Journal of Economic Interaction and Coordination
https://doi.org/10.1007/s11403-020-00301-6

REGULAR ARTICLE

Modeling maladaptation in the inequality–environment nexus

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Received: 25 April 2020 / Accepted: 15 September 2020

Abstract

Adaptation against environmental degradation has the potential to generate further environmental pressures. Does this aspect of adaptation affect the inequality–environment link? To answer this question, we develop a one-sector and one-input model which integrates threats to social and environmental sustainability posed by feedback effects of agents' adaptation strategies. We distinguish between income inequality and welfare inequality with the latter depending on environmental quality, leisure time, income level and allocation of income to consumption or adaptation. Despite its parsimony, the model describes the conditions for the existence of different inequality–environment dynamic regimes. The model confirms the standard view that environmental degradation exacerbates welfare inequality, but it also produces non-trivial and surprising insights. It illustrates that income inequality affects the type of dynamic regime followed by the economy. High-income economies and economies with high-income inequality are most at risk of following a pattern of maladaptive growth with increasing welfare inequality and environmental pressure.

Keywords Environmental degradation · Poverty · Maladaptation · Welfare inequality

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The authors would like to express their sincere appreciation to two anonymous reviewers for their very useful suggestions.

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1 Introduction

The literature on the nexus between inequality and environment agrees that climate change, pollution and other environmental hazards exacerbate inequalities (Olsson et al. 2014; UN/DESA 2016), not only because socially and economically disadvantaged people usually live in more marginal and exposed areas, but also because they are less able to cope and adapt due to poor access to knowledge, resources, technologies, credit and insurance markets (Barbier 2010, 2015). These facts are widely documented in advanced (Brooks and Sethi 1997; Bell and Ebisu 2012; Clark et al. 2014; Zwickl et al. 2014), middle-income (Schoolman and Ma 2012; Zhao et al. 2014) and low-income countries (Barbier 2010; Barbier and Hochard 2018), but also at global level we can observe that estimated impacts of climate change on regions are stronger and more severe in low- and middle-income regions compared to richer regions (IPCC 2007; De Cian and Sue Wing 2019; Diffenbaugh and Burke 2019). Looking at the other direction of causality, the impact of inequality on environment and pollution is usually explained via technological progress, consumers' preferences for environmental intensive goods or social groups' political demands for environmental quality and their ability to get a political response (for a review see Cushing et al. 2015; Berthe and Elie 2015). To the best of our knowledge, the literature on the inequality–environment nexus has overlooked the role of environmental changes produced by adaptation initiatives. Recent empirical research has instead introduced the notion of maladaptation,[1] showing that not all adaptation strategies are sustainable. IPCC (2001) warned against the risk of maladaptation to climate change almost 20 years ago and it has expressed increasing concern over time (Klein et al. 2014; de Coninck et al. 2018), while UNEP (2019) identifies maladaptation as one of the emerging environmental problem facing our planet.

This work investigates whether and how adaptation to pollution and environmental risks represent an additional mechanism by which inequality can affect environmental degradation. To this purpose, this paper employs a nonlinear dynamic model in which agents have exogenous and heterogeneous labor productivity, i.e., capacity to carry out income generating production activities. We denote this heterogeneity in potential income as economic or income inequality. Production activities cause damages to public environmental goods, and agents can use their income to purchase a consumption good or to finance defensive expenditures which counteract the effect of environmental degradation. The actual environmental quality experienced by each agent therefore is due to the combined role of public environmental goods and private adaptation. Agents' utility depends on leisure time, consumption level of the final good and environmental quality. While economic inequality is exogenous, inequality in terms of utility (hereafter referred as welfare inequality) is endogenously determined by dynamic and uncoordinated interactions between agents' decisions on time allocation between leisure and labor activity and income allocation between adaptation and consumption. Since agents with higher capacity to produce—the "rich"—are able to finance higher levels of defensive expenditure than those with lower capacity

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[1] Barnett and O'Neill define maladaptation as "action taken ostensibly to avoid or reduce vulnerability to climate change that impacts adversely on, or increases the vulnerability of other systems, sectors or social groups" (2010: 211).

to produce—the “poor”—the model reproduces the standard view that pollution exacerbates welfare inequality. This mirrors the recurrent fact that all agents want to react against a threat to their welfare, but some tactics and projects are affordable only by richer agents.[2]

What is less trivial in this context is the impact of income inequality on environmental outcomes. The model supports the hypothesis of a negative effect of inequality on environment through the maladaptation channel. The analysis demonstrates that environmental dynamics can detach potential income inequality from welfare inequality. It identifies the conditions under which the economy can enter what we have called a vicious circle of maladaptive growth, namely a self-reinforcing process of output growth, environmental deterioration and increasing welfare inequality but constant income inequality. The model reveals that low- and middle-income countries with high economic inequality and rich countries, regardless of level of income concentration, are most at risk of falling into a vicious circle of maladaptive growth.

The intuition behind this result is based on the fact that the need to finance adaptation fosters production growth which, in turn, increases environmental degradation creating an endogenous process which can be triggered more easily in economies where at least a part of population can afford adaptation expenditures. This holds even using a conservative notion of maladaptation which does not refer to direct increased environmental damages or vulnerability, but it indirectly originates from continuing to produce without any change in production technique.

The paper is structured as follows. Section 2 discusses the related literature. Section 3 presents the model; Sect. 4 analyzes agents’ choices. Section 5 identifies and describes the main scenarios of welfare and environmental dynamics generated by the model; Sect. 6 discusses how the model takes a fresh look at the nexus between welfare inequality and environment. Section 7 concludes.

2 Related literature

2.1 The literature on the relationship between inequality and environmental degradation

The impact of inequality on pollution and environmental degradation is usually explained through its direct effect on the individual consumption choices (i.e., preferences for environmental intensive goods compared to green substitutes) and its dynamic impact on environmental policy formulation or technological change. A crucial factor influencing the environmental consequences of inequality is the relation between individual income and environmental pressure which can be convex, concave or linear, as demonstrated by the vast literature on environmental Kuznets curve (EKC) constellated with results of different signs (see, for instance, the literature reviews by

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[2] An illustrative and simple example is provided by a study on private defensive expenditure against exposure to outdoor air pollution in China which finds that all groups respond to severe pollution alerts, but richer people are more likely to invest in air filters, which are much more expensive, and more effective, than masks (Sun et al. 2017). In other words, differences in private self-protection investment exacerbates air pollution exposure inequality.

Dinda 2004, Borghesi 2006 and Stern 2017). The type of marginal propensity to pollute - increasing, decreasing or stable in income level - by means of simple aggregation of individual consumption choices, determines if a redistribution of income from lower-income households to upper-income households translates, respectively, in increased, reduced or constant consumption of pollution-intensive goods (Scruggs 1998; Heerink et al. 2001; Berthe and Elie 2015). In addition, aggregated individual consumption choices and preferences for environment can affect social groups' demands for environmental quality, green policies and environmental regulation as well as development and deployment of environmentally friendly innovative products. The political economy and social choice argument focuses on the role of income concentration in shaping the distribution of power and, consequently, the introduction and the enforcement of environmental policies. If income elasticity of the demand for environmental quality is high a certain level of inequality may be beneficial for environmental protection as the rich may be more interested in environmental amenities than the poor. Nevertheless, high inequality may imply a low median income which, in turn, under a median voter rule, reduces willingness to finance public environmental expenditure (Magnani 2000). An alternative explanation supporting the view that inequality negatively affects the environment is provided by the framework of asymmetry in the power-weighted social decision rule adopted by Boyce (1994): The poor may be those who bear the costs of and are more vulnerable to pollution, who are more voiceless or have less political power, while the wealthy are more likely to influence policy decisions, to gain profits from environmentally degrading activities and therefore take advantage of weak environmental policies (Boyce 1994, 2018). Finally, inequality may produce environmental consequences through its influence on trust and collective action. Inequality can affect incentives and ability to cooperate in managing local common resources but in ambiguous ways (Baland et al. 2007) or may reduce environmental group membership (Sonderskov 2008).

A number of contextual elements shape the sign and composition of all these dynamics creating a nonlinear and not uniquely determined relationship. The types of environmental impact—localized or global (Heerink et al. 2001; Clément and Meunié 2010)—can influence exposure to losses from pollution and therefore public opinion interests in environmental protection, but also ability, for the richest, to defend themselves by substituting public environmental goods with private environmental goods and moving away from polluting sites. The political economy mechanism can operate differently depending on the degree of international trade integration in combination with the source of the inequality—concentrated ownership of clean as opposed to dirty factors of production (McAusland 2003). Level of income and the stage in industrialization/deindustrialization process can be other important factors: In rich or poor countries, inequality growth is more likely to occur during a phase of, respectively, declining or rising industrial sector leading to different correlation between inequality and environmental degradation (Gassebner et al. 2008; Grunewald et al. 2017). In addition, the effects of inequality may be contingent to the strength of democracy (Kashwan 2017). The technological channel can also be heterogeneous across country income groups. Vona and Patriarca (2011), for instance, find that, in advanced economies, high level of inequality can hamper the emergence of a sizeable internal demand of green products, which can stimulate environmental innovation, while in poor countries what really counts is per capita income.

