CHAOTIC OSCILLATIONS OF A SINGLE CAVITATING BUBBLE IMMERSED IN A MAXWELL LIQUID

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ABSTRACT

In the present work we develop a mathematical model for a bubble immersed in a Maxwell fluid insonified by an ultrasonic field. As in previous works we wanted to see the effect of the viscoelasiticy in the overall behavior of the bubble in time. With a modified Rayleigh-Plesset equation we obtain the evolution of radius in time, for different Deborah and Reynolds Numbers. Of interest the phase space graphs that show the chaotic oscillations for different values of the parameters, and how these parameters can be used as controllers of the oscillations. For a more complete analysis we perform a Perturbation Multiple Scale technique to get a frequency analysis, which can demonstrate how the parameters might modulate the resonance.

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INTRODUCTION

With increasing engineering applications that use inertial cavitation, new kinds of fluids are needed to fulfill the properties required. Thus experiments with polymeric liquids in [1], showed the necessity of rheological models that focused on the elasticity properties. In [2] the elasticity in the liquid delay the collapse of the bubble and in [3] the rheological parameters are fundamental for the oscillations and collapse of bubbles. In [4] display the frequency response curves as function of the maximum pressures and the initial radiuses of bubbles. In [5] taking into account the compressibility of the bulk liquid together the viscoelastic properties of the liquid, new regimes of oscillations are found for low Reynolds numbers. The works [6, 7] show two viscoelastic models for the oscillations of a bubble with the Maxwell and Jeffries liquids, they provide a perturbation analysis obtaining the frequency response curves. First in [8] for an Oldroyd viscoelastic model and then in [9] for an Upper Convective Maxwell model, chaotic oscillations are present for Deborah numbers higher than 4, regardless of the damping factors. A complete account of all the previous works and applications can be found in [7].

With this background we propose a modified Rayleigh-Plesset equation for a single bubble immersed in a simple Maxwell fluid. This model can be set for different Deborah and Reynolds Numbers to first obtain the evolution of radius and its chaotic oscillations numerically and second to find the frequency response curves using the Multiple Scale Analysis.

NOMENCLATURE

a	[-]	Non dimensional amplitude
a^*	[-]	Real part in perturbation analysis
A	[-]	Constant for the solution proposed
De	[-]	Deborah Number
f	[-]	Non dimensional frequency
p_l	[N/m]	Liquid pressure
p_A	[N/m]	Driving pressure
$p\infty$	[N/m]	Ambient pressure
p_g	[N/m]	Gas pressure inside the bubble
p_{g0}	[N/m]	Initial gas pressure inside the bubble
P_A	[-]	Non dimensional driving pressure
R	[m]	Bubble radius
r	[m]	Radial coordinate
Re	[-]	Reynolds Number
$\bar{R}$	[-]	Non dimensional bubble radius
s	[-]	Non dimensional spatial integral over radial stress
S	[Pa/m^2]	Spatial integral over radial stress
t	[s]	time
T_n	[s]	Characteristic time
u	[-]	Time scales
$u´$	[-]	Perturbation variable
W_e	[m]	Weber number
x	[-]	Relative amplitude in perturbation analysis

Special characters

α	[-]	Non dimensional thermal penetration length
β	[-]	Shift in time
$\dot{\gamma}$		
γ	[-]	Adiabatic index
$\bar{\gamma}$	[-]	Detuned frequency
σ	[N/m]	Surface tension
$\bar{\sigma}$	[-]	Detuning parameter
τ	[-]	Non dimensional time
τ_{rr}	[N/m^2]	Radial stress component
$\tau_{\theta\theta}$	[N/m2]	Angular stress component
λ	[-]	Relaxation time
ρ	[kg/m^3]	Liquid density
χ	[-]	Non dimensional thermal ratio
ξ	[-]	Perturbation variable
Ω	[s^{-1}]	Driving Frequency
ω_0	[-]	Non dimensional natural frequency
η	[Pas]	Liquid viscosity
η_A	[-]	Non dimensional driving pressure

