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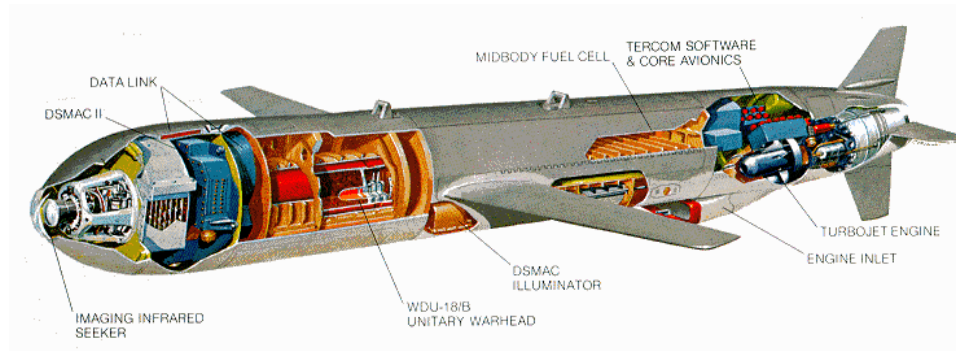
Optimizing Tomahawk Strikes

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The Tomahawk land attack missile (TLAM) is the Navy's weapon of choice for striking shore targets from the sea. A TLAM launched from a surface combatant or a submarine is a reliable, unmanned, long-range, accurate weapon with sufficient payload to threaten almost any shore target. The Operations Research Department at the Naval Postgraduate School has developed optimization-based decision-support tools to optimize TLAM strikes from single firing units or entire battle groups. The idea is to execute each strike efficiently while retaining residual firepower, and while considering a number of other essential details. By applying mathematical modeling, the result is the ability to plan fleet and theater-wide strikes in seconds.

When the U.S. National Command Authority authorizes a Tomahawk land attack missile (TLAM) strike, its aim points pass down through the chain of command via a regional Commander-in-Chief and thence to the Battle Group Tomahawk Strike Coordinator (TSC). The TSC predesignates these aim points to firing platforms. Predesignation considers geographic proximity of candidate platforms to aim points, the inventory and location of TLAMs aboard each platform, engineering limitations on the way in which and the rate at which a platform can prepare and fire particular missiles, flight route coordination among TLAMs, and other tactical concerns. The TSC must also take care to leave his combat units with maximal residual firepower after the strike, individually and as a battle group. Once a firing platform receives from the TSC its designated aim points and told what TLAMs to use, the actual selection of which particular TLAMs to fire may be adjusted by its Combat Information Center based on the last-minute status of the platform and its individual missiles.

Predesignation is a complicated decision problem, and an important one. TLAMs are expensive, about \$600,000 each. They come equipped with several variations in guidance, propulsion, and payload, and we anticipate new variations with even more options. Surface combatants deploy with a variety of TLAMs preloaded in canisters. A missile salvo can be prepared and fired only if the designated TLAMs are located in canister cells that do not interfere with one another during preparation for launch.



Tomahawk (BGM-109C) land attack cruise missile. Its turbojet propulsion can accurately deliver a 450 kg warhead more than 1,300 km. Guidance options include inertial, terrain contour match (TERCOM), digital scene matching area correlation (DSMAC), and global positioning satellite (GPS).

Predesignation considers more than just a particular salvo or mission at hand. We must keep track of where candidate platforms are, what they are doing, and when they are scheduled to leave the theater of operations. We want to preserve as much residual firepower—defined here as the remaining salvo size of each missile type—as possible on the combat platforms that will be remaining with the battle group in theater, avoid redesignations that interfere with other duties of the firing platforms, and expend TLAMs from platforms that will soon be departing the theater.

The TSC currently redesignates by hand. There are no Tactical Decision Aids, Naval Warfare Publications, or Tactical Memoranda to guide this complex set of decisions.

Optimal decision making is the central theme of operations research. And, the Operations Research (OR) Department at the Naval Postgraduate School is always looking for important, fleet-relevant problems for which it can develop and apply appropriate solution technologies, and integrate these into officer-student education. OR has been supported by Naval Surface Warfare Center Dahlgren Division and the Office of Naval Research to develop an automated decision-support tool to optimally redesignate TLAM strikes.

