Historical Forecasting of Interest Rate Mean and Volatility of the United States: Is there a Role of Uncertainty?

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Abstract

Uncertainty is known to have negative impact on financial markets through its effects on investors' decisions. In the wake of the Great Recession, quite a few recent studies have highlighted the role of uncertainty in predicting in-sample movements of interest rates. Since in-sample predictability does not guarantee out-of-sample forecasting gains, in this paper we used historical daily and monthly data for the US to forecast mean and volatility of interest rate. The results show that, changes in uncertainty measure movements fails to add any significant statistical gains to the forecast of interest rates for the US. In other words, policy makers in the US are not likely to improve their accuracy of future movements of the policy rate's mean and volatility by incorporating information derived from changes in metrics of uncertainty.

Keywords: Interest Rate; Metrics of Uncertainty; Mean and Volatility Forecasting

1. Introduction

Theoretically, uncertainty is known to have a negative impact on the real economy and financial markets via postponement of investment and consumption decisions (see for example, Bernanke [1], Dixit and Pindyck [2], and more recently Bloom [3]). Following the Great Moderation, the world economy experienced a substantial increase in financial and macroeconomic volatility as a result of the global financial crisis starting in the summer of 2007, followed by a major global recession (the Great Recession) between 2008 and 2009, and regional crises such as the European sovereign debt crisis starting in 2010. As a result, the analysis of the role of volatility and uncertainty in the macroeconomy has regained a prominent role in recent years (see, Bloom [4], Chuliá et al. [5] and Gupta et al.[6; 7; 8] for detailed reviews of this literature), with the majority of these studies concluding that unexpected large changes in uncertainty (or the closely related concepts of risk and volatility)

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represent an important source of macroeconomic (and financial markets) fluctuations. Not surprisingly, large number of recent studies have also highlighted the fact that, central banks around the world respond to such uncertainties by changing their interest rate decisions to nullify the recessionary impact (see for example, Çekin et al. [9], Christou et al.[10; 11] and references cited there in). Given this in-sample evidence, and realizing the fact that, the ultimate test of any predictive model (in terms of the econometric methodologies and the predictor(s) used) is in its out-of-sample performance [12], we aim to analyse the ability of movements of uncertainty in forecasting the policy rate of the United States. In this regard, we look at linear and nonlinear models over the longest possible historical sample of daily and monthly data covering 6th January 1927 to 30th November 2016, and July 1889 to March 2016, respectively. This way, we are able to not only analyse the importance of data frequency in interest rate decisions, but also avert the problem of sample-selection bias. In addition, we go beyond the impact of uncertainty on the first-moment of interest rate, and build on the in-sample evidence provided by Valeraet al. [13] and Donzwa et al. [14] on the second-moment, i.e., the volatility of interest rates. In other words, we use alternative models of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-family to forecast interest rate volatility as well based on the information content of uncertainty movements.

To the best of our knowledge this is the first attempt to provide historical evidence of the possible role of uncertainty movements in improving forecasts of the first and second-moments associated with the monetary policy decisions of the US. Having said that, large number of studies have looked into forecasting monetary policy decisions of the US, as well as the volatility of interest rate using a wide array of linear and nonlinear frameworks. In terms of the first moment, reliance has been placed on variations of the Taylor-rule used independently or within dynamic stochastic general equilibrium, as well as large-scale atheoretical macro models, with the reader is referred to [15; 16; 17; 18] for detailed reviews of this literature. As far as volatility, there is an extensive literature focused on modelling interest rate volatility. This requires (as with most financial series) taking into account volatility clustering, asymmetric effects, excess kurtosis, time-varying volatility, long-memory or leverage effects [19; 20]. Chan et al. [21] assumes a level effects in their study of UK interest rate on interest rate volatility, while Brenner et al. [22] and Koedijk et al. [23] propose a model that considers conditional heteroscedasticity in a GARCH type models [24; 25] in addition to the level effects of Chan et al. [21]. This explains the extensive use of GARCH models for this application [26], however, traditional GARCH models are unable to account for asymmetries, which led to the development of the Exponential GARCH (EGARCH) models [27], Asymmetric Power ARCH (APARCH) models [28], or GJR-GARCH models

[29] used to model and forecast interest rates. Drost and Klaassen [30] show that the distributions are not normal and usually unknown, however, semiparametric techniques can be used to model financial variables. For example, in their study to forecast short-term interest rates, Hou and Saurdi [31] applies a semiparametric smoothing to a GARCH model of short rate volatility. They obtain more accurate forecasts of interest rate volatility as opposed to parametric models. TIan and Hamori [32] use a Realized GARCH (RGARCH) to capture volatility clustering and the mean-reverting nature of interest rates in a model of the Euro-Yen short-term interest rate, and find that it performs better than traditional GARCH-type models.

