INQUIRY-BASED MATHEMATICS TEACHING, HISTORY OF MATHEMATICS AND NEW COMMUNICATION TOOLS : AN EXCITING CHALLENGE !

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ABSTRACT

Following a European project on the relationships between inquiry based science teaching (IBST), internet and communication tools (ICT) and a third topic, history of science, this article deals with some of the new questions raised by the recent introduction of epistemology, history of science and technology (EHST) in the school curricula. The definition of the inquiry and the role that can be played by epistemological thinking in science education are especially analysed in this text. Based on the analysis of historical examples and many on-line resources, as a first result, a new framework for the elaboration of new resources in IBST, ICT and EHST is given and some opening questions have been pointed out.

1 Context : IBST in Europe

All over Europe, the lack of student interest in science or in scientific careers has been noticed for years. This situation has led the European Community to launch a call for research projects in science education (the FP7 Science in Society program) and the publication of the Rocard Report about Science Education [1]. One of the main recommendations was to promote the advancing of teaching methods toward Inquiry Based Science Teaching (IBST) and the request for international comparisons. Engaged in 2008 [2], the European research project Mind the Gap was one of the answers. Mind the Gap was a large didactic program in which about 15 European universities worked on inquiry based science teaching in order to develop useful scientific tools within this topic. Many historian of science wished to join in this program. In a first meaning, IBST consists in learnings based on an open problem in which the student has to propose experiment or use instruments in order to find a solution. Such inquiry based teaching can be divided into different aims : an understanding of the articulation between empirical evidences and concepts (e. g. testing hypothesis, modelling, results

1. http://ec.europa.eu/research/science-society/index.cfm?fuseaction=public.topic&id=1100
2. The program ended in 2010, website : http://www.uv.uio.no/english/research/projects/mindingthegap/index.html

evaluation), the practice of hands-on activities comprising an informations quest or not (experimentations), the introduction to a specific scientific language (argumentation, debate), the enhancement of students' autonomy ... Mind the Gap was a place wherein to think about the interaction between IBST, history of science and technology (HST) and internet and communication tools (ICT). In this article, we would like to render an account of one of these possible interactions and to show thereafter how historical on-line resources can constitute well-adapted references of authentic problems.

2 IBST in Mathematics : a historical answer

According to Pr. Martin Andler (University of Versailles – France), a contemporary mathematical research activity comprises : 45% devoted to observation, 45% to experiment and only 10% to demonstration [3]. In the field of mathematics learning, the inquiry-based style often claims to have been inspired by the "scholar at work". In mathematics as in the other fields of science, inquiry takes up a large part of the research process. There is no doubt about the fact that the job has changed throughout the ages. However, by rendering an account of these changes, the epistemology and history of mathematics can help to explain what this inquiry could be.

2.1 First example

In ancient times in favouring results to reasoning mathematicians rarely expressed themselves on their relationships to the experiments. However when they did so, they gave us the opportunity to see the complexity of the links between theory and the use of technical instruments. The history of science assures us that : mathematical theories never emerge from nothingness. The scientist describes, builds and explores multiple examples before proposing an analysis or a system. On this subject, the work of the Arabic scholar al-Sijzī is a model of such a process. Ahmad ibn Muhammad ibn `Adb al-Jalīl al-Sijzī was born and lived in Iran. Son of a mathematician, he worked between 969 and 998 and he wrote exclusively books on geometry. In all, he has written approximately fifty treatises and lots of letters to his contemporaries. Al-Sijzī was working within a very specific scientific context. Since the ninth century, the development of algebra in the Arabic world has created new types of questions on the fundaments of this field. For instance, the point-by-point construction of conics has been well known since Antiquity (see the Apollonius' book entitled the *Conics*, for example), and that method is efficient enough for the analysis of the main properties of those curves. But during the ninth century, as the major part of algebraic equations studied during that period can be solved by intersecting conics curves (ellipsis, parabola, and hyperbola), the necessary taking into account of these intersections creates new difficulties. Indeed this possibility is based on the continuity of the different curves which is difficult to "prove". The solution that has therefore been chosen is to associate the curve with a tool that enables a real construction. As the ruler and the compass allow straight lines or circles to be drawn and so justify their continuity, a new tool had to be invented to draw all the conics. Not only interesting from a mathematical point of view, such a technical development is also useful in technological areas such as the construction of

3. Colloque Mathématiques, Sciences exprimentales et d'observation l'école primaire, Table ronde La démarche d'investigation en mathématiques , Chairman : P. Léna, École Normale Supérieure de Paris, September 28th 2005 : http://www.diffusion.ens.fr/index.php?res=conf&idconf=882

astrolabes and sundials where conics are essential.