An automated tool must consider all the details governing the preparation and firing of every missile in every launcher on every platform. A significant part of the research effort has been devoted to capturing all the engineering details and merging these with Naval tactics.

Kuykendall [1998] breaks ground with the first comprehensive operations analysis of the TLAM redesignation problem. He states the problem and how the Navy addresses it manually, assesses the capabilities of TLAM missile variants, illustrates the physical

layout of TLAM launch cells, and expresses their engineering peculiarities. Kuykendall then proposes an optimization model to predesignate aim points to TLAMs to maximize residual firepower for either a battle group or a single platform.

In the best spirit of operations research, Kuykendall conjures examples to illustrate why this problem is non-trivial, and why using only common sense and thumb rules can lead to bad tactics. Consider the following trivial attack plan requiring one missile of type “A” and one of type “B.” The firing platform has its missiles stowed in rows. For engineering reasons, a missile salvo can include at most one missile from each row:

B A C B
C B A A

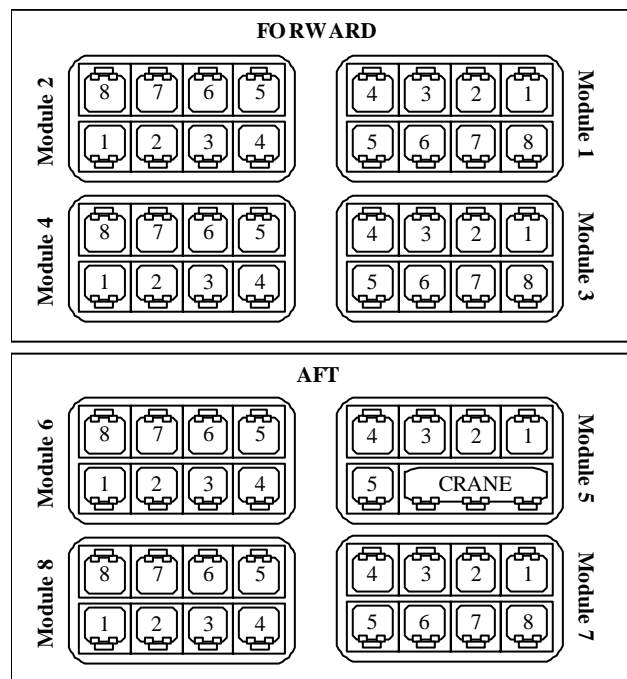
If we shoot “A” from the first row, and “B” from the second, the attack mission is satisfied.

However, we leave ourselves with insufficient residual firepower if the next mission calls for two “A's” or two “B's.” On the other hand, had we chosen to shoot:

B A C B
C B A A

then we would have residual capability to fire any requested combination of two missiles in the next mission. You may have quickly seen the better salvo for this trivial example, but picture yourself having to solve the same problem with an attack plan calling for 100 predesignations to be chosen from 250 missiles on seven platforms, instead of these two predesignations from eight missiles on just one platform.

This trivial example is important for two reasons. First, Kuykendall shows the subtlety of the problem with elegant simplicity. Second, he points to what may have been a flaw in an early Navy attempt at automating missile predesignation. The first solution in the example typifies the inferiority of applying a simple rule of thumb, such as: Use the first “A” you find; then use the first “B.”



A Mark 41 Vertical Launching System. The 61 missiles are identified by *module* (F1,F2,..., F8) and *cell number* (1,2,...,8). Each row of four cells is called a *half-module*. Engineering restrictions prevent simultaneous preparation of two missiles from the same half-module. A Ticonderoga Class cruiser has two full-size launchers, a Spruance Class destroyer has one, and an Arleigh Burke Class destroyer has a full-size launcher and a half-size launcher (modules 5-8 of this diagram).

