Parameter estimation for heat transfer analysis during casting processes based on ensemble Kalman filter

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Abstract:

It is very important for production of casts with high quality to predict and control the solidification processes of the alloy. Heat transfer analysis has been utilized for understanding solidification processes. However, it is often difficult to obtain values of all input parameters such as thermal conductivity and heat transfer coefficient precisely. In this study, a parameter estimation method in heat transfer analysis is developed based on data assimilation. In the authors' previous study, particle filter, a method of data assimilation, was applied to estimation of thermal conductivity and heat transfer coefficient in heat transfer analysis for mold casting, and its applicability was systematically investigated. It was shown that particle filter is very effective in estimating these parameters. However, particle filter suffers from a shortcoming called sample degeneracy which often prevents accurate estimation of parameters in phenomena of interest. The present study focuses on a different method of data assimilation called ensemble Kalman filter and its applicability to the estimation of heat transfer coefficient and thermal conductivity is investigated based on twin experiments. It is shown that thermal conductivity and constant or time-dependent heat transfer coefficient can be accurately estimated independently with three and two cooling curves, respectively. Furthermore, the thermal conductivity and time-dependent heat transfer coefficient can be estimated simultaneously with high accuracy.

Key Words: casting, data assimilation, parameter estimation, heat transfer coefficient, thermal conductivity

1. Introduction

It is important to prevent formation of casting defects such as porosity, hot tears, residual stresses and cracks in production of high quality casts. Solidification processes during casting must be understood in detail to predict the formation of casting defects. Heat transfer analyses offer an effective way of understanding of solidification processes [1-3].

Validity of the heat transfer analysis is entirely determined by accuracy of the input parameters. However, it is not always straightforward to obtain the values of input parameters such as thermal conductivity k and heat transfer coefficient h with high accuracy especially when new alloys and newly designed processes are considered [4]. During solidification processes of the alloy, the fluid flow takes place in the liquid phase and moreover the concentration distribution changes due to the solute redistribution at the solid/liquid interface [5]. Therefore, the fluid dynamics and time change of concentration field should be described in combination with the heat transfer analysis. However, such an analysis generally requires high calculation cost and, accordingly, the heat transfer analysis is often carried out without explicitly describing fluid dynamics and the time change of concentration field. In such a case, k used in the heat transfer analysis corresponds to an apparent value unique to each casting problem, which implicitly includes effects of the fluid flow and concentration change. However, it is difficult to evaluate the apparent value easily and accurately because it depends on several factors such as alloy system and casting conditions. It is important to develop the simple method to determine the apparent value of k. It is also important to point out that even the true value of k is not readily available especially in multi-component alloys.

The heat transfer coefficient h is usually estimated by trial and error based on measured cooling curves [6] and its estimated value generally involves large uncertainty. Furthermore, h is often time-dependent [3, 7-12] and its estimation is more complicated when the time dependence of h must be taken into account. In many cases, the approximation is a priori introduced into the form of the time dependence, while its validity is not always assured.

It is very important to develop a reliable and simple estimation method for k and h in the heat transfer analysis. It is also desirable that such a method is applicable to various kind of casting processes without a prior knowledge of time dependences of parameters. Attempts have been made to develop methods for parameter estimation in heat transfer analysis. For example, there are several effective methods such as a method for simultaneously estimating k and h in a steady-state based on Bayesian inference [13], a method for estimating h during sand mold casting processes of stainless steel based on genetic algorithm [7], a method for estimating h during casting processes of single crystal blade based on sensitivity analysis and convergent calculation [6]. However, further efforts need to be devoted to development of a method having wider range of applications and, importantly, it is still very difficult to estimate time-dependent parameters without a prior knowledge. In this study, we focused on data assimilation as a method for estimating parameters in heat transfer analysis.

Data assimilation is a method for incorporating measured (observed) values into simulations. It has been developed in the field of data science based on Bayesian inference, and it has been successfully applied to estimations of state and/or parameters in many problems in the fields of oceanography and meteorology [14-16]. In the previous study [17], we developed the estimation method for k and h based on particle filter which is one of the methods for data assimilation. The particle filter is based on a Monte Carlo method consisting of many particles. One particle represents the dynamics of one system (time evolution of a set of state variables and/or parameters), and likelihood of the particles is calculated using measured values, and distribution of particles in the sampling space spanned by state variables and/or parameters is updated according to likelihood. The estimated value can be obtained from the expectation value calculated from the distribution of particle. It was shown that the particle filter was very effective in estimating these parameters. However, there is a critical issue in particle filter called sample degeneracy [18]. The sample degeneracy is a problem that appropriate sampling of state and/or parameters is hampered by localization of the particles in the sampling space and, accordingly, the estimation of parameters and/or state becomes impossible. Although this problem was not observed in our previous study on parameter estimation in one-dimensional heat transfer analysis, the sample degeneracy will generally become significant when the scale of simulation becomes large.

Ensemble Kalman filter (hereafter abbreviated as EnKF) is one of data assimilation methods which corresponds to a Monte Carlo approximation of Kalman filter. EnKF generally offers a reasonable balance between the accuracy and computational cost. Importantly, the sample degeneracy can be suppressed in EnKF. Hence, it is very important to investigate the effectiveness of EnKF for the parameter estimation in heat conduction problem. This is tackled in this study. EnKF is applied to (i) individual estimation of constant k and constant h, (ii) estimation of time-dependent h and (iii) simultaneous estimation of constant k and time-dependent h. The feasibility of these estimations is investigated in detail.

2. Calculation procedure

2.1 Heat transfer analysis and twin experiment

In this study, we focus on a one-dimensional system which consists of a mold on the left-hand side and the cast on the right-hand side. The lengths of mold and cast are 30 and 33 mm, respectively. The schematic figure of calculation area is shown in Figure 1. This is the same system used in the previous study [17]. The heat conduction in this system is described by the following equation,

$$\rho_i C_{p,i}(\partial T/\partial t) = k_i(\partial^2 T/\partial x^2) + \rho_i \Delta H \frac{\partial f_s}{\partial T}\frac{\partial T}{\partial t} \tag{1}$$

where T is the temperature, ρ_i is the density in i region with i = mold or alloy, $C_{p,i}$ is the specific heat in i region, ΔH is the latent heat, f_s is the fraction of solid and k_i is the heat conduction coefficient in i region. Note that the second term on the right-hand side represents release of latent heat associated with solidification and, thus, it is included only in the calculation of alloy region. This equation is discretized in a standard finite different scheme with second order accuracy in the space and it was solved in an explicit Euler scheme. The heat flux across the mold-alloy interface was calculated by the following heat transfer equation.

