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DEPARTMENT OF ECONOMICS

Essays on Fiscal Policy and the Macroeconomy

Lukas Mayr

Thesis submitted for assessment with a view to obtaining the degree of Doctor of Economics of the European University Institute

Florence, 14 June 2018

European University Institute
Department of Economics

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Abstract

The three chapters of this thesis provide a theoretical and quantitative analysis of three elements of the tax-transfer system which recently received increased attention in both the academic and the public policy debate in the United States and Europe.

In the first chapter, I show theoretically that optimal taxes on business owners are not entirely characterized by the usual, well established, equity-efficiency trade-off but that additional "trickle down" effects reduce optimal tax rates. As taxes on business income reduce investment, the demand for labor declines, which results in lower wages. I show quantitatively, on the basis of a dynamic general equilibrium model calibrated to US data, that these effects substantially reduce the optimal progressivity of the income tax code.

In the second chapter, joint with Fabian Kindermann and Dominik Sachs, we argue that the taxation of bequests can have a positive impact on the labor supply of heirs through wealth effects. On the basis of a life-cycle model calibrated to the German economy we show that for each Euro of bequest tax revenue that the government mechanically generates, it can expect another 7.6 Cent through higher labor income taxes of heirs. We show theoretically and quantitatively that a proper modeling of - empirically hard to identify - anticipation effects is crucial to obtain this result.

Finally, in the third chapter, joint with Árpád Ábrahám, João Brogueira de Sousa and Ramon Marimon, we assess the benefits of a potential European Unemployment Insurance System using a multi-country dynamic general equilibrium model with search frictions. In spite of substantial heterogeneity of labor market institutions across Europe, we find that a harmonized benefit system with a low replacement rate but an unlimited duration of eligibility is welfare improving in all countries as long as country specific contribution payments eliminate persistent transfers.

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In the third year of my PhD, Dominik joined the department, which proved as a door opener to the field of Public Economics, a field I had little knowledge about before. He helped me to explore an understudied area in the intersection of Macroeconomics and Public Economics, an area all the three papers of this thesis belong to. He taught me how to communicate my research more effectively. His guidance sometimes reached levels that made me feel as if my Job Market paper was as much his as it was mine. He helped me to make connections I didn't see and pushed me to levels I didn't think I could reach. I could not have had a better couple as advisors than Árpád and Dominik. I hope very much that I will keep on working together with them in the future.

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Contents

Chapter 1

Taxing Entrepreneurial Income in the Presence of Trickle Down Effects

1.1 Introduction

Entrepreneurship is one of the main drivers of inequality and contributes substantially to the high concentration of wealth in the developed world. For example, only about 10% of the US population are active business owners but these individuals own more than 40% of total US wealth.[1] Among the richest percentile of Americans about two thirds are entrepreneurs, between six and seven times their share in the overall population.[2]

The taxation of these individuals is a reoccurring theme in the political debate. Proponents of redistributive policies argue that the most well off in a society, many of whom are business owners, should contribute more to the financing of public expenditures and the welfare state. Opponents, on the other hand, stress the distortionary effects of these policies on investment and labor demand. It is often argued that high tax rates on entrepreneurs "trickle down" to the working poor, to whom part of the tax burden is passed on through lower wages.

Indeed recent empirical research on the incidence of corporate taxation finds significant negative effects of these taxes on wages. Exploiting regional and time variation in local business taxes in the United States, Suarez Serrato and Zidar (2016) estimate that workers bear about one third of the tax incidence. With a similar approach and German data Fuest et al. (2017) find that even half of the tax burden is passed onto workers. Yet, from a normative point of view it remains an open question how redistributive a tax system should optimally be in light of these "trickle down" effects. The goal of this paper is to provide an answer.

My first result is a theoretical one. In a general equilibrium setting where entrepreneurs produce output using their invested capital and hired labor as inputs I derive an intuitive formula that expresses the optimal income tax rate as a function of estimable sufficient statistics. I show that the classical linear income tax formula,[3] which solves for the

[1] Data from the Survey of Consumer Finances (SCF) 2013. Active business owner is defined as an individual who owns a private business and has an active management interest in this business. Throughout this paper we use the terms "active business owners" and "entrepreneurs" interchangeably.

[2] Similar figures are reported in Cagetti and De Nardi (2006) and Quadrini (2000) based on earlier waves of the SCF.

[3] The derivation of this formula goes back to Sheshinski (1972).

equity-efficiency trade-off, extends by a "trickle down" term.[4] This term is the product of (i) the elasticity of the wage with respect to the aggregate capital stock and (ii) the ratio of labor to capital income. The reason for this result is that an increase in the tax rate reduces investment. When capital and labor are complements this reduces the productivity of labor and therefore wages. In the presence of such a general equilibrium effect, the government does not want to tax income too strongly. Simple back-of-the-envelope calculations suggest that this effect might reduce the optimal tax rate by more than one half.

I then move on to a full-blown quantitative model that successfully matches relevant patterns of US data. In this framework, I compute the optimal one-time tax reform within a restricted class of income tax functions, the "constant rate of progressivity" tax schedule of Heathcote et al. (2017). I find that the optimal tax code is much more progressive than the status quo. I quantify the impact of trickle down effects in two different ways. First, I compute the optimal tax reform from the perspective of a naive policy maker, who wrongly assumes that prices are fixed at the benchmark level. The tax schedule obtained in this way is even more progressive with high incomes taxed at much higher rates. For example, earnings of 100,000 USD would be taxed at an average rate of 37%, almost twice the optimal rate in the general equilibrium economy. Second, I decompose the welfare gains of the optimal reform into a partial and a general equilibrium component. I find that general equilibrium effects reduce the overall gain by about 2% CEV. The reduction in investment induced by the more progressive tax schedule increases the productivity of capital and decreases the productivity of labor causing redistribution in the "wrong" direction from poor households with mostly wage income to rich ones with mostly capital income.

The policy reform induces an economic transition to a new stationary equilibrium, which I explicitly take into account in the welfare analysis. This turns out to be quantitatively important. A planner who maximizes steady state welfare only, would choose a less progressive system than the fully optimal one. For example, earnings of 100,000 USD would be taxed at an average rate of 10%, about the same rate as in the status quo, while with the fully optimal reform this rate would double to about 20%. The reason for this finding is that such a planner does not take into account that a more progressive tax system causes a gradual decummulation of the capital stock, which is associated with higher average consumption in the short run.

In my framework the market structure is incomplete and the laissez-faire equilibrium is *constrained inefficient*. Following Davila et al. (2012), I show that a constrained planner, whose only power is to dictate investment but who is prevented from direct redistribution, would choose a higher capital stock than the one in the laissez-faire equilibrium. This increases the marginal productivity of labor and therefore *indirectly* insures and redistributes through general equilibrium effects on wages. In theory, this form of second best allocation could be decentralized via subsidies on entrepreneurial investment financed by individualized lump-sum taxes, which ensure that the net transfer between the government and any agent is zero. In practice, however, such a policy is difficult to implement. Even if it was implementable it would not be desirable from the perspective of a policy maker with equity concerns. Tax and transfer systems of any modern welfare state are designed to *directly* insure and redistribute. A lesson from this exercise, though, is that

[4]In this set up the tax code is progressive in the sense that average tax payments (accounting for the transfers from the government) are decreasing in income.

subsidizing entrepreneurial investment has an important advantage over subsidizing entrepreneurial income. While both instruments encourage investment, the latter has less desirable insurance properties as it benefits lucky agents more than unlucky ones.

Motivated by this result I then extend the set of policy instruments for the Ramsey planner by linear investment subsidies. I derive an analytical formula for the optimal linear subsidy as a function of the income tax rate. This formula balances a trade-off. On the one hand subsidies alleviate tax distortions and by encouraging investment lead to positive "trickle down" effects. On the other hand subsidizing investment redistributes in the "wrong direction" from poor to rich households with higher levels of investment. I show that the inclusion of investment subsidies allows for a higher optimal income tax rate but this increase in the tax rate is bounded by the degree of initial wealth inequality. Introducing this policy instrument also in the quantitative framework I find that for each dollar an entrepreneur invests in his firm the government should optimally contribute 10 cent. Further, the associated increase in investment results in an even more progressive optimal income tax schedule.

This paper contributes to two strands of the literature. First, it contributes to the theoretical Public Finance literature on optimal income taxation. Specifically, I show that the optimal linear income tax rate depends negatively not only on a *behavioral* elasticity, here the elasticity of investment with respect to the net-of-tax rate, but also on the elasticity of the equilibrium wage with respect to the capital stock, a *price* elasticity.

The inverse relation between optimal tax rates and behavioral elasticities is well established in the literature on optimal taxation. This is particularly true for traditional Ramsey models that constrain tax instruments to be linear. Keeping the restriction on linear instruments but otherwise already adopting all model assumptions of Mirrlees (1971), Sheshinski (1972) establishes this relationship too. The seminal paper of Saez (2001) further shows how fully optimal, non-linear, tax schedules can be expressed as functions of these behavioral elasticities.

However, only recently this literature attributes increased importance to the fact that tax policy has welfare consequences also indirectly, through general equilibrium effects on prices.[5] Sachs et al. (2017) consider a Mirrleesian economy, in which output is produced by a continuum of workers with heterogeneous skills. These skills are complementary and thus the labor supply response to a tax change of one agent affects the wages of all other individuals as well. Similarly, and more closely related to the present paper, Scheuer (2014) characterizes certain properties of the set of Pareto optimal tax systems in a Mirrleesian economy with occupational choice, where output is produced by the joint effort of entrepreneurs and workers. He shows that when firm profits and labor income are subject to the same income tax schedule, a restriction that is employed also here, the planner exploits general equilibrium effects on wages in order to indirectly redistribute across occupations. My approach differs as I restrict the set of tax instruments in the tradition of Ramsey, which allows me to derive an explicit expression in terms of estimable sufficient statistics for the optimal tax rate.

Second, I contribute to the quantitative dynamic optimal taxation literature. For a quantitative policy analysis to be credible it is crucial for the model to match relevant features of the data. My framework therefore builds on Quadrini (2000) and Cagetti and De Nardi (2006), who, by introducing an entrepreneurial sector are able to generate the high concentration of wealth observed in the data. As in their model, some households in

[5] A notable early contribution on this topic is Stiglitz (1982).

my framework become rich because they obtain higher returns on their wealth by investing it in their business. Thus not only does the model a good job in replicating the wealth distribution, it also generates the observed over-representation of entrepreneurs among the rich.

Further, I introduce idiosyncratic investment risk as in Angeletos (2007).[6] The presence of such risks is widely documented in the empirical literature and seems to be a relevant factor for entrepreneurial investment decision making. For example, Moskowitz and Vissing-Jorgenson (2002) find that about 75 percent of all private equity is owned by households for whom it constitutes at least half of their total net worth and that households with entrepreneurial equity invest on average more than 70 percent of their private holdings in a single private company for which they have an active management interest. It has further been shown that these risks are one of the main drivers of wealth inequality (Piketty (2014), Benhabib et al. (2011)).[7]

Several papers analyze the effect of tax policy reform in this or a similar type of framework. For instance Cagetti and De Nardi (2009) analyze the effect of an elimination of estate taxes on wealth accumulation and welfare and Kitao (2008) analyzes the effects of capital taxation on investment. Yet, none of these papers calculates the optimal income tax code when firm profits and labor income are subject to the same income tax schedule.[8] This, however, is a sensible restriction. Not only because it is the status quo in the United States, where profits of privately owned businesses are in general taxed as part of the personal income of their respective owners.[9] But also because a tax code that distinguishes between sources of income is prone to tax shifting. This is especially the case for private businesses, the owners of which are typically employed in their own firm and thus able to declare a composition of wage and profit income that minimizes tax payments.[10]

Outline. The remainder of this paper is organized as follows. Section 1.2 introduces the two period model, derives the constrained efficient allocation and compares it to the laissez-faire equilibrium. Section 1.3 derives the optimal Ramsey plan. First, I consider the case where the only policy instruments are linear income taxes and anonymous lump-sum transfers. I then extend the set of instruments by linear investment subsidies. In section 1.4, I calibrate a dynamic quantitative version of the model to US data. The optimal income tax reform is computed in section 1.5. I again consider first the case with progressive income taxes only. I quantify the impact of "trickle down" effects on

[6]Other papers along these lines are Quadrini (2000), Meh and Quadrini (2006), Covas (2006), Angeletos and Calvet (2006).

[7]Aside from entrepreneurship, the focus of this paper, it has been shown that these returns can result from different positions in agents' financial portfolios. For example, some households invest only in riskless bonds, while others also hold stocks, which - while being risky - deliver higher returns on average (Guvenen (2006), Guvenen (2009)).

[8]Papers that do study the optimal progressivity of the income tax code but in a framework without an entrepreneurial sector are Conesa and Krueger (2006), Conesa et al. (2009)) and Heathcote et al. (2017).

[9]To be precise this restriction holds only for so called "flow through" entities such as partnerships, sole proprietorships, limited liability corporations and S corporations. The exception are businesses with the legal form of a C corporation. Profits of these businesses are subject to double taxation, i.e. they are taxed first at the corporate level and again, once distributed to its shareholders, on the personal income levels. However, according to Gentry and Hubbard (2004) the fraction of entrepreneurs choosing this legal form is only about 14%.

[10]See Slemrod (1995) and Gordon and Slemrod (2000) for historical evidence on tax shifting between corporate and personal income.

the optimal policy and on welfare. I then introduce linear investment subsidies in this framework and again calculate the optimal reform. Finally, section 3.6 concludes.

1.2 Two Period Model

The main mechanisms can be explained by means of a simple two period model. There is a continuum of agents of measure one. Each agent is the manager of a firm but also a worker. Equivalently, one can think of an agent as a household consisting of managers and workers who pool their income. Initially, in period $t = 0$, agents are endowed with a certain amount of the only consumption good. I will use the terms endowments and assets interchangeably. Assets a are distributed over agents according to some distribution function $G(a)$. There is no production in the initial period. Agents consume part of their assets and invest the other part in their firm. In the second period, $t = 1$, the invested capital is used together with labor input for production and output is consumed.

Preferences. Agents value consumption in both periods. The expected lifetime utility of each agent is given by

$$u(c_0) + \beta \mathbb{E} u(c_1)$$

where $u(.)$ is increasing, concave and satisfies typical Inada conditions: $u'(c) > 0$, $u''(c) < 0$, $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$.

Technology. The output of each firm is produced according to a constant returns to scale neoclassical production function

$$y = F(\theta k, l),$$

where k and l are the production inputs capital and labor, and θ is an idiosyncratic productivity (managerial ability) shock. This should capture the fact that entrepreneurial investments are risky. Each agent supplies his unit labor endowment inelastically and thus receives a riskless wage. The production function features typical concavity assumptions. The marginal products of capital and labor are both positive, $F_k > 0$ and $F_l > 0$, but decreasing, $F_{kk} < 0$ and $F_{ll} < 0$. Importantly, in line with empirical evidence it is assumed that capital and labor are complements, $F_{kl} > 0$, implying that labor is more productive when the capital stock is high.

For now I assume that all agents are ex ante identical with respect to their ability, i.e. θ is distributed iid across agents. It can take on N different values $\theta \in \Theta = \{\bar{\theta}_1, \bar{\theta}_2, ..., \bar{\theta}_N\}$, where $\bar{\theta}_n < \bar{\theta}_{n+1}$. The capital stock k needs to be installed in order for the entrepreneur to learn her productivity while labor input l is chosen after learning θ.

Budget Constraints and Market Structure. In this simplest version of the model, agents only differ with respect to their initial wealth holdings. I therefore denote all idiosyncratic variables as functions of a. The budget constraint in the initial period for an agent with endowment a is given by

$$c_0(a) + k(a) = a,$$

the sum of consumption $c_0(a)$ and capital investment $k(a)$ equals assets a. In the second period, an agent with initial assets a, who experienced productivity shock θ consumes

$$c_1(a, \theta) = \pi(k(a), \theta) + w,$$

the sum of his firm profits $\pi(k(a), \theta)$, which depend on the investment in the first period and the realization of the shock, and his riskless wage income w. Markets are incomplete. In particular, agents cannot insure against adverse realizations of the idiosyncratic productivity shock θ. The labor market, however, clears competitively.

1.2.1 Laissez-Faire Economy

In this section I characterize the equilibrium in the laissez-faire economy. Each agent makes an investment decision and plans on hiring labor in the next period contingent on the realization of the shock. These decisions induce consumption levels in both periods. Formally, a household with initial wealth a solves

$$\max_{k(a)\geq 0, \{l(a,\theta)\}_{\theta\in\Theta}} \quad u(a - k(a)) + \beta \mathbb{E}_\theta \Big[u\big(\pi(k(a), l(a,\theta), \theta) + w\big) \Big] \tag{H1}$$
$$\text{s.t.} \qquad \pi(k(a), l(a,\theta), \theta) = F(\theta k(a), l(a,\theta)) - w l(a,\theta).$$

We now have all the ingredients to define a general equilibrium.

Definition 1. *A general equilibrium in the laissez-faire economy is defined by the wage w, a collection of individual investment decisions $\{k(a)\}$ and a collection of contingent plans on labor demand $\{\{l(a,\theta)\}_{\theta\in\Theta}\}$, such that given w the individual policies $\{k(a), \{l(a,\theta)\}_{\theta\in\Theta}\}$ solve problem* (H1) *and the labor market clears, $\int l(a,\theta) dG(a) = 1$.*

To characterize the equilibrium it is useful to define the aggregate capital stock as

$$K = \int k(a) dG(a).$$

I solve for the equilibrium recursively. In a first step I consider an arbitrary collection of investment decisions $\{k(a)\}$ and derive second period labor input, firm profits and wage as functions of these investment decisions.

Lemma 1. *Let $\{k(a)\}$ be an arbitrary collection of (non-negative) investment decisions. Optimizing behaviour in the second period implies that labor demand is given by*

$$l(k(a), \theta) = \frac{\theta}{\mathbb{E}[\theta]} \frac{k(a)}{K}, \tag{1}$$

individual firm profits are given by

$$\pi(a, \theta) = r^k \theta k(a), \tag{2}$$

where

$$r^k = F_k(\mathbb{E}[\theta] K, 1) \tag{3}$$

denotes the return on effective capital, and the equilibrium wage is given by

$$w = F_l(\mathbb{E}[\theta] K, 1). \tag{4}$$

Proof. See Appendix 1.A.1. □

In the second step I solve for agents' optimal investment decision in the initial period. Using Lemma 1 I can reduce problem (H1) to the one dimensional optimization problem

$$\begin{aligned} \max_{k(a)\geq 0}\ & u(c_0(a)) + \beta\mathbb{E}_\theta\Big[u\big(c_1(a,\theta)\big)\Big] \\ \text{s.t.} \quad & c_0(a) = a - k(a) \\ & c_1(a,\theta) = r^k\theta k(a) + w, \end{aligned} \tag{H2}$$

where the only choice variable is capital investment $k(a)$.

The necessary and sufficient first order condition of this problem is given by the Euler equation

$$u'(c_0(a)) = \beta\mathbb{E}_\theta\Big[u'(c_1(a,\theta))\theta\Big]r^k + \lambda(a), \tag{EE1}$$

where the Lagrange multiplier $\lambda(a) \geq 0$ is equal to zero whenever the non-negativity condition on capital investment is not binding. This conditions implicitly defines the equilibrium collection of investment decisions $\{k(a)\}$ and thus completes the characterization of the laissez-faire equilibrium.

1.2.2 Constrained Efficiency

The laissez-faire equilibrium is *constrained inefficient*. The concept of a constrained efficient allocation goes back to Diamond (1967). It is the solution to the problem of a planner, who must not overcome the frictions implied by missing markets. The planner is not allowed to *directly* redistribute across agents, neither before nor after the shocks realize. She only chooses investment on behalf of each agent, whose budget constraint she has to satisfy.

I show that the planner's allocation obtained in this way features a higher capital stock than the laissez-faire equilibrium. There are two reasons for this finding. First, the planner achieves a better allocation of risk because the combination of market incompleteness and complementarity of factor inputs induces a pecuniary externality. Agents do not internalize the effect of their savings choice on wages, while the planner does. A higher capital stock increases equilibrium wages and thus the riskless part of agents' income while reducing firm profits and thus the risky part. Second, this change in the wage results in indirect redistribution from rich agents with mostly capital income to poor agents with mostly labor income. Closely related is the paper of Davila et al. (2012), who study constraint efficieny in a similar framework as mine with the main difference that that labor- instead of capital income is risky.[11]

Throughout this section I impose the following assumption on technology.

Assumption 1. *The production function is given by the constant elasticity of substitution (CES) function*

$$F(K,L) = \Big(\alpha K^\psi + (1-\alpha)L^\psi\Big)^{\frac{1}{\psi}}$$

where $\alpha \in (0,1)$ denotes the capital share and $\psi \in (-\infty, 1)$.

[11]The more general framework of Gottardi et al. (2016) captures both cases. The authors analyze welfare effects of price changes locally around the laissez-faire equilibrium but do not characterize the constrained efficient allocation as here or in Davila et al. (2012).

Ex Ante Identical Agents

I first investigate the case in which all agents have the same initial wealth $\bar{a}$ and only differ in their idiosyncratic returns to capital. In this case we can set $k(\bar{a}) = K$. The constrained planner solves

$$\begin{aligned} \max_{K \geq 0} u(c_0) + \beta \mathbb{E}_\theta \Big[u\big(c_1(\theta)\big) \Big] \\ \text{s.t.} \qquad c_0 =& \bar{a} - K \\ c_1(\theta) =& r^k \theta K + w, \end{aligned} \tag{P1}$$

where r^k and w are given by, respectively, (3) and (4).

It is useful to define the marginal change of the wage induced by an increase in the capital stock as

$$\frac{\partial w}{\partial K} = F_{kl}(\mathbb{E}[\theta]K, 1)\mathbb{E}[\theta] > 0, \tag{5}$$

and the marginal change of the average return on capital by

$$\frac{\partial r^k}{\partial K} = F_{kk}(\mathbb{E}[\theta]K, 1)\mathbb{E}[\theta] < 0. \tag{6}$$

Euler's homogeneous function theorem tells us that

$$F_{kk}(\mathbb{E}[\theta]K, 1)\mathbb{E}[\theta]K + F_{kl}(\mathbb{E}[\theta]K, 1) = 0. \tag{7}$$

We therefore can write the marginal change in the average return on capital as

$$\frac{\partial r^k}{\partial K} = -\frac{1}{\mathbb{E}[\theta]K}\frac{\partial w}{\partial K} < 0, \tag{8}$$

allowing us to express the first order condition that characterizes the constrained efficient allocation as function of the wage change only.

Proposition 1. *With ex-ante identical agents the constrained efficient capital stock K is implicitly given by the Euler equation*

$$u'(c_0) = \beta \mathbb{E}_\theta \Big[u'\big(c_1(\theta)\big)\theta \Big] r^k + \mu, \tag{EE2}$$

where

$$\mu = \beta \frac{\partial w}{\partial K} \mathbb{E}_\theta \left[u'\big(c_1(\theta)\big) \left(1 - \frac{\theta}{\mathbb{E}[\theta]}\right) \right] > 0. \tag{9}$$

K is higher than the capital stock in the laissez-faire equilibrium. Further, the consumption allocation in the constrained optimum strictly Pareto dominates the one obtained in the laissez faire equilibrium.

Proof. See Appendix 1.A.1. □

The combination of market incompleteness and complementarity of factor inputs induces a *pecuniary externality.* Agents do not internalize the effect of their savings decision on equilibrium prices. The term μ can be interpreted as the marginal social benefit of increasing the capital stock. It is positive because labor and capital are complements ($F_{kl} > 0$) and marginal utility $u'(c_1(\theta))$ is decreasing in θ. Intuitively, a higher capital stock increases the marginal product of labor (hence wages) but reduces the marginal product of effective capital (hence returns on capital). As the former income is riskless while the latter is risky this has desirable insurance properties.

Ex Ante Heterogeneous Agents

I now move to the general case where agents differ with respect to their initial wealth holdings and the planner is choosing $k(a)$ for each initial wealth endowment a. The constrained utilitarian planner's problem is given by

$$\max_{\{k(a)\geq 0\}} \int \left(u(c_0(a)) + \beta \mathbb{E}_\theta \left[u(c_1(a,\theta)) \right] \right) dG(a) \tag{P2}$$

$$\text{s.t.} \qquad c_0 = a - k(a)$$

$$c_1(a,\theta) = r^k \theta k(a) + w,$$

where r^k and w are again given by, respectively, (3) and (4).

Proposition 2. *In the constrained optimum the collection of capital investments* $\{k(a)\}$ *is implicitly given by the Euler equations*

$$u'(c_0(a)) = \beta \mathbb{E}_\theta \left[u'\big(c_1(a,\theta)\big)\theta \right] r^k + \lambda(a) + \mu \qquad \forall a, \tag{EE3}$$

where $\lambda(a)$ *is the Lagrange multiplier on the non-negativity condition for capital investment and*

$$\mu = \beta \frac{\partial w}{\partial K} \int \mathbb{E}_\theta \left[u'\big(c_1(a,\theta)\big) \left(1 - \frac{\theta}{\mathbb{E}[\theta]} \frac{k(a)}{K}\right) \right] dG(a) > 0. \tag{10}$$

Proof. See Appendix 1.A.1. □

Following Gottardi et al. (2016) I decompose the integrand of μ into a set of insurance components $I(a)$ and redistribution components $D(a)$,

$$\int \mathbb{E}_\theta \left[u'\big(c_1(a,\theta)\big) \left(1 - \frac{\theta}{\mathbb{E}[\theta]} \frac{k(a)}{K}\right) \right] dG(a) =$$

$$\int \underbrace{\mathbb{E}_\theta \left[u'\big(c_1(a,\theta)\big) \right] \left(1 - \frac{k(a)}{K}\right)}_{D(a)} dG(a) + \int \frac{k(a)}{K} \underbrace{\mathbb{E}_\theta \left[u'\big(c_1(a,\theta)\big) \left(1 - \frac{\theta}{\mathbb{E}[\theta]}\right) \right]}_{I(a)} dG(a).$$

The whole term can then be written as

$$\mu = \beta \frac{\partial w}{\partial K} \int \left[D(a) + \frac{k(a)}{K} I(a) \right] dG(a). \tag{11}$$

It is easy to see that if agents are ex-ante identical the redistribution component is zero. With initial wealth heterogeneity the term $I(a)$ is positive for all a, while the sign of $D(a)$ switches from positive to negative exactly at that a, where $k(a) = K$. Hence it is positive for agents with less than average capital investment, $k(a) < K$.

Contrary to the case with ex-ante identical agents, here the constrained efficient allocation does not Pareto dominate the laissez-faire allocation. Richer agents are better off in the laissez-fare allocation. Consider for simplicity the situation without production risk, i.e. $\theta = \mathbb{E}[\theta]$. In this case μ is given only by the distribution component,

$$\mu = \beta \frac{\partial w}{\partial K} \int \left(1 - \frac{k(a)}{K}\right) u'(c_1(a)) dG(a) > 0.$$

On average the term in round brackets is equal to zero. But since the marginal utility of consumption $c_1(a)$ is decreasing in a, while capital investment $k(a)$ is increasing in a the whole term is positive. Hence the utilitarian social planner would invest more in capital than agents in the laissez-faire allocation even when there is no idiosyncratic investment risk. An increase in the capital stock increases wages, the main income source of poor agents, and decreases capital returns, the main income source of rich agents. Hence, the planner *indirectly* redistributes through general equilibrium effects on wages. In Appendix 1.A.2 I show the comparison between the laissez-faire and the constrained efficient allocation graphically on the basis of a numerical example.

Comparison to the Case with Idiosyncratic Wage Risk. It is worth comparing these results with those in Davila et al. (2012), who performed a similar analysis with idiosyncratic labor income risk instead of investment risk. In this case two opposing forces determine whether there is over- or under-accumulation of capital. First, in the two period model with ex-ante identical agents a reduction of capital investment is optimal as it downscales risky wage income and it upscales the riskless return on capital. Instead, with no risk but heterogeneous initial wealth the same analysis as here goes through. Since consumption poor agents derive their income mainly from wages, the utilitarian planner would increase capital. In the multiple period version of their model with homogeneous initial assets, a dispersion of wealth endogenously emerges as a consequence of risky wage income and the two effects are counteracting each other. Which one dominates crucially depends on the calibration of the stochastic labor productivity process. Instead, in the present framework, both effects work in the same direction. Risky capital income as well as the fact that consumption poor agents obtain most of their income from labor call for an increase in wages, which is achieved by a higher capital stock.

Decentralization of the Constrained Optimum

It turns out that a tax and transfer system which decentralizes the constrained optimum is unlikely to be implementable in reality. Such a system must ensure that each individual's Euler equation is given by (EE3). In theory, this could be achieved via a set of wealth dependent subsidies on capital investment

$$s(a) = \frac{\mu}{\beta \mathbb{E}_\theta \left[u'\big(c_1(a, \theta)\big) \right]} > 0, \tag{12}$$

with μ given by (10). In order to neutralize transfers between each agent and the government these subsidies would have to be financed via lump sum, but again wealth dependent, taxes

$$T(a) = s(a)k(a). \tag{13}$$

The partial confiscation of initial wealth raises concerns regarding the political feasibility already in this stylized two period model. Additionally, in a multi-period version of the model, wealth becomes endogenous as it depends on agents' investment decisions. In such a setting, the government cannot implement the constrained efficient allocation by only conditioning its policy instruments on wealth. It would require both the subsidy and the lump sum transfer to depend on the full history of exogenous shocks.[12] Such a requirement is informationally very demanding. This kind of implementation is therefore of only limited relevance for practical tax policy. For this reason I from now on restrict attention to simpler, more realistic, policy instruments which have the power to redistribute directly as in any modern welfare state.

There are two main takeaways from the present section, though. First, general equilibrium effects are relevant for welfare. Even in the extreme case considered here, where policy is by construction prevented from direct redistribution, it can improve welfare by affecting prices. Policies which redistribute directly need to take these effects into account. In the present framework a higher capital stock is desirable because it increases wages. A tax and transfer system that discourages investment reduces wages and a trade-off emerges.

Second, contrary to Davila et al. (2012), who implemented the constrained efficient allocation via a tax/subsidy on capital *income*, I subsidize capital *investment*. The reason is intuitive. While a subsidy on capital income also encourages investment it has less desirable insurance properties as it benefits lucky agents more than unlucky ones. Investment subsidies hence seem a powerful policy instrument in the present framework.

1.3 Ramsey Problem

In this section I derive my main theoretical result. I show that in the presence of general equilibrium effects the classical linear income tax formula that solves for the equity-efficiency trade-off extends by a "trickle down" term. This term reduces the optimal tax rate and is the product of (i) the elasticity of the equilibrium wage with respect to the capital stock and (ii) the ratio of aggregate labor- to capital income. The more elastic the wage and the higher the fraction of labor income, the lower the optimal tax rate. I first derive this formula for a Ramsey planner who has access to linear income taxes and anonymous lump-sum transfers only. Inspired by the implementation of the constrained efficient allocation, I then augment the set of policy instruments by linear investment subsidies. In Appendix 1.A.3, I show that the formulas derived here by and large extend to an infinite horizon economy and the inclusion of a riskless asset.

1.3.1 Linear Income Taxes

The set of policy instruments is very simple. A linear tax τ on total income raises revenues which are distributed to the households via anonymous lump sum transfers T.

[12]See section 4.2 in Davila et al. (2012)

The Ramsey planner solves

$$\max_{\tau} \int \Big\{ u(c_0(a)) + \beta \mathbb{E}_\theta \big[u(c_1(a,\theta)) \big] \Big\} dG(a) \tag{R1}$$

$$\begin{aligned}
\text{s.t.} \qquad c_0(a) =& a - k(a) \\
c_1(a,\theta) =& (1-\tau)\big[r^k k(a)\theta + w \big] + T \\
u'(c_0(a)) =& \beta \mathbb{E}_\theta \Big[\big[(1-\tau) r^k \theta \big] u'(c_1(a,\theta)) \Big] \\
T =& \tau \big[\mathbb{E}[\theta] r^k K + w \big],
\end{aligned}$$

as well as (3) and (4). The first three constraints describe agents' optimization behavior given prices and policies. The fourth constraint guarantees that the government budget clears. Finally, the last two constraints define equilibrium prices.

Exogenous labor supply and riskless wage income imply that the problem is equivalent to the problem where only firm profits are taxed. Only the lump sum transfer would be reduced by τw in this case.

The first order condition with respect to τ is given by

$$\int \mathbb{E}_\theta \left[u'(c_1(a,\theta)) \frac{\partial c_1(a,\theta)}{\partial \tau} \right] dG(a) = 0, \tag{14}$$

where the partial derivatives entering these equations are

$$\frac{\partial c_1(a,\theta)}{\partial \tau} = \underbrace{\mathbb{E}[\theta] r^k K \left(1 - \frac{\theta k(a)}{\mathbb{E}[\theta] K} \right)}_{\text{Mechanical Tax (MT) Effect}} + \underbrace{\frac{\partial K}{\partial \tau} \tau \mathbb{E}[\theta] r^k}_{\text{Behavioral (BH) Effect}} + \underbrace{\frac{\partial w}{\partial K} \frac{\partial K}{\partial \tau} (1-\tau) \left(1 - \frac{k(a)\theta}{K \mathbb{E}[\theta]} \right)}_{\text{General Equilibrium (GE) Effect}}.$$

A marginal increase in τ has three effects on consumption of an agent with initial wealth a and productivity θ: First, and most straight forward, a marginal increase in the tax rate reduces net firm profits and increases lump sum transfers. Absent any change in behavior an increase in the tax rate by a marginal unit increases total revenue - which is rebated lump sum to all agents - by $\mathbb{E}[\theta] r^k K$ units. However, net capital income of an agent with initial assets a and productivity draw θ is reduced by $\theta r^k k(a)$ unit. In the spirit of Saez (2001), I call this effect the *mechanical tax effect*.

Second, a change in the tax rate influences the investment behavior of agents, which in turn affects taxable income and hence revenues. A marginal increase in the tax rate changes tax revenues by $(\partial K/\partial \tau)\tau \mathbb{E}[\theta] r^k$ units. This term is typically negative. Lower tax revenues go hand in hand with lower government transfers, reducing agents' consumption in the second period. Again following Saez (2001), I call this effect the *behavioral effect*.

Finally, the change in investment behavior influences equilibrium prices,

$$GE = \underbrace{(1-\tau)\frac{\partial w}{\partial \tau}}_{<0} + \underbrace{(1-\tau) k(a) \theta \frac{\partial r^k}{\partial \tau}}_{>0} + \underbrace{\tau \frac{\partial w}{\partial \tau} + \mathbb{E}[\theta] K \frac{\partial r^k}{\partial \tau}}_{?}.$$

Complementarity of capital and labor implies that $\partial w/\partial\tau$ and $\partial K/\partial\tau$ have the same (typically negative) sign. If an increase in the tax rate leads to a reduction in investment, it also reduces wages. The net labor income of agents is hence reduced by $(1-\tau)\partial w/\partial\tau$, the first term. But the reduction in K also increases capital returns by $\partial r^k/\partial\tau$. The second term denotes the increase in net capital income of an agent with investment $k(a)$ and productivity draw θ. Finally, the change in prices affects tax revenues, the last term. These revenues are rebated lump sum and hence affect all agents' consumption. Using Euler's homogeneous function theorem it can be expressed in the concise form above. I call this effect the *general equilibrium effect*.

Some Notation. In order to obtain an intuitive formula for the optimal tax rate it is useful to define a few objects. First, the elasticity of capital investment with respect to the net-of-tax rate is defined as

$$\epsilon_{K,1-\tau}(\tau) = \frac{\partial K(\tau)}{\partial(1-\tau)}\frac{1-\tau}{K(\tau)} > 0 \tag{15}$$

and the elasticity of the equilibrium wage with respect to aggregate capital as

$$\epsilon_{w,K}(K) = \frac{\partial w(K)}{\partial K}\frac{K}{w(K)} > 0. \tag{16}$$

Second, define the "composite redistribution term" as

$$D = \frac{1}{\int \mathbb{E}_\theta\big[u'(c_1(a,\theta))\big]dG(a)} \int \left(1 - \frac{k(a)}{K}\right)\mathbb{E}_\theta\big[u'(c_1(a,\theta))\big]dG(a) \geq 0. \tag{17}$$

It captures the variation in expected marginal utilities induced by the distribution of initial wealth. Note that the term in round brackets is zero on average. Moreover, both the term in round brackets and the expected marginal utility are decreasing is a. Hence the overall term is positive. The term D is increasing in the dispersion of initial wealth. If assets were initially equally distributed across agents the term would be zero.

Third, define the "composite insurance term" as

$$I = \frac{1}{\int \mathbb{E}_\theta\big[u'(c_1(a,\theta))\big]dG(a)} \int \frac{k(a)}{K}\mathbb{E}_\theta\left[u'(c_1(a,\theta))\left(1 - \frac{\theta}{\mathbb{E}[\theta]}\right)\right]dG(a) > 0. \tag{18}$$

This term captures the risk in the economy. It is a weighted average of individual insurance terms. Again note that the term in round brackets is zero in expectation. Moreover, both this term and marginal utility are decreasing in the realization θ. The term in square brackets hence captures the variation in marginal utilities for a particular agent a. A mean preserving spread in θ increases this term. These individual insurance terms are weighted by the degree the respected agents invest in capital.

Both, the redistribution and the insurance term are normalized by the average marginal utility in the second period. Further, note that

$$D + I = \frac{1}{\int \mathbb{E}_\theta\big[u'(c_1(a,\theta))\big]dG(a)} \int \mathbb{E}_\theta\left[u'(c_1(a,\theta))\left(1 - \frac{\theta}{\mathbb{E}[\theta]}\frac{k(a)}{K}\right)\right]dG(a) > 0.$$

The Optimal Tax Formula. Using these definitions and rearranging terms in (14) then gives an intuitive expression for the optimal linear income tax rate,

$$\frac{\tau}{1-\tau} = \big(I + D\big)\left(\frac{1}{\epsilon_{K,1-\tau}(\tau)} - \epsilon_{w,K}(K)\frac{w}{r^k\mathbb{E}[\theta]K}\right). \tag{19}$$

This formula directly affiliates to the traditional optimal income taxation literature. Let us assume for the moment that $\epsilon_{w,K} = 0$.[13] Then the formula simply balances an equity-efficiency trade-off. The more elastic agents react to changes in tax rates the lower the optimal tax rate, the efficiency concern. The more unequal income is distributed, the higher the optimal tax rate, the equity concern. Further, income can be unevenly distributed for two reasons, a high dispersion of initial wealth (summarized by D) that leads to a high dispersion of investment and therefore income, and high dispersion of idiosyncratic realizations of income shocks (summarized by I). As in Sheshinski (1972) or Saez (2001) the statistic that governs the optimal tax rate is the aggregate earnings elasticity with respect to the net-of-tax rate.[14] Here an increase in the tax rate reduces capital investment and hence the tax base. The benefit of distributing income more evenly comes at the cost of having less aggregate income to redistribute.

Let us now assume that $\epsilon_{w,K} > 0$. Then an additional force comes into play. Now a tax increase not only reduces taxable income, the reduction in investment also reduces wages, the main income source of poor households. The total "trickle down" effect is the product of (i) the wage elasticity with respect to the capital stock and (ii) the ratio of aggregate labor- to capital income. The more elastic the wage reacts to changes in investment and the higher aggregate labor- relative to capital income, the more important is this "trickle down" effect.

Back-of-the-Envelope Calculation. A simple back of the envelope calculation already provides an indication that this effect might be quantitatively important. Consider the standard parameterization of the neoclassical growth production function, in which the ratio of labor- to capital income is around two and the elasticity of wages with respect to capital is 1/3, implying that the second term is 2/3. In a recent paper Devereux et al. (2014) find that the elasticity of taxable income with respect to the corporate income tax rate is between 0.13 and 0.17 for companies with profits around 300,000 pounds and between 0.53 and 0.56 for companies with profits around 10,000 pounds. For simplicity let us take the somewhat medium value of 1/3. In my model this elasticity corresponds to

$$-\frac{\partial \ln \mathbb{E}[\theta] r^k K}{\partial \ln \tau} = \frac{1}{3}.$$

Again with the standard parameterization of the neoclassical production function one obtains $\partial \ln r^k / \partial \ln K = -2/3$, implying

$$-\frac{\partial \ln \mathbb{E}[\theta] r^k K}{\partial \ln \tau} = -\left(\frac{\partial \ln r^k}{\partial \ln K} + 1\right)\frac{\ln K}{\ln \tau} = -\frac{1}{3}\frac{\ln K}{\ln \tau} = \frac{1}{3}.$$

[13] This would be the case, for example, if $Y = \alpha\mathbb{E}[\theta]K + (1-\alpha)L$.

[14] To be precise it is the aggregate investment elasticity that shows up in the formula but since earnings depend positively on investment the statement is true.

Hence our estimate for the elasticity is

$$\epsilon_{K,1-\tau} = -\frac{\ln K}{\ln \tau} = 1.$$

Therefore,

$$\frac{1}{\epsilon_{K,1-\tau}(\tau)} - \epsilon_{w,K}(K)\frac{w}{\mathbb{E}[\theta]r^k K} \approx 1 - \frac{2}{3}$$

This implies that general equilibrium effects on wages influence the optimal tax rate substantially. If, for example, the optimal tax rate in partial equilibrium (without responses in wages) is 50%, it would be reduced by half to 25% when accounting for the reaction on wages. Moreover, the general equilibrium effect becomes relatively larger, the smaller the partial equilibrium tax. If, for example, the optimal partial equilibrium tax rate is only 25%, it would be reduced by more than half to 10%.

1.3.2 Investment Subsidies

For two reasons I now incorporate investment subsidies into the Ramsey problem: (i) they counteract the discouraging effect of income taxes on investment; (ii) these instruments were required in the decentralization of the constrained optimum in section 1.2.2. The Ramsey planner now solves

$$\max_{\tau,s} \int \left\{ u(c_0(a)) + \beta \mathbb{E}_\theta \big[u(c_1(a,\theta)) \big] \right\} dG(a) \tag{R2}$$

$$\begin{aligned} \text{s.t.} \qquad c_0(a) =& a - k(a) \\ c_1(a,\theta) =& (1-\tau)\big[r^k k(a)\theta + w\big] + sk(a) + T \\ u'(c_0(a)) =& \beta \mathbb{E}_\theta \Big[\big[(1-\tau)r^k\theta + s\big] u'(c_1(a,\theta)) \Big] \\ T =& \tau \big[\mathbb{E}[\theta] r^k K + w\big] - sK, \end{aligned}$$

as well as (3) and (4).

We obtain two first order conditions, one with respect to the tax rate τ (where we plugged in the envelope condition)

$$\int \mathbb{E}_\theta \left[u'(c_1(a,\theta)) \frac{\partial c_1(a,\theta)}{\partial \tau} \right] dG(a) = 0,$$

and one with respect to the subsidy s,

$$\int \mathbb{E}_\theta \left[u'(c_1(a,\theta)) \frac{\partial c_1(a,\theta)}{\partial s} \right] dG(a) = 0.$$

The partial derivative of $c_1(a,\theta)$ with respect to τ is given by

$$\frac{\partial c_1(a,\theta)}{\partial \tau} = \underbrace{\mathbb{E}[\theta]r^k K\left(1 - \frac{\theta k(a)}{\mathbb{E}[\theta]K}\right)}_{\text{Mechanical Tax Effect}} + \underbrace{\frac{\partial K}{\partial \tau}\left[\tau \mathbb{E}[\theta]r^k - s\right]}_{\text{Behavioral Effect}} + \underbrace{\frac{\partial w}{\partial K}\frac{\partial K}{\partial \tau}(1-\tau)\left(1 - \frac{k(a)\theta}{K\mathbb{E}[\theta]}\right)}_{\text{General Equilibrium Effect}}.$$

Given s, a marginal increase in τ has the same three effects as above. Only the behavioral effect is lower now. As before the reduction investment reduces taxable income and therefore tax revenue, but now this saves the government subsidy payments so that the overall effect on the government budget is ambiguous.

The partial derivative of $c_1(a,\theta)$ with respect to s is given by

$$\frac{\partial c_1(a,\theta)}{\partial s} = \underbrace{-K\left(1-\frac{k(a)}{K}\right)}_{\text{“Wrong Redistribution”}} + \underbrace{\frac{\partial K}{\partial s}\left[\tau\mathbb{E}[\theta]r^k - s\right]}_{\text{Behavioral Effect}} + \underbrace{\frac{\partial w}{\partial K}\frac{\partial K}{\partial s}(1-\tau)\left(1-\frac{k(a)\theta}{K\mathbb{E}[\theta]}\right)}_{\text{General Equilibrium Effect}}.$$

Given τ, a marginal increase in s has three effects: (i) it redistributes in the "wrong" direction, from poor to rich, as rich agents have higher investment and thus benefit more from the subsidy; (ii) it increases investment, hence tax revenues and subsidies paid by the government; and (iii) the increase in investment increases wages, which distributes in the "right" direction, from high to low earners.

Defining the elasticity of aggregate capital investment with respect to the investment subsidy as

$$\epsilon_{K,s} = \frac{\partial K}{\partial s}\frac{s}{K}.$$

and using the definitions (17) and (18) of the composite redistribution and insurance terms, we obtain a formula of the optimal subsidy s as a function of the tax rate τ,

$$s(\tau) = \frac{1}{1+\frac{D}{\epsilon_{K,s}}}\left[\tau\mathbb{E}[\theta]r^k + (1-\tau)\epsilon_{w,K}\frac{w}{K}\big(D+I\big)\right]. \tag{20}$$

How can we interpret this condition? Let us restrict attention to the case where $\tau > 0$ and let us first consider the case where $D = 0$, i.e. all agents are ex-ante identical. In this case the optimal subsidy is

$$s(\tau) = \tau\mathbb{E}[\theta]r^k + (1-\tau)\epsilon_{w,K}\frac{w}{K}I.$$

The first term neutralizes expected tax payments. If there was no risk, i.e. $I = 0$, this would guarantee that capital investment is not distorted. However, if investment is risky, it is beneficial to further increase the subsidy as a higher capital stock increases wages, the riskless part of agents income, and reduces capital returns, the risky part. Obviously the subsidy is increasing in risk (I) and in the wage elasticity $\epsilon_{w,K}$. Moreover, the less the planner uses taxes to redistribute from lucky to unlucky, i.e. the lower τ, the more she relies on this general equilibrium effect. Also note that $s(\tau) > \tau\mathbb{E}[\theta]r^k$ implies that with $D = 0$ the lump sum transfer is negative.

Let us now consider the case of initial wealth heterogeneity in which case $D > 0$. The term in square brackets changes only insofar as the general equilibrium effect gets bigger. The change in wages induced by a higher capital stock not only provides insurance, it also redistributes from agents who rely mostly on capital income to agents who rely mostly on labor income. However, an increase in the subsidy has also adverse redistribution effects. Rich agents with high investment benefit more from an increase in the subsidy than poor agents with little investment. Remember that anonymous lump sum transfers/taxes imply that all agents share the burden of financing entrepreneurial subsidization equally. The multiplicative term in front of the term in square brackets captures this effect. It reduces the subsidy for two reasons. First, with wealth heterogeneity it is not optimal to fully neutralize expected tax payments as positive expected income taxes are a way to redistribute from rich to poor. Second, it is also not optimal to fully exploit the general equilibrium effect. The indirect redistribution and insurance through wages now comes at the cost of direct redistribution in the "wrong direction". The more inelastic capital investment is, i.e. the lower $\epsilon_{K,s}$, the lower the subsidy. In the limit, i.e. as $\epsilon_{K,s} \to 0$ the subsidy is zero. If capital investment was not affected by the subsidy, insurance and redistribution (directly through taxes and indirectly through wages) are achieved without incurring redistribution in the "wrong" direction by subsidizing rich entrepreneurs.

Similarly we can obtain a formula for the optimal tax rate given s using the first order condition with respect to τ:

$$\frac{\tau(s)}{1-\tau(s)} = (D+I)\left(\frac{1}{\epsilon_{K,1-\tau}} - \epsilon_{w,K}\frac{w}{\mathbb{E}[\theta]r^k K}\right) + \frac{s}{(1-\tau(s))\mathbb{E}[\theta]r^k}$$

This formula differs from the one obtained in the Ramsey problem without subsidies only with respect to the last term. A positive subsidy increases the optimal tax rate. This is natural as a subsidy counteracts the discouragement in investment from taxation. Combining the results yields the optimal tax formula,

$$\frac{\tau}{1-\tau} = (D+I)\left[\frac{1}{\epsilon_{K,1-\tau}}\left(1+\frac{\epsilon_{K,s}}{D}\right) - \epsilon_{w,K}\frac{w}{r^k\mathbb{E}[\theta]K}\right]. \tag{21}$$

Note that with ex-ante identical agents the first best allocation is achieved with linear capital income taxes and linear investment subsidies. The optimal tax rate in this case is $\tau = 1$ and the optimal subsidy is $s = \mathbb{E}[\theta]r^k$.

1.4 Quantitative Dynamic Model

In this section I describe the quantitative model. The two most important additional ingredients are a corporate sector and a riskless asset. In reality not all production takes place in private businesses and therefore not all investment is subject to the same kind of risk. In line with Quadrini (2000) or Cagetti and De Nardi (2006) I therefore consider a framework in which a corporate sector and an entrepreneurial sector coexist. Further, in reality agents cannot only save by investing in their own firm. In practice there is a variety of financial assets that households can use to transfer ressources over time. In the model I include the most common one, a riskless bond. I assume that these bonds are issued by corporates whose production is riskless or - equivalently - who are able to perfectly diversify risk.

1.4.1 Model Set Up

Demografics and Preferences. Agents are infinitiely lived and derive utility from consumption only. Preferences are time separable with isoelastic instantaneous utility

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of relativ risk aversion.

Technology-Corporate Sector. Production takes place in two sectors, a conventional corporate sector as in the neoclassical growth model or the incomplete markets model with idiosyncratic labor risk of Aiyagari (1994) and an entrepreneurial sector similar to the one in the two period model above. Output in the corporate sector is produced according to the Cobb-Douglas production function

$$Y_c = K_c^{\alpha_c} L_c^{1-\alpha_c},$$

where K_c and L_c denote the capital, respectively labor, employed in the corporate sector and α_c is the capital share in this sector.

Technology-Entrepreneurial Sector. The technology operated by entrepreneurs is now given by

$$F(\theta, k, l) = \left((\theta k)^{\alpha} l^{1-\alpha}\right)^{\nu}, \tag{22}$$

where $\alpha, \nu \in (0, 1)$. Contrary to the two-period model above it exhibits decreasing returns to scale. I choose this specification for two reasons. The first reason is methodological. As discussed in Angeletos (2007) when entrepreneurs' production function exhibits constant returns to scale, there is ever increasing wealth inequality, i.e. a stationary distribution does not exist. With decreasing returns to scale, however, there is an optimal size of the firm beyond which the entrepreneur does not want to expand and a stationary wealth distribution exists. The second reason is that in this way I capture the fact that individual agents have limited capacity to oversee and manage production, i.e. individuals have limited "span of control" as in Lucas (1978).

As before I denote by θ an individual's entrepreneurial productivity, which can take on values from the finite set $\Theta = \{\bar{\theta}_1, ..., \bar{\theta}_{N_\theta}\}$ and follows a first order Markovian process with transition matrix P_θ. This is a parsimonous way to model two facts: (i) entrepreneurship is risky as in the two period version of the model; (ii) agents differ in their ability to run enterprises, i.e. more able entrepreneurs (with high current θ) draw future productivity θ from a more advantagous distribution.

Firm Profits and Optimal Labor Input. For a given capital investment k and a realization of the shock θ an entrepreneur's profits are given by

$$\pi(\theta, k) = \max_{l} \left((\theta k)^{\alpha} l^{1-\alpha}\right)^{\nu} - wl - \kappa \mathbb{I}_{k>0}.$$

The first term denotes total production, the second wage payments and the third are fix costs of production κ. Optimal labor input is given by

$$l(\theta, k) = \left[\frac{(1-\alpha)\nu}{w} (\theta k)^{\alpha\nu}\right]^{\frac{1}{1-(1-\alpha)\nu}}. \tag{23}$$

Substituting out labor allows me to reduce the number of choice variables in the individual optimization problem in a similar way as in the two period problem above.

Labor Productivity Process. While entrepreneurs face on average higher risks than workers, labor income is risky too. I am accounting for this and choose a specification standard in the literature on uninsurable idiosyncratic labor income risk. As in Aiyagari (1994) I assume that log labor productivity η follows an AR(1) process with autocorrelation ρ_η and a standard deviation of the innovation term of σ_η,

$$\log(\eta_t) = \rho_\eta \log(\eta_{t-1}) + \epsilon_\eta, \qquad \epsilon_\eta \sim \mathcal{N}(0, \sigma_\eta).$$

I approximate this process by a Markov transition matrix P_η with a finite number of states $\mathbb{E} = \{\bar{\eta}_1, \bar{\eta}_2, ..., \bar{\eta}_{N_\eta}\}$ using the Tauchen method.