Note that, quite a few recent studies have analysed with success the importance of uncertainty in forecasting movements of macroeconomic variables [33; 34; 35], asset prices [36; 37; 38], and commodity markets [39; 40]. The remainder of the paper is organized as follows: Section 2 outlines the empirical methodologies, while Section 3 discusses the data, and Section 4 presents the forecasting results, with Section 4 concluding the paper.

2. Methods

Following the method employed by Hassani et al. [41] and [42] we used out-of-sample forecasts to test for uncertainty's movements effect on forecasting both mean and volatility of interest rate. Half of the data is used for model selection and parameter estimation. The h step ahead out-of-sample forecasts are calculated with adding one observation at a time (from the other half) and without updating the estimated parameters. For interest rate mean forecasting, the nonparametric Functional Coefficient Autoregressive model (FAR) of Chen and Tsay [43] and Cai et al. [44] is used, as well as linear $ARMA$ model, both with and without including the uncertainty measure as predictor. The $ARMA$ model makes it possible to test for linear dependencies between uncertainty movements and interest rate mean, as it is a simple base for many statistical tests in econometrics. Since $ARMA$ only tests for linear dependencies, we are using nonlinear and nonparametric FAR model as well, to test nonlinear relations between uncertainty movements and interest rate mean.

In volatility forecasting, we are using method employed by Hassani et al. [45]; which used a variation of $GARCH$ type models with different error distributions to forecast UK's interest rate. In this research the Family Nesting $GARCH$ ($FNGARCH$) model of Hentschel [46] with different parametric error distributions is used with and without uncertainty's first difference as predictor. The $FNGARCH$ model is has a general formulation of $GARCH$ family and contains all of the well performed model in [45]. Uncertainty's first difference is considered as exogenous variable. FAR, $ARMA$ and random walk are employed to forecaster the uncertainty (random walk is applied to uncertainty rather than its first difference, since the uncertainty's first difference is stationary). The accuracy of fitted models are compared using Mean Square Error (MSE) and Kolmogorov-Smirnov Predictive Accuracy ($KSPA$) nonparametric test [47]. In order to increase $KSPA$ test's reliability, we also used a bootstrap with sample size of 10000 to estimate the $KSPA$'s P-Value.

2.1. FAR model

The Functional-Coefficient Autoregressive with Exogenous variables ($FARX$) formulates the time series conditional expectation of y_t as follows [44; 43]:

$$y_t = \sum_{i=1}^{p} f_i(y_{t-d})y_{t-i} + \sum_{i=1}^{q} g_i(y_{t-d})x_{t,i} + \varepsilon_t, \tag{1}$$

where ε_t is white noise and $x_i(i = 1, \ldots, q)$ are exogenous variables (and may contain the exogenous variables' lags). The nonlinear functions $f_i(y_{t-d})$ and $g_i(y_{t-d})$ are estimated using local linear regression [44].

2.2. FNGARCH model

Suppose ε_t be a time series with zero mean and variance σ_t^2. The $FNGARCH$ models σ_t^2 using following functional form [46]:

$$\sigma_t^\lambda = \left(\beta_0 + \sum_{i=1}^{q} \beta_i x_{t,q}\right) + \sum_{i=1}^{k} \alpha_k \sigma_{t-i}^\lambda \left\{|z_{t-i} - \eta_{2,i}| - \eta_{1,i}\left(z_{t-i} - \eta_{2,i}\right)\right\}^\delta + \sum_{i=1}^{p} \gamma_p \sigma_{t-i}^\lambda, \tag{2}$$

where $z_t = \frac{\varepsilon_t}{\sigma_t}$ and $x_i(i = 1, \ldots, q)$ are exogenous variables. The $FNGARCH$ model can be used as a general functional form for volatility forecasting, since many common GARCH models can be formulated as special parameterization of $FNGARCH$. For instance $GARCH$ [25], $AVGARCH$ [48], $GJR - GARCH$ [49], $TGARCH$ [50], Nonlinear $GARCH$ [51], Nonlinear Asymmetric $GARCH$ [52], $A - PARCH$ [28] and $EGARCH$ [27] models are sub-models of $FNGARCH$.

3. Data Description

As far as the interest rate variable is concerned, we use the risk-free rate from 6th January 1927 to 16th March, 1936 from Professor Kenneth R. Frenchs database[53], the 3-month Treasury bill rate over the period of 17th March, 1936 till 30th June, 1954, the effective Federal funds rate from July 1st, 1954 to 15th December, 2008 and then from 16th December 2015 to 30thNovember, 2016, with all the latter set of data derived from the FRED database. For the time period of 16thDecember, 2008 till 15th December, 2015, which corresponds to the zero lower bond (ZLB)scenario, we used the shadow short rate developed by Krippner [54; 55] based on models of term structure (The data is available for download from [56]).