Kirk [1999] isolates more of the nuances guiding predesignation. The most daunting challenge we face with automated decision support is inducing an objective assessment of the quality of a proposed decision. Kirk develops multiple, hierarchical objectives to assess effective TSC attack planning. While these objectives are not official doctrine, they have been reviewed by knowledgeable authorities and found “acceptable.” In descending priority, the TSC should:

- maximize the number of targets designated to missiles, then
- minimize the use of firing platforms performing other duties in other areas, then
- maximize the use of missiles from firing platforms that will soon leave a theater of operations, then
- minimize the deviation from the mean on each firing platform of the residual missile inventories among firing platforms that remain in a theater of operations, then
- if desired, maximize the number of firing platforms with predesignations, then
- minimize the use of “over-qualified” missiles for a predesignation, and finally

- maximize residual firing capability.

Kirk develops a number of ambitious, extremely detailed mathematical optimization models and tests them with scenarios provided by the research sponsor. His solutions and analysis establish that the essence of Tomahawk strike planning has been captured, and that the strike plans can be optimized. Kirk's objectives can be reordered, redefined, prioritized, or softened with the use of aspiration levels that seek most of the optimal value of each function, but not all of it, thus providing more flexibility for lower-level considerations.

Hodge [1999] develops a prioritized target list that he uses to mimic the optimal decisions of Kirk's most comprehensive model with a fast heuristic algorithm that selects firing platforms, and then predesignated targets from the list in a single pass. When the target ordering priorities are well stated for the scenario at hand, this one-pass heuristic suggests good strike plans very quickly. To prove this, Hodge uses Kirk's much slower, but optimal, results for qualitative assessment. Hodge's heuristic takes less than a minute to deliver strike plans good enough for operational use.



Flying her battle flag, USS Shiloh (CG-67) fires a Tomahawk land attack missile in Operation Desert Strike, 3 September 1996.

Arnold [2000] improves Hodge's strike-plan heuristic, and adds assurance that a recommended strike plan cannot be improved by any simple adjustment. This is key to retaining the hard-earned confidence of planners who might otherwise lose faith if some

minor blemish were to be discovered in a near-optimal heuristic solution. Arnold also accommodates:

- submarine launch platforms with their unique capabilities,
- the restriction of individual aim-point assignments to a single firing platform to minimize collateral damage,
- the assignment of special types of redundant predesignations, and
- manual prioritization of the targets.

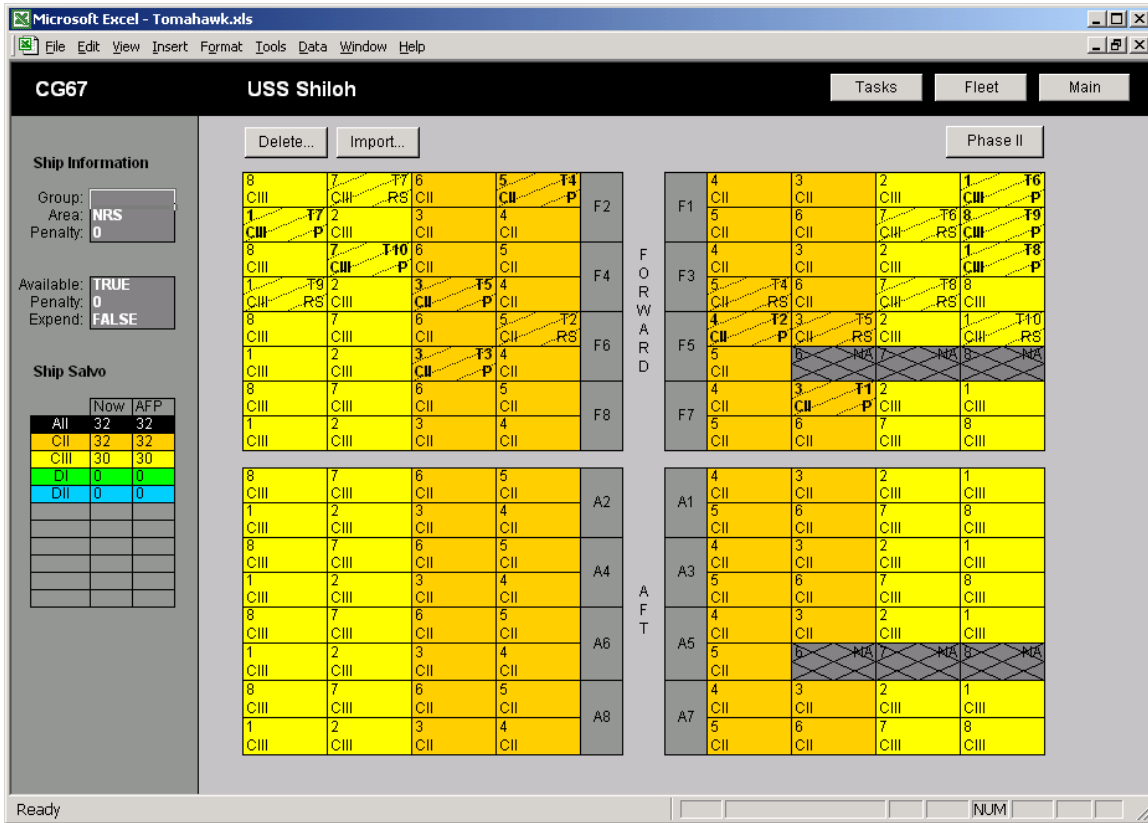
For a scenario with 104 targets and seven firing platforms, the Arnold heuristic delivers a complete strike plan in less than ten seconds. Equally important, we can forecast the necessary computation time for this heuristic, which is key for real-time decision support.