Market Structure. The market structure is incomplete. There are no state contingent claims to insure against adverse draws of θ or η. The only traded financial asset is a riskless bond. The labor market clears competitively.

Individual Optimization. The assumptions on the productivity processes imply that the agents' optimization problem has a recursive representation. An agent's value in period t depends on current working- and managerial ability and on cash-at-hand a and can be described by the Bellman equation

$$V_t(\eta_t, \theta_t, a_t) = \max_{k_{t+1}\geq 0, b_{t+1}\geq 0} u(a_t - k_{t+1} - b_{t+1}) + \mathbb{E}_t\Big[V_{t+1}(\eta_{t+1}, \theta_{t+1}, a_{t+1})\Big] \tag{24}$$

subject to the budget constraint

$$c_t + k_{t+1} + b_{t+1} = a_t, \tag{25}$$

where

$$a_t = \pi_t(\theta_t, k_t) + \eta_t w_t + (1-\delta)k_t + (1+r_t)b_t - \mathcal{T}(\mathcal{I}_t) \tag{26}$$

denotes "cash-on-hand".

The budget constraint says that the sum of consumption, capital investment and bond holdings is equal to cash-on-hand a_t, which is given by the sum of total current period income

$$I_t = \pi_t(\theta_t, k_t) + \eta_t w_t - \delta k_t + r_t b_t, \tag{27}$$

net of tax payments $\mathcal{T}(\mathcal{I}_t)$ and the stock of financial wealth $k_t + b_t$.

Note that the formulation allows for self-employment. This becomes clear when rewriting the sum of firm and labor income as

$$\pi_t(\theta_t, k_t) + \eta_t w_t = \max_l \left((\theta_t k_t)^\alpha l^{1-\alpha}\right)^\nu + w_t(\eta_t - l) - \kappa \mathbb{I}_{k_t>0}..$$

The case $l > \eta_t$ can be interpreted as the entrepreneur working full time in her own firm and additionally hiring $l-\eta$ units of effective labor from outside. Similarly, the case $l \leq \eta$ corresponds to a self-employed entrepreneur, who does not hire labor from outside and works part-time for another firm. To be precise she works $(\eta_t - l)/\eta$ of her disposable time for another firm.

Taxable Income. I follow Kitao (2008) and Cagetti and De Nardi (2009) in the sense that in my model all income is subject to the same progressive personal income tax schedule. In particular, taxable personal income is the sum of business income (where depreciation is deductable), labor income and capital gains from bond holdings. Only positive income is taxable,

$$\mathcal{I}_t = \max\{0, I_t\}. \tag{28}$$

While the reality consists of a more complicated tax-transfer system with many exemptions, this provides a very good approximation, at least for the United States. In particular, most businesses that correspond to the non-corporate sector in my model, are so called "flow-through" entities, income from which is added for tax purposes to the personal income of the investor or owner. These entities include partnerships, sole proprietorships, S corporations and limited liability companies. Only incorporated businesses with the legal form of a C corporation are subject to so called "double taxation", where firm profits are taxed first at the corporate level, and, once distributed to shareholders, again as part of personal income. In practice, however, owners who are employed at the firm, are able to avoid this double taxation by classifying business income as wages, which are deductable for the porpose of taxation on the corporate level. Further, capital gains, or interest income from bonds in terms of our model, are taxed under the personal income tax if they result from a short term investment of less than a year. Longer term capital gains are subject to a lower, though also progressive tax schedule. But for simplicity we abstract from this in the model.

Government. The government runs a balanced budget and needs to raise income tax revenues in order to finance an exogenous stream $\{G_t\}_{t=0}^{\infty}$ of government expenditures. The applied literature suggests several tax functions to approximate the US income tax code. The comprehensive study of Guner et al. (2014) shows that most of the proposed functions - properly calibrated - provide a very good fit to the actual tax schedules. In particular, they capture the degree of progressivity remarkably well. In this paper I use the specification used in Heathcote et al. (2017), often referred to as "HSV tax function",

$$\mathcal{T}(\mathcal{I}) = \mathcal{I} - \tau_0 \mathcal{I}^{1-\tau_1}. \tag{29}$$

The parameter τ_1 captures the progressivity of the tax code. Note that

$$\frac{1-\mathcal{T}'(\mathcal{I})}{1-\mathcal{T}'(\mathcal{I})/\mathcal{I}} = 1-\tau_1.$$

Thus the ratio of marginal to average tax rates is larger than one exactly when $\tau_1 > 0$. If it is zero the income is taxed at the flat rate τ_0. If it is positive (negative) the income tax code is progressive (regressive).

Stationary Competitive Equilibrium. For notational convenience I summarize the individual state by $x = (\eta, \theta, a) \in \mathbb{X} = \mathbb{E} \times \Theta \times \mathbb{R}^+$.

Definition 2. *A stationary competitive general equilibrium is given by a constant stream of government expenditures G, a tax policy (τ_0, τ_1), prices (w, r), individually optimal policies $(k(x), b(x), c(x))$, value functions $V(x)$, capital K^c and labor L^c in the corporate sector, the distribution of agents $\zeta(x)$ over the state space $\mathbb{X}$, and a transition operator $\mathbb{P}$ such that*

(i) Given prices (w, r) and tax policy (τ_0, τ_1), the individual policy functions $(k(x), b(x), c(x))$ and value functions $V(x)$ solve the Bellman equation (24).

(ii) Corporates make zero profit. The wage is given by the marginal product of labor in the corporate sector, $w = (1-\alpha_c)\left(\frac{K_c}{L_c}\right)^{\alpha_c}$, and the riskfree interest rate is given by the marginal product of capital in the corporate sector, $r = \alpha_c\left(\frac{K_c}{L_c}\right)^{\alpha_c-1} - \delta$.

(iii) The government budget is balanced,

$$\int \mathcal{T}(\mathcal{I}(x, \eta', \theta')) \sum_{\eta'} P_\eta(\eta'|\eta) \sum_{\theta'} P_\theta(\theta'|\theta) d\zeta(x) = G,$$

(iv) The market for riskless bonds clears,

$$\int b(x)d\zeta(x) = K^c,$$

(v) The labor market clears,

$$\int l(\theta', k(x)) \sum_{\theta'} P_\theta(\theta'|\theta)d\zeta(x) + L^c = \int \eta d\zeta(x),$$

where $l(\theta, k)$ is given by equation (23).

(vi) The distribution is time invariant, $\zeta = \mathbb{P}(\zeta)$.

1.4.2 Parameterization and Model Fit

I calibrate the model in order to match a number of targets and evaluate its performance by comparing certain other, untargeted, moments of the data with the model counterparts. The parameters are summarized in table 1.1.

Table 1.1: Parameters.

Parameter	Value	Description
Preference		
β	0.94	Time discount factor
γ	1.5	Coefficient of risk aversion
Technology		
α_c	0.36	Capital share in corporate sector
α	0.36	Capital share in entrepreneurial sector
ν	0.915	Returns to scale
δ	0.06	Capital depreciation
κ	0.3	Fix costs of production
θ	see text	Enrepreneurial productivity states
P_θ	see text	Transition matrix of entrepreneurial productivity
Labor Productivity		
ρ_η	0.94	Persistence of labor income process
σ^2_η	0.02	Variance of innovation term
Policy		
τ_0	0.96	Level parameter in income tax schedule
τ_1	0.08	Progressivity parameter in income tax schedule

Preferences. Some of these parameters are directly taken from the related literature. As for the preference parameters, the coefficient of relative risk aversion γ is set to 1.5 (as in Cagetti and De Nardi (2006)) while the discount factor β is chosen such that in the benchmark equilibrium a capital output ratio of 2.65 is attained (as in Quadrini (2000) and Kitao (2008)).

Labor Productivity. Following Kitao (2008) the AR(1) labor productivity process is discretized by a five state Markov process using the Tauchen method. For the autocorrelation ρ_η and the variance of the corresponding continuous time process I use values in the range of empirical estimates (see, for example Storesletten et al. (2004).

Technology. Again following Kitao (2008) I set the share of output that goes to capital, α to 0.36 and the capital depreciation rate to 6% both for the entrepreneurial and the corporate sector. A three state Markov chain determines the process of entrepreneurial ability. I set the lowest productivity shock to zero in order to have a substantial share in the population who does not run a firm. Following Quadrini (2000) I assume that entrepreneurial productivity changes only gradually, i.e. at most one state up or down each period.

The three productivity states are given by

$$\theta = \begin{bmatrix} 0.0 & 4.5 & 5.9 \end{bmatrix}$$

and the Markov transition matrix by

$$P_\theta = \begin{bmatrix} 0.95 & 0.05 & 0 \\ 0.34 & 0.64 & 0.02 \\ 0 & 0.055 & 0.945 \end{bmatrix}.$$

Policy Parameters. The progressivity parameter τ_1 of the income tax code is taken from from Feenberg et al. (2017). It is between the estimates reported in Guner et al. (2014) and Heathcote et al. (2017). The other parameter τ_0 is adjusted in order to match the ratio of income tax revenue over GDP reported in the OECD revenue statistics 2016. This revenue is used to finance government expenditures G, which from now on I fix at this level.

Calibrated Parameters and Targeted Moments. The Markov chain process for entrepreneurial ability has six free parameters (two states, and four transition probabilities). Together with the returns to scale parameter ν, the fix costs of production κ, and the time discount factor β this gives nine free parameters, which I calibrate in order to match the targets summarized in table 1.2. The moments are the capital output ratio, the share of active business owners, the yearly exit rate from entrepreneurship, the share of capital in the entrepreneurial sector, the wealth Gini index, the share of income earned by entrepreneurs, the share of wealth owned by entrepreneurs, the average tax rate and the income tax revenue as a fraction of GDP. All of the targets are matched well.

Table 1.2: Calibrated Moments

Moment	Data Value	Model Value
Capital-Output Ratio	2.65	2.64
Share of active business owners	10%	10%
Yearly exit rate from entrepreneurship	20%	20%
Share of capital in entrepreneurial sector	35%	36%
Wealth Gini index	0.82	0.83
Share of entrepreneurs' income	27%	27%
Share of wealth owned by active business owners	43%	41%
Population weighted average tax rate	7%	7%
Income tax revenue / GDP	10%	10%

Match of Untargeted Moments. Table 1.3 compares the wealth distribution in the US with the model implied stationary wealth distribution.

The model generates the high concentration of wealth observed in the data at least up to the wealthiest decile. It somewhat underestimates the share of wealth onwed by the

Table 1.3: Wealth Distribution

Top	1%	5%	10%	20%	50%
US data	35%	63%	75%	87%	99%
Model	24%	57%	72%	86%	99%

richest percentile (24% in the model vs. 35% in the data). Table 1.4 compares the share of entrepreneurs in the respective top wealth percentiles.

Table 1.4: Entrepreneurs in Top Wealth Percentiles

Top	1%	5%	10%	20%	50%
US data	65%	46%	36%	26%	16%
Model	62%	48%	40%	31%	21%

The model qualitatively and to a large extent quantitatively generates the overrepresentation of entrepreneurs among the wealthiest. The pattern is qualitatively the same, quantitatively the share of entrepreneurs is diminishing a bit less than in the data as you move down the distribution. While in the data 65% of the wealthiest percentile are active business owners and this reduces to 16% in the top half of the distribution, in the model 62% of the wealthiest percentile are entrepreneurs and this reduces to 21% in the top half.

1.5 Policy Reform

This section reforms the tax and transfer system. I first restrict the Ramsey planner to only change the income tax code. Later, I add linear investment subsidies as another policy instrument. In either case the economy is originally, at $t = 0$, in its benchmark steady state computed above. At the beginning of period $t = 1$, the government announces the change in policy. This change is unexpected by agents. However, the government can credibly commit not to perform any changes again. Thus agents have perfect foresight regarding aggregate variables from time $t = 1$ onward.

1.5.1 The Optimal Tax Reform

In the first policy experiment only the parameters (τ_0, τ_1) change. All other parameters are set to the benchmark values summarized in the previous section. This includes the constant stream of government expenditures G. Accounting for transitional dynamics requires assumptions on how the tax code adjusts as the economy moves from the old to the new steady state. I assume that the government's only action is to change the progressivity parameter τ_1 once and for all. Along the path to the new steady state the government budget needs to clear period by period, implying that the other tax parameter $\{\tau_{0,t}\}_{t=1}^{\infty}$ is time varying. Some notation is useful for a formal description of the government's optimization problem. Denote by $\eta^t = \{\eta_1, \eta_2, ..., \eta_t\}$ and $\theta^t = \{\theta_1, \theta_2, ..., \theta_t\}$ the histories of shock realizations up to time t. As before I summarize the initial state by $x_0 = (\eta_0, \theta_0, a_0)$. Further, for a particular tax reform τ_1 denote by $c_t(x_0, \eta^t, \theta^t; \tau_1)$ the equilibrium consumption level in period t of an agent with initial state x_0 and history (θ^t, η^t).

The Objective. The utilitarian government solves

$$\max_{\tau_1} SW(\tau_1) = \max_{\tau_1} \int \sum_{t=1}^{\infty} \beta^t \sum_{(\eta^t,\theta^t)} p(\eta^t, \theta^t | x_0) u\big(c_t(x_0, \eta^t, \theta^t; \tau_1)\big) d\zeta(x_0),$$

where $p(\eta^t, \theta^t|x_0)$ is the probability of history (η^t, θ^t) given initial state x_0. The optimal tax reform is defined by

$$\tau_1^R = \arg\max_{\tau_1} SW(\tau_1)$$

and the corresponding equilibrium consumption levels by $c_t^R(x_0, \eta^t, \theta^t)$. Similarly, $c_t^{NR}(x_0, \eta^t, \theta^t)$ are the consumption levels if the system was not reformed.

The Naive Planner. Further, in order to assess the importance of general equilibrium effects I solve the problem of a naive policy maker. This policy maker takes into account behavioral responses of agents to tax changes but wrongly assumes that prices were unaffected by these behavioral responses and fixed at their benchmark steady state level (w^{NR}, r^{NR}), i.e. I solve for the optimal tax schedule in partial equilibrium.

Results. The tax schedule obtained in this way ($\tau_1^{PE} = 0.97$) is much more progressive than the optimal one that takes into account general equilibrium effects ($\tau_1^R = 0.49$). The optimal tax system, in turn, is much more progressive than the status quo ($\tau_1^{NR} = 0.08$). The three (steady state) tax schedules are depicted in Figure 1.1. The solid blue line depicts the status quo, the dased red line the optimal tax schedule and the dash-dotted yellow line the suboptimal tax schedule of the naive policy maker.

According to the model no agent with an income of less than 65,000 USD pays positive income taxes after the system is optimally reformed. Instead most of this part of the population receives a substantial transfer from the government. These transfers are even higher in the partial equilibrium case. In contrast, high earners are taxed at much higher rates after the reform. For example, earnings of 100,000 USD are taxed at an average rate of 9%-10% in the status quo. Under the optimal reform this rate about doubles to 19% and the naive policy maker, who ignores general equilibrium effects, would tax them at even 37%.

The average welfare gain Δ of the reform is defined as the constant percentage increase in consumption for each agent, in each period $t \in \{1, 2, ...\}$, and at each possible state in the benchmark economy necessary to equate social welfare. Formally, it is given by

$$\int \sum_{t=1}^{\infty} \beta^t \sum_{(\eta^t,\theta^t)} p(\eta^t, \theta^t|x_0) u\Big(c_t^{NR}(x_0, \eta^t, \theta^t)(1 + \Delta)\Big) d\zeta(x_0) = SW(\tau_1^R).$$

The CRRA specification of preferences allows for an analytical expression for the welfare gain,

$$\Delta = \left(\frac{SW(\tau_1^R)}{SW(\tau_1^{NR})}\right)^{\frac{1}{1-\gamma}} - 1.$$

I find very high aggregate welfare gains of $\Delta = 6.9\%$. The approval rate of the reform is almost two thirds (65.9%).

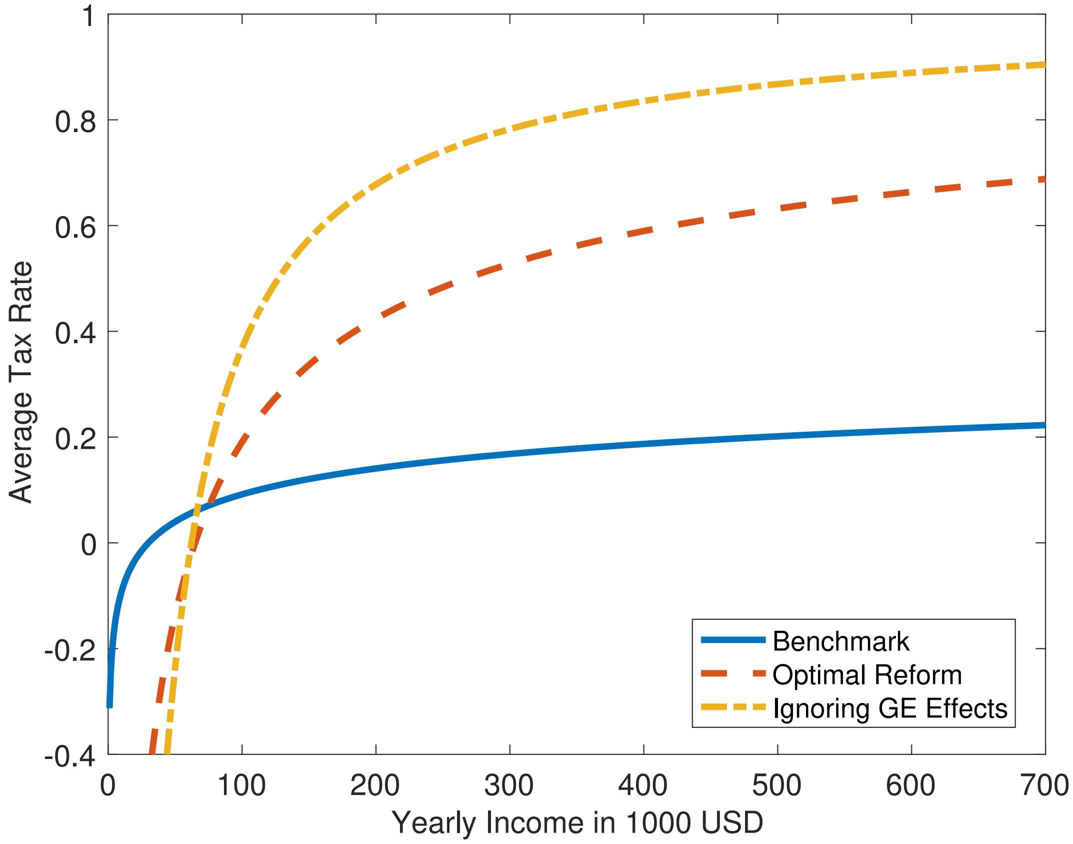


Figure 1.1: Benchmark and Reform Tax Schedules

1.5.2 Decomposition of Welfare Gains

A decomposition of welfare gains proves particularly insightful for understanding the aggregate and distributional consequences of the reform and its induced general equilibrium effects. Following Domeij and Heathcote (2004), I decompose the total welfare gain into an aggregate and a distributional component.

Level and Distributional Component. For a household with initial state x_0 and history (η^t, θ^t) denote by $\hat{c}_t^R(x_0, \eta^t, \theta^t)$ the hypothetical value of consumption in case of reform if the household got to consume the same share of aggregate consumption as in the case without reform,

$$\hat{c}_t^R(x_0, \eta^t, \theta^t) = \frac{c_t^{NR}(x_0, \eta^t, \theta^t)}{C_t^{NR}} C_t^R,$$

where C_t^R (C_t^{NR}) denotes aggregate consumption in the case of reform (no reform). The *level component* Δ^L is defined in an analogous way as the average welfare gain, namely as the percentage increase in consumption compared to the benchmark that satisfies

$$\int \sum_{t=1}^{\infty} \beta^t \sum_{(\eta^t,\theta^t)} p(\eta^t, \theta^t | x_0) u\Big(c_t^{NR}(x_0, \eta^t, \theta^t)(1+\Delta^L) \Big) d\zeta(x_0)$$
$$= \int \sum_{t=1}^{\infty} \beta^t \sum_{(\eta^t,\theta^t)} p(\eta^t, \theta^t | x_0) u\Big(\hat{c}_t^{R}(x_0, \eta^t, \theta^t) \Big) d\zeta(x_0).$$

The *distributional component* Δ^D is then implicitly given by

$$(1+\Delta) = (1+\Delta^L)(1+\Delta^D).$$

Partial and General Equilibrium Component. In a second step I decompose the total welfare gain into a partial equilibrium and a general equilibrium component. For this purpose I solve the individually optimal decision rules when prices would be fixed at their pre-reform values, i.e. if for all t

$$(w_t^R, r_t^R) = (w^{NR}, r^{NR}).$$

I further compute the evolution of the distribution over time that is implied by these decision rules and the shock processes. Denote by $\tilde{c}_t^R(x_0, \eta^t, \theta^t)$ the individually optimal consumption levels for this case. The *partial equilibrium component* Δ_{PE} of the welfare gain is defined as

$$\int \sum_{t=1}^{\infty} \beta^t \sum_{(\eta^t,\theta^t)} p(\eta^t, \theta^t | x_0) u\Big(c_t^{NR}(x_0, \eta^t, \theta^t)(1+\Delta_{PE}) \Big) d\zeta(x_0)$$
$$= \int \sum_{t=1}^{\infty} \beta^t \sum_{(\eta^t,\theta^t)} p(\eta^t, \theta^t | x_0) u\Big(\tilde{c}_t^{R}(x_0, \eta^t, \theta^t) \Big) d\zeta(x_0).$$

The *general equilibrium component* Δ_{GE} is given by the residual to the overall welfare gain,

$$(1+\Delta) = (1+\Delta_{PE})(1+\Delta_{GE}).$$

Sub-Components. In an analogous way to above we can compute the level sub-component of the partial equilibrium component. Denote by $\hat{\tilde{c}}_t^R(x_0, \eta^t, \theta^t)$ the hypothetical value of consumption in case of reform in the partial equilibrium economy if the household got to consume the same share of aggregate consumption as in the case without reform

$$\hat{\tilde{c}}_t^R(x_0, \eta^t, \theta^t) = \frac{\tilde{c}_t^{NR}(x_0, \eta^t, \theta^t)}{C_t^{NR}} \tilde{C}_t^R,$$

where $\tilde{C}_t^R$ denotes aggregate consumption in the partial equilibrium economy after the reform.[15] The *PE-level component* Δ^L_{PE} is then implicitly given by

$$\int \sum_{t=1}^{\infty} \beta^t \sum_{(\eta^t,\theta^t)} p(\eta^t,\theta^t|x_0) u\Big(c_t^{NR}(x_0,\eta^t,\theta^t)(1+\Delta^L_{PE})\Big) d\zeta(x_0)$$
$$= \int \sum_{t=1}^{\infty} \beta^t \sum_{(\eta^t,\theta^t)} p(\eta^t,\theta^t|x_0) u\Big(\hat{\tilde{c}}_t^{R}(x_0,\eta^t,\theta^t)\Big) d\zeta(x_0).$$

We can back out the other three sub-components, namely the *GE-level component* Δ^L_{GE}, the *PE-distributional component* Δ^D_{PE} and the *GE-distributional component* Δ^D_{GE} in the following way,

$$\begin{aligned}(1+\Delta) &= \underbrace{(1+\Delta^L_{PE})(1+\Delta^L_{GE})}_{(1+\Delta^L)}\underbrace{(1+\Delta^D_{PE})(1+\Delta^D_{GE})}_{(1+\Delta^D)} \\ &= \underbrace{(1+\Delta^L_{PE})(1+\Delta^D_{PE})}_{(1+\Delta_{PE})}\underbrace{(1+\Delta^L_{GE})(1+\Delta^D_{GE})}_{(1+\Delta_{GE})}.\end{aligned}$$

Results. The results of this decomposition are summarized in Table 1.5. The level component is negative, about -2.7% in consumption equivalent variation. Decomposing the level effect further into a partial and a general equilibrium component reveals an important mechanism. If prices were not changing after the reform the welfare loss due to lower aggregate consumption would be much bigger, about 9.4%. The reason is that prices serve an allocative purpose. Rigid wages (interest rates) amplify the disincentives to invest (save) induced by an increase in tax progressivity. In general equilibrium the reduction in labor productivity associated with a decline in the capital stock is accompanied by a reduction in wages and an increase in the rate of return both in the entrepreneurial and the corporate sector. These price effects mitigate the behavioral responses and therefore the drop in output and consumption so that most of the partial equilibrium losses are compensated.

Table 1.5: Decomposition of Welfare Gains

CEV (in %)	Level	Distribution	Total
Partial Equilibrium	$\Delta^L_{PE} = -9.4$	$\Delta^D_{PE} = +20.4$	$\Delta_{PE} = +9.2$
General Equilibrium	$\Delta^L_{GE} = +7.4$	$\Delta^D_{GE} = -8.8$	$\Delta_{GE} = -2.0$
Total	$\Delta^L = -2.7$	$\Delta^D = +9.9$	$\Delta = +6.9$

With a total welfare gain of 6.9% in CEV but negative losses of 2.7% from lower average consumption it is clear that the distributional component must be positive and large. It is about 9.9%. A more progressive tax code mechanically redistributes from high to low earners. With fixed prices the distributional component is as high as 20.4%. In general equilibrium, however, the reduction in investment decreases wages and increases capital returns. As wage income is the primary income source of consumption poor agents, who have higher marginal utility and therefore enter the social welfare function with higher weight, this implies sizable distributional losses of 8.8%. Thus almost half of the distributional gains are lost through general equilibrium or "trickle down" effects.

[15] Note that in the case of no reform partial and general equilibrium allocation coincide and is equal to the one in the benchmark steady state, therefore $\tilde{C}_t^{NR} = C_t^{NR} = C^{NR}$.

In total, the partial equilibrium welfare gain is about 9.2%. The opposite sign of the two sub-components Δ^L_{PE} and Δ^D_{PE} illustrates a standard equity-efficiency trade-off. A more progressive tax system leads to more redistribution (equity) but reduces economic activity and hence average consumption (efficiency). Hence, the direct redistributional gains of 20.4% are partially offset. The total general equilibrium component is -2.0%. Its two sub-components, however, are much bigger in absolute value. They have the opposite sign as the partial equilibrium components. A reduction in the capital stock decreases wages and increases capital returns both in the entrepreneurial and the corporate sector. These price effects make it more attractive to invest, dampening the negative partial equilibrium level effect. However, the price responses are not desirable from a redistributional perspective as they increase the income of rich agents, who have mostly capital income, and decrease the income of poor agents, who have mostly wage income.

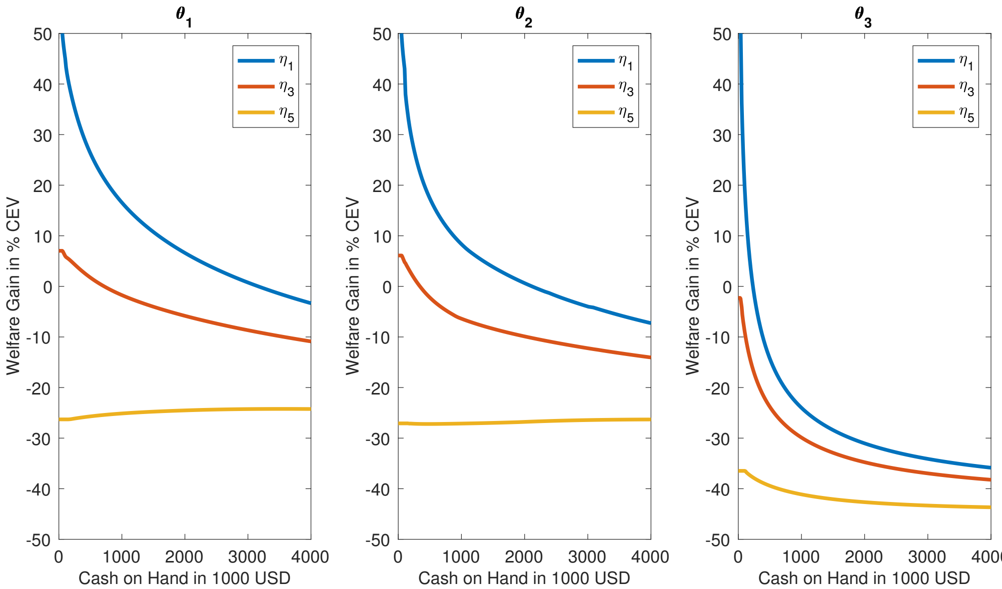


Figure 1.2: Heterogeneity of Welfare Effects

Heterogeneity of Welfare Effects. The three panels of Figure 1.2 depict the welfare gains of agents with low, medium and high entrepreneurial ability, respectively, as a function of cash on hand a. Each panel in turn consists of three lines representing low, medium and high working ability, respectively. The general picture from this graph is that more able and richer agents like the reform less. This is natural as these are the agents with high (expected) gross income, whom the government now taxes at higher rates. For agents with high entrepreneurial ability ($\bar{\theta}_3$) but low working ability ($\bar{\eta}_1$) the welfare gain is decreasing particularly strongly in wealth. Poor but able entrepreneurs benefit from high transfers which they can directly invest in their firm, while rich ones suffer from high tax payments. In contrast able entrepreneurs with high working ability dislike the reform also if they are poor. These entrepreneurs don't get transfers as they have higher wage income. Interestingly, for agents with low and medium entrepreneurial but high working ability the (substantial) welfare loss is basically independent of wealth. In fact, the losses for these agents are even a bit smaller if they are richer. Poor agents with high labor earnings suffer the most from the reform because they are taxed at much higher rates despite being poor and thus cannot accumulate wealth as easy as prior to the reform. The discrepancy to poor agents with high entrepreneurial ability comes from the fact that high entrepreneurial ability only translates into high income if also investment is high while high working ability translates directly into high income independent of wealth.

1.5.3 Steady State Comparison

What are the effects of the reform on macroeconomic aggregates? Table 1.6 compares the calibrated moments of the benchmark economy with their post reform steady state counterparts. Table 1.7 summarizes the changes in output and prices. The column 'No Subsidy' shows the numbers for the optimal tax reform just described, where the Ramsey planner only changes the income tax code but does not have investment subsidies at her disposal. The increase in tax progressivity discourages investment and saving and as a result total output decreases by about 24%. We observe a reallocation of production from the entrepreneurial sector to the corporate sector. The share of capital employed in the entrepreneurial sector decreases from 36% to 27%. As a consequence, output in the entrepreneurial sector declines much more than in the corporate sector (-35% vs. -13%). Since labor supply is exogenous relatively more labor is used in production after the reform and the capital output ratio decreases by more than one third.

Table 1.6: Calibrated Moments after Reform

Moment	Benchmark	No Subsidy	Subsidy
Capital-output ratio	2.64	1.67	1.28
Share of active business owners	10%	17%	7%
Yearly exit rate from entrepreneurship	20%	25%	13%
Share of capital in entrepreneurial sector	36%	27%	54%
Wealth Gini index	0.83	0.34	0.93
Share of entrepreneurs' income	27%	27%	29%
Share of wealth owned by active BO	41%	29%	56%
Population weighted average tax rate	7%	-1%	-29%
Income tax revenue / GDP	10%	13%	14%

General Equilibrium Effects. The decline in the capital-labor ratio implies a reduction in wages and an increase in the risk free interest rate. These general equilibrium effects are substantial. Wages decrease by about 23% and the riskfree rate increases by 6 percentage points. Why do we observe an increase in welfare although wages, the main income source of poor agents, decrease so drastically? Because the direct or mechanical tax effects turn out to overcompensate for this decline.

Table 1.7: Changes in Macroeconomic Aggregates

Variable	No Subsidy	Subsidy
Output in corporate sector	-12.7%	-52.1%
Output in entrepreneurial sector	-35.3%	-9.9%
Output total	-24.1%	-30.8%
Wages	-22.5%	-31.6%
Risk free interest rate	+6.1%	+10.3%

Mechanical Tax Effects. While the average (population weighted) tax rate in the benchmark is about 7%, after the reform it is slighly negative. There is a substantial shift in the tax burden to high earners and as a consequence a large share of the population does not pay any income taxes, in fact many even receive transfers. Since GDP declines after the reform but by assumption the government needs to finance the same amount of expenditures, the share of income tax revenues to GDP increases slightly.

Distributional Effects. The reform induces a more equal wealth distribution as can be seen by the decline in the wealth Gini index from 0.83 to 0.34. Table 1.8 compares the wealth distributions before and after reform. The share of wealth held by the richest percentile decreases by more than two thirds from 24% to 7%. In contrast, the share of wealth owned by the poorer half of the population increases drastically from 1% to 28%.

Table 1.8: Wealth Distribution

Top	1%	5%	10%	20%	50%
Benchmark	24%	57%	72%	86%	99%
Reform without subsidy	7%	22%	33%	45%	72%
Reform with subsidy	50%	82%	91%	97%	100%

The reform also induces a redistribution from entrepreneurs to workers. It is important to note that the share of active business owners increases by about 70% from 10% to 17% of the overall population. While entrepreneurial investment on the intensive margin decreases substantially, more agents decide to start a business. The reason is that those on the margin to enter entrepreneurship face lower tax rates. After the reform it pays off for more agents to incur the fixed costs of running a business. Although the share of entrepreneurs in the population increases by 70%, their share of income remains at the pre-reform level of from 27%. Their share of wealth even decreases from 41% to 29%. After the reform the tax rates on capital returns increase more dramatically in (both entrepreneurial and corporate) investment than before the reform. Those agents with the highest ability to become wealthy, entrepreneurs with high θ, find it less attractive to do so and thus own a smaller share of aggregate wealth. As a consequence the over-representation of entrepreneurs in the top wealth percentiles is less pronounced after the reform. For example, before the reform, the share of entrepreneurs in the top wealth percentile was more than six times the share in the overall population (62% vs. 10%), while after the reform this factor decreases to five (85% vs. 17%).

Table 1.9: Entrepreneurs in Top Wealth Percentiles

Top	1%	5%	10%	20%	50%
Benchmark	62%	48%	40%	31%	21%
Multiple of share in total population	6.2	4.8	4.0	3.1	2.1
Reform without subsidy	85%	63%	51%	40%	25%
Multiple of share in total population	5.0	3.7	3.0	2.4	1.5
Reform with subsidy	62%	58%	51%	34%	13%
Multiple of share in total population	8.5	8.0	7.0	4.6	1.8

The Role of Inelastic Labor Supply. It is worth discussing the normative consequences of two possible deviations from the present modeling framework. The first deviation would be to relax the assumption of exogenous labor supply. In the present framework income taxes distort investment and savings but not labor supply. This is likely to bias the optimal tax code towards more progressivity. With elastic labor supply large transfers for low incomes and increasingly high tax rates for higher incomes will discourage work. This in turn will reduce the productivity of capital and therefore discourage investment even more. Hence, one can expect that endogenizing labor supply decreases the optimal progressivity of the tax code.

The Role of Entrepreneurship. Second, it is instructive to analyze the discrepancies between the policy implications of the present framework and the ones of the standard incomplete markets model of Aiyagari (1994), where all production takes place in the riskless corporate sector. It is well known that standard labor income processes alone are not able to generate the concentration of earnings and wealth observed in the data. A reason is that in the standard Aiyagari model precautionary savings decrease with wealth. Once agents are rich enough they start dissaving. Castañeda et al. (2003) overcome this problem by calibrating a labor productivity process that features very high risk for the highest earners. In Appendix 1.A.4, I shut down entrepreneurship in my model and perform a similar exercise. I show that this feature makes the investment (savings) decision of these agents very inelastic to changes in the tax code. As a consequence the optimal tax code is much more progressive than in the present framework.[16] The question is then which of the two frameworks is more realistic. The rationale behind the calibration of Castañeda et al. (2003) was that earnings processes are typically estimated on top coded data sets that miss the highest earners. In the absence of this data calibrating the income process to match certain cross-sectional moments of the income and wealth distribution seemed to be a way out. However, De Nardi et al. (2016) use richer tax data and do not find evidence for the high risk process of high earners. In a very recent paper De Nardi and Fella (2017) write: "[E]arnings data for non-entrepreneurs do not feature sufficient downward risk to generate a long right-tail in the wealth distribution as a result of precautionary saving. To the extent that this kind of risk is confined to entrepreneurs or business owners, it should not just be modeled as an exogenous shock, but should, rather be endogenous to entrepreneurial decision about savings, labor hiring, and portfolio choice." All the suggested model ingredients are present in my framework.

1.5.4 Transitional Dynamics

Accounting for transitional dynamics in the optimization problem turns out quantitatively important both for the optimal tax schedule and for the size of the welfare gain. A simple steady state comparison can be misleading for policy analysis as the economy will reach the new steady state only asymptotically. Figure 1.3 depicts the tax schedule that maximizes steady state utilitarian welfare only (dash-dotted yellow line). It is much less progressive than the optimal one that accounts for the transition (dashed red line). For example earnings of 100,000 USD would be taxed at an average rate of 10% with the former but at 19% with the latter tax schedule. The welfare gain from the steady state welfare maximizing reform would be 5.4% in consumption equivalent variation,[17] less than two thirds of the gain from the optimal reform.

The reason for this finding is that along the transition agents decumulate assets. Average consumption on the whole transition path is therefore higher than in the new

[16] Kindermann and Krueger (2017) compute the optimal tax rate for the highest earners in a setting with endogenous labor supply and a calibration along the lines of Castañeda et al. (2003). The authors find optimal marginal tax rates for the top earners of around 90%. The reason is a similar one. High risk of losing the high productivity makes high productive agents work a lot even if they are taxed at such a high rate.

[17] Here the transition is taken into account.

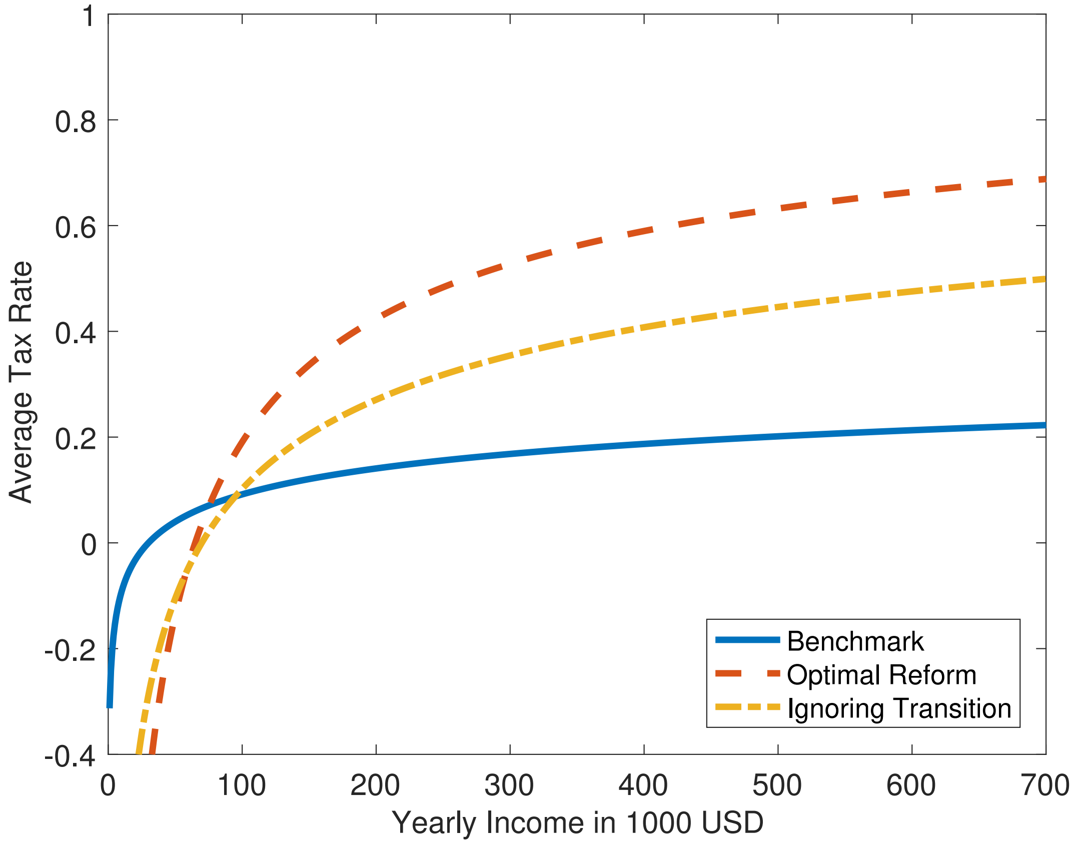


Figure 1.3: Benchmark and Reform Tax Schedules

steady state. In the first years it is even higher than in the original steady state. Figure 1.4 depicts the transitional path of aggregate consumption along with the average (income weighted) tax rate and prices. Consumption and wages are normalized such that their value in the benchmark steady state is equal to one.

The capital labor ratio gradually decreases as agents decumulate the capital stock. As a result wages decrease and the interest rate increases. Total taxable income along the transition is higher than in the new steady state. Hence, in order to generate the same revenue, it can be taxed at a lower rate.

1.5.5 Investment Subsidies

Sections 1.2.2 and 1.3.2 showed theoretically that investment subsidies can be an effective policy instrument to counteract the discouraging effect of income taxes on investment. In the following I assess quantitatively how the inclusion of this instrument affects the optimal tax reform and welfare. There are only two differences compared to the model set up described in section 1.4. First, agents' "cash-on-hand" is now given by

$$a_t = \pi_t(\theta_t, k_t) + \eta_t w_t + (1 - \delta + s)k_t + (1 + r_t)b_t - \mathcal{T}(\mathcal{I}_t),$$

where s is the investment subsidy. Second, the government now needs to raise tax revenue in order to finance both, the exogenous stream of government expenditures and the subsidy

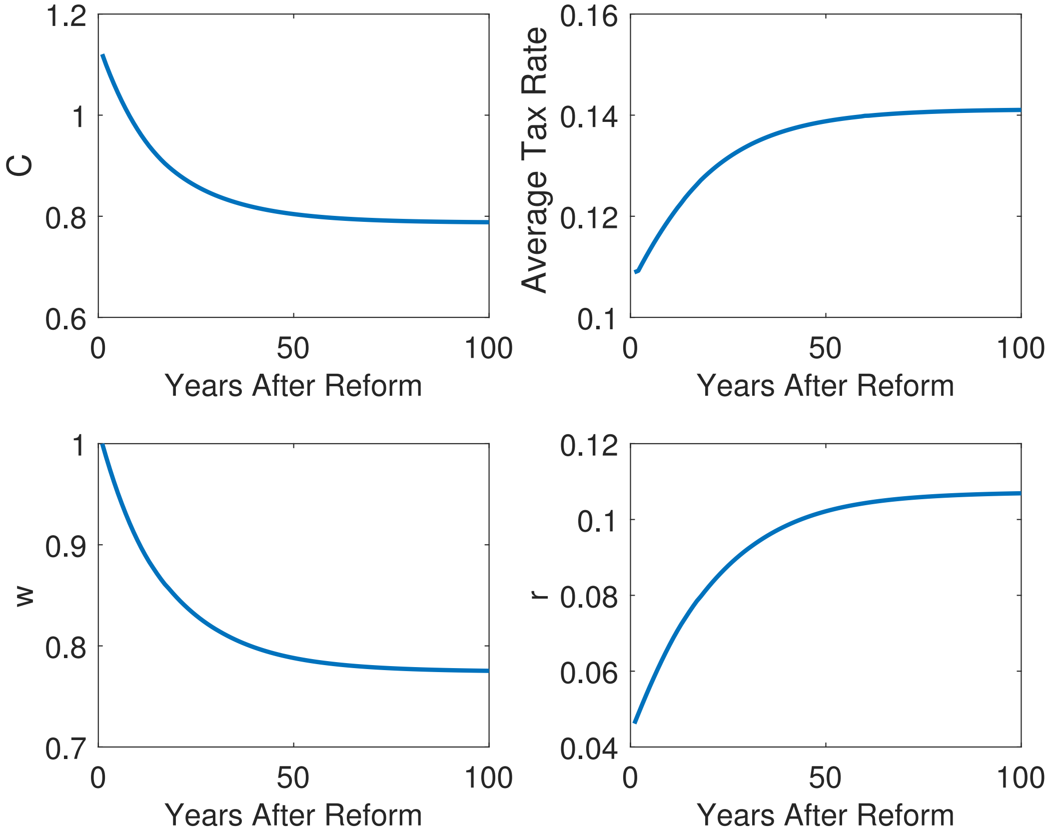


Figure 1.4: Transitional Dynamics

payments,

$$\int \mathcal{T}(\mathcal{I}(x,\eta',\theta'))\sum_{\eta'} P_\eta(\eta'|\eta)\sum_{\theta'} P_\theta(\theta'|\theta)d\zeta(x) = G + s\int k(x)d\zeta(x).$$

In this set up, a policy reform is given by the two dimensional vector (τ_1, s). The other tax parameter τ_0 adjusts to clear the government budget. Along the transition to the new steady state this parameter is again time varying.

Results. I find that the welfare optimizing values are $\tau_1 = 0.8$ and $s = 0.1$.[18] Thus for each dollar an entrepreneur invests in her firm, the government optimally contributes 10 cent. This counteracts the discouraging affect of income taxes on investment and hence allows for a more progressive tax system. The additional welfare gains of including subsidies as a policy instrument are large, around 2% CEV.

Figure 1.5 includes the optimal tax code when the Ramsey planner has access to linear investment subsidies (dotted green line). The inclusion of this policy instrument increases the optimal progressivity of the tax code. Table 1.10 compares the welfare effects of the two considered reforms. The more progressive tax code leads to much higher redistributional gains but also to higher level losses. Investment subsidies counteract the discouraging effect of a more progressive tax system on investment but only to a limited

[18] I optimized over a relatively course grid with increments of 0.05 in the τ_1 dimension and 0.025 in the s dimension.

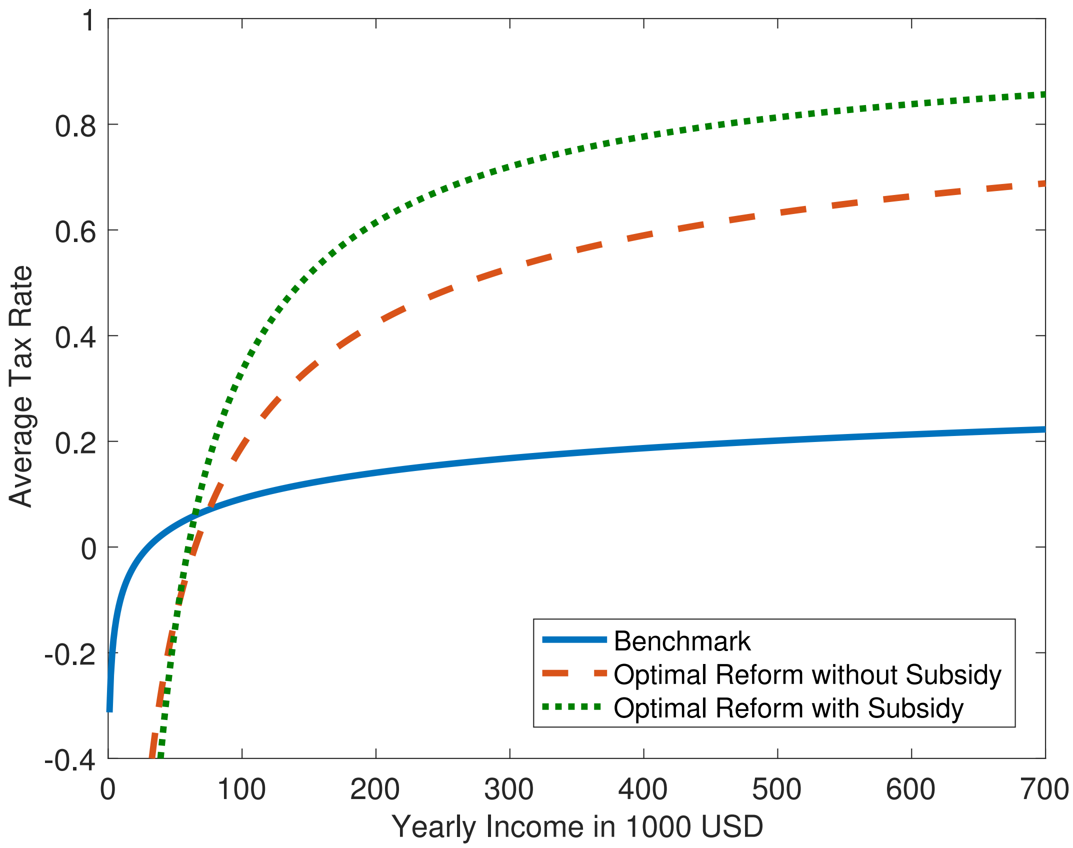


Figure 1.5: Benchmark and Reform Tax Schedules

extent. In fact, Table 1.7 shows that output declines by about 6.7% more than in the reform without subsidies. Further, now there is a reallocation of production from the corporate to the entrepreneurial sector. While the decline in corporate production is about four times higher, the decline in entrepreneurial production is less than one third compared to the reform without investment subsidies.

Table 1.10: Welfare Effects

	No Subsidy	Subsidy
Total welfare tain in CEV	+6.9%	+8.9%
Level component	-2.7%	-9.1%
Distributional component	+9.9%	+19.7%
Approval of Reform	65.9%	66.0%

The two reforms have strikingly different implications on inequality. Progressive income taxes redistribute from rich to poor. Hence, one could expect that the more progressive tax code considered here leads to less inequality. The opposite is the case. In fact, inequality rises even compared to the benchmark. The wealth Gini index in the status quo is 0.83. In the reform without subsidies it declines to 0.34, whereas with investment subsidies it increases to 0.93 (see Table 1.6). With this new reform in the long run the richest percentile owns half of total financial wealth and the poorest 80% own only 3% (see Table 1.8). One reason for this finding that I identified already in the two period model above is that investment subsidies redistribute from poor to rich households since the latter have in general higher levels of investment. In particular, there is a redistribution to entrepreneurs and as a consequence an even more pronounced over-representation of active business owners in the top of the wealth distribution (see Table 1.9). The dynamic model structure adds another reason for the increase in inequality: Large transfers provide little inventive for low earners to accumulate assets. Note that the population weighted average tax rate is -29%. Hence the majority of agents receive substantial transfers financed by the the richest in the population. Further the reform decreases the need for precautionary savings.

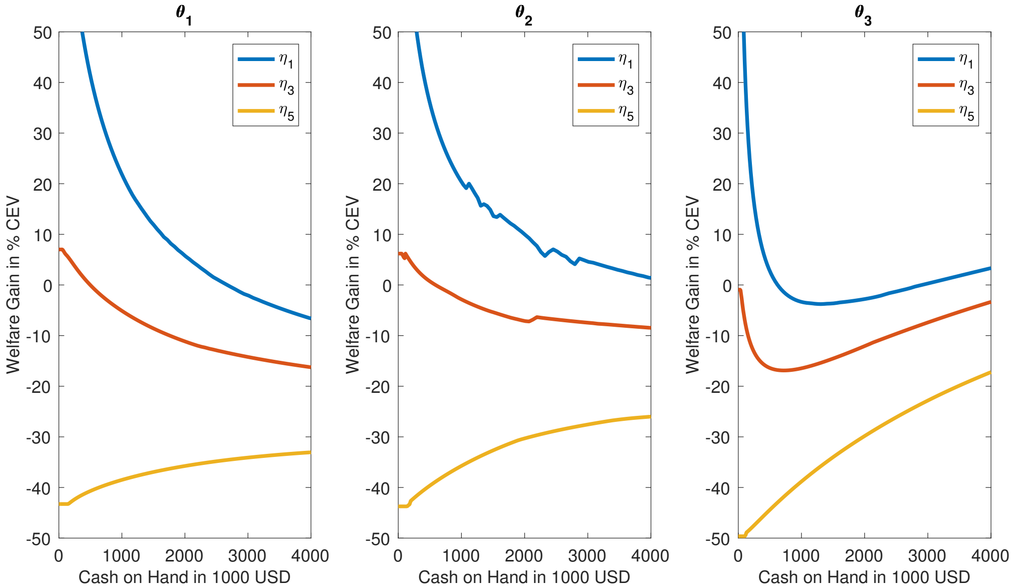


Figure 1.6: Heterogeneity of Welfare Effects with Investment Subsidy

This last observation also explains why despite a large decline in output *and* an increase in wealth inequality, the welfare gains from this reform are positive and large. Wealth inequality does not translate into consumption inequality. The standard equity-efficiency trade-off suggests that a more progressive tax code will reduce the dispersion of consumption at the cost of having lower consumption on average. This holds also here as can be seen by the opposite sign of the level and distributional component of the welfare gain. The subsidy stabilizes investment of rich entrepreneurs, the proceeds of which are to a large extent redistributed to the poor. The poor agents in turn do not mind staying poor as the welfare state is taking care of them.

Heterogeneity of Welfare Effects. The optimal reform with investment subsidies again induces large heterogeneity of welfare effects. The main difference to the reform without subsidies is that before the gains were in general decreasing in wealth, while now this is not the case anymore. For rich agents with high entrepreneurial ability the gains are decreasing in wealth. The reason is simply that subsidization of investment redistributes to these agents undoing part of the losses from higher tax rates.