Subscripts

0	Initial
g	Gas
go	Gas initial

THEORETICAL MODEL

Consider a single spherical bubble immersed in an infinite non-Newtonian liquid described as a Maxwell rheological model. In this model the surface stresses in the bubble are substantially modified, therefore the Rayleigh-Plesset equation is modified. The detailed derivation of this model can be traced in [6], [8], [9] and [11].

$$\rho\left[R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2\right]=p_g-p_\infty\left(1-\eta_A\cos\Omega t\right)-\frac{2\sigma}{R}+S \quad (1)$$

In the above equation, p_g represents the pressure of the gas inside the bubble, the driving pressure p_A can be defined as a factor of the ambient pressure $\eta_A p_\infty$, the equilibrium radius of the bubble R_0 is determined by $p_{g0}-p_\infty=2\sigma/R_0$ and the last term S on the right-hand side represents the integral contribution of the non-Newtonian stress defined by

$$S=-2\int_R^\infty\frac{\tau_{rr}-\tau_{\theta\theta}}{r}dr=-3\int_R^\infty\frac{\tau_{rr}}{r}dr \quad (2)$$

For spherical symmetry all the components outside the main diagonal are zero and the deviatoric condition states $\tau_{rr}+\tau_{\theta\theta}+\tau_{\phi\phi}=0$ and also by symmetry $\tau_{\phi\phi}=\tau_{\theta\theta}=-\tau_{rr}/2$ these stresses must satisfy the Maxwell relations

$$\tau_{rr}+\lambda\frac{\partial\tau_{rr}}{\partial t}=-\eta\dot{\gamma}_{rr}=4\eta\frac{R^2}{r^3}\frac{dR}{dt}$$

$$\tau_{\theta\theta}+\lambda\frac{\partial\tau_{\theta\theta}}{\partial t}=-\eta\dot{\gamma}_{\theta\theta}=-2\eta\frac{R^2}{r^3}\frac{dR}{dt}$$

$$\tau_{\phi\phi}+\lambda\frac{\partial\tau_{\phi\phi}}{\partial t}=-\eta\dot{\gamma}_{\phi\phi}=-2\eta\frac{R^2}{r^3}\frac{dR}{dt} \quad (3)$$

These last three equations represent an integro-differential problem, following the articles [6-8] and a methodology well described in [9] for an Upper convective Maxwell, the present article can be a particular case of a more general such as that described in [9]. Under the aid of the Leibniz Rule described in [11], the integral function (2) of the stress taking into account (3) and the respective considerations in the stresses can be written as

$$\frac{dS}{dt}=-4\frac{\eta}{\lambda R}\frac{dR}{dt}-\left(\frac{1}{\lambda}+\frac{3}{R}\frac{dR}{dt}\right)S \quad (4)$$

Considering the model of Prosperetti [12] for the pressure of gas inside the bubble, in which an almost a quasi-isothermal compression is assumed during the collapse

$$p_g=p_{g0}\left(\frac{R_0}{R}\right)^3\left(1-\frac{\gamma-1}{5\gamma}\frac{R_0^3}{\alpha R^2}\frac{dR}{dt}\right) \quad (5)$$

where γ represents the adiabatic index. In addition, eqs. (1) and (4) must satisfy the following initial conditions,

$$R(0)=R_0 \quad , \quad \frac{dR}{dt}=0 \quad \text{and} \quad S(0)=0 \quad (6)$$

The last set of equations can be written in dimensionless form, introducing the characteristic scales respect to time and radius we have

$$\tau=\frac{t}{\sqrt{\frac{R_0^2\rho}{p_{g0}}}} \quad , \quad \bar{R}=\frac{R}{R_0} \quad \text{and} \quad s=\frac{S}{\eta_p}\sqrt{\frac{R_0^2\rho}{p_{g0}}} \quad (7)$$

therefore Eq. (1) takes non-dimensional form

$$\bar{R}\frac{d^2\bar{R}}{d\tau^2}+\frac{3}{2}\left(\frac{d\bar{R}}{d\tau}\right)^2=F(\tau)-\left(1-We\right)\left(1-\eta_A\cos\Omega\tau\right)-\frac{We}{\bar{R}}+\frac{s}{\mathrm{Re}} \quad (8)$$

while Eq. (4) is given by,

$$\frac{ds}{d\tau}=-\frac{4}{\bar{R}De}\frac{d\bar{R}}{d\tau}-\left(\frac{1}{De}+\frac{3}{\bar{R}}\frac{d\bar{R}}{d\tau}\right)s \quad . \quad (9)$$