The purpose of this paper is to extend the analysis of the relationship between income inequality and environmental degradation to other unexplored mechanisms. For this reason, our model nullifies the political economy argument, the aggregation argument, as well as the dynamic innovation channel. In the proposed model, the marginal impact of production and consumption choices is constant (the aggregation argument would lead to no impact of inequality on environment), agents are exposed to uniform environmental pressure, there is no innovation, and there is no trade and lobbying activity for policy changes. In this simplified context, can we uncover additional channels of interaction through which inequality affects environmental quality? This is the key question addressed in the model. We show that under these conditions, (i) adaptation becomes maladaptation and (ii) highly unequal economies are likely to converge to poorer environmental outcomes. In other words, uncoordinated and myopic private defensive strategies combined with substitutability between public and private environmental goods can represent an additional argument to explain the role of inequality for the environmental quality. As discussed in the following section, this exercise might be worthwhile as empirical research has documented instances of maladaptation in several sectors, policy and economic decisions.

2.2 Empirical evidence on maladaptation

This study borrows from the distinction proposed by Shogren and Crocker (1991) between self-protection which transfers externalities to another agent and self-protection which filters or dilutes them. It also builds on the works of Antoci and Bartolini (1999, 2004), and Antoci and Borghesi (2012), which highlight some mechanisms according to which the negative esternalities generated by self-protection choices may be an engine of welfare-reducing economic growth. However, only more recently a substantial literature on maladaptation has emerged providing examples of negative environmental feedbacks generated by adaptation strategies in several human–environmental interactions, starting with climate change responses. Individual agents, but not infrequently also governments (Bird 2014), prefer adaptation to mitigation action against global warming. However, adaptation strategies may add further environmental risks and vulnerability. One telling example of maladaptation is represented by energy-intensive adaptation actions that create a feedback effect by increasing emissions of greenhouse gases, thereby inducing further adaptation to climate change. Global warming effects, for instance, may lead to a growth in electricity consumption if the increase in cooling demand outweighs the decline in heating electricity demand. Indeed, some studies (Deschênes and Greenstone 2011; Auffhammer and Aroonruengsawat 2011) find a positive relationship between electricity consumption and temperature shocks (one of the effects of climate change) in the US residential sector also after controlling for reduced energy demand for warmer winters, while Cohen et al. (2014) predict an increase in carbon emissions, despite a reduction in gas consumption, due to climate shocks. Similarly, Davis and Gertler (2015), studying data on Mexico, find large increases in electricity consumption on hot days, with essentially no offsetting impact from reduced heating on cold days. They also observe that the expected global impact from air conditioning is enormous, since the potential demand for air conditioning[3] is extremely high in middle- and low-income countries with warm climates (India, Philippines, Nigeria, etc.) where income growth can have a dramatic impact on air conditioning demand. In line with this prediction, more recently, van Ruijven et al. (2019) estimate that warming increases global climate-exposed energy demand around 2,050 between 11 and 58% on top of a growth of 170–280% due to socioeconomic developments. Other adaptation initiatives that can require a large amount of energy and cause additional greenhouse gas emissions are also snow making (Abegg et al. 2007), desalination, cross-basin water transfer projects (Barnett and O'Neill 2010) or water efficiency schemes based on pumping (Beilin et al. 2012) and climate-change-induced agriculture's demand for irrigation. In other cases, feedback mechanisms do not cause carbon emissions, but do affect other types of emission or environmental risks. Empirical case studies provide a great variety of examples, from the use of sandbags to reduce coast erosion in Cape Town with consequent release of plastic into the sea and loss of recreational value of beaches (Magnan et al. 2016) to the more severe and widespread phenomenon of shrimp farming development in Bangladesh. Indeed, several coastal villages in Bangladesh share the experience of the rural village of Subarnabad documented by Pouliotte et al. (2009), where the combined effect of climate change and environmental changes due to infrastructure megaprojects has increased salt water intrusion, floods and waterlogging. Over the past decades, residents have implemented adaptation strategies, but with differences across income groups and with important negative feedbacks. The wealthier groups started shrimp farming or other types of aquaculture, either converting their lands from crop production and purchasing additional lands. In contrast, the poorest and subsistence farmers were forced off the land and to had to become wage earners. At the same time, the conversion of lands from crop and fodder cultivation to saline ponds have further increased salinity, with negative consequences for fresh water, commons, land fertility, health and livelihood diversification.

The contribution by Klein et al. (2014) to the fifth assessment report of the IPCC illustrates other cases of externalities or possible environmental risks associated with some adaptation options: Biotechnology and genetically modified crops, for instance, can enhance yields as well as drought and resistance to pests, but environmentalists and consumers may view it as a risk to public health and safety, ecosystems equilibrium and food security. Similarly, chemical fertilizers and pesticides can be used to maintain crop yields and suppress agricultural pests and invasive species, but they also increase chemical pollution of the environment and emissions of greenhouse gases. Hamin and Gurran (2009) instead analyze land use plans in the USA and Australia, showing that half of the identified actions contain potential conflicts to achieving climate change adaptation and mitigation simultaneously. Fezzi et al. (2015) find that, in Great Britain, the expected changes in land use in the farming sector due to climate change increase the estimated area of land at risk of high nitrate and high phosphate concentrations. In other words, data show a problem of adaptation-induced deterioration of river water quality.

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[3] Total potential demand for cooling due to cooling degree days per person.

We can extend this line of argument in a context in which the various agents are represented by countries. Governments can try to solve environmental problems by introducing stricter regulations, but firms can react by adopting private self-protection which shifts pollution damages elsewhere and increase the severity of the environmental risks (an inefficient overprotection as in the examples made by Shogren and Crocker 1991). A growing number of studies corroborate the hypothesis of pollution haven behavior in firms' location choices, especially in highly polluting industries, namely the role of weak environmental policies as a source of comparative advantage (Levinson and Taylor 2008; Waldkirch and Gopinath 2008; Dean et al. 2009; Mulatu et al. 2010; Broner et al. 2012; Candau and Dienesch 2017; Sapkota and Bastola 2017; Zugravu-Soilita 2017). A potential corollary is that polluting industries in countries with strict environmental regulations increasingly invest in outsourcing and off-shoring in countries with weak environmental regulation and enforcement. If polluting activities tend to be concentrated in countries with a weak environmental policy, the outcome is likely to be an increase in world pollution. Several studies on consumption-based accounting of emissions and resource use (Peters et al. 2011; Lenzen et al. 2013; Wiedmann et al. 2015; Teixidó et al. 2016; Yang et al. 2016; Caro et al. 2017) suggest that advanced economies might improve their environmental performance as a response to internal and international green agendas by increasing the use of non-domestic resources, imports of industrial goods and commodities, delocalization and emission shifts (i.e., self-protection which transfers externalities to agents, to use Shogren and Crocker 1991) terminology rather than by promoting a real improvement in resource productivity, decoupling or dematerialization. In the case of carbon dioxide, it is estimated that this process can lead to increased global emissions due to production shift in countries with greater carbon intensity and relatively less efficient production (Li and Hewitt 2008).

The existence of self-reinforcing negative mechanisms is also consistent with the literature on the energy and environmental rebound, suggesting that improved energy efficiency results in more energy use. Indeed, the existing literature agrees that the rebound effect is positive with a very high likelihood, albeit with quite large differentials in magnitudes across studies, sectors and technologies (van den Bergh 2011). This implies that even potential positive initiatives against climate warming such as energy conservation can have unintentional negative effects.

We do not intend to model all these complex dynamics, but we instead propose a stylized model which can be adapted to mirror more sophisticated settings. The aim is a model which reflects the underlying mechanism shared by these diverse examples: Differentials in defensive strategies due to inequality and the perverse effects of uncoordinated self-protection decisions can increase both environmental degradation and inequality.

3 Setup of the model and inequality dimensions

We study the dynamics of a one-sector economy with two interacting populations of economic agents: the rich and the poor, namely R-agents and P-agents. Each population is constituted by a very high number of agents. The representative *R*-agent and *P*-agent produce the output according to, respectively, the production functions:

$$Y_R = A_R \cdot L_R$$
$$Y_P = A_P \cdot L_P$$

where L_R and L_P represent the labor inputs of the (representative) *R*-agent and the *P*-agent, respectively; A_R and A_P, with $A_R > A_P > 0$, are parameters representing labor productivity.

We assume that the utility of *R*-agents and *P*-agents depends on the consumption of the produced output, C_R and C_P, respectively, and on the quality of the environmental goods to which each agent has access, which may be different for the two types of agent. More specifically, we assume that each agent can have access to environmental goods of quality $Q \geq 0$ without sustaining any cost (Q is an index representing the quality of common access environmental resources). However, agents can make defensive expenditures D_i, $i = R, P$, to obtain the access to environmental goods with higher quality Q_i, $i = R, P$, according to the following technology:

$$Q_i := Q + dD_i.$$

Welfare is measured by the Cobb–Douglas utility function:

$$U_i(C_i, Q, D_i, L_i) = C_i^{\alpha} \cdot \left(\overline{L} - L_i\right)^{\beta} \cdot Q_i^{\gamma}, \quad i = P, R. \tag{1}$$

The strictly positive parameter $\overline{L}$ measures the time endowment of each agent; the other parameters satisfy the conditions: $\alpha, \beta, \gamma > 0$, $\alpha + \beta + \gamma = 1$.

Finally, we assume the time evolution of the variable Q to be described by the equation:

$$\begin{aligned} &\dot{Q} = q\,(Q_w - Q) - e\left(\overline{Y}_P + \overline{Y}_R\right) \quad \text{if } Q > 0 \\ &\dot{Q} = 0 \quad \text{if } Q = 0 \end{aligned} \tag{2}$$

where $\dot{Q}$ represents the time derivative of Q, $\mathrm{d}Q(t)/\mathrm{d}t$; Q_w, q, and e are strictly positive parameters; and $\overline{Y}_R$ and $\overline{Y}_P$ represent, respectively, the average values of the outputs Y_R and Y_P. The parameter Q_w is the value that the variable Q would approach in the absence of economic activity (i.e., if $\overline{Y}_P = \overline{Y}_R = 0$ for every $t \in [0, +\infty)$) the parameter q measures the speed of adjustment of Q in approaching Q_w, while the parameter e measures the negative impact of $\overline{Y}_P$ and $\overline{Y}_R$ on Q. The representative agents consider the environmental impact of their decisions as negligible, and therefore, they consider the economy-wide average values $\overline{Y}_P$ and $\overline{Y}_R$ as exogenously determined. In this context, the trajectories generated by Eq. (2) do not maximize social welfare. However, they represent Nash equilibria: Along the trajectories agents do not change their choices if also the others do not modify theirs.