Following his predecessors (Banū Mūsā brothers, Ibrāhīm ibn Sinān...) from whom he quoted in a precedent book on the description of the conic sections, al-Sijzī engages himself too in a treatise specifically on the *Construction of the perfect compass which is the compass of the cone*[4]. In this book, he studies a new tool, made-up a while ago by al-Qūhī : the perfect compass. As he announces in the first pages of his treatise, al-Sijzī wants to "build a compass which enables him to obtain the three sections of the cone." He first notes that all the conics can be obtained from the right cone (depending on the position of the cutting plane), and afterwards he proposes three possible structures for the perfect compass. The beginning of the study is technical, "We must now show how to shape a compass by which we can trace these sections. Shaping a stalk or AB. We put on the top tube, or NA. We link another tube to the end of it. [...]", and these instructions should enable the reader to build such a compass. But for al-Sijzī, the aim of his work on the perfect compass is not only to draw conics. The end of the text shows that this compass is also a theoretical tool and a tool for the discovery of new concepts. Al-Sijzī explains that the link between the circle and the ellipsis is quite obvious. Indeed, the construction of the ellipsis by orthogonal affinity and the formula for the area are both well known. But what are the links between the circle and the parabola or the hyperbola ? Now oriented towards the exploration and the solving of new problems, the practical tool becomes an instrument of discovery and as stated by al-Sijzī himself : "I always thought that there was a relationship between these two figures and the circle and their similarities and tried to get it but the knowledge of this has only become possible to me once I had learned how to turn the perfect compass following the positions of the plans."

In this example, the comings and goings between theory and practice appear clearly. Confronted with the theoretical problem of the continuity of curves, the scientist suggests the use of a new instrument. The experimentation with this instrument creates new theoretical results that create new questions and so on and so forth. In this text, al-Sijzī clarifies the role of mathematical instruments. They are objects as much as models and this dual status facilitates the theory-experiment passage.

2.2 Another example

Pierre de Fermat (1601-1665), the French lawyer who works during his free time on mathematics, is well known and his name is associated with the famous theorem which was only demonstrated in the 1990's. Nowadays, we can find [5] some parts of his mathematical works on the theory of numbers on the Web. Even if he has not published any treatise on this subject, some elements can be found in quotations of his Diophantus' book and included in his correspondence especially with Marin Mersenne, Huygens or Carcavi.

In a letter Fermat sent around august 1659 to Pierre de Carcavi (1600-1684), another amateur French mathematician, he tried to demonstrate some properties with, according to him, a new kind of demonstration he called "la descente infinie ou indefinie". This kind of demonstration is also used in the margin of his Diophantus' volume in

4. The texts are available in a French translation in *Œuvre mathématique d'al-Sijzi. Volume 1 : Géométrie des coniques et théorie des nombres au Xe siècle*, Trad. R.Rashed, Les Cahiers du MIDEO, 3, Peeters, 2004.

5. http://www.archive.org/details/oeuvresdefermat942ferm

order to prove that the area of rectangular triangle with integer sides (i.e. ones which are measured by integers) cannot be a square. Up to 1659, Fermat only uses this method to prove some negative results. He proposes a positive result, for the first time, in his letter to Carcavi, : any prime number such as $4k + 1$ is the sum of two squares, and this form is unique. For instance, $5^2 + 1$, $13 = 3^2 + 2^2, \cdots$. The demonstration is based on a reduction *per absurdum.* For instance, to prove the first above proposition, he presupposes that this kind of triangle exists. Then he shows that if it is true, we can find another triangle with shorter sides which validates the assertion, and so on. Since the sides must be integers he arrives at a contradiction through the method of infinite descent. It is quite easy to prove negative assertions, but its harder to show positive assertions. In his letter, Fermat announces that he only demonstrated that any prime number which could be written as $4k + 1$ could be decomposed as the sum of two squares of integers and that this decomposition was unique but, unfortunately he did not give any demonstration. The first proof of this assertion seems to appear in Leonard Euler's (1707-1783) works under the Latin title of *Demonstratio theorematis Fermantiani omnem numerum primum formae* $4n + 1$ *esse summam duorum quadratorum* [Demonstration of the Fermat's theorem "All prime number like $4n + 1$ is sum of two squares"] [6], in which he strictly follows the Fermat's method.