The yield curve-based framework developed by Krippner [54; 55] essentially removes the effect that the option to invest in physical currency (at an interest rate of zero) has on yield curves. This results in a hypothetical shadow yield curve that would exist if the physical currency were not available (The process allows one to answer the question: What policy rate would generate the observed yield curve if the policy rate could be taken as negative? The shadow policy rate generated in this manner, therefore, provides a measure of the monetary policy stance after the actual policy rate reaches zero). The monthly version of the interest rate covers July 1889 to March 2016, with data on risk-free interest rate from July, 1889 to December, 1933 obtained from the website of Professor Amit Goyal, and for the rest of the period we use the same sources as the daily data [57]. Note the coverage of our sample periods is purely driven by the availability of the daily and monthly measures of uncertainties, which we discuss next.

The daily measure of uncertainty associated with financial market uncertainty is measured using the metric developed by Chuliá et al. [58], who uses 25 portfolios of stocks belonging to the NYSE, AMEX, and NASDAQ, which comprises the CRSP (Center for Research in Security Prices) stock index, sorted according to size and their book-to-market value. These authors follow a two-step process for the construction of their uncertainty index. First, they remove the common component of the series under study and calculate their idiosyncratic variation by filtering the original series using a generalized dynamic factor model (GDFM). Second, these authors calculate the stochastic volatility of each residual in the previous step using Markov Chain Monte Carlo (MCMC) techniques. Then, Chuliá et al. [58] obtain a single index of uncertainty for the stock market by average the series (The data can be downloaded from the website of Professor Jorge M. Uribe at [59]). The monthly data on uncertainty is text-based and includes the title and abstract of all front-page articles of the Wall Street Journal, as developed by Manela and Moreira [60]. These authors focus on front-page titles and abstracts in order to ensure feasibility of data collection, and also because these are manually edited and corrected following optical character recognition, which in turn, improves their earlier sample reliability. The News Implied Volatility Index (NVIX) data is found to peak during stock market crashes, times of policy-related uncertainty, world wars, and financial crises. The reader is referred to Manela and Moreira [60] for further details.

The PELT (Pruned Exact Linear Time) algorithm [61] is employed to test for structural change in the first difference of interest rate. The results of the changepoint test shows significant structural change in variance of the interest rate's first difference, on Friday, 22nd November 1985 (the R package "changepoint" [62] is used to apply the algorithm). The same test is applied to first difference of monthly data which shows significant structural change in first difference of interest rate's monthly average on November 1972. Original time series and their differences in new timeline (after the structural change) are shown in Figures 1 and 2. As it can be seen, the mean of the original time series are not stable. After removing the time series before the structural changes, the Augmented Dickey-Fuller (ADF) test is applied to test stationarity of both interest rate and uncertainty measure in new timeline (Ng and Perron [63] method is used for lag selection in ADF). Results (Table 1) shows that none of four time series (daily and monthly interest rate and uncertainty measure) are stationary, however, their first differences (daily and monthly changes) are stationary.

4. Empirical Results

As mentioned before, FAR, linear $ARMA$, $FARX$ (FAR with uncertainty's first difference as predictor) and $ARMAX$ ($ARMA$ with uncertainty's first difference as predictor) are used to forecast first difference of interest rate's daily and monthly mean. Moreover, the Random Walk (RW) model for interest rate daily and monthly mean is fitted as well. Half of the available data (after structural change) are used for model fitting and the rest of the data are used for accuracy comparison. The Mean Square Error (MSE)s for out-of-sample mean forecasting are given in Tables 2 and 3 for different forecasting horizons.

As it is shown by $KSPA$ P-Values in Tables 2 and 3, none of the models with uncertainty's first difference as predictor outperformed the models without predictor (The uncertainty forecast with significantly higher accuracy is used to forecast interest rate). According to these results, random walk can be used to forecast the mean of both daily and monthly uncertainty measure. One can forecast the mean of the monthly interest rate in short, medium and long forecasting horizons ($h = 1, 3, 6$ and 12) using random walk as well. However, in daily interest rate forecasting, the $ARMA$ ($AR(2)$) model has significantly better out-of-sample accuracy in very short forecasting horizon ($h = 1$) whilst in medium and long term forecasting, none of the models outperform the random walk. The estimated models for uncertainty and interest rate forecasting are presented in Tables 7 and 8. Since the mean interest rate follows a random walk model in monthly data (and consequently its first difference is a has constant mean), we removed the constant mean from first difference of monthly interest rates difference and test for ARCH effects in new time series. In Daily data the test is performed on the residuals of $AR(2)$, since none of the estimated models outperform the $AR(2)$. The $ARCH - LM$ test results (Table 4) show presence of $ARCH$ effect in first difference of both daily and monthly interest rates.