In this model, we can identify two types of inequality. The standard concept of inequality refers to agents' heterogeneity in potential income and can be measured by the difference $A_R - A_P$. It can be referred to as economic inequality and can be interpreted as inequality in a hypothetical initial status without environmental dynamics (in which negative externalities do not affect agents' utility and agents do not consider the possibility to adopt defensive strategies). This is the standard vision of inequality in contexts that completely overlook the role of natural resources. Indeed, the quality of environmental goods to which agents have access and the composition of the produced output by destination (for defensive expenditure D_i or consumption C_i) do not affect the difference $A_R - A_P$. In our setting, however, these factors have an impact on the utility of P- and R-agents. Therefore, in a context of multidimensional welfare as that considered here, a more relevant measure of inequality is represented by the following difference:

$$\Delta U = U_R(C_R, Q, D_R, L_R) - U_P(C_P, Q, D_P, L_P). \tag{3}$$

4 Agents' choices and dynamics

4.1 Agents' choices

In each instant of time t, the representative i-agent, $i = P, R$, chooses the values of the control variables L_i, C_i and D_i that solve the following optimization problem:

$$\max_{C_i, L_i, D_i} U_i(C_i, Q, D_i, L_i) = C_i^{\alpha} \cdot \left(\overline{L} - L_i\right)^{\beta} \cdot Q_i^{\gamma} \tag{4}$$

subject to the constraints:

$$A_i L_i = C_i + D_i \tag{5}$$

$$Q_i = Q + dD_i \tag{6}$$

$$C_i \geq 0, \quad 0 \leq L_i \leq \overline{L}, \quad D_i \geq 0. \tag{7}$$

Straightforward computations lead to the solutions C_P^*, D_P^*, L_P^*, C_R^*, D_R^*, L_R^* of problem (4)–(7) illustrated in Table 1. The solutions depend on the position of the quality index Q with respect to the following two threshold values:

$$\overline{Q}_R := \frac{\gamma d A_R \overline{L}}{1-\gamma} \tag{8}$$

$$\overline{Q}_P := \frac{\gamma d A_P \overline{L}}{1-\gamma} \tag{9}$$

with $\overline{Q}_R > \overline{Q}_P > 0$. By reading Table 1 by row, we can observe how the solutions C_P^*, D_P^*, L_P^*, C_R^*, D_R^*, L_R^* of problem (4)–(7) change as Q crosses the thresholds $\overline{Q}_i$ which separate the three intervals $\left(0, \overline{Q}_P\right]$, $\left(\overline{Q}_P, \overline{Q}_R\right)$, and $[\overline{Q}_R, \infty)$. If the value

Table 1 The solutions of problem (4)–(7), varying the quality index Q

	$Q \le \overline{Q}_P = \frac{\gamma d A_P \overline{L}}{1-\gamma}$	$\overline{Q}_P < Q < \overline{Q}_R$	$Q \ge \overline{Q}_R = \frac{\gamma d A_R \overline{L}}{1-\gamma}$
C_P^*	$\frac{\alpha}{d}(A_P d\overline{L} + Q)$	$\frac{A_P \alpha \overline{L}}{1-\gamma}$	$\frac{A_P \alpha \overline{L}}{1-\gamma}$
C_R^*	$\frac{\alpha}{d}(A_R d\overline{L} + Q)$	$\frac{\alpha}{d}(A_R d\overline{L} + Q)$	$\frac{A_R \alpha \overline{L}}{1-\gamma}$
D_P^*	$\gamma A_P \overline{L} - \frac{1-\gamma}{d} Q$	0	0
D_R^*	$\gamma A_R \overline{L} - \frac{1-\gamma}{d} Q$	$\gamma A_R \overline{L} - \frac{1-\gamma}{d} Q$	0
L_P^*	$(1-\beta)\overline{L} - \frac{\beta}{A_P d} Q$	$\frac{\alpha}{1-\gamma}\overline{L}$	$\frac{\alpha}{1-\gamma}\overline{L}$
L_R^*	$(1-\beta)\overline{L} - \frac{\beta}{A_R d} Q$	$(1-\beta)\overline{L} - \frac{\beta}{A_R d} Q$	$\frac{\alpha}{1-\gamma}\overline{L}$

of the quality index Q goes below the threshold $\overline{Q}_i$, then the representative i-agent, $i = P, R$, chooses to defend herself against environmental deterioration: $D_i > 0$; otherwise, she chooses $D_i = 0$. So, as first result of the model, we can observe that the higher labor productivity of the R-agent leads to a higher ability to react and greater responsiveness to environmental degradation compared to the P-agent. In fact, when Q decreases, R-agents start before P-agents to defend themselves against environmental degradation.

Comparisons of the cells of Table 1 by column, however, show how solutions C_P^*, D_P^*, L_P^*, C_R^*, D_R^*, L_R^* of problem (4)–(7) change as Q moves within each interval. It is easy to check that, within each interval, a reduction in the quality index Q negatively affects the utility of both agents resulting from the solution of their optimization problem, since it reduces at least one component of the utility function. If the representative i-agent, $i = P, R$, defends herself, a decline in Q is associated with a decrease in C_i^* and $\overline{L} - L_R^*$, so the net impact is a reduction in U_i^*. If the representative i-agent does not defend herself, a decline in Q (within the interval $[\overline{Q}_R, \infty)$) does not affect C_i^* neither $\overline{L} - L_R^*$, so two factors of utility function remain unchanged and one (Q) decreases: The net impact is again a reduction in U_i^*. Moreover, when at least one agent implements self-protection actions, the model can generate a self-reinforcing process of environmental degradation, output growth, increasing defensive expenditure and decline in utility of both agents. We shall call it a *vicious circle of maladaptive growth*. Indeed, when the representative i-agent starts to defend herself from environmental degradation, her labor input L_i^* and the produced output $Y_i^* = A_i \cdot L_i^*$ are inversely related to environmental quality Q. Thus, environmental degradation may fuel a self-reinforcing mechanism according to which a reduction in Q generates an increase in L_i^* and in output Y_i^* which, in its turn, generates [(according to (2)] a further reduction in Q, and so on. Notice that consumption C_i^* and defensive expenditure D_i^* are, respectively, positively and negatively correlated with environmental quality Q: Environmental deterioration has the effect to increase the output employed in defensive expenditures and to reduce the output consumed for non-defensive purposes.

4.2 Dynamics

The equation describing the time evolution of Q is obtained taking into account that, ex post, the average values $\overline{Y}_R$ and $\overline{Y}_P$ coincide with the outputs $Y_R^* = A_R \cdot L_R^*$ and $Y_P^* = A_P \cdot L_P^*$ produced by the representative R-agent and P-agent, respectively. So, by substituting $A_R \cdot L_R^*$ and $A_P \cdot L_P^*$ (Table 1), respectively, in place of $\overline{Y}_R$ and $\overline{Y}_P$, in Eq. (2), we obtain the piecewise equation of motion:

$$\dot{Q} = \begin{cases} 0, & \text{if } Q = 0 \\ -a_1 Q + b_1, & \text{if } Q \in (0, \overline{Q}_P] \\ -a_2 Q + b_2, & \text{if } Q \in (\overline{Q}_P, \overline{Q}_R] \\ -a_3 Q + b_3, & \text{if } Q \in (\overline{Q}_R, \infty) \end{cases} \tag{10}$$

where we have posed:

$$\begin{aligned} a_1 &:= \left(q - \frac{2\beta}{d} e\right) \\ a_2 &:= \left(q - \frac{\beta}{d} e\right) \\ a_3 &:= q \\ b_1 &:= q Q_w - e(A_P + A_R)(1 - \beta)\overline{L} \\ b_2 &:= q Q_w - e\left[A_P \frac{\alpha}{1-\gamma} + A_R(1-\beta)\right]\overline{L} \\ b_3 &:= q Q_w - e(A_P + A_R)\frac{\alpha}{1-\gamma}\overline{L}. \end{aligned}$$

Remark 1 Notice that for any choice of the parameters of the model, $a_1 < a_2 < a_3$ and $b_1 < b_2 < b_3$ hold. So, a strict positivity of the couple (a_k, b_k), $k = 1, 2, 3$, ensures that the stationary states $Q_j^* > 0$, $k \leq j \leq 3$ are attractive. In mathematical appendix, we also find the conditions to ensure that these stationary states belong to the corresponding regions in the parameters plane (A_P, A_R).