The method of "infinite descent" has profoundly renewed the theory of numbers. Even if, as usual, Fermat does not demonstrate what he announces, he suggests to the other mathematicians that they can easily find the demonstration of the properties he gives. For instance, in the above mentioned letter, he does not give any demonstration, however, he asks his contemporaries to do so : "je serai bien aise que les Pascal et les Roberval et tant d'autres savants la cherchent sur mon indication" (*op. cit.* p. 432). In this letter, he shows two important aspects of mathematics. First, some methods of proof are not able to solve a problem and newer ones must be invented and then reused to solve other mathematical enigmas. Creativity can go through a new method and a new approach. Secondly, this letter suggests to the other mathematicians to deal with those kinds of problems. This demonstrates how mathematical progress can be transmitted.

2.3 Many types of inquiry

The reading of the ancient texts is always interesting. Both examples presented above are only a small part of the historical resources dealing with inquiry, but they are still meaningful. What the historical approach shows is that *inquiry* cannot be reduced to a single aspect of the research process. Depending on the situation, each scientist engages himself in a different type of inquiry. A theoretical question does not require the same method as an experimental one, etc. IBST is clearly inspired by the work of the scientist as a professional. Nonetheless, the way this one has been understood is sometimes a bit caricatural. The history of science can prevent us from simplifying to such an extent and can restore the wealth of the research process.

3 History of mathematics and on-line resources

Nowadays, on the web, anyone can find many websites, pages or documents related to the epistemology, history of science and technology (EHST). Some sites like "Internet

6. http://math.dartmouth.edu/~euler/docs/originals/E241.pdf

Resources for History of Science and Technology"[7] have even been created to help users to find the right resources. More and more primary sources are also available on "Google books" or on the French project, "Gallica"[8]. It is relatively easy to read and to study ancient texts on science and technology. But in order to avoid mistakes or misinterpretations, it has been demonstrated that the texts must be contextualized. This is one of the main results of the research on the history of science. Now, due to the development of the new technology and the easiness of the on-line publication, some websites are devoted to IBST but they do not include any historical aspects. Is it possible to conciliate the two aspects : IBST with a historical approach ? In the Mind the Gap research program, the study of the relationships between inquiry and the history of science has shown that science and investigation are very close in different ways. In every scientific field, the scholars have to elaborate a way of questioning their object. In each case, new instruments should be built, and new experiments should be elaborated in order to be able to ask the right questions and finally to answer them. This first part of scientific activity is very similar to a second one which is dealing with theories and models. When their pertinence has been proved, the new concepts are applied to other situations or fields and they become a part of common knowledge. These newly born theories have still to be discussed and they constitute a third part of scientific activity. Communication between scholars often needs to create a suitable language and is essential in the building process of knowledge. An on-line publication should take into account all this wealth that gives many opportunities to the enlightening of historical documents and in making it suitable for IBST in multiple ways.

4 Digital document for EHST

According to Michael Shepherd and Livia Polanyi[9] quoted by Ioannis Kanellos[10], the genre of digital documents can be characterized through three constitutive elements :

- The *contents* (information, ...), organized following a *material* structure (disposition, page setting,...) which is often enough for a first and quick reading, and a *logical* structure (title, author, date, abstract,...) which brings some information on the intellectual organisation of the document.
- The *container* (support, medium), which determines the manner in accessing the information.
- The *context of production*, which relates the publication design. This context plays an essential role in the reading process of the document and it can be found as much in the content than in the container.