In order to test the uncertainty first difference's effect on volatility forecasting, the $FNGARCH$ model is fitted to the daily and monthly interest rate first difference (after removing constant mean) with different error distributions. Again half of the available data (after changepoints) are used for model fitting and the rest are used for accuracy comparison. $KSPA$ test is employed to compare fitted models (with and without uncertainty's firs difference as predictor). The out-of-sample MSEs, minimum MSE models and KSPA test for comparing each model with minimum MSE model, are given in Tables 5 and 6 for daily and monthly time series. Estimated models for daily and monthly interest rate volatility forecasting are given in Tables 7 and 8. As it can be seen in estimated models for volatility forecasting, the best models to forecast volatility has asymmetric behaviour (evident from estimated parameters η_1 and η_2 in (2) formulation) which shows the asymmetric response of the interest rate volatility to previous large and small volatilities.

As it can be seen in Tables 5 and 6, none of the $FNGARCH$ models with uncertainty's first difference as predictor, outperformed the $FNGARCH$ models without predictors. In other word, adding the uncertainty's first difference does not improve the out-of-sample forecasting accuracy of interest rate volatility.

5. Conclusion

In the wake of the Great Recession, quite a few recent studies have highlighted the role of uncertainty in predicting in-sample movements of both first- and second moments of interest rates. Since in-sample predictability does not guarantee out-of-sample forecasting gains, in this paper we used historical daily and monthly data for the US covering 6th January 1927 to 30th November 2016, and July 1889 to March 2016, respectively, to forecast mean and volatility of interest rate based on linear and nonlinear frameworks. Our results show that, changes in uncertainty measure movements, as defined above, fails to add any significant statistical gains to the forecast of interest rates for the US. In other words, policy makers in the US are not likely to improve their accuracy of future movements of the first and second moments of the policy rate by incorporating information derived from changes in metrics of uncertainty. While there is lack of forecastability of uncertainty's movements for the interest rate setting behaviour of the US, it would be interesting to extend our work in future analysis to other developed and emerging market economies.

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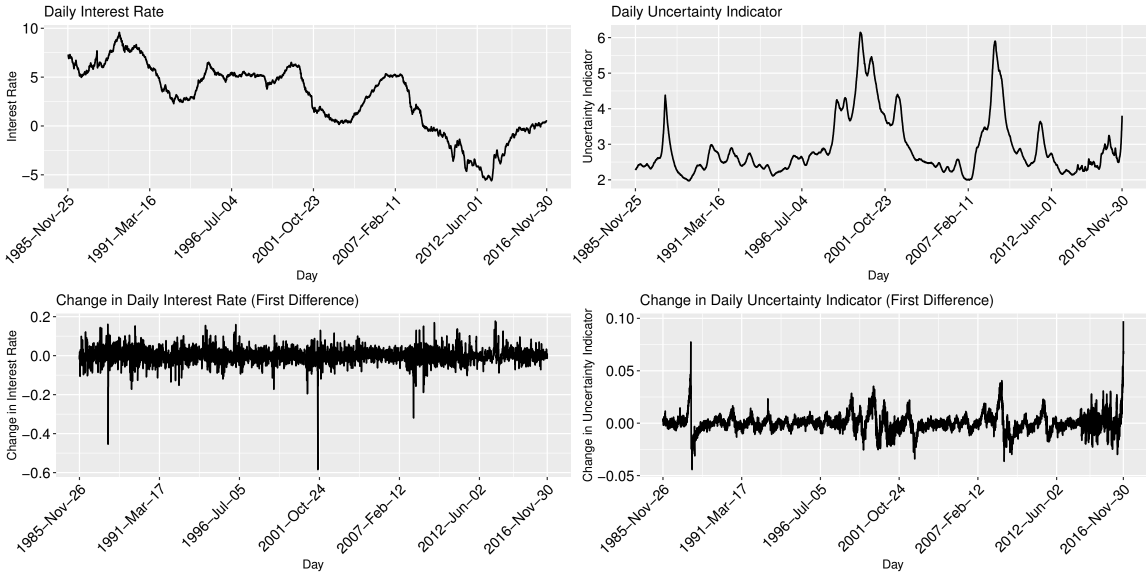

Figure 1: Daily interest rate and uncertainty time series and their first difference after the changepoint (Friday, 22nd November 1985)