With the following set of non-dimensional numbers:

$$\eta_A=\frac{p_A}{p_\infty}, We=\frac{\sigma}{p_{g0}R_0}, Re=\frac{\sqrt{\rho p_{go}}R_0}{\eta_p}, De=\frac{\lambda}{\sqrt{\frac{R_0^2\rho}{p_{g0}}}} . \quad (10)$$

First the leading parameter for the Maxwell fluid should be the Deborah Number, which typically represents the ratio of the relaxation time to the characteristic time both govern the phenomenon, second the Reynolds Number defined using dynamical parameters. The Weber number taken as a damping parameter for the oscillations and η_A that represents the forcing amplitude times the equilibrium pressure. Finally the compression model $F(\tau)$ that represents the gas inside the bubble

$$F(\tau)=\left(1-\frac{\chi}{\bar{R}^2}\frac{d\bar{R}}{d\tau}\right)/\bar{R}^3 \quad (11)$$

While $\chi=(1-\gamma)R_0/\gamma\alpha$ is another parameter that consider the quasi isothermal compression.

The initial conditions in non-dimensional form can be written as,

$$\bar{R}(0)=1 \quad , \quad \left.\frac{d\bar{R}}{d\tau}\right|_{\tau=0}=0 \quad \text{and} \quad s(0)=0 \quad . \quad (12)$$

Perturbation Method

In previous works [6] and [13-16] the linearization of eqs. (8)-(9) gives us an expression to obtain the natural frequency and the role of many damping parameters as a function of frequency. In [6], [14], [17] and [18] the frequency resonance curves for the involved parameters, using multiple scale analysis was obtained.

In this work following the references [20] and [21] we develop an analysis to obtain the frequency response involving all of the damping and driving parameters. For small oscillations around of an equilibrium radius we have

$$\bar{R}=1+\xi x \tag{13}$$

Expanding up to third order and substituting in (14), (15) and (17), we obtain the following equations

$$\begin{aligned}&\ddot{x}+\frac{3}{2}\dot{x}^2\xi\left(1-\xi x+(\xi x)^2-(\xi x)^3\right)=\\&=\frac{1}{\xi}\left[1-4\xi x+10(\xi x)^2-20(\xi x)^3-\chi\xi\left(1-6\xi x+21(\xi x)^2-56(\xi x)^3\right)\right]\\&-\frac{We}{\xi}\left(1-2\xi x+3(\xi x)^2-4(\xi x)^3\right)-\frac{s}{\mathrm{Re}\,\xi}\left(1-\xi x+(\xi x)^2-(\xi x)^3\right)\\&-\frac{(1-We)}{\xi}\left(1-\eta_A\cos(\tau)\right)\left(1-\xi x+(\xi x)^2-(\xi x)^3\right)\end{aligned} \tag{14}$$

$$\begin{aligned}&\frac{ds}{d\tau}=-\frac{4\dot{x}}{De}\left(1-\xi x+(\xi x)^2-(\xi x)^3\right)-\\&\left(\frac{1}{De}+3\dot{x}\xi\right)\left(1-\xi x+(\xi x)^2-(\xi x)^3\right)s\end{aligned} \tag{15}$$