5 Environment and dynamic regimes

Equation (10) admits the closed form solution:

$$Q_k(t) = \frac{b_k}{a_k} + \left[Q(0) - \frac{b_k}{a_k}\right] e^{-a_k t}, \quad k = 1, 2, 3 \tag{11}$$

where $Q(0)$ is the initial value of Q. After straightforward computations, we find that:

1. The state $Q_0^* = 0$ is always a stationary state of Eq. (10).

2. At most one stationary state Q_1^* can exist in the interval $(0, \overline{Q}_P)$ (where both types of agent defend themselves against environmental degradation, that is $D_i > 0$, $i = R, P$) and, when existing, it is given by:

$$Q_1^* = \frac{b_1}{a_1} = \frac{qQ_w - e(A_P + A_R)(1-\beta)\overline{L}}{\left(q - \frac{2\beta}{d}e\right)}. \tag{12}$$

3. At most one stationary state Q_2^* can exist in the interval $[\overline{Q}_P, \overline{Q}_R)$ (where R-agents defend themselves against environmental degradation, $D_R > 0$, while P-agents do not, $D_P = 0$) and, when existing, it is given by:

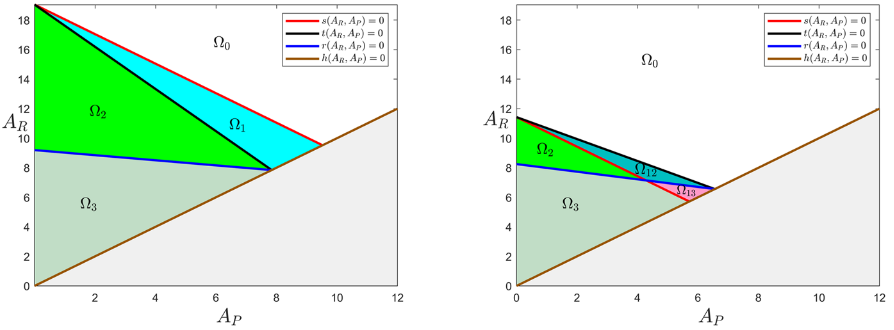
$$Q_2^* = \frac{b_2}{a_2} = \frac{qQ_w - e\left[A_P\frac{\alpha}{1-\gamma} + A_R(1-\beta)\right]\overline{L}}{\left(q - \frac{\beta}{d}e\right)}. \tag{13}$$

4. At most one stationary state Q_3^* can exist in the interval $[\overline{Q}_R, \infty)$ (where both types of agent do not defend themselves against environmental degradation, $D_R = D_P = 0$) and, when existing, it is given by:

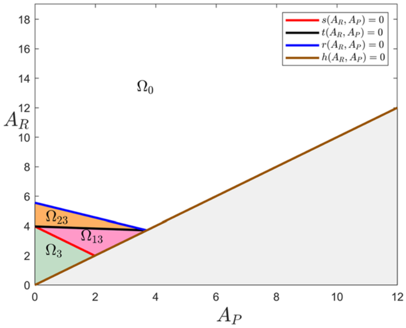
$$Q_3^* = \frac{b_3}{a_3} = \frac{qQ_w - e(A_P + A_R)\frac{\alpha}{1-\gamma}\overline{L}}{q}. \tag{14}$$

The complete taxonomy of dynamic regimes that can be observed under dynamics (10) is illustrated in Fig. 1d (see the mathematical appendix). In this figure, attractive stationary states are indicated by a full dot, while repulsive ones are represented by an empty dot. Notice that at most one attractive stationary state with $Q > 0$ can exist. The regions in the parameter plane (A_P, A_R) in which each regime occurs are shown in Fig. 1a–c. In these figures, the symbol Ω_i, $i = 1, 2, 3,$ indicates the set of the plane (A_P, A_R) in which there exists one stationary state with $Q > 0$, Q_i^*; the symbol Ω_{ij}, $i = 1, 2$ and $i < j \leq 3$, indicates the set in which there exist two stationary states with $Q > 0$, Q_i^* and Q_j^*. The analytical conditions on parameters to obtain each regime are given in the mathematical appendix.

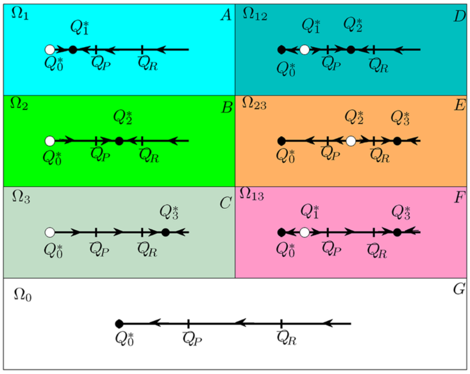
In the dynamic regimes illustrated in Fig. 1d, the aggregated output is inversely correlated with environmental quality Q, and the above-mentioned vicious circle of maladaptive growth may be observed: A reduction in Q generates an increase in aggregated output (via the increase in defensive expenditures), which in its turn generates a further reduction in Q, and so on. When Q increases, constraint (6) is relaxed. This implies that, along the trajectories in Fig. 1d in which the value of Q is decreasing, the economy experiences a growth process of the aggregated output accompanied by a reduction in welfare of both types of agent; furthermore, in bistable regimes (see point 2 below), in the attracting stationary state $Q_0^* = 0$ the welfare of each type of agent is strictly lower than in the other attracting stationary state (either Q_2^* or Q_3^*). Based on this analysis, we can distinguish three main welfare and environmental dynamics:



(a) $e < \dfrac{dq}{2\beta}$ (i.e. $0 < a_1 < a_2 < a_3$) **(b)** $\dfrac{dq}{2\beta} < e < \dfrac{dq}{\beta}$ (i.e. $a_1 < 0 < a_2 < a_3$)



(c) $e > \dfrac{dq}{\beta}$ (i.e. $a_1 < a_2 < 0 < a_3$)



(d)

Fig. 1 Possible dynamic regime: *dynamic regime A*: There exists only the attractive stationary state Q_1^*; *dynamic regime B*: there exists only the attractive stationary state Q_2^*; *dynamic regime C*: there exists only the attractive stationary state Q_3^*; *dynamic regime D*: there exist the attractive stationary states Q_2^* and Q_0^*, and the repulsive stationary state Q_1^*; *dynamic regime E*: there exist the attractive stationary states Q_3^* and Q_0^*, and the repulsive stationary state Q_2^*; *dynamic regime F*: there exist the attractive stationary states Q_3^* and Q_0^*, and the repulsive stationary state Q_1^*; *dynamic regime G*: there exists the attractive stationary states Q_0^*

1. *Environmental collapse*: In region Ω_0 of Fig. 1a–c, we have that the stationary state $Q_0^* = 0$ is globally attractive. Along the trajectory approaching it, the outputs $Y_R^* = A_R \cdot L_R^*$ and $Y_P^* = A_P \cdot L_P^*$ grow and reach their maximum value when the stationary state $Q_0^* = 0$ is reached. The economy falls into the trap of maladaptive growth: The dynamic system follows undesirable trajectories, approaching $Q_0^* = 0$, which are environmentally and socially unsustainable in that they lead the economy toward a complete collapse of free access environmental resources and are characterized by a progressive decline in utility for both agents. Figure 1a–c also shows that the region of environmental collapse expands as the parameter representing the marginal pollution of production (i.e., parameter e) increases.
2. *Path-dependent equilibrium*: In regions Ω_{12}, Ω_{13}, Ω_{23} of Fig. 1a–c, a bistable regime is observed. It is characterized by the existence of two attracting stationary states: The stationary state $Q_0^* = 0$ (where environmental quality reaches its minimum level and both types of agent defend themselves against environmental degradation) and the state Q_2^* (where only R-agents defend themselves) or the state Q_3^* (where no agent defends herself). In this case, $Q(0)$ (the initial value of Q) plays a critical role in the selection of the path followed by the economy: sufficiently low values of $Q(0)$ push the economy toward the vicious circle of maladaptive growth; alternatively, sufficiently high values of $Q(0)$ facilitate the convergence to positive levels of environmental quality.
3. *Safety net against environmental collapse*: In regions Ω_1, Ω_2, Ω_3 of Fig. 1a–c, a globally stable regime is observed, characterized by the existence of a unique globally attracting stationary state with $Q > 0$: Q_1^*, Q_2^*, or Q_3^*. Parameters of the model determine which stationary state is admissible among Q_1^*, Q_2^* or Q_3^*. It is worth recalling that the welfare of both agents improves as the economy converges to stationary states with higher values of Q and $Q_1^* < Q_2^* < Q_3^*$. In this sense, from a social point of view, the sets of parameters which are preferable are, in this order, those who make Q_3^*, Q_2^*, Q_1^* admissible. Figure 1a–c suggests that the regions of convergence to Q_3^* and to Q_2^* widen as parameter e declines.

These results suggest that a wide range of social and environmental dynamics are admissible. In order to gain some insights on the nexus between inequality and environment, it is therefore important to identify under what conditions each of them is more likely to occur. Four main dynamic regimes can be classified according to the productivity parameter A_P, and the difference between A_R and A_P which can be considered, respectively, a proxy of initial poverty and income inequality:

1. *Economies with high level of poverty and low economic inequality* (i.e., low A_P and low $A_R - A_P$). These are economies where labor productivity of both the P-agent and R-agent is low. These poor economies are likely to belong to regions Ω_3 with regimes offering a safety net against environmental collapse. The economy converges to Q_3^*, a stationary state with high environmental quality and absence of defensive strategies since low labor productivity implies low environmental externalities due to production and to defensive strategies.
2. *Economies with high level of poverty and high economic inequality* (i.e., low A_P, and high $A_R - A_P$). In this case, A_R and A_P tend to put the economy into the regions Ω_0 (Fig. 1a, b, d) characterized by environmental collapse. This dynamic

regime is less likely if the rate of environmental impact e is very low (i.e., $e < \frac{dq}{2\beta}$, Fig. 1a): In this case, the economy tends to be located in region Ω_2 in which the system converges to Q_2^*. For intermediate levels of pollution intensity (Fig. 1b), these economies can prevent complete environmental degradation if inequality is high but not excessively so that the economy belongs to the region Ω_{23} (Ω_{13}) in which the economy converges to the state Q_3^*. The convergence to Q_3^*, however, occurs only under strict conditions on the initial value of environmental quality ($Q(0) > Q_2^*(Q_1^*)$).