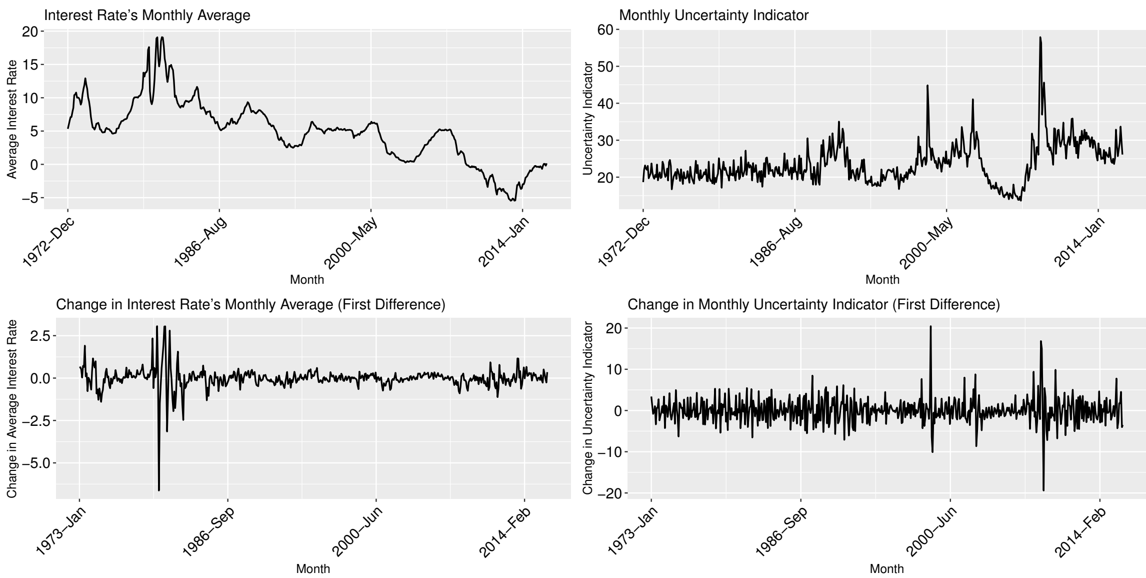

Figure 2: Monthly interest rate and uncertainty time series and their first difference after the changepoint (November 1972)

Table 1: Augmented Dickey-Fuller test Results on original series and their differences, after changepoints

Series' Names	Time Series' Variance	Parameter (Lag order)	ADF Test Statistic	P-Value
Daily Interest Rate	12.18	70	-1.44	0.5616
Daily Interest Rate's First Difference	0.0011	70	-7.93	8.637e-13
Monthly Interest Rate	21.49	10	-1.72	0.4186
Monthly Interest Rate's First Difference	0.36	10	-7.29	1.399e-10
Daily Uncertainty	0.72	70	0.0086	0.6854
Daily Uncertainty's First Difference	7.6428e-05	70	-7.80	1.677e-13
Monthly Uncertainty	29.08	10	-0.35	0.5591
Monthly Uncertainty's First Difference	10.09	10	-8.45	5.841e-15

Table 2: Daily out-of-sample mean forecast MSEs and Bootstrap $KSPA$ P-Value for comparing each model with minimum MSE model. P-Values are presented in parenthesis.

Forecasting Model	MSEs and Bootstrap $KSPA$ (P-Values) for daily interest rate's first differene forecasting $h=1$	$h=5$	$h=10$	$h=20$	MSEs and Bootstrap $KSPA$ (P-Values) for daily uncertainty's first differene forecasting $h=1$	$h=5$	$h=10$	$h=20$
RW	0.0011	0.0011	0.0011	0.0011	0.00009	0.00009	0.00009	0.00009
	(0.0000)	(0.9970)	-	-	(0.1811)	(0.1811)	(0.2406)	(0.4005)
$ARMA$	0.0007	0.0011	0.0011	0.0011	0.00002	0.00003	0.00004	0.00006
	-	-	(0.9108)	(0.9038)	-	-	-	-
FAR	0.0009	0.0011	0.0011	0.0011	0.00125	0.01962	1.493E+13	5444.99
	(0.0000)	(1.0000)	(0.9037)	(0.9032)	(0.4318)	(0.0000)	(0.0000)	(0.0000)
$ARMAX$	0.0007	0.0011	0.0011	0.0011	-	-	-	-
	(1.0000)	(0.9973)	(1.0000)	(1.0000)	-	-	-	-
$FARX$	0.0016	0.0023	0.0022	0.0022	-	-	-	-
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	-	-	-	-
Min. MSE Model	$ARMA$	$ARMA$	RW	RW	$ARMA$	$ARMA$	$ARMA$	$ARMA$

Table 3: Monthly out-of-sample mean forecast MSEs and Bootstrap $KSPA$ P-Value for comparing each model with minimum MSE model. P-Values are presented in parenthesis.