For equations with cubic nonlinearities [19] the forcing, damping and the nonlinearity must appear at the same order, therefore we define in terms of the small parameter ξ

$$(1-We)\eta_A=\xi^3 f\ ;\ \frac{1}{\mathrm{Re}}=\xi^2\ ;\ x=u$$

To implement the method of multiple scales we must define

$$\begin{aligned}&u=u_0\left(T_0,T_1,T_2\right)+\xi u_1\left(T_0,T_1,T_2\right)+\xi^2u_2^2\left(T_0,T_1,T_2\right)\\&s=s_0\left(T_0,T_1,T_2\right)+\xi s_1\left(T_0,T_1,T_2\right)+\xi^2 s_2^2\left(T_0,T_1,T_2\right)\\&T_n=\xi^n\tau;\ n=0,1,...\end{aligned} \tag{16}$$

The same takes for the derivatives with more detail in [6], taking these expansions to eqs. (14) and (15) gives a system of equations for different powers of ξ we have

ξ^0

$$D_0^2u_0+\omega_0^2u_0=0 \tag{17}$$

$$S_0+De\left(D_0S_0\right)=0 \tag{18}$$

Where the linear natural frequency is

$$\omega^2{}_0=3-We \tag{19}$$

In eq. (18) we need to have $S_0=0$ because the MSA requires the damping to be of higher order.

The following ordering equations can be expressed as

ξ^1

$$D_0^2u_1+\omega_0^2u_1=-2D_0D_1u_0-\frac{3}{2}\left(D_0u_0\right)^2+u_0^2\left[9-2We\right]+6u_0\chi D_0u_0 \tag{20}$$

$$S_1+De\left(D_0S_1\right)=-4D_0S_0 \tag{21}$$

For the second order equation we have

ξ^2

$$\begin{aligned}&D_0^2u_2+\omega_0^2u_2=-2D_0D_1u_1-\left(D_1^2+2D_0D_2\right)u_0+\\&\quad+3\left(D_0u_0\right)\left(D_0u_1\right)+3\left(D_0u_0\right)\left(D_1u_0\right)-\frac{3}{2}\left(D_0u_0\right)^2u_0+\\&+\left[\begin{aligned}&u_0\left(-21u_0{}^2+22u_1\right)+11u_1{}^2-\\&-21\chi u_0{}^2\left(D_0u_0\right)+6\chi\left(u_0\left(D_0u_1+D_1u_1\right)+u_1D_0u_0\right)\end{aligned}\right]+\\&\quad+We\left(5u_0{}^3-8u_0u_1-4u_1{}^2\right)+f\cos\left(\omega_0T_0+\sigma T_2\right)+\frac{S_1}{\mathrm{Re}}\end{aligned} \tag{22}$$

We define a complex solution and the dimensionless detuning frequency

$$\begin{aligned}&u_0=A\left(T_1,T_2\right)e^{i\omega_0T_0}+c.c.\\&\Omega=\omega_0+\xi^2\sigma\end{aligned} \tag{23}$$

Where the last term represents the complex conjugate, A represents an unknown complex function that should be evaluated by substituting in eq. (22)

$$\begin{aligned}&D_0^2u_1+\omega_0^2u_1=-2D_1Ai\omega_0e^{i\omega_0T_0}+e^{2i\omega_0T_0}A^2\left(\frac{3\omega_0^2}{2}+(9-We)\right)\\&+A\tilde{A}\left(-3\omega_0^2+(9-We)\right)+c.c.\end{aligned} \tag{24}$$

$$S_1+De\left(D_0S_1\right)=-4D_0u_0 \tag{25}$$

Here the symbol “~” denotes the complex conjugate and the *c.c.* represents all the complex conjugate terms generated by the solution and that might give a redundant solution. As in reference [19] the secular terms are the solution of the homogeneous equation, in this case the first term on the right hand side of eq. (24) is equal to zero, and implies that

$$A=A\left(T_2\right) \tag{26}$$

In this scheme we propose the solution of (24) as

$$u_1=c_1A^2e^{2i\omega_0T_0}+c_2A\tilde{A}+c.c. \tag{27}$$

Where

$$c_1=\frac{1}{3}\left(\frac{3}{2}+\frac{9-2We}{\omega_0^2}+\frac{6}{\omega_0}\chi i\right)\quad c_2=\frac{9-2We}{\omega_0^2}-\frac{3}{2} \tag{28}$$