Kubu [2001] diagnoses conflicts in mission plans that inhibit a complete predesignation and prescribes suggested modifications (e.g., shifting the launch time of a missile). These prescriptions show the TSC how to make minor changes to the mission plan so that more of the desired targets can be struck, especially with heavy strikes involving scores of missiles and targets. The modifications are proposed if and when an initial application of the heuristic deems certain missile-to-target predesignations infeasible based on missile inventories, the number, type and location of missiles required for the strike, and target attributes (e.g., the launch time for a missile to ensure on-time arrival at the target).

Wingart [2001] compares our automated strike planner with actual fleet exercise decisions. The idea is to reconcile our results with those of experienced decision makers to ensure that the automated tool captures subtle human reasoning that may have been overlooked. He has developed an interactive mode through which the TSC can manually control all or part of the heuristic, while continually receiving guidance from the heuristic on the influence his changes have on overall strike efficiency. There will be extenuating circumstances that cannot be anticipated and incorporated in advance by the automated heuristic, and we want an expert human in the loop.

We have developed a graphical user interface to maintain and display the state of every combatant, every launcher, and every missile. The interface has drill-down and fly-over features to permit arbitrary navigation among all the data elements and displays: This is important, because the TSC needs a global view of his battle group. It is vital to try

to display not only which TLAM to fire from which platform and position, but visible reckoning of why. We also display aggregate statistics that measure the state of battle group firepower as a consequence of any action taken. This graphical user interface is the vehicle Wingart's manual method will use to accept guidance directly from the TSC.



User Interface View of USS Shilo (CG-67) Tomahawk Status. This screen highlights with cross hatching the predesignation of a salvo of Tomahawk missiles. Primary assignments are displayed in bold. Redundant designations—used in case a primary fails—are shown in standard font. Codes shown in each cell indicate the assigned task number (T1,T2,...), primary or redundant (P, RS) and type of tomahawk missile (CII, CIII). The type of missile is also indicated by the color of the cell. The left-hand side of the screen shows summary information about the battle group and the Shilo's missile load before and after the designated missiles are fired. Each ship has such a display; companion displays (not shown) detail tasking and other details. One button on this dashboard optimizes an entire fleet-wide Tomahawk strike.

Research on optimization of TLAM strikes continues with collaboration among our faculty, a succession of surface naval officer graduate students, and fleet experts at Dahlgren. The faculty provide guidance and continuity, the students are highly motivated by their anticipation of actually using their tools when they get back to sea, and Dahlgren is testing, documenting, and issuing a product to our fleet.