$$q_{i.j} = h_{i/j}(T_j - T_i) \tag{2}$$

where $q_{i,j}$ represents the heat flux across i/j interface with i = mold and j = alloy, $h_{i/j}$ is the heat transfer coefficient, T_j and T_i are the temperatures in i and j regions at i/j interface, respectively. In addition, the temperature at the surface of mold was calculated using Eq. (2) with i = air and j = mold where the temperature of the air is assumed to be constant. The mold and the alloy were supposed to be cast iron and Al-4 wt. % Cu, respectively. Input parameters are presented in Table 1. In reality, the physical properties such as thermal conductivity and density should be temperature dependent. However, such dependences are neglected for simplicity in this study. Note that the inclusion of temperature dependence of physical parameters are not indispensable in the present approach. EnKF provides the estimated values of heat transfer coefficient and thermal conductivity that allow for the accurate reproduction of the measured data by the heat transfer simulation regardless of consideration of the temperature dependences of physical quantities. When the temperature dependences are not considered, the estimated value of thermal conductivity should be different from its true value and it corresponds to "apparent value". Importantly, the apparent value thus obtained will enable accurate reproduction of the measured data by the simulation with assumption of no temperature dependences. This is one of advantageous features in the present approach. The inclusion of the temperature dependences of parameters will be considered in a future work.

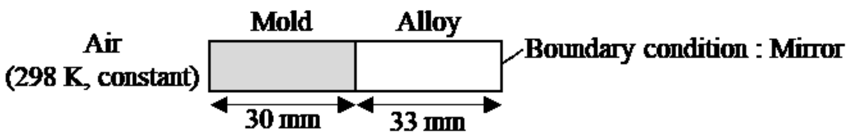

Figure 1 One-dimensional calculation area consisted of the mold and the alloy

Table 1 Parameters employed for heat transfer analysis

Parameter	Value	
	Mold (Hyper-eutectic gray cast iron)	Alloy (Al – 4.0 mass% Cu)
Initial temperature (T_0 /K)	298	950
Liquidus temperature (T_l /K)	-	924
Solidus temperature (T_s /K)	-	905
Density (ρ /kgm^{-3})	7000 [19]	2800
Thermal conductivity (k_i /Wm^{-1}K^{-1})	65.4 [19]	Estimated
Heat transfer coefficient Mold/Alloy ($h_{mold/alloy}$ /Wm^{-2}K^{-1})	Estimated	
Heat transfer coefficient Air/Mold ($h_{air/mold}$ /Wm^{-2}K^{-1})	120	
Air temperature (T_{air} /K)	298	
Spatial grid spacing (Δx /mm)	1	
Time step (Δt /s)	0.1	

As mentioned in the introduction, we focus on the estimation of k_{alloy} and $h_{\mathrm{mold/alloy}}$ based on EnKF. Hereafter, k_{alloy} is denoted as k. In practice, the estimation of EnKF needs the measured data. The measured data must be the quantity sensitive to the change of k and $h_{\mathrm{mold/alloy}}$. The time dependences of temperatures measured at fixed positions in the mold and the alloy, i.e., cooling curves are the data suitable to the present purpose. In this study, for the sake of convenience, the twin experiments were conducted to evaluate the accuracy of EnKF. The twin experiment is an effective method for evaluating accuracy and efficiency of data assimilation technique. In the twin experiment, the measured data are not real data obtained by experimental measurement but data obtained by a heat transfer simulation using prescribed (or assumed) values of input parameters of interest. Then, the prescribed values are regarded as unknown values and the results of the simulation, i.e., the “measured data” are utilized for estimating the unknown parameters of interest based on EnKF. Therefore, the accuracy of estimation can be clearly evaluated by comparing the estimated values and the prescribed values of parameters of interest. In the present case, the measured data correspond to the temperatures at different time and positions that were obtained by the following procedure. The casting process of Al-Cu alloy for 30 seconds was simulated by solving Eqs. (1) and (2) with the parameters listed in Table 1 and prescribed values of k_i and $h_{i/j}$. Then, the temperatures at different positions at an interval of 0.1 seconds with addition of observation noise were regarded as the measured data. The observation noise was given as the Gaussian noise with the mean value 0 K and the standard deviation σ_T K.

2.2 Ensemble Kalman filter (EnKF)

The procedure of parameter estimation based on EnKF is explained below, taking estimation of constant $h_{\mathrm{mold/alloy}}$ as an example. Hereafter, $h_{\mathrm{mold/alloy}}$ is denoted as h. EnKF is a method for estimating state and/or parameters based on the Monte Carlo method where a lot of simulations are performed simultaneously under different conditions. When the estimation of h is considered, a lot of heat transfer simulations for different value of h are simultaneously performed. Each simulation is called the particle which is characterized by a state vector $\boldsymbol{x}_{t,i} = (\{a_{t,i}\}, h_{t,i})^{\mathrm{T}}$ where $\{a_{t,i}\}$ represent a set of state variables, i.e., temperature in a particle i at time t and $h_{t,i}$ is h in the particle i at time t. The superscript T denotes the transposition of vector. The simulations are conducted until the time at which the measured data (measured temperatures) are available and, then, the filtering is performed. In the filtering, $\{\boldsymbol{x}_{t,i}\}$ are updated according to the following equation,

$$\boldsymbol{X}_{t,i} = \boldsymbol{x}_{t,i} + \boldsymbol{K}_t (\boldsymbol{y}_t - \boldsymbol{H}\boldsymbol{x}_{t,i}) \tag{3}$$

where $\boldsymbol{X}_{t,i}$ is the updated state vector, $\boldsymbol{y}_t = (T_{t,1}\ T_{t,2}\ \ldots T_{t,m})^{\mathrm{T}}$ is the measured data at the time t and the position m, $\boldsymbol{H}$ is the observation matrix that are multiplied by $\boldsymbol{x}_{t,i}$ to obtain the observable quantities corresponding to $\boldsymbol{y}_t$. Note that $\boldsymbol{y}_t - \boldsymbol{H}\boldsymbol{x}_{t,i}$ is the difference between the result of the simulation of particle i and the measured data $\boldsymbol{y}_t$. $\boldsymbol{K}_t$ is called the Kalman gain given as

$$\boldsymbol{K_t} = \widehat{\boldsymbol{V}}_{\boldsymbol{t}}\boldsymbol{H}'(\boldsymbol{H}\widehat{\boldsymbol{V}}_{\boldsymbol{t}}\boldsymbol{H}' + \boldsymbol{R_t}) \tag{4}$$

where $\widehat{\boldsymbol{V}}_t$ is the variance-covariance matrix of the ensemble and $\boldsymbol{R}_t$ is the matrix of observation noise. $\boldsymbol{H'}$ is the transposed matrix of $\boldsymbol{H}$. $\widehat{\boldsymbol{V}}_t$ is described by the expression

$$\tilde{x}_{t,i} = x_{t,i} - (1/N)\sum_{j=1}^{N} x_{t,j} \tag{5}$$

$$\hat{V}_t = (1/(N-1))\sum_{j=1}^{N} \tilde{x}_{t,j}\tilde{x}_{t,j}{}' \tag{6}$$

where N is total number of particles.

