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"EXTRA!"

USING THE Newsvendor Model TO OPTIMIZE WAR RESERVE STORAGE



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and Joseph T. Klamo*

The United States Marine Corps (USMC) Installation and Logistics Command requested a study for determining appropriate inventory levels of war reserve materiel to meet future operational needs under surge demands in uncertain environments. This study sought to explore a potential approach by using the common newsvendor model, but modified for a military scenario. The authors' novel version of this core concept considers the purchase and storage costs of an item and proposes an intangible cost function to capture the consequences of a shortage. Further, they show a sample application of the model using a ubiquitous military item—the BA-5590/U battery. The output of the model provides USMC with a new tool to optimize inventory levels of a given item of interest, depending on scenario inputs.

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Keywords: *Inventory Level, Newsvendor Model, Military Cost of Shortage, War Reserve Materiel*

The objective of this study was to support recent United States Marine Corps (USMC) efforts to modernize approaches used to decide the optimal levels within their War Reserve Materiel (WRM). The acquisition of materiel for the WRM is unique compared to other acquisition conditions since it is not buy-as-you-need. Instead, enough inventory must already exist so that surged troops are not experiencing a shortage of items needed during combat. On the other hand, unused and discarded items erode the overall benefit of having the WRM. Although we focused on the specific requirements of the USMC and their typical deployment units and timelines, we presented an approach in a general framework that could be adjusted to meet the WRM storage requirements of other branches of military service as well. This expository piece explores a new method—the newsvendor model—that has not yet been fully evaluated for military applications. The primary research focus was to determine appropriate modifications to the newsvendor model for adaptation to military scenarios for predicting an appropriate WRM inventory level.



How Much Supply Is Enough Without Buying Excess Capacity?

Marine Air-Ground Task Forces (MAGTFs) are typically employed by the USMC to serve as a unified arms organization for military operation missions. They consist of air, ground, command, and logistics elements and come in battalion, brigade, and larger force sizes. In the event of a surge deployment requiring supplies beyond the stock on hand, the Marine Corps will need additional supplies from the WRM inventory, with immediate availability to support theater operations of MAGTFs until the expected long-term sustainment is established.

To make optimized decisions on WRM levels, the USMC must consider logistical capabilities and capacity for theater-level sustainment. It must also identify items that may have limited suppliers, lengthy production lead times, or both. However, the definitive features of the USMC WRM problem are uncertainties surrounding such issues as no restock opportunity, seemingly random demand, and the ultimate discarding of any unused expendable items. While multiple purchases may be made to stock the WRM, the no-restocking constraint refers to the fact that during the typical 60-day mission surge window, the USMC will be unable to replenish the WRM when inventory runs low. The demand has large uncertainty because, while surges have occurred before, databases tracking required demand were not kept until recently. Even now, the method used for estimating the WRM is lacking tracking systems to measure the materiel demand during conflicts and a process for keeping track of changes made to storage levels during peacetime. Given these circumstances, the USMC has significant concerns regarding its ability to accurately predict and store appropriate levels of WRM inventory that can support its next surge demand. This is likely not a problem confined to the USMC, but extends to rapid-deploying units of the other Services as well.

“ Few demand-tracking systems exist, and logisticians struggle to account for item additions, deletions, and usage over many past years. Given these circumstances, the USMC has significant concerns regarding its ability to accurately predict and store appropriate levels of WRM inventory that can support its next surge demand.

Department of Defense Instruction 3110.06 War Reserve Materiel (2019) establishes policy and provides guidance to Military Departments for computing WRM levels of inventory. However, this policy guidance provides the military services a great deal of latitude without specifying what methods should be used to determine the appropriate WRM inventory level. Currently, the Marine Corps utilizes the legacy War Reserve System (WRS) software program to aid in WRM decisions. WRS uses inputs such as unit size, operational plans, temperature zone, tempo of combat, estimated number of days for the mission plan, classes of supply requirements, and several other factors. The USMC WRS approach has had issues in the past though, highlighted by the fact that some critically important items were understocked during the opening days of Operation Iraqi Freedom in 2003. For example, BA-5590/U batteries were severely understocked. Navy



CAPT Clark Driscoll, liaison to the Joint Staff for the Defense Contract Management Agency said, “We literally [came] within days of running out of these batteries—where major combat operations would either have ceased or changed in their character because of the lack of battery support” (Fein 2003). Although 180,000 batteries were maintained as a reserve in the period leading up to the beginning of the conflict, initial demand for the batteries was nearly 620,000—far exceeding the ability of the reserve inventory to supply them (Government Accountability Office, 2005).