Forecasting Model	MSEs and Bootstrap $KSPA$ (P-Values) for monthly interest rate's first differene forecasting				MSEs and Bootstrap $KSPA$ (P-Values) for monthly uncertainty's first differene forecasting			
	$h = 1$	$h = 3$	$h = 6$	$h = 12$	$h = 1$	$h = 3$	$h = 6$	$h = 12$
RW	0.0866	0.0866	0.0866	0.0866	12.5340	12.5340	12.5340	12.5340
	(0.5880)	(0.9990)	(0.9970)	(0.9570)	-	(0.9741)	(0.8351)	(0.8851)
$ARMA$	0.0719	0.0867	0.0867	0.0867	13.8963	12.5174	12.5333	12.5340
	-	(0.9390)	(0.9970)	(0.7390)	(0.0000)	-	(0.6947)	(0.7547)
FAR	0.0725	0.0859	0.0859	0.0871	1429.0423	1437.8198	8.2976	8.1777
	(0.9810)	-	(0.9980)	(0.9830)	(0.0004)	(0.6274)	-	-
$ARMAX$	0.0734	0.0880	0.0880	0.0880	-	-	-	-
	(1.0000)	(0.9610)	(0.9970)	(0.8940)	-	-	-	-
$FARX$	0.0757	0.0862	0.0855	0.0866	-	-	-	-
	(0.9930)	(1.0000)	-	-	-	-	-	-
Min. MSE Model	$ARMA$	FAR	$FARX$	$FARX$	RW	$ARMA$	FAR	FAR

Table 4: ARCH-LM test Results for first difference of daily and monthly interest rate

Series' Name	Parameter (Lag order)	$ARCH - LM$ Test Statistic	P-Value
Daily Interest Rate's First Difference	20	986.9186	0.0000***
Monthly Interest Rate's First Difference	20	192.5429	0.0000***

***. The null hypothesis of ARCH-LM test (H_0: There is no existing ARCH up to specified lag order) is rejected at 0.001 significance level.

Table 5: Daily out-of-sample volatility forecast MSEs and Bootstrap $KSPA$ P-Value for comparing each model with minimum MSE model. P-Values are presented in parenthesis.

Forecasting Model	Error Dist.	Daily MSEs and Bootstrap $KSPA$ (P-Values) for different forecasting horizons (h) $h=1$	$h=5$	$h=10$	$h=20$
	N^1	0.00064(0.03)*	0.00069(0.016)*	0.0007(0.028)*	0.00068(0.016)*
	SN^2	0.00063(0.086)†	0.00067(0.084)†	0.00068(0.078)†	0.00067(0.098)†
	T^3	0.00057(0.000)*	0.00062(0.000)*	0.00064(0.000)*	0.00063(0.000)*
$FNGARCH$	ST^4	0.00051(0.000)*	0.00055(0.000)*	0.00058(0.000)*	0.0006(0.000)*
witout	GED[5]	0.00049	0.00052	0.00054(0.998)†	0.00052
predictors	SGED[6]	0.0005(0.996)†	0.00052(1.000)†	0.00053	0.00052(0.99)†
	IG^7	0.00052(0.000)*	0.00056(0.000)*	0.00057(0.000)*	0.00055(0.000)*
	GH^8	0.00052(0.000)*	0.00056(0.000)*	0.00058(0.000)*	0.00056(0.000)*
	JSU^9	0.00052(0.000)*	0.00056(0.000)*	0.00057(0.000)*	0.00056(0.000)*
	N^1	0.00061(0.066)†	0.00064(0.052)†	0.00065(0.014)*	0.00064(0.046)†
$FNGARCHX$	SN^2	0.00054(0.000)*	0.00059(0.002)*	0.00058(0.000)*	0.00056(0.000)*
($FNGARCH$	T^3	0.00054(0.000)*	0.00059(0.000)*	0.0006(0.000)*	0.00058(0.000)*
with	ST^4	0.00054(0.000)*	0.00059(0.000)*	0.0006(0.000)*	0.00058(0.000)*
Uncertainty	GED[5]	0.00054(0.000)*	0.00058(0.000)*	0.00057(0.000)*	0.00056(0.000)*
as	SGED[6]	0.00061(0.001)*	0.00067(0.004)*	0.00069(0.002)*	0.0007(0.000)*
predictor)	IG^7	0.00053(0.000)*	0.00058(0.000)*	0.00059(0.000)*	0.00057(0.000)*
	GH^8	0.00055(0.000)*	0.0006(0.000)*	0.00058(0.000)*	0.00057(0.000)*
	JSU^9	0.00053(0.000)*	0.00058(0.000)*	0.00059(0.000)*	0.00057(0.000)*
Min MSE Model (Error Distribution)		$FNGARCH$ GED^5	$FNGARCH$ GED^5	$FNGARCH$ $SGED^6$	$FNGARCH$ GED^5

[1]. Normal; [2]. Skew Normal; [3] t-student; [4]. Skew t-student;

[5]. Generalized Error Distribution; [6]. Skew-Generalized Error Distribution;

[7]. Invers Gaussian Distribution; [8]. Generalized Hyperbolic Distribution

[9]. Johnsons SU Distribution[64];

*. Accuracy of the model is significantly lower than the minimum MSE model (at $\alpha = 0.05$ level);

†. Accuracy of the model is the same as minimum MSE model (at $\alpha = 0.05$ level);

Table 6: Monthly out-of-sample volatility forecast MSEs and $KSPA$ P-Value for comparing each model with minimum MSE model. P-Values are presented in parenthesis.