For (24) the solution might be

$$S_1=ACe^{i\omega_0T_0}+ED+c.c. \tag{29}$$

ED stands for the exponentially decaying terms that depend on the initial conditions and does not contribute to the steady-state state analysis. The constant C can be written in its real and imaginary components as

$$C = c_R + ic_i = -\frac{4\omega_0^2 De}{1+\omega_0^2 De^2} + \frac{4i\omega_0}{1+\omega_0^2 De^2} \qquad (30)$$

As indicated in [6] the real part is related to the viscous damping and the imaginary represent the elastic component of the liquid. For Newtonian fluids $De = 0$, there is only one viscous loss term proportional to its natural frequency.

Using (23), (27), (29) and (30) we can solve the second order equation (22)

$$D_0^2 u_2 + \omega_0^2 u_2 = \left[-2i\omega_0 D_2 A + \frac{c_R + ic_i}{\mathrm{Re}}\right] e^{i\omega_0 T_0} + A^2 \tilde{A} e^{i\omega_0 T_0} \begin{Bmatrix} \left(\frac{21}{2} + 22(c_1 + c_2)\right)\omega_0^2 - 63 + \\ \chi i \omega_0^2 (21 + 18c_1) + \\ We\left(15 + 8(c_1 + c_2)\right) \end{Bmatrix} + \frac{1}{2} f \exp(i\sigma T_2) e^{i\omega_0 T_0} + nst + c.c. \qquad (31)$$

Disregarding the non-secular terms *nst* and the *c.c.* ones and equating the secular terms to zero one obtains the proposed solution as:

$$A = \frac{a(T_2)}{2} e^{i\beta(T_2)} \qquad (32)$$

And a set of equations separating the real and imaginary part as

$$a\frac{d\beta}{dT_2}\omega_0 + \frac{a^3}{8}\begin{Bmatrix} \begin{pmatrix} \frac{21}{2}\omega_0^2 + 15We - 63 + \\ \left(22\omega_0^2 + 8We\right)\left(\frac{36-5We}{3\omega_0^2} - 1\right) \end{pmatrix} - \frac{6(\chi)^2}{\omega_0} + \\ \frac{We}{\omega_0}\left(15 + 8\left(\frac{36-5We}{3\omega_0^2} - 1\right)\right) \end{Bmatrix} + f\cos(\bar{\sigma}T_2) - \frac{2aA\omega_0 De}{\left(1+\omega_0^2 De^2\right)\mathrm{Re}} = 0 \qquad (33)$$

$$-\frac{da}{dT_2}\omega_0 + f\sin(\bar{\sigma}T_2) + \frac{6\chi}{\omega_0}\left(\frac{7}{2}\omega_0^2 + 22\omega_0^2 + 18 + 4We\right) - \frac{2a\omega_0}{\left(1+\omega_0^2 De^2\right)\mathrm{Re}} = 0 \qquad (34)$$

With

$$\bar{\gamma} = \bar{\sigma}T_2 - \beta \qquad (35)$$

Finally for the solution for the steady-state response is:

$$a\bar{\sigma} + \frac{a^3}{8}\begin{Bmatrix} \left(\frac{21}{2} + 22\left(\frac{36-5We}{3\omega_0^2} - 1\right)\right)\omega_0 - \frac{63}{\omega_0} + \\ \frac{We}{\omega_0}\left(15 + 8\left(\frac{36-5We}{3\omega_0^2} - 1\right)\right) \end{Bmatrix} + \frac{1}{2\omega_0} f\cos(\bar{\gamma}) + \frac{2aA\omega_0 De}{\left(1+\omega_0^2 De^2\right)\mathrm{Re}} - \frac{108(\chi)^2}{\omega_0} \qquad (36)$$

$$\frac{1}{2\omega_0} f\sin(\bar{\gamma}) + \chi\left(30 + \frac{27-6We}{3\omega_0^2}\right) - \frac{2a}{\left(1+\omega_0^2 De^2\right)\mathrm{Re}} = 0 \qquad (37)$$