3. *Economies with low level of poverty* (i.e., high A_P). These are rich economies with high levels of A_R and A_P that are likely to belong to regions Ω_0 (Fig. 1a, b, d) characterized by maladaptive growth and environmental collapse or, at most (when the rate of environmental impact e is sufficiently low, i.e., $e < \frac{dq}{2\beta}$) to region Ω_1 with convergence to Q_1^*, a stationary state with positive but low environmental quality.
4. *Economies with intermediate level of poverty and low economic inequality*. These are economies with medium levels of A_P and close to the line $A_R = A_P$. They are likely to belong to regions Ω_3, Ω_{13} (or Ω_{12}) which are characterized by the existence of an attractive stationary state Q_3^* (Q_2^*) with high level of environmental quality and in which nobody (only the R-agent) adopts negative defensive strategies. This equilibrium can be globally attractive (if e or labor productivity of both agents is sufficiently low) or can be reachable if the initial environmental quality is $Q(0) > Q_1^*$, namely is above a fairly low threshold level.
5. *Economies with intermediate level of poverty and high economic inequality*. These are economies with medium levels of A_P and high levels of A_R which are characterized by the same dynamic regimes as economies with low poverty levels. Thus, they are likely to fall into the vicious circle of maladaptive growth (i.e., to be in region Ω_0 with convergence to Q_0^*).

6 From income inequality to welfare inequality through environmental quality

The model allows considering two types of inequality. As mentioned above, the difference $A_R - A_P$ is exogenous and can be considered as potential income inequality. Welfare inequality ΔU [see (3)], instead, is endogenously determined by interactions of economic agents' choices and their effect on environment and on labor incentives.

By substituting the solutions C_i^*, D_i^*, L_i^*, $i = P, R$, of problem (4)–(7) (Table 1) in the utility functions $U_i(C_i, Q, D_i, L_i)$, $i = P, R$, we obtain that:

- For $Q < \overline{Q}_P$ [(both types of agent defend themselves; Table 1 and (9)], the utility of P- and R-agents is represented by the formula:

$$U_i(C_i^*, Q, D_i^*, L_i^*) = \left[\frac{\alpha}{d}\left(A_i d\overline{L} + Q\right)\right]^{\alpha} \cdot \left(\beta\overline{L} + \frac{\beta}{A_i d}Q\right)^{\beta} \cdot \left(d\gamma A_i\overline{L} + \gamma Q\right)^{\gamma}, \quad i = P, R. \tag{15}$$

- For $Q \geq \overline{Q}_R$ (no agent defends herself; Table 1 and (8), the utility functions can be written as follows:

$$U_i(C_i^*, Q, D_i^*, L_i^*) = \left(\frac{A_i\alpha\overline{L}}{1-\gamma}\right)^{\alpha} \cdot \left[\frac{(1-\alpha-\gamma)\overline{L}}{1-\gamma}\right]^{\beta} \cdot Q^{\gamma}, \quad i = P, R. \tag{16}$$

- For $\overline{Q}_P \leq Q < \overline{Q}_R$ (only the R-agent defends herself), the utility functions of R- and P-agents are, respectively, given by formulas (15) and (16).

Notice that inequality (3), in the context in which no agent defends herself (i.e., when $Q \geq \overline{Q}_R$), can be written as follows:

$$\Delta U = (A_R^{\alpha} - A_P^{\alpha}) \cdot \left(\frac{\alpha\overline{L}}{1-\gamma}\right)^{\alpha} \left(\frac{1-\alpha-\gamma}{1-\gamma}\overline{L}\right)^{\beta} \cdot Q^{\gamma} \tag{17}$$

while, in the context in which all agents defend themselves (i.e., when $Q < \overline{Q}_P$), it is expressed by:

$$\Delta U = N\left(M - \frac{A_R^{\beta} - A_P^{\beta}}{A_R^{\beta}A_P^{\beta}}Q\right) \tag{18}$$

where we have posed:

$$N := \frac{\beta^{\beta}\gamma^{\gamma}\left(\frac{\alpha}{d}\right)^{\alpha}}{d^{\beta}} > 0 \quad \text{and} \quad M := \overline{L}d\left(A_R^{1-\beta} - A_P^{1-\beta}\right) > 0.$$

The model therefore identifies two possible relationships between welfare inequality and environment.

1. For high levels of environmental quality ($Q \geq \overline{Q}_R$), namely in the context in which agents do not need to defend themselves, the connection between environment and welfare inequality is positive. As it is easy to check from formula (17), a decrease in environmental quality Q reduces welfare inequality between R- and P-agents. In fact, the utility functions of both types of agent [see (16)] tend to zero as Q tends to zero, and consequently, the inequality measure (3) does the same. It is worth noting that in this context, the dependence of the difference (17) on Q is

actually due to a scale effect, in that the rate U_P/U_R does not depend on Q [see (16)]. This is not the case when at least one agent defends her/himself.

2. For low levels of environmental quality ($Q \le \overline{Q}_P$), that is in the context in which both types of agent defend themselves against degradation of free access natural resources (Q), the relationship between environment and welfare inequality is negative: A decrease in environmental quality Q increases welfare inequality [see (18)].
3. For intermediate levels of environmental quality ($\overline{Q}_P < Q < \overline{Q}_R$), the relationship between environment quality and welfare inequality may be either positive or negative, according to the value of Q. More specifically, the relationship is negative (respectively, positive) when Q is near enough to $\overline{Q}_P$ (respectively, $\overline{Q}_R$). This result follows from the continuity, with respect to Q, of the function defined by formulas (16)–(18).

These results suggest that when environmental stress becomes significant (in mathematical terms when $Q < \overline{Q}_P$ or $Q \in (\overline{Q}_P, \overline{Q}_P + \delta)$), environmental degradation exacerbates welfare inequality.

Figure 2a shows the time evolution of U_R and of U_P along a trajectory approaching the stationary state Q_1^* (where all agents defend themselves) starting from an initial value $Q(0) > Q_1^*$ following a path of decreasing environmental quality (Fig. 1 Ω_1 in d). Figure 2b, c illustrates, respectively, the time evolution of the difference $U_R - U_P$ and of outputs Y_R, Y_P along the same trajectory. Notice that the increase in outputs is associated to a decrease in utility, for both types of agents, and to an increase in inequality measured in terms of welfare but R-agents start earlier than P-agents to increase the produced income in order to finance higher D. P-agent, in contrast, initially chooses a constant labor and production. Figure 2b also shows how the impact of environmental degradation on inequality is nonlinear. Initially, as the environmental quality is still high, environmental degradation leads to a reduction in welfare inequality. However, as the convergence to Q_1^* proceeds leading to a decline in Q, the relationship is reversed and environmental degradation is associated with a worsening of welfare inequality. The initial decline in the $U_R - U_P$ difference, therefore, may be due to the fact that, at first, P-agents do not increase their labor effort to finance D. Consequently, they do not suffer from the disutility deriving from a reduction in their leisure time, while the R-agents defend themselves, and are therefore initially affected by a greater decline in utility. Over time goes, the environmental negative effects due to R-agents' defensive expenditures generate a significant reduction in Q that affects P-agents more than R-agents, and consequently welfare inequality begins to rise.

These results clearly show two things. First, agents' differentiation in defensive capacity plays a crucial role in shaping the nexus between welfare inequality and environment: To the extent the poorer agents have lower ability to implement self-protection expenditure, environmental degradation may be an amplifier mechanism for inequality. Second, the introduction of maladaptive choices (i.e., the idea that defensive expenditure can produce negative environmental externalities) in the model lies at the core of the perverse mechanism which self-fuels a cycle of environmental degradation, welfare-reducing growth of output and welfare inequality.