Forecasting Model	Error Dist.	Monthly MSEs and Bootstrap $KSPA$ (P-Values) for different forecasting horizons (h) $h = 1$	$h = 3$	$h = 6$	$h = 12$
	N^1	$0.06269(1.000)^\dagger$	$0.06591(1.000)^\dagger$	$0.06949(1.000)^\dagger$	$0.07336(1.000)^\dagger$
	SN^2	$0.06303(0.998)^\dagger$	$0.06594(0.996)^\dagger$	$0.06915(1.000)^\dagger$	$0.0726(0.994)^\dagger$
	T^3	0.06021	0.06314	0.06628	0.06956
$FNGARCH$	ST^4	$0.06296(0.998)^\dagger$	$0.06565(1.000)^\dagger$	$0.06852(1.000)^\dagger$	$0.07149(1.000)^\dagger$
witout	GED^5	$0.06094(0.000)^*$	$0.07412(0.000)^*$	$0.07608(0.000)^*$	$0.07621(0.000)^*$
predictors	SGED^6	$0.06094(0.000)^*$	$0.07412(0.000)^*$	$0.07608(0.000)^*$	$0.07621(0.000)^*$
	IG^7	$0.06267(1.000)^\dagger$	$0.06527(1.000)^\dagger$	$0.06803(1.000)^\dagger$	$0.07091(1.000)^\dagger$
	GH^8	$0.06259(1.000)^\dagger$	$0.06518(1.000)^\dagger$	$0.06797(1.000)^\dagger$	$0.07089(1.000)^\dagger$
	JSU^9	$0.0631(1.000)^*$	$0.06576(1.000)^\dagger$	$0.0686(1.000)^\dagger$	$0.07156(1.000)^\dagger$
	N^1	$0.06425(0.020)^*$	$0.07152(0.018)^*$	$0.08019(0.020)^*$	$0.09025(0.042)^*$
$FNGARCHX$	SN^2	$0.06498(0.102)^\dagger$	$0.07145(0.112)^\dagger$	$0.07906(0.092)^\dagger$	$0.08776(0.116)^\dagger$
($FNGARCH$	T^3	$0.06137(0.408)^\dagger$	$0.06593(0.416)^\dagger$	$0.07096(0.398)^\dagger$	$0.0764(0.394)^\dagger$
with	ST^4	$0.06379(0.063)^\dagger$	$0.06885(0.988)^\dagger$	$0.07442(0.990)^\dagger$	$0.08042(0.982)^\dagger$
Uncertainty	GED^5	$0.06094(0.000)^*$	$0.07412(0.000)^*$	$0.07608(0.000)^*$	$0.07621(0.000)^*$
as	SGED^6	$0.06094(0.000)^*$	$0.07412(0.000)^*$	$0.07608(0.000)^*$	$0.07621(0.000)^*$
predictor)	IG^7	$0.06297(1.000)^\dagger$	$0.06591(1.000)^\dagger$	$0.06908(1.000)^\dagger$	$0.0724(1.000)^\dagger$
	GH^8	$0.06306(0.308)^\dagger$	$0.06747(0.302)^\dagger$	$0.07233(0.288)^\dagger$	$0.0776(0.278)^\dagger$
	JSU^9	$0.06363(0.898)^*$	$0.06676(0.892)^\dagger$	$0.07011(0.890)^\dagger$	$0.07361(0.886)^\dagger$
Min MSE Model (Error Distribution)		$FNGARCH$ T^3	$FNGARCH$ T^3	$FNGARCH$ T^3	$FNGARCH$ T^3

1. Normal; 2. Skew Normal; 3 t-student; 4. Skew t-student;

5. Generalized Error Distribution; 6. Skew-Generalized Error Distribution;

7. Invers Gaussian Distribution; 8. Generalized Hyperbolic Distribution;

9. Johnsons SU Distribution[64];

*. Accuracy of the model is significantly lower than the minimum MSE model (at $\alpha = 0.05$ level)

†. Accuracy of the model is the same as minimum MSE model (at $\alpha = 0.05$ level)