A REPRESENTATIVE PREDESIGNATION FORMULATION

The following model, adapted from Kirk [1999], represents predesignation of a set of Tomahawk missions to be fired from Vertical Launch Systems aboard surface combatants in a dispersed battle group. Variations include launching missiles from submarine torpedo tubes and/or launch canisters, and newer Tomahawk types. Optimization models like this are solved with mixed integer programming to evaluate the efficacy of fast heuristics applied to the same problem.

Indices:

w	Tomahawk type (e.g., block and variant: CIII, DII, ...).
f	firing platform (e.g., DDG-57, CG-73, DD-997).
h	half-module, dependent on type of ship (e.g., h1-h24 for DDG-57).
c	cell, each half-module contains four cells (c1-c4) or (c5-c8).
t	target task number (e.g., t1, t2, ...); each target task corresponds to a mission. Each mission may include multiple target tasks.
p	task part; each task may consist of a primary part, a ready-spare part, and a back-up part indexed as follows: 1 = primary, 2 = ready-spare, 3 = back-up.
a	launch area (e.g., N. RED SEA, EAST MED).
i	instance of a set of conflicting tasks (e.g., {t1, t2, t3})

Index Set:

con_i	set of tasks that is to be executed within an epoch such that all tasks in the set conflict, i.e., at most one task per set can be assigned to any one half-module.
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Data:

$inArea_{ta}$	1 if task t is in launch area a , 0 otherwise
$geoFe_{ft}$	1 if platform f can attain firing position for task t , 0 otherwise
$isReq_{tp}$	1 if task t , part p requires a missile, 0 otherwise
$expend_f$	1 if platform f needs to expend missiles, 0 otherwise
$priSpread_a$	1 if primary missiles should be spread across as many platforms in launch area a as possible, 0 otherwise

$buSpread_a$	1 if back-up missiles should be spread across as many platforms in launch area a as possible, 0 otherwise
$load_{wfhc}$	1 if a weapon of type w is loaded in location (f,h,c) , 0 otherwise
$emPen_f$	1 if firing platform f should continue its current mission instead of being used for TLAM tasking, 0 otherwise
$mCubePri_p$	priority weight of task part p
$levelPlat$	total number of firing platforms in the operational theater minus the number of “expend platforms,” i.e., the number of platforms for each of which the number of missiles remaining after a strike should be equalized
$mCubePos_{wt}$	priority of Tomahawk type w for task t , or 0 if this weapon type is not applicable for this task
$value_w$	relative value of Tomahawk type w (e.g., a CIII is more valuable than a CII, which in turn is more valuable than DIII)
$avail_{fhctp}$	equals 1 if firing platform f , half-module h , cell c contains a TLAM available for assignment to meet part p of task t , if $geoFe_{ft}=1$, and if $isReq_{tp}=1$, 0 otherwise

Decision Variables:

X_{fhctp}	defined only for $avail_{fhctp}=1$, this variable equals 1 if the weapon in location (f,h,c) is selected for task t , part p , 0 otherwise
$UNABLE_{tp}$	1 if missile cannot be allocated for task t , part p , 0 otherwise

$SHIPREQ_{fa}$	1 if platform f is required to be in area a for firing, 0 otherwise
$SALVO_{wfh}$	1 if one or more weapons of type w remain on platform f in half-module h after firing all primary missiles, 0 otherwise
$NUMPLATPR_f$	1 if platform f is assigned one or more primary task parts, 0 otherwise
$NUMPLATBU_f$	1 if platform f is assigned one or more back-up task parts, 0 otherwise
$NUMEXPEND$	number of missiles pre-designated from platforms designated to receive as much tasking as possible
$NUMREMAIN_f$	number of missiles on platform f that are not selected for a primary task part from non-expend platforms
$AVGREMAIN$	mean number of missiles of all types on board “non-expend” firing platforms in theater, after all missiles have been selected for the given strike
$DIFFMEAN_f$	absolute difference between the residual number of missiles on platform f and $AVGREMAIN$
$MCUBESUM_p$	sum of M^3 list positions for missiles that are selected for part p