1.6 Conclusion

How redistributive should a tax and transfer system be? Politicians and economists alike disagree over the answer to this question. Often, a stumbling block is the argument that high taxes on the rich discourage investment and therefore reduce labor demand and wages. Recent empirical research on the incidence of corporate taxation finds evidence for this theory, but leaves the stated normative question unanswered.

In this paper, I show theoretically that "trickle down" effects reduce the optimal progressivity of the income tax code. I affiliate to the classical literature on optimal income taxation by deriving an intuitive formula for the optimal income tax rate, which is solely expressed in terms of sufficient estimable statistics. I show that the optimal tax rate depends negatively on the wage elasticity with respect to the capital stock. In a quantitative experiment, I compute the optimal reform of the income tax code in a rich model economy that replicates relevant patterns of US data. I find that the optimal income tax code is much more progressive than the status quo although general equilibrium effects substantially reduce the optimal progressivity. A unique decomposition of welfare gains into partial- and general equilibrium as well as into level- and distributional components reveals that general equilibrium forces mitigate both redistributional welfare gains but also level losses of moving towards a more progressive tax system. Finally, I show that investment subsidies effectively counteract the discouraging effect of progressive income taxes on investment. The inclusion of this policy instrument leads to an even more progressive optimal income tax code. Moreover, such a reform optimally creates high wealth inequality with low earners having little incentive to build up assets as they are protected by the welfare state.

1.A Appendix to Chapter 1

1.A.1 Proofs

Proof of Lemma 1

Let an $\{k(a)\}$ be an arbitrary collection of investment decisions. A firm with investment $k(a)$ and productivity draw θ solves

$$\pi(k(a),\theta) = \max_{l}\{F(\theta k(a), l) - wl\}. \tag{30}$$

Because of constant returns this problem is equivalent to

$$\pi(k(a),\theta) = \theta k(a) \max_{\tilde{l}}\{F(1,\tilde{l}) - w\tilde{l}\},$$

where $\tilde{l} \equiv \frac{l}{\theta k(a)}$. This implies that the ratio of labor to effective capital is the same for each firm. Market clearing then implies

$$\int l(a,\theta)dG(a) = \int \sum_{i=1}^{N} p_i \bar{\theta}_i k(a)\tilde{l}dG(a) = \mathbb{E}[\theta]K\tilde{l} = 1.$$

The definition of $\tilde{l}$ then immediately gives (1). By concavity of F, deriving (30) with respect to $\tilde{l}$ gives the necessary and sufficient first order condition

$$F_l(\theta k(a), l(a,\theta)) = F_l\left(\theta k(a), \frac{\theta k(a)}{\mathbb{E}[\theta]K}\right) = F_l\big(\mathbb{E}[\theta]K, 1\big) = w,$$

where the second equality follows from the fact that F_l is homogeneous of degree zero.

What is left to show is that profits are given by

$$\pi(a,\theta) = F_k(\mathbb{E}[\theta]K, 1)\theta k(a).$$

Plugging the expressions (1) and (4) into (30) gives

$$\pi(k(a),\theta) = F\left(\theta k(a), \frac{\theta k(a)}{\mathbb{E}[\theta]K}\right) - F_l(\mathbb{E}[\theta]K, 1).$$

Using the fact that F is homogeneous of degree one and that F_l is homogeneous of degree zero this is equivalent to

$$\pi(k(a),\theta) = \left[F(\mathbb{E}[\theta]K, 1) - F_l(\mathbb{E}[\theta]K, 1)\right]\frac{\theta k(a)}{\mathbb{E}[\theta]K}.$$

Using Euler's homogeneous function

$$F(\mathbb{E}[\theta]K, 1) = F_k(\mathbb{E}[\theta]K, 1)\mathbb{E}[\theta]K + F_l(\mathbb{E}[\theta]K, 1)$$

we obtain

$$\pi(k(a),\theta) = F_k(\mathbb{E}[\theta]K, 1)\frac{\theta k(a)}{\mathbb{E}[\theta]K}.$$

This holds for any arbitrary $k(a)$, hence also for the optimal one. This completes the proof.

□

Proof of Proposition 1

Note first that the non-negativity condition on the capital stock will not be binding as this would imply zero consumption in the second period, ruled out by the Inada condition. This holds both for the constrained optimum as for the laissez-faire equilibrium.

The first order condition with respect to capital is given by

$$u'(c_0) = \beta \mathbb{E}_\theta \left[u'\big(c_1(\theta)\big) \frac{\partial c_1(\theta)}{\partial K} \right], \tag{31}$$

where

$$\frac{\partial c_1(\theta)}{\partial K} = r^k \theta + \theta K \frac{\partial r^k}{\partial K} + \frac{\partial w}{\partial K}$$

The equilibrium wage is given by (4), implying

$$\frac{\partial w}{\partial K} = F_{kl}(\mathbb{E}[\theta]K, 1)\mathbb{E}[\theta].$$

We further use Euler's homogeneity theorem

$$F_{kk}(\mathbb{E}[\theta]K, 1)\mathbb{E}[\theta]K + F_{kl}(\mathbb{E}[\theta]K, 1) = 0,$$

implying

$$\frac{\partial r^k}{\partial K} = F_{kk}(\mathbb{E}[\theta]K, 1)\mathbb{E}[\theta] = -\frac{F_{kl}(\mathbb{E}[\theta]K, 1)}{K} = -\frac{1}{\mathbb{E}[\theta]K} \frac{\partial w}{\partial K}. \tag{32}$$

Hence, we can write the first order condition (31) as

$$u'(c_0) = \beta \mathbb{E}_\theta \left[u'\big(c_1(\theta)\big) \left(r^k \theta + \frac{\partial w}{\partial K} \left(1 - \frac{\theta}{\mathbb{E}[\theta]} \right) \right) \right] \tag{33}$$

which is equivalent to (EE2).

To see that μ is positive note that $\partial w / \partial K$ is postive by complementarity of capital and labor and that also the expectation term is positive. To see this note that both $u'(c_1(\theta))$ and the term in round brackets are decreasing in θ. The rest follows from $u' > 0$ and the fact that in expectation the term in round brackets is zero.

We need to show that the first order condition indeed gives the global maximum of (1). First, note that the left hand side of (31) is strictly increasing in K. Hence a sufficient condition for a global maximum is that the right hand side is strictly decreasing in K. This is equivalent in showing

$$\frac{\partial}{\partial K} \mathbb{E}_\theta \left[u'(c_1) \frac{\partial c_1(\theta)}{\partial K} \right] < 0.$$

Using the product rule this can be written as[19]

$$\mathbb{E}_\theta \left[u''(c_1(\theta)) \left(\frac{\partial c_1(\theta)}{\partial K} \right)^2 + u'(c_1(\theta)) \Big\{ \mathbb{E}[\theta] F_{kk} \theta + F_{kkl}(\mathbb{E}[\theta] - \theta) \Big\} \right].$$

[19]For notational convenience we use the shortcut $F = F(\mathbb{E}[\theta]K, 1)$, and similarly for the respective derivatives.

The first term in squared brackets and the first term in curly brackets are clearly negative as $u'', F_{kk} < 0$ and $u' > 0$. Moreover note that $\mathbb{E}_\theta[u'(c_1(\theta))(\mathbb{E}[\theta] - \theta)] > 0$ as $u'(c_1(\theta))$ is positive and decreasing in θ. Hence a sufficient condition for negativity of the whole term is $F_{kkl} < 0$. One can check that with a CES production function

$$F_{kl} = \frac{F_k F_l}{F},$$

and therefore

$$F_{kkl} = \frac{(F_{kk}F_l + F_k F_{ll})F - F_k^2 F_l}{F2} < 0.$$

Hence the second order condition is satisfied and the capital stock implicitly given by (EE2) is indeed the unique global maximum of (P1).

What is left to show is that this capital stock is indeed higher than the capital stock in the laissez-faire equilibrium. For this purpose, remember that the first order condition for the household in the laissez faire equilibrium (with homogeneous initial wealth) is given by

$$u'(c_0) = \beta \mathbb{E}_\theta \Big[u'(c_1(\theta))\theta \Big] r^k.$$

The left hand side of this equations equals the left hand side of (31). Assume by contradiction that the capital stock in the laissez faire equilibrium, $\tilde{K}$, is not less than the capital stock in the cosntrained optimum. Since the left hand side of the Euler equation is increasing in K, it then must be the case that

$$\mathbb{E}_\theta \Big[u'(\tilde{c}_1(\theta))\theta \Big] F_k(\mathbb{E}[\theta]\tilde{K}) \geq \mathbb{E}_\theta \Big[u'\big(\tilde{c}_1(\theta)\big)\theta \Big] F_k(\mathbb{E}[\theta]\tilde{K}) + \underbrace{F_{kl}(\mathbb{E}[\theta]\tilde{K}, 1)\mathbb{E}_\theta \Big[u'\big(\tilde{c}_1(\theta)\big)(\mathbb{E}[\theta] - \theta) \Big]}_{>0},$$

a contradiction.

As the laissez-faire allocation is feasible but the unique global maximum of (P1) differs and all agents are ex-ante identical, the constrained efficient allocation strictly Pareto dominates the allocation in the laissez-faire equilibrium. This completes the proof.

□

Proof of Proposition 2

The Lagrangian of problem (P2) is given by

$$L(\{a, k(a), \lambda(a)\}) = \int \Big[u(c_0(a)) + \beta \mathbb{E}_\theta \Big[u(c_1(a, \theta)) \Big] + \lambda(a)k(a) \Big] g(a)da.$$

The first order condition with respect to capital investment $k(a)$ of an agent with initial wealth a is given by

$$\begin{aligned} u'(c_0(a))g(a) =& \beta \mathbb{E}_\theta \Big[u'\big(c_1(a, \theta)\big)\theta \Big] r^k g(a) + \lambda(a)g(a) \qquad (34) \\ &+ \beta \int \mathbb{E}_\theta \Big[u'(c_1(\tilde{a}, \theta)) \Big(\theta k(\tilde{a}) \frac{\partial r^k}{\partial K} \frac{\partial K}{\partial k(a)} + \frac{\partial w}{\partial K} \frac{\partial K}{\partial k(a)} \Big) \Big] g(\tilde{a})d(\tilde{a}), \end{aligned}$$

The definition of the aggregate capital stock (??) gives $\partial K/\partial k(a) = g(a)$. Further, using equation (32) from the proof of Proposition 1 we get

$$\begin{aligned} u'(c_0(a)) =& \beta \mathbb{E}_\theta \Big[u'\big(c_1(a,\theta)\big)\theta \Big] r^k + \lambda(a) \\ &+ \beta \frac{\partial w}{\partial K} \int \mathbb{E}_\theta \Big[u'(c_1(\tilde{a},\theta)) \Big(1 - \frac{\theta}{\mathbb{E}[\theta]} \frac{k(\tilde{a})}{K} \Big) \Big] dG(\tilde{a}), \end{aligned}$$

equivalent to (EE3).

To show that μ is positive it is sufficient to show that $k(a)$ is non-decreasing in a. Then $u'(c_1(a,\theta))$ and the term in round brackets are both decreasing θ and non-increasing in a, and the same argument as in the proof of proposition 1 can be applied. Assume by contradiction that there exists a pair $a_l < a_h$ such that $k(a_l) > k(a_h) \geq 0$. Then clearly $u'(c_0(a_l)) > u'(c_0(a_h))$, implying

$$\beta \mathbb{E}_\theta \Big[u'(c_1(a_l,\theta))\theta \Big] r^k > \beta \mathbb{E}_\theta \Big[u'(c_1(a_h,\theta))\theta \Big] r^k + \lambda(a_h) \geq \beta \mathbb{E}_\theta \Big[u'(c_1(a_h,\theta))\theta \Big] r^k,$$

or equivalently

$$\mathbb{E}_\theta \Big[u'(c_1(a_l,\theta))\theta \Big] > \mathbb{E}_\theta \Big[u'(c_1(a_h,\theta))\theta \Big].$$

But with $k(a_l) > k(a_h)$, consumption in the second period satisfies $c_1(a_l,\theta) > c_1(a_h,\theta)$ for all possible realizations of θ. Hence

$$\mathbb{E}_\theta \Big[u'(c_1(a_l,\theta))\theta \Big] \leq \mathbb{E}_\theta \Big[u'(c_1(a_h,\theta))\theta \Big],$$

a contradiction. Hence capital investment in non-decreasing in a and μ is positive.

To show that (EE3) indeed gives a global maximum of (P2) we again check the seond order conditions. For this purpose we use

$$\begin{aligned} c_1(a,\theta) &= r^k \theta k(a) + w \\ &= F_k \theta k(a) + F_l \\ &= F + F_k(\theta k(a) - \mathbb{E}[\theta]K). \end{aligned}$$

Hence, we can write the partial derivative of $c_1(a,\theta)$ with respect to $k(a)$ as

$$\begin{aligned} \frac{\partial c_1(a,\theta)}{\partial k(a)} &= F_k \mathbb{E}[\theta] g(a) + F_{kk} \mathbb{E}[\theta] g(a)(\theta k(a) - \mathbb{E}[\theta]K) + F_k(\theta - \mathbb{E}[\theta] g(a)) \\ &= F_k \theta + F_{kl} \mathbb{E}[\theta] g(a) \Big(1 - \frac{\theta}{\mathbb{E}[\theta]} \frac{k(a)}{K} \Big), \end{aligned}$$

where we used Euler's homogeneous function theorem twice.

It is again easy to see that the left hand side of (EE2) is increasing in $k(a)$. A unique global maximum is hence achieved if we can show that the right hand side is decreasing in $k(a)$. This is equivalent to showing that the following term is negative,

$$\mathbb{E}_\theta\left[u''(c_1(a,\theta))\theta F_k\left\{F_k\theta + F_{kl}\mathbb{E}[\theta]g(a)\left(1-\frac{\theta}{\mathbb{E}[\theta]}\frac{k(a)}{K}\right)\right\}\right]+$$

$$\mathbb{E}_\theta\left[u'(c_1(a,\theta))\theta\right]\mathbb{E}[\theta]g(a)F_{kk} + \mathbb{E}[\theta]g(a)F_{kkl}\int \mathbb{E}_\theta\left[u'(c_1(\tilde{a},\theta))\left(1-\frac{\theta}{\mathbb{E}[\theta]}\frac{k(\tilde{a})}{K}\right)\right]dG(\tilde{a})$$

$$+F_{kl}\mathbb{E}_\theta\left[u''(c_1(a,\theta))\left\{F_k\theta + F_{kl}\mathbb{E}[\theta]g(a)\left(1-\frac{\theta}{\mathbb{E}[\theta]}\frac{k(a)}{K}\right)\right\}\right]g(a)$$

$$+F_{kl}\mathbb{E}_\theta\left[u'(c_1(a,\theta))\left(-\frac{\theta}{\mathbb{E}[\theta]}\frac{1}{K}\right)\right]g(a) < 0.$$

This is the case because $u' > 0$, $u'' < 0$, $F_k, F_{kl} > 0$, $F_{kk} < 0$, and, as has been shown in the proof of Proposition 1 $F_{kkl} < 0$.

□

1.A.2 Numerical Example for Constraint Efficient Allocation

Figure 1.7 illustrates the difference between the laissez-faire and the constrained efficient allocation via a numerical example. In this example only two outcomes of the investment are possible. It can be successful, in which case $\bar{\theta}_g = 10$, or it can be unsuccessful, in which case $\bar{\theta}_b = 0$, i.e. all invested capital is lost. The coefficient of relative risk aversion is set to $\gamma = 2$ and the discount factor is set to $\beta = 1$. For the production function we used the Cobb-Douglas case ($\psi \to 0$) with the capital share $\alpha = 0.34$. Initial wealth is assumed to be equally distributed over the interval $[0, 10]$.

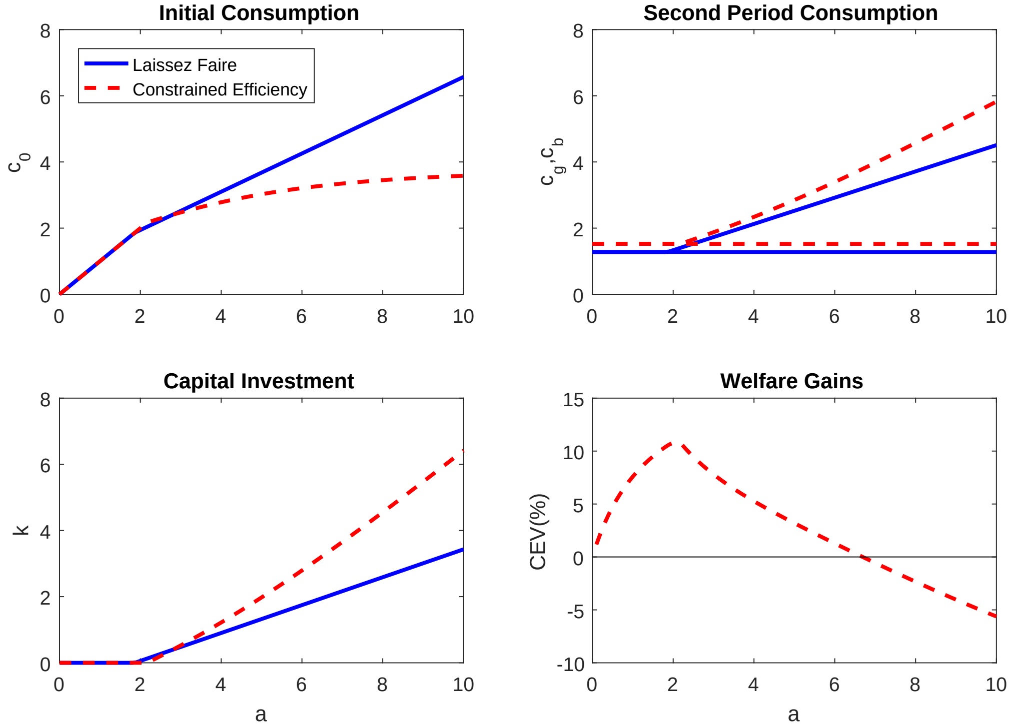


Figure 1.7: Laissez-Faire vs. Constrained Efficient Allocation

In the south-west panel we observe that the planner in general dictates higher levels of investment than in the laissez-faire. This is financed by a reduction in initial consumption (north-west panel). As a result not only income from investment increases but also the wage. The level of consumption in the second period is hence higher for all agents and with both realizations of shocks (north-east panel). As discussed above, the very rich agents are worse off in the constrained efficient allocation, while most of the population is better off (south-east panel). We also observe that the welfare gains reach the maximum around the point where the non-negativity constraint ceases to bind.

1.A.3 Analytical Ramsey Problem with Infinite Horizon and Riskless Asset

I extend the analysis of section 1.3 in two dimensions. First, the time horizon is infinite. I denote the realization of a shock in a particular period t by the subscript t and the history of realisations up to period t by superscript t, i.e. $\theta^t = \{\theta_1, \theta_2, ..., \theta_t\}$. I further assume that the shock is identically and independently distributed not only across agents but also across time. Second, I allow agents to invest in two assets: risky firm capital as in the two period model and a riskless bond that pays the exogenous interest rate $R < 1/\beta$.

Agents maximize their expected discounted lifetime utility,

$$\mathbb{E}_{\{\theta_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Consumption, firm investment and bond holdings in each period are financed by current assets,

$$c_t + k_t + b_t = a_t.$$

Current assets in turn are the sum of an agent's (net) capital- bond- and wage income as well as the lump sum transfer,

$$a_t = \big[(1-\tau) r_t^k \theta_t + s\big] k_t + R b_t + w_t + T_t.$$

Following Angeletos (2007) it is useful to define the net present value of future riskless income by

$$H_t = \sum_{t=1}^{\infty} \frac{w_t + T_t}{R^t}. \tag{35}$$

Agents are allowed to borrow up to the natural borrowing limit,

$$b_t \geq -H_t. \tag{36}$$

This problem has a recursive representation. To be specific, an agent's value in period t is given by

$$V_t(a_t) = \max_{k_t \geq 0, b_t \geq -H_t} u(c_t(a_t)) + \beta \mathbb{E}_\theta V_{t+1}(a_{t+1}),$$

where

$$c_t(a_t) + k_{t+1}(a_t) + b_{t+1}(a_t) = a_t \tag{37}$$

and

$$a_{t+1} = \big[(1-\tau_{t+1}) r_{t+1}^k \theta_{t+1} + s\big] k_{t+1}(a_t) + R b_{t+1} + w_{t+1} + T_{t+1}. \tag{38}$$

Note that value and policy functions have a time subscript to make clear that both agents' value- and policy functions depend on time varying wages, capital returns and transfers.

The agents' first order condition with respect to capital investment is then given by

$$u'(c_t(a_t)) = \beta \mathbb{E}_\theta \Big[u'(c_{t+1}(a_{t+1})) \big((1-\tau_{t+1}) r_{t+1}^k \theta_{t+1} + s \big) \Big]. \tag{39}$$

Further, the Euler equation with respect to bond holdings is

$$u'(c_t(a_t)) = \beta R \mathbb{E}_\theta [u'(c_{t+1}(a_{t+1}))]. \tag{40}$$

The Ramsey Problem with Linear Capital Income Taxes

As in section 1.3 I first consider the case without investment subsidies, i.e. $s = 0$. The Ramsey planner is only allowed to choose a single tax rate τ for every future period and is required to balance the budget period by period. She thus solves

$$\max_{\tau} \sum_{t=0}^{\infty} \int \sum_{\theta^t} p(\theta^t) u\big(c_t(a_0, \theta^t)\big) dG(a_0) \tag{41}$$

s.t. $\forall t \geq 0$

$$\begin{aligned}
&c_t(a_0, \theta^t) = a_t(a_0, \theta^t) - k_{t+1}(a_0, \theta^t) - b_{t+1}(a_0, \theta^t) \\
&a_{t+1}(a_0, \theta^{t+1}) = r^k_{t+1}(1-\tau)\theta_{t+1} k_{t+1}(a_0, \theta^t) + R b_{t+1}(a_0, \theta^t) + w_{t+1} + T_{t+1} \\
&u'(c_t(a_0, \theta^t)) = \beta \mathbb{E}_{\theta_{t+1}} \big[u'(c_{t+1}(a_0, \theta^{t+1}))\big] R \\
&u'(c_t(a_0, \theta^t)) = \beta \mathbb{E}_{\theta_{t+1}} \big[u'(c_{t+1}(a_0, \theta^{t+1})) r^k (1-\tau)\theta_{t+1}\big]
\end{aligned}$$

and $\forall t \geq 1$

$$\begin{aligned}
&K_t = \int \sum_{\theta^{t-1}} p(\theta^{t-1}) k_t(a_0, \theta^{t-1}) dG(a_0) \\
&w_t = F_k(\mathbb{E}[\theta] K_t, 1) \\
&r^k_t = F_l(\mathbb{E}[\theta] K_t, 1) \\
&T_t = \tau r^k_t \mathbb{E}[\theta] K_t
\end{aligned}$$

Note that whenever we multiplicatively separate $\mathbb{E}[\theta]$ and the capital stock K_t we are allowed to do so by the assumption that the shock is identically and independently distributed across agents and time.

The first order condition with respect to τ is given by

$$\int \sum_{t=1}^{\infty} \beta^t \sum_{\theta^t} p(\theta^t) u'(c_t(a_0, \theta^t)) \frac{\partial c_t(a_0, \theta^t)}{\partial \tau} dG(a_0) = 0, \tag{42}$$

where $p(\theta^t)$ denotes the probability of a particular history of shocks up to time t, where the partial derivatives entering this condition are

$$\frac{\partial c_t(a_0, \theta^t)}{\partial \tau} = \frac{\partial K_t}{\partial \tau} \tau r^k_t \mathbb{E}[\theta] + \left[r^k_t \mathbb{E}[\theta] K_t + \frac{\partial w_t}{\partial K_t} \frac{\partial K_t}{\partial \tau}(1-\tau) \right] \left(1 - \frac{k_t(a_0, \theta^{t-1})\theta_t}{K_t \mathbb{E}[\theta]}\right).$$

I define the dynamic composite redistribution and insurance terms analogously to their static counterparts (17) and (18). The "dynamic composite redistribution term" is

$$D_t = \frac{1}{\int \sum_{\theta^t} p(\theta^t) u'(c_t(a_0, \theta^t)) dG(a_0)} \times \int \sum_{\theta^{t-1}} p(\theta^{t-1}) \left(1 - \frac{k_t(a_0, \theta^{t-1})}{K_t}\right) \mathbb{E}\big[u'(c_t(a_0, \theta^t)) | \theta^{t-1}\big] dG(a_0) \geq 0 \tag{43}$$

and the "dynamic composite insurance term" is

$$I_t = \frac{1}{\int \sum_{\theta^t} p(\theta^t) u'(c_t(a_0, \theta^t)) dG(a_0)} \times \int \sum_{\theta^{t-1}} p(\theta^{t-1}) \frac{k(a_0, \theta^{t-1})}{K_t} \mathbb{E}\left[u'(c_t(a_0, \theta^t)) \left(1 - \frac{\theta_t}{\mathbb{E}[\theta]}\right) \Big| \theta^{t-1} \right] dG(a_0) > 0. \tag{44}$$

These terms have the same interpretation as their analogues in the two period model. The term D_t captures the wealth inequality at the beginning of period t. Contrary to the two period case it is now an endogenous object (for $t \geq 2$). The insurance term is again an average of individual risk terms weighted by the degree of capital investment. Note that

$$I_t + D_t = \frac{1}{\int \sum_{\theta^t} p(\theta^t) u'(c_t(a_0, \theta^t)) dG(a_0)} \times \int \sum_{\theta^t} p(\theta^t) u'(c_t(a_0, \theta^t)) \left(1 - \frac{\theta_t k_t(a_0, \theta^{t-1})}{\mathbb{E}[\theta] K_t}\right) dG(a_0) > 0.$$

Further, it is useful to define

Using these definitions one can show that the first order condition is equivalent to the intuitive tax formula[20]

$$\frac{\tau}{1-\tau} = \sum_{t=1}^{\infty} \omega_t \big(I_t + D_t\big) \left(\frac{1}{\epsilon_{K_t, 1-\tau}} - \epsilon_{w_t, K_t} \frac{w_t}{r_t K_t \mathbb{E}[\theta]} \right). \tag{45}$$

This formula is the infinite horizon analogue of equation (19). It is a weighted average of period terms that are identical to the two period case. The period t effect enters with weight

$$\omega_t = \frac{R^{-t} \epsilon_{K_t, 1-\tau} r_t^k K_t}{\sum_{j=1}^{\infty} R^{-j} \epsilon_{K_j, 1-\tau} r_j^k K_j}.$$

The weight ω_t is higher the smaller t (the bigger R^{-t}), the higher aggregate capital income $\mathbb{E}[\theta] r_t^k K_t$ in t and the more elastic the capital stock in this period.

The Ramsey Problem with Linear Capital Income Taxes and Investment Subsidies

I now incorporate investment subsidies. The Ramsey planner solves

[20] See the Appendix for a detailed derivation.

$$\max_{\tau,s}\sum_{t=0}^{\infty}\int\sum_{\theta^t}p(\theta^t)u\big(c_t(a_0,\theta^t)\big)dG(a_0) \tag{46}$$

$$\begin{aligned}
\text{s.t.}\quad &\forall t\geq 0\\
&c_t(a_0,\theta^t)=a_t(a_0,\theta^t)-k_{t+1}(a_0,\theta^t)-b_{t+1}(a_0,\theta^t)\\
&a_{t+1}(a_0,\theta^{t+1})=\big(r^k_{t+1}(1-\tau)\theta_{t+1}+s\big)k_{t+1}(a_0,\theta^t)+Rb_{t+1}(a_0,\theta^t)+w_{t+1}+T_{t+1}\\
&u'(c_t(a_0,\theta^t))=\beta\mathbb{E}_{\theta_{t+1}}\big[u'(c_{t+1}(a_0,\theta^{t+1}))\big]R\\
&u'(c_t(a_0,\theta^t))=\beta\mathbb{E}_{\theta_{t+1}}\big[u'(c_{t+1}(a_0,\theta^{t+1}))\big(r^k(1-\tau)\theta_{t+1}+s\big)\big]\\
\text{and}\quad &\forall t\geq 1\\
&K_t=\int\sum_{\theta^{t-1}}p(\theta^{t-1})k_t(a_0,\theta^{t-1})dG(a_0)\\
&w_t=F_k(\mathbb{E}[\theta]K_t,1)\\
&r^k_t=F_l(\mathbb{E}[\theta]K_t,1)\\
&T_t=\big(\tau r^k_t\mathbb{E}[\theta]-s\big)K_t
\end{aligned}$$

The first four constraints are the necessary and sufficient conditions for agents' optimization given policies $\{\tau,s,\{T_t\}_{t=1}^{\infty}\}$ and prices $\{w_t,r^k_t\}_{t=1}^{\infty}$. The fifth constraint simply defines the aggregate capital stock in period t. The sixth and seventh constraint denotes the equilibrium wage and capital return given capital investment. Finally, the last constraint requires the government budget to clear period by period.

The first order conditions of this problem are

$$\sum_{t=1}^{\infty}\beta^t\int\sum_{\theta^t}p(\theta^t)u'(c_t(a_0,\theta^t))\frac{\partial c_t(a_0,\theta^t)}{\partial\tau}dG(a_0)=0$$

and

$$\sum_{t=1}^{\infty}\beta^t\int\sum_{\theta^t}p(\theta^t)u'(c_t(a_0,\theta^t))\frac{\partial c_t(a_0,\theta^t)}{\partial s}dG(a_0)=0,$$

where the partial derivatives of consumption with respect to the tax rate and with respect to the subsidy that enter this equations are

$$\frac{\partial c_t(a_0,\theta^t)}{\partial\tau}=\frac{\partial K_t}{\partial\tau}\big(\tau r^k_t\mathbb{E}[\theta]-s\big)+\left[r^k_t\mathbb{E}[\theta]K_t+\frac{\partial w_t}{\partial K_t}\frac{\partial K_t}{\partial\tau}(1-\tau)\right]\left(1-\frac{k_t(a_0,\theta^{t-1})\theta_t}{K_t\mathbb{E}[\theta]}\right)$$

and

$$\frac{\partial c_t(a_0,\theta^t)}{\partial s}=\frac{\partial K_t}{\partial s}\big(\tau r^k_t\mathbb{E}[\theta]-s\big)-K_t\left(1-\frac{k_t(a_0,\theta^{t-1})}{K_t}\right)+\frac{\partial w_t}{\partial K_t}\frac{\partial K_t}{\partial s}(1-\tau)\left(1-\frac{k_t(a_0,\theta^{t-1})\theta_t}{K_t\mathbb{E}[\theta]}\right).$$

From the first order condition with respect to s we obtain the optimal subsidy as a function of τ. Denoting

$$\chi_t=R^{-t}\epsilon_{K_t,s}K_t,$$

we get

$$s(\tau) = \frac{1}{\sum_{j=1}^{\infty} \chi_j \left(1 + \frac{D_j}{\epsilon_{K_j,s}}\right)} \sum_{t=1}^{\infty} \chi_t \left(\tau \mathbb{E}[\theta] r_t^k + (1-\tau)\epsilon_{w_t,K_t} \frac{w_t}{K_t}(I_t + D_t)\right). \tag{47}$$

Plugging this into the first order condition with respect to τ and rearranging terms yields a formula for the optimal tax rate,

$$\frac{\tau}{1-\tau} = \frac{1}{\sum_{j=1}^{\infty} r_j^k \zeta_j \varphi_j} \sum_{t=1}^{\infty} r_t^k \zeta_t \varphi_t (I_t + D_t) \left(\frac{1}{\epsilon_{K_t,1-\tau}} \frac{1}{\varphi_t \sum_{j=1}^{\infty} \zeta_j} - \epsilon_{w_t,K_t} \frac{w_t}{\mathbb{E}[\theta] r_t^k K_t} \right), \tag{48}$$

where

$$\zeta_t = R^{-t} \epsilon_{K_t,1-\tau} K_t.$$

and

$$\varphi_t = \frac{1}{\sum_{j=1}^{\infty} \zeta_j} - \frac{\epsilon_{K_t,s}}{\epsilon_{K_t,1-\tau}} \frac{1}{\sum_{j=1}^{\infty} \zeta_j \left(1 + \frac{D_j}{\epsilon_{K_j,s}}\right)}.$$

Derivations of Infinite Horizon Tax Formulas

The first order condition with respect to s is given by

$$\sum_{t=1}^{\infty} \beta^t \int \sum_{\theta^t} p(\theta^t) u'(c_t(a_0,\theta^t)) \left\{ \frac{\partial K_t}{\partial s}(\tau r_t^k \mathbb{E}[\theta] - s) - K_t \left(1 - \frac{k_t(a_0,\theta^{t-1})}{K_t}\right) + \right.$$
$$\left. + \frac{\partial w_t}{\partial K_t} \frac{\partial K_t}{\partial s}(1-\tau) \left(1 - \frac{k_t(a_0,\theta^{t-1})\theta_t}{K_t \mathbb{E}[\theta]}\right) \right\} dG(a_0) = 0.$$

This can be rewritten in terms of elasticities instead derivatives as

$$\sum_{t=1}^{\infty} \beta^t \int \sum_{\theta^t} p(\theta^t) u'(c_t(a_0,\theta^t)) \left\{ \epsilon_{K_t,s} \tau r_t^k \mathbb{E}[\theta] K_t - \epsilon_{K_t,s} K_t s - K_t s \left(1 - \frac{k_t(a_0,\theta^{t-1})}{K_t}\right) + \right.$$
$$\left. + \epsilon_{w_t,K_t} \epsilon_{K_t,s} w_t (1-\tau) \left(1 - \frac{k_t(a_0,\theta^{t-1})\theta_t}{K_t \mathbb{E}[\theta]}\right) \right\} dG(a_0) = 0.$$

We consider the partial sum until period T and use

$$\lim_{T\to\infty} \sum_{t=1}^{T} \beta^t X_t = \sum_{t=1}^{\infty} \beta^t X_t.$$

Using the fact that the Euler equation for riskless bonds implies

$$\sum_{\theta^t} p(\theta^t) u'(c_t(a_0,\theta^t)) = \beta^{T-t} R^{T-t} u'(c_T(a_0,\theta^T))$$

we can rewrite the first order condition as

$$\lim_{T\to\infty}\Bigg\{\beta^T R^T \mathbb{E}[u'(c_T)]\sum_{t=1}^{T}\beta^T R^{-t}\Big(\epsilon_{K_t,s}\tau r_t^k \mathbb{E}[\theta]K_t - \epsilon_{K_t,s}K_t s\Big)+$$

$$\sum_{t=1}^{T}\beta^t\sum_{\theta^t}p(\theta^t)u'(c_t(a_0,\theta^t))\Bigg[-K_t s\Bigg(1-\frac{k_t(a_0,\theta^{t-1})}{K_t}\Bigg)+$$

$$+\epsilon_{w_t,K_t}\epsilon_{K_t,s}w_t(1-\tau)\Bigg(1-\frac{k_t(a_0,\theta^{t-1})\theta_t}{K_t\mathbb{E}[\theta]}\Bigg)\Bigg]\Bigg\}=0,$$

where $\mathbb{E}[u'(c_T)]$ denotes the average marginal utility of consumption in period T.[21] Dividing by $\beta^T R^T\mathbb{E}[u'(c_T)]$ and using

$$\beta^T R^{T-t}\mathbb{E}[u'(c_T)] = \beta^t \mathbb{E}[u'(c_t)] \quad \forall t,$$

we obtain

$$\lim_{T\to\infty}\Bigg\{\sum_{t=1}^{T}R^{-t}\Big(\epsilon_{K_t,s}\tau r_t^k\mathbb{E}[\theta]K_t-\epsilon_{K_t,s}K_t s\Big)+$$

$$\sum_{t=1}^{T}R^{-t}\Bigg[-K_t s D_t+\epsilon_{w_t,K_t}\epsilon_{K_t,s}w_t(1-\tau)\big(I_t+D_t\big)\Bigg]\Bigg\}=0.$$

Taking the limit and rearranging terms gives the desired expression for the subsidy s.

$$s=\frac{1}{\sum_{j=1}^{\infty}R^{-t}\epsilon_{K_t,s}K_t\Big(1+\frac{D_j}{\epsilon_{K_j,s}}\Big)}\sum_{t=1}^{\infty}R^{-t}\epsilon_{K_t,s}K_t\Big(\tau\mathbb{E}[\theta]r_t^k+(1-\tau)\epsilon_{w_t,K_t}\frac{w_t}{K_t}(I_t+D_t)\Big).$$

To calculate the optimal tax rate plug this expression into the first order condition with respect to τ, which is given by

$$\sum_{t=1}^{\infty}\beta^t\int\sum_{\theta^t}p(\theta^t)u'(c_t(a_0,\theta^t))\Bigg\{\frac{\partial K_t}{\partial\tau}(\tau r_t^k\mathbb{E}[\theta]-s)+\Bigg[r_t^k\mathbb{E}[\theta]K_t+\frac{\partial w_t}{\partial K_t}\frac{\partial K_t}{\partial\tau}(1-\tau)\Bigg]\times$$

$$\times\Bigg(1-\frac{k_t(a_0,\theta^{t-1})\theta_t}{K_t\mathbb{E}[\theta]}\Bigg)\Bigg\}dG(a_0)=0.$$

We further rewrite this condition (with the optimal subsidy plugged in) in terms of elasticities and obtain

$$\sum_{t=1}^{\infty}\beta^t\int\sum_{\theta^t}p(\theta^t)u'(c_t(a_0,\theta^t))\Bigg\{\epsilon_{K_t,1-\tau}K_t\frac{\tau}{1-\tau}\mathbb{E}[\theta]\frac{\sum_{j=1}^{\infty}\chi_j r_j^k}{\sum_{j=1}^{\infty}\Big(1+\frac{D_j}{\epsilon_{K_j,s}}\Big)}+$$

$$\epsilon_{K_t,1-\tau}K_t\frac{\sum_{j=1}^{\infty}R^{-j}\epsilon_{K_j,s}\epsilon_{w_j,K_j}w_j(I_j+D_j)}{\sum_{j=1}^{\infty}\chi_j\Big(1+\frac{D_j}{\epsilon_{K_j,s}}\Big)}-\epsilon_{K_t,1-\tau}r_t^k\mathbb{E}[\theta]K_t\frac{\tau}{1-\tau}+$$

$$\Big(r_t^k\mathbb{E}[\theta]K_t-\epsilon_{w_t,K_t}\epsilon_{K_t,1-\tau}w_t\Big)\Bigg(1-\frac{k_t(a_0,\theta^{t-1})\theta_t}{K_t\mathbb{E}[\theta]}\Bigg)\Bigg\}dG(a_0)=0,$$

[21] We use this notation as it also denotes the unconditional expectation. Unconditional in the sense that it does not condition on initial assets.

where χ_j is defined as in the main text. Analogous to above we write this sum as the limit of the partial sum, i.e. we use $\sum_{t=1}^{\infty} \beta^t X_t = \lim_{T\to\infty} \sum_{t=1}^{T} \beta^t X_t$ and we use $\beta^t \mathbb{E}[u'(c_t)] = \beta^T R^{T-t} \mathbb{E}[u'(c_T)]$ to obtain a multiplicative constant in front of the sum, by which we divide. Taking the limit and rearranging terms then gives

$$\frac{\tau}{1-\tau}\mathbb{E}[\theta]\left(\sum_{t=1}^{\infty} R^{-t}\epsilon_{K_t,1-\tau}K_t r_t^k - \sum_{t=1}^{\infty} R^{-t}\epsilon_{K_t,1-\tau}K_t \frac{\sum_{j=1}^{\infty}\chi_j r_t^k}{\sum_{j=1}^{\infty}\left(1+\frac{D_j}{\epsilon_{K_j,s}}\right)}\right) =$$

$$\sum_{t=1}^{\infty} R^{-t}\epsilon_{K_t,1-\tau}K_t \frac{\sum_{j=1}^{\infty} R^{-j}\epsilon_{K_j,s}\epsilon_{w_j,K_j}w_j(I_j+D_j)}{\sum_{j=1}^{\infty}\chi_j\left(1+\frac{D_j}{\epsilon_{K_j,s}}\right)} +$$

$$+\sum_{t=1}^{\infty} R^{-t}\left(r_t^k \mathbb{E}[\theta]K_t - \epsilon_{w_t,K_t}\epsilon_{K_t,1-\tau}w_t\right)(I_t+D_t).$$

Dividing by $\sum_{t=1}^{\infty} R^{-t}\epsilon_{K_t,1-\tau}K_t$ gives

$$\frac{\tau}{1-\tau}\sum_{t=1}^{\infty} R^{-t}K_t r_t^k \epsilon_{K_t,1-\tau}\varphi_t =$$

$$\sum_{t=1}^{\infty} R^{-t}K_t r_t^k \epsilon_{K_t,1-\tau}(I_t+D_t)\left(\frac{1}{\epsilon_{K_t,1-\tau}}\frac{1}{R^{-j}\epsilon_{K_j,1-\tau}K_j} - \epsilon_{w_t,K_t}\frac{w_t}{r_t^k\mathbb{E}[\theta]K_j}\varphi_t\right),$$

where φ_t is defined as in the main text. This is equivalent to

$$\frac{\tau}{1-\tau} = \frac{1}{\sum_{j=1}^{\infty} r_j^k R^{-j}\epsilon_{K_j,1-\tau}K_j\varphi_j}\times$$

$$\sum_{t=1}^{\infty} r_t^k R^{-t}\epsilon_{K_t,1-\tau}K_t\varphi_t(I_t+D_t)\left(\frac{1}{\epsilon_{K_t,1-\tau}}\frac{1}{\varphi_t\sum_{j=1}^{\infty} R^{-j}\epsilon_{K_j,1-\tau}K_j} - \epsilon_{w_t,K_t}\frac{w_t}{\mathbb{E}[\theta]r_t^k K_t}\right),$$

the desired formula.

1.A.4 Comparison to Standard Incomplete Markets Model

In this section I compare the policy prescriptions implied by my model with the ones implied by the standard incomplete markets model of Aiyagari (1994), where all production takes place in a riskless corporate sector. This model is nested in the present framework ($\theta = 0$ for all agents). Davila et al. (2012) show that in this setup the constrained efficient capital stock can, in principle, be higher or lower than the capital stock in the laissez-faire equilibrium. For a calibration of the labor income process that is able to generate realistic wealth heterogeneity a higher capital stock is desirable as the implied price changes (higher wages, lower interest rate) induce redistribution from rich agents, whose main income source are capital returns to poor agents with mostly wage income. Thus, as in my model, one would not want to tax rich agents (with high capital income) too much as this discourages investment, which in turn reduces wages. However, with 5% of agents owning about 60% of the capital stock the modeling of how these agents become, and stay, rich is crucial in order to assess the quantitative significance of trickle down effects. The calibration in Davila et al. (2012) that generates the high wealth dispersion of the data follows a simplified version of Castañeda et al. (2003). There, the highest productivity state is more than 100 times bigger than the second highest and agents in this high state face a risk of about 20% of being more than 100 times less productive the following period.[22] This high risk of the high ability households make them save at very high rates for precautionary reasons. This feature of being in an extremely high state for a short period of time makes saving very inelastic to changes in the tax rate.

In contrast, in my model high ability entrepreneurs are more likely to stay productive. High wealth holdings are to most extent not a result of precautionary savings of the highest earners but due to a persistent high ability of managing a firm. Thus, agents react much more elastically to changes in the tax code. In order to assess the quantitative impact of including entrepreneurship in the model I re-calibrate the nested standard incomplete markets model (with $\theta = 0$ for all agents) and find the optimal tax reform for this model. The re-calibrated model should (i) match all targeted moments of section 1.4.2 that are not related to entrepreneurship (see Table 1.2) and (ii) generate a wealth distribution that is approximately the same as in my benchmark model.

Re-Calibrating the Model. I achieve this goal by calibrating a three state Markov chain for working ability. Thereby I impose that (1,3) and the (3,1) element of the transition matrix are equal to zero. This leaves me with seven free parameters. The three productivity states are given by

$$\eta = \begin{bmatrix} 0.5 & 2.1 & 41.0 \end{bmatrix}$$

and the Markov transition matrix by

$$P_\eta = \begin{bmatrix} 0.91 & 0.09 & 0 \\ 0.015 & 0.98 & 0.005 \\ 0 & 0.35 & 0.65 \end{bmatrix}.$$

Additionally, I recalibrate the time discount factor $\beta = 0.915$ in order to again match the capital output ratio. All other parameters (including the benchmark policy parameters) are the same as in section 1.4.2.

[22]The authors use a simplified three state version of the process taken from Diaz et al. (2003). There the process is a bit less extreme. Still the highest productivity state is about 9 times the middle one, and the probability of leaving the highest state is around 8%.

Table 1.11: Calibrated Moments in Model without Entrepreneurs

Moment	Benchmark	Without Entrepreneurs
Capital-Output Ratio	2.64	2.66
Wealth Gini index	0.83	0.83
Population weighted average tax rate	7%	6%
Income tax revenue / GDP	10%	11%

Table 1.11 compares the targeted moments and Table 1.12 the wealth distribution of the benchmark economy to the economy without entrepreneurship.

Table 1.12: Wealth Distribution in Model without Entrepreneurs

Top	1%	5%	10%	20%	50%
Benchmark	24%	57%	72%	86%	99%
Withouth Entrepreneurs	21%	57%	76%	88%	99%

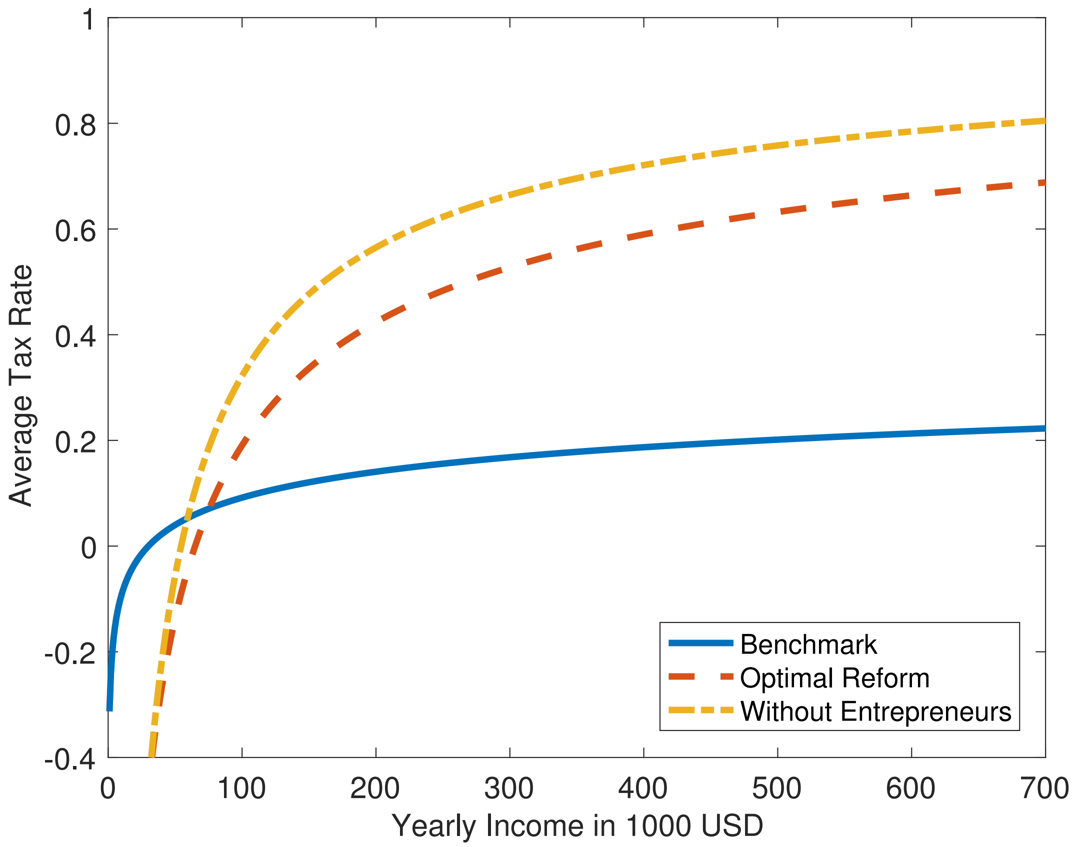


Figure 1.8: Benchmark and Reform Tax Schedules

The Optimal Tax Reform. I then perform the same optimization exercise as in section 1.5.1, i.e. I find the tax progressivity τ_1, which maximizes utilitarian social welfare, again accounting for the transitional dynamics to the new steady state. I find that for this re-calibrated model the optimum would be $\tau_1 = 0.64$, compared to $\tau_1 = 0.49$ for the benchmark model. The two tax codes are depicted in Figure 1.8. The highest productivity state is about 20 times higher than the second one and there is a risk of around one third of leaving this high state. This feature results in very inelastic savings for the highest earners. Thus their income can be taxed at much higher rates than in the economy with entrepreneurs.

Chapter 2

Inheritance Taxation and Wealth Effects on the Labor Supply of Heirs

with Fabian Kindermann and Dominik Sachs

2.1 Introduction

Inheritances are of growing importance for Western economies. Using data from France, Piketty (2011) shows that since the 1950s the annual flow of inheritances has been ever increasing, so that in 2010 it amounted to roughly 15 percent of national income. He also predicts that this share could become as large as 25 percent in the mid 21st century. Following his theoretical arguments, it is quite likely that a similar (and potentially even stronger) trend should be observed in other countries with low economic and population growth such as Spain, Italy and Germany (Piketty, 2011, p.1077). This development clearly highlights the increasing power of an inheritance tax in raising revenue.[1]

Despite the apparent importance of the topic, the incentive costs of inheritance taxation are not very well understood (Kopczuk, 2013). Measuring them empirically is a complicated task, because wealth transfers "are infrequent (at the extreme, occurring just at death), thereby allowing for a long period of planning, making expectations about future tax policy critical and empirical identification of the effect of incentives particularly hard" (Kopczuk, 2013, p.330). Furthermore, inheritances shape incentives along various dimensions, like wealth accumulation, labor supply and entrepreneurship.

In this paper we make progress on understanding and quantifying the revenue effects of inheritance taxation by elaborating one particular channel: labor supply of heirs. Concretely, we tackle the following policy question: For each Euro of revenue raised directly through inheritance taxes, how much additional labor income tax revenue from heirs can the government expect to obtain?

Answering this question purely empirically is problematic because it is difficult to identify the impact of inheritances on the earnings of heirs. One reason for this is that inheritances can be (imperfectly) anticipated and therefore already shape labor earnings prior to receipt. Further, settings with exogenous variation in inheritances are rare.[2] By

[1] We use the terms bequest taxes and inheritance taxes interchangeably in this paper albeit the fact that they might have different effects once an individual bequeathes to more than one heir and tax schedules are not proportional. For the experiments carried out in this paper, such a distinction, however, plays no role.

[2] There exists a small empirical literature on that issue to which we relate below in the literature

contrast, there exists quasi-experimental evidence regarding the wealth effect of lottery gains on labor income (Imbens et al., 2001; Cesarini et al., 2017). Our methodological approach is to calibrate a version of the workhorse life-cycle model of the macroeconomics literature to be consistent with this quasi-experimental evidence and then answer our policy question through the lens of this model.

As a theoretical warm-up, we first set up a simple two-period overlapping generations framework with stochastic bequests to analyze the tax revenue effects of a change in the bequest tax rate. We formally isolate the revenue effect that is due to the labor supply of (potential) heirs. We show that the marginal propensity to earn out of unearned income is not a sufficient statistic for the change in their lifecycle labor supply (and therefore labor tax revenue) because an increase in the bequest tax is not an unanticipated reduction in wealth. Due to anticipation, two effects arise on top of the simple standard wealth effect: (i) Individuals do form expectations about the inheritances they will receive and accordingly adjust their labor supply. (ii) If inheritances are stochastic, even individuals that do not inherit but did assign a positive probability on receiving an inheritance also adjust their life-cycle labor supply.

We then study the quantitative importance of all these effects in a state of the art life-cycle labor supply model that accounts for expectations. We build such a model that features consumption, labor supply and savings decisions, heterogeneous labor productivity profiles and realistic expectations about the size and timing of bequests. We calibrate it to the German economy, most importantly to match the joint distribution on the size and timing of inheritances as well as labor earnings profiles.[3] To achieve credible magnitudes for the implied wealth effects, we target quasi-experimental evidence on wealth effects based on lottery gains (Cesarini et al., 2017). Specifically, we distribute lottery gains of different sizes among individuals of different ages in our model in the same way as they are distributed in the data set of Cesarini et al. (2017). We then measure the resulting impulse response function for labor earnings and vary preference parameters until the model predicted impulse response matches the empirical one.