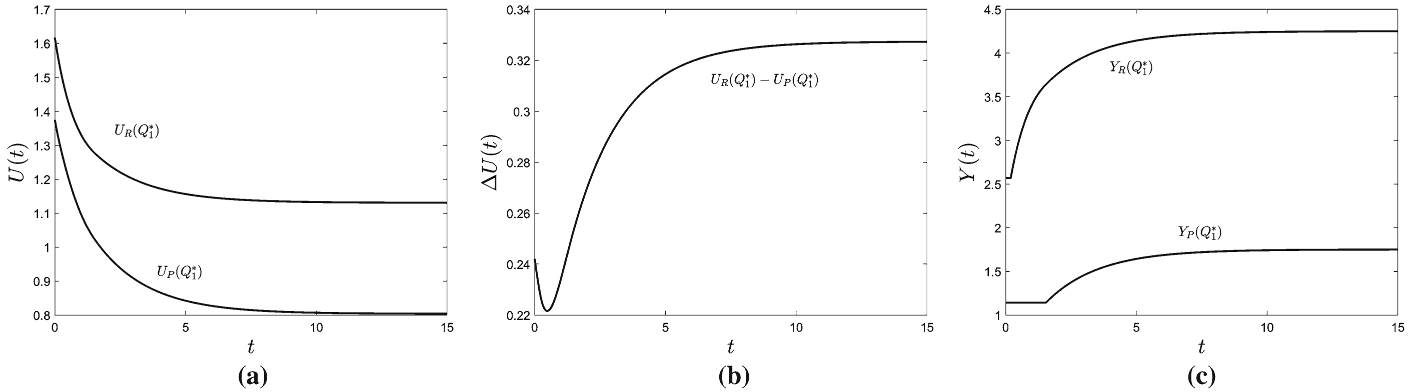
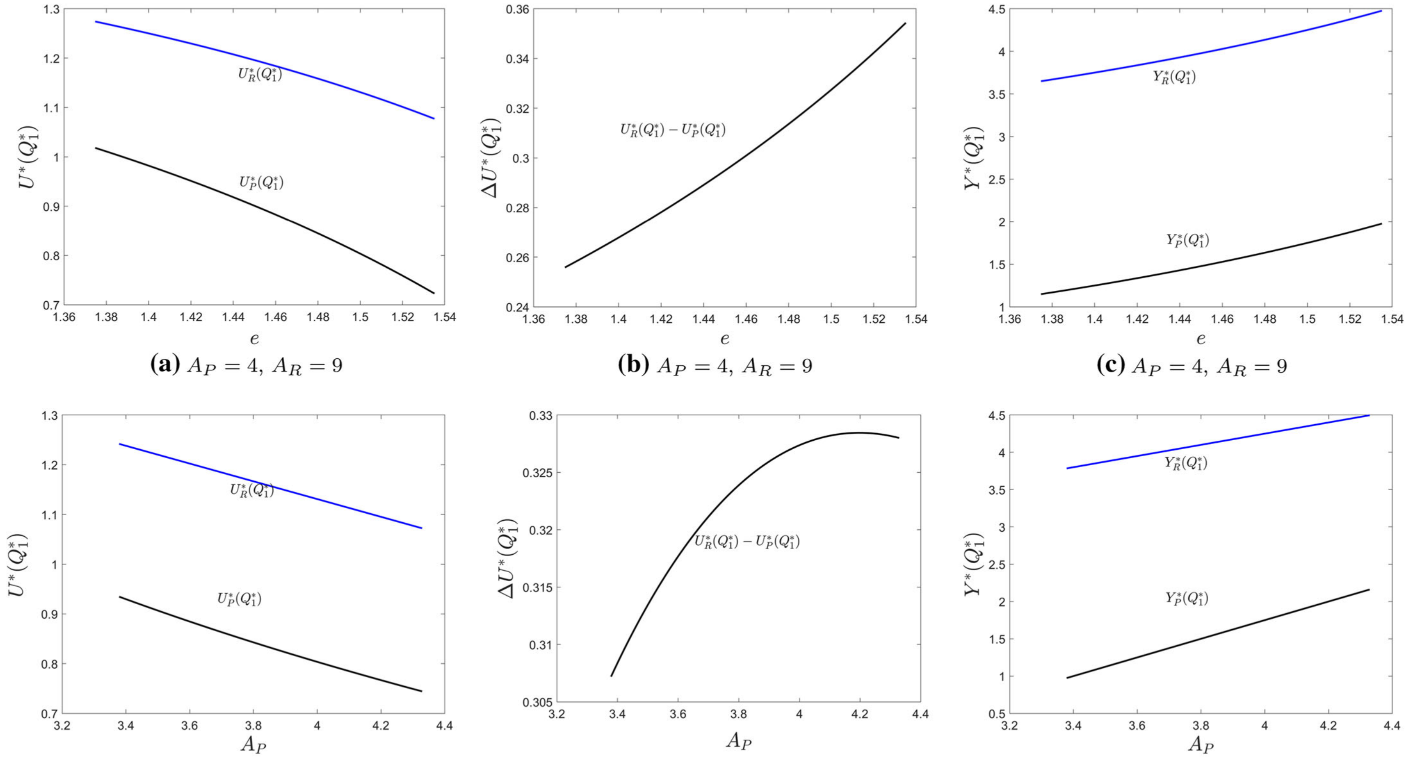


Fig. 2 Case $e < \frac{qd}{2\beta}$, (*dynamic regimes* A, B, C, in Fig. 1d). The parameter values are: $\alpha = 2/10$, $\beta = 5/10$, $\gamma = 1 - \alpha - \beta$, $\overline{L} = 1$, $Q_w = 5$, $e = 1.5$, $q = 2$, $A_R = 9$, $A_P = 4$



(a) $A_P = 4,\ A_R = 9$ **(b)** $A_P = 4,\ A_R = 9$ **(c)** $A_P = 4,\ A_R = 9$

(d) $e = 1.5,\ A_R = 9$ **(e)** $e = 1.5,\ A_R = 9$ **(f)** $e = 1.5,\ A_R = 9$

Fig. 3 Case $e < \frac{qd}{2\beta}$ (*dynamic regime* A in Fig. 1d). Values of U^*, ΔU^*, Y^* evaluated at the attractive stationary state Q_1^* (where both types of agents defend themselves), varying the parameters e (panels a–c), A_P (panels d–f). The remaining parameters are fixed at the values $\alpha = 1/5,\ \beta = 1/2,\ \gamma = 1 - \alpha - \beta, q = 2, \overline{L} = 1,\ Q_w = 5$

These results are confirmed by comparative statics illustrated in Fig. 3 displaying the effects on the stationary state Q_1^* of variations in the parameters representing the labor productivity of the P-agent (A_P) and the pollution rate (e). Recalling the assumption $A_R > A_P$, a rise in A_P can be interpreted as a reduction in income inequality and as an improvement in income-generating capacity of the group of economically disadvantaged people.

Figure 3a–c clearly shows the fact that a lower labor productivity for the P-agent than for the R-agent translates into lower defensive capacity to environmental pressure and, in turn, greater vulnerability. An increase in e, i.e., the pollution rate of economic activities (which may be due to relaxed environmental regulations), in fact, leads not only to a deterioration of environmental quality and utility losses for both agents, but also to increased inequality. Interestingly, these effects are also associated with an increase in level of output produced by both agents, meaning that production growth can also be accompanied by welfare decline due to environmental degradation.

Figure 3d–f simulates the effect of an increase in productivity of the P-agent, which reduces (*ceteris paribus*) income inequality. They show an initial phase in which the environmental impact of a growing productivity offsets welfare improvements associated with a higher potential income of the P-agent. Such a result is explained by the fact that in our model economic agents' choices are not coordinated by a policy maker (or any other coordination factor), and therefore, they produce negative environmental externalities. In this context, an increase in A_P allows P-agents to better adapt to environmental degradation: Their production activity and adaptation expenditure increase. The consequent negative impact of environmental quality prompts R-agents to increase their defensive expenditures. This mechanism may drive the economy toward declining utility for both types of agent (as shown in panel d) and widening welfare inequality, as shown in panel e. Income and welfare inequalities ($A_R - A_P$ and $U_R - U_P$) therefore are not necessarily correlated. Policy interventions aimed at reducing $A_R - A_P$ (e.g., through an increase in A_P) may have an opposite effect on $U_R - U_P$. Finally, the upside-down U shape of inequality (see panel e) is due to the fact that once a certain threshold is exceeded, the increase in A_P causes the two types of agents to become more and more similar (i.e., the $A_R - A_P$ difference tends to zero) and therefore the inequality starts to decrease also in terms of welfare.

7 Concluding remarks

Existing analytical frameworks explain the impact of economic inequality on environmental degradation by the mediating role of the environmental intensity of consumption, asymmetries in political power, innovation dynamics and cooperation in management of local resources. Our model excludes all these mechanisms to explore whether, ceteris paribus, the aggregation of adaptation choices represents a new channel of interaction between economic inequality and environmental pressure. The model supports the idea of a negative impact of inequality on environmental quality for low- and middle-income countries. We find that wealthy societies and inegalitarian economies, where the population (or at least the richest part of it) can afford adaptation strategies which may be detrimental to common access environmental resources are at risk of falling into a vicious circle of maladaptive growth characterized by environmental degradation, increasing production, defensive actions and welfare inequality. In other words, if common sense argues that affluent economies are usually in a better position to tackle environmental problems, our analysis suggests that, in the absence of counterbalancing actions, such as a reduction in the pollution rate of production (i.e., the parameter e), they are also more likely to be affected by maladaptive choices. The same risk affects low- and middle-income countries with high-income inequality where the environmental impact of maladaptation might be particularly serious as the poorest part of the population has a limited capacity to defend itself. The analysis of the model, indeed, confirms the view that environmental degradation has negative consequences on welfare equality.

The analysis of the model also suggests that the region of environmental collapse becomes wider as the pollution intensity of economic activities (i.e., the parameter e) increases. In rich and in inegalitarian economies, therefore, the balance between defensive and mitigation strategies is particularly important: While government actions that encourage adaptation represent a first response to protect welfare, especially for the most vulnerable groups, policies to reduce the emission and pollution rate of economic activities should be a priority for both social and environmental sustainability. This line of reasoning can be extended to the global economy in which the agents referred to in the model are single countries which are not able to coordinate their actions. High-income inequality between countries, according to our analysis, represents a threat of environmental pressure and further inequality in welfare. In other words, the model offers an integrated approach showing that policies to reduce pollution and resource use, i.e., mitigation policies, and to reduce income inequality, both at national and global level, should be conceived, not as two parallel actions, but as two essential components of the *same* agenda.

Funding Open access funding provided by Università degli Studi di Sassari within the CRUI-CARE Agreement. The research of Angelo Antoci and Paolo Russu was supported by the Italian Ministry of Education (MIUR) ("Dipartimenti di Eccellenza" Program 2018-2022), and by the University of Sassari ("Finanziamento straordinario una tantum per la ricerca 2019").

8 Mathematical appendix

Let us define the following functions:

$$
\begin{aligned}
h(A_P, A_R) &:= A_R - A_P \\
s(A_P, A_R) &:= A_R + A_P - \frac{qQ_w}{e(1-\beta)\overline{L}} \\
t(A_P, A_R) &:= A_R + \left[1 + \frac{\gamma(dq - 2\beta e)}{e(1-\beta)(1-\gamma)}\right] A_P - \frac{qQ_w}{e(1-\beta)\overline{L}} \\
r(A_P, A_R) &:= A_R + \frac{e\alpha}{\gamma qd + e\alpha} A_P - \frac{qQ_w(1-\gamma)}{(\gamma qd + e\alpha)\overline{L}}
\end{aligned}
$$

defined in the set:

$$\Omega := \{(A_R, A_P) \in \mathcal{R}_+^2 |\ A_R - A_P > 0\}. \tag{19}$$

Then, we can enunciate the following proposition about the existence of stationary states $Q_i^* > 0$, $i = 1, 2, 3$. This proposition distinguishes among three different dynamic regimes according to the values of the parameter e, which represents the environmental impact of the production processes.