Taking squares and adding (36) and (37)

$$\frac{f^2}{4\omega_0^2} = \left(a\bar{\sigma} + \frac{a^3}{8}\begin{Bmatrix} \frac{21\omega_0}{2} + \frac{15We-63}{\omega_0} + \\ \left(22\omega_0 + \frac{8We}{\omega_0}\right)\left(\frac{36-5We}{3\omega_0^2} - 1\right) - \\ \frac{6(\chi)^2}{\omega_0} \end{Bmatrix} - \frac{2a\omega_0 De}{\mathrm{Re}\left(1+De^2\omega_0^2\right)} \right)^2 + \left[\frac{6\chi}{\omega^2{}_0}\left(\frac{7\omega^2{}_0}{2} + (22\omega_0 - 18) + 4We\right) - \frac{2a}{\mathrm{Re}\left(1+De^2\omega_0^2\right)}\right]^2 \qquad (38)$$

RESULTS AND DISCUSSION

For the system (8), (9), (11) and (12) we propose a Runge Kutta fourth order with typical values taken from references [20] and [21] where the ultrasonic bandwidth involves medical applications, therefore the values of the parameters are around the same order of magnitude.

In Figs.1 and 2 the radius versus time for different Weber and Deborah numbers. The Weber number is a clear damping for the amplitude while the Deborah slightly promotes bigger radiuses. In Fig. 3 all parameters fixated except the amplitude, we thought Deborah might be a parameter that showed the nonlinearity and chaos beyond the value of 4. But with these examples it is shown that the increase in the driving amplitude leads to chaos as shown in Fig. 4, and also the elastic part of the Maxwell model dominates increasing dramatically the amplitude of oscillation that was not seen in the previous cases.

For the Reynolds number its influence is marginal we didn´t included the charts here.

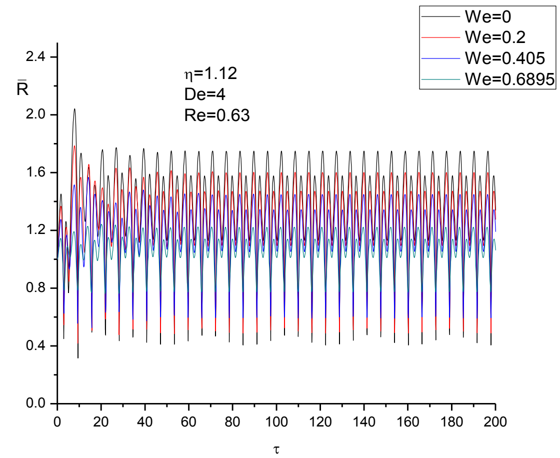

Figure 1 Radius versus time for different Weber numbers and De=4, all the rest fixated.

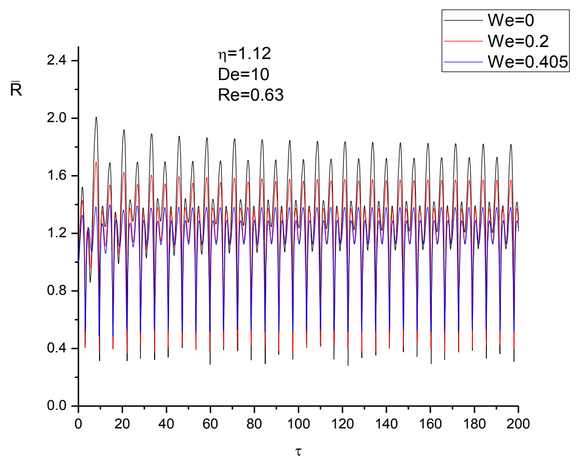

Figure 2 Radius versus time for different Weber numbers and De=10, all the rest fixated.