The only feature of our model, for which neither quasi-experimental evidence nor the used survey data provide us with clear guidance on how to calibrate it, are expectations about the size of inheritances. Different assumptions on rational expectations can be consistent with the cross-sectional distribution of inheritances and earnings of the heirs. We therefore consider a class of expectations, that captures two special cases as polar outcomes: (i) Conditional on the age and the earnings profile, all individuals draw their inheritance from the estimated cross-sectional distribution. (ii) Conditional on the bequeather dying at a certain time, the heir knows for sure how much she inherits. Besides these two polar cases, we consider linear combinations of the two that are all consistent with the cross-sectional joint distribution of inheritances and earnings of the heirs.

Equipped with this quantitative model, we conduct the following policy experiment: We let the government levy a proportional tax of 1 percent on all bequests and calculate the resulting change in lifetime income and income tax payments for the total population

section.

[3]Whereas the quasi-experimental evidence that we target was obtained for Sweden (Cesarini et al., 2017), we currently don't have access to Swedish data on earnings and bequests and therefore calibrate the model be consistent with German survey data. We will soon have access to the cross sectional features of the joint distribution of inheritances, income and age when inheriting for Sweden (based on administrative data), which will then allow us to conduct the quantitative analysis for Sweden in future versions of this paper.

of our model. For our benchmark calibration, we find that any Euro of bequests that is taken away from heirs increases their lifetime income by around 18.50 cent in net present value, that is discounted to the year of inheritance receipt. In terms of income tax payments this means that any Euro of revenue directly obtained through bequest taxes leads to additional tax revenues of around 7.64 cents (in net present value).

We decompose this number in two different ways. First, we show that anticipation effects constitute approximately half of it. This highlights the importance of considering a model with expectations and not only relying on a simple back-of-the envelope calculation, where one would focus on post-inheritance earnings of heirs only. More generally, our approach quantifies the bias that would occur in an estimation where only the labor supply changes of heirs after the inheritance would be taken into account and neither anticipation effects nor labor supply changes of non-heirs would be considered. Second, we consider heterogeneity in effects and answer our policy question for different earnings levels. We find that the number is increasing in earnings of the heirs, which simply reflects that a decrease in leisure is associated with a higher earnings gain for individuals with higher productivity.

Lastly, these policy implications are rather insensitive to the assumptions that we make about how informed individuals are with respect to their inheritances. Only in the polar case (or close to it) that there is no uncertainty about the size of the size of the inheritance (but not about the timing) does this number change significantly: it increases to 9.5 Cents.

We conclude that the additional labor tax revenue of heirs is likely to be of sizable magnitude and should be taken into account in fiscal planning (dynamic scoring).[4]

Related Literature. The paper is related to and motivated by a small but growing quasi-experimental literature of wealth effects on labor supply. Imbens et al. (2001) is the first paper to use lottery data to estimate the impact of wealth on labor supply. They document that on average a one dollar wealth increase triggers a decrease in earnings of 11 Cents. Cesarini et al. (2017) use a similar setting in Sweden and obtain surprisingly similar numbers. Picchio et al. (2015) study lottery winners in the Netherlands. While they find no effects along the extensive margin, the impact along the intensive margin is a bit smaller than in Imbens et al. (2001) and Cesarini et al. (2017). Gelber et al. (2017) analyze the wealth effect for individuals who receive disability insurance. The individuals they consider receive around $1,700 of DI benefits per month. The sample is particular in the sense that monthly income among the studied subjects is very low, on average around $200 per month. The authors have a very clean identification strategy (regression-kink design) and find an income effect from one dollar of additional unearned income of about 20 Cents.[5]

Further, our paper is related to the literature that estimates the impact of inheritances on labor supply of the heirs. Papers along these lines include Holtz-Eakin et al. (1993), who document the effect of bequests on labor force participation, and Brown et al. (2010), who investigate retirement choices. In a recent study Doorley and Pestel (2016) use the German Socio-Economic Panel (SOEP) to analyze the effect of inheritances on (actual

[4]To put this number into perspective, note that Saez et al. (2012) report the marginal excess burden per dollar of federal income tax raised to be below 20 cents.

[5]Another recent related study is Bick et al. (2018), who document differences in hours worked across countries at different development stages. They find that both labor force participation (extensive margin) and hours worked conditional on employment (intensive margin) are lower in high income countries. This pattern is very much in line with wealth effects on labor supply.

and desired) hours worked, self-employment and hiring of entrepreneurs. The authors find that women who receive an inheritance reduce their labor supply by about 1.5 hours a week, while men's labor supply is by and large unaffected.

More relatedly, Elinder et al. (2012) study the impact of inheritances on earnings of the heirs and use variation in the size of inheritances for identification. The sample they consider is very small, however. They do find effects on earnings that are significantly larger than the one implied by our model. More recently, Bø et al. (2018) study this impact with Norwegian administrative data using a propensity score matching approach. The authors also find significantly larger effects. Whereas our model is consistent with wealth effects that are measured in the perhaps cleanest experimental studies one can think of (lotteries), their approach, while relying on less clean identification, has the advantage to rely directly on inheritance data. In that sense, we consider the two approaches of quantifying earnings changes as complementary. More importantly, our contribution is not to per se measure these labor supply effects, but to elaborate the implications for public finances in a transparent way. Changing our calibration such that the model is consistent with this alternative empirical evidence would significantly strengthen our policy implications that inheritance taxes have significant positive implications for labor tax revenue from heirs. In that sense, our numbers can be interpreted as lower bounds.

A recent related public economics paper is Koeniger and Prat (2018), who analyze the policy implications of wealth effects. In a dynastic Mirrleesian environment, they find that such wealth effects create a force for less educational investment of children from wealthy families.

The remainder of this paper is organized as follows. In section 2.2 we illustrate the main mechanisms within a tractable two-period OLG model. In section 2.3 we present the full life-cycle model. We present our parameterization of expectations in Section 2.4. The calibration is explained in section 2.5. In section 2.6 we summarize the results and perform several robustness checks. Section 2.7 concludes.

2.2 Two-Period OLG Framework

In this section, we illustrate our general ideas using a simple two-period overlapping generations framework. At each point in time $t \in \{0, 1, \ldots, \infty\}$, there are two generations alive, the sizes of which we normalize to one without loss of generality. From one period to the next, the older of the two generations dies, the younger generation turns old and a new generation is born. We denote by $j = 1, 2$ the age of a generation. Members of each generation have to decide about how much to consume c and how much effort l to put into working. When old, they might receive an inheritance b with a certain probability π from their parent generation.[6] In addition, they can choose themselves how much of a bequest to leave to their descendants. For the sake of simplicity and in line with the recent literature (see e.g. Piketty and Saez (2013)), we focus on the case where net bequests of descendants directly enter the utility function instead of considering a dynastic Barro-Becker model.

[6] In the following, we use the words bequest and inheritance synonymously.

Life-time utility of a household is given by

$$U_t = u(c_{1t}, l_{1t}) + \beta\Big[\pi \cdot v\left(c^I_{2t+1}, l^I_{2t+1}, (1-\tau_b)b^I_{t+2}\right) \\ + (1-\pi)\cdot v\left(c^N_{2t+1}, l^N_{2t+1}, (1-\tau_b)b^N_{t+2}\right)\Big], \quad (1)$$

where I denotes the case in which the agent receives an inheritance and N the case in which she does not. The instantaneous utility functions u and v are assumed to be strictly increasing and concave in c and $(1-\tau_b)b$ as well as strictly decreasing and convex in l.

The agent maximizes her life-time utility given the budget constraint

$$c_{1t} + a_{t+1} \leq (1-\tau_l)w_1 l_{1t} + T_1 \quad (2)$$

in the first period and the state-dependent constraints

$$c^K_{2t+1} + b^K_{t+2} \leq (1-\tau_l)w_2 l^K_{2t+1} + (1+r)a_{t+1} \\ + \mathbb{1}_{K=I}(1-\tau_b)b_{t+1} + T_2 \quad \text{for} \quad K = I, N. \quad (3)$$

In the first period, households use their labor earnings net of proportional labor taxes τ_l as well as (potential) lump-sum transfers from the government to either consume or save into the next period. When they are old, they split their net labor earnings, gross savings, potential net bequest and the lump-sum transfer received between own consumption and bequests to their descendants. Note that we assume prices to be constant over time, but allow wages to be age dependent, reflecting potential wage growth over the life cycle. For the sake of simplicity, we assume that all bequests a generation leaves to their descendants are pooled and then distributed evenly across the group of heirs of the subsequent cohort.[7] In order to guarantee that all bequests are transferred to the descendant generation we require

$$\pi b_{t+2} = \pi b^I_{t+2} + (1-\pi)b^N_{t+2}, \quad (4)$$

which directly follows from the fact that only a share π of the population receives an inheritance.

Let us finally define the expected lifetime tax payments of the generation that is born at time t in present value terms as

$$R_t = \tau_l \cdot \left[y_{1t} + \frac{\pi y^I_{2t+1} + (1-\pi)y^N_{2t+1}}{1+r}\right] + \frac{\pi\tau_b b_{t+1}}{1+r} - T_1 - \frac{T_2}{1+r}. \quad (5)$$

Before thinking about how tax revenues change when bequest tax rates vary, let us first define what an equilibrium and a steady state of the above model are.

Definition 3. *Given an initial level of bequests b_1, an equilibrium allocation is a set of household decision rules $\{c_{1t},\ a_{t+1},\ c^I_{2t+1},\ c^N_{2t+1},\ b^I_{t+2},\ b^N_{t+2}\}^\infty_{t=0}$ that maximize the household's utility function* (1) *subject to the budget constraints* (2) *and* (3)*, a set of bequest levels $\{b_t\}^\infty_{t=2}$ that is consistent with* (4) *and a set of lifetime tax revenues $\{R_t\}^\infty_{t=0}$ derived from* (5).

A steady state is an equilibrium allocation in which all variables are constant over time. We denote a steady state allocation as $\{c_1, a, c^I_2, c^N_2, b^I, b^N, b, R\}$.

[7] An alternative would be to create a dynastic model in which there is a direct link between parents and children. Yet, in such a model, we would need another mechanism that can account for the fact that some children receive an inheritance and others don't. Since in a long-run equilibrium, this might result in a continuous distribution of bequests over households, it is beyond the scope of our theoretical analysis.

2.2.1 The Effect of Changes in Bequests on Household Labor Earnings

In our modeling framework, we now want to work towards clarifying what the effect of a change in the proportional bequest tax τ_b on the life-time tax revenue R_t of a generation born in t is. Before, we however need to define how labor earnings of a household change with respect to exogenous variations in unearned income.

Definition 4. *Let us define*

$$\eta_1 = -\left.\frac{dy_1}{dT_1}\right|_{da=0} \quad , \quad \eta_2^K = -\left.\frac{dy_2^K}{dT_2}\right|_{da=0} \quad \text{and} \quad \alpha = -(1+r)\cdot\frac{da}{d\left[(1-\tau_b)b\right]}. \tag{6}$$

η_1 and η_2^K denote the instantaneous wealth effects on labor earnings, meaning the decline in labor earnings as a result of an exogenous increase in lump-sum transfers under the assumption that savings are kept constant. α is the reaction in savings to an exogenous increase in the amount of bequests heirs receive at old age.

The following proposition summarizes the impact of a change in the net-of-tax-rate $1-\tau_b$ on household labor earnings in different periods of life, evaluated and linearized around a steady state with a constant tax rate τ_b.

Proposition 3. *A change in the net-of-tax rate on bequests $1-\tau_b$ leads to a total labor earnings reaction of*

$$\frac{dy_1}{d(1-\tau_b)\cdot b} = -\eta_1\cdot(1+\varepsilon)\cdot\frac{\alpha}{1+r} \quad \text{and} \tag{7}$$

$$\frac{dy_2^K}{d(1-\tau_b)\cdot b} = \eta_2^K\cdot(1+\varepsilon)\left[-\mathbb{1}_{K=I}+\alpha\right]+\eta_2^K\cdot\xi_\tau^K, \tag{8}$$

where

$$\varepsilon = \frac{db}{d(1-\tau_b)}\cdot\frac{1-\tau_b}{b}$$

is the elasticity of bequests the household receives with respect to the net-of-tax rate $1-\tau_b$. ξ_τ^K measures the effect of a change in the net-of-tax-rate $1-\tau_b$ on the willingness of a household of type $K=I,N$ to bequeath to her own descendants.

Proof: see Appendix 2.A.1. □

Proposition 3 tells us that upon an exogenous change in the net-of-tax-rate, the household labor earnings reaction has three components. First, there is a direct wealth effect on the earnings y_2^I of those who inherit some bequests. Second, in anticipation of a change in future bequest levels, the household can adjust her savings behavior in period one, which influences labor supply in period 1 as well as labor supply of both household types in period 2. Note that the intensity of the wealth effect on labor supply is itself due to two components: On the one hand, a net-of-tax rate increase leads to a mechanical wealth effect, on the other hand, the change in the net-of-tax rate might induce some behavioral reactions on the parent's bequeathing behavior. The sum of the two effects is captured in the term $1+\varepsilon$, where ε measures the elasticity of gross bequests a household receives from her parents with respect to the net-of-tax rate. Finally, when the tax rate on bequests declines, leaving bequests to her own descendants becomes more attractive to the household. Note that owing to our specification of utility, this argument holds for net bequests. A change in $1 - \tau_b$, however, already mechanically leads to a rise in net bequests. The extent to which this influences the gross bequest level b^K is measured by the parameter ξ_τ^K, the sign of which is ambiguous. In any case, whether gross bequests increase ($\xi_\tau^K > 0$) or decrease ($\xi_\tau^K < 0$), labor supply will have to adjust accordingly, which is captured by $\eta_2^K \cdot \xi_\tau^K$. As we explain further below, however, our focus is on wealth effects and not on the price effect $\eta_2^K \cdot \xi_\tau^K$.

The following corollary shows we can put a lot of structure on these wealth effects, if we impose the assumption that all goods are normal goods.

Corollary 1. *If consumption and leisure in both periods as well as bequests are normal goods, we have*

$$\eta_1 \geq 0 \quad , \quad \eta_2^K \geq 0 \quad \textit{and} \quad \alpha \geq 0.$$

Hence, if the assumptions in the preceding corollary hold, we can expect that upon an increase in expected net bequests in the second period:

(i) The household generates less labor earnings in the case she receives an inheritance in period two owing to the direct wealth effect.

(ii) In order to smooth consumption and leisure over time, she also lowers her savings.

(iii) The savings reaction leads to lower labor earnings in period 1, it dampens the labor earnings reaction of those who inherit in period 2, and implies an increase in labor earnings for those who did not inherit in period 2.

(iv) Finally, the household either increases (or decreases) gross bequests to her descendants, which has an additional positive (or negative) effect on labor supply.

2.2.2 Bequest Taxes and Cohorts' Life-Time Tax Payments

Knowing what happens to labor earnings when bequest levels change, we can now look at how a cohort's life-time tax payment changes upon the increase of bequest taxes. We therefore conduct the following thought experiment. We assume that our model is in a steady state. At some date s, the government changes the level of the bequest tax by a (marginal) amount $d\tau_b$. This change is not anticipated by households. Hence, the old generation at time s – the one born in $s-1$ – is surprised by this change. Since bequests are predetermined by the decisions of the generation born at date $s-2$, the change in bequest received by generation $s-1$ is

$$d\left[(1-\tau_b)\cdot b_s\right] = d(1-\tau_b)\cdot b_s = -d\tau_b \cdot b,$$

where b is the level of bequest in the steady state prior to the tax reform. Now as a result to this change in net bequests, the old households in period s adapt the amount of bequests they leave to their descendants, such that under the assumption of normal goods we should expect $b_{s+1} \leq b_s$. Having received a smaller amount of inheritance, the next generation then again changes its bequeathing behavior etc., which leads us to a series of new bequest levels

$$b = b_s \geq b_{s+1} \geq b_{s+2} \geq \dots \quad \text{or in differences} \quad 0 \geq db_{s+1} \geq db_{s+2} \geq \dots$$

until bequests finally converge to a new steady state value. Let us again define the elasticity of bequests that a household receives from her parent's generation at time t with respect to the net-of-tax-rate $1-\tau_b$ as[8]

$$\varepsilon_t = \frac{db_t}{d(1-\tau_b)} \cdot \frac{1-\tau_b}{b} \geq 0.$$

With this elasticity definition, we can obviously write

$$db_t = \varepsilon_t \cdot \frac{b}{1-\tau_b} \cdot d(1-\tau_b) \quad \text{where} \quad \varepsilon_s = 0.$$

Proposition 4. *The change in life-time tax payments of a cohort born at time $t \geq s$ to a change in bequest taxes $d\tau_b$ – which comes surprisingly at a date s – is given by*

$$dR_t = \pi \cdot \frac{d\left[\tau_b b_{t+1}\right]}{1+r} \cdot \left\{ 1 + \frac{\tau_l}{\pi \left[1 - \frac{\tau_b}{1-\tau_b} \cdot \varepsilon_{t+1}\right]} \cdot \left\{ (1+\varepsilon_{t+1}) \cdot \left[\alpha\eta_1 + \pi\left[\eta_2^I - \alpha\eta_2^I\right] + (1-\pi)\left[-\alpha\eta_2^N\right]\right] - \left[\pi\eta_2^I\xi_\tau^I + (1-\pi)\eta_2^N\xi_\tau^N\right] \right\}\right. . \quad (9)$$

For the cohort born at date $s-1$ we have

$$dR_{s-1} = \pi \cdot \frac{d\tau_b \cdot b_s}{1+r} \cdot \left\{ 1 + \frac{\tau_l}{\pi} \cdot \left[\pi\eta_2^I - \left[\pi\eta_2^I\xi_\tau^I + (1-\pi)\eta_2^N\xi_\tau^N\right]\right]\right\}.$$

Proof: see Appendix 2.A.1. □

Before we interpret these equations, note that the total revenue effect of a change in bequest taxes has a direct component[9] as well as an additional component through changes in labor supply behavior and a corresponding impact on labor tax revenue. In order to isolate the latter and explore by how much life-time tax payments of a cohort rise in excess of the bequest taxes it pays, we normalize R_t by the expected bequest tax payment of the generation born in period t.

Corollary 2. *The change in life-time tax payments in excess of the bequest tax revenue effect is*

$$dE_t = \frac{\tau_l}{\pi \left[1 - \frac{\tau_b}{1-\tau_b} \cdot \varepsilon_{t+1}\right]} \cdot \left\{ (1+\varepsilon_{t+1}) \cdot \left[\alpha\eta_1 + \pi\left[\eta_2^I - \alpha\eta_2^I\right] + (1-\pi)\left[-\alpha\eta_2^N\right]\right] - \left[\pi\eta_2^I\xi_\tau^I + (1-\pi)\eta_2^N\xi_\tau^N\right] \right\} \quad (10)$$

for all generations born in period $t \geq s$ and

$$dE_{s-1} = \frac{\tau_l}{\pi} \cdot \left\{\pi\eta_2^I - \left[\pi\eta_2^I\xi_\tau^I + (1-\pi)\eta_2^N\xi_\tau^N\right]\right\}. \quad (11)$$

[8] In the same way as in Proposition 3.

[9] Reflected in the term 1 in parenthesis and simply indicating that higher bequest taxes will (at least on the upward sloping part of the Laffer curve) lead to higher bequest tax revenues.

Note that this corollary directly follows from

$$dE_t = \frac{dR_t}{\pi \cdot \frac{d\left[\tau_b b_{t+1}\right]}{1+r}} - 1.$$

Equations (10) and (11) tell us that for each dollar of bequest tax revenue the government receives (in present value terms) from a generation that is affected by an increase in proportional bequest taxes $d\tau_b$, we can expect dE_t additional cents of labor tax revenue. The effect dE_t thereby consists of multiple components. Starting with the old generation at the time of the bequest tax increase in equation (11), we can directly see two effects at work. All households of this generation are surprised by the change in taxes. Since they are already old, the only margin by which they can react to this change is by adjusting their current consumption and labor earnings as well as the amount of bequest they leave to their descendants. All households of type i who receive an inheritance therefore experience a negative *wealth effect* of $d\tau_b \cdot b$,[10] which directly translates into higher labor earnings. The size of this wealth effect is given by η_2^I, which measures the households willingness to earn out of unearned income, holding fix life cycle savings. Non-heirs, of course, experience no wealth effect.

Yet, an increase in bequest taxes also induces a *price effect*, which impacts on the households' willingness to leave bequests to their own descendants. This channel is summarized in the second term of equation (11). ξ_τ^K measures the extent to which households of type $K = I, N$ adjust their *gross bequests* to a change in the tax rate $d\tau_b$. Note that ξ_τ^K itself is a result of two effects. On the one hand, an increase in the tax rate τ_b makes bequeathing to the descendants less attractive, which is why – if all goods are normal – households want to reduce their level of *net bequests*. However, at the same time, the tax change $d\tau_b$ already mechanically reduces net bequest by an amount of $d\tau_b \cdot b_t^K$, where b_t^K is the level of *gross bequests*. If $d\tau_b \cdot b_t^K$ is smaller (larger) than the household's desired decline in net bequests, then the agent will also lower (increase) her gross bequest level b_t^K. As a result, she will require less (more) labor earnings which mitigates (reinforces) the wealth effects.

With these effects in mind, let us turn to the excess tax revenue of all generations born at time $t \geq s$ in equation (10). We can immediately see that the same effects are at work for this generation. However, the *wealth effect* is now a product of three subcomponents:

$$\underbrace{\alpha\eta_1}_{\text{Anticipation Effect}} + \underbrace{\pi\left[\eta_2^I - \alpha\eta_2^I\right]}_{\text{Effect on Heirs}} + \underbrace{(1-\pi)\left[-\alpha\eta_2^N\right]}_{\text{Effect on Non-Heirs}} \tag{12}$$

The term $\pi\eta_2^I$ again covers the direct wealth effect that we would observe if a generation was hit by the tax change unexpectedly in the middle of their life. Since all households born at a time $t \geq s$ however observe the increased bequest tax rate already in the first period of life, there is an *anticipation effect*. Specifically, all members of a cohort will try to smooth the impact of a smaller expected inheritance over their life cycle. As a result, if all goods are normal, they lower consumption in period one in order to increase savings into the next period. This leads labor earnings to already increase prior to a (potential) bequest tax receipt ($\alpha\eta_1$). The savings increase in turn induces an additional positive

[10]Recall that bequest b_s are predetermined by the old generation in period $s-1$, which was totally unaffected by the tax increase

wealth effect on households when old. Hence, it mitigates the labor earnings reaction of heirs and induces non-heirs' labor earnings to even fall below their steady state earnings level.

Over and above the three labor supply effects discussed so far, there is a fourth effect in equation (10), which relates to the impact the tax increase $d\tau_b$ has on the equilibrium bequests received by generation t. By definition, bequests in the period of the reform are predetermined, i.e. $\varepsilon_s = 0$. The old generation at time s will, however, adjust their bequest level both owing to the wealth effect induced by a lower amount of inheritance as well as to the price effect. This induces aggregate bequest of the next generation to fall, which is why we should expect $\varepsilon_{s+1} \geq 0$. The old generation in period $s+1$ hence does not only experience a mechanical wealth effect due to the change in the tax rate, but an additional wealth effect owing to the decline in bequests. As a result, their bequest will fall even below the amount they received, leading to an elasticity of $\varepsilon_{s+2} \geq \varepsilon_{s+1}$. Following this logic period by period yields a sequence of elasticities

$$0 = \varepsilon_s \leq \varepsilon_{s+1} \leq \varepsilon_{s+2} \leq \ldots,$$

which converges to some steady state level. Summing up, the factor $1+\varepsilon_{t+1}$ measures the *exposure* or *equilibrium effect* of each generation that results from intertemporal spill-overs through the bequest channel. A greater ε_{t+1}, hence, leads to a stronger decline in the net bequests the generation born at time t receives and therefore induces stronger wealth effects on labor earnings. Note that the price effect does not depend on ε_{t+1}, as it is merely a consequence of the change in the price of bequests $d\tau_b$, where this price change is constant across all affected cohorts.

Summing up, we have shown that by increasing bequest taxes in our model, the government not only receives additional bequest tax revenue, it can also expect a rise in labor taxes paid by each generation. The extend by which labor earnings actually increase is the product of

1. a *direct wealth effect* on heirs through a fall in net inheritances,
2. an *anticipation effect* leading to a smoothing of labor earnings (also for individuals that are ex-post non-heirs) over the life cycle and therefore changes in savings,
3. a *price effect* associated with the behavioral reaction to a change in the price of net bequests, and
4. an *equilibrium effect* that results from intergenerational spill-overs and that leads to a different extent of the wealth and anticipation effect for generations born at different points in time.

In the following analysis, we concentrate on the first two effects, since they can be traced by suitably calibrating a quantitative model to quasi-experimental evidence on the wealth effects on labor earnings. The price and equilibrium effects, on the other hand, require a careful specification of bequest motives and the sensitivity of bequests with respect to tax rates. Since evidence on the effects of bequest taxes on intergenerational bequeathing behavior is scarce, we will leave these channels to future research. In terms of our model, one can interpret this exercise as setting $\xi_\tau^K = \varepsilon_t = 0$ for all $t = 0, 1, \infty$. In this case, the excess tax payments associated with a change in proportional bequest taxes can be summarized as

$$dE_t = \frac{\tau_l}{\pi} \cdot \left[\alpha\eta_1 + \pi \left[\eta_2^I - \alpha\eta_2^I\right] + (1-\pi)\left[-\alpha\eta_2^N\right] \right] \quad \text{and} \quad dE_{s-1} = \tau_l \cdot \eta_2^I.$$

2.3 Quantitative Life-Cycle Model

Our previous theoretical analysis has revealed that the anticipation of bequests plays a crucial role in determining the labor supply response to a change in bequest taxes. In the following sections we construct and calibrate a full life-cycle model that accounts for proper expectations and allows us to realistically quantify the effect of a change in bequest taxes on the labor supply of heirs.

Timing and endowments Time $t \in \{1, \ldots, T\}$ is discrete and period length is one year. The economy is populated by a continuum of mass one of heterogeneous households. Households enter the economy at age 20 (model age $t = 1$). At this point in time, they are endowed with an earnings ability level $e \in \{1, \ldots, E\}$ and a signal $s \in \{0, \ldots, n\}$ about the amount of inheritance they might receive. Agents work until they reach the (exogenous) retirement age t_r. They die with certainty at age T.

Bequest and expectations Throughout their life-cycle, households might receive a bequest. Bequests are stochastic both with respect to timing and size. We assume, for the sake of simplicity, that a household can only inherit once in her lifetime – at the age at which her ancestors pass away. Denote by $\{p_t^e\}_{t=1}^T$ the unconditional probability distribution of ancestors passing away when a household of ability e is of age t. We assume that the chance of parents surviving their children is zero, i.e. $\sum_{t=1}^T p_t^e = 1$.

When a household's parents die at time t, their bequest can take one of $n + 1$ different levels $\{b_{it}^e\}_{i=0}^n$, where $b_{0t}^e = 0$. We call $i \in \{0, \ldots, n\}$ a bequest class and assume that the conditional probability of the household's inheritance falling into such a class is time invariant. Agents form expectations about the class their inheritance will belong to according to the signal s they received at the beginning of their life cycle. A signal of perfect quality would imply that a household falls into inheritance class $i = s$ with certainty. We will also consider less precise signals and will be more specific about how we formalize the quality of the signal in the next section. For now, we just denote by π_{is}^e the time invariant probability that a household with signal s and earnings capacity e attaches to receiving an inheritance of class i. The probability that an individual of type (e, s) receives a bequest at age t that falls into class i is then given by $p_t^e \cdot \pi_{is}^e$.

While the probability distribution over bequest classes i is time invariant, bequest levels b_{it}^e in each class are allowed to vary over time t. This reflects, for example, that ancestors might run down their wealth throughout a prolonged retirement phase. The bequests levels b_{it}^e as well as depend on the individual earnings capacity e which can account for the empirical fact that higher earning children tend to have richer parents.

Preferences At any age t, households decide about how much to consume c_t, how much to work l_t and how much to save a_t. They have preferences over consumption and labor supply

$$U_0(e,s) = \mathbb{E}\left[\sum_{t=1}^T \beta^{t-1} \left(\frac{c_t^{1-\gamma}}{1-\gamma} - \lambda \frac{l_t^{1+\chi}}{1+\chi} \right) \;\middle|\; e, s \right]$$

and form expectations about inheritances according to the above probabilities. We assume utility of consumption and disutility of labor to be additively separable. The parameter λ denotes the relative weight of labor in the agent's utility, χ is the inverse of the Frisch elasticity of labor supply, β is the time discount factor, and γ is risk aversion.

Budget constraint The budget constraint is given by

$$c_t + a_{t+1} = w_t^e l_t - \mathcal{T}\left(w_t^e l_t\right) + \mathcal{P}_t^e + W_t.$$

Consumption and savings into the next period are financed out of gross labor income $w_t^e l_t$ minus taxes $\mathcal{T}\left(w_t^e l_t\right)$, pension income $\mathcal{P}_t^e$ and net wealth W_t. Gross labor income is the product of the wage rate w_t^e and labor effort l_t. The function $\mathcal{T}(.)$ maps gross labor income into a tax payment and is specified in more detail in the calibration section of this paper. Throughout retirement, the household receives pension income $\mathcal{P}_t^e$, which we assume to be constant and conditional on the household's earnings capacity. In particular, we set

$$\mathcal{P}_t^e = \begin{cases} 0 & \text{if } t < t_r \\ \mathcal{P}^e > 0 & \text{if } t \geq t_r. \end{cases}$$

Net wealth is a composite of both individual savings a_t and (potential) bequests b_{it}^e received

$$W_t = \left[1 + (1 - \tau_k) r\right] a_t + (1 - \tau_b) b_{it}^e,$$

where $(1 - \tau_k)r$ is the net-of-tax interest rate on savings and τ_b is a proportional tax rate on bequests.

Finally, each household faces a borrowing constraint

$$a_{t+1} \geq a_{min},$$

with the minimal asset level being a number $a_{min} \in (-\infty, 0]$. Retirement at age t_r is mandatory. Hence labor supply needs to satisfy

$$l_t = 0 \quad \text{for all} \quad t \geq t_r.$$

Dynamic optimization problem The state space of the household optimization problem contains the individual's earnings capacity e, the signal about the size of bequests s as well as net wealth W_t. Since households only inherit once in their life time, the state space further contains an indicator $h_t \in \{0, 1\}$ for whether the agent's parents already passed away prior to or at date t. The dynamic optimization problem of the household hence reads

$$V_t(e, s, h_t, W_t) = \max_{c_t, l_t, a_{t+1}} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} - \lambda \frac{l_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}\left[V_{t+1}\left(e, s, h_{t+1}, W_{t+1}\right) \middle| e, s, h_t \right] \right\}.$$

If the household's parents are still alive, expectations are formed according to

$$\mathbb{E}\left[V_{t+1}\left(e, s, h_{t+1}, W_{t+1}\right) \middle| e, s, h_t = 0 \right] = \tilde{p}_{t+1}^e \cdot \sum_{i=0}^{n} \pi_{is}^e \cdot V_{t+1}\left(e, s, 1, W_{t+1,i}\right) + \left[1 - \tilde{p}_{t+1}^e\right] V_{t+1}\left(e, s, 0, W_{t+1}\right),$$

where

$$W_{t+1,i} = [1 + (1-\tau_k)r]\, a_{t+1} + (1-\tau_b) b^e_{i,t+1} \quad \text{and}$$
$$W_{t+1} = [1 + (1-\tau_k)r]\, a_{t+1}.$$

Furthermore,

$$\tilde{p}^e_{t+1} = \frac{p^e_{t+1}}{1 - \sum_{s=1}^{t} p^e_s}$$

is the conditional probability of receiving an inheritance at age $t+1$, given that one hasn't received an inheritance yet. In case the agent's ancestors already deceased, all uncertainty has been revealed and we can simply write

$$\mathbb{E}\left[V_{t+1}\left(e, s, h_{t+1}, W_{t+1}\right) \middle| e, s, h_t = 1\right] = V_{t+1}\left(e, s, 1, W_{t+1}\right).$$

2.4 Parameterizing expectations about bequests

One crucial element of our life cycle model is the probability distribution π^e_{is} according to which a household forms expectations about the class i her inheritance can fall into, including the case where no inheritance is received $i = 0$. Measuring expectations about inheritances is complicated if one can only observe actual cases of inheritances. Whereas our data allows us to estimate the distribution of inheritances conditional on age and earnings of the heirs, this does not inform us about the expectations heirs in that age-earnings class actually had. We therefore suggest different parameterizations of the signal quality. We only require that they are all consistent with the conditional cross-sectional distribution of inheritances. On the one extreme, we will consider signals of perfect quality: conditional on the parents dying, heirs know for sure how much they inherit. On the other extreme, the signal contains no information at all: heirs just draw their inheritance from the estimated cross-sectional distribution. To elaborate how our results depend on expectations, we consider both extreme cases as well as intermediate ones.

More formally, the signal $s \in \{0, \ldots, n\}$ an agent receives is a discrete number that contains information about which class i her inheritance will fall into. The parameter $\sigma \in [0, 1]$ is an indicator for the quality of this signal. If $\sigma = 0$, the signal contains no information at all, while for $\sigma = 1$ the household knows with certainty that $i = s$. At the beginning of the life cycle, a fraction φ^e_s of households of ability e is equipped with the signal s. We now have to make a distinction between the individual specific probability distribution π^e_{is}, which depends on the individual signal s, as well as the population wide (cross-sectional) distribution ω^e_i of households of earnings class e over different bequest levels i. In order for the individual probability distributions to be consistent with the cross-sectional distribution, we require

$$\forall i, e: \sum_{s=0}^{n} \varphi^e_s \cdot \pi^e_{is} = \omega^e_i. \tag{13}$$

Note that when the signal is fully informative about the household's bequest class ($\sigma = 1$), the individual probability distribution is

$$\pi^e_{is} = \begin{cases} 1 & \text{if } i = s \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

On the other hand, if the signal contains no information ($\sigma = 0$), the best forecast a household can make about the class her inheritance will fall into is the cross-sectional distribution over all households of the same earnings level ω_i^e, meaning that $\pi_{is}^e = \omega_i^e$ for all $s = 0, \ldots, n$. For any intermediate signal quality, we let the individual probability distribution be a convex combination of the two. Hence, we have

$$\pi_{is}^e = (1-\sigma)\omega_i^e + \sigma \cdot \mathbb{1}(i = s) \quad \text{for} \quad \sigma \in [0, 1],$$

where $\mathbb{1}(i = s)$ is an indicator function that takes a values of 1 if i is equal to s and 0 otherwise. For any $\sigma > 0$, equation (13) implies $\varphi_s^e = \omega_s^e$, meaning that the distribution of the population of an earnings level e over different signals s has to exactly equal the cross-sectional distribution of this population over inheritance levels i.

2.5 Calibration

We calibrate our model in three steps:

1. We first estimate labor earnings profiles $y_t^e = w_t^e l_t$, the probability of ancestral death p_t^e, the cross-sectional distribution ω_i^e as well as bequest levels b_{it}^e using data from the *German Socio-Economic Panel (GSOEP)*.

2. In a second step, we parameterize further model parameters, prices and government policies.

3. Finally, we jointly pin down both the labor supply elasticity parameter χ and risk aversion γ such that our model is consistent with recent empirical evidence on the effects of lottery wins on labor earnings provided in Cesarini et al. (2017).

2.5.1 Labor earnings and bequests

Our main data source is the GSOEP, an annual panel survey on German households.[11] We use data on age, education, labor income and inheritances on the household level in between the years 2000 and 2014, and pool together all data from these 15 different waves into one cross-section.[12] We assume that a household consists of either one or two persons, meaning that we abstract from the presence of children or any other relative or non-relative household members. For two person households we identify the household head as the primary earner and use the head's age and education level in all further calculations. We define household labor income as the sum of labor earnings, public transfers (such as social assistance) and pension payments. In addition to age, GSOEP provides data on whether the household has received an inheritance in a respective survey year and if yes, about its size. To account for different household sizes, we divide gross labor income and inheritances of two person households by 1.5, which equals the common scale parameter used by the OECD. Finally, we drop all observations for which information on either age, education level, labor income or inheritances are missing as well as all households aged 19 and below. This leaves us with a total of 163,369 observations.

[11] For detailed information about the GSOEP, see Wagner et al. (2007).

[12] Note that we can not use data on the individual level, as the household is the only unit on which inheritance data can be observed in the GSOEP.

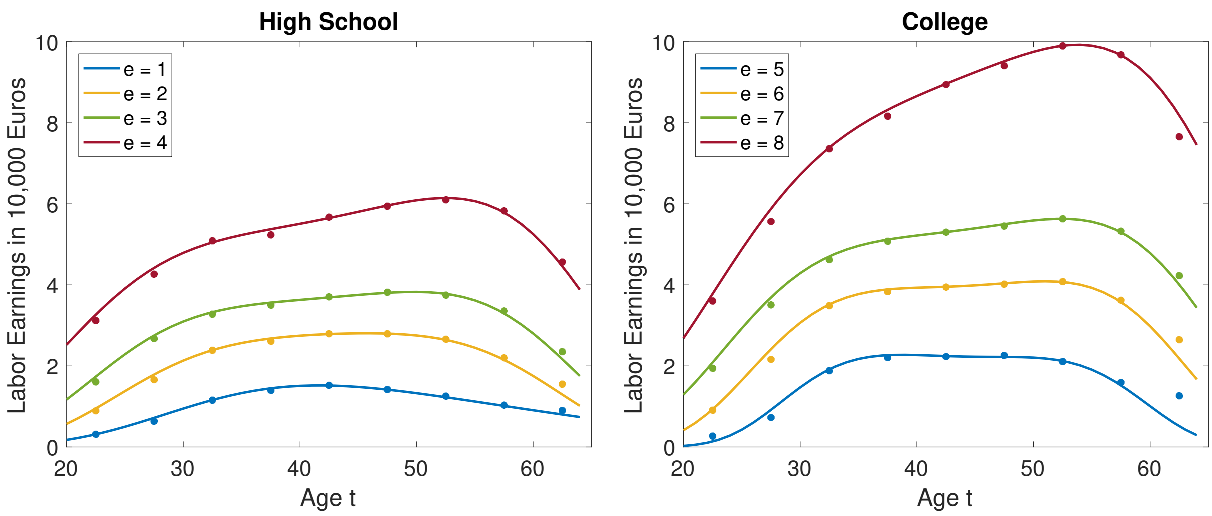
Labor earnings classes

We define a total of $E = 8$ different earnings classes, which results as a combination from two education levels and four income groups per education levels. We first stratify our sample according to the education level of the household. We say that a household has a low education, if the highest educational degree of the household head is a secondary or lower degree according to the ISCED97 education classification standard. All households with household head holding a tertiary education degree are considered highly educated. We assign households with low education into earnings classes $e = 1, 2, 3, 4$ and those with high education into $e = 5, 6, 7, 8$. We then group all households of an education level according to five year age bins, that is 20-24, 25-29, ..., 60-64, and pool all observations aged 65 and above into one bin. Within each education-age group, we separate households into four quartiles according to their labor income, leading to 4 earnings classes within each educational group. Table 2.6 in Appendix 2.A.2 summarizes mean earnings of the 8 earnings classes at different ages derived from the GSOEP. The last row of this table shows the shares of households in each earnings class in the total population. This shows that in our sample 28.4 percent of household heads hold a higher education degree. In order to feed our model with annual data, we fit polynomials of the form

$$y_t^e = \exp\left(\kappa_0^e + \kappa_1^e \cdot t + \kappa_2^e \cdot t^2 + \kappa_3^e \cdot t^3 + \kappa_4^e \cdot t^4\right) \tag{14}$$

for each earnings class e to our data. We derive the polynomial coefficients by minimizing a simple residual sum of squares between the data reported in Table 2.6 and the corresponding moments derived from the polynomial. Figure 2.1 shows the resulting age-earnings profiles.

Figure 2.1: Estimated age-earnings profiles for different earnings classes



Our model features endogenous labor supply decisions. Hence, labor earnings – the product of labor effort l_t and productivity w_t^e – are an endogenous object. In order to back out labor productivity profiles that lead to the labor earnings profiles shown in Figure 2.1, we follow the strategy proposed by Saez (2001). Note that, in our model, labor productivity is assumed to be deterministic over the life cycle and utility from consumption and disutility from labor are additively separable. In order to be able to apply the strategy of Saez (2001), we have to make on additional simplifying assumption, namely that instead of receiving bequests according to the risk process outlined above, households of each earnings class e receive a lump-sum transfer in each period of life that is equal to the average amount of bequest for this group, that is

$$\mathcal{Z}_t^e = p_t^e \cdot \sum_{i=0}^{n} \omega_i^e \cdot b_{it}^e.$$

In doing so, we eliminate all uncertainty from our model,[13] which allows us to write the household optimization problem as

$$\max_{c_t^e, y_t^e, a_{t+1}^e} \sum_{t=1}^{T} \beta^{t-1} \left(\frac{(c_t^e)^{1-\gamma}}{1-\gamma} - \lambda \frac{\left[\frac{y_t^e}{w_t^e}\right]^{1+\chi}}{1+\chi} \right)$$

$$\text{s.t.} \quad c_t^e + a_{t+1}^e = y_t^e - \mathcal{T}\left(y_t^e\right) + \mathcal{P}_t^e + \mathcal{Z}_t^e + (1+r)a_t^e \quad \text{and} \quad a_{t+1}^e \geq a_{min}.$$

The first order conditions of this problem read

$$(c_t^e)^{-\gamma} = \beta(1+r)\left(c_{t+1}^e\right)^{-\gamma} + \alpha_t \quad \text{with} \quad a_{t+1} \cdot \alpha_t = 0 \quad \text{and}$$
$$(w_t^e)^{1+\chi} = \frac{\lambda}{1-\mathcal{T}'(y_t^e)} \cdot \frac{(y_t^e)^{\chi}}{(c_t^e)^{-\gamma}},$$

where α_t is the Lagrangean multiplier on the minimum asset constraint in instantaneous utility values. Given a government policy $\mathcal{T}(\cdot)$ and $\mathcal{P}_t^e$, a set of lump sum transfers $\mathcal{Z}_t^e$ and a deterministic earnings path y_t^e, we can use the Euler equation together with the household budget constraint to calculate the deterministic consumption path c_t^e. We can then use the intra-period first order condition to back out the corresponding labor productivity profile w_t^e for households of earnings class e. Note that the resulting productivity profile is only approximately correct, owing to the assumption we made. However, comparing the model simulated average earnings path including bequest uncertainty for each earnings class to the earnings profiles estimated from the data showed only minor differences.

Probabilities of ancestral death and receiving and inheritance

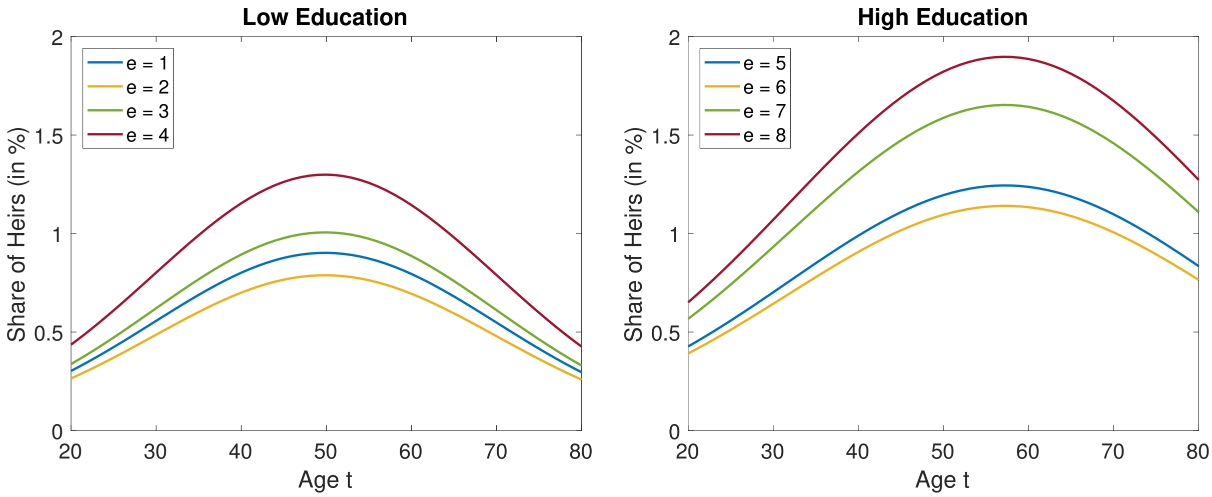
Having grouped our observations into suitable earnings classes, we next have to estimate the age-dependent probability of ancestral death for members of each of these earnings groups. As inheritances arrive typically only onces or twice in a life-time, receiving an inheritance is an infrequent event in our data. Hence, albeit the fact that we have 163,369 observations, only 2,394 observed households (1.47 percent of our sample) received an inheritance in the sample period. In order to guarantee somewhat reliable estimates, we therefore use a coarser definition of age groups, namely 20-34, 35-44, 45-54, 55-64 and 65+ in what follows. For each earnings class e and age group, we calculate the fraction of the observed population in the GSOEP that actually received an inheritance. The results are shown in Table 2.7 and 2.8 in Appendix 2.A.2. We again fit this data using cubic log-polynomials

$$q_t^e = \exp\left(\kappa_0^e + \kappa_1^e \cdot t + \kappa_2^e \cdot t^2 + \kappa_3^e \cdot t^3\right). \tag{15}$$

[13]Note that we only do this for the purpose of calibration, not in our main simulations.

We weigh each moment in the residual sum of squares with the inverse of its standard error in order to control for the varying precision of our estimates. In addition, to reduce the degrees of freedom, we assume that polynomials across households of different earnings classes, but within the same education level (low or high), are only allowed to vary in the intercept κ_0. All other polynomial coefficients need to be identical for households of the same education level. Finally, we have to control for the fact that a large number of households in our sample is composed of a head and a spouse, and such couples tend to receive an inheritance twice in their lifetime, once from the head's parents and once from the spouse's parents. In order to make the estimated polynomials consistent with our model, we therefore standardize them with a factor of $1 + \varsigma^e$, where ς^e is the fraction of two-person households in each earnings class e in the GSOEP data. Figure 2.2 shows the resulting polynomials. The share of heirs in a cohort is the highest around ages 50 to 60,

Figure 2.2: Estimated age-inheritance relationship for different earnings classes



which is consistent with a roughly 30 year age difference between parents and children as well as a life expectancy of around 80 years. Higher educated households are more likely to receive an inheritance and tend to get it later in life, mirroring a higher average life expectancy of their (potentially high skilled) parents.

Note that the estimated polynomials represent the share of a cohort that receives an inheritance. In terms of our model, this share is a combination of the probability of the parents deceasing and the likelihood that they pass a positive inheritance to their offspring. Consequently, the polynomials identify

$$q_t^e = p_t^e \cdot \sum_{i=1}^{n} \omega_i^e = p_t^e \cdot (1 - \omega_0^e).$$

Using our structural assumption that parents cannot outlive their children, we immediately get

$$\sum_{t=1}^{T} q_t^e = (1 - \omega_0^e) \sum_{t=1}^{T} p_t^e \quad \Leftrightarrow \quad \omega_0^e = 1 - \sum_{t=1}^{T} q_t^e.$$

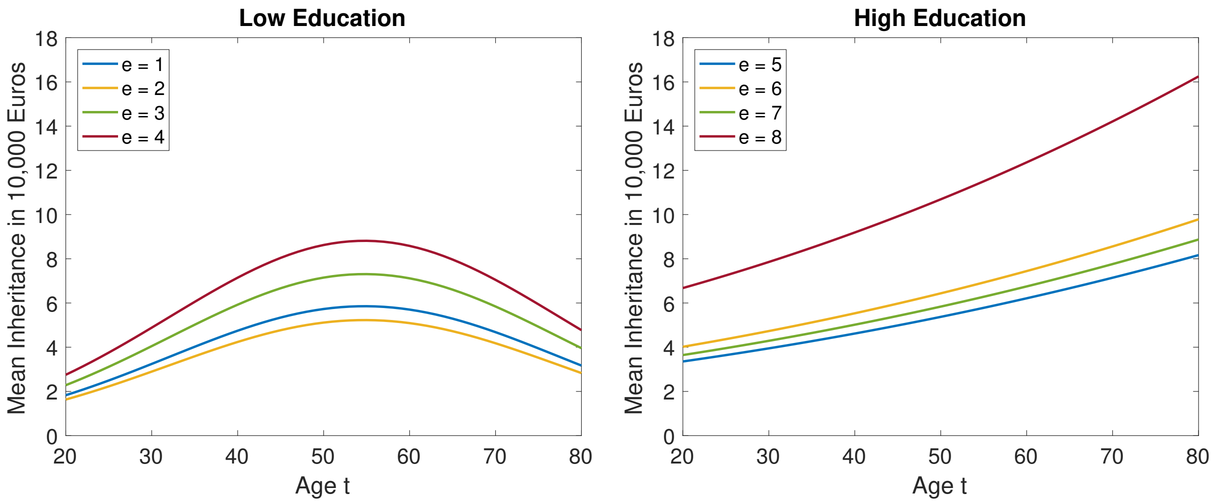
Furthermore, the probabilities of ancestral death are consequently given by

$$p_t^e = \frac{q_t^e}{\sum_{t=1}^{T} q_t^e}.$$

Bequest classes and bequest levels

In a last step, we have to determine the cross-sectional distribution over (positive) bequest classes ω_i^e, $i \in \{1, \dots, n\}$ as well as the average bequest levels b_t^i. To this end, we first calculate mean bequests of households who received a positive inheritance for each age group and earnings class in the GSOEP, see Tables 2.7 and 2.8 in Appendix 2.A.2. We again fit this data with cubic log-polynomials using the same methodology as described in the previous section. Figure 2.3 shows the resulting mean bequest profile by age and earnings level. Interestingly, the mean bequest profiles of the lower skilled are hump-

Figure 2.3: Estimated mean bequest profiles for different earnings classes



shaped over the life cycle, while those of the high skilled are strictly upward sloping. This could indicate that bequests of parents of lower skilled households tend to be accidental. Hence, if parents follow a regular life-cycle savings pattern and successively outlive their wealth at very high ages, bequests fall again. On the other hand, that bequests of parents of higher skilled households increase with the heirs' age indicates that parents consume less than their income speaking in favor of an active bequest motive. This is in line with the view of De Nardi et al. (2010), who model bequests as a luxury good.

In order to determine bequest levels in each bequest class i, we standardize the amount of inheritance of each household in the GSOEP who received a positive bequest by the age group and earnings class specific mean bequest level as reported in Tables 2.7 and 2.8. We then pool together all data for households of one education level, separate the data into quartiles and calculate the mean standardized bequest level for each of these quartiles. The resulting quartile means by education level are shown in Table 2.1. The table reveals that the distribution of bequests within the group of heirs is very skewed. While the lowest quartile of heirs receives an average inheritance that amounts to 7 percent of the mean bequest level, the upper quartile's inheritance ranges around three times the mean. The distribution does not differ substantially across households of different education levels. We multiply the mean bequest profiles in Figure 2.3 with the factors in the above

Table 2.1: Standardizes bequest quartile means by education

Education	Q1 ($i = 1$)	Q2 ($i = 2$)	Q3 ($i = 3$)	Q4 ($i = 4$)
Low	0.070	0.232	0.611	3.095
High	0.070	0.258	0.704	2.971

table in order to construct the bequest levels in each bequest class b^e_{it}. Since we divided bequests into quartiles, we set the cross-sectional distribution of households with positive inheritances over bequest classes to $\omega^e_i = 0.25 \cdot (1 - \omega^e_0)$.

2.5.2 Parameters, prices and government policy

Table 2.2 summarizes our choices for parameters, prices and government policy. Starting their life by the age of 20 ($t = 1$) we let households live with certainty up to age 80 ($t = 61$), which corresponds to the average life expectancy at birth of the German population. Retirement is mandatory at age 65.

Table 2.2: Parameters, prices and government policy

Parameter	Value	Note
T	61	Age of death = 80
t_r	46	Retirement age = 65
β	0.98	Time discount factor
λ	1	Coefficient for disutility of work
γ	1	Coefficient of risk aversion
χ	4.37	Frisch elasticity = 0.23
σ	0.75	Signal quality (benchmark)
r	2%	Interest rate
a_0	0	No initial wealth
a_{min}	$-\infty$	Only natural borrowing limit
$\mathcal{P}$	0.40	Pension = 40% of average gross income
τ_0	0.679	Average labor earnings tax rate
τ_1	0.128	Progressivity of labor tax
τ_k	0.25	Linear capital income tax
τ_b	0.00	Linear inheritance tax

We choose a time discount factor for the household of $\beta = 0.98$, such that the time preference rate is equal to the gross interest rate. We normalize $\lambda = 1$, which only has an implication for the endogenous labor productivity profiles w^e_t we estimate in the simplified model version in Section 2.5.1, but doesn't have an impact on our simulation results otherwise. We set the coefficient of risk aversion to $\gamma = 1$ and the labor supply elasticity parameter to $\chi = 4.37$, implying a Frisch elasticity of labor supply of 0.23. Section 2.5.3 provides more details on how we jointly pin down the two. Finally, we set the signal quality to $\sigma = 0.75$ in our benchmark scenario. We, however, consider various other scenarios for σ in a sensitivity analysis.