Numerous other approaches exist to achieve optimized inventory levels. One such approach is Economic Order Quantity (EOQ), which determines the number of items that should be purchased given the demand, cost to place an order, and the cost of holding the items. The method balances the costs of placing orders and storing the items in conditions of constant usage rates. Although useful in some applications, this approach is not applicable to WRM planning for two reasons. First, the EOQ model simply assumes restocking will take place when an item inventory level decreases and does not capture the consequences of running out of an item. Second, the EOQ model assumes a constant demand, but for WRM inventory the demand is uncertain.

Our Approach: Adopting the Newsvendor Model Framework

The newsvendor model is based on assumptions that are consistent with WRM modeling constraints. Porteus (2008) noted that the newsvendor model provides a methodology for solving the problem of how much inventory to purchase for the economical storage of a perishable item. Other considerations include solving the problem within an applicable time frame

when the actual demand of the item is unknown, and when the economic consequences of having “too much” and “too little” are known. Moreover, our interest in the newsvendor model is motivated by the fact that many others experiencing similar inventory-level problems, where the items in inventory are not sold for profit, have used the model. Olivares et al. (2008) applied the newsvendor model to estimate how many operating rooms should be reserved for cardiac surgery cases. Arikan and Deshpande (2012) used the approach in airline flight scheduling to estimate the impact that airport operational factors have on airline block-time scheduling. Hadas and Herbon (2014) applied a generalized newsvendor model to a public transportation operation to estimate the balance between the physical size of the fleet and the frequency of certain routes taken. Chen et al. (2017) used the same approach to estimate purchasing humanitarian relief items as a secondary sourcing option to monetary donations. Likewise, Mallidis et al. (2018) estimated the amount of perishable inventory that should be donated in humanitarian efforts to achieve an ethical goal to reduce food waste to best assist those in need of food. Thus, the newsvendor model as a core concept appealed to us as a potentially viable new approach to the USMC’s problem.

“ The EOQ model simply assumes restocking will take place when an item inventory level decreases and does not capture the consequences of running out of an item.

News vendor Model Explained

The newsvendor model optimizes the inventory level by balancing the expected marginal costs of both excess and shortage. The expected marginal cost of excess at a particular inventory level is simply the product of the marginal cost of excess and the probability of demand being less than that level. Similarly, the expected marginal cost of shortage is the product of the marginal cost of shortage and the probability of demand being greater than that level. The basic idea is that when the expected marginal cost of excess is less than the expected marginal cost of shortage, the inventory level should be increased. The optimal inventory level occurs when these two expected marginal costs are equal. Adelman et al. (1999) demonstrated the process of determining the optimal inventory level in detail for the nonmilitary applications of a fashion store and an individual selling newspapers.

Marginal Cost of Excess and Shortage Formulation

To perform this optimization, we must first determine the marginal cost of excess and the marginal cost of shortage. The marginal cost of excess is simply the additional cost incurred if we were to have one more item in our inventory and it ended up not being used. The marginal cost of shortage is the additional cost that we would incur if we had decided not to stock that extra item in our inventory, even though it would have been used if available. The formulation to estimate the marginal cost of excess is usually straightforward. Since it captures the cost of increasing our inventory by one additional item, the marginal cost of excess, ΔC_e , is calculated as

$$\Delta C_e(Q) = P(Q) + H(Q) - R_u \tag{1}$$

where $P(Q)$ is the unit cost to purchase one additional item when purchasing a total of Q items for a given inventory, $H(Q)$ is the unit holding, or storage, cost of that one additional item, and R_u is the unit resale value of that one item. Since items are typically sold individually or in batches much smaller than one’s inventory size, we consider this value to be a constant independent of the inventory level, and therefore it is not a function of Q . Certain items, while stored, could create additional costs, such as maintenance.