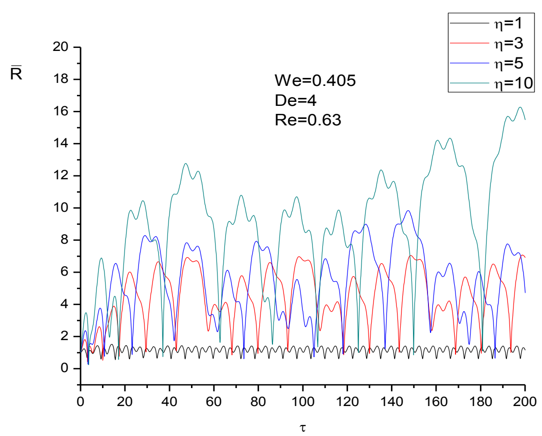

Figure 3 Radius versus time for different driving amplitudes and De=4, all the rest fixed.

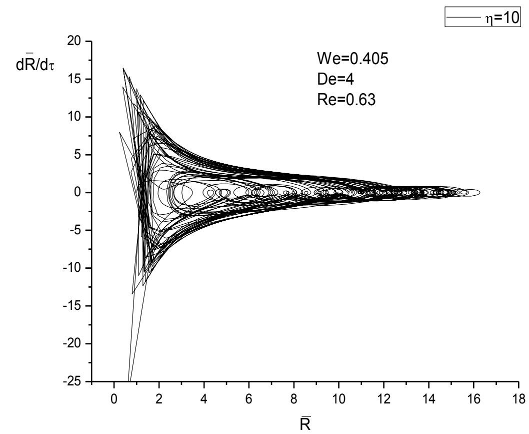

Figure 4 Phase space for fig. 3 largest forcing amplitude, chaotic oscillation.

Fig. 5 shows equation (38) for different Deborah numbers and $f = \mathrm{Re} = 1$, this frequency response curves show that the damping effects do not allow the system to resonate but shows as expected that for the increasing Deborah the amplitude also increases, but also show a clear shift of phase. This shift of phase is more evident in the backbone curves in fig. 5, the shift of phase occurs since the very start.

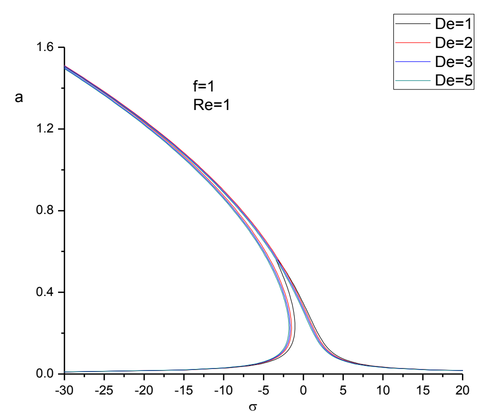

Figure 5 Frequency response curves for different Deborah numbers.

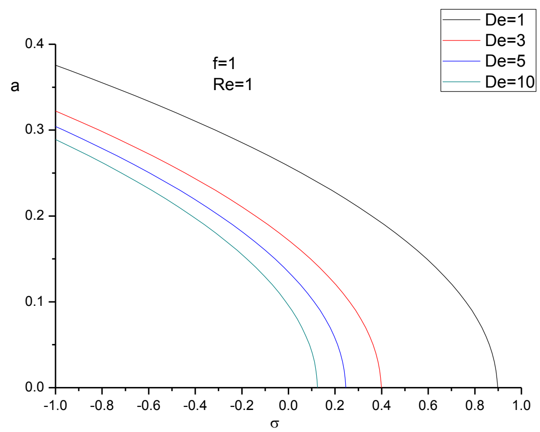

Figure 1 Back bone curves for fig. 5 showing different Deborah numbers.

CONCLUSION

For the numerical results of the differential equations system we can conclude that the Weber number can be a damping parameter that modulates the oscillation. The influence of an increasing Deborah numbers might result in larger radiuses because the cumulative effect of the viscoelastic nonlinear oscillations. In fig. 3 for small driving amplitude the oscillations keep monotonous and around moderate values, with all other values different from the unity the chaotic behaviour is evident and larges radiuses can always be expected with increasing time. This is a combination of different effects that need more study for each and every case; the nonlinear oscillations can be modulated with the damping parameters. This is shown in the frequency response curves that detune the oscillation, and show how these damping parameters can modulate the amplitude and the phase.

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