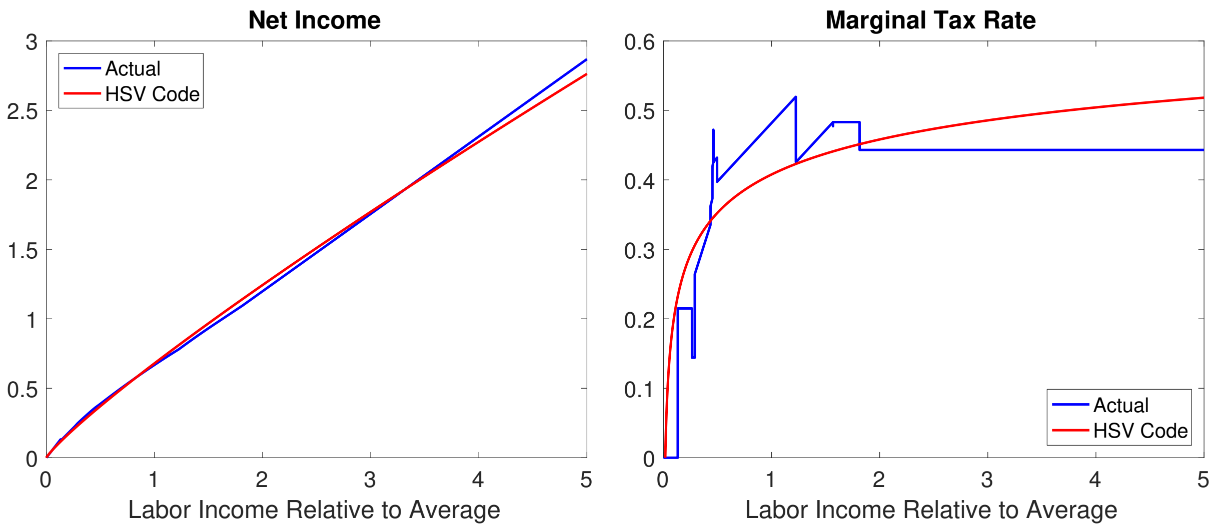
Taking a longer run perspective on savings, we take the annual interest rate to be 2%, which is equal to the current interest rate on outstanding household deposits with an agreed maturity of over two years. We furthermore assume that households start their life with zero own wealth. However, they might of course receive an inheritance early in life. Finally, we assume that the only borrowing limit the household faces is the natural borrowing limit, meaning that $a_{min} = -\infty$. We show in Appendix 2.A.3 that this choice provides the best fit for impulse responses to lottery gains, which we use to calibrate the extent of wealth effects on labor earnings in Section 2.5.3. This appendix also reveals that the policy implications do not depend starkly on this assumption.

Finally, we have to specify the tax and pension policy of the government. Starting with the latter, we set the replacement rate of pensions to 40% of average gross labor earnings over the life cycle, which matches the replacement rate reported by the OECD (2017). We calculate pension payments separately for households of different earnings classes, such that higher earners also receive a higher pension. With regard to labor income taxes, we use data on the mapping from gross into net income provided by Lorenz and Sachs (2016). We fit this data in a least squares sense using a functional form that was first proposed by Benabou (2002) and more recently applied by **?**. We therefore write net income as a function of gross income as

$$y_{net} = y - \mathcal{T}(y) = (1 - \tau_0) \cdot y^{1-\tau_1},$$

where τ_0 roughly captures the average tax rate of the system and τ_1 is an index for its progressivity. Figure 2.4 shows our original data as well as the fitted tax schedule. The parameter set that yields the best match is $\tau_0 = 0.321$ as well as $\tau_1 = 0.128$ with an R^2 value of 0.998. Last but not least, we set the flat capital income tax rate at $\tau_k = 0.25$,

Figure 2.4: Net Income and Marginal Tax Rates



which is equal to the statutory tax rate in Germany, and assume that in our benchmark simulation bequests are not taxed, which reflects very high exemption levels (400 000 Euro) for inheritances received from parents.

2.5.3 Pinning down wealth effects on labor earnings

In our model, the elasticity of labor earnings of a household with respect to an exogenous and unexpected change in wealth is given by

$$\eta_{y,t} = -\frac{W_t - a_{t+1} \cdot \eta_{a,t+1}}{\frac{\chi+\tau_1}{\gamma} \cdot c_t + (1-\tau_1) \cdot [y_t - \mathcal{T}(y_t)]},$$

where $\eta_{a,t+1}$ is the elasticity of savings into the next period with respect to current wealth, see Appendix 2.A.4 for a proof. With consumption in each period being a normal good, we can expect $a_{t+1}\eta_{a,t+1} < W_t$. Hence, labor earnings of a household unambiguously decline upon exogenous wealth changes. The extent of this decline depends both on the progressivity of the labor earnings tax schedule – measured by τ_1 – as well as on the preference parameters χ and γ. The greater is their ratio $\frac{\chi}{\gamma}$, the smaller we can expect the wealth effect on labor earnings to be.[14] Since we estimated τ_1 from the data, the only thing that remains to pin down the wealth effects on labor earnings are the preference parameters. Note that, if labor taxes were proportional ($\tau_1 = 0$), then the wealth effect on labor earnings would be solely identified by their ratio $\frac{\chi}{\gamma}$, which is not exactly true under a progressive tax system.

As outlined in the introduction, estimating the impact of inheritances on labor earnings is empirically difficult, as studies can be expected to produce only biased results. In particular, in the data – as in our model – inheritances are not a random and unexpected treatment. Instead, agents rater adjust their economic decisions (such as saving, consumption and labor supply) prior to their arrival, owing to an anticipation effect. A more reliable and convincing source of data comes from a recent study by Cesarini et al. (2017). They evaluate the effect of winning the lottery on individual labor earnings using a rich administrative data set of over 250,000 lottery winners in Sweden. Their empirical estimates indicate a marginal propensity to earn out of unearned income of -0.11 before labor taxes and social security contributions of employers. When including employer contributions this number declines to -0.14.[15]

In order to pin down the wealth effect on labor earnings in our model determined by the ratio between χ and γ, we directly use the evidence from Cesarini et al. (2017). More specifically, we randomly pay out lottery gains to our model households, using exactly the lottery size and age distribution provided in their Computational Online Appendix. We then calculate the reduction in labor earnings of all households in the first five years after they won the lottery, measured as a fraction of the amount gained. We target an average annual reduction in labor earnings of -1.07% of the lottery win. Our preferred choice of parameter that matches these targets is $\gamma = 1$ and $\chi = 4.37$. In our preference specification, this implies a value for the Frisch elasticity of labor supply of 0.23. This is within the range of estimates provided in MaCurdy (1981) and Altonji (1986) for prime

[14]Note that at this point it is easy to see that our choice of the level parameter λ does not influence the extend of labor earnings reactions to wealth changes, but only the level of labor effort l_t, which we do not necessarily have to interpret as labor hours.

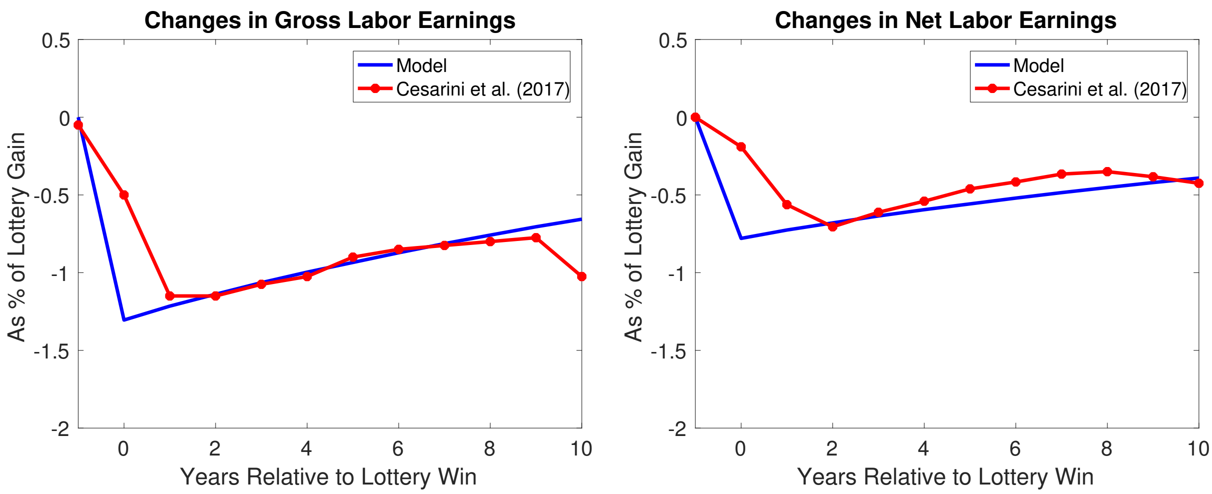
[15]One concern of lottery studies typically is external validity, meaning that lottery players might be systematically different from the Swedish population at large. Cesarini et al. (2017) address this issue by pulling a random sample from the entire Swedish population, which can be done in Swedish register data. After reweighing this random sample to match the demographic characteristics of the sample of lottery winners, the authors find no significant difference in observable labor market characteristics between lottery players and the general population.

age males. Blundell et al. (2016) find slightly higher values for the Frisch labor supply elasticity of males using a sample of married couples and values of around 1 for married females. Fiorito and Zanella (2012) reconcile the consistency between micro- and macro-level estimates.

A risk aversion of 1 and a Frisch labor supply elasticity of 0.23 both range at the lower end of the spectrum typically found in the life cycle and the macroeconomic literature. However, increasing both risk aversion and the Frisch labor supply elasticity to higher values would significantly increase the wealth effect on labor earnings, which would strongly enforce the labor tax revenue response to an increase in bequest taxes. However, this wealth effect would be inconsistent with empirical evidence. Yet, we provide some sensitivity checks with respect to our parameter choices in Section 2.6, where we set γ at a value smaller than 1, which directly implies a higher Frisch elasticity as well as a value of $\gamma = 4$, which implies a high risk aversion.

Figure 2.5 reports the average impulse response functions of gross and net labor earnings in our model for the first 10 years after a lottery win. Although we only targeted the average gross labor earnings response of households in the first five years after a lottery win to calibrate χ, both the gross as well as the (untargeted) net labor earnings response functions show a remarkably good fit with the impulse response data provided in Cesarini et al. (2017). This is of course only true starting from year one, the year after the lottery

Figure 2.5: Impulse Response Functions in Data and Model



gain, since lotteries are paid out at some date throughout year 0, which creates an upward bias in the labor supply response in the data. If at all, we slightly overestimate the net earnings response of individuals, indicating that either our average tax rate is too small or the employed tax code is not progressive enough. In either case, this only enforces our simulation results in the next section. Note further that, albeit the fact that we paired lottery evidence from Sweden with labor earnings data from Germany, we do get a good fit for both impulse responses in Figure 2.5, which makes us confident that we do provide valid estimates even with such a mixture of different data sources.[16]

[16]In future work, we plan to also estimate our model using register data on labor earnings and bequests from Sweden, which at the time this paper was written was not yet available to us.

2.6 Results

The policy experiment in our numerical simulation model is very similar to the one in the theoretical analysis. Specifically, we assume that the government unexpectedly increases the (proportional) tax rate on bequests by one percentage point. We start from a case without any inheritance taxes which reflects the large exemption levels for inheritance taxes in Germany. We, for now, focus on the effect such a tax increase has on the life cycle behavior of a generation that lives under the new bequest tax rate for all their life. In Section 2.6.4, we illustrate how to measure the effects on short-run generations, who get surprised by a bequest tax change at some date in the middle of their life cycle.

The column *Total* of Table 2.5 shows the effect of a one percentage point bequest tax increase on the labor earnings and labor tax payments of one cohort. In particular, we evaluate the change in the expected present value of labor earnings and labor tax payments of one generation and relate it to the change in this generation's expected present value of bequest tax payments. The resulting number can be interpreted as the excess tax revenue effect of a change in the bequest tax rate in the spirit of Corollary 2. We find that a one percentage point bequest tax increase leads to an increase in gross earnings of 18.5 cents for each Euro of additional bequest tax payments. This results in a labor tax revenue increase of more than 7.5 cents.

Table 2.3: Effect of a 1% increase in bequest taxes

		Decomposition		
	Total	*Anticipation*	*Heirs*	*Non-Heirs*
Gross Earnings	18.50	9.57	9.60	-0.67
Labor Taxes	7.64	3.93	3.98	-0.27

Effects are measured as fraction of change in bequest tax revenue.

Our theoretical analysis has shown that the present value of labor earnings and labor tax changes can be decomposed into three components, confer (12):

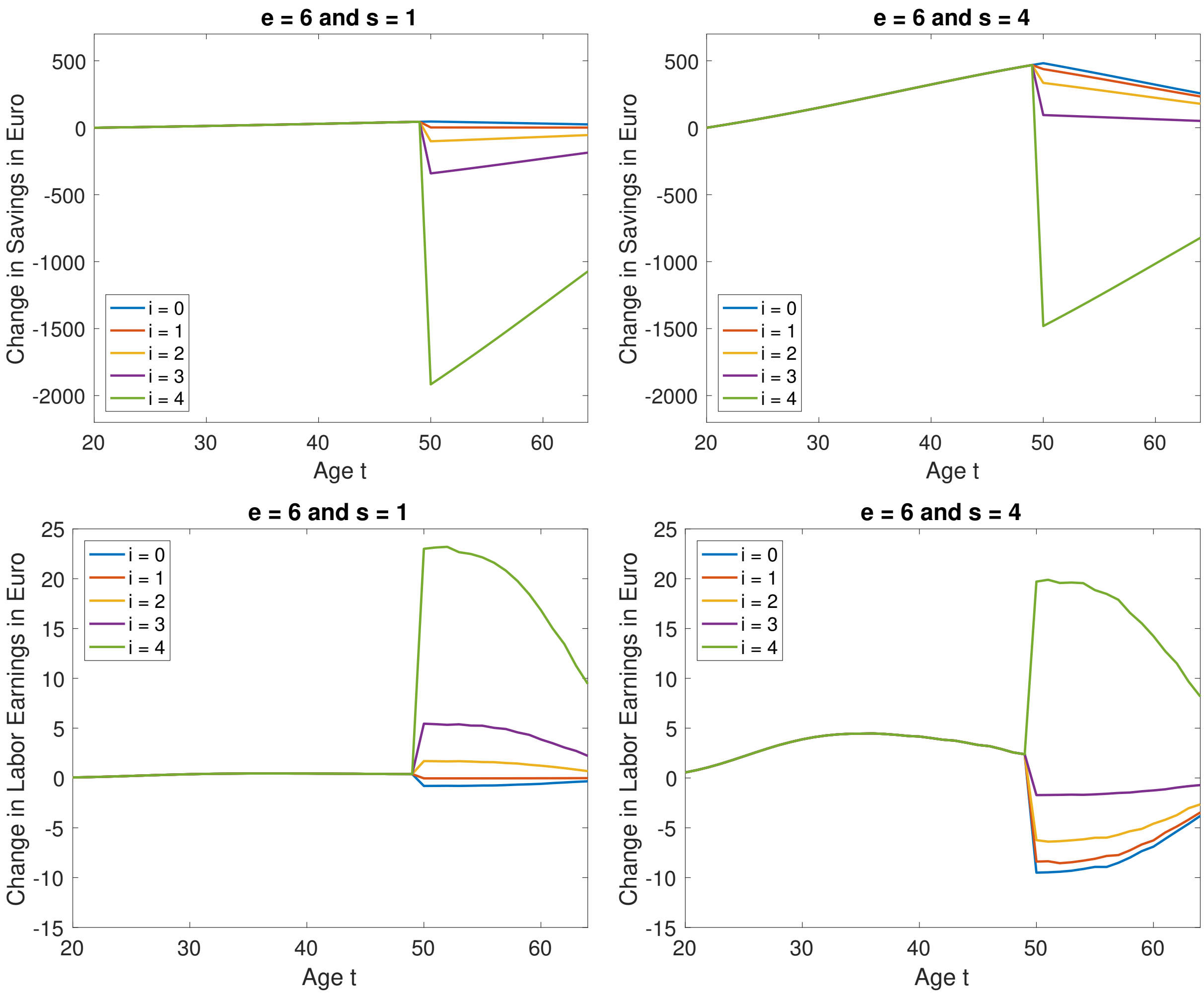
1. Labor supply of heirs increases owing to the direct negative wealth effect induced by a bequest tax increase.
2. The anticipation effect causes households to smooth their labor earnings reaction over the life cycle and leads to higher labor earnings and tax payments already prior to the arrival of an inheritance.
3. As the anticipation effect involves an increase in savings, the resulting negative wealth effect on older cohorts mitigates the earnings reaction fo heirs and leads to a decline in labor earnings for non-heirs.

The extent of these effects is shown in the last three columns of Table 2.5. Both in terms of labor earnings as well as in terms of tax payments the anticipation effect plays an equally important role as the wealth effect on heirs. From this follows that, if we were to treat changes in bequests and bequest taxes as totally exogenous and unanticipated and would therefore only look at the impact on heirs, we would suffer from a serious downward bias in the tax revenue effect, leading us to an estimate of around 4 cents instead of 7.5 for excess labor tax revenue. The impact on non-heirs, which is a result of increased savings, is only modest and reduces the overall excess labor tax payment by only around 0.3 cents.

2.6.1 Illustrating the Mechanism

We now want to elaborate a bit more on the mechanism at work. To this end, Figure 2.6 shows the change in life cycle savings (upper panels) and earnings (lower panels) in Euro values that results from the one percentage point increase in bequest taxes. As an example, we picked households from a moderate earning class ($e = 6$), who's parents die at the age of 50. On the left hand side, we plot life-cycle graphs for agents who are endowed with a signal of $s = 1$ at the beginning of the life cycle, and therefore only expect a very small inheritance. The right hand side shows the same plots for households with a signal of $s = 4$, who consequently expect their inheritance to fall into class $i = 4$ with probability 0.78 (for a signal quality of $\sigma = 0.75$). The different lines denote the actual inheritance the household receives $i = 0, \ldots, 4$.

Figure 2.6: Change in life-cycle behavior of different households



The figure shows that upon the increase in bequest taxes, both household types – those with a low and those with a high signal – increase their savings throughout the life cycle, up to the point where they receive an inheritance. Since households with a high signal expect a larger inheritance and therefore experience a greater wealth effect (at least in expectation), their savings reaction is much more pronounced than for the low signal households. Once the inheritance is received, on the other hand, savings typically drop below steady state levels, which is a direct result of the negative wealth effect induced by the bequest tax.

The lower panels of Figure 2.6 illustrate the importance of the anticipation effect, which first and foremost causes labor earnings to already increase prior to the date at which the household receives an inheritance. As with life-cycle savings, for individuals who expect a large inheritance ($s = 4$), this effect is much more pronounced than for agents with a low signal. Yet. the anticipation effect has a second component: It dampens the labor earning reaction in case the agent receives an inheritance that is greater than her expected inheritance level and causes labor earnings to fall below initial steady state levels in case the expected inheritance is small. Of course, the household endowed with signal $s = 4$ has a much higher expectation than the one with $s = 1$. Hence, labor earnings of the former fall for all inheritance levels but $i = 4$.

2.6.2 Heterogeneity of Effects

Table 2.4 shows the effects of a one percentage point increase in the bequest tax for households of different earnings classes. In order to control for differences in expected bequests, we normalize the earnings and labor tax effects using the expected present value of bequest tax payments for each earnings level. We find a substantial amount of heterogeneity across labor productivity groups. Specifically, within each education group, higher earnings class households exhibit a greater reaction in labor supply. This

Table 2.4: Effect of a 1% increase in bequest taxes by Earnings-Class

	Low Education				*High Education*			
$e =$	1	2	3	4	5	6	7	8
Earnings	12.99	17.90	18.66	21.11	13.31	16.60	19.69	20.36
Taxes	3.99	6.59	7.38	9.12	4.64	6.63	8.37	9.43

Effects are measured as fraction of change in bequest tax revenue by earnings class.

relationship can be understood by realizing that the intratemporal first order condition in our model implies

$$y_t = \left[\frac{1 - \mathcal{T}'(y_t)}{\lambda}\right]^{\frac{1}{\chi}} \cdot w_t^{1+\frac{1}{\chi}} \cdot (c_t)^{-\frac{\gamma}{\chi}},$$

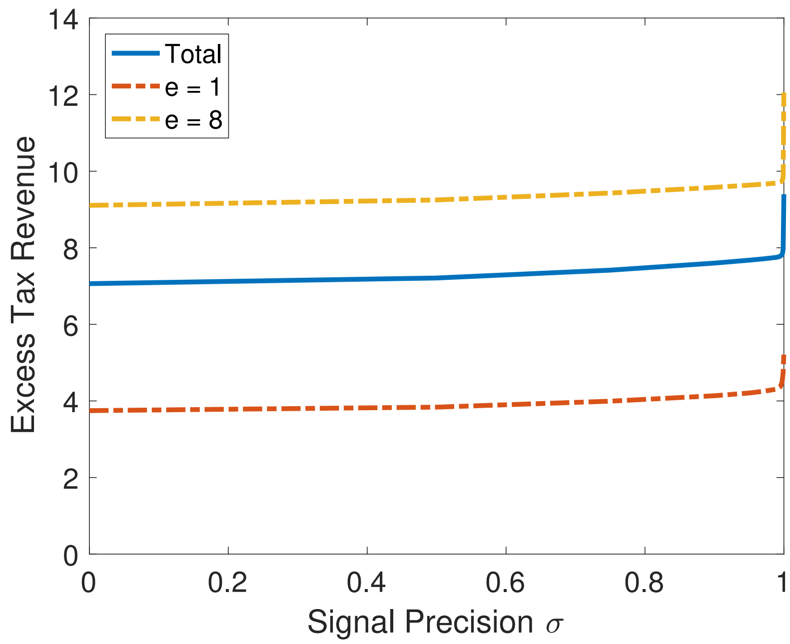
see (16) in Appendix 2.A.4. From this follows that for any decline in consumption c_t (which would be the result of a bequest tax increase), a household with a higher labor productivity will always increase her labor earnings to a greater extent than an agent with low labor productivity.

In economic terms, a higher labor productivity allows a household to counteract changes in exogenous income much easier than an agent with low labor productivity, since a one unit change in labor hours just leads to a much higher change in earnings for the former than for the latter. Or put it differently, a one hour reduction in leisure due to lower wealth translates into a larger increase in earnings and therefore consumption the larger the hourly wage is. Note that the heterogeneity in labor tax changes is larger than the heterogeneity in earnings effects across earnings classes. The reason is that, owing to the progressive labor tax schedule, households with higher labor productivity face much higher marginal tax rates.

2.6.3 The Role of Signal Quality

In our benchmark simulation, we chose a signal quality of $\sigma = 0.75$. Figure 2.7 shows the sensitivity of our results with respect to this signal quality.[17] Recall that for $\sigma = 0$, the

Figure 2.7: Varying Signal Quality



signal contains no information and all households use the cross-sectional distribution of bequests in their earnings class to forecast the size of their inheritance. For $\sigma = 1$, the signal is fully informative and households know exactly in which class their inheritance is going to fall. On the vertical axis of the figure, we again report the excess labor tax effect per unit of additional bequest tax revenue, when we increase the bequest tax rate by one percentage point. We find that, for any $\sigma \ll 1$, labor taxes increase by the same amount of roughly 7.5 cents per Euro of additional bequest tax revenue, regardless of the quality of the signal.

Only when the signal quality approaches 1, this suddenly changes and the excess labor tax revenue increases to almost 10 cents. The reason for this can be found in the natural borrowing constraint (?) of a household. Whenever the signal is less than fully informative, a household can make some forecast about her future inheritance. Yet, there still is the possibility that the agent ends up inheriting nothing. Households would obviously like to distribute the benefits of the expected bequest (that are typically received around the age of 50 to 60) evenly over the life cycle. Those with a higher expected inheritance might therefore even run into debt against future bequest transfers. The amount of debt they can hold is limited by the natural borrowing constraint. In case there is even a slight chance of inheriting nothing, the agent has to make sure that she can still service her debt in case she gets no bequest from her parents. Hence, her natural borrowing limit is relatively tight, even if on average she expects a large bequest. This

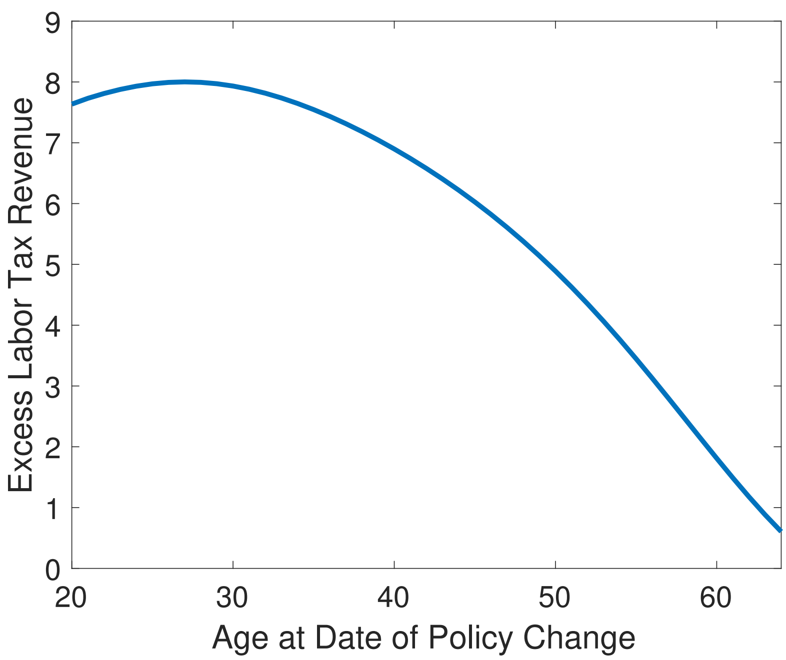
[17]Note that we only vary signal quality and do not recalibrate the labor supply elasticity parameter χ. We however checked for certain combinations that our results also hold under recalibration of χ.

only changes with a fully informative signal. In this case, the only remaining uncertainty is the uncertainty about timing. But eventually, every household with a positive signal will receive a positive bequest. Hence, life-cycle smoothing works much better in this scenario, as the natural borrowing constraint is relaxed. As a result, agents who have a high expectation about bequests will also react much stronger to changes in bequest taxes. In Figure 2.7 this fact can be seen when comparing the change in excess labor taxes for households from a low earnings class, who on average have low expectations about inheritances, with those from a high earnings class.

2.6.4 The Short vs. the Long Run

So far, we only focused on the effect of a change in the bequest tax rate on a cohort that has lived under the new bequest tax rate for their whole life. However, as already pointed out in the theoretical analysis, there is a difference between such cohorts and generations that are surprised by a change in bequest taxes at some date in the middle of their life cycle. In the following, we therefore conduct the same thought experiment as in our theoretical analysis. We assume that the economy is in a steady state with a bequest tax rate of 0%. Then, the government surprisingly increases the bequest tax rate by one percentage point. Figure 2.8 then shows the excess labor tax effect on cohorts with different ages at the time of the reform. Of course, for the cohort aged 1, we again get the very same number as in previous sections, as this cohort is the one that lives under the new tax system for their whole life span.

Figure 2.8: Short-run vs. Long-run Effects



The older a cohort is at the time the bequest tax rate changes, the less years of work remain to react to the tax change. Consequently, the excess labor tax effect declines in a cohort's age almost everywhere. Only for very young cohorts, we see a slight increase in excess tax revenue, which is due to a denominator effect. Since bequests are most likely to arrive at later ages, the labor earnings effect for cohorts between ages 20 and 30 at the time of the reform is almost identical. However, as some inheritances do arrive at these ages, the present value of bequest tax revenue (the denominator in the excess tax revenue effect) decreases in age, which causes the overall excess labor tax effect to increase slightly.

2.6.5 Sensitivity Analysis

As discussed in section 2.5.3, we have two parameters, the coefficient of relative risk aversion γ and the inverse of the Frisch elasticity of labor supply χ to match one target, the propensity to earn out of lottery gains in the five years following the lottery win. Our benchmark calibration of $\gamma = 1$ and $\chi = 4.37$ implies that both risk aversion and the Frisch elasticity of labor supply are in the range of empirical estimates, even though both are at the lower end of this range. In this section we provide robustness checks to this choice. Specifically, we consider the case of a relatively high Frisch elasticity of 0.5 ($\chi = 2.0$). In order for the model to match the lottery evidence on labor earnings, this yet implies that risk aversion needs to be extremely low ($\gamma = 0.475$). Similarly, we consider the other extreme case of a high risk aversion ($\gamma = 4.0$), even though this implies an extremely low Frisch labor supply elasticity of 0.06 ($\chi = 17.9$). For each of these calibrations we compute the effect of a marginal increase in bequest taxes on labor earnings and excess labor income taxes. Table 2.5 summarized the results.

Table 2.5: Effect of a 1% increase in bequest taxes

	$\gamma = 0.475$ *and* $\chi = 2.0$			
	Total	*Anticipation*	*Heirs*	*Non-Heirs*
Gross Earnings	19.36	10.75	9.37	-0.77
Labor Taxes	7.98	4.41	3.88	-0.31
	$\gamma = 4.0$ *and* $\chi = 17.9$			
[0.5ex]	*Total*	*Anticipation*	*Heirs*	*Non-Heirs*
Gross Earnings	15.79	5.60	10.57	-0.38
Labor Taxes	6.54	2.30	4.39	-0.16

Effects are measured as fraction of change in bequest tax revenue.

Despite the very different parameterizations, our number of interest is affected only modestly in both cases. In the case of a high labor supply elasticity and very low risk aversion, it increases by less than half a cent to 7.98, while in the case of high risk aversion and very low labor supply elasticity, it decreases by a bit more than one cent to 6.54. In general, the more elastic labor supply the higher the overall effect on labor earnings and hence tax revenue. We further observe that for the parameterization with high risk aversion, anticipation effects decline while post receipt effects increase. Under a high γ, a larger amount of savings is due to a precautionary motive. An increase in the bequest tax rate reduces not only the expected value of future bequests (increasing savings) but also the variance of potential bequests (reducing savings). With higher precautionary savings this second effect is more important. Lower precautionary savings prior to the receipt of inheritances in turn are financed with lower labor earnings. This, on the other hand, decreases wealth at the time of inheritance receipt further, triggering stronger responses in labor supply thereafter.

2.7 Conclusion

In this paper we elaborate one particular channel of how a change in inheritance taxes affects tax revenue: labor supply of the heirs. We quantify this effect through the lens of a state of the art life-cycle labor supply that is calibrated to match clean quasi-experimental evidence on wealth effects on labor supply. We show that this effect is positive and likely to be sizable and should therefore be taken into account in dynamic scoring exercises where revenues of such tax changes are simulated.

One margin that we were not accounting for and which could make the effect stronger is the education margin. It is likely that individuals do not only make their labor supply decisions conditional on their expectations about inheritances but also their education decisions. In that sense, an increase in inheritance taxes could also imply a positive effect on education of heirs which would imply another positive effect on labor income tax revenue

As we illustrated in our theoretical analysis, inheritance taxes are also likely to affect of labor supply of the bequeathers. This is an other channel that should be studied in future research.

2.A Appendix to Chapter 2

2.A.1 Proofs for 2 Period OLG model

Proof of Proposition 1

Let us assume that our model is in a steady state, meaning that all variables are constant over time. We will work ourselves backwards through the model, starting with period 2 of the household choice problem.

The household problem in period 2 Given a certain level of household savings a, a household of type $K = I, N$ maximizes her remaining life time utility given her instantaneous budget constraint. It is useful to write the optimization problem in terms of labor earnings $y_2^K = w_2 l_2^K$ as

$$\max_{c_2^K, y_2^K, b^K} \quad v\left(c_2^K, \frac{y_2^K}{w_2}, (1-\tau_b)b^K\right)$$
$$\text{s.t.} \quad c_2^K + b^K \leq (1-\tau_l)y_2^K + (1+r)a + \mathbb{1}_{K=I}(1-\tau_b)b + T_2$$

Let's for expositional purposes write the net bequest level a household leaves to her descendants as $b_{net}^K = (1-\tau_b)b^K$. The first order conditions of the optimization problem then read

$$-\frac{v_l\left(c_2^K, \frac{y_2^K}{w_2}, b_{net}^K\right)}{w_2(1-\tau_l)} = v_c\left(c_2^K, \frac{y_2^K}{w_2}, b_{net}^K\right) = (1-\tau_b)v_b\left(c_2^K, \frac{y_2^K}{w_2}, b_{net}^K\right).$$

Using the implicit function theorem, we get

$$\left[v_{cc} + \frac{v_{lc}}{w_2(1-\tau_l)}\right] dc_2^K + \left[v_{cb} + \frac{v_{lb}}{w_2(1-\tau_l)}\right] db_2^K$$
$$= -\left\{ \left[\frac{v_{cl}}{w_2(1-\tau_l)} + \frac{v_{ll}}{[w_2(1-\tau_l)]^2}\right](1-\tau_l)dy_2^K + \left[v_{cb}\cdot\frac{b^K}{b} + \frac{v_{lb}\cdot\frac{b^K}{b}}{w_2(1-\tau_l)}\right] d(1-\tau_b)\cdot b \right\}$$

as well as

$$\left[(1-\tau_b)v_{bc} + \frac{v_{lc}}{w_2(1-\tau_l)}\right] dc_2^K + \left[(1-\tau_b)^2 v_{bb} + \frac{(1-\tau_b)v_{lb}}{w_2(1-\tau_l)}\right] db_2^K$$
$$= -\left\{ \left[\frac{(1-\tau_b)v_{bl}}{w_2(1-\tau_l)} + \frac{v_{ll}}{[w_2(1-\tau_l)]^2}\right](1-\tau_l)dy_2^K \right.$$
$$\left. + \left[(1-\tau_b)v_{bb}\cdot\frac{b^K}{b} + \frac{v_{lb}\cdot\frac{b^K}{b}}{w_2(1-\tau_l)} + v_b\cdot\frac{b^K}{b}\right] d(1-\tau_b)\cdot b \right\}.$$

Note that we use v_{xy} as abbreviation for $v_{xy}\left(c_2^K, \frac{y_2^K}{w_2}, b_{net}^K\right)$.

These two equations constitute a linear equation system in dc_2^K and db_2^K, which (under some regularity assumptions) has a unique solution

$$\begin{bmatrix} dc_2^K \\ db^K \end{bmatrix} = -\begin{bmatrix} \xi_{2cy}^K & \xi_{2c\tau}^K \\ \xi_{2by}^K & \xi_{2b\tau}^K \end{bmatrix} \cdot \begin{bmatrix} (1-\tau_l)dy_2^K \\ d(1-\tau_b)\cdot b \end{bmatrix}$$

Assuming that no resources are put to waste, total differentiation of the budget constraint yields

$$dc_2^K + db^K = (1-\tau_l)dy_2^K + (1+r)da + \mathbb{1}_{i=k} \cdot d\left[(1-\tau_b)b\right] + dT_2$$

which under substitution of the above relationships brings us to

$$dy_2^K = \frac{-(1+r)da - \mathbb{1}_{i=k} \cdot d\left[(1-\tau_b)b\right] - dT_2 - \left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right) \cdot d(1-\tau_b) \cdot b}{(1-\tau_l)\left[1 + \xi_{2cy}^K + \xi_{2by}^K\right]}.$$

From this relationship, we directly see that the labor earnings reaction to a pure change in exogenous income dT_2, keeping savings da, bequests received $d\left[(1-\tau_b)b\right]$ and the net-of-tax rate $d(1-\tau_b)$ fixed, is

$$\left.\frac{dy_2^K}{dT_2}\right|_{da=0} = -\frac{1}{(1-\tau_l)\left[1 + \xi_{2cy}^K + \xi_{2by}^K\right]} =: -\eta_2^K.$$

At the same time, we immediately get with $dT_2 = 0$ that

$$dy_2^K = \eta_2^K \cdot \left\{-\mathbb{1}_{i=k} \cdot d\left[(1-\tau_b)b\right] - (1+r)da - \left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right) \cdot d(1-\tau_b) \cdot b\right\}$$

from which follows that

$$\begin{aligned}\frac{dy_2^K}{d(1-\tau_b) \cdot b} &= \eta_2^K \cdot \frac{d\left[(1-\tau_b)b\right]}{d(1-\tau_b) \cdot b}\left\{-\mathbb{1}_{i=k} - \frac{(1+r)da}{d\left[(1-\tau_b)b\right]}\right\} - \eta_2^K \cdot \left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right) \\ &= \eta_2^K \cdot (1+\varepsilon) \cdot \left[-\mathbb{1}_{i=k} + \alpha\right] - \eta_2^K \cdot \left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right),\end{aligned}$$

with ε being the elasticity of total bequests b received by the household with respect to the net of tax rate $1-\tau_b$. Let us further define $\xi_\tau^K = -\left(\xi_{2c\tau}^K + \xi_{2b\tau}^K\right)$, which measures the effect of a change in the net-of-tax-rate $1-\tau_b$ on the willingness of a household to bequeath to her own descendants. Then by substituting ξ_τ^K into the above equation, we obtain the second part of (7).

The household problem in period 1 Let us define

$$V(a) = \pi \cdot \max_{c_2^I, y_2^I, b^I} \; v\left(c_2^I, \frac{y_2^I}{w}, b^I\right) + (1-\pi) \max_{c_2^N, y_2^N, b^N} \; v\left(c_2^N, \frac{y_2^N}{w}, b^N\right)$$

subject to the second period budget constraints. Then, using Bellman's principle of optimality, we can write the first period optimization problem as

$$\max_{c_1, y_1, a} \; u\left(c_1, \frac{y_1}{w_1}\right) + \beta V(a) \quad \text{s.t.} \quad c_1 + a = (1-\tau_l)y_1 + T_1.$$

The first order conditions with respect to c_1 and y_1 read

$$-\frac{u_l\left(c_1, \frac{y_1}{w}\right)}{w(1-\tau_l)} = u_c\left(c_1, \frac{y_1}{w}\right).$$

Using the implicit function theorem yields

$$dc_1 = -\underbrace{\frac{u_{ll} + \left[w_1(1-\tau_l)\right] \cdot u_{cl}}{\left[w_1(1-\tau_l)\right]^2 \cdot u_{cc} + \left[w_1(1-\tau_l)\right] \cdot u_{lc}}}_{=:\xi_{c1}} \cdot (1-\tau_l)dy_1.$$

Assuming that no resources are put to waste, total differentiation of the budget constraint yields

$$dc_1 + da = (1 - \tau_l)dy_1 + dT_1$$

which under substitution of the above relationships brings us to

$$dy_1 = -\frac{dT_1 - \frac{(1+r)da}{1+r}}{(1-\tau_l)\left[1+\xi_{c1}\right]}.$$

From this relationship, we directly see that the labor earnings reaction to a pure change in exogenous income is

$$\left.\frac{dy_1}{dT_1}\right|_{da=0} = -\frac{1}{(1-\tau_l)\left[1+\xi_{c1}\right]} =: -\eta_1.$$

At the same time, we immediately get with $dT = 0$ that

$$\begin{aligned}\frac{dy_1}{d(1-\tau_b)\cdot b} &= -\frac{\eta_1}{1+r}\cdot\frac{d\left[(1-\tau_b)b\right]}{d(1-\tau_b)\cdot b}\cdot\left[-\frac{(1+r)da}{d\left[(1-\tau_b)b\right]}\right]\\ &= -\frac{\eta_1}{1+r}\cdot(1+\varepsilon)\cdot\alpha.\end{aligned}$$

□

Proof of Proposition 2

The total differential of the life time tax revenue (5) of a generation born at date t is

$$dR_t = \tau_l\cdot\left[dy_{1t} + \frac{\pi dy^I_{2t+1} + (1-\pi)dy^N_{2t+1}}{1+r}\right] + \frac{\pi d\left[\tau_b b_{t+1}\right]}{1+r}.$$

Note that we made the assumption that neither the labor earnings tax rate nor lump-sum transfers are affected by the change in $d\tau_b$. We can write this equation as

$$dR_t = \frac{\pi d\left[\tau_b b_{t+1}\right]}{1+r}\cdot\left\{1 + \tau_l\cdot\frac{d(1-\tau_b)\cdot b_{t+1}}{d\left[\tau_b b_{t+1}\right]}\cdot\frac{1+r}{\pi}\cdot\left[\frac{dy_{1t}}{d(1-\tau_b)\cdot b_{t+1}} + \frac{\pi\frac{dy^I_{2t+1}}{d(1-\tau_b)\cdot b_{t+1}} + (1-\pi)\frac{dy^N_{2t+1}}{d(1-\tau_b)\cdot b_{t+1}}}{1+r}\right]\right\}.$$

We then obtain

$$
\begin{aligned}
&\frac{1+r}{\pi}\cdot\left[\frac{dy_{1t}}{d(1-\tau_b)\cdot b_{t+1}}+\frac{\pi\frac{dy^I_{2t+1}}{d(1-\tau_b)\cdot b_{t+1}}+(1-\pi)\frac{dy^N_{2t+1}}{d(1-\tau_b)\cdot b_{t+1}}}{1+r}\right]\\
&=\frac{1+r}{\pi}\cdot\left[-\frac{\eta_1(1+\varepsilon_{t+1})}{1+r}\cdot\alpha\right.\\
&\qquad\left.+\frac{\pi\left[\eta^I_2(1+\varepsilon_{t+1})\left[-1+\alpha\right]+\eta^I_2\cdot\xi^I_\tau\right]+(1-\pi)\left[\eta^N_2(1+\varepsilon_{t+1})\alpha+\eta^N_2\xi^N_\tau\right]}{1+r}\right]\\
&=-\frac{1}{\pi}\cdot\left\{(1+\varepsilon_{t+1})\left[\eta_1\cdot\alpha+\pi\left[\eta^I_2-\alpha\eta^I_2\right]+(1-\pi)\left[\alpha\eta^N_2\right]\right]\right.\\
&\qquad\qquad\left.-\left[\pi\eta^I_2\chi^I_\tau+(1-\pi)\eta^N_2\chi^N_\tau\right]\right\}
\end{aligned}
$$

Furthermore we get

$$
\begin{aligned}
\frac{d(1-\tau_b)\cdot b_{t+1}}{d\left[\tau_b b_{t+1}\right]}&=\frac{d(1-\tau_b)\cdot b_{t+1}}{\tau_b db_{t+1}+d\tau_b b_{t+1}}=\frac{d(1-\tau_b)\cdot b_{t+1}}{\tau_b db_{t+1}-d(1-\tau_b)b_{t+1}}\\
&=\frac{1}{\frac{\tau_b}{1-\tau_b}\cdot\frac{(1-\tau_b)db_{t+1}}{d(1-\tau_b)b_{t+1}}-1}=-\frac{1}{1-\frac{\tau_b}{1-\tau_b}\cdot\varepsilon_{t+1}}.
\end{aligned}
$$

Putting all of this together yields (9).

The equation for the cohort born at time $s-1$, i.e. right before the bequest tax is increased, then simply follows from the fact that this cohort has – by definition – a savings reaction of $\alpha=0$ and at the same time bequests are predetermined $\varepsilon_s=0$. □

2.A.2 Calibration data extracted from GSOEP

Table 2.6: Mean labor earnings in different earnings classes

	Low Education				*High Education*			
Age	$e = 1$	$e = 2$	$e = 3$	$e = 4$	$e = 5$	$e = 6$	$e = 7$	$e = 8$
20-24	3126	8947	16061	31182	2676	9070	19407	36026
25-29	6342	16614	26748	42639	7274	21607	35064	55638
30-34	11544	23854	32762	50884	18828	34868	46228	73596
35-39	13965	26082	34988	52340	22071	38341	50761	81618
40-44	15216	27946	37049	56708	22313	39453	53004	89428
45-49	14184	27929	38173	59408	22582	40171	54511	94091
50-54	12547	26578	37469	60999	21083	40803	56316	98965
55-59	10328	22015	33568	58279	15927	36203	53249	96778
60-64	9002	15500	23521	45613	12640	26474	42283	76568
65+	8527	13122	16634	28023	10756	16888	22562	45823
Share	0.179	0.179	0.179	0.179	0.071	0.071	0.071	0.071

Table 2.7: Fraction of heirs and mean bequest level by earnings class (low education)

Age	Frac. Heirs (in %)	Mean Bequest	Frac. Heirs (in %)	Mean Bequest
	$e = 1$		$e = 2$	
20-34	0.84	26579	0.61	53812
	(0.14)	(9659)	(0.11)	(16780)
35-44	0.81	39176	1.19	31761
	(0.13)	(10543)	(0.15)	(6165)
45-54	1.11	68150	1.08	49147
	(0.15)	(15992)	(0.15)	(8699)
55-64	1.25	52864	1.17	51501
	(0.18)	(10495)	(0.16)	(8282)
65+	0.60	46869	0.52	46197
	(0.09)	(9562)	(0.08)	(11311)
	$e = 3$		$e = 4$	
20-34	1.43	23577	1.20	73607
	(0.17)	(5573)	(0.16)	(18286)
35-44	0.92	73587	1.37	52417
	(0.14)	(20388)	(0.16)	(15080)
45-54	1.89	63092	1.92	131542
	(0.19)	(18833)	(0.17)	(26858)
55-64	1.54	93182	2.51	70160
	(0.18)	(16922)	(0.21)	(10216)
65+	0.58	47055	1.04	62391
	(0.09)	(9451)	(0.11)	(17901)

Standard errors are reported in parenthesis.

Table 2.8: Fraction of heirs and mean bequest by earnings class (high education)

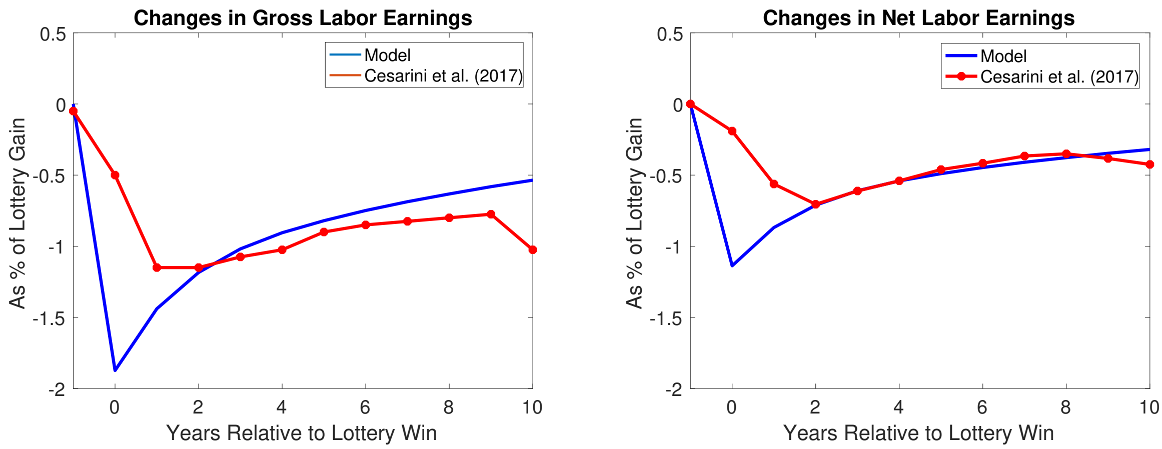
Age	Frac. Heirs (in %)	Mean Bequest	Frac. Heirs (in %)	Mean Bequest
	$e = 5$		$e = 6$	
20-34	1.73	72007	1.14	33552
	(0.34)	(28507)	(0.26)	(10246)
35-44	0.81	46598	1.22	35946
	(0.18)	(16519)	(0.21)	(9806)
45-54	2.38	54616	1.67	68809
	(0.31)	(10300)	(0.24)	(18128)
55-64	2.04	55539	3.11	94364
	(0.31)	(12675)	(0.36)	(16702)
65+	1.13	69136	0.88	103915
	(0.21)	(15121)	(0.17)	(26950)
	$e = 7$		$e = 8$	
20-34	2.03	281532	2.05	81609
	(0.36)	(107188)	(0.38)	(22610)
35-44	1.47	31910	1.85	95899
	(0.23)	(5146)	(0.25)	(16113)
45-54	2.68	55250	2.50	112098
	(0.28)	(11426)	(0.25)	(24719)
55-64	2.75	97200	3.87	127256
	(0.33)	(16277)	(0.33)	(38036)
65+	2.33	76044	2.52	133747
	(0.28)	(12190)	(0.27)	(22585)

Standard errors are reported in parenthesis.

2.A.3 Borrowing limits

In our benchmark calibration we assumed that $a_{min} = -\infty$, i.e. as long as households can service their debt until they die, they can run into debt as much as they wish. In this section we perform the analysis for the other extreme case when no borrowing is allowed ($a_{min} = 0$). We again fix the parameter $\gamma = 1$ and re-calibrate χ such that the model replicates the evidence on earnings responses of lottery winners (Cesarini et al., 2017). Specifically, we need to increase the inverse of the Frisch elasticity to $\chi = 4.50$ such that in the five years following the lottery win, agents reduce their gross earnings on average by 1.07% of the lottery amount. Figure 2.9 again compares the average impulses of gross and net earnings in data and the model.

Figure 2.9: Impulse Response Functions in Data and Model with Strict Borrowing Limit



We observe that in the case of a strict no-borrowing limit the response is too strong in the first years following the lottery win, and too weak in later years (at least for gross earnings). The reason for the difference to the natural borrowing limit (compare Figure 2.5) is that in the no-borrowing case agents at the beginning of their economic life are (without a lottery win or inheritance) not able to borrow against their future income in order to smooth consumption. Instead they finance their whole consumption via labor earnings. Hence, labor earnings are higher than in the case without a constraint on borrowing. If an agent now receives an early lottery, it is not only an income effect that makes her work less but also the relaxation of the borrowing constraint. Further, in the no-borrowing case the limited ability to postpone labor to later periods (when wages are higher), not only makes them work more in early periods but also less in later periods compared to the case of unlimited borrowing. As a consequence with a strict no-borrowing limit a lottery win makes - on average - labor earnings drop a bit less in the years 4-10 after the lottery win than in the case of unlimited borrowing. Overall, this analysis suggests that borrowing constraints do not play a big role or, at least, that a strict no-borrowing limit is a too extreme assumption.

Since it seems to explain the data better, we chose the case of $a_{min} = -\infty$ in our standard parameterization.[18] Nevertheless, as another robustness check we present the results of our policy experiment (increasing the bequest tax from zero to 1%) for $a_{min} = 0$ in Table 2.9.

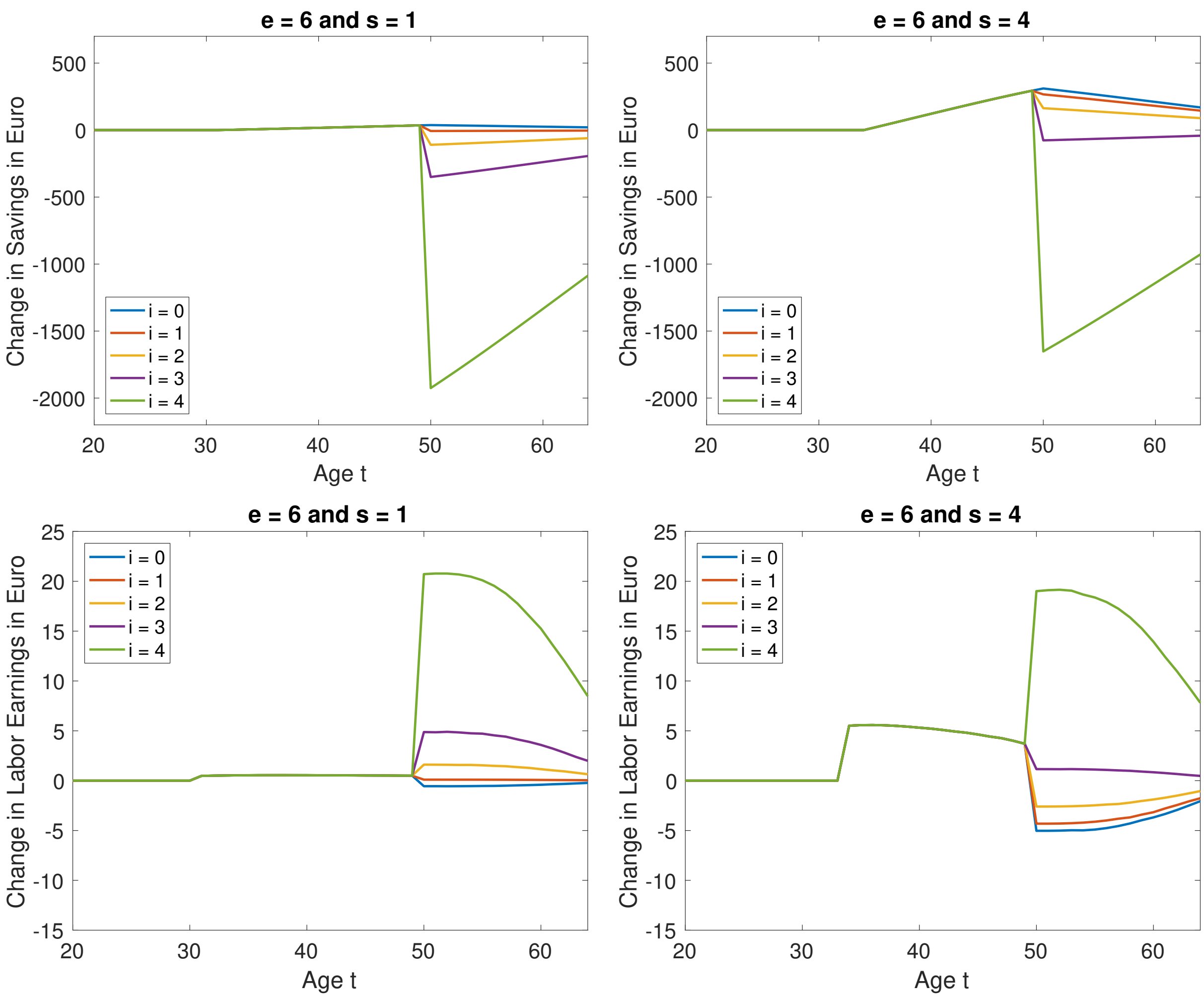
[18] Note that in practice individuals may not only borrow through bank loans but also from their parents.