Proposition 1 (Existence of stationary states)

1. *If* $e < \dfrac{qd}{2\beta}$ *(Fig.* 1a*), the differential equation* (10) *admits:*

   (1.1) *the stationary state* Q_1^**, if and only if* $h(A_P, A_R) > 0$, $s(A_P, A_R) \le 0$ *and* $t(A_P, A_R) > 0$ *(these conditions are satisfied in the set* Ω_1 *in Fig.* 1a*);*

   (1.2) *the stationary state* Q_2^**, if and only if* $h(A_P, A_R) > 0$, $t(A_P, A_R) \le 0$ *and* $r(A_P, A_R) > 0$ *(these conditions are satisfied in the set* Ω_2 *in Fig.* 1a*);*

   (1.3) *the stationary state* Q_3^**, if and only if* $h(A_P, A_R) > 0$ *and* $r(A_P, A_R) \le 0$ *(these conditions are satisfied in the set* Ω_3 *in* Fig. 1a*).*

   Furthermore, it holds $\Omega_{i\ j} := \Omega_i \bigcap \Omega_j = \emptyset$*, for every* $i = 1, 2, 3$*, and* $j = 1, 2, 3$*, with* $i \neq j$*.*

2. *If* $\dfrac{qd}{2\beta} < e < \dfrac{qd}{\beta}$ *(Fig.* 1b*), the differential equation* (10) *admits:*

   (2.1) *the stationary state* Q_1^**, if and only if* $h(A_P, A_R) > 0$, $t(A_P, A_R) < 0$ *and* $s(A_P, A_R) \ge 0$ *(these conditions are satisfied in the set* $\Omega_{12} \cup \Omega_{13}$ *in Fig.* 1b*);*

   (2.2) *the stationary state* Q_2^**, if and only if* $h(A_P, A_R) > 0$, $t(A_P, A_R) \le 0$ *and* $r(A_P, A_R) > 0$ *(these conditions are satisfied in the set* $\Omega_{12} \cup \Omega_2$ *in Fig.* 1b*);*

   (2.3) *the stationary state* Q_3^**, if and only if* $h(A_P, A_R) > 0$ *and* $r(A_P, A_R) \le 0$ *(these conditions are satisfied in the set* $\Omega_3 \cup \Omega_{13}$ *in Fig.* 1b*).*

   Furthermore, it holds $\Omega_{1\ j} \neq \emptyset$*,* $j = 2, 3$*.*

3. *If* $e > \dfrac{qd}{\beta}$ *(Fig.* 1c*), the differential equation* (10) *admits:*

- (3.1) *the stationary state* Q_1^**, if and only if* $h(A_P, A_R) > 0$*,* $t(A_P, A_R) < 0$ *and* $s(A_P, A_R) \geq 0$ *(these conditions are satisfied in the set* Ω_{13} *in Fig.* 1c*);*
- (3.2) *the stationary state* Q_2^**, if and only if* $h(A_P, A_R) > 0$*,* $t(A_P, A_R) \leq 0$ *and* $r(A_P, A_R) < 0$ *(these conditions are satisfied in the set* Ω_{23} *in Fig.* 1c*);*
- (3.3) *the stationary state* Q_3^**, if and only if* $h(A_P, A_R) > 0$ *and* $r(A_P, A_R) \leq 0$ *(these conditions are satisfied in the set* $\Omega_3 \cup \Omega_{13}$ *in Fig.* 1c*).*

Furthermore, it holds $\Omega_{i\,3} \neq \emptyset$*,* $i = 2, 3$*.*

4. *Moreover, the stationary state* $Q_0^* = 0$ *always exists in the set* Ω*.*

Proof By straightforward calculations, it is easy to prove that the inequalities [see (12)–(14)]:

$$0 < \frac{b_1}{a_1} \leq \overline{Q}_P, \quad \overline{Q}_P < \frac{b_2}{a_2} < \overline{Q}_R, \quad \frac{b_3}{a_3} \geq \overline{Q}_R$$

are satisfied in the subsets of Ω bounded by the straight lines of equations:

$$s(A_P, A_R) = 0, \ t(A_P, A_R) = 0, \ r(A_P, A_R) = 0.$$

□

Under the conditions of existence of stationary states (see Proposition 1), we can state the following proposition:

Proposition 2 *The differential equation* (10) *admits at most three stationary states. More precisely (Fig.* 1*):*

1. *in the sets* Ω_i*,* $i = 1, 2, 3$*, two stationary states exist,* $Q_0^* = 0$ *and* Q_i^**; the former is repulsive, the latter is (globally) attractive (see dynamic regimes A, B, C in Fig.* 1d*);*
2. *in the sets* $\Omega_{i,j}$*,* $i = 1, 2$ *and* $i < j \leq 3$*, three stationary states exist,* $Q_0^* = 0$*,* Q_i^* *and* Q_j^**; the stationary states* $Q_0^* = 0$ *and* Q_j^* *are (locally) attractive, while the stationary state* Q_i^* *is repulsive (see dynamic regimes D, E, F in Fig.* 1d*);*
3. *in the set* Ω_0*, a unique (globally) attractive stationary state* $Q_0^* = 0$ *exists (see dynamic regime G in Fig.* 1d*).*

Proof According to the differential equation (2), $\dot{Q} < 0$ holds for high enough values of Q, therefore the stationary state Q_i^*, $i = 0, 1, 2, 3$, with the highest value of i (with the highest value of Q) is always attractive. Since the differential equation (2) is a continuous function of Q, for $Q > 0$, we have that: (1) when two stationary states $Q_0^* = 0$ and Q_i^* exist, Q_0^* is repulsive while Q_i^* is (globally) attractive; (2) when three stationary states exist, $Q_0^* = 0$, Q_i^* and Q_j^*, the stationary states $Q_0^* = 0$ and Q_j^* are (locally) attractive, while the stationary state Q_i^* is repulsive; and (3) when only the stationary state $Q_0^* = 0$ exists, then it is globally attractive. □

References

Abegg B, Agrawala S, Crick F, Montfalcon A (2007) Climate change impacts and adaptation in winter tourism. In: Shardul A (ed) Climate Change in the European Alps: Adapting Winter Tourism and Natural Hazards Management. OECD, Paris, France, pp 27–60

Antoci A, Bartolini S (1999) Negative externalities as the engine of growth in an evolutionary context. Nota di lavoro 83.99, Fondazione Eni Enrico Mattei, Milano. Available at http://hdl.handle.net/10419/155036. Accessed Sept 2020

Antoci A, Bartolini S (2004) Negative externalities, defensive expenditures and labour supply in an evolutionary context". Environ and Develop Econ 9(5):591–612

Antoci A, Borghesi S (2012) Preserving or escaping? The welfare effects of environmental self-protective choices". J Socio-econ 41:248–254

Auffhammer M, Aroonruengsawat A (2011) Simulating the impacts of climate change, prices and population on California's residential electricity consumption. Clim Change 109:191–210

Baland J-M, Bardhan P, Bowles S (2007) Inequality, cooperation, and environmental sustainability. Princeton University Press, Princeton

Barbier EB (2010) Poverty, development and environment. Environ Dev Econ 15:635–660

Barbier EB (2015) Nature and wealth overcoming environmental scarcity and inequality. Palgrave Macmillan, London

Barbier EB, Hochard JP (2018) Poverty, rural population distribution and climate change. Environ Dev Econ 23(3):234–256

Barnett J, O'Neill S (2010) Maladaptation. Glob Environ Change 20:211–213

Beilin R, Sysak T, Hill S (2012) Farmers and perverse outcomes: the quest for food and energy security, emissions reduction and climate adaptation. Glob Environ Change 22:463–471

Bell ML, Ebisu K (2012) Environmental inequality in exposures to airborne particulate matter components in the United States. Environ Health Perspect 120:1699–1704

Berthe A, Elie L (2015) Mechanisms explaining the impact of income inequality on environmental deterioration. Ecol Econ 116:191–200

Bird N (2014) Fair share: climate finance to vulnerable countries, ODI Briefing papers, September 2014, available at https://www.odi.org/publications/8517-fair-share-climate-finance-vulnerable-countries. Accessed July 2019

Borghesi S (2006) Income inequality and the environmental Kuznets curve. In: Basili M, Franzini M, Vercelli A (eds) Environment, inequality and collective action. Routledge, London

Boyce JK (1994) Inequality as a cause of environmental degradation. Ecol Econ 11:169–178

Boyce JK (2018) The environmental cost of inequality. Sci Am 319(5):72–77

Broner F, Bustos P, Carvalho VM (2012) Sources of comparative advantage in polluting industries. NBER working paper no. w18337. Available at SSRN: https://ssrn.com/abstract=2136006. Accessed July 2019

Brooks N, Sethi R (1997) The distribution of pollution: community characteristics and exposure to air toxics. J Environ Econ Manag 32:233–250

Candau F, Dienesch E (2017) Pollution haven and corruption paradise. J Environ Econ Manag 85:171–192

Caro D, Pulselli FM, Borghesi S, Bastianoni S (2017) Mapping the international flows of GHG emissions within a more feasible consumption-based framework. J Clean Prod 147:42–151