The estimated value of h at the time t can be determined by averaging values of $h_{t,i}$ after filtering. In this study, the noise called the system noise was added to $h_{t,i}$ after updating of particles and it was given as the Gaussian noise. The mean value of the system noise was set to zero and the standard deviation was set to σ_h % of the estimated value of h where σ_h is the input parameter. In the case of estimation of k, the Gaussian noise with the mean value of zero and the standard deviation of σ_k% was added to k as the system noise after updating of the particles. The filtering thus described is repeated every time when the measured data are available. It is expected that the estimated value should approach the true value by repeating the filtering. When k and h are simultaneously estimated, each particle has the different values of k and h and both parameters are simultaneously updated in the filtering.

3. Results and discussion

3.1 Estimation of heat transfer coefficient

3.1.1 Effects of estimation conditions on the estimation accuracy

We first focus on the estimation of only heat transfer coefficient h. The true value of h was set to a constant value of 600 $Wm^{-2}K^{-1}$. The standard deviations of observation noise and system noise were given as σ_T=1 K and σ_h=10 %, respectively. The number of particles was set to 128. The thermal conductivity was set to k = 87 $Wm^{-1}K^{-1}$. The initial value of h in the particles were given by the random numbers generated in the range of 0 to 6000 $Wm^{-2}K^{-1}$.

Figure 2 (a) shows time change of h estimated using the measured data from only single measurement position. Specifically, the temperature changes at every 0.1 s in the alloy at the position 1 mm away from the mold were used as the measured data. The dotted line is the true value of h, while the solid line is the estimated value. Note that this true value is not a realistic one but the one prescribed (assumed) in this twin experiment. The distance between the solid and dashed line corresponds to the standard deviation of the estimated h. The standard deviation keeps increasing and the estimated value of h is not in agreement with the true value the whole time. That is, the estimation is not successful. Figure 2 (b) shows time change of h estimated using the measured data from two different positions. The one measurement position is located in the alloy and the other is in the mold. These measurement positions were located at x_{alloy} and x_{mold} away from the alloy/mold boundary. Here, x_{alloy} and x_{mold} indicates the absolute distances from the alloy/mold boundary in the alloy and mold, respectively. In the case of Figure 2 (b), the measured data were obtained at the positions x_{alloy} = x_{mold} = 20 mm away from the alloy/mold boundary. The estimated values approached at early time periods and, thereby, we considered that the estimation is successful in this case. As exemplified in Figure 2, we found that the data measured in both the alloy and mold are required for accurate estimation of h.

Figure 3 shows the dependence of estimation time on the measurement position. The estimation time is defined as the time required for the estimated value to coincide with the true value within 10 % error. In Figure 3 (a), when x_{alloy} = 1 mm, the estimation time is very short regardless of value of x_{mold}. On the other hand, when x_{alloy} = 10 or 30 mm, x_{mold} should be less than 10 mm to realize the short estimation time. Similarly, in Figure 3 (b), one can find that the accurate estimation can be achieved in very short time when x_{alloy} is less than 5 mm regardless of value of x_{mold}. Therefore, either of x_{alloy} and x_{mold} must be set to less than 5 mm for estimating h immediately.

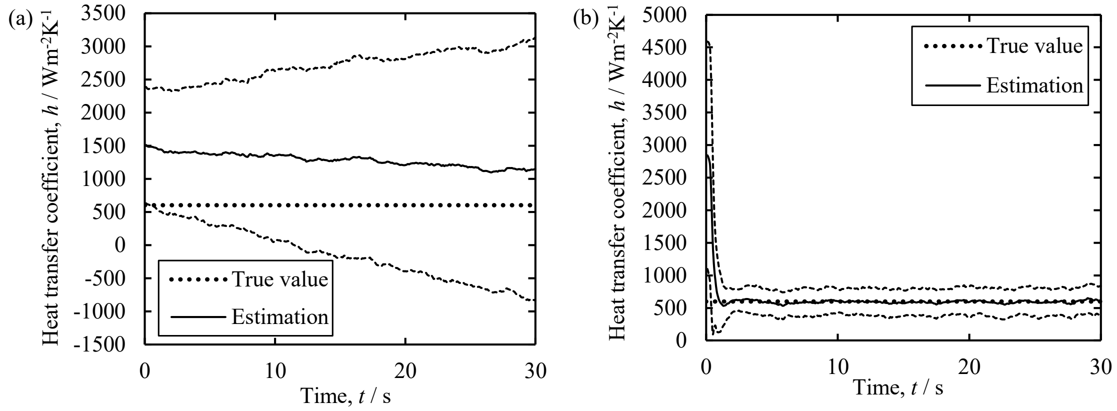

Figure 2 Time dependence of heat transfer coefficient estimated with measured data from (a) one measurement position in the alloy 1 mm away from the alloy/mold boundary and (b) two positions in both the alloy and mold at 20 mm away from the boundary.