Table 2.9: Effect of a 1% increase in bequest taxes

		Decomposition		
	Total	*Anticipation*	*Heirs*	*Non-Heirs*
Gross Earnings	16.27	6.29	10.13	-0.31
Labor Taxes	6.83	2.66	4.29	-0.13

Effects are measured as fraction of change in bequest tax revenue.

Figure 2.10: Change in life-cycle behavior of different households



We observe that our number of interest decreases slightly by about 0.8 cents to 6.83. Furthermore, anticipation effects are lower, while post-receipt effects are higher than with $a_{min} = -\infty$. To understand this result, it is again useful to look at the change in life-cycle profiles of savings and labor earnings depicted in Figure 2.10. The strict no-borrowing limit allows agents less to smooth consumption and labor supply over the life-cycle. While in the case of $a_{min} = -\infty$ the increase in the bequest tax resulted in an increase of earnings and savings already from the first year, now this is true only from the age onwards, at which the borrowing constraint seizes to bind (30-35 years). As agents save less early in their life at the time they receive an inheritance their wealth is lower than in the case with borrowing, triggering stronger responses thereafter.

2.A.4 Wealth effect on labor earnings

The dynamic household optimization problem in our model reads

$$V_t(e,s,h_t,W_t) = \max_{c_t,l_t,a_{t+1}} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} - \lambda \frac{l_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}\left[V_{t+1}\left(e,s,h_{t+1},W_{t+1}\right) \middle| e,s,h_t \right] \right\}$$

subject to the budget constraint

$$c_t + a_{t+1} = w_t^e l_t - \mathcal{T}\left(w_t^e l_t\right) + \mathcal{P}_t^e + W_t,$$

where $\mathcal{P}_t^e = 0$ for all workers. We can write the Lagrangean for a working age household as

$$\mathcal{L} = \frac{c_t^{1-\gamma}}{1-\gamma} - \lambda \frac{l_t^{1+\chi}}{1+\chi} + \beta \mathbb{E}\left[V_{t+1}\left(e,s,h_{t+1},W_{t+1}\right)\right] + \mu\left[w_t^e l_t - \mathcal{T}\left(w_t^e l_t\right) + W_t - c_t - a_{t+1}\right].$$

First order conditions with respect to consumption and labor effort are

$$(c_t)^{-\gamma} - \mu = 0 \quad \text{and} \quad \lambda (y_t)^{\chi} = \mu \cdot [1-\mathcal{T}'(y_t)] \cdot (w_t^e)^{1+\chi}. \tag{16}$$

Together with the budget constraint, this leads to

$$F(y_t,W_t,a_{t+1}) := \lambda (y_t)^{\chi} - \left[y_t - \mathcal{T}\left(y_t\right) + W_t - a_{t+1}\right]^{-\gamma} \cdot [1-\mathcal{T}'(y_t)] \cdot (w_t^e)^{1+\chi} = 0,$$

which implicitly defines labor earnings. The implicit function theorem then implies

$$\begin{aligned}
& \frac{\partial F}{\partial y_t} \cdot dy_t + \frac{\partial F}{\partial W_t} \cdot dW_t + \frac{\partial F}{\partial a_{t+1}} \cdot da_{t+1} = 0 \\
\Leftrightarrow \quad & \left[\chi \lambda (y_t)^{\chi-1} + \gamma (c_t)^{-\gamma-1} \cdot [1-\mathcal{T}'(y_t)]^2 \cdot (w_t^e)^{1+\chi} - (c_t)^{-\gamma} \cdot (-\mathcal{T}''(y_t)) \cdot (w_t^e)^{1+\chi}\right] \cdot dy_t \\
& \quad + \left[\gamma (c_t)^{-\gamma-1} \cdot [1-\mathcal{T}'(y_t)] \cdot (w_t^e)^{1+\chi}\right] \cdot dW_t \\
& \quad - \left[\gamma (c_t)^{-\gamma-1} \cdot [1-\mathcal{T}'(y_t)] \cdot (w_t^e)^{1+\chi}\right] \cdot da_{t+1} = 0 \\
\Leftrightarrow \quad & \frac{\chi \lambda (y_t)^{\chi-1} + \gamma (c_t)^{-\gamma-1} \cdot [1-\mathcal{T}'(y_t)]^2 \cdot (w_t^e)^{1+\chi} + (c_t)^{-\gamma} \cdot \mathcal{T}''(y_t) \cdot (w_t^e)^{1+\chi}}{\gamma (c_t)^{-\gamma-1} \cdot [1-\mathcal{T}'(y_t)] \cdot (w_t^e)^{1+\chi}} \cdot dy_t \\
& \quad = -dW_t \cdot \left[1 - \frac{da_{t+1}}{dW_t}\right] \\
\Leftrightarrow \quad & \left[\frac{\chi}{\gamma} \cdot \frac{c_t}{y_t} \cdot \frac{\lambda (y_t)^{\chi}}{(c_t)^{-\gamma} \cdot [1-\mathcal{T}'(y_t)] \cdot (w_t^e)^{1+\chi}} + 1 - \mathcal{T}'(y_t) + \frac{c_t}{\gamma} \cdot \frac{\mathcal{T}''(y_t)}{1-\mathcal{T}'(y_t)}\right] \frac{dy_t}{dw_t} \\
& \quad = -\left[1 - \frac{da_{t+1}}{dW_t}\right]
\end{aligned}$$

From the first order conditions of the household problem, we directly get

$$\frac{\lambda (y_t)^{\chi}}{(c_t)^{-\gamma} \cdot [1-\mathcal{T}'(y_t)] \cdot (w_t^e)^{1+\chi}} = 1.$$

Furthermore, using the functional form of our tax function yields

$$1 - \mathcal{T}'(y_t) = (1-\tau_1) \cdot \frac{y_t - \mathcal{T}(y_t)}{y_t} \quad \text{and} \quad \frac{\mathcal{T}''(y_t)}{1-\mathcal{T}'(y_t)} = -\frac{\tau_1}{y_t}.$$

Hence, we obtain

$$\frac{dy_t}{dW_t} = -\frac{1 - \frac{da_{t+1}}{dW_t}}{\frac{\chi+\tau_1}{\gamma} \cdot \frac{c_t}{y_t} + (1-\tau_1) \cdot \frac{y_t - \mathcal{T}(y_t)}{y_t}}.$$

Consequently, we can write the wealth effect on labor earnings in form of an elasticity as

$$\eta_{y,t} = \frac{dy_t}{dW_t} \cdot \frac{W_t}{y_t} = -\frac{W_t - a_{t+1} \cdot \eta_{a,t+1}}{\frac{\chi+\tau_1}{\gamma} \cdot c_t + (1-\tau_1) \cdot [y_t - \mathcal{T}(y_t)]},$$

with $\eta_{a,t+1}$ being the elasticity of savings into the next period with respect to current wealth.

□

Chapter 3

On the Design of a European Unemployment Insurance System

with Árpád Ábrahám, João Brogueira de Sousa and Ramon Marimon

3.1 Introduction

The recent financial and sovereign debt crises have affected European labour markets asymmetrically both in terms of duration and severity of unemployment. In particular, stressed countries - such as Greece, Portugal and Spain - have experienced high levels of unemployment, making it very difficult, if not impossible, to provide adequate insurance for the unemployed and, at the same time, to satisfy the low-deficit (Fiscal Compact) commitments. This has raised interest in proposals for Europe-wide, or Euro-Area-wide, Unemployment Insurance schemes.[1]

Given the asymmetries and lack of perfect coordination of real business cycles across European countries,[2] a European Unemployment Insurance System (EUIS) can efficiently provide risk-sharing across national labour markets and, at the same time, reduce the countercyclical impact of unemployment expenditures on national budgets. Furthermore, it can provide three additional important benefits for the participant states. First, it can reduce the lasting recessionary effects which follow severe crisis, as it has happened in the euro crisis and recession; second, it can develop a much needed solidarity across national labour markets and, third, it can improve labour mobility and market integrations, since unemployment benefits, and the corresponding active policies of surveillance, do not need to be tied to a specific location.

However, the same asymmetries show that implementing a European Unemployment Insurance scheme may not be easy - or politically feasible - if it implies large and 'persistent transfers' across countries. In fact, these 'persistent transfers' are a good indicator of pending structural reforms; therefore, it is not just an issue of redistribution, it can also be a moral hazard problem: 'persistent transfers' may further delay costly, but needed,

[1] In this paper we abstract from specific legal and institutional requirements; we will therefore refer to a European Unemployment Insurance System (EUIS) in reference to any possible transnational scheme that addresses the type of diversities which are present in the EU.

[2] For an overview on business cycles in the Euro Area see, for example, Böwer (2006), Giannone et al. (2009) and Saiki and Kim (2014).

reforms.

Therefore, to assess the need, viability and possible design of an EUIS one needs to take into account its potential effects: on individual agents' employment and savings decisions; on the aggregate distribution of employment, unemployment and inactivity; on national budgets, in particular taxes to finance unemployment benefits; on insurance transfers across countries; on aggregate savings and investment and, ultimately, on social welfare. In other words, one needs to address these interrelated effects in order to answer a basic question: which unemployment risks need and should – and, if so, how they should – be shared across European countries?

This is a conceptual question that requires a quantitative answer. Unfortunately, with the exception of the works of Dolls et al. (2015) and Beblavy and Maselli (2014), there is very little quantitative evaluation of European Unemployment Insurance schemes. In particular, there is no modelling framework to analyse the key trade-offs of such schemes. In this paper we develop and calibrate – to European countries – a dynamic model to study these effects and provide a set of policy experiments and an implementable proposal.

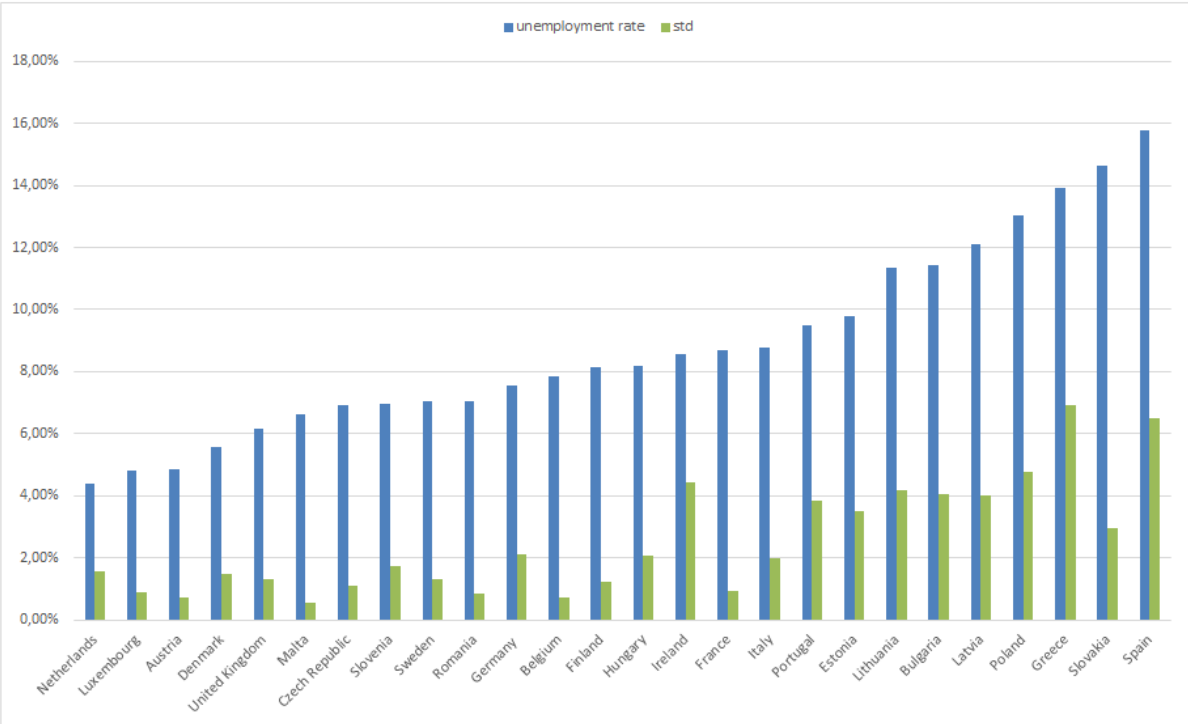


Figure 3.1: Average European Unemployment Rates: 2001-2014

Any model requires an adequate level of abstraction, in our case we need to effectively compare labour markets and unemployment policies of different countries. Regarding labour markets, Figure 1 ranks European countries using Eurostat data on average unemployment rates (and their variability) for different European countries (2001-2014). This is informative of the 'European labour market diversity' but it is too partial and crude an approximation to build a model just based on these statistics. Alternatively, a very detailed description of countries' labour markets and unemployment policies can be very informative but dilutes the main tradeoffs that should be at the core of a dynamic equilibrium model. Our approach is to study worker flows across the three states of employment, unemployment and inactivity. The corresponding transition matrices, and associated steady-state distributions, are the pictures that describe our different economies. For example, using Eurostat quarterly data on worker flows (2010Q2-2015Q4), Figure 2 shows similarities and differences in terms of 'persistency flows': Employment to Employment ("E to E", denoted E-E) versus Unemployment to Unemployment ("U to U", denoted U-U). With the exception of three countries (Spain, Portugal and Slovenia), these 'persistency flows' show a strong correlation among European labour markets, with more important differences on U-U. The corresponding ranking, across this E-E vs. U-U axis (of all but three countries), is not the same as the ranking of unemployment rates of Figure 1. In steady-state, the transition matrix of flows for a given country defines its stationary distribution of employment, and the corresponding Figures 1 and 2 are just two snapshots of European labour markets. Behind the scattered plots lie possible differences in preferences, technologies and market institutions, and labour policies. We will assume that across EU countries citizens share (almost) the same preferences and that labour mobility is relatively low across countries (we assume it is nil) but that EU countries still differ in the other aspects – mainly, market institutions and labour policies.

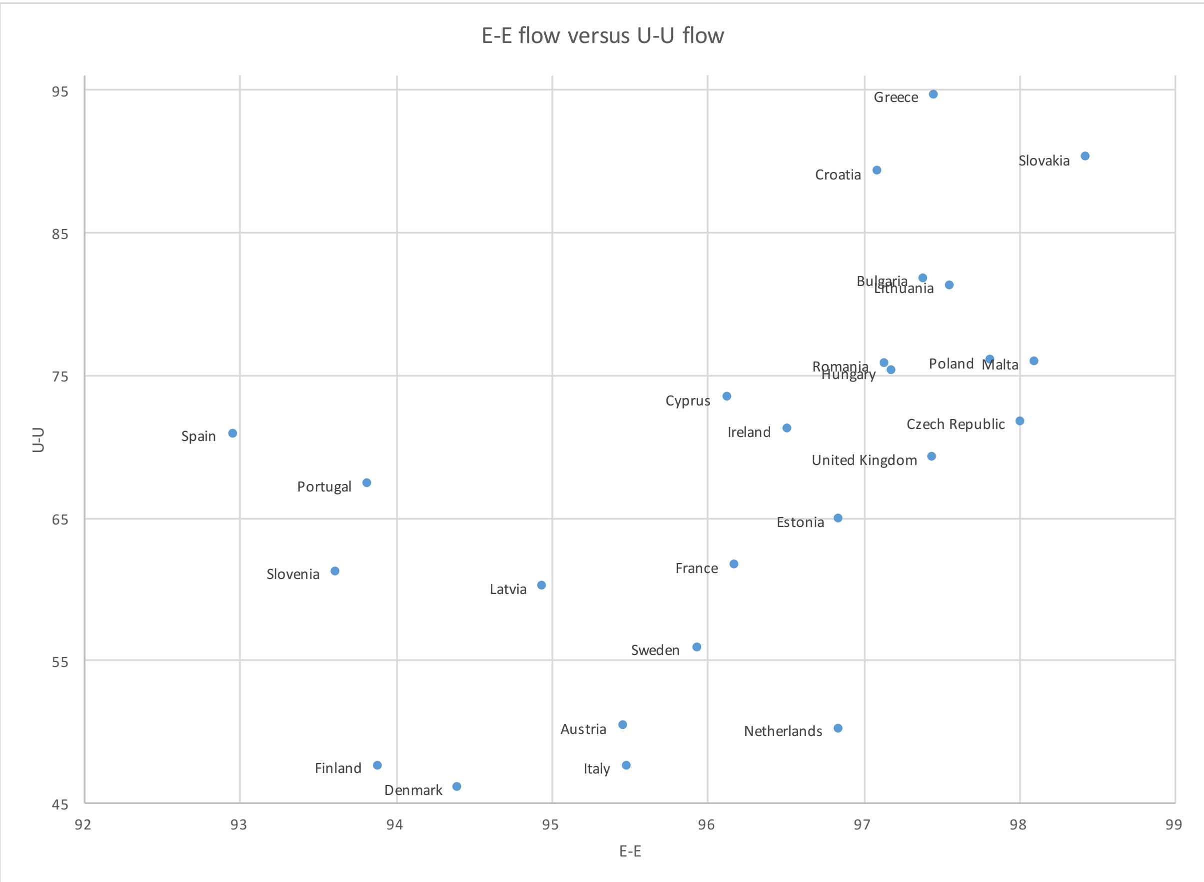


Figure 3.2: Persistence of Employment and Unemployment

We build on the work of Krusell et al. (2011) and Krusell et al. (2015), who calibrate the U.S. three-states flows with a dynamic general equilibrium model with labour market frictions, to analyse the diverse European labour markets. As in their calibration analysis, we generate worker-flows transition matrices and distributions across the three states as the outcome of a dynamic general equilibrium. This requires us to set a few parameters on preferences and technology, and calibrate others to match flows and stocks, consistently with observed time series and the existing unemployment policies of a country. More specifically, our model economies are characterised by three sets of parameters: (*i*) generic parameters of preferences and technologies common to all economies – agents' discount factors, idiosyncratic productivity shock, etc.; (*ii*) country-specific structural parameters of their economies - for example, the job-separation and job-finding rates, which in turn are a summary of different factors determining job creation, destruction and matching, and (*iii*) the country-specific unemployment insurance policies, summarized in two – plus one – parameters; the two are the replacement ratio (unemployment benefits to wages) and the duration of unemployment benefits; the third is the unemployment payroll tax rate needed to balance the budget within a period. Section 3.3 describes our model.

Our calibration is a contribution in itself: it provides a novel diagnosis of the European labour markets, since it reveals the key parameters that explain their different performance – in terms of unemployment (or employment) and its persistence. Country-specific structural parameters – in particular, job-separation and job-finding rates – and not UI policy parameters, are the key parameters. Not surprisingly, the job-finding rates for unemployed and for inactive are aligned, but their ranking, while very significant to explain persistence, provides a partial picture of labour market performance: one needs to account for the job-separation rate – for example, the very high job-separation rate of Spain – to get a more accurate one. In contrast, the 'technological' dimension in which we allow countries to differ – the total factor productivity – is not a key parameter to account for labour market differences, it mostly accounts for average wage differences. The fact that differences in UI policy parameters do not correspond to differences in labour market performance does not mean they are not relevant: they are, for two related reasons. First, because they show interesting patterns: for example, countries with high unemployment rates –say, Spain, Portugal, Greece and Slovakia – have low replacement rates but, among them, only those with high job-separation rates have long average duration of unemployment benefits (Spain and Portugal), while long average duration of unemployment benefits and high job-separation rates are also characteristic of countries with low unemployment rates (Denmark and Finland). Second, they are relevant because different UI policies – and/or different distributions of employment – result in different payroll taxes, since in our calibration all national budgets balance. These tax differences also determine the desirability of UI policy changes, at the national or at the EU – or some other – level. It should be noted that our UI policy parameters are related, but not on a one-to-one basis, with reported replacement and duration rates. We account for the reported eligibility rates, but then we let the reported benefits and the existing unemployment rates and flows determine our calibrated UI parameters. Section 3.4 provides a more detailed description of our calibration procedures and results.

Our model and its calibration provide the framework for our policy experiments, the main goal and contribution of this paper. Perhaps the most frequently used argument in favor of an EUIS is that it may provide insurance against country specific large fluctuations in unemployment, which with limited fiscal capacity result in fluctuations in the tax burden associated with its financing. Our first experiment therefore targets a quantitative evaluation of the potential pure risk sharing benefits of an EUIS when one country suffers a severe negative shock. To this end, we compute the labour market and welfare consequences of a deep recession in two alternative scenarios: (i) the government is in financial autarky and needs to raise taxes on the employed in order to maintain a balanced UB budget; (ii) the country is insured against increased unemployment and can go through the recession without raising taxes. Otherwise, we assume that the unemployment insurance system remains the same in all remaining countries in both cases. We find that the risk sharing benefits resulting from the welfare differences of the second scenario with respect to the first one are small, and marginally higher for the employed, whose taxes are smoother, than for the unemployed, whose benefits have not changed. This experiment implies that although insurance benefits exist, their small size, questions the rationale for a EUIS as a "rainy day fund", unless it rains very often.

In light of this result, one may doubt the desirability of a European unemployment insurance system. Even more so as the observed heterogeneity in labour market institutions suggests that the optimal benefit systems could differ substantially across European countries, making it difficult for governments to reach a common ground. To evaluate this claim, we compute the optimal unilateral reform of the unemployment benefit system (financed at the national level), separately for each country. We perform this exercise in partial equilibrium assuming that a single country does not affect equilibrium prices. We find that the optimal mix of replacement rate, and duration of unemployment benefits, is surprisingly similar across the countries studied. In all countries it is optimal to provide an unlimited duration of eligibility and the optimal replacement rates vary between 20% and 45%.

Despite similar optimal national unemployment insurance policies one may still argue that the small difference suffice to let countries reform their systems by themselves rather than to force them into a common European benefit scheme. We show that this argument is flawed because individual national governments do not internalize general equilibrium effects of their reforms on citizens in other European countries. In particular, we show that if all European countries would reform their system simultaneously and the capital market is required to clear at the union level, i.e. in general equilibrium, the very same UI benefit systems that seem optimal in partial equilibrium, are in fact welfare reducing in most of the countries. If national governments are benevolent but only towards the citizens of their own country, they would reform the benefit system towards a more generous one than what is optimal from a collective European perspective. Increasing the generosity of the UI benefit system in some European countries, reduces private savings and hence the aggregate, European, capital stock. As a consequence the marginal product of labour declines everywhere. This redistributes from poor agents, who derive most of their income from wages to rich agents with mainly capital income. Importantly, the common European capital market implies that this redistribution happens across all Europe.

The final contribution is to provide a better alternative: a common European Unemployment Insurance System (EUIS). We first show that a fully harmonized system which is jointly financed at the European level is unlikely to achieve unanimous support across member states as it would result in transfers from countries with structurally low unemployment to high unemployment countries. Interestingly, for some of the net payers, the welfare gains of such a reform are positive, suggesting that in these countries the current unemployment benefit systems are far from optimal. We then neutralize transfers through varying contribution payments across countries. We find that an EUIS with an unlimited duration and a replacement rate of 15% is welfare improving in *all* countries and almost unanimous. The unlimited duration insures agents against the risk of losing eligibility before the receipt of unemployment benefits ends. At the same time the low replacement rate stabilizes incentives to work and save, keeping the European capital stock and therefore wages high. A positive side effect of such a system with tax differences that eliminate cross-country transfers is that these differences may serve as an incentive device for individual countries to structurally reform weak labour market institutions.

Implementation

Although it is not the focus of this paper, it is worth to briefly consider how this EUIS proposal could be implemented. The basic idea is that it can be implemented through the existing national Unemployment Insurance Systems, it is for this reason we have only considered the common form of unemployment benefits defined by their 'replacement and duration'. If the national funds had enough borrowing capacity, to provide the unemployment benefits without increasing the taxes in times of crisis, and enough commitment, to properly accumulate funds in normal and good times, the EUIS would only require policy commitment and coordination. However, not all (if any) existing national systems satisfy these requirements, in which case a mixed solution between the national UI funds and a central EU fund is in order.

The EUIS central fund can be hosted in the *European Stability Fund*[3] which would have contracts with participating countries stipulating (unemployment) countercyclical transfer between the national fund and the central fund as to guarantee the uniform unemployment benefits preserving smooth taxes within the limited borrowing capacity of the national fund. In other words, as with other *ESF* contracts, first there must be a country-risk assessment (an improved version of our calibration) to assess the country referential stable payroll tax rate and unemployment rate, as well as the thresholds unemployment rates determining country transfers to and from the central fund. The contract should be designed, as other ESF contracts, to guarantee that these transfers do not become permanent transfers. In fact, a stable system of payroll taxes and benefits results in fluctuating net revenues at the country level when, in additional to agents' idiosyncratic risk, there is also country risk (as in our first experiment in Section 3.5).

The mixed design of the EUIS means that the central fund absorbs these fluctuations beyond certain limits (given by unemployment rate thresholds), acting as a safe deposit when unemployment is relatively low and providing insurance when it is relatively high. Our reported structural differences across countries imply that constrained efficient contracts between participating countries and the fund should be country specific, but based on the same common principles. On a periodic basis – say, every seven years – the country risk assessment should be updated and the referential rates adapted accordingly, to make sure that transfers fulfil their stabilisation role without becoming persistent inflows or outflows, to or from the fund.

The remainder of this paper is organized as follows. The next section briefly discusses the current literature on the topic. In section 3.3 we present the model and in section 3.4 our calibration, which provides the basis for our policy experiments in section 3.5. Finally, section 3.6 concludes.

3.2 Literature Review

There are a few recent papers that also study different aspects of the design of a EUIS coming both from academic scholars and from policy institutions. In this section, we review briefly some of the most recent and relevant papers on this issue.

[3] See the ESF ADEMU proposal in Marimon (2018) based on Ábrahám et al. (2018) characterization of ESF constrained efficient contracts.

On the hand, Ignaszak et al. (2018) study the optimal provision of unemployment insurance in a federal state containing atomistic (and symmetric) regions. The focus of their paper is different from ours in three important dimensions. First, in their environment, the regions are ex ante identical, hence they cannot study the asymmetric effect of a EUIS on the different participating nations as we do. At the same time, their model allows for a rich interaction between federal and local policies as regional governments have a wide set of instruments, that they can use to respond to the introduction of new federal policies. Their main focus is indeed to study the crowding out of regional incentives due to generous federal insurance schemes (moral hazard). The third difference is that their model does not allow for an intertemporal saving technology for any agents (households, regions or the union altogether). Our results show that general equilibrium effects of different unemployment insurance policies through the savings channel can be quantitatively very important.

On the other hand, Claveres and Clemens (2017) and Moyen et al. (2016) study unemployment insurance and international risk sharing in a two-region DSGE model with frictional labour markets and calibrate their model to the core and the periphery of the Euro-zone. In both papers, a supranational agency runs an unemployment insurance scheme that triggers transfers to recessionary countries but has zero transfers in expectation. Such a scheme allows recessionary countries to maintain unemployment benefits and simultaneously reduce taxes, thus dampening recessionary effects. Our model differs in many dimensions from these papers. First, our model features a higher degree of heterogeneity both across and within countries. In particular, our policy experiments are performed with ten countries of the Euro area instead of two regions. As we show, labour market institutions and consequently flows across employment, unemployment and inactivity are as heterogeneous across countries within the core (and the periphery) as across the core and the periphery. For example, we found that certain implementations of an EUIS have significantly different effects on Belgium and Germany, two core countries. In addition, the combination of endogenous savings decisions and idiosyncratic productivity shocks result in a non-degenerate distribution of wealth in our model. We show that this within country wealth heterogeneity is a key determinant for both the welfare effects of UI policies and for determining the general equilibrium channel of policies through precautionary savings. Finally, our paper provides an extensive welfare evaluation (across countries, employments states and wealth levels) of different EUIS implementations both with business cycle fluctuations and by studying the transition to a new steady states after a policy reform.

In contrast with the previous papers, Dolls et al. (2015) and Beblavy and Lenaerts (2017) take into account the rich heterogeneity within the Euro area. They provide quantitative exercises that measure the possibilities for intertemporal and interregional smoothing of unemployment benefits and social security contributions under different versions of a EUIS. Both papers present a set of counterfactual scenarios where household income and the evolution of labour markets are kept fixed during the period of study, and different specifications of a EUIS are considered. As in our paper, both studies find considerable interregional and intertemporal smoothing possibilities. In contrast with our paper, the lack of individual responses does not allow them to evaluate the effects of different insurance systems on labour markets, household consumption, individual savings and welfare. In addition, this implies that there are no equilibrium adjustments either and no effect on aggregate savings and capital accumulation.

Finally, Dullien et al. (2018) provide a concrete proposal to be discussed at the European Parliament following a similar approach as the two papers above. In contrast with our work, they only focus on the fund-contract aspect, applying the self-insurance and the reinsurance principles to the design of a EUIS which operates national funds and a joint 'stormy day fund' that is operational only when the country is hit by a severe crisis. Similarly to ours, their scheme is intended to be implemented on a voluntary basis and it has interesting countercyclical features, which can improve upon the current situation. However, the national contracts are not based on a country-specific risk-assessment, the final destination of the funds is not guaranteed and similarly the above papers the methodology does not allow to evaluate the impact on individual decisions and on equilibrium outcomes.

3.3 Model

Our model economy consists of a union of $I \in \mathbb{N}$ countries. We assume that the population in each country $i \in \{1, ..., I\}$ is fixed and that there is no migration across countries. This implies that labor markets clear country by country. Capital, on the other hand, is perfectly mobile across countries. We assume that the union as a whole is a closed economy such that the (weighted) sum of the capital stocks in all countries equals the savings of all citizens in the union.

Each country is modeled along the lines of Krusell et al. (2011) and Krusell et al. (2015). Their model captures key economic decisions of agents regarding their labour market behaviour and is therefore suited to think about unemployment policy. In particular, in the model, given labour income taxes and unemployment benefits, agents with an opportunity to work are able to choose whether or not they work and agents currently not employed are able to choose whether or not to actively search for a job.

Timing and Preferences. Time $t \in \{0, 1, 2, ...\}$ is discrete. Each country is populated by a continuum of agents of measure n^i, where $\sum_{i=1}^{N} n^i = 1$. Preferences over consumption, labour supply and job search are given by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big[\log(c_t) - \alpha w_t - \gamma^i s_t \Big]. \tag{1}$$

Agents derive utility from consumption c_t and disutility from employment w_t and job search s_t. The parameter α captures the disutility of work and is assumed to be the same in each country. The parameter γ^i denotes the disutility of active job search and is varying across countries. In this way we capture that the governments' assistance in the search for a job differs across countries. The time discount factor $\beta \in (0, 1)$ is the same for all citizens in the union. Workers can only choose to supply labor on the extensive margin, i.e. $w_t \in \{0, 1\}$. Additionally, the search decision is also discrete: $s_t \in \{0, 1\}$.

Markets and Technology. The production sector is competitive. Firms, who produce according to a constant returns to scale technology, hire labour from the domestic labour market and pay a wage per efficiency unit of labour that equals the marginal product of labour. They rent capital from the international capital market at a price r_t and pay for the depreciation of capital; the total rental price equals the marginal product of capital, which is the same across countries. Workers supply labour in the domestic market. This market is characterized by frictions that affect workers' separations from jobs, and workers' access to a job opportunity. In what follows, these frictions are described in detail.

In the beginning of every period, agents who were employed in the previous period can loose their job with probability σ^i. The probability of finding a job while not employed depends on the search effort. An agent who is actively searching during period t finds an employment opportunity for period $t+1$ with probability λ_u^i; an agent who is not actively searching, with probability $\lambda_n^i < \lambda_u^i$. After loosing a job, agents who search may be eligible for unemployment benefits. The process that determines eligibility for unemployment benefits is described below. Note that the job arrival rates and the job separation rate are country specific. In this way we capture the heterogeneity in labour market institutions across Europe.

Agents are heterogeneous with respect to their labour productivity, denoted by $z_t \in Z = \{\bar{z}_1, \bar{z}_2, ..., \bar{z}_{n_z}\}$. Idiosyncratic productivity follows a first order Markov chain with transition probabilities $p(z'|z)$. This process is assumed to be the same in each country.

Agents cannot directly insure themselves against the idiosyncratic productivity risk, however they can save using a risk-free bond. The risk-free return is given by the international real interest rate r_t.

Production is given by the Cobb-Douglas technology:

$$F^i(K_t^i, L_t^i) = A_t^i (K_t^i)^\theta (L_t^i)^{1-\theta}, \tag{2}$$

where A_t^i denotes total factor productivity in country i, K_t^i the aggregate capital stock in country i and θ the capital share of output. L_t^i is aggregate labour in country i, measured in efficiency units. In what follows, we generally assume no aggregate (country-specific) shocks, i.e. $A_t^i = A^i$.[4]

Individual Labour Market States. An agent can be employed, unemployed or inactive. The difference between unemployed and inactive agents is that the former exert search effort while the latter do not. Further, if an agent is unemployed he can either be eligible for unemployment benefits, in which case he receives a certain fraction of his potential income as a wage worker or he can be non-eligible, in which case he does not receive benefits and hence solely lives from his savings. This gives a total of four possible individual labor market states that an agent can attain, $x_t \in \{e, u^e, u^n, n\}$: employed, unemployed eligible, unemployed non-eligible, non-participating;

Unemployment Benefits. Eligibility for unemployment benefits is partially determined by agent's endogenous decisions, partially by exogenous shocks. Only agents who are exogenously separated from their job are eligible for unemployment benefits, while agents who quit their job themselves are not eligible. Further, in order to maintain eligibility agents have to continuously exert search effort. Once an agent stops searching, she is non-eligible even if at some later time she starts searching again. Finally, in every period with some probability μ^i agents loose eligibility even if they search for a job. This is a parsimonious way to capture limited (and country-specific) duration of unemployment benefit receipt.[5] Non-eligibility is an absorbing state. The only way to regain eligibility is to find a job, be employed for some time and then be exogenously separated again.

[4]We deviate from this assumption only in subsection 3.5.1.

[5]In reality this duration is not stochastic but fixed. However, implementing a fixed duration is computationally expensive as it requires to keep track of the periods that each unemployed agent already receives benefits. To economize on the state space we hence use this stochastic process as in Krusell et al. (2011) and Krusell et al. (2015).

An eligible unemployed agent in country i receives unemployment benefits $b_t^i(z_t)$ according to

$$b_t^i(z_t) = \bar{b}_t^i \omega_t^i z_t \tag{3}$$

where $\bar{b}_t^i$ is the replacement rate in country i, ω_t^i is the wage per efficiency unit of labour and z_t is the agent's current productivity level. The formula in (3) implies that an agent receives unemployment benefits according to his current labor market productivity. A more realistic assumption would be to have unemployment benefits depend on past labour earnings. We choose (3) to economize in the dimension of the state space of the model (avoiding the need to keep track of past productivity of currently unemployed agents), and because the process z_t is persistent, implying that current productivity is a good proxy for previous labor earnings.

Budget Sets. In every period t, each agent in country i chooses a pair of consumption and savings from a budget set $B_t^i(a, z, x)$ that depends on his current assets, productivity and employment state as well as on current prices r_t and ω_t^i. The budget set of an agent who is employed in period t ($x_t = e$) is given by

$$B_t^i(a, z, e) = \Big\{(c, a') \in \mathbb{R}_+^2 : c + a' \geq (1 + r_t)a + (1 - \tau_t^i)\omega_t^i z\Big\}. \tag{4}$$

An employed agent finances consumption c and savings a' with current period's asset a inclusive of interest income $r_t a$ and income from work, net of the tax rate τ_t^i. An unemployed agent who is eligible for unemployment benefits faces the budget set

$$B_t^i(a, z, u^e) = \Big\{(c, a') \in \mathbb{R}_+^2 : c + a' \geq (1 + r_t)a + b_t^i(z)\Big\}. \tag{5}$$

He does not have wage income but receives some fraction of his potential income as unemployment benefits.

Finally, both unemployed non-eligible and non-active agents finance consumption and next period's assets exclusively from savings:

$$B_t^i(a, z, u^n) = B_t^i(a, z, n) = \Big\{(c, a') \in \mathbb{R}_+^2 : c + a' \geq (1 + r_t)a\Big\}. \tag{6}$$

Labor Market Decisions and Value Functions. The individual optimization problem has a recursive representation. Denote the value of an individual in country i, period t, and state (a, z, x), by $V_t^i(a, z, x)$. The time index of the value function captures in a simple way that the current value depends on current and future prices and government policies, which both vary over time. Then the value of an agent in employment is given by

$$\begin{aligned} V_t^i(a, z, e) = \max_{(c,a') \in B_t^i(a,z,e)} \Bigg\{ & \log(c) - \alpha + \beta \sum_{z' \in Z} p(z'|z) \Big[(1 - \sigma^i) \max_{x' \in \{e, u^n, n\}} V_{t+1}^i(a', z', x') \\ & + \sigma^i \Big(\lambda_u^i \max_{x' \in \{e, u^e, n\}} V_{t+1}^i(a', z', x') + (1 - \lambda_u^i) \max_{x' \in \{u^e, n\}} V_{t+1}^i(a', z', x') \Big) \Big] \Bigg\}. \end{aligned} \tag{7}$$

The Bellman equation reflects the dynamics of the labour market. In the present period the worker derives utility from consumption but disutility of work. The continuation value takes into account that with probability $1-\sigma^i$ the agent will not be separated from the job. In this case he can choose between staying employed or to quit the job. In the latter case he can choose to stay inactive or to search for a new job. He will, however, not be eligible for benefits as he decided to leave the firm himself. Hence, if the worker does not get separated from his job he has three choices, $x' \in \{e, u^n, n\}$. With probability σ^i the worker is separated from his job. Then with probability λ_u^i he immediately gets matched with a new firm, in which case he again can choose between employment, unemployment and inactivity. If he chooses unemployment he is eligible for benefits since he was exogenously separated from the job. With probability $1 - \lambda_u^i$ he does not immediately find a new job. In this case he can only choose between eligible unemployment and inactivity, i.e. $x' \in \{u^e, n\}$. Note that a worker who was separated from his job will get unemployment benefits for one period with certainty as long as he searches for a new job during this period.

Similarly, the value of an eligible unemployed agent in country i satisfies:

$$V_t^i(a, z, u^e) = \max_{(c,a')\in B_t^i(a,z,u^e)} \Bigg\{ \log(c) - \gamma^i + \beta \sum_{z'\in Z} p(z'|z) \Bigg[\lambda_u^i \Bigg((1-\mu^i) \max_{x'\in\{e,u^e,n\}} V_{t+1}^i(a', z', x') + \mu^i \max_{x'\in\{e,u^n,n\}} V_{t+1}^i(a', z', x') \Bigg) + (1-\lambda_u^i) \Bigg((1-\mu^i) \max_{x'\in\{u^e,n\}} V_{t+1}^i(a', z', x') + \mu^i \max_{x'\in\{u^n,n\}} V_{t+1}^i(a', z', x') \Bigg) \Bigg] \Bigg\}. \quad (8)$$

In the present period an unemployed agent incurs the utility cost of searching γ^i. While searching, a job offer for next period arrives with probability λ_u^i, in which case the agent can choose between employment, unemployment and inactivity. With the remaining probability $1-\lambda_u^i$ the agent does not receive a new offer and thus can only choose between unemployment and inactivity. Further the unemployed looses eligibility for benefits with probability μ^i and keeps eligibility with the remaining probability $1 - \mu^i$.

The value of the non-eligible unemployed is very similar. The only exception is that he will not be eligible for benefits next period with certainty,

$$V_t^i(a, z, u^n) = \max_{(c,a')\in B_t^i(a,z,u^n)} \Bigg\{ \log(c) - \gamma^i + \beta \sum_{z'\in Z} p(z'|z) \Bigg[\lambda_u^i \max_{x'\in\{e,u^n,n\}} V_{t+1}^i(a', z', x') + (1-\lambda_u^i) \max_{x'\in\{u^n,n\}} V_{t+1}^i(a', z', x') \Bigg] \Bigg\}. \quad (9)$$

Finally, the value for non-active (i.e. not actively searching) agents in country i is given by

$$V_t^i(a, z, n) = \max_{(c,a')\in B_t^i(a,z,n)} \Bigg\{ \log(c) + \beta \sum_{z'\in Z} p(z'|z) \Big[\lambda_n^i \max_{x'\in\{e,u^n,n\}} V_{t+1}^i(a', z', x') + (1-\lambda_n^i) \max_{x'\in\{u^n,n\}} V_{t+1}^i(a', z', x') \Big] \Bigg\}. \quad (10)$$

The value of the non-active is similar to the non-eligible unemployed. The difference is that a non-active does not suffer the disutility of search and has a lower probability of a receiving a job offer next period, i.e. $\lambda_n^i < \lambda_u^i$.

Definition of Partial and General Equilibrium. We will now define two equilibria: (i) the partial equilibrium for a specific country i, which takes the union interest rate r_t as given; (ii) the general equilibrium for the union, for which the interest rate r_t is required to adjust such that aggregate savings equal aggregate capital in the union.

Individual state variables are assets $a \in \mathbb{R}_+$, idiosyncratic productivity $z \in Z$, and employment status $x \in \{e, u^e, u^n, n\}$. The aggregate state in country i is described by the joint measure ζ_t^i over assets, labor productivity status and employment status. Let $\mathcal{B}(\mathbb{R}_+)$ be the Borel σ-algebra of $\mathbb{R}_+$, $\mathcal{P}(Z)$ the power set over $Z = \{\bar{z}_1, \bar{z}_2, ..., \bar{z}_{n_z}\}$ and $\mathcal{P}(X)$ the power set over $X = \{e, u^e, u^n, n\}$. Further, let $\mathcal{M}$ be the set of all finite measures over the measurable space $\{(\mathbb{R}_+ \times Z \times X), \mathcal{B}(\mathbb{R}_+) \times \mathcal{P}(Z) \times \mathcal{P}(X)\}$.

Definition 5. *Partial equilibrium in country i: Given sequences of interest rates $\{r_t\}_{t=0}^{\infty}$ and unemployment benefit policies $\{(\bar{b}_t^i, \mu_t^i)\}_{t=0}^{\infty}$ and given an initial distribution ζ_0^i, a partial equilibrium in country i is defined by a sequence of value functions $\{V_t^i\}_{t=0}^{\infty}$, consumption and savings decisions $\{c_t^i, a_{t+1}^i\}_{t=0}^{\infty}$, firm production plans $\{K_t^i, L_t^i\}_{t=0}^{\infty}$, payroll taxes $\{\tau_t^i\}_{t=0}^{\infty}$, wages $\{\omega_t^i\}_{t=0}^{\infty}$ and measures $\{\zeta_t^i\}_{t=1}^{\infty}$, with $\zeta_t^i \in \mathcal{M}$ $\forall t$, such that:*

(i) Agents optimize: Given prices, unemployment benefit policies and tax rates, the value function V_t^i and the policy functions for consumption c_t^i and savings a_{t+1}^i satisfy the Bellman equations (7), (8), (9) *and* (10) *with equality for each $t \geq 0$.*

(ii) Firms optimize: Prices satisfy $r_t = F_K^i(K_t^i, L_t^i) - \delta$ and $\omega_t^i = F_L^i(K_t^i, L_t^i)$ for each $t \geq 0$.

(iii) The labour market clears:

$$L_t^i = \sum_{z \in Z} z \int_0^{\infty} \zeta_t^i(a, z, e) da \quad \forall t \geq 0 \tag{11}$$

(iv) The government budget clears:

$$\tau_t^i \omega_t^i L_t^i = \sum_{z \in Z} b_t^i(z) \int_0^{\infty} \zeta_t^i(a, z, u^e) da \quad \forall t \geq 0 \tag{12}$$

(v) The law of motion $\zeta_{t+1}^i = H_t^i(\zeta_t^i)$ holds for each $t \geq 0$: Thereby the function H_t^i : $\mathcal{M} \to \mathcal{M}$ can be explicitly written as follows:

$$\zeta_{t+1}^i(\mathcal{A} \times \mathcal{Z} \times \mathcal{X}) = \sum_{x \in X} \sum_{z \in Z} \int_0^{\infty} T_t^i((a, z, x); \mathcal{A} \times \mathcal{Z} \times \mathcal{X}) \zeta_t^i(a, z, x) da,$$

where $T_t^i((a, z, x); \mathcal{A} \times \mathcal{Z} \times \mathcal{X})$ describes the transition probability of moving from state (a, z, x) in period t to any state (a', z', x') such that $a' \in \mathcal{A} \subset \mathbb{R}_+$, $z' \in \mathcal{Z} \subset Z$, $x' \in \mathcal{X} \subset X$ in period $t + 1$.[6]

Definition 6. *General equilibrium in the union of countries: Given a collection of sequences of unemployment benefit policies $\{\{(\bar{b}_t^i, \mu_t^i)\}_{t=0}^{\infty}\}_{i=1}^{I}$ and given a collection of initial distributions $\{\zeta_0^i\}_{i=1}^{I}$, a general equilibrium in the union of countries is defined by sequences of value functions $\{\{V_t^i\}_{t=0}^{\infty}\}_{i=1}^{I}$, policy functions $\{\{c_t^i, a_{t+1}^i\}_{t=0}^{\infty}\}_{i=1}^{I}$, firm production plans*

[6]The description of the transition function T_t^i is quite involved and therefore deferred to the appendix.

Parameter	Definition
θ	Capital share of output
δ	Capital depreciation rate
β	Discount factor
ρ_z	Persistence of productivity
σ_z^2	Variance of prod. shock
α	Utility cost of labor
A^i	Total factor productivity
γ^i	Utility cost of search
σ^i	Job separation rate
λ_u^i	Job finding rate for unemployed
λ_n^i	Job finding rate for inactive
μ^i	Prob. of loosing UB eligibility
$\bar{b}^i$	UB replacement rate

Table 3.1: Model parameters.

$\{\{L_t^i, K_t^i\}_{t=0}^{\infty}\}_{i=1}^{I}$, *payroll taxes* $\{\{\tau_t^i\}_{t=0}^{\infty}\}_{i=1}^{I}$, *wages* $\{\{\omega_t^i\}_{t=0}^{\infty}\}_{i=1}^{I}$, *measures* $\{\{\zeta_t^i\}_{t=1}^{\infty}\}_{i=1}^{I}$, *with* $\zeta_t^i \in \mathcal{M}$, *and by a sequence of interest rates* $\{r_t\}_{t=0}^{\infty}$ *such that all conditions of definition 5 are satisfied for each country* $i \in \{1, 2, ..., I\}$ *and in addition the capital market clears at the union level, i.e.*

$$\sum_{i=1}^{I} n^i K_{t+1}^i = \sum_{i=1}^{I} n^i \sum_{x \in X} \sum_{z \in Z} \int_0^{\infty} a_{t+1}^i(a, z, x)\zeta_t^i(a, z, x)da \tag{13}$$

holds.

Definition 7. *Stationary general equilibrium: A stationary general equilibrium is a general equilibrium in which all government policies, decision rules, value functions, aggregate variables and prices are constant in all countries of the union.*

3.4 Calibration

We calibrate the model assuming that in $t = 0$ the union of I countries is in a stationary general equilibrium (see Definition 7 above). Hence, we assume that the Euro-Zone as a whole is a closed economy with no net capital in- or outflows. However, we want to note here that the structural calibrated parameters are not sensitive to this choice. In particular, if we do not require capital market clearing at the union level and consider any world interest rate within a reasonable range it does not affect the overall calibration much. Currently the countries we consider are the eleven countries that formed the original Euro-Zone in 1999 plus Estonia, Greece, Latvia, Slovenia and Slovakia.

The model presented in the previous section has three sets of parameters, which correspond to the three panels of Table 3.1. The upper panel describes technological and preference parameters that are common to all countries. In particular, we assume that in all countries the capital share of production θ, the depreciation of capital δ, the time discount factor β and the utility cost of work α is the same. Further, we assume that idiosyncratic productivity follows the same Markov process, for which we use a discretized version of an AR(1) process with persistence ρ_z and variance σ_z^2.

The middle and lower panels display parameters that are specific to each country. The middle panel includes parameters that capture - in a reduced form - different labour market institutions: total factor productivity A^i (which affects wage differences across countries), the cost of job search γ^i, the exogenous job separation rate σ^i, as well as the job arrival rates λ_u^i and λ_n^i. The lower panel contains parameters that define country specific unemployment benefit policies $(\mu^i, \bar{b}^i)$.

In total our model has $6 + I \times 7$ parameters.The three sets of parameters constitute a hierarchical structure in the degree to which policy can influence them. The unemployment benefit policy parameters $(\mu^i, \bar{b}^i)$ can be changed relatively easy by governments, while it takes more complex labour market reforms to change the institutional parameters $(A^i, \gamma^i, \sigma^i, \lambda_u^i, \lambda_n^i)$ and it is very hard, if not impossible, to change the parameters of the first panel. Given the scope of this paper, in the policy experiments below we only vary unemployment benefit policies (and how these are financed), though we want to explicitly mention here that the institutional parameters are not set in stone and can be changed through structural labour market reforms.

A central aspect of our analysis are the transitions between employment, unemployment and inactivity. Flow statistics are a useful measure since they quantify the aggregate transitions between labour market states in the data. In order to calibrate the model, we therefore use estimated quarterly transition probabilities, and the corresponding three average labour market stocks, generously provided by Etienne Lalé. Lalé and Tarasonis (2017) estimate these transition probabilities using quarterly data on prime-age workers (25-54) in the EU countries, from 2004 until 2013. [7] Data on unemployment benefits in EU Member States is taken from Esser et al. (2013), and data on population and average labour earnings from Eurostat.

3.4.1 Calibration strategy

We now describe in detail how the model is calibrated. First, we set the technological parameters θ, δ, ρ_z and σ_z to the quarterly counterparts of Krusell et al. (2015), who use monthly data for the US economy to estimate them. We discretize the AR(1) process for individual productivity process by 5 different productivity states using the Tauchen method. We set the discount factor β to 0.99, implying a subjective discount rate close to one percent per quarter.

The policy related parameters are chosen as follows. The parameter μ^i, which is the conditional probability of remaining eligible for UB next period, is also the inverse of the average duration of unemployment benefits in the model. We therefore set $1/\mu^i$ to the maximum duration of eligibility acoording to the law in country i. As described above, we model the eligibility process in this way because it allows for a simpler representation and a reduction in the dimensionality of the state space. For the unemployment benefit replacement rates, we set $\bar{b}^i$ to the data equivalents in Esser et al. (2013).

The remaining five country specific parameters A^i, γ^i, σ^i, λ_u^i and λ_n^i are calibrated in order to match the following five data moments: the differentials of average wages across countries,[8] the share of unemployed individuals in the population, the employment-to-employment, the unemployment-to-employment, and the non-active to employment flows. Finally, we set the common utility cost of work parameter α such that the population-weighted average of the fraction of employed agents in the Union matches the data.

[7] The underlying data is from the EU-SILC dataset, except Germany which comes from the GSOEP.

[8] We picked Germany, the largest country in the European Union, as our reference country. So TFP in Germany is equal to one and for the other countries it is calibrated in order to match wages relative to German wages.

Parameter	Definition	Value
θ	Cobb-Douglas capital weight	0.3
δ	Capital depreciation rate	0.01
ρ_z	Persistence of individual productivity	0.89
σ_z^2	Variance of individual productivity	0.08
α	Utility cost of work	0.89
β	Discount factor	0.99

Table 3.2: Common Parameters. Time period is 1 quarter; r clears the EU capital market.

Table 3.2 lists the common parameters, and table 3.3 contains the country specific parameters for the calibrated European countries. We also report the tax rates τ^i that clear the government budget in each country.