Formulation:

LEXICOGRAPHICALLY MINIMIZE

1a) $+\sum_{tp} UNABLE_{tp}$

1b) $+\sum_{fa} emPen_f SHIPREQ_{fa}$

1c) $-NUMEXPEND$

1d) $+\sum_f DIFFMEAN_f$

1e) $-\sum_f NUMPLATPR_f$

1f) $-\sum_f NUMPLATBU_f$

1g) $+\sum_p mCubePri_p MCUBESUM_p$

1h) $-\sum_{wfh} value_w SALVO_{wfh}$

2) $\sum_{f,h,c|avail_{fhctp}=1} X_{fhctp} + UNABLE_{tp} = 1 \quad \forall t, p | isReq_{tp} = 1$

3) $SHIPREQ_{fa} \geq X_{fhct'1'} \quad \forall f, h, c, t | avail_{fhct'1'} = 1$
 $a | inArea_{ta} = 1$

4) $SHIPREQ_{fa} \geq X_{fhct'3'} \quad \forall f, h, c, t | avail_{fhct'3'} = 1$
 $a | inArea_{ta} = 1$

5) $\sum_a SHIPREQ_{fa} \leq 1 \quad \forall f$

6) $NUMEXPEND = \sum_{\substack{f,h,c,t|avail_{fhct'1'}=1 \\ \cap expend_f=1}} X_{fhct'1'}$

7) $NUMREMAIN_f = \sum_{whc} load_{wfhc} - \sum_{h,c,t|avail_{fhct'1'}=1} X_{fhct'1'} \quad \forall f | expend_f = 0$

8) $AVGREMAIN = \frac{1}{levelPlat} \sum_f NUMREMAIN_f$

9) $DIFFMEAN_f \geq +(NUMREMAIN_f - AVGREMAIN) \quad \forall f | expend_f = 0$

10) $DIFFMEAN_f \geq -(NUMREMAIN_f - AVGREMAIN) \quad \forall f | expend_f = 0$

11) $NUMPLATPR_f \leq \sum_{\substack{a|priSpread_a=1 \\ h,c,t|avail_{fhct'1'}=1 \\ \cap inArea_{ta}=1}} X_{fhct'1'} \quad \forall f$

- 12) $NUMPLATBU_f \leq \sum_{\substack{a|buSpread_a=1 \\ h,c,t|avail_{fhct'3}=1 \\ \cap inArea_a=1}} X_{fhct'3}, \quad \forall f$
- 13) $MCUBESUM_p = \sum_{\substack{f,h,c,t|avail_{fhctp}=1 \\ w|load_{wfhc}=1}} mCubePos_{wt} X_{fhctp}, \quad \forall p$
- 14) $SALVO_{wfh} \leq \sum_{c|avail_{fhct'1}=1} load_{wfhc} - \sum_{c,t|avail_{fhct'1}=1} X_{fhctp}, \quad \forall w, f, h | \sum_c load_{wfhc} \geq 1$
- 15) $\sum_w SALVO_{wfh} \leq 1, \quad \forall f, h | \sum_{cw} load_{wfhc} \geq 1$
- 16) $\sum_{h,c|avail_{fhct'1}=1} X_{fhct'1} \geq \sum_{h,c|avail_{fhct'2}} X_{fhct'2}, \quad \forall f, t | geoFe_{ft} = 1$
- 17) $\sum_{c|avail_{fhct'1}=1} X_{fhct'1} + \sum_{c|avail_{fhct'2}} X_{fhct'2} \leq 1, \quad \forall f, h, t | [\sum_{cw} load_{wfhc} \geq 1 \\ \cup t | isReq_{t'1} + isReq_{t'2} \geq 1]$
- 18) $\sum_{h,c|avail_{fhct'1}=1} X_{fhct'1} + \sum_{h,c|avail_{fhct'3}} X_{fhct'3} \leq 1, \quad \forall f, t | geoFe_{ft} = 1$
- 19) $\sum_{t,p|avail_{fhctp}=1} X_{fhctp} \leq 1, \quad \forall f, h, c | \sum_w load_{wfhc} \geq 1$
- 20) $\sum_{f,h,c|avail_{fhctp}=1} X_{fhctp} \leq 1, \quad \forall t, p | isReq_{tp} = 1$
- 21) $\sum_{\substack{c,p|avail_{fhctp}=1 \\ t \in con_i}} X_{fhctp} \leq 1, \quad \forall i, f, h | \sum_{cw} load_{wfhc} \geq 1$
- 22) $X_{fhctp} \in \{0,1\} \quad \forall fhctp$
 $UNABLE_{tp} \in \{0,1\} \quad \forall tp$
 $SHIPREQ_{fa} \in \{0,1\} \quad \forall fa$
 $SALVO_{wfh} \in \{0,1\} \quad \forall wfh$
 $NUMPLATPR_f \in \{0,1\} \quad \forall f$
 $NUMPLATBU_f \in \{0,1\} \quad \forall f$
 $NUMEXPEND \geq 0$
 $NUMREMAIN_f \geq 0 \quad \forall f$
 $AVGREMAIN \geq 0$
 $DIFFMEAN_f \geq 0 \quad \forall f$
 $MCUBEMSUM_p \geq 0 \quad \forall p$

