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Notes on Profile Inversion and Closed Form Formulation of Compact GRIN Lenses

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ABSTRACT This note is an addendum to the paper Profile Inversion and Closed Form Formulation of Compact GRIN Lenses [1] and summarizes a few comments obtained after publication of [1].

INDEX TERMS Lens-antennas, GRIN lenses, inhomogeneous lenses.

IN [1], the authors focused on the derivation of closed form formulas based on Geometrical Optics of tapering and spill-over efficiency for a radial refractive index profile of a GRIN lens with integrated feeder, namely a zero focal-length lens. The exact closed form of the radial refractive index is well known; but its derivation is not easily found in the literature and the authors have proposed their own derivation in [1]. They state in the introduction of [1].

Assuming that the focus is inside the medium, the radially varying refractive index profile is derived in analytical form via the inverse truncated Abel transform. This differs from the conventional Abel transform since a finite limit integration is used to define the transform. Although the GRIN solution for infinite cylinder and radial graded index exists in the literature, it is mostly used in the framework of fiber optics [2], [3]. The application to antennas has not been investigated for these zero focal-length lenses, in contrast with the more popular GRIN lens with non-zero focal length (without an integrated feeder). To the authors knowledge the above integration with a clear explanation and the analytical derivation for solving the non-linear integral equation required to describe the problem cannot be found in the literature. Therefore, additional information is presented here to render this derivation complete.

The authors have been aware that the solution of the problem existed in the GO-ray assumption and they have indeed cited [2], [3]. However, they did not find a derivation of this solution in the open literature, and therefore they have thought it is appropriate to show a derivation by Abel-truncated transform, which incidentally is also the method used by Luneburg to derive its famous lens solution.

After the publication of [1] the authors received a few comments from Sergei Skobelev that are summarized here, since they appear important to give suitable justification to the derivation of the solution studied in [1].

According to Skobelev the exact refractive index profile was found by Mikaelian in 1951 ([4], in Russian). However, in [4], Mikaelian obtained this exact solution empirically. The first rigorous derivation was probably given by Fletcher *et al.* [5]. They provided in 1954 a rigorous analytical solution of the problem, formulating it in terms of an Abel integral equation, by using a technique available for the Abel integrals. The solution was also successively described in Russian antenna textbooks [6, pp. 470–476] and [7, pp. 130–133].

The comparison of the derivation of the results in [1] appears similar to the one in [5] since it is based on the same process that Luneburg used to find his famous solution. The authors of [1] wish to apologize for this oversight; the literature review in their paper was conducted in good faith. They have derived the solution themselves, being aware (and declaring it in the paper), that could have been done before, since the solution appears several times in the literature concerning fiber optics (but always without appropriate references or applications to antennas). The authors believe that showing the derivation in [1] should have been beneficial for the antennas and propagation community.

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