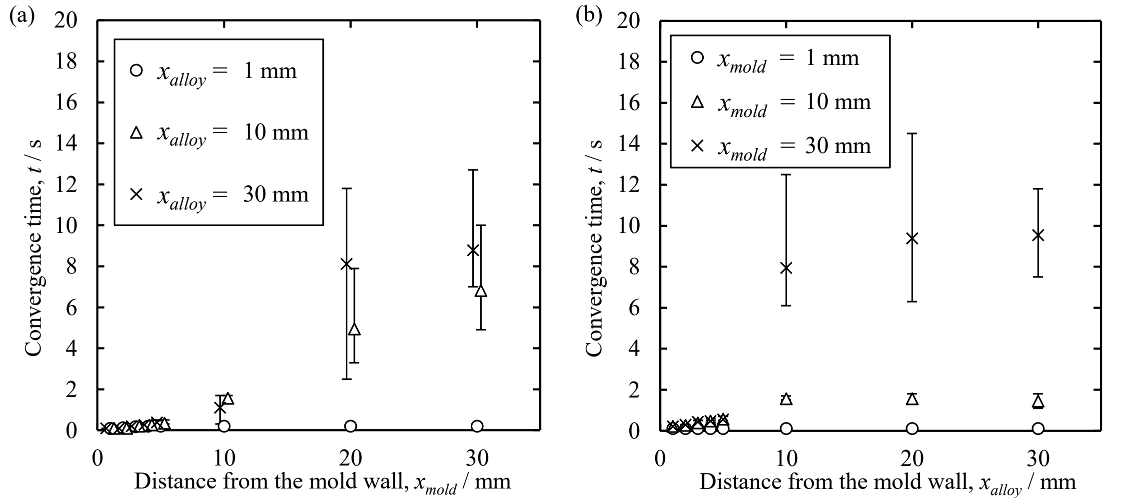

Figure 3 Dependences of estimation time on the measurement position. (a) The dependence on x_{mold} for three different value of x_{alloy}. (b) The dependence on x_{alloy} for three different value of x_{mold}.

Figure 4 (a) and (b) shows the relation between the estimation accuracy and the two measurement positions. The estimation value corresponds to the value averaged from t = 15 to 30 seconds. Each estimation was performed five times and the average value of five values are shown as plots. The minimum and maximum values among five values are shown as error bars. It is seen that h can be estimated with high accuracy independently of the measurement positions. This is because that the estimated value converged to the true value in 15 seconds.

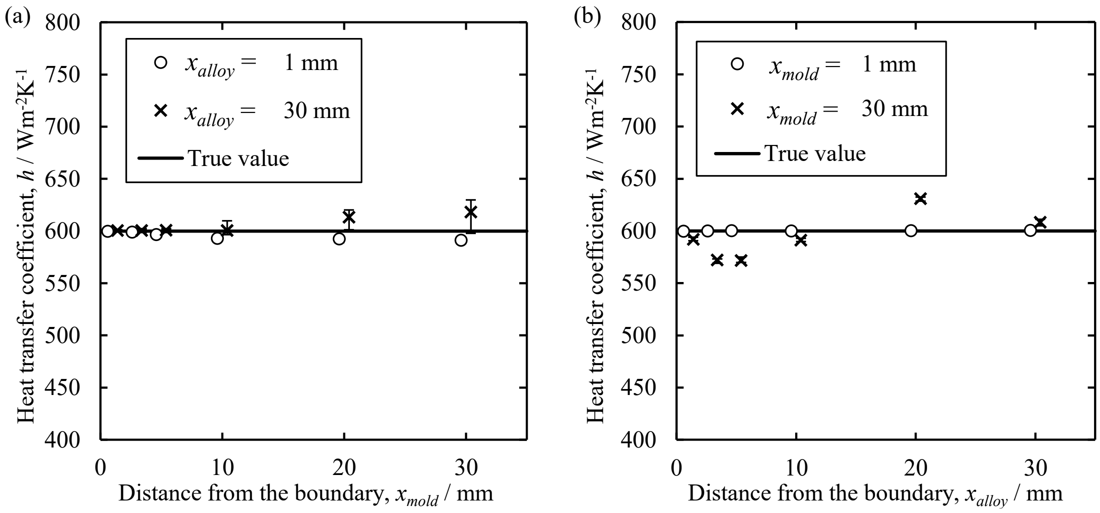

Figure 4 Dependence of estimation accuracy of heat transfer coefficient on the measurement positions of temperature. (a) The dependence on x_{mold} for two different value of x_{alloy}. (b) The dependence on x_{alloy} for two different value of x_{mold}.

Figure 5 (a) shows the dependence of estimation accuracy on σ_T and two measurement positions. The horizontal axis represents the distance from the alloy/mold boundary, $x = x_{\mathrm{alloy}} = x_{mold}$. Although the accuracy is slightly low when x = 30 mm, h is estimated with high accuracy regardless of σ_T. Figure 5 (b) shows the dependence of estimation accuracy on σ_h and measurement positions. h can be estimated with high accuracy regardless of σ_h when x = 1 mm and 10 mm. However, the estimation accuracy is remarkably low when x = 30 mm and σ_h = 20 %. In short, it was found that h can be estimated with high accuracy regardless of σ_h and σ_T when the measurement positions are set close to the alloy/mold boundary.

Figures 6 and 7 shows the results when the true value of h was set to 3000 $\mathrm{Wm^{-2}K^{-1}}$ and 120 $\mathrm{Wm^{-2}K^{-1}}$, respectively. In both cases, either of x_{alloy} and x_{mold} must be close to the alloy/mold boundary in order to estimate h with high accuracy.

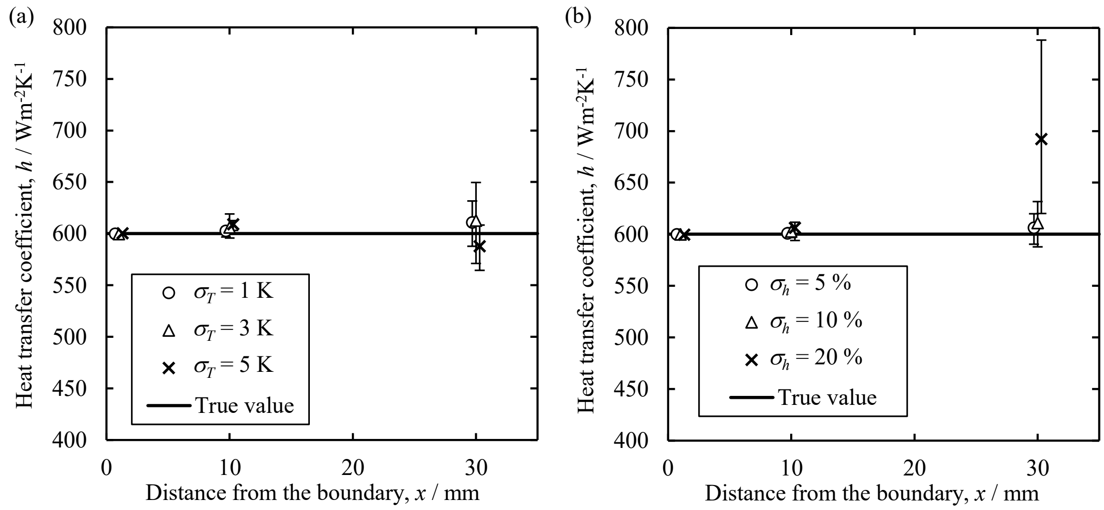

Figure 5 Dependence of estimation accuracy on the measurement positions with $x = x_{alloy} = x_{mold}$ for different standard deviation of observation noise σ_T and σ_h. (a) σ_T = 1, 3 or 5 K and σ_h = 10 %. (b) σ_h = 5, 10 or 20 % and σ_T = 1 K.