	A^i	γ^i	σ^i	λ_u^i	λ_n^i	$\bar{b}^i$	$1/\mu^i$	$\tau^i(\%)$
Austria	0.92	0.63	0.04	0.25	0.08	0.28	2.27	0.92
Belgium	1.01	0.60	0.02	0.10	0.06	0.37	19.70	2.13
Germany	1.00	0.01	0.01	0.10	0.10	0.23	3.94	0.45
Estonia	0.57	0.35	0.03	0.17	0.10	0.46	3.86	2.94
Spain	0.81	0.68	0.06	0.17	0.04	0.33	7.80	4.43
Finland	0.97	0.40	0.05	0.21	0.18	0.36	7.58	3.75
France	0.93	0.30	0.02	0.16	0.06	0.35	7.88	1.90
Greece	0.82	0.90	0.04	0.17	0.03	0.65	3.94	5.60
Ireland	1.05	0.55	0.03	0.13	0.05	0.36	3.94	1.97
Italy	0.92	0.48	0.03	0.13	0.04	0.09	2.58	0.25
Luxembourg	1.15	0.95	0.02	0.17	0.04	0.27	3.94	0.53
Latvia	0.45	0.30	0.04	0.16	0.07	0.57	2.95	1.60
Netherlands	0.87	0.03	0.01	0.14	0.13	0.35	3.50	1.00
Portugal	0.69	0.49	0.06	0.15	0.09	0.36	5.91	4.98
Slovenia	0.77	0.14	0.01	0.14	0.05	0.65	1.97	0.98
Slovakia	0.54	0.25	0.03	0.13	0.07	0.08	1.97	0.16

Table 3.3: Country specific parameters.

3.4.2 Quality of the Fit

In this section we investigate how well the model fits the European labour markets. In the calibration described above, several labour market moments were targeted. These are shown in Figures 3.3 to 3.6. In Figure 3.3 we observe that the average unemployment rate in Spain, Greece, Latvia and Portugal is much higher than the European average, while in Austria, Germany, Luxembourg and the Netherlands it is lower. The persistence of employment (Figure 3.4) is high in almost all countries. The exceptions are Spain, Finland and Portugal who have substantial flows out of employment in each quarter. The flows from unemployment to employment (Figure 3.5) are quite heterogeneous across European countries. Interestingly, it is lowest in Germany, a country with rather low structural unemployment. By contrast, Austria, which has similar average unemployment rates as Germany, has the highest flow from unemployment to employment. We observe substantial heterogeneity also in the flows from inactivity to employment (Figure 3.6). For example, in Finland this flow is much higher than in the other countries.

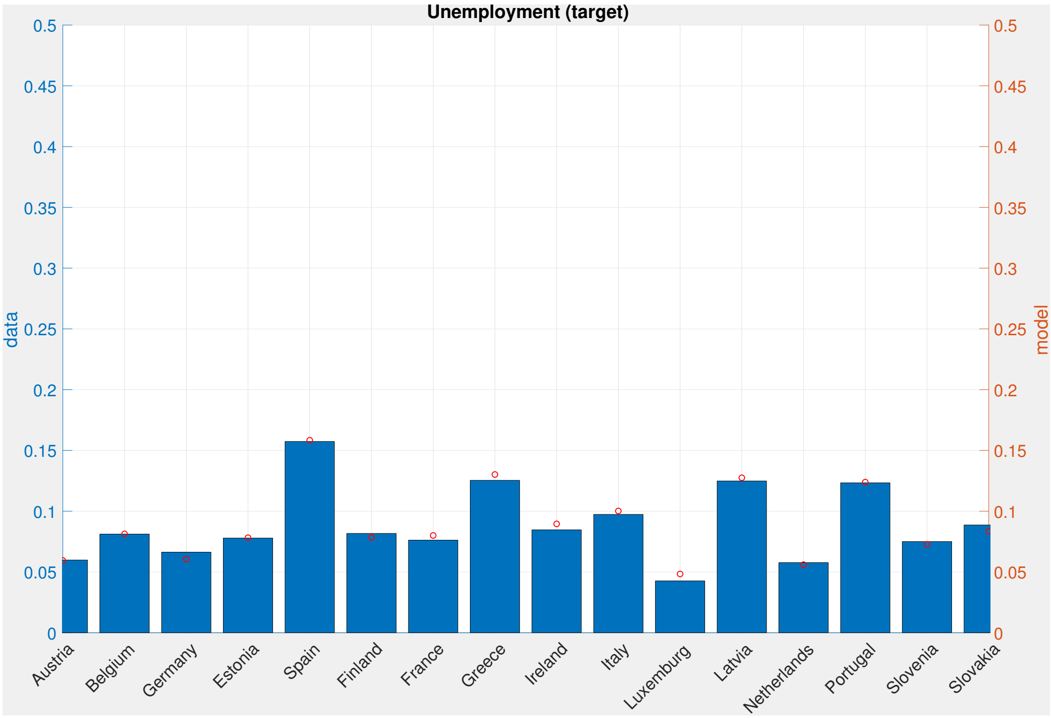


Figure 3.3: Unemployment.

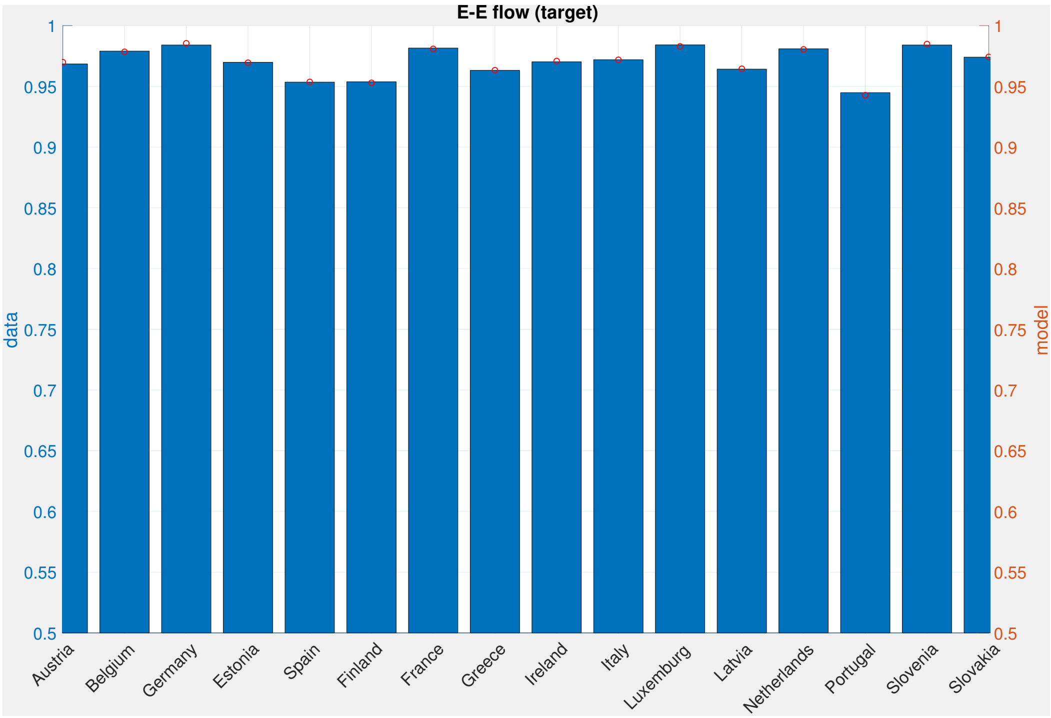


Figure 3.4: Employment-Employment Flows.

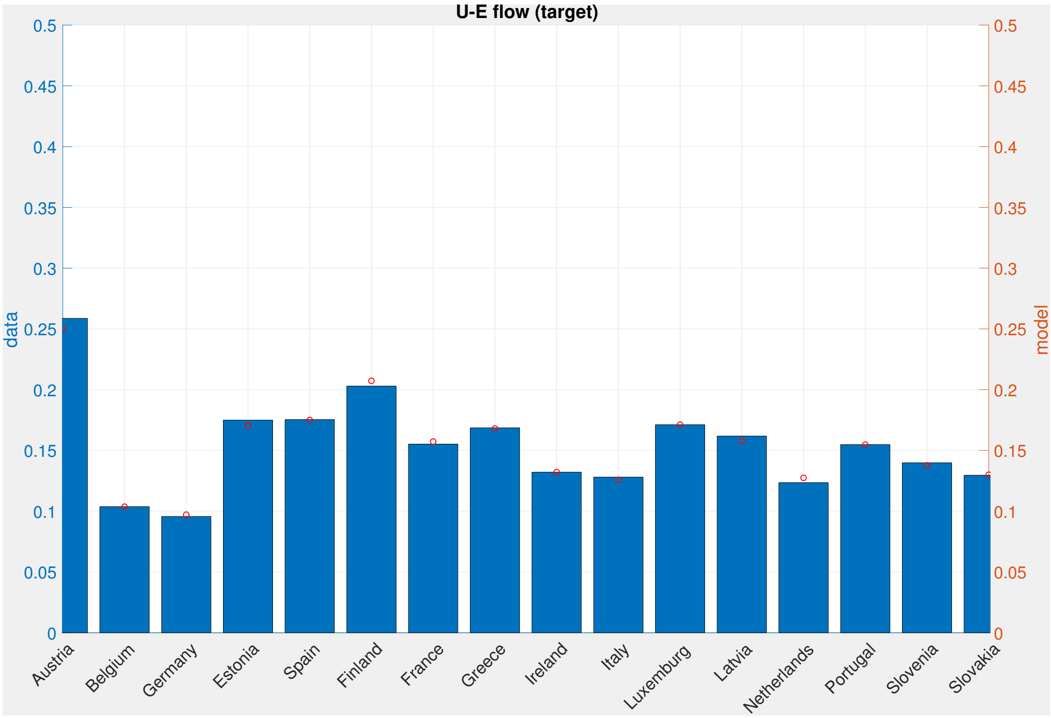


Figure 3.5: Unemployment-Employment Flows.

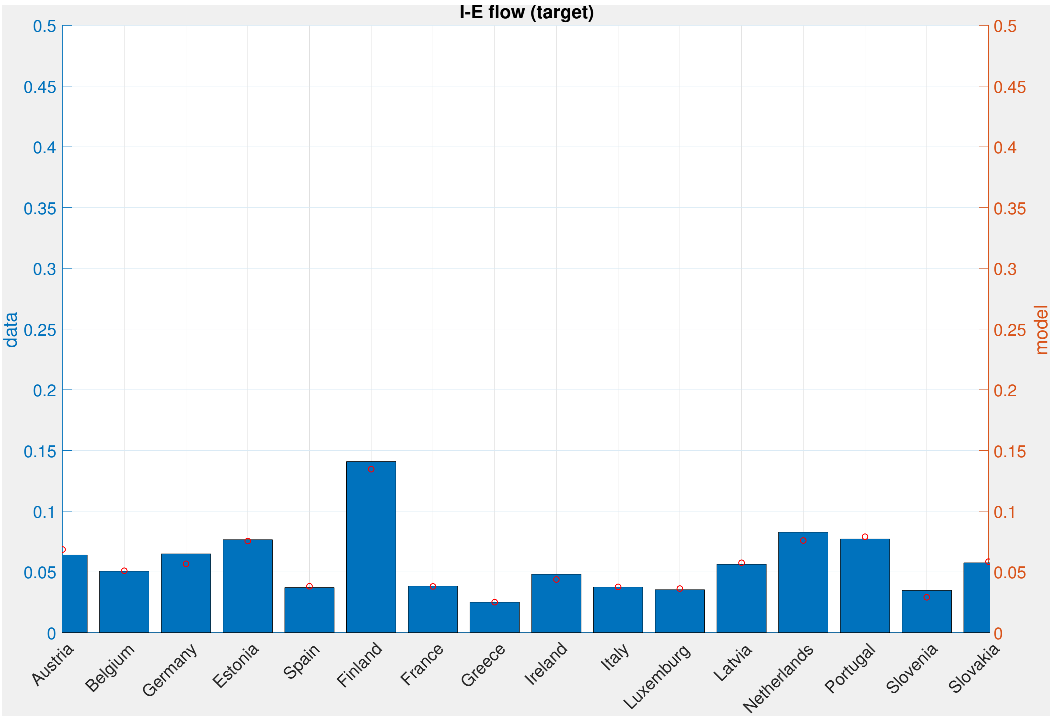


Figure 3.6: Inactivity-Employment Flows.

The employment ratios were not targeted country by country, but the union average was. At the country level, the comparison with the data is shown in Figure 3.7. The model does very well in replicating the heterogeneity in stocks of employment that we observe in the data.

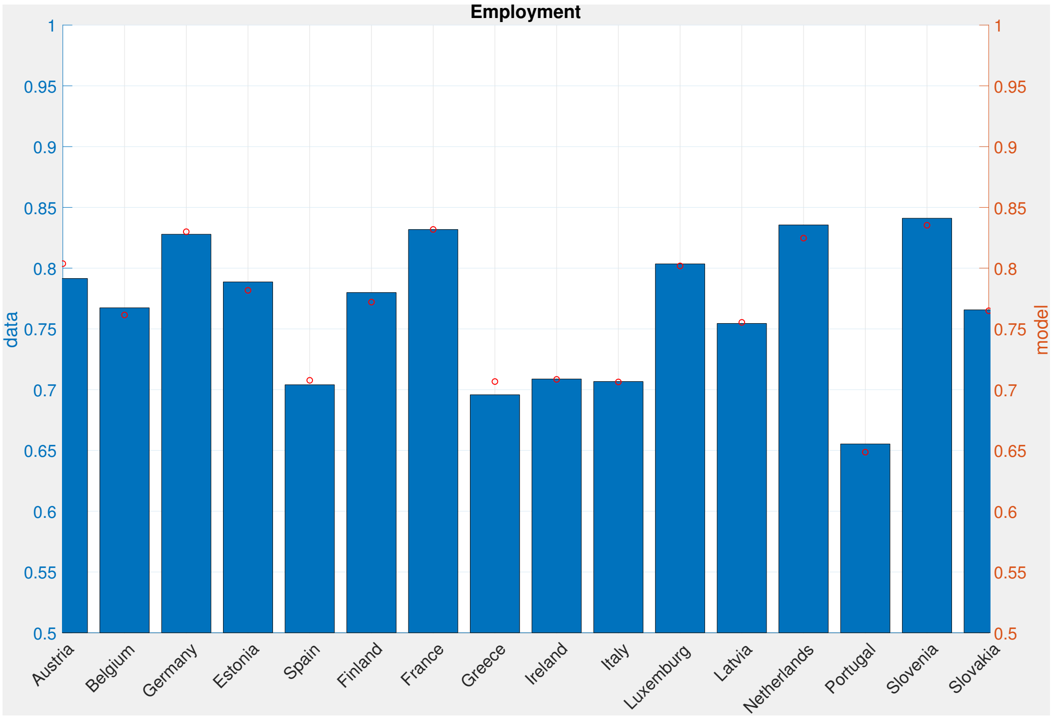


Figure 3.7: Employment.

Given that the model shares of employed and unemployed agents is in line with the data counterparts, the model unemployment rate is also as in the data (Figure 3.8). Another important moment in the model is the average persistence of an unemployment spell in each country, which is not targeted directly by the calibration. The model predictions is shown in Figure 3.9. By and large the model performs well also along this dimension, though there are some cases where it underpredicts (Estonia, Greece, Portugal), and one where it overpredicts (Slovenia) the average duration of an unemployment spell by more than one quarter.

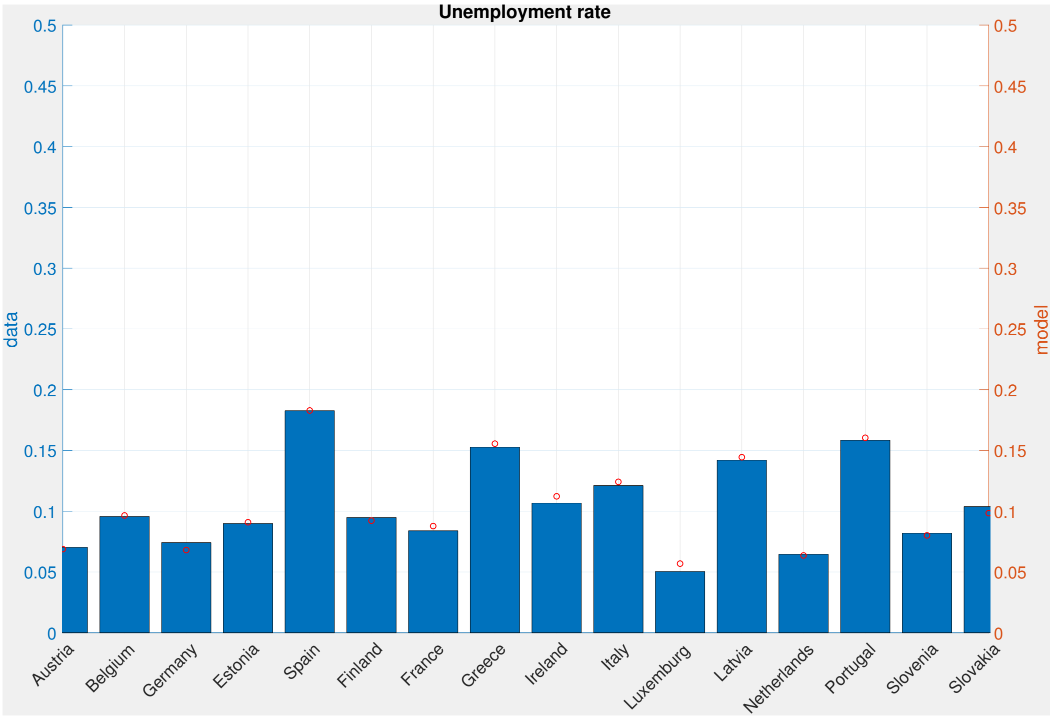


Figure 3.8: Unemployment Rate.

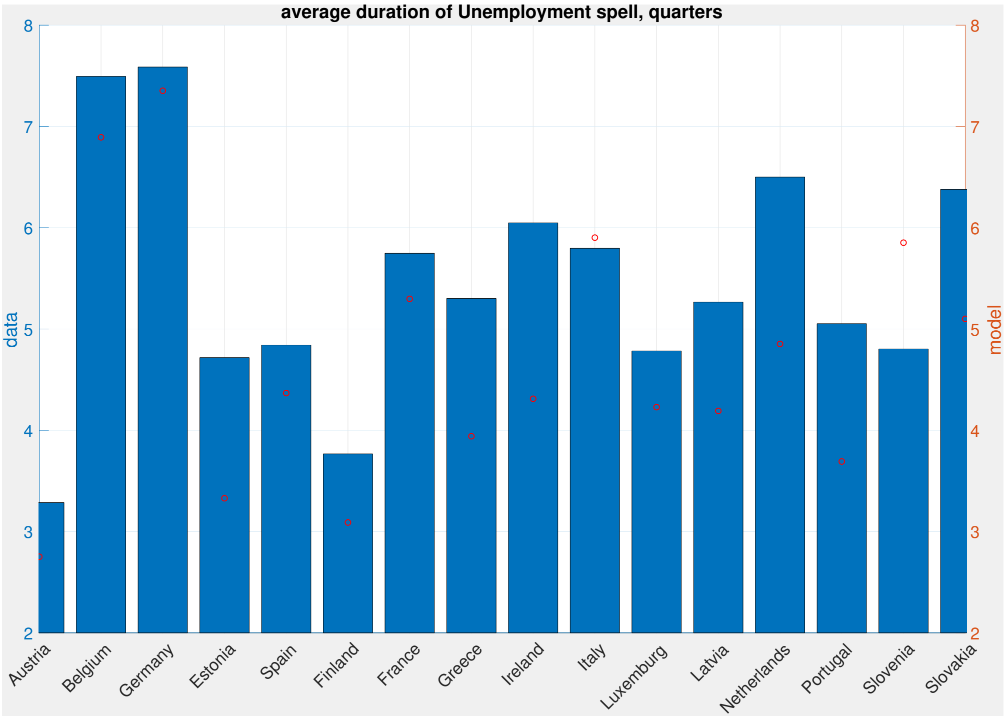


Figure 3.9: Average Unemployment Duration.

For completeness, the persistence of inactivity is shown in Figure 3.10 below. Again, the model does a good job in replicating the data with only minor deviations.

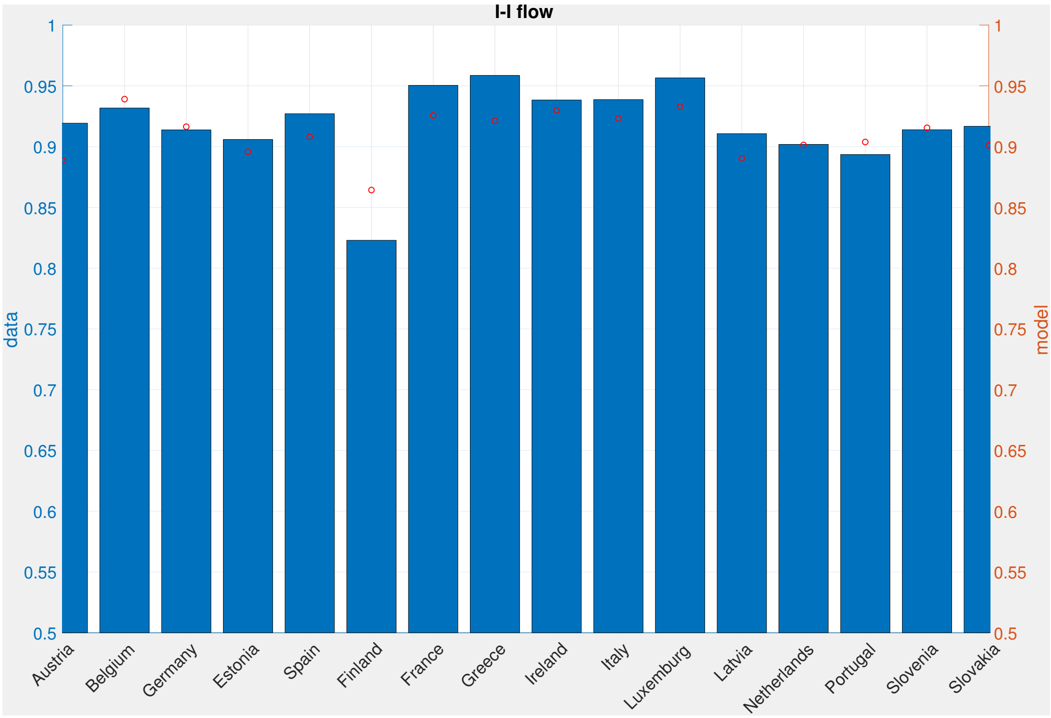


Figure 3.10: Inactivity to Inactivity Flows.

3.4.3 Diversity of Labour Market Institutions

Our calibration makes apparent that labour market institutions vary substantially across countries. We visualize this in the Figures 3.11 to 3.13. Figure 3.11 shows the job arrival rate for non-searchers (λ_n^i, horizontal axis) and searchers (λ_u^i, vertical axis) for each of the calibrated economies. We observe that these two rates are correlated but their values differ substantially across countries. For example, in Finland, the Netherlands and Estonia the job finding rate for not actively searching individuals is higher than 10%, while in Greece, Italy, Spain and Luxembourg it is lower than 5%.

Figure 3.12 plots average the job finding rate for non-employed on the x-axis, but this time against the job separation rate σ^i on the y-axis. It hence gives an idea of the rigidity of the respective labour markets. Countries in the southeast have the high job finding and low separation rates. Countries in the northwest corner have high job destruction risk and low chances of finding a job while not working. Note for instance that while Germany and Spain have similar job finding rates for the non-employed, job destruciton in Spain in roughly 5 times higher, contributing to higher unemployment in Spain.

Finally, Figure 3.13 shows that the countries also differ substantially with respect to their unemployment benefit system. It plots the replacement rate vs. the average duration for which unemployed are eligible to receive benefits.

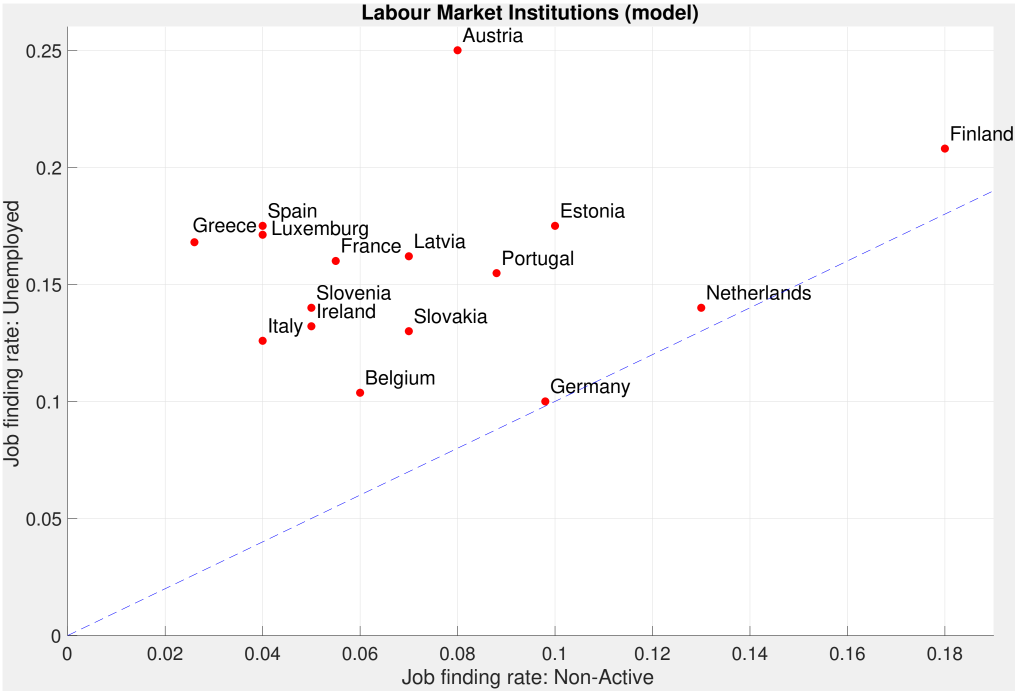


Figure 3.11: Job Arrival Rates

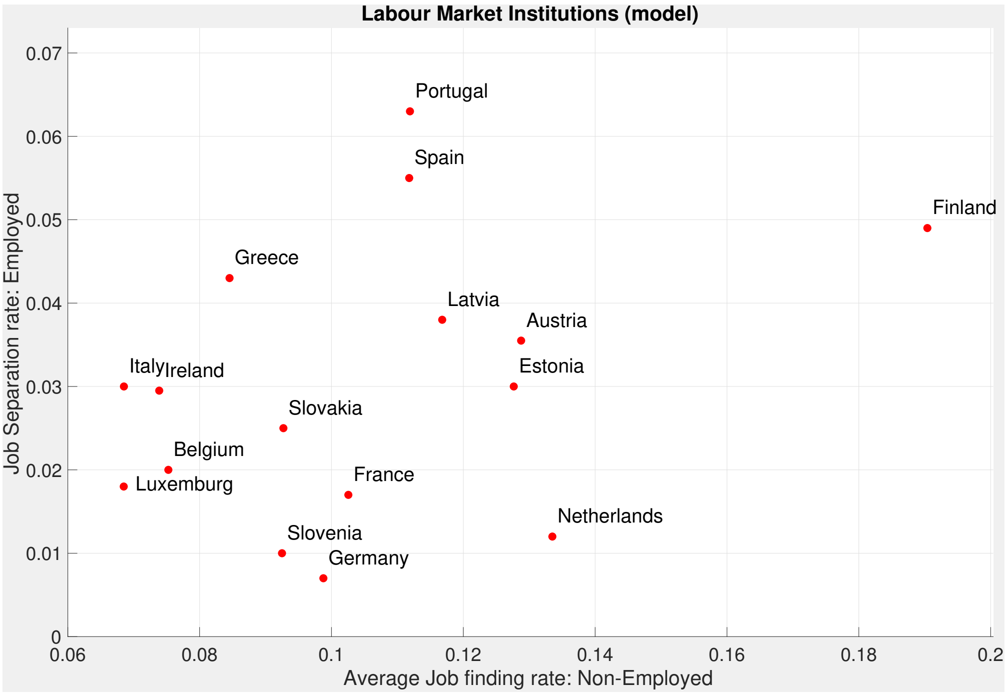


Figure 3.12: Labour Market Rigidity

3.5 Policy Experiments

Based on our calibration, which initializes the economy in $t = 0$, we are now able to perform several experiments and analyze the evolution of countries' labour markets and other macroeconomic variables under different configurations of unemployment policy

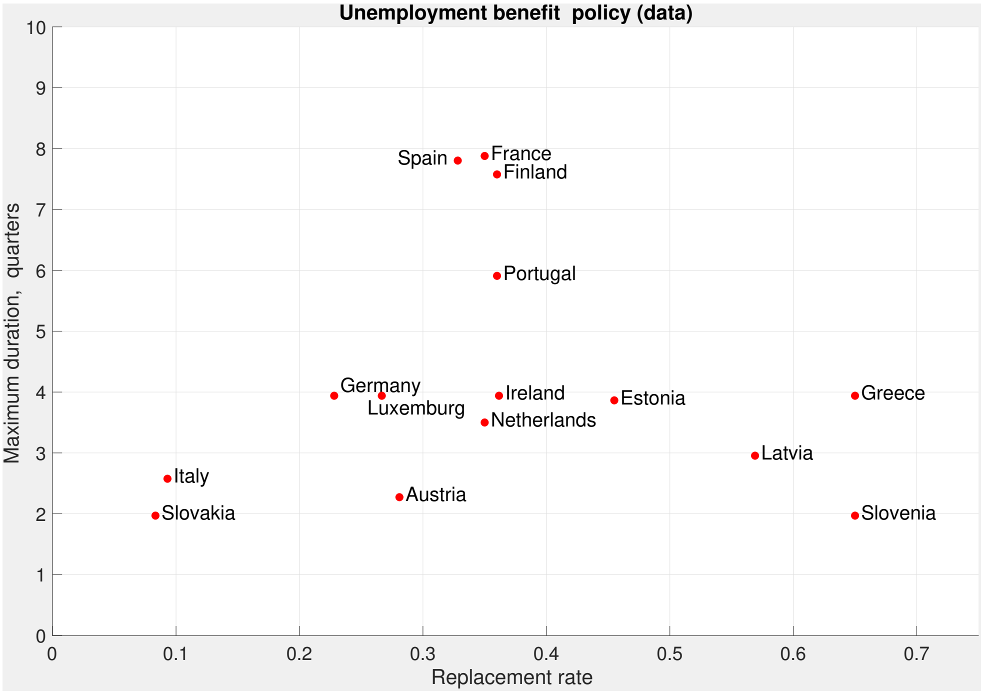


Figure 3.13: National Unemplyoment Benefit Systems.

for $t \geq 1$. In this part of the analysis we restrict the set of countries to a number of ten: Austria, Belgium, Germany, Spain, Finland, France, Ireland, Italy Luxembourg and Netherlands. These countries account for more than 90% of the Euro-Zone's total GDP.

As we pointed out earlier, a key motivation for an EUIS would be that it may provide insurance against country specific fluctuations in unemployment and, in particular, against fluctuations in the tax burden associated with its financing. In Subsection 3.5.1, we directly study the pure insurance effects of such policy from the viewpoint of individual European countries. In this experiment, the EUIM only insures countries against country-specific fluctuations in unemployment, while keeping the benefit systems at the status quo. We will see that the welfare benefits of such an insurance system are very small.

As we have seen above, the ten countries are quite heterogeneous in their labour market institutions, suggesting that the individually optimal benefit systems could vary substantially across countries. If this was the case and in light of the result that the benefits of insuring against country specific shocks are small, one may doubt the desirability of a common European unemployment insurance system. In Subsection 3.5.2, we ask what the countries' individually optimal benefit systems are and whether an agreement to a common harmonized European benefit system could actually be achieved. For this means we compute the optimal unilateral reform separately for each country (in partial equilibrium). We find that the optimal unemployment benefit systems are surprisingly similar. In all countries it is optimal to provide an unlimited duration of eligibility and the optimal replacement rates vary between 20% and 45%.

Abstracting from other political constraints, one may argue that these slight differences are sufficient to implement reforms at the country level rather than at the union level. We show that such an argumentation is only true if one assumes that the Euro-Zone as a whole is a small open economy. If, on the other hand, prices are affected by unemployment policy, it neglects spill over effects across countries. In particular, we show that if all

European countries would reform their system simultaneously and the capital market is required to clear at the union level, i.e. in general equilibrium, the very same UI benefit systems that are optimal in partial equilibrium turn out to be actually welfare worsening in most of the countries. The reason for this result is that increasing the generosity of the UI benefit system in almost all countries, reduces private savings and hence the aggregate, European, capital stock. As a consequence the marginal product of labour declines everywhere. This is bad for poor agents, who, irrespective whether they are currently employed or not, derive most of their lifetime income from wages.

In reality, the responsiveness of prices to policy changes is likely in between the two extreme cases we consider - the Euro-Zone as small open vs. a closed economy. The point we want to make is that if prices are at least to some degree affected by unemployment policy, lack of coordination across European member states may result in detrimental reforms, thus providing a rationale for centralizing unemployment policy at the European level. In the last subsection 3.5.3, we therefore search for a common harmonized European benefit system that is welfare improving in all countries of the union in general equilibrium. We consider two versions of this experiment that differ in the way the governments' budgets clear. First, we consider a fully harmonized system, where the tax rate in each country is the same and the ten individual government budget constraints (12) are replaced by a single European one. Such a system, by construction, results in transfers from countries with structurally low unemployment rates to countries with high unemployment and thus does not achieve unanimous agreement among member states. Interestingly, for some of the net payers, the welfare gains of such a reform are positive, suggesting that in these countries the current unemployment benefit system is far from optimal. In the second version of this experiment the tax rates which finance the harmonized system vary across countries such that each government's budget constraint (12) is satisfied. Such a system eliminates cross-country transfers and we show that with a replacement rate of 15% all countries are better off than in the status quo.

3.5.1 Insuring Country Level Fluctuations Only

As we mentioned above, the main argument for an EUIM can be that it may provide insurance against country level fluctuations in unemployment. This insurance might be very valuable as European countries (especially recently after the crisis) have a hard time to finance the increasing fiscal burden of unemployment using debt because of tighter deficit requirements. In the following experiment we provide the "best chance" for these insurance benefits to realise as we assume that individual countries have no access to any debt or savings to smooth out unemployment fluctuations.

The experiment is constructed as follows. At time $t = 0$ the country is in its steady state. At the end of this period, when all decisions are already made, it becomes aware that at $t = 1$ it is hit by a completely unanticipated negative shock. After the shock hits the country returns back to its steady state in a deterministic and gradual way.

Similarly to Krusell et al. (2015), we model shocks as hitting simultaneously TFP (A) and exogenous labor market flows (σ, λ_u and λ_n).[9] In particular, a deep recession will be modelled as a drop in TFP and job arrival rates and a rise in the separation rate. We model economic fluctuations in this way, because it is well-known that fluctuations of

[9]Note that in order to economize on notation we suppressed the time subscript in these parameters in the description of our model. In most of our analysis these parameters are indeed treated as constant. Only in the present subsection we deviate from this assumption.

TFP alone are not able to generate large enough fluctuations of unemployment if output fluctuations are reasonable. This issue is amplified in our framework by the fact that job creation and job destruction are not modelled endogenously.

Given all these assumptions, note that after the shock is realised the economy is following a deterministic pattern and eventually converges back to its steady state. Hence, after the realisation of the shock agents have perfect foresight when solving their dynamic optimisation problems. We consider two cases: financial autarky and insurance through the EUIS. In financial autarky, along the transition the tax rate needs to adjust to balance the government budget constraint every period. In the case of the EUIS, we assume that countries can get full insurance against the rise in unemployment expenditure and thus can leave the tax rate at its steady state value. We assume that the shock is a zero probability event and therefore comes at a complete surprise to agents. This assumption also implies that leaving the tax rate with the EUIS at the steady state value is actuarially (from an ex-ante perspective) fair.

We want to note here that the zero probability assumption serves one purpose: To calculate an *upper bound* for the actual welfare gains that a EUIS would achieve when its sole purpose was to insurance against country level fluctuations in unemployment expenditures. If we relax this assumption and assume that the shock happens with some positive probability, an actuarially fair EUIS would imply a higher tax rate than the steady state tax rate, i.e. countries would have to pay an insurance premium. This reduces consumption and thus welfare. It also would imply that agents would prepare themselves for the possibility of such a shock through higher savings, in which case the smoothing of taxes is less valuable than in case of the fully unanticipated shock.

We calculate the welfare effect of the introduction of this EUIS by comparing the welfare of autarky and the EUIS of each individual at the end of period $t = 0$, i.e. after learning that shocks will occur next period. That we calculate the welfare gains conditional on the negative shock happening again provides an upper bound for the actual insurance gains.

The argument that we want to make with this exercise is that even in a highly stylized scenario which in several dimensions is constructed in a way to increase welfare gains of an EUIS relative to what we would expect in reality, the gains of insuring country level fluctuations are small.

The Shocks. The combination of shocks has the following structure. Consider first total factor productivity in country i. At $t = 0$ the country is in steady state, i.e. $A_0^i = A^i$. At $t = 1$ a negative shock of size ϵ_A hits,

$$\log(A_1^i) = (1 - \epsilon_A)\log(A^i).$$

The shock has persistence ρ_A and moves back to the steady state in a gradual and deterministic way,

$$\log(A_t^i) = \rho_A \log(A_{t-1}^i) + (1 - \rho_A)\log(A^i) \quad \text{for } t \geq 1.$$

Similarly, the job separation rate and the job arrival rates are hit in $t = 1$,

$$\begin{aligned} \sigma_1^i &= (1 + \epsilon_\sigma)\sigma^i \\ \lambda_{u,1}^i &= (1 - \epsilon_{\lambda_u})\lambda_u^i \\ \lambda_{n,1}^i &= (1 - \epsilon_{\lambda_n})\lambda_n^i. \end{aligned}$$

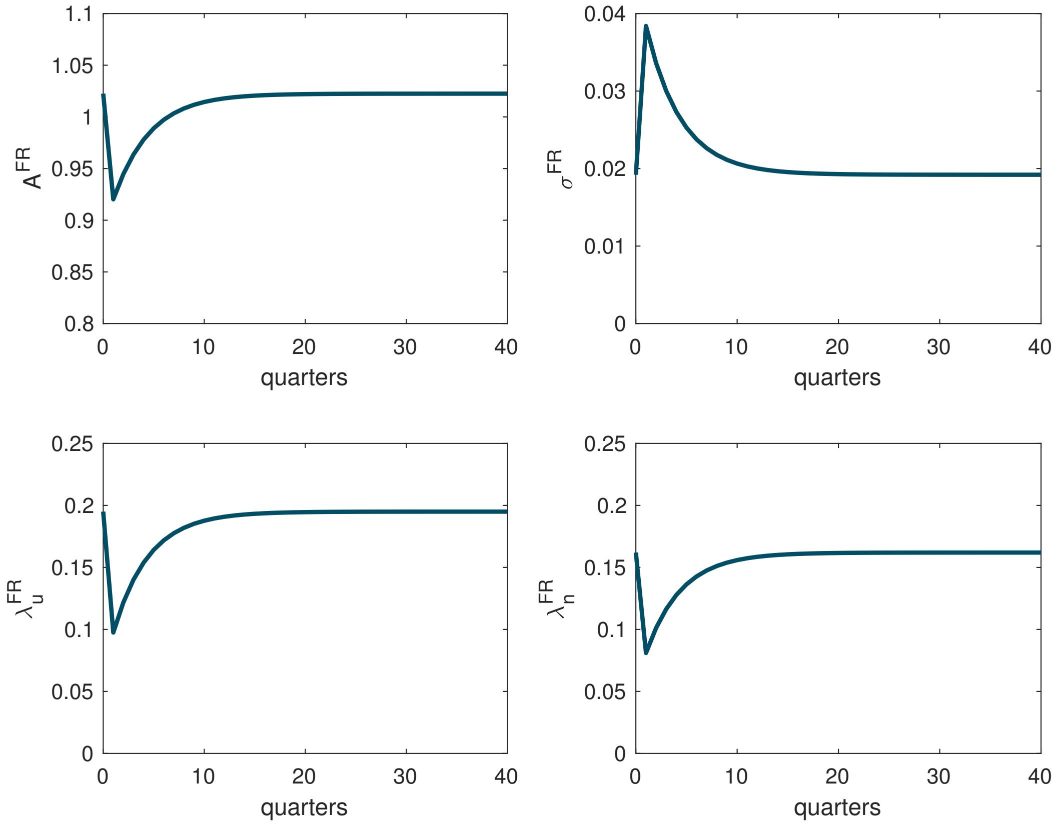


Figure 3.14: Shock process in France

After that they gradually return back to their steady state values, i.e. for $t \geq 1$

$$\sigma_t^i = \rho_\sigma \sigma_{t-1}^i + (1-\rho_\sigma)\sigma^i$$
$$\lambda_{u,t}^i = \rho_{\lambda_u} \lambda_{u,t-1}^i + (1-\rho_{\lambda_u})\lambda_u^i$$
$$\lambda_{n,t}^i = \rho_{\lambda_n} \lambda_{n,t-1}^i) + (1-\rho_{\lambda_n})\lambda_n^i$$

holds.

We consider a deep recession with TFP dropping by 10% ($\epsilon_A = 0.1$), the job separation rate doubling ($\epsilon_\sigma = 1$), and the job finding rates being reduced by half ($\epsilon_{\lambda_u} = \epsilon_{\lambda_n} = 0.5$). We further assume that $\rho_A = \rho_\sigma = \rho_{\lambda_u} = \rho_{\lambda_n} = 0.75$. Figure 3.14 depicts the evolution of the shock for the case of France.

The shock induces changes in labour markets, which are depicted in Figure 3.15. To some extent these responses are driven directly by the exogenous shock. For example a higher separation rate reduces employment by construction. But to a substantial degree they result from endogenous decisions of agents. For example, we observe that unemployment decreases at impact and only later rises above its steady state value (middle panel) and that at the same time inactivity increases at impact and gradually decreases later (lower panel). The reason is that because of lower wages and a lower likelihood to find a job even when searching, many agents are not willing to incur the utility loss of searching and instead decide not to participate. Only later, when economic conditions improved, they start searching for a job again. Further, many not separated agents decide to quit

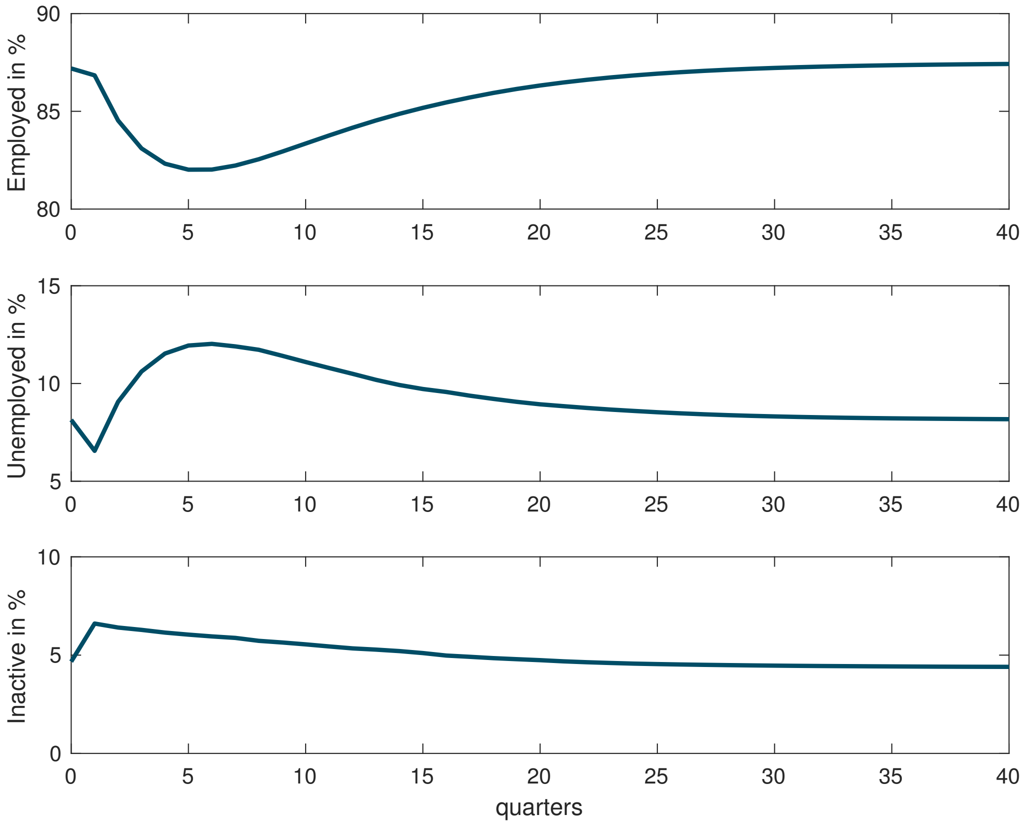


Figure 3.15: Labor Market in France

working because of the reduction in wages. In the upper panel we see that already during the first period (before the shock hits but after agents learned about the coming recession) employment declines.

If the country is in financial autarky, this mechanism is amplified through a rise in taxes, distorting incentives to (try to) work further. Figure 3.16 shows how taxes in France would evolve under autarky (solid line) as opposed to the case in which the country is fully insured against fluctuations in benefit expenditures (dashed line). In France such a shock would result in a gradual increase in the payroll tax that is from 1.9% to about 3.3% at the peak of the recession.

We performed this very same exercise for all our ten countries. The tax rates in autarky for all countries are shown in Figure 3.17. We saw before that the steady state taxes which finance the country specific unemployment benefit systems differ substantially across country. This is a consequence of both different unemployment benefit policies and different labour market institutions that determine job creation and destruction. As we can see in Figure 3.17 these difference not only affect the steady state level of taxes but also their responses to shocks.

Table 3.4 shows the welfare gains of insuring against country level fluctuations in taxes. Remember that they are computed conditional on the shock happening. One obvious feature of these results is the very small magnitude of the average welfare gains. This is due to the fact that most welfare gains come from the small improvement of consumption smoothing for the employed. In fact, the only reason why also unemployed

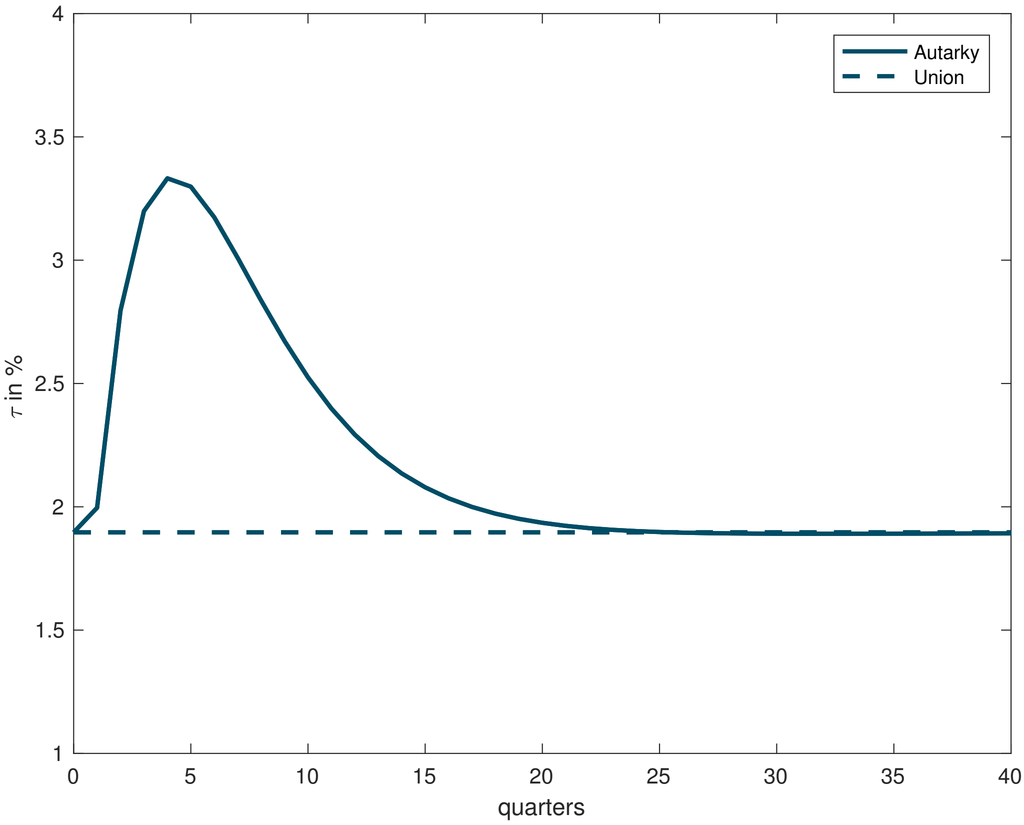


Figure 3.16: Taxes in France

and non-active have positive welfare gains is because they may be employed, and thus paying taxes, in the future.

This exercise shows that risk-sharing by itself does not provide a strong rationale for the introduction of an EUIS. In light of this result one may doubt the desirability of a common European unemployment benefit scheme. Especially, since the observed heterogeneity in labour market institutions (see section 3.4) suggests that the optimal benefit system could differ substantially across countries, making it difficult to reach a consensus across Europe. In the next section we want to evaluate this claim and analyze how different optimal benefit systems are across countries.

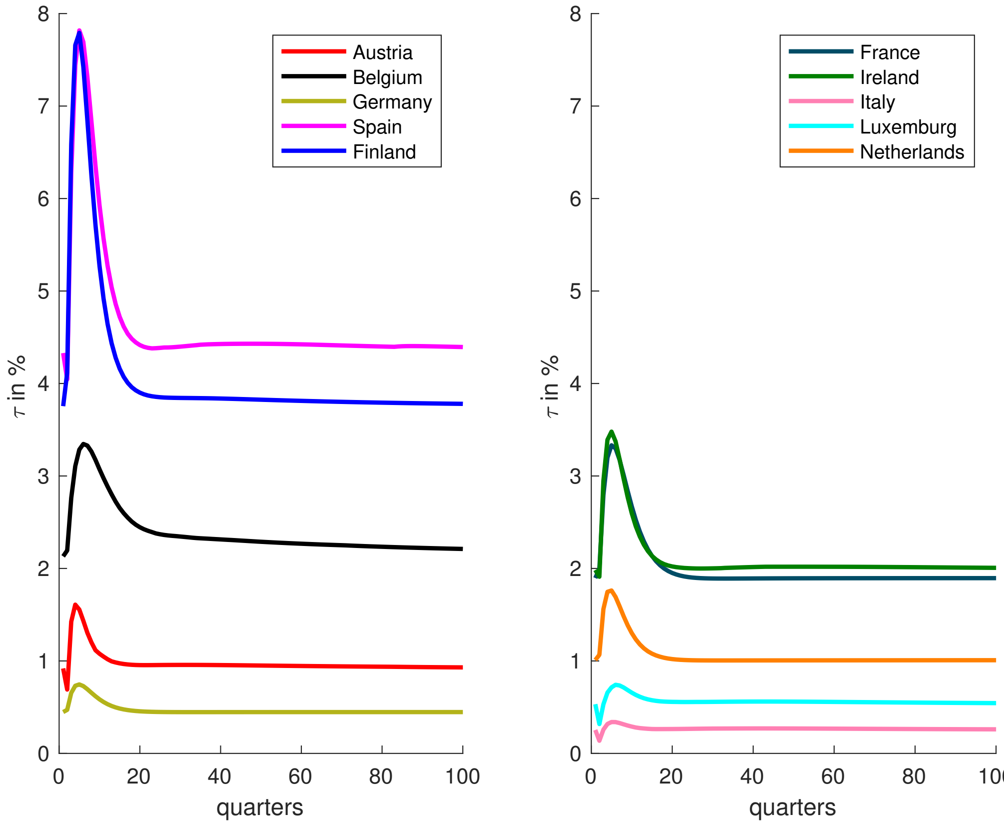


Figure 3.17: Taxes in Autarky

	Employed	Un. Eligible.	Un. Non-Elig.	Non-Active	Total
Austria	0.05	0.04	0.04	0.03	0.04
Belgium	0.18	0.14	0.17	0.10	0.15
Germany	0.02	0.01	0.01	0.00	0.02
Spain	0.26	0.18	0.20	0.09	0.21
Finland	0.27	0.20	0.27	0.16	0.24
France	0.10	0.06	0.06	0.02	0.09
Ireland	0.11	0.06	0.08	0.04	0.09
Italy	0.01	0.01	0.01	0.01	0.01
Luxembourg	0.02	0.02	0.02	0.01	0.02
Netherlands	0.04	0.02	0.02	0.01	0.04

Table 3.4: Welfare gains (in % CEV) of insuring country level fluctuations

3.5.2 National Reforms of the Unemployment Benefit System

Before trying to harmonize European unemployment insurance systems, it is worthwhile to compute the optimal unemployment insurance system individually for each country. This gives us an idea whether the substantial heterogeneity in labour market institutions across Europe actually allow for such a harmonization or whether they make it impossible to reach a consensus across Europe. In the following subsection 3.5.2 we therefore compute the optimal national unemployment benefit reforms separately for each country, in partial equilibrium. It turns out that optimal national benefit systems are similar but not identical. To be specific, all countries find an unlimited duration of benefit receipt optimal while optimal replacement rates vary between 20% and 45%.