“ In our military application, where the inventory level is quite large, it is possible that as more items are produced, the supplier can produce them more cheaply. This is the concept of a learning curve. These savings could be partially passed on to the buyer by lowering the purchase price accordingly as more items are produced.

These costs would need to be captured as well, most naturally as part of the holding term. Although the formulation is straightforward, the actual estimation of the value of each term could be difficult and, in some situations, require best engineering judgments to be made.

In the typical newsvendor problem, the unit cost to purchase an additional item is not a function of the quantity. However, in our military application, where the inventory level is quite large, it is possible that as more items are produced, the supplier can produce them more cheaply. This is the concept of a learning curve. These savings could be partially passed on to the buyer by lowering the purchase price accordingly as more items are produced. In that case the purchase price of an additional item would depend on quantity.



The analysis also requires that the marginal cost of shortage be estimated. To properly capture it, we need to take into account that not incurring the cost of purchasing and storing an item in the first place partially offsets the cost of revenue lost from being unable to sell the item. Therefore, the cost of shortage, ΔC_s , for an item is expressed as

$$\Delta C_s(Q) = S_u + \Delta I(Q) - P(Q) - H(Q) \quad (2)$$

where S_u is the sale price of a single item and $\Delta I(Q)$ is the marginal intangible costs that result from not having the additional item. Like Equation (1), the sale price is assumed independent of the quantity in inventory. The marginal intangible cost attempts to quantify, in monetary terms, the future costs that will occur due to not having an additional item available for a customer. In typical newsvendor applications, this is normally the loss of future revenue from customers that do not return after being unable to purchase the item the first time.

Occasionally, newsvendor models contain an alternative source term to capture the situation, where the vendor quickly obtains additional items from a back-up source when item demand exceeds the inventory level purchased from the original source. However, since we assume that logisticians have no option to quickly procure additional items from an alternate source during the surge, we do not include an alternative source term in our formulation. If alternative sourcing were considered, it could be from a domestic producer of the item or an allied nation.

Expected Marginal Cost Analysis

The optimal inventory level is determined by increasing the inventory level, Q , while the expected marginal cost of excess is less than the expected marginal cost of shortage. This condition, where the inventory should be increased, is shown mathematically as

$$\Delta C_e(Q) P[d \leq Q] < \Delta C_s(Q) P[d > Q] \tag{3}$$

where d is the unknown future demand and $P[\]$ is the probability that the condition inside $[\]$ is true. The inventory level should be increased until the expected marginal cost from excess is greater than the expected marginal cost of shortage. Therefore, we can define the optimal inventory quantity, Q^* , that occurs when the following expression is satisfied

$$\Delta C_e(Q) P[d \leq Q^*] = \Delta C_s(Q) (1 - P[d \leq Q^*]) \tag{4}$$

while acknowledging that due to the discrete nature of inventory levels, the equality may not be realized, in which case the difference between the two expected marginal costs should be minimized.



Military Application of the Newsvendor Model

To develop our model, we focused on the unique aspects of military operations. Our analysis considered a Marine Expeditionary Brigade (MEB) as a typical force that would need to be supported by supplies from the WRM. Our results could be scaled to account for a different force size, for instance a Marine Expeditionary Force that planners wish to support with their WRM. Our model was based on the expectation of a 60-day surge deployment. After that time, we assumed the USMC units would be re-supplied through theater sustainment operations. We also assumed a useful shelf life of 10 years for the military item. This contributes to the cost of excess by establishing the total holding cost of the item. Finally, we only considered materiel from the inventory control stocks, which consists of materiel stored centrally at the wholesale level using USMC logistics bases, held within the DoD supply system, or positioned around the globe.

In military inventory scenarios, the possibility of revenue does not exist since the inventory item is never intended to be sold. Instead, the item is stored for the sole purpose of using it to achieve military objectives. This situation simplifies the cost of excess so that Equation (1) reduces to

$$\Delta C_e(Q) = P(Q) + H(Q) \quad (5)$$

since $R_u = 0$ because the item has no resale value. This expression becomes our military scenario marginal cost of excess function.