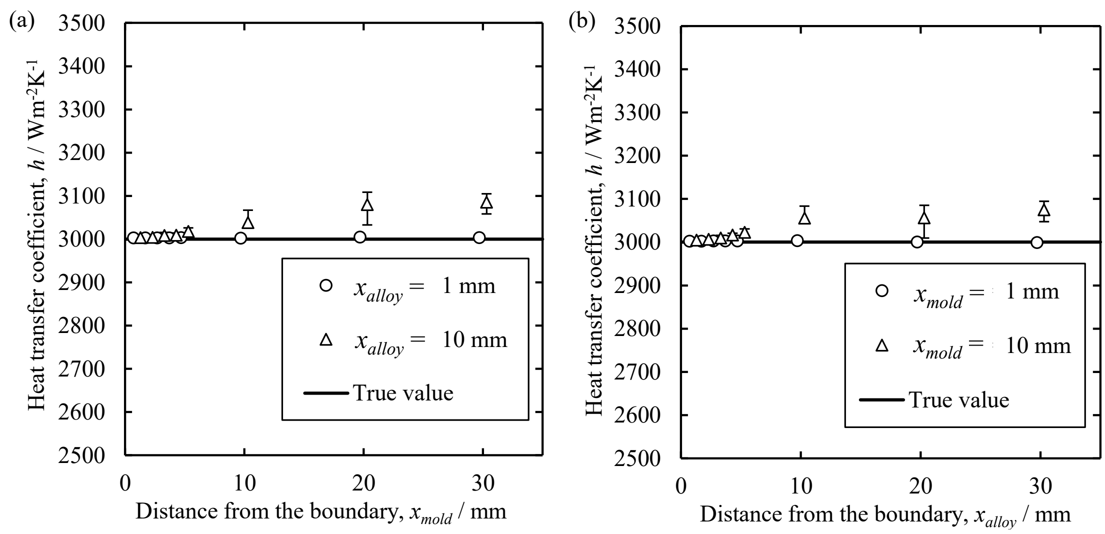

Figure 6 Dependence of estimation accuracy of heat transfer coefficient on the measurement positions of temperature when the true value of h = 3000 Wm^{-2}K^{-1}. (a) The dependence on x_{mold} for two different value of x_{alloy}. (b) The dependence on x_{alloy} for two different value of x_{mold}.

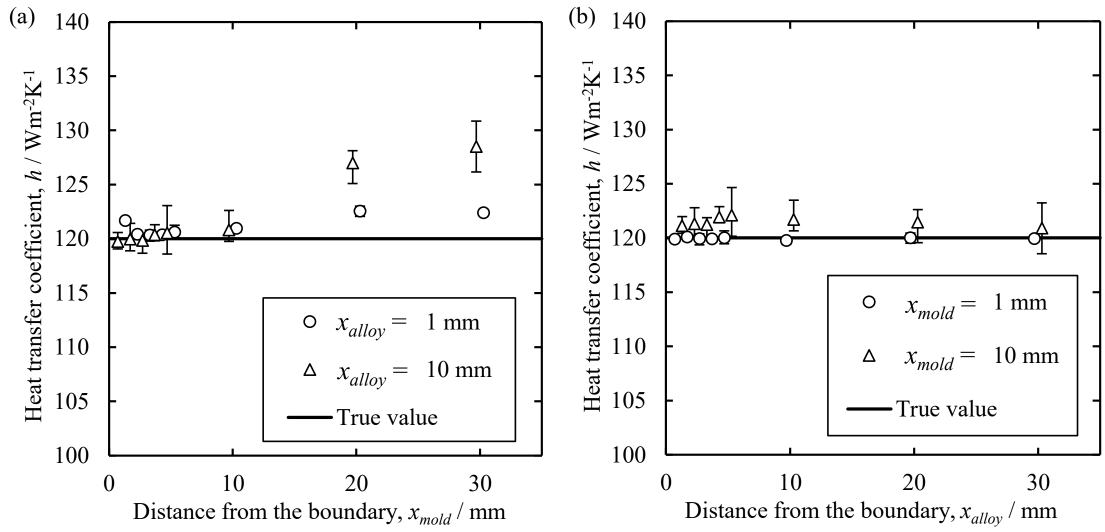

Figure 7 Dependence of estimation accuracy of heat transfer coefficient on the measurement positions of temperature when the true value of h = 120 Wm^{-2}K^{-1}. (a) The dependence on x_{mold} for two different value of x_{alloy}. (b) The dependence on x_{alloy} for two different value of x_{mold}.

3.1.2 Estimation of time-dependent heat transfer coefficient

The heat transfer at a boundary between the mold wall and cast depends on several factors such as fluid flow and concentration change near the boundary. Also, the formation of air gap between the mold wall and solidifying shell has a significant influence on the heat transfer. These effects must be considered in the heat transfer coefficient, h. In general, therefore, h should be time-dependent. We conducted the estimation of time-dependent h. According to Ref. 6 and 12, the true value was set to $h(t) = h_0\, t^{-0.5}$ Wm^{-2}K^{-1} with a constant h_0. The standard deviations of observation and system noises were σ_T=1 K and σ_h=10 %, respectively. The number of particles was set to 128. The measured data from two measurement positions were utilized in the estimation. The one is located in the alloy, while the other is located in the mold.

Figure 8 shows the time change of estimated values of h when h_0 = 6000, 3000 or 600. The measured data with $x_{alloy} = x_{mold}$ = 1 mm were utilized. The dotted line indicates the true value, while the solid line is the estimated results. The distance between the solid and the dashed line corresponds to the standard deviation of h. The estimated results coincide well with the true value for different values of h_0. Hence, it is demonstrated that the time dependent h can be accurately estimated by EnKF without a priori information about its dependence.

