed, the section is controlled by STSMC algorithm

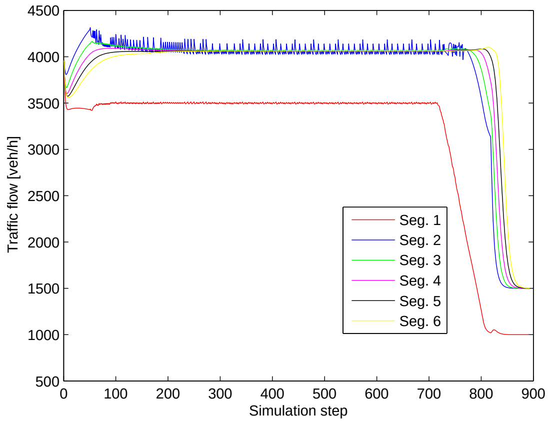

Figure 13: Traffic flow, the section is controlled by STSMC algorithm

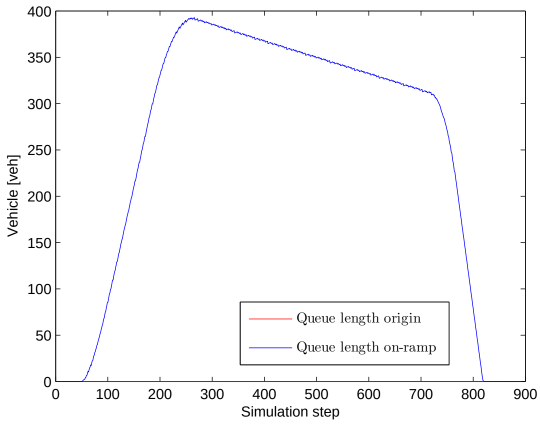

Figure 14: Queue length at the origins: STSMC algorithm case

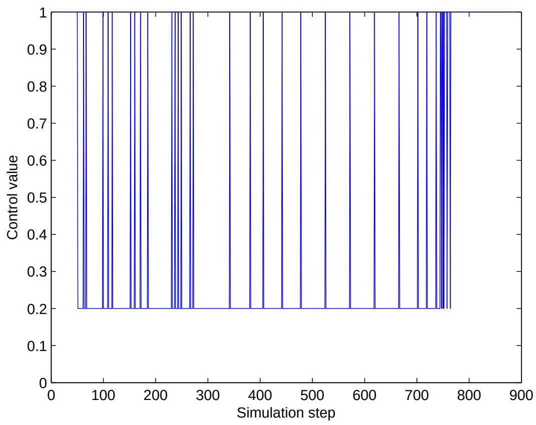

Figure 15: control signal (flow rate): STSMC algorithm case

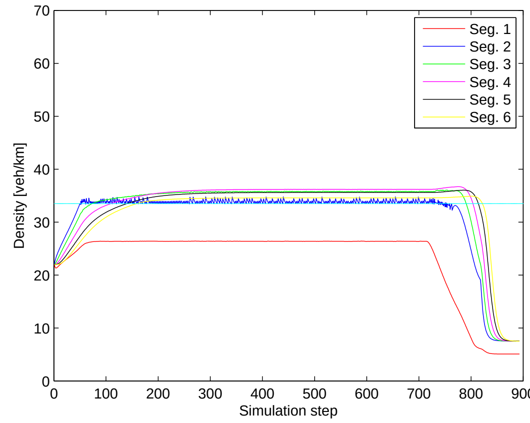

Figure 16: Traffic density, the section is controlled by ALINEA algorithm

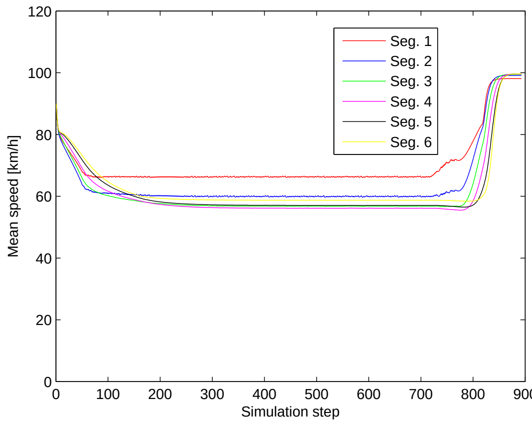

Figure 17: Traffic mean speed, the section is controlled by ALINEA algorithm

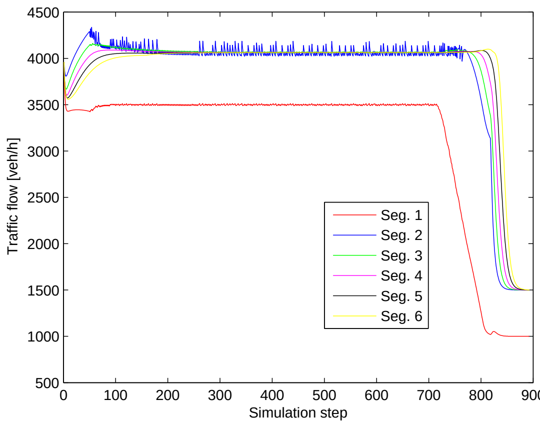

Figure 18: Traffic flow, the section is controlled by ALINEA algorithm

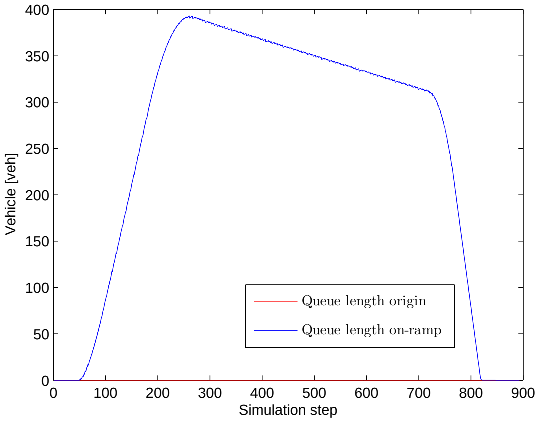

Figure 19: Queue length at the origins: ALINEA algorithm case

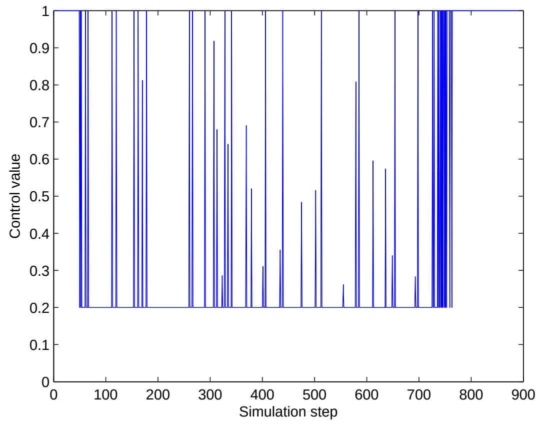

Figure 20: control signal (flow rate): ALINEA algorithm case

uncertainties, and its fast reaction to perturbation. The approach was validated via numerical simulations using the well known METANET model and then compared to the ALINEA strategy. Although, in our case study the two algorithms are relatively comparable, ALINEA (as a linear integral controller) needs a linearisation of the system around the equilibrium points. The advantages of STSMC are the robustness and insensitivity with respect to parameters and the model uncertainty. The effect of robustness in this paper does not quite appear, because we use a good model (METANET). The effect of robustness appears when we use a linear model (for example, LWR[1] model or CTM[2] models) which can not describe all the behavior of traffic flow in which the STSMC can give a better result compared to ALINEA. As a perspective and future research, we are implementing this new algorithm to coordinate ramp metering through a freeway section and for control integration between different control tools as well.

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