Similarly, in the marginal cost of shortage expression, the item has no sale price. Unlike the typical newsvendor problem, where the cost of an inventory shortage is lost profit, in a military application the cost of a shortage must be captured by quantifying the intangible cost of not achieving the objectives of the operation due to the shortage. The resultant marginal cost of shortage expression from Equation 2 becomes

$$\Delta C_s(Q) = \Delta I(Q) - P(Q) - H(Q) \quad (6)$$

since there is no sale of the item. The lack of revenue makes the intangible cost portion of the marginal cost of shortage critical for the analysis. If no marginal intangible cost component can be determined, then the cost of shortage is negatively valued. This negative cost represents a *benefit* to being short of inventory. In such a situation, since it would be beneficial to have a shortage, the analysis would provide $Q^* = 0$ as the optimized inventory. Therefore, the intangible cost term creates a penalty for being short and an incentive to hold a certain minimum inventory level.

To construct our marginal intangible cost function, we first note that for a military scenario, the marginal intangible cost of shortage depends on how many items were available in inventory. This is because the ability to achieve the objectives of the operation will depend on the warfighting capability of the unit. As the warfighting capability increases, the ability to achieve a larger number of the mission objectives increases and therefore the intangible cost of not having an additional item should decrease.

“ Our model was based on the expectation of a 60-day surge deployment. After that time, we assumed the USMC units would be re-supplied through theater sustainment operations. ”

To better illustrate this, consider a small unit in an isolated engagement with only one battery. The soldiers would have to choose between using it to fire a weapons system to defend themselves or to power a radio to call for support or an evacuation. In such a situation, having an additional battery would have allowed the soldiers to do both, greatly increasing the soldier's warfighting capability. On the other hand, consider the same situation but with enough batteries to power primary weapons and communications systems as well as multiple back-up systems with corresponding batteries. If the soldiers are short one battery, due to doctrine that requires back-up

systems to be powered and available, then the main consequence is that one of multiple back-up systems cannot be operated. But since the primary systems, and at least one back-up system, are operational, little, if any, of the unit’s warfighting capability decreases.

Mathematically, this behavior translates to a function that has a larger value for small inventory levels, monotonically decreases, and approaches zero for large inventory levels. The exponential function with a negative argument has those characteristics. Further, through the use of two coefficients, which we will call the value-to-cost ratio and the inverse rate, we can control how large the initial cost is for small inventory levels and the rate at which the penalty for being short decreases. The use of these two coefficients provides a large amount of freedom to specify the shape of the exponential function.

We define our marginal intangible cost function, $\Delta I(Q)$, found in Equation (6), as

$$\Delta I(Q) = \alpha P(Q) e^{-\left(\frac{Q}{\gamma}\right)} \tag{7}$$

where α is the value-to-cost ratio coefficient and γ is the inverse rate coefficient. Since we were interested in only the first 60 days of a surge deployment, we assumed that the capability, or value, of the item would not decrease over this timeframe. We also assumed that the value of the item would not decrease while in storage.

“ As the warfighting capability increases, the ability to achieve a larger number of the mission objectives increases and therefore the intangible cost of not having an additional item should decrease. ”

The value-to-cost ratio coefficient, α , can be used to adjust the procurement cost to better reflect the value of a single item in terms of achieving military objectives. Figure 1(a) shows how changing the value-to-cost ratio coefficient alters the marginal intangible cost function when the inverse rate coefficient is held fixed. In the figure, the inverse rate coefficient is set at 500,000. As the value-to-cost ratio coefficient increases, the marginal intangible cost increases for a given inventory level. Since no single value can be determined for all the items that might be stored in WRM, the appropriate value for the value-to-cost ratio coefficient will depend on the item of interest.