One might think that even if optimal policies are similar, countries are still better off by reforming the system to their individual optimum rather than to join a common European scheme. In subsection 3.5.2 we show that in such a scenario it is important to take into account that the European Union has a common capital market. In particular, we show that sticking to the partial equilibrium analysis would result in large imbalances between capital and savings across Europe, unlikely to be sustained in reality. The reason is that the optimal partial equilibrium reforms are rather generous, resulting in a large decline in precautionary savings. In section 3.5.2 we show that in general equilibrium this would result in a substantial decline in the capital stock. The induced changes in equilibrium prices, in particular a substantial decrease in Europe-wide wages would render seven out of the ten reforms that seem optimal in partial equilibrium, actually welfare worsening. This suggests that optimal unemployment insurance should be designed at the broader European level.

Optimal Unilateral Reform

In this section we compute the optimal reform for each country separately. For each country i we ask the question: What is the optimal unilateral once-and-for-all change in $(\bar{b}^i, \mu^i)$ if only country i was to change its benefit system and the other countries would stick to the status quo? This analysis is done in partial equilibrium, i.e. we assume that a single country does not affect the equilibrium interest rate when changing its unemployment benefit policy even thought the savings decisions of its citizens change. This implies that the marginal product of capital and hence the capital-labour ratio is pinned down by the interest rate and as a consequence also wages are unaffected by the change in policy.

We assume that the government maximizes the utilitarian welfare of its citizens. Formally, the government in country i chooses a pair of policy parameters $(\bar{b}_1^i, \mu_1^i)$ with $\bar{b}_t^i = \bar{b}_1^i$ and $\mu_t^i = \mu_1^i$ for all $t \geq 1$ such that social welfare is maximized,[10]

$$\max_{(\bar{b}_1^i, \mu_1^i)} SW(\bar{b}_1^i, \mu_1^i) = \max_{(\bar{b}_1^i, \mu_1^i)} \sum_{x \in X} \sum_{z \in Z} \int_0^\infty V_0^i(a, z, x; \bar{b}_1^i, \mu_1^i) \zeta_0^i(a, z, x) da.$$

Thereby, individually optimal policies, firm production plans and taxes adjust such that all equilibrium conditions in Definition 5 are satisfied. Note that for each individual we compute the value in the initial period and therefore take into account the whole transitional dynamics to the new steady state.

In order to be able to interpret the welfare gains associated with the policy reform, we translate them into consumption equivalent variation. In particular, $\Delta^i(a, z, x)$ defines the per period percentage increase in consumption that you would need to give an individual with initial state (a, z, x) when the benefit system is kept at the status quo such that he is indifferent between this status quo and the optimal reform. The aggregate welfare gain

[10] Here we add the policy parameters as arguments in the value function to make it explicit that the values depend on policy parameters.

is then defined as

$$\Delta^i = \sum_{x \in X} \sum_{z \in Z} \int_0^\infty \Delta^i(a,z,x)\zeta^i(a,z,x)da.$$

Similarly, we define the aggregate welfare gain of the employed, unemployed eligible, unemployed non-eligible and inactive as

$$\Delta^i_x = \frac{\sum_{z \in Z} \int_0^\infty \Delta^i(a,z,x)\zeta^i(a,z,x)da}{\sum_{z \in Z} \int_0^\infty \zeta^i(a,z,x)da} \quad \text{for } x \in \{e, u^e, u^n, n\}.$$

Table 3.5 shows the current benefit policy and the optimal reform in each country along with the taxes that finance this policy. For the optimal reform we report the new steady state taxes τ^i_∞. Note, however, that along the transition taxes vary in order to clear the government budget period by period.

We see that despite the substantial heterogeneity in labour market institutions, optimal unemployment benefit policies are surprisingly similar. In particular, in all countries an unlimited duration of eligibility is optimal. This policy eliminates the risk of not finding a job before losing eligibility. There is some variation in optimal replacement rates but all are in the range of 20 to 45 percent.[11] The main difference are the tax rates that finance the rather similar benefit policies.

Country	Status Quo			Optimal Reform			Δ
	$1/\mu^i_0$	b^i_0	$\tau^i_0(\%)$	$1/\mu^i_1$	b^i_1	$\tau^i_\infty(\%)$	
Austria	2	0.28	0.92	∞	0.45	4.61	1.51
Belgium	20	0.37	2.13	∞	0.25	1.36	0.16
Germany	4	0.23	0.45	∞	0.25	3.15	1.40
Spain	8	0.33	4.43	∞	0.45	11.26	2.27
Finland	8	0.36	3.75	∞	0.20	0.90	0.33
France	8	0.35	1.90	∞	0.35	3.75	1.18
Ireland	4	0.36	1.97	∞	0.45	8.37	1.72
Italy	3	0.09	0.25	∞	0.45	9.22	3.23
Luxembourg	4	0.27	0.53	∞	0.45	3.52	1.07
Netherlands	4	0.35	1.00	∞	0.25	4.53	0.89

Table 3.5: Optimal National Reforms of the Benefit System

For example, there are five countries for which a replacement rate of 45% is optimal. In Ireland, Italy and Spain the more generous benefit scheme comes along with large increases in the tax rate, which eventually reach 8.4%, 9.2%, and 11.3%, respectively. In Austria and in Luxembourg, on the other hand, the same benefit system is optimal but the tax rates only reach values of 4.6% and 3.5%, respectively. The reason for these are the structurally different labour market institutions. In particular, in section 3.4 we saw that in Spain the job separation rate is much higher, while in Ireland and Italy the job finding rates are much lower then in the other countries.

The last column shows the aggregate welfare gains. In Belgium this gain is less than 0.2% of consumption equivalent variation as the current benefit system is not too far from

[11] Our optimization routine optimized over increments of 0.05 in the replacement rate dimension.

the optimal one. In particular the duration of eligibility is much higher than in all the other countries. Still there are some gains from extending the duration to infinity and slightly reducing the replacement rate. On the other extreme the welfare gains for Italy are large, about 3.2% CEV. According to our model the current benefit system in Italy with an average duration of eligibility of 3 quarters and a replacement rate of less than 10% is way too restrictive.

Of course, not every individual in a country benefits from the reform in the same way. Table 3.6 shows the welfare gains at a more dis-aggregated level. We see that in countries where the unemployment system becomes more generous (Austria, Spain, France, Ireland, Italy, Luxembourg) the main beneficiaries are the eligible unemployed as they are the ones who are most directly affected. However, even if the gains are smaller, also the other agents benefit from the reform as they might be eligible unemployed in the future.

	Employed	Un. Eligible.	Un. Non-Elig.	Non-Active	Total
Austria	1.40	3.06	2.62	1.57	1.51
Belgium	0.13	0.28	0.70	0.18	0.16
Germany	1.35	5.20	1.42	1.11	1.40
Spain	2.07	4.16	3.87	1.74	2.27
Finland	0.34	0.14	0.51	0.37	0.33
France	1.11	2.63	1.44	0.84	1.18
Ireland	1.54	4.52	4.37	1.40	1.72
Italy	3.06	9.42	6.15	2.50	3.23
Luxembourg	0.94	4.05	2.56	1.12	1.07
Netherlands	0.85	2.98	0.79	0.71	0.89

Table 3.6: Welfare gains (in % CEV) of individually optimal reforms

Finland has the most restrictive reform. Even if the duration of eligibility is extended, the replacement rate is substantially reduced to 20%. As a consequence the eligible unemployed benefit the least from the reform. The other agents benefit more through the reduction in taxes. This is directly true for the employed. But also the non-eligible unemployed and the inactive like the reform more than the eligible unemployed. The reason is that these agents will sooner earn wage income and pay taxes than they will receive unemployment benefits. Remember that the only way to gain eligibility for benefits for these agents is by going through employment and being exogenously separated from the job.

Simultaneous Reforms - The Euro-Zone as a Small Open Economy

Remember that we computed the optimal reform above sequentially, each time from the perspective of an individual country, which takes interest rates and wages as given. We treated this single country as a small open economy and assumed that eventual capital in- or outflows resulting from the reform are absorbed from outside the country.

We now ask the question: What happens if all countries would simultaneously reform their system to the individually optimal reforms computed above?

To answer this question an assumption on how the capital market clears is necessary. An extreme assumption would be that the Euro-Zone as a whole is a small open economy and therefore prices would not change even if all countries would change their

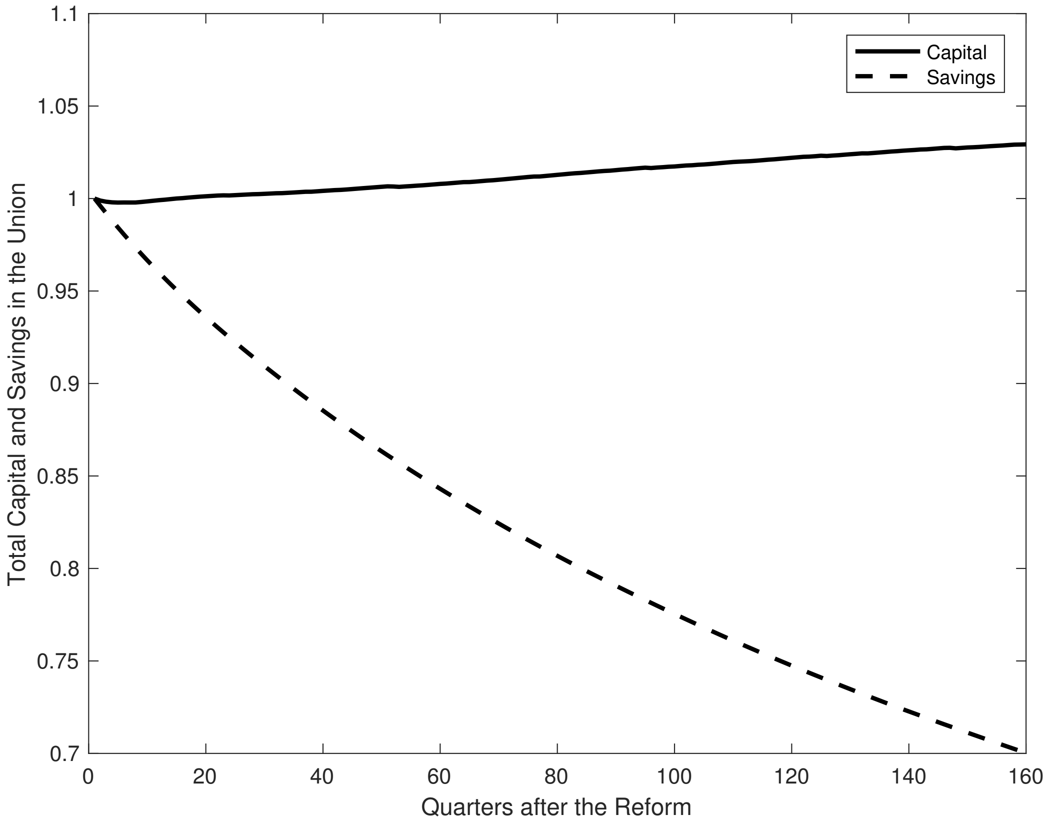


Figure 3.18: Capital and Savings in the Union under Open Economy Assumption

unemployment benefit systems at the same time.[12] In such a case the answer is simple: The same set of reforms as the one computed above (Table 3.5) is optimal and it will result in the same welfare gains (Table 3.6). However, the reforms would imply large imbalances between total European savings and total European capital. Figure 3.18 shows that in this scenario private savings decline substantially while the capital stock actually rises as a result of the reforms. Forty years after the reform about one third of the total European capital stock is financed from outside the union. The unlimited duration of eligibility reduces the risk of individuals substantially and therefore leads to a reduction in precautionary savings. On the other hand, Figure 3.19 shows that the new policies, on aggregate, induce many inactive agents to start searching. As some of them eventually find jobs also employment and thus aggregate effective labour increases. As a consequence the marginal product of capital increases and firms demand more capital.

Simultaneous Reforms - The Euro-Zone as a Closed Economy

While one can challenge the view that the Euro-Zone as a whole is a closed economy, one can certainly reject the hypothesis that it is too small to have any impact on the interest rate. In particular, the imbalances in savings and capital which we see in Figure 3.18 are way beyond what we observe in the data.

[12]Note that in such a case, also at $t = 0$ the capital market need not clear and one could instead pick another world interest rate. We experimented with interest rates in a reasonable range and this did not affect optimal policies much.

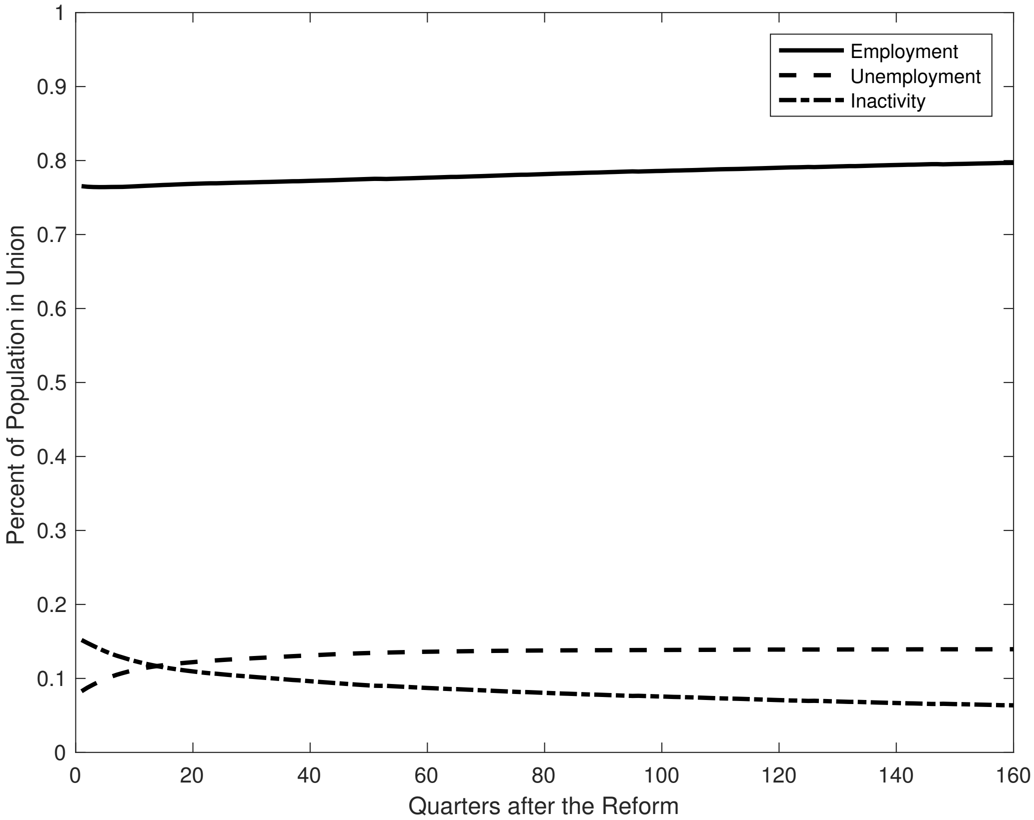


Figure 3.19: Aggregate Labour Market States under Open Economy Assumption

Therefore, we now assume that there cannot be imbalances in capital and savings for the union as a whole. In particular, we solve for the path of interest rates $\{r_t\}_{t=1}^{\infty}$ that clears the capital market at the union level (condition (13)). This change in the interest rates will change optimal firm production plans and optimal individual behaviour and it will change the equilibrium path of wages in all countries.

The resulting welfare effects are strikingly different. In Table 3.7 we observe that general equilibrium effects not only have negative effects in eight countries, in seven out of the ten countries they even reverse the sign of the welfare gains. Only in Belgium and in Finland agents benefit from general equilibrium effects. In Luxembourg the gains are still positive but substantially lower than under the open economy assumption.

	Employed	Un. Eligible.	Un. Non-Elig.	Non-Active	Total
Austria	-0.39	1.12	0.43	-0.21	-0.30
Belgium	1.16	0.96	1.19	1.03	1.12
Germany	-0.82	2.72	-1.20	-0.82	-0.77
Spain	-0.39	1.43	0.72	-0.36	-0.17
Finland	1.83	1.53	1.96	1.69	1.78
France	-0.44	0.98	-0.33	-0.51	-0.37
Ireland	-0.31	2.32	1.55	-0.21	-0.12
Italy	-0.32	5.05	1.56	-0.32	-0.10
Luxembourg	-0.01	2.76	1.08	0.24	0.10
Netherlands	-3.09	-1.48	-3.95	-3.15	-3.09

Table 3.7: Welfare gains (in % CEV) of Reforms under Closed Economy Assumption

How can these results be explained? First, note that the aggregate capital stock is now substantially lower than in the open economy case. Capital is now solely financed by the saving of Euro-Zone citizens and as mentioned above the reforms on average reduce savings. Figure 3.20 depicts the evolution of capital (savings) when the capital market clears at the union level.

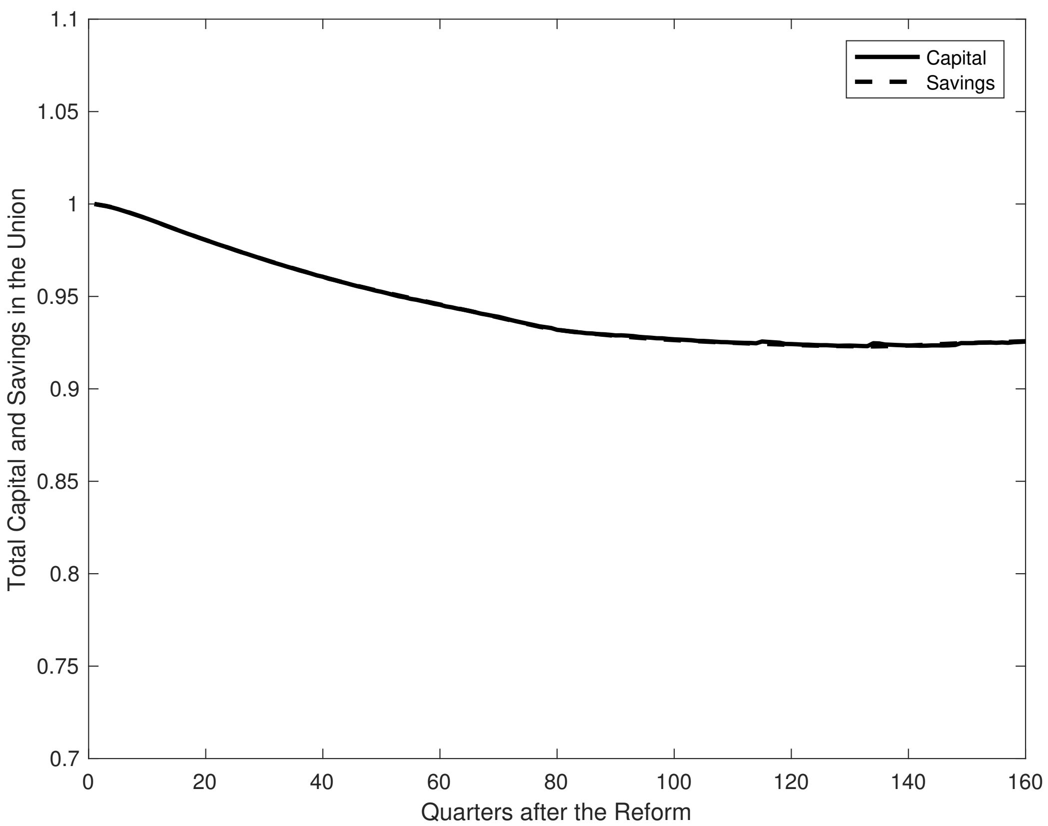


Figure 3.20: Capital and Savings in the Union under Closed Economy Assumption

This reduction in the capital stock causes an increase in the marginal product of capital but reduces the marginal product of labour. As a consequence, interest rates rise but wages decline after the reform (see Figure 3.21). Note that wages vary across countries but their relative proportions are unaffected. To be specific, for any pair $i, j \in \{1, 2, ..., I\}$ and any $t \geq 0$ it holds that $w_t^i / w_t^j = A^i / A^j$. This implies that wages decline in all countries of the union.

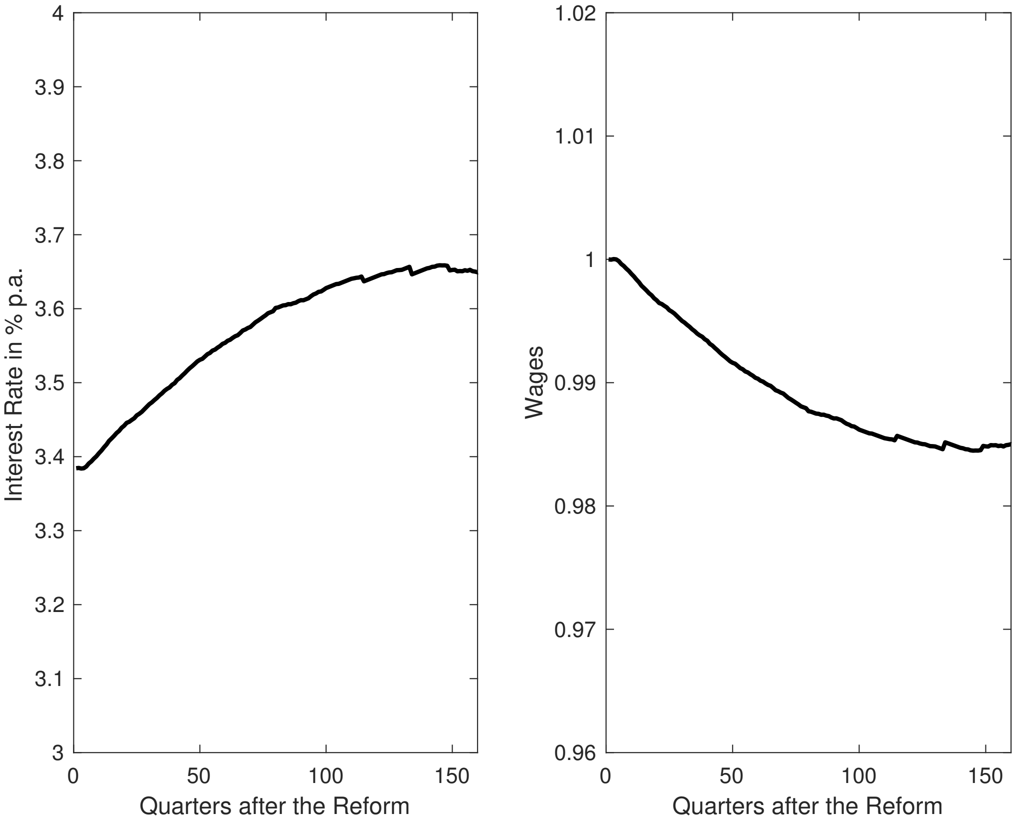


Figure 3.21: Equilibrium Prices under Closed Economy Assumption

The movements in prices are good for agents who derive most of their net present live time income from capital but bad for those whose main income source are wages. Welfare gains are computed in percent of initial consumption implying that if an initially poor and an initially rich agent experience the same consumption increase in absolute value, the welfare gain for the poor agent will be higher. Hence, what matters mostly for the aggregate welfare effects are the changes in the income composition of poor agents. Agents with little assets do not gain much from the higher interest rate but they lose substantially from lower wages. In the right panel of Figure 3.21 we see that in the long run wages decrease by about 1.5%, the same order of magnitude as the average decline in welfare through GE effects. It is important to note that by and large all groups in a country lose or benefit from general equilibrium effects in a similar way (compare Tables 3.6 and 3.7). At first glance this seems surprising since non-eligible unemployed and non-active agents currently do not have wage income while the other two groups do, the employed directly and the eligible unemployed indirectly as their unemployment benefits are proportional to wages. The reason for this result is that mobility across labour market states is higher than across wealth. For example, a currently poor non-eligible unemployed will sooner be employed and get wage income than he will be rich enough to derive most of his lifetime income from capital returns.

Now why are there two countries, Belgium and Finland, who benefit from the general equilibrium effects? Because these two countries are relatively rich on average and have a relatively equal distribution of wealth. This means that the wealth owned by consumption poor agents is higher than in the other countries and therefore their consumption losses are not big enough to offset the overall consumption gains.

The general equilibrium feedback effects from prices in turn change the behaviour of agents. For example, in Figure 3.22 we observe that the decline in inactivity is muted compared to the open economy case and that employment declines, while before it was increasing. Higher interest income allows inactive agents to run down their assets at a lower pace and at the same time lower wages make employment less attractive.

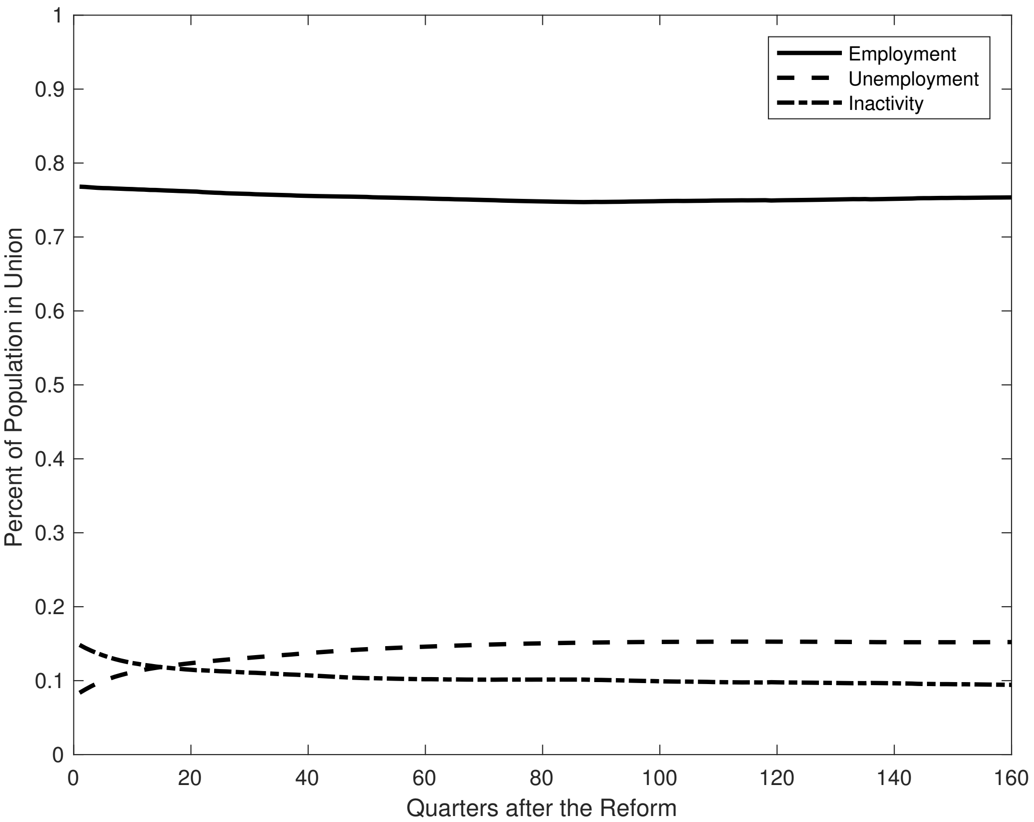


Figure 3.22: Aggregate Labour Market States under Closed Economy Assumption

In sum, the results of this section are twofold: (i) According to our model current national unemployment benefit systems are not optimal and for some countries there is a large potential for welfare improvement through policy reform; (ii) An integrated European capital market causes unemployment policies in some European countries to affect the citizens of other European countries even under the assumption that labor is not mobile. The latter implies that lack of coordination across member states may result in detrimental reforms and thus provide a rationale to centralize unemployment policy at the European level. This is the direction we explore next.

3.5.3 Harmonized European Unemployment Insurance Scheme

Can we find a harmonized European unemployment benefit system that is welfare improving in all countries of the union? We find that the answer to this question depends on how such a system is financed. If contribution payments vary such that cross-country transfers are eliminated, we find that a harmonized benefit system with an unlimited duration and a replacement rate of 15% is welfare improving in all countries. On the other hand, we could not find such a system, when it is financed jointly at the union level.

Joint Financing on the Union Level

Let us first consider jointly financed benefit systems. In this experiment, we replace individual countries' budget constraints with a common European one. Instead of I government budget constraints (equation (12)) which solve for I different tax rates, there is only one tax rate that clears the union budget constraint

$$\tau_t \sum_{i=1}^{I} \omega_t^i L_t^i = \bar{b} \sum_{i=1}^{I} \omega_t^i \sum_{z \in Z} z_t \int_0^{\infty} \zeta_t^i(a, z, u^e) da \quad \forall t \geq 0. \tag{14}$$

Note that both the tax rate and the replacement rate are independent of i. As we mentioned above not every country would agree to this form of financing no matter what the benefit system is.

In Table 3.8 we show the results for the benefit system with an unlimited duration of eligibility and a common replacement rate of 15%. We see that while most countries gain from the reform, there are three exceptions, all of whom are net payers: Austria, France and the Netherlands. Interestingly, not all countries which are net payers lose from the reform. In particular the welfare gains in Belgium, Germany, Finland and Ireland are positive even though these countries pay substantial transfers, between 0.14% and 0.87% of their respective GDP. This result is another indicator that current unemployment benefit policies are far from optimal in some of the countries.

Country	Status Quo			Optimal Reform			Δ	Transfer/GDP
	$1/\mu_0^i$	$\bar{b}_0^i$	$\tau_0^i(\%)$	$1/\mu_1^i$	$\bar{b}_1^i$	$\tau_\infty^i(\%)$		
Austria	2	0.28	0.92	∞	0.15	1.47	-0.29	-0.51
Belgium	20	0.37	2.13	∞	0.15	1.47	0.25	-0.71
Germany	4	0.23	0.45	∞	0.15	1.47	0.20	-0.14
Spain	8	0.33	4.43	∞	0.15	1.47	1.45	0.74
Finland	8	0.36	3.75	∞	0.15	1.47	1.04	-0.87
France	8	0.35	1.90	∞	0.15	1.47	-0.06	-0.16
Ireland	4	0.36	1.97	∞	0.15	1.47	0.68	-0.09
Italy	3	0.09	0.25	∞	0.15	1.47	0.87	0.29
Luxembourg	4	0.27	0.53	∞	0.15	1.47	-0.40	-0.64
Netherlands	4	0.35	1.00	∞	0.15	1.47	0.17	0.07

Table 3.8: Harmonized Benefit System Financed Jointly

Country-Specific Contribution Payments

Let us next consider the case of varying contribution payments across countries, which clear each country's government budget constraint separately. To be specific, in this experiment we require that condition (12) holds for each $i \in \{1, 2, ..., I\}$. Table 3.9 shows that in this case the very same benefit system (unlimited duration, replacement rate of 15%) that we considered above is welfare improving in all countries of the union.

To understand why this mix of taxes and benefits is welfare improving in all countries it is worthwhile to remember three findings of our analysis so far: (i) European countries have structurally different labour market institutions which as a consequence lead to very different long term averages in employment, unemployment and inactivity (section 3.4); (ii) Despite these differences, the optimal unilateral reforms are surprisingly similar with an unlimited duration of unemployment benefits and replacement rates between 20% and 45% (section 3.5.2). (iii) If all countries were to reform their benefit systems by themselves to their respective individual optima this would result in a substantial decline in the capital stock. As a consequence wages across Europe would be lower and especially poor agents, who derive most of their lifetime income from labour, would experience large welfare losses (section 3.5.2).

Country	Status Quo			Optimal Reform			Δ
	$1/\mu_0^i$	b_0^i	$\tau_0^i(\%)$	$1/\mu_1^i$	b_1^i	$\tau_\infty^i(\%)$	
Austria	2	0.28	0.92	∞	0.15	0.73	0.23
Belgium	20	0.37	2.13	∞	0.15	0.45	0.92
Germany	4	0.23	0.45	∞	0.15	1.27	0.38
Spain	8	0.33	4.43	∞	0.15	2.53	0.62
Finland	8	0.36	3.75	∞	0.15	0.22	2.03
France	8	0.35	1.90	∞	0.15	1.23	0.11
Ireland	4	0.36	1.97	∞	0.15	1.34	0.79
Italy	3	0.09	0.25	∞	0.15	1.90	0.60
Luxembourg	4	0.27	0.53	∞	0.15	0.55	0.32
Netherlands	4	0.35	1.00	∞	0.15	1.57	0.08

Table 3.9: Optimal Harmonized Benefit System Financed at the Country Level

These three findings help in the design of a collectively optimal unemployment benefit system. The first finding says that for a system to be politically sustainable we need varying contribution payments across countries. As we have seen, a common, jointly financed, system would result in transfers from countries with structurally low unemployment to countries with structurally high unemployment. The former, at least some of them, would not participate in such a scheme. We therefore demand from the system that it eliminates cross-country transfers. This is achieved through varying tax rates that clear government budgets at the national level, i.e. that satisfy condition (12). We again observe that in countries with structurally weak labour market institutions such as Spain or Italy these tax rates are substantially higher than in other countries with more efficient institutions such as Belgium or Finland.

The second finding suggests that it makes sense to harmonize the benefit system and we indeed find that an unlimited duration of eligibility and a replacement rate of 15% is welfare improving in all countries. As before the unlimited duration eliminates the risk of losing eligibility before finding a job. However, the replacement rate is lower than in any of the unilaterally optimal reforms computed above, where the minimal replacement rate was the Finnish one with 20%. The third finding explains why this is the case. We have seen that a too generous benefit system discourages savings, which in general equilibrium reduces wages, the main income source of poor agents. In Figure 3.23 we see that in the long-run the capital stock is reduced by only about 2% and 3%, while before it was between 7% and 8%. As a consequence the effects on prices are muted as can be seen in Figure 3.24. In particular, the wage decline is only about half to what we observed in Figure 3.21.

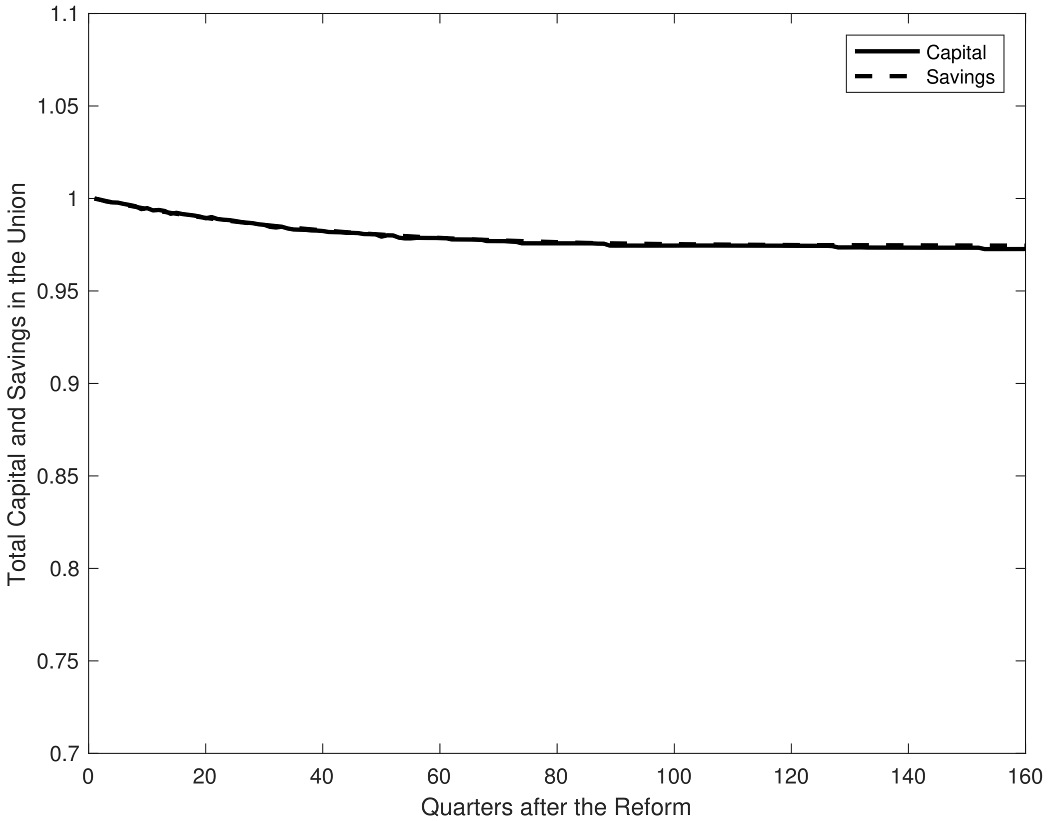


Figure 3.23: Capital and Savings with Harmonized UI Benefit System

Both, less generous unemployment benefits and higher wages, make employment more and inactivity less attractive and as a consequence aggregate employment is stabilized (Figure 3.25).

As before, the welfare effects are not only heterogeneous across countries but also across different groups within each country. This is shown in Table 3.10. We see that while the size varies, the sign is positive almost everywhere. In fact, only one single group does not benefit from the reform, the eligible unemployed in Belgium. The reason is that the current benefit system in Belgium has already a rather high duration of eligibility (20 quarters) and its current replacement rate is much higher than after the reform (37% vs. 15%). What stands out are the high welfare gains in Finland. In Table 3.10 we see that the tax rate after the reform is much lower than before. Together with strong labour market institutions that make the duration of non-employment very short, at least for people who are willing to work, these tax reductions lead to substantial welfare gains for all groups.

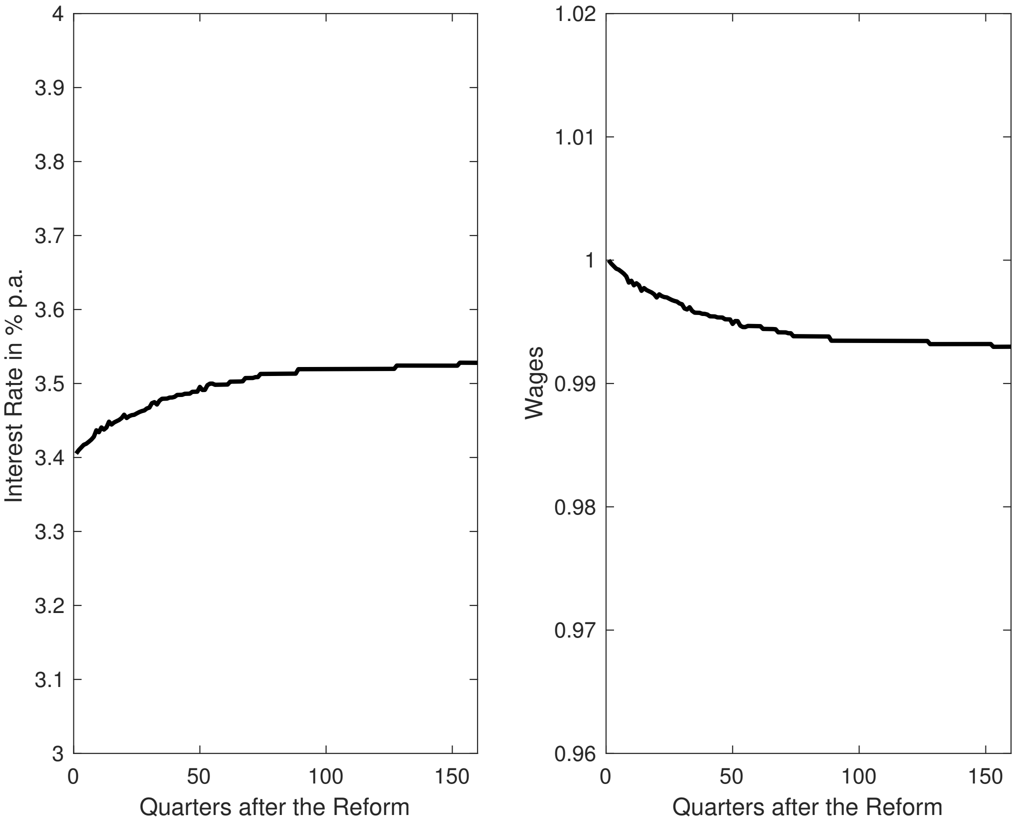


Figure 3.24: Equilibrium Prices with Harmonized UI Benefit System

	Employed	Un. Eligible.	Un. Non-Elig.	Non-Active	Total
Austria	0.21	0.56	0.47	0.24	0.23
Belgium	1.05	-0.35	0.15	0.81	0.92
Germany	0.34	2.34	0.44	0.28	0.38
Spain	0.65	0.44	0.65	0.58	0.62
Finland	2.09	1.64	1.83	1.90	2.03
France	0.11	0.15	0.18	0.09	0.11
Ireland	0.77	1.07	1.31	0.73	0.79
Italy	0.48	2.73	1.58	0.51	0.60
Luxembourg	0.30	0.85	0.51	0.33	0.32
Netherlands	0.06	0.91	0.13	0.03	0.08

Table 3.10: Welfare gains in (in % CEV) with Harmonized UI Benefit System

3.6 Conclusion

This paper is aimed at assessing the value of a European Unemployment Insurance System (EUIS) and, in particular, how it should be designed as a constrained efficient mechanism. We take as a constraint the current labour market institutions which determine differences in job destruction and the likelihood to receive offers by the unemployed (searching for a job) and the inactive (not actively searching), we also limit the scope of unemployment

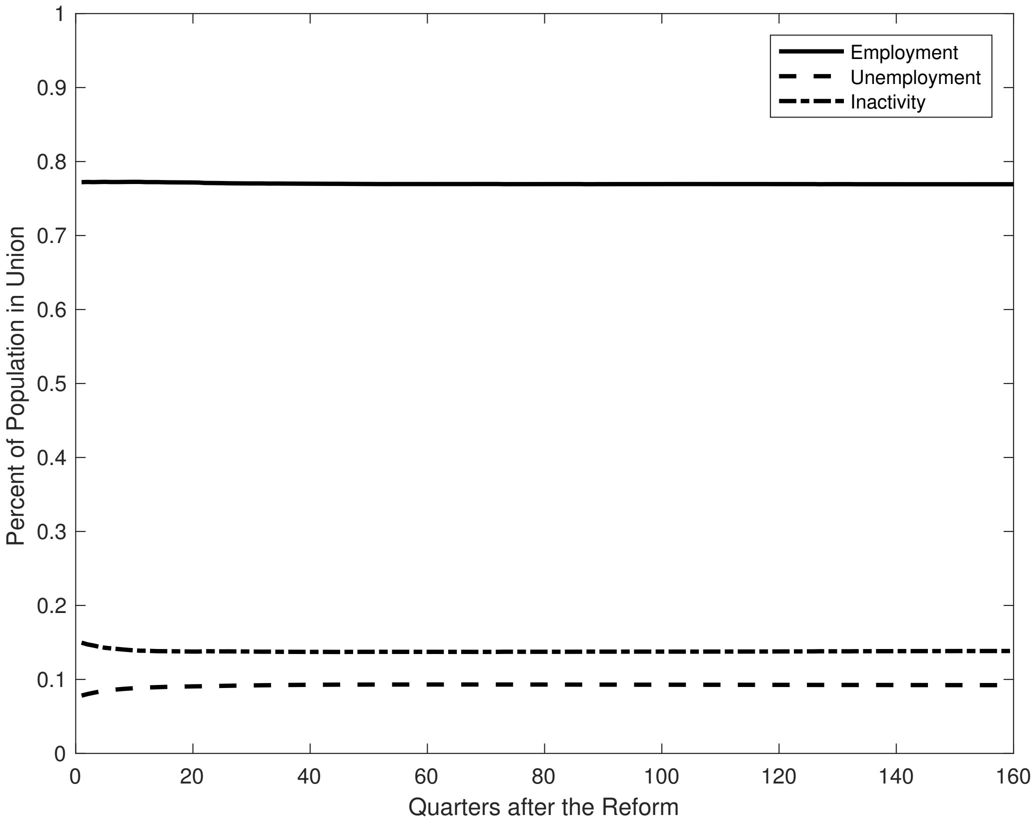


Figure 3.25: Aggregate Labour Market States with Harmonized UI Benefit System

insurance contracts to contracts defined by their coverage duration and their replacement rate. Our work provides a quantitative proof of the potential gains that market reforms – not just labour market reforms – can achieve in many European countries. In fact, the first contribution of this paper is to provide a novel diagnosis of European labour markets. The second, which is almost a corollary of the first, is to show quantitatively that country-specific structural parameters play a determinant role in explaining the different performance of labour markets across the EU.

Based on this calibration we perform a set of policy experiments. We show that the gains from pure risk-sharing (i.e. absent UI reforms) are very limited but that substantial welfare gains can be achieved by reforming the existing UI systems within European countries. Even if, as we document, labour markets are very different, almost surprisingly the (parameterised) UI systems that maximise welfare are very similar: unemployment benefits duration should be unlimited and replacement rates more similar across countries than what they are now. We show that unemployment benefit reforms in some European countries affect the citizens of other countries through general equilibrium effects which result from a common capital market. In particular, the decline in precautionary savings associated with more generous unemployment benefit systems causes a reduction in the aggregate, European, capital stock. This in turn causes a decline in wages all over Europe. This means that reforms which may seem optimal at the national level can be detrimental at the European level once general equilibrium effects are taken into account.

Finally, we show that a common, less generous, European benefit system can tackle this problem. We find that a harmonized benefit system with an unlimited duration and a replacement rate of 15% is welfare improving in all countries, when it is financed by country specific contribution payments. The welfare gains are relatively large for all countries, and almost unanimous within countries, even without accounting for the risk-sharing gains that countries would have if, in addition to agents' idiosyncratic risk we also had aggregate country risks. That is, we required that each country runs a balanced budget, thereby eliminating permanent cross-country transfers. With country risks and no aggregate European risk, at the steady-state constant – but differential – taxes would also provide risk-sharing, with short-run cross-country transfers across the EUIS, possibly with the support of a centralized fund as we discussed in the Introduction. Even with European aggregate risk the EUIS would play a major stabilising role: taxes would not be constant, unless the fund has borrowing capacity (which it should), but still provide risk-sharing across countries and agents. In any case, the resulting tax differences across countries reflect their structural labour market differences, in terms of job creation and destruction. These tax differences also provide clear incentives for labour market reforms.

In sum, by increasing welfare across European citizens the proposed EUIS can also be an important cohesive EU institution. There is no need to wait for European labour markets to converge to implement the EUIS. In fact, it can promote national labour market reforms and European labour market integration.

3.A Appendix to Chapter 3

3.A.1 Transition Function

The transition function $T_t^i((a,z,x);\mathcal{A}\times\mathcal{Z}\times\mathcal{X})$ describes the probability that an agent, who is in state (a,z,x) in period t, is in any state $\{(a',z',x') : a'\in\mathcal{A}, z'\in\mathcal{Z}, x'\in\mathcal{X}\}$ in period $t+1$. This function is quite involved as it captures exogenous shocks and endogenous decisions of the agent. Next period's assets $a'(a,z,x)$ are purely endogenous as they are chosen from the agent in period t and not subject to any shock. Next period's productivity level z' is purely exogenous and depends on the Markov transition probabilities. Next period's employment state $x'\in\{e,u^e,u^n,n\}$ depends on a combination of exogenous shocks (job separation, job finding) and endogenous decisions (work, search), which in turn depend on assets and individual productivity.

We can write the transition function as

$$\begin{aligned}
&T_t((a,z,x);\mathcal{A}\times\mathcal{Z}\times\mathcal{X}) = \\
&\mathbb{1}_{a^i_{t+1}(a,z,x)\in\mathcal{A}}\cdot\sum_{z'\in\mathcal{Z}} p(z'|z)\Big\{\mathbb{1}_{e\in\mathcal{X}}\cdot xe\big(a^i_{t+1}(a,z,x),z'\big)+\mathbb{1}_{u^e\in\mathcal{X}}\cdot xu^e\big(a^i_{t+1}(a,z,x),z'\big)+ \\
&\qquad\qquad \mathbb{1}_{u^n\in\mathcal{X}}\cdot xu^n\big(a^i_{t+1}(a,z,x),z'\big)+\mathbb{1}_{n\in\mathcal{X}}\cdot xn\big(a^i_{t+1}(a,z,x),z'\big)\Big\},
\end{aligned}$$

where $xe\big(a^i_{t+1}(a,z,x),z'\big)$ describes the probability of moving from labor market state $x\in\{e,u^e,u^n,n\}$ into employment, conditional on saving $a^i_{t+1}(a,z,x)$ and on drawing productivity shock z'. Similarly, $xu^e(.)$ is the conditional probability of moving into unemployment and being eligible for benefits, and so on.

It is useful to define the decision to search for a job next period, conditional on being not eligible for unemployment benefits by

$$s_{t+1}(a',z',0)=\begin{Bmatrix} 1 & \text{if arg} \max\limits_{x'\in\{u^n,n\}} V^i_{t+1}(a',z',x')=u^n \\ 0 & \text{else} \end{Bmatrix}$$

and the decision to search for a job conditional on being eligible for unemployment benefits by

$$s(_{t+1}a',z',1)=\begin{Bmatrix} 1 & \text{if arg} \max\limits_{x'\in\{u^e,n\}} V^i_{t+1}(a',z',x')=u^e \\ 0 & \text{else} \end{Bmatrix}$$

Similarly, define the decision to work next period, conditional on being not eligible for unemployment benefits by

$$w_{t+1}(a',z',0)=\begin{Bmatrix} 1 & \text{if arg} \max\limits_{x'\in\{,e,u^n,n\}} V^i_{t+1}(a',z',x')=e \\ 0 & \text{else} \end{Bmatrix}$$

and the decision to work conditional on being eligible for unemployment benefits by

$$w_{t+1}(a',z',1)=\begin{Bmatrix} 1 & \text{if arg} \max\limits_{x'\in\{,e,u^e,n\}} V^i_{t+1}(a',z',x')=e \\ 0 & \text{else} \end{Bmatrix}$$

The conditional transition probability from employment into employment is then given by

$$ee\big(a^i_{t+1}(a,z,x),z'\big) = (1-\sigma^i)w^i_{t+1}(a^i_{t+1}(a,z,e),z',0) + \sigma^i\lambda^i_u w^i_{t+1}(a^i_{t+1}(a,z,e),z',1).$$

There are two possibilities how an agent, who is employed in period t, is also employed in $t+1$: (i) the agent does not get separated, which happens with probability $1-\sigma^i$ and does not quit his job, which is the case if the work decision $w^i_{t+1}(a^i_{t+1}(a,z,e),z',0) = 1$. Since job quitters are not eligible for benefits the last entry of the work decision is zero; (ii) the agent gets separated from his job (with probability σ^i) but immediately finds a new job (with probability λ^i_u) and decides to work. In case of exogenous separation the agent would be eligible for unemployment benefits, therefore the last entry in the work decision is equal to one. One can observe that this conditional probability is a mixture of exogenous probabilities and endogenous decisions.

Similarly, we can define the other conditional probabilities: The probability of moving from employment to unemployment and being eligible for benefits is

$$eu^e\big(a^i_{t+1}(a,z,x),z'\big) = \sigma^i s^i_{t+1}(a^i_{t+1}(a,z,e),z',1)\Big[(1-\lambda^i_u) + \lambda^i_u\big(1-w^i_{t+1}(a^i_{t+1}(a,z,e),z',1)\big)\Big].$$

Eligibility next period requires that the worker is exogenously separated, which happens with probability σ^i and that the agent is actively searching for a job, i.e. $s^i_{t+1}(.) = 1$. There are again two possibilities to be unemployed next period: (i) With probability $1-\lambda^i_u$ the agent does not immediately find a new job (ii) with probability λ^i_u the agent immediately finds a new job but he decides not to accept the offer ($w^i_t(.) = 0$).

The conditional probability of moving from employment into unemployment and being eligible for benefits is equal to the probability of not being separated (once you are separated you are automatically eligible for benefits), given that the agent decides to quit $w(.) = 0$ and to search for a new job $s(.) = 1$:

$$eu^n\big(a^i_{t+1}(a,z,x),z'\big) = (1-\sigma^i)\big(1-w^i_{t+1}(a^i_{t+1}(a,z,e),z',0)\big)s^i_{t+1}(a'(a,z,e),z',0).$$

Finally, the conditional probability of moving from employment into inactivity is given by

$$\begin{aligned} eu^n\big(a^i_{t+1}(a,z,x),z'\big) &= (1-\sigma^i)\big(1-w^i_{t+1}(a^i_{t+1}(a,z,e),z',0)\big)\big(1-s^i_{t+1}(a^i_{t+1}(a,z,e),z',0)\big)+ \\ &\sigma^i\Big(1-\lambda^i_u + \lambda^i_u\big(1-w^i_{t+1}(a^i_{t+1}(a,z,e),z',1)\big)\Big)\big(1-s^i_{t+1}(a^i_{t+1}(a,z,e),z',1)\big). \end{aligned}$$

The agent can become inactive either if he does not get exogenously separated but decides to quit working and searching (fist line) or if he gets separated and does not search for a new job (second line).

We now described all possibly cases for an agent who is employed in period t, i.e. $x_t = e$. In an analogous way this can be done for all other initial labor market states, i.e. for $x_t \in \{u^e, u^n, n\}$.

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