NOTES

- 1a) This term expresses the number of unmet tasking requirements;
- 1b) this expression sums employment penalties incurred by using platforms to fire Tomahawks rather than continuing some other mission they are performing;
- 1c) this gives the number of missiles predesignated from “expend” platforms;
- 1d) this totals the absolute differences between the number of missiles remaining on non-expend platforms after tasking, and the average number of missiles remaining on non-expend platforms after tasking;
- 1e) this sums the number of platforms given some primary task part to perform;
- 1f) this sums the number of platforms given some back-up task part to perform;
- 1g) this assesses a penalty for selecting Tomahawks that have low desirability for each respective type of task part; and
- 1h) this evaluates the total residual salvo capability.
- 2) If a task t and task part p require a missile, a corresponding constraint here determines whether or not some missile is predesignated. A candidate predesignation variable X_{fncp} is only “counted” here if the firing platform can attain a launch position and if a missile is loaded in the referent location.
- 3) A firing platform f is required to be in area a if a missile on that firing platform is selected for a primary task in launch area a .
- 4) A firing platform f is required to be in area a if a missile on that firing platform is selected for a back-up task in launch area a .
- 5) A platform f can launch from at most one launch area.
- 6) The variable $NUMEXPEND$ equals the sum of all missiles predesignated to expend platforms for primary task parts.
- 7) Assuming that all primary task part missiles are fired, the variable $NUMREMAIN_f$ counts the missiles remaining aboard each non-expend platform.
- 8) This constraint computes the average number of missiles remaining on non-expend platforms in the theater.
- 9-10) These constraints determine for each non-expend platform f the absolute difference between its remaining number of missiles and the average of this across all non-expend platforms.

- 11) For each platform f , a constraint here determines whether or not a primary task part has been predesignated.
- 12) For each platform f , a constraint here determines whether or not a back-up task part has been predesignated.
- 13) For each task part p , a constraint here totals the priorities of all missiles predesignated to this type of task part.
- 14) Assuming all primary task part predesignations will be fired, a constraint here determines whether or not a missile of type w will remain on platform f in and half-module h .
- 15) The residual salvo size for each firing platform f and half-module h is either 0 or 1. Due to half-module power constraints, a platform can only fire a missile from one cell per half-module in any given salvo.
- 16) Ready-spare missiles for a task must be assigned to the same firing platform as the primary missile for that task.
- 17) Ready-spare missiles for a task cannot be assigned to the same half-module as the primary missile for that task.
- 18) Back-up missiles for a task must be assigned to a firing platform other than that assigned the primary missile for that task.
- 19) At most one missile may be selected per half-module on a given firing platform.
- 20) At most one missile may be selected per task part.
- 21) At most one missile from a set of conflicting tasks may be selected per half-module on each firing platform.
- 22) These are variable bounds and an additional specification that some variables can only assume binary values.

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