Clark LP, Millet DB, Marshall JD (2014) National patterns in environmental injustice and inequality: outdoor NO2 air pollution in the United States. PLoS One 9(4):e94431. https://doi.org/10.1371/journal.pone.0094431

Clément M, Meunié A (2010) Is inequality harmful for the environment? An empirical analysis applied to developing and transition countries. Rev Soc Econ 68:413–445

Cohen F, Glachant M, Söderberg M (2014) Adaptation to climate change and US residential energy use—does adaptation reduce greenhouse gas emissions?. In: Paper presented at the 1st annual conference of the French association of environmental and resource economists, Montpellier, 11–12 September 2014

Cushing L, Morello-Frosch R, Wander M, Pastor M (2015) The haves, the have-nots, and the health of everyone: the relationship between social inequality and environmental quality. Ann Rev Pub Health 36:193–209

Davis LW, Gertler PJ (2015) Contribution of air conditioning adoption to future energy use under global warming. Proc Natl Acad Sci USA 112(19):5962–5967

Dean J, Mary M, Lovely E, Wang H (2009) Are Foreign investors attracted to weak enviromental regulations? Evaluating the evidence from China. J Dev Econ 90:1–13

De Cian E, Sue Wing I (2019) Global energy consumption in a warming climate. Environ Resour Econ 72:365–410

de Coninck H, Revi A, Babiker M, Bertoldi P, Buckeridge M, Cartwright A, Dong W, Ford J, Fuss S, Hourcade JC, Ley D, Mechler R, Newman P, Revokatova A, Schultz S, Steg L, Sugiyama T (2018) Strengthening and Implementing the Global Response. In Masson-Delmotte V, Zhai P, Portner HO, Roberts D, Skea J, Shukla PR, Pirani A, Moufouma-Okia W, Pean C, Pidcock R, Connors S, Matthews JBR, Chen Y, Zhou X, Gomis MI, Lonnoy E, Maycock T, Tignor M, Waterfield T (eds): Global Warming of 1.5°C. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty. In press. Available at https://www.ipcc.ch/site/assets/uploads/sites/2/2019/02/SR15_Chapter4_Low_Res.pdf. Accessed Sept 2020

Deschênes O, Greenstone M (2011) Climate change, mortality, and adaptation: evidence from annual fluctuations in weather in the US. Am Econ J: Appl Econ 3:152–185

Diffenbaugh NS, Burke M (2019) Global warming has increased global income inequality. PNAS 116(20):9808–9813. https://doi.org/10.1073/pnas.1816020116

Dinda S (2004) Environmental Kuznets curve hypothesis: a survey. Ecol Econ 49(4):431–455

Fezzi C, Harwood AR, Lovett AA, Bateman IJ (2015) The environmental impact of climate change adaptation on land use and water quality. Nat Clim Change 5:255–260

Gassebner M, Gaston N, Lamla MJ (2008) Relief for the environment? The importance of an increasingly unimportant industrial sector. Econ Inq 46(2):160–78

Grunewald N, Klasen S, Martínez-Zarzoso I, Muris C (2017) The trade-off between income inequality and carbon dioxide emissions. Ecol Econ 142:249–256

Hamin EM, Gurran N (2009) Urban form and climate change: balancing adaptation and mitigation in the US and Australia. Habitat Int 33:238–245

Heerink N, Mulatu A, Bulte E (2001) Income inequality and the environment: aggregation bias in environmental Kuznets curves. Ecol Econ 38:359–367

IPCC (2001) Climate change 2001: impacts, adaptation, and vulnerability. In: McCarthy D, Canziani OF, Leary NA, Dokken DJ, White KS (eds) Contribution of working group II to the 3rd assessment report of the intergovernmental panel on climate change. Cambridge University Press, Cambridge

IPCC (2007) Summary for policymakers. In: Parry ML, Canziani OF, Palutikof JP, van der Linden PJ, Hanson CE (eds) Climate change 2007: impacts, adaptation and vulnerability. Contribution of working group II to the 4th assessment report of the intergovernmental panel on climate change. Cambridge University Press, Cambridge, pp 7–22

Kashwan P (2017) Inequality, democracy, and the environment: a cross-national analysis. Ecol Econ 131:139–151

Klein RJT, Midgley GF, Preston BL, Alam M, Berkhout FGH, Dow K, Shaw MR (2014) Adaptation opportunities, constraints, and limits. In: Field CB, Barros VR, Dokken DJ, Mach KJ, Mastrandrea MD, Bilir TE, Chatterjee M, Ebi KL, Estrada YO, Genova RC, Girma B, Kissel ES, Levy AN, MacCracken S, Mastrandrea PR, White LL (eds) Climate change 2014: impacts, adaptation, and vulnerability. Part A: global and sectoral aspects. Contribution of working group II to the 5th assessment report of the intergovernmental panel on climate change. Cambridge University Press, Cambridge, New York, pp 899–943

Lenzen M, Moran D, Bhaduri A, Kanemoto K, Bekchanov M, Geschke A, Foran B (2013) International trade of scarce water. Ecol Econ 94(C):78–85

Levinson A, Taylor MS (2008) Unmasking the pollution haven effect. Int Econ Rev 49(1):223–254

Li Y, Hewitt C (2008) The effect of trade between China and the UK on national and global carbon dioxide emissions. Energy Policy 36:1907–1914

Magnan AK, Schipper ELF, Burkett M, Bharwani S, Burton I, Eriksen S, Gemenne F, Schaar J, Ziervogel G (2016) Addressing the risk of maladaptation to climate change. WIREs Clim Change 7:646–665

Magnani E (2000) The Environmental Kuznets Curve, environmental protection policy and income distribution. Ecol Econ 32(3):431–443

McAusland C (2003) Voting for pollution policy: the importance of income inequality and openness to trade. J Int Econ 61:425–451

Mulatu A, Gerlagh R, Rigby D, Wossink A (2010) Environmental regulation and industry location in Europe. Environ Resour Econ 45(4):459–479

Olsson L, Opondo M, Tschakert P, Agrawal A, Eriksen SH, Ma S, Perch LN, Zakieldeen SA (2014) Livelihoods and poverty. In: Field CB, Barros VR, Dokken DJ, Mach KJ, Mastrandrea MD, Bilir TE, Chatterjee M, Ebi KL, Estrada YO, Genova RC, Girma B, Kissel ES, Levy AN, MacCracken S, Mastrandrea PR, White LL (eds) Climate change 2014: impacts, adaptation, and vulnerability. Part A: global and sectoral aspects. Contribution of working Group II to the 5th assessment report of the intergovernmental panel on climate change. Cambridge University Press, Cambridge, New York, pp 793–832

Peters G, Minx J, Weber CL, Edenhofer O (2011) Growth in emission transfers via international trade from 1990 to 2008. Proc Natl Acad Sci USA 108:8903–8908

Pouliotte J, Smit B, Westerhoff L (2009) Adaptation and development: livelihoods and climate change in Subarnabad. Bangladesh. Clim Dev 1(1):31–46

Sapkota P, Bastola U (2017) Foreign direct investment, income, and environmental pollution in developing countries: panel data analysis of Latin America. Energy Econ 64(C):206–212

Schoolman ET, Ma C (2012) Migration, class and environmental inequality: exposure to pollution in China's Jiangsu Province. Ecol Econ 75:140–151

Scruggs LA (1998) Political and economic inequality and the environment. Ecol Econ 26:259–275

Shogren JF, Crocker TD (1991) Cooperative and noncooperative protection against transferable and filterable externalities. Environ Resour Econ 1:195–214

Sonderskov KM (2008) Environmental group membership, collective action and generalised trust. Environ Politics 17(1):78–94

Stern DI (2017) The environmental Kuznets curve after 25 years. J Bioecon 19:7–28

Sun C, Kahn ME, Zheng S (2017) Self-protection investment exacerbates air pollution exposure inequality in urban China. Ecol Econ 131:468–474

Teixidó J, Steinberger J, Haberl H, Krausmann F, Peters G, Wiedmann T, Duro J, Kastner T (2016) International inequality of environmental pressures: decomposition and comparative analysis. Ecol Indic 62:163–173

UN, DESA, (2016) Climate change resilience: an opportunity for reducing inequalities. United Nations, New York

UNEP (2019) Frontiers 2018/19: emerging issues of environmental concern. Available at https://www.unenvironment.org/resources/frontiers-201819-emerging-issues-environmental-concern. Accessed July 2019

van den Bergh JCJM (2011) Energy conservation more effective with rebound policy. Environ Resour Econ 48:43–58

van Ruijven BJ, De Cian E, Sue Wing I (2019) Amplification of future energy demand growth due to climate change. Nat Commun 10:2762

Waldkirch A, Gopinath M (2008) Pollution control and foreign direct investment in Mexico: An industry-level analysis. Environ Resour Econ 41:289–313

Wiedmann TO, Schandl H, Lenzen M, Moran D, Suh S, West J, Kanemoto K (2015) The material footprint of nations. Proc Natl Acad Sci 112(20):6271–6276

Yang Z, Wei T, Moore JC, Chou J, Dong W, Dai R, Yang S, Ban J (2016) A new consumption-based accounting model for greenhouse gases from 1948 to 2012. J Clean Prod 133:368–377

Zhao X, Zhang S, Fan C (2014) Environmental externality and inequality in China: current status and future choices. Environ Pollut 190:176–179

Zugravu-Soilita N (2017) How does foreign direct investment affect pollution? Toward a better understanding of the direct and conditional effects. Environ Resour Econ 66(2):293–338

Zwickl K, Ash M, Boyce JK (2014) Regional variation in environmental inequality: industrial air toxics exposure in US cities. Ecol Econ 107:494–509