Table 7: Estimated models for daily interest rate and uncertainty forecasting

Uncertainty forecasting models	
Forecasting Model	Description
FAR	Bandwidth: 0.2884, Kernel Function: *Gaussian*, $p^{\dagger} = 1$, $d^{\dagger} = 1$
ARMA	$dx_t = 0.9767 dx_{t-1} - 0.5126\omega_{t-1} + \omega_t$
RW	$dx_t = 0.0005 + \eta_t$
Interest rate mean models	
Forecasting Model	Description
FAR	Bandwidth: 0.2884, Kernel Function: *Gaussian*, $p^{\dagger} = 1$, $d^{\dagger} = 1$
FARX	Bandwidth: 0.2884, Kernel Function: *Gaussian*, $p^{\dagger} = 1$, $d^{\dagger} = 1$
ARMA	$dy_t = 0.7009 dy_{t-1} - 0.1189 dy_{t-2} + \nu_t$
ARMAX	$dy_t = 0.7005 dy_{t-1} - 0.1189 dy_{t-2} + 0.0452 dx_t + \epsilon_t$
RW	$dy_t = -0.00087 + \zeta_t$
Interest rate volatility model with minimum out-of-sample forecasting MSE	
Forecasting Model	Description
$FNGARCH$ with GED error dis.	$\sigma_t^{0.9966} = 0.006 + 0.3822\sigma_{t-1}^{0.9966}\left\{\lvert z_{t-1} + 0.0325\rvert - 0.0789\left(z_{t-1} + 0.0.0325\right)\right\}^{1.3756} + 0.022\sigma_{t-2}^{0.9966}\left\{\lvert z_{t-2} + 1.3033\rvert - 0.2078\left(z_{t-2} + 1.3033\right)\right\}^{1.3756} + 0.017\sigma_{t-3}^{0.9966}\left\{\lvert z_{t-3} - 5.9897\rvert - 0.3994\left(z_{t-3} - 5.9897\right)\right\}^{1.3756} + 0.032\sigma_{t-4}^{0.9966}\left\{\lvert z_{t-4} + 7.2959\rvert + 0.5174\left(z_{t-4} + 7.2959\right)\right\}^{1.3756}$
$FNGARCH$ with $SGED$ error dis.*	$\sigma_t^{1.0163} = 0.0054 + 0.4001\sigma_{t-1}^{1.0163}\left\{\lvert z_{t-1} + 0.0325\rvert - 0.122\left(z_{t-1} + 0.0325\right)\right\}^{1.3619} + 0.0193\sigma_{t-2}^{1.0163}\left\{\lvert z_{t-2} + 1.3033\rvert - 0.2397\left(z_{t-2} + 1.3033\right)\right\}^{1.3619} + 0.0145\sigma_{t-3}^{1.0163}\left\{\lvert z_{t-3} - 5.9897\rvert + 0.2461\left(z_{t-3} - 5.9897\right)\right\}^{1.3619} + 0.0287\sigma_{t-4}^{1.0163}\left\{\lvert z_{t-4} + 7.2959\rvert + 0.4946\left(z_{t-4} + 7.2959\right)\right\}^{1.3619}$

.† for description of p and d see equation (1)

dx_t and dy_t are first difference of monthly uncertainty and interest rate, respectively.

ω_t, η_t, ν_t, ϵ_t, ζ_t and z_t are white noises.

.* Estimated skewness parameter of $SGED$ distribution is 1.0509.

Table 8: Estimated models for monthly interest rate and uncertainty forecasting

Forecasting Model	Description
Uncertainty forecasting models	
FAR	Bandwidth: 0.3291, Kernel Function: *Gaussian*, $p^{\dagger} = 1$, $d^{\dagger} = 1$
ARMA	$dx_t = 0.1981dx_{t-1} - 0.8045\omega_{t-1} + \omega_t$
RW	$dx_t = 0.00068 + \eta_t$
Interest rate mean models	
Forecasting Model	Description
FAR	Bandwidth: 0.3291, Kernel Function: *Gaussian*, $p^{\dagger} = 1$, $d^{\dagger} = 1$
FARX	Bandwidth: 0.3291, Kernel Function: *Gaussian*, $p^{\dagger} = 1$, $d^{\dagger} = 1$
ARMA	$dy_t = 0.449\nu_{t-1} + \nu_t$
ARMAX	$dy_t = 0.4509\epsilon_{t-1} + 0.0055dx_t + \epsilon_t$
RW	$dy_t = -0.00243 + \zeta_t$
Interest rate volatility model with minimum out-of-sample forecasting MSE	
Forecasting Model	Description
$FNGARCH$	$\sigma_t^{2.5842} = 0.0685\sigma_{t-1}^{2.5842}\left\{\lvert z_{t-1} + 0.1878\rvert + 0.6253\left(z_{t-1} + 0.1878\right)\right\}^{6.0549} + 0.7683\sigma_{t-1}^{2.5842}$

.[†] for description of p and d see equation (1)

dx_t and dy_t are first difference of monthly uncertainty and interest rate, respectively.

ω_t, η_t, ν_t, ϵ_t, ζ_t and z_t are white noises.