Contents as well containers, as such, do not enable the easy expressing of the context of production, and therefore the genre (historical or not), of a document. Only published information in the frame of the digital document identifies and determines the context

7. http://www2.lib.udel.edu/subj/hsci/ (last accession 16th November 2009)

8. http://books.google.com/ and http://gallica.bnf.fr/

9. Michael Shepherd, Livia Polanyi, Genre in digital documents , 33rd Hawaii International Conference on System Sciences, Volume 3, 2000, pp. 3010, http://doi.ieeecomputersociety.org/10.1109/HICSS.2000.926693.

10. See Ioannis Kanellos, Thomas Le Bras, Frédéric Miras, Ioana Suciu, « Le concept de genre comme point de départ pour une modélisation sémantique du document électronique », *Actes du colloque International sur le Document Électronique (CIDE'05)*/, Beyrouth, Liban, avril 2005, pp.201-216

of production and the genre. Sometimes the context of production is difficult to define and so the best way to enlighten it is to refer to a community of practices [11].

With this definition of the digital document, the context of production reveals the genre and shows the quality of the documents dedicated to EHST. We will thus give some criteria :

- The first and major one is the availability of primary sources. If there is no primary source, it becomes difficult to ensure that the document is within the frame of the history of science. The sources have to be contextualised and explained.
- The second one is the use of secondary sources. These documents may help to contextualise and to understand the scientific problem.
- The kind of media : texts, pictures, audio, video, ...
- The possibility to make a simulation, to create experiments
- The opening to a new view point and to other historical facts or problems (with hypertexts, links,...).

Then, these criteria can be connected with the aims of EHST digital documents. Is the document about the nature of science ? Does it refer to the macro or the micro history ? Is it based on the history of a concept or does it link science and society ? Does it deal with scientists' biographies or controversies, and so on and so forth.

5 Digital documents in HST for the use in IBST

All these questions opens interesting research ways on the elaboration of EHST documents suitable to a use in IBST. Some works have already been engaged, especially at the European level, and reader will find more detailed results (guidelines for digital documents building, content and form analysis grid, examples, ...) in our publications on this specific topic [12].

6 Conclusion

Nowadays, on-line publication is quickly increasing quantitatively and it gives many opportunities to create innovative learning sessions. Nonetheless, quantity is not quality and the new technologies such as web 3.0 already point out the risk of losing oneself in an ocean of data. Facing to this situation, historians of science should not stand back. The examples above show that a little vigilance enables us to make a document suitable for IBST with all its historical wealth. The task is not so heavy (3 or 4 paragraphs are often enough) and the community of historians of science should be aware of these questions. IBST is one of the main active topics in didactic research and, in this article, we have tried to show that historians of science have many things to say on this subject.

11. A community of practices is defined through three aspects : the borders of its application field, its social existence, its language and the documents used and shared by the members of this community. It is also a group with interactions and learning that develop a feeling of belonging and a mutual engagement. See, for instance, Etienne Wenger, *Communities of Practice : Learning, Meaning, and Identity*, Cambridge University Press, 1998 or his website : http://www.ewenger.com/theory/

12. See for instance the European collective book *The Usage of ICT and IBST by the History of Science and Technology*, O.Bruneau (Ed.),T.de Vittori Thomas (Ed.), P.Grapi (Ed.), P.Heering (Ed.), S.Laubé (Ed.), M.Massa (Ed.), Frank & Timme, Berlin, to be published in 2011.

Some new perspectives of collaboration have to be opened, and finally, as the situation does not concern only one country, a European network has to be enhanced in order to share all the experiences.

REFERENCES

- Shepherd M., Polanyi L. (2000) "Genre in digital documents", 33rd Hawaii International Conference on System Sciences, Vol. 3. http://doi.ieeecomputersociety.org/10.1109/HICSS.2000.926693.
- Kanellos I., Le Bras T., Miras F., Suciu I. (2005) "Le concept de genre comme point de départ pour une modélisation sémantique du document électronique", *Actes du colloque International sur le Document Électronique (CIDE'05)*, Beyrouth, Liban, pp.201-216.
- Wenger E. (1998) *Communities of Practice : Learning, Meaning, and Identity*, Cambridge University Press.
- Europan Research Programs http://ec.europa.eu/research/science-society/
- Mind the Gap http://www.uv.uio.no/english/research/projects/mindingthegap/