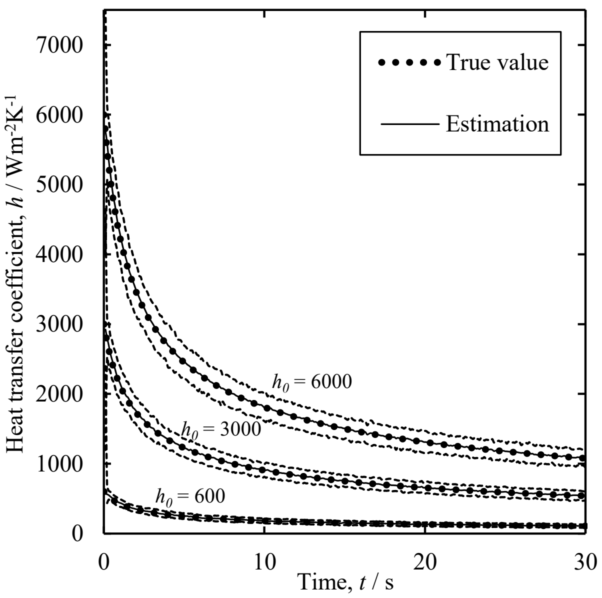

Figure 8 Estimated results of time-dependent heat transfer coefficient. The true value is given as $h = h_0 t^{-0.5}$ Wm^{-2}K^{-1}. The measured data with $x_{alloy} = x_{mold} = 1$mm were utilized.

In order to understand the dependence of the estimation conditions on the accuracy, the estimation accuracy was evaluated by the relative-root-mean-square error (*RRMSE*) given as follows:

$$RRMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{h_{True} - h_{estimated}}{h_{True}}\right)^2} \quad (7)$$

where n indicates the total number of filtering during the whole process (n = 300), h_{True} and $h_{estimated}$ are the true value of h and the estimated value at each time, respectively. Figure 9 (a) shows the dependence of *RRMSE* on x_{mold} for different x_{alloy}, while Figure 9 (b) represents the dependence on x_{alloy} for different x_{mold}. As is similar to the case of constant h discussed in the previous sub-section, accurate estimation of time-dependent h can be achieved when either of x_{alloy} and x_{mold} is set close to 0.

It was shown that time-dependent h can be estimated with two measurement positions when one of these measurement positions was located in a vicinity of the alloy/mold boundary.

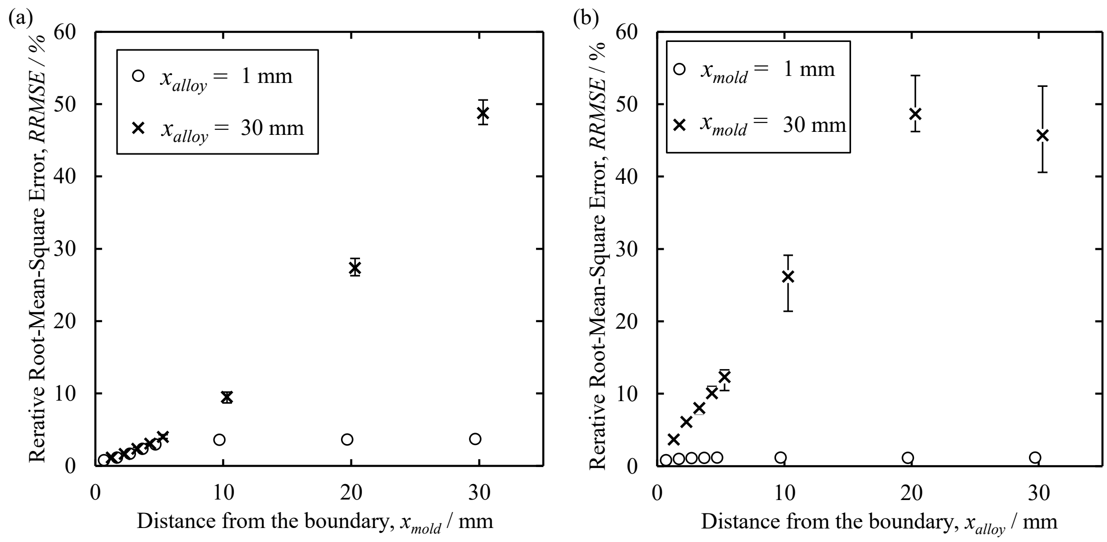

Figure 9 Dependence of RRMSE for time-dependent heat transfer coefficient on (a) x_{mold} for two different x_{alloy} and (b) x_{alloy} for x_{mold}.

3.2 Estimation of thermal conductivity

We carried out the estimation of the thermal conductivity k. The true value was set to 87 $Wm^{-1}K^{-1}$. The measured data from three measurement positions were utilized in this case, because the estimation was unsuccessful when the number of measurement positions is one and two. One measurement position was in the mold, and the other two were in the alloy. The distance between two measurement positions in the alloy was fixed to 5 mm. The standard deviations of observation noise and system noise were σ_T=1 K and σ_k=10 %, respectively, unless stated otherwise. The number of particles is 128. The initial values of k were given by random numbers generated in the range of 0 to 150 $Wm^{-1}K^{-1}$. The heat transfer coefficient h was set to 600 $Wm^{-2}K^{-1}$.

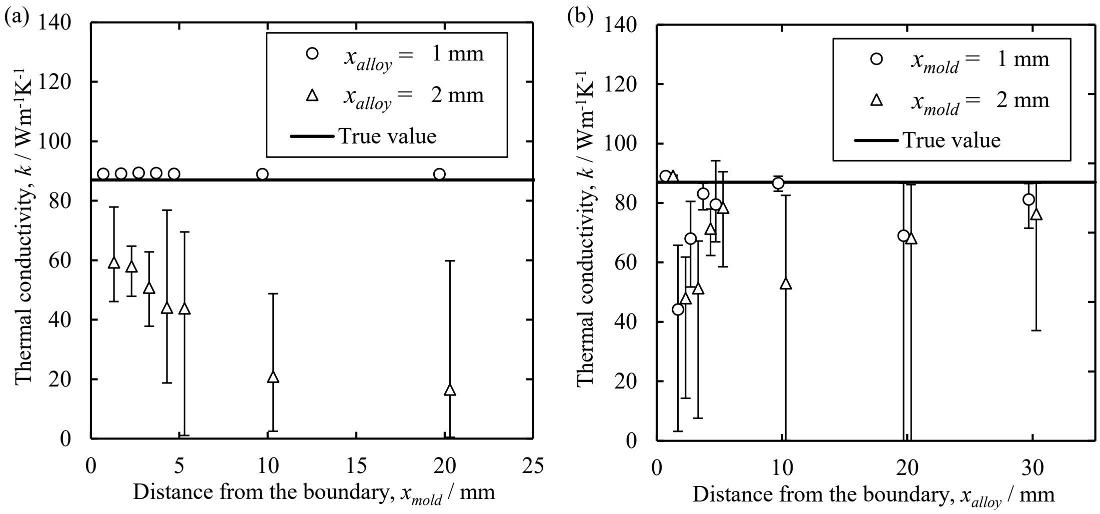

Figure 10 Estimation results of thermal conductivity. (a) Dependence on x_{mold} for two different values of x_{alloy}. (b) dependence on x_{alloy} for two different values of x_{mold}. Here, x_{alloy} indicates the measurement position in the alloy closer to alloy/mold boundary.

Figures 10 (a) and 10 (b) shows the relation between the measurement positions and the estimation accuracy. Here, x_{alloy} indicates the measurement position in the alloy closer to the alloy/mold boundary. From both figures, it is understood that k can be estimated with high accuracy when x_{alloy} = 1 mm. Figure 11 (a) shows the effect of x_{mold} on the estimation accuracy for different values of σ_T. x_{alloy} was set to 1 mm. The estimation accuracy does not substantially depend on σ_T and x_{mold} in Fig. 11 (a). On the other hand, Figure 11 (b) demonstrates that σ_k must be lower than 20% for accurate estimation of k. Figures 12 (a) and (b) shows the estimation results when the true value of k are 44 $Wm^{-1}K^{-1}$ and 130 $Wm^{-1}K^{-1}$, respectively. x_{alloy} was set to 1 mm. The former case can be accurately estimated for any value of x_{mold}, while the latter case cannot be estimated with high accuracy in this estimation condition. Here, we examined effect of σ_T on the estimation accuracy when the true value of k is 130 $Wm^{-1}K^{-1}$. The result is shown in Figure 13. It is seen that k can be accurately estimated only when σ_T = 0.1 K. When k is large, the value of k does not significantly change the cooling curves in the present casting process. Therefore, no substantial difference appears in the temperature distribution of particles with different value of k and thereby in the likelihood of particles, which makes it difficult to estimate the true value of k. Although not shown here, it was found that the accurate estimation of k needs small observation noise, i.e., σ_T = 0.1 K, when the true value of k is larger than 100 $Wm^{-2}K^{-1}$.

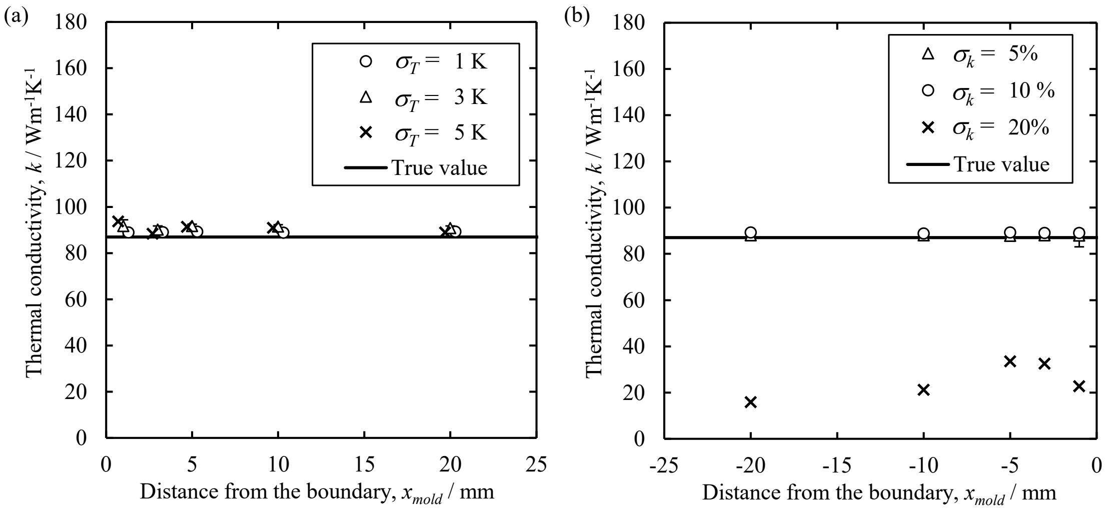

Figure 11 (a) Dependence of the estimation accuracy of k on x_{mold} with σ_k = 10% for σ_T = 1, 3 and 5 K. (b) Dependence of the estimation accuracy of k on x_{mold} with σ_T = 1 K for σ_k = 5, 10 and 20 %.

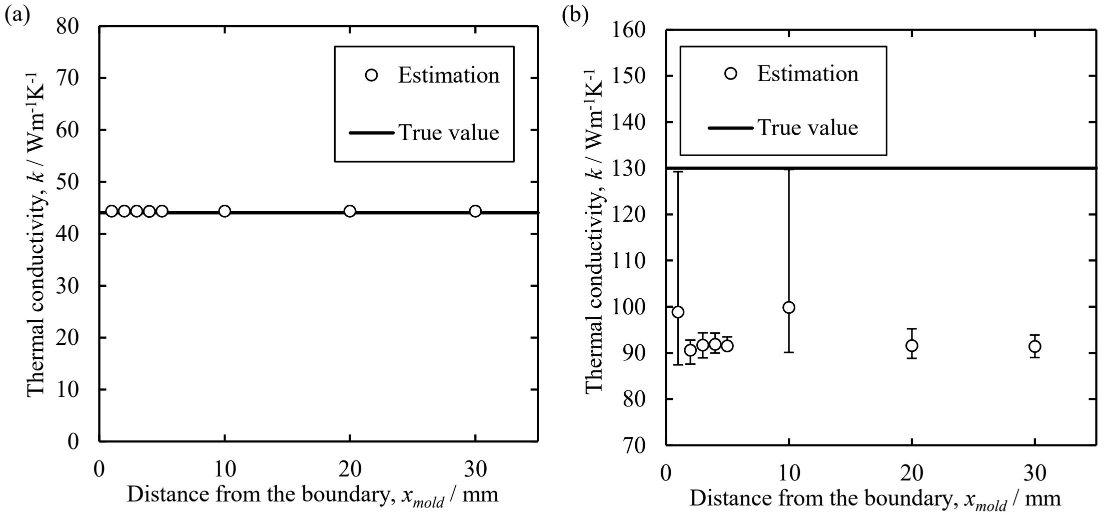

Figure 12 Estimation results of thermal conductivity when its true value was (a) 47 $Wm^{-1}K^{-1}$ and (b) 130 $Wm^{-1}K^{-1}$. x_{alloy} is 1 mm. σ_T=1 K and σ_k=10 %.

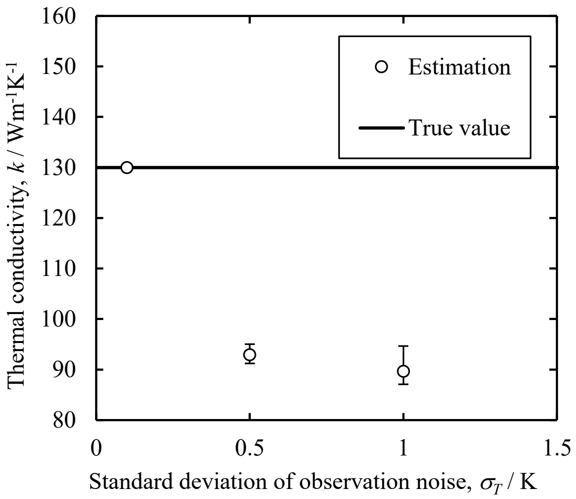

Figure 13 Relation between estimation accuracy and σ_T when the true value was 130 Wm^{-1}K^{-1}.

3. 3 Simultaneous estimation of thermal conductivity and time-dependent heat transfer coefficient

In simultaneous estimation of k and time-dependent h, the true value of k and time-dependent h were set to 87 Wm^{-1}K^{-1} and $h(t)$=3000$t^{-0.5}$ Wm^{-2}K^{-1}, respectively. The estimation results of k was determined by averaging the values estimated at the time from 15 to 30 seconds. The estimation error of h was evaluated by *RRMSE* described by eq. (7). The standard deviation of the observation noise was set to σ_T=1 K, while the standard deviations of the system noise were set to σ_k =10 % and σ_h =20 %. The number of particles was set to 128. The initial values of k and h in particles were given by random numbers generated in a range from 0 to 150 Wm^{-1}K^{-1} and from 0 to 6000 Wm^{-2}K^{-1}, respectively. Four measurement positions were utilized. Two positions were located in the alloy at 1 mm and 6 mm away from the alloy/mold boundary. The other two measurement positions were located in the mold, and the distance between these measurement positions was fixed to 5 mm. Figures 14 (a) and (b) shows the relation between the distance of the measurement position in the mold closer to the alloy/mold boundary x_{mold} and the estimation accuracy of h and k, respectively. It is first important to point out that both k and time-dependent h can be simultaneously estimated with high accuracy for the value of x_{mold} investigated here. In addition, the accuracy is improved when x_{mold} is closer to 0.

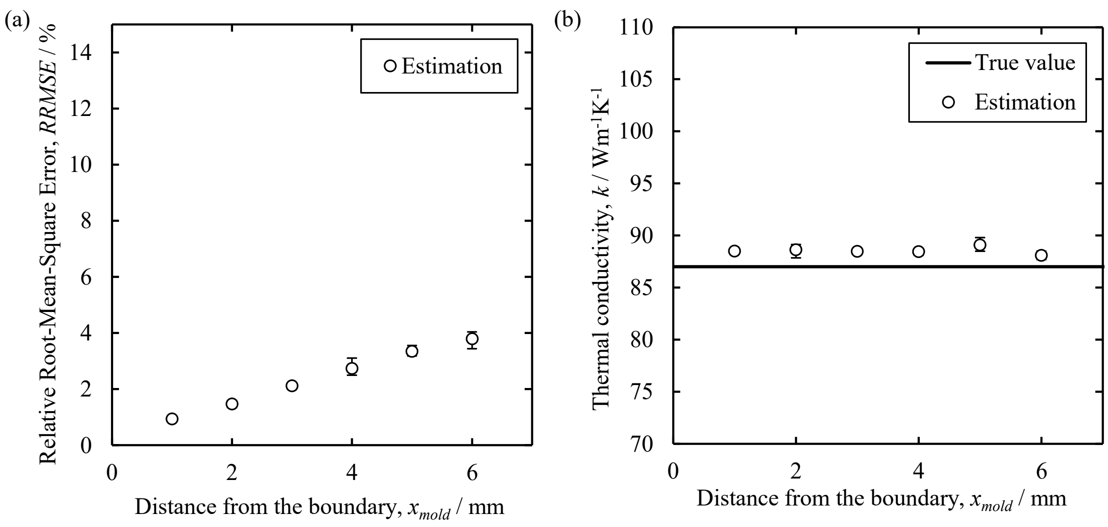

Figure 14 (a) Estimation error of heat transfer coefficient and (b) estimated value of thermal conductivity in simultaneous estimation.

As described above, the present investigation has demonstrated EnKF is very effective in estimating the h and k by using the cooling curves in the casting process. Here it would be worth comparing effectiveness of EnKF and particle filters for the parameter estimation in casting process. The number of measurement positions utilized to the estimation based on EnKF is many more than particle filter. For example, h and k were estimated simultaneously with two measurement positions when particle filter is utilized, while four measurement positions when EnKF was utilized. However, the number of particles necessary for simultaneous estimation based on EnKF is less than particle filter. 128 particles are utilized for the individual estimation of h and k based on both EnKF and particle filter. On the other hand, in simultaneous estimation, h and k can be estimated with 128 particles based on EnKF, while 128^2 particles based on particle filter.

4. Conclusions

In this study, EnKF was applied to the parameter estimation in heat conduction problem during casting process. EnKF was applied to (i) the individual estimation of k and h, (ii) estimation of time-dependent h, (iii) simultaneous estimation of k and time-dependent h, and the effectiveness of EnKF was investigated. The constant and time-dependent h was estimated with high accuracy with measured data from two measurement positions when one of these measurement positions was located in the vicinity of the mold/alloy boundary. k was estimated with high accuracy with measured data from three measurement positions when one of measurement positions in the alloy was located in the vicinity of the mold/alloy boundary. Moreover, k and time-dependent h were estimated simultaneously with high accuracy with measured data from four measurement positions based on EnKF. In summary, it was shown that EnKF is an effective method for estimating the important parameters in heat conduction problem during the casting process.

In this study, we have focused on a simple system, i.e., one-dimensional system for casting of Al-Cu alloy. It is expected that the present method can be applied to three-dimensional simulations for more complex casting of various alloys such as continuous casting processes of steels and Al alloys and a variety of ingot casting processes as long as the temperature measurement in mold and/or alloy is possible. It is very important to investigate the applicability of the present method in a future work.

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