FIGURE 1(A). DEPENDENCE OF THE PROPOSED MARGINAL INTANGIBLE COST FUNCTION ON THE VALUE-TO-COST RATIO COEFFICIENT, α , FOR AN INVERSE RATE COEFFICIENT, γ , OF 500,000

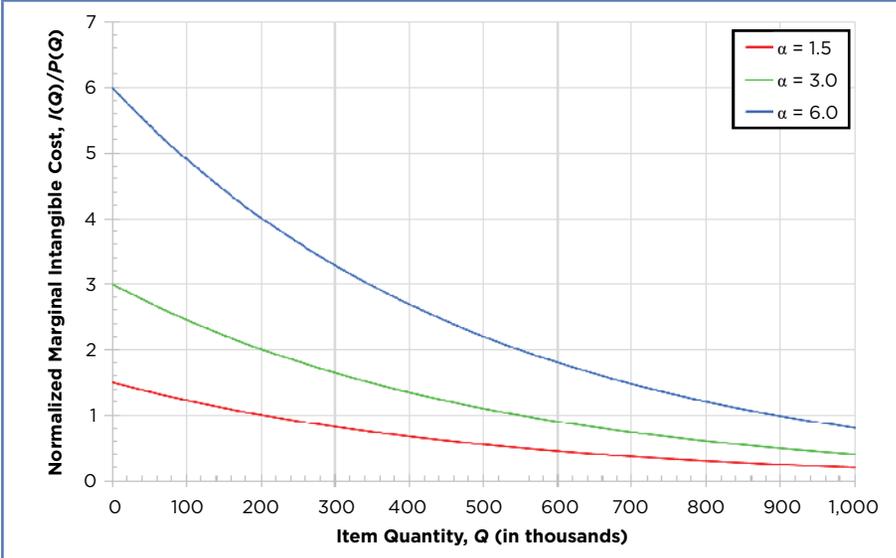
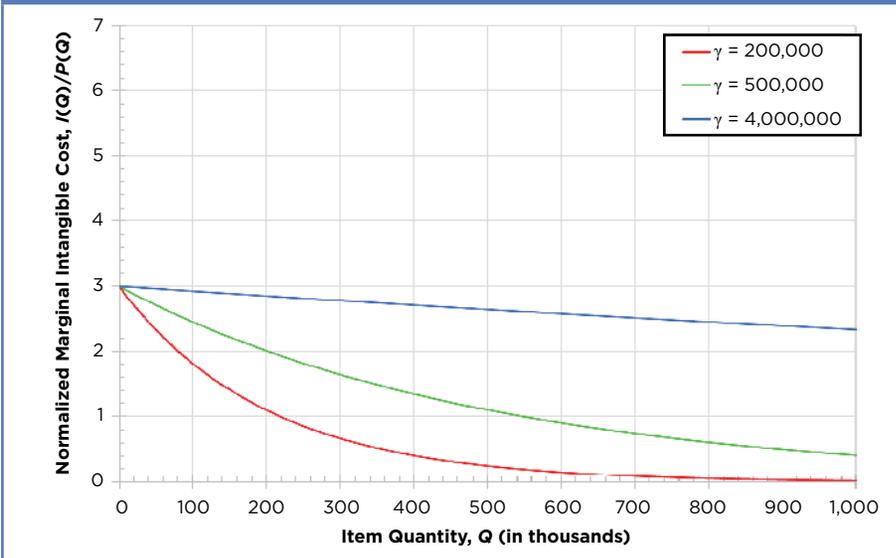


FIGURE 1(B). DEPENDENCE OF THE PROPOSED MARGINAL INTANGIBLE COST FUNCTION ON THE INVERSE RATE COEFFICIENT, γ , FOR A VALUE-TO-COST RATIO COEFFICIENT, α , OF 3.0



The inverse rate coefficient, γ , can be described as the rate at which the intangible cost function penalty decreases as the inventory level increases. We named it the inverse rate because as its value gets larger, the rate at which the intangible cost function approaches zero decreases. Figure 1(b) shows how changing the inverse rate coefficient alters the marginal intangible cost function when the value-to-cost ratio is held fixed. At a given inventory level, as the inverse rate coefficient is decreased, the marginal intangible cost decreases. Mathematically, when the inventory level is equal to the value of the inverse rate coefficient, the marginal intangible cost will be 36.8% of its value at zero inventory. Therefore, the value of the inverse rate coefficient should typically be on the order of the median from the expected item demand curve or larger. If the value is too small, then the marginal intangible cost function goes to zero too rapidly, before even reaching the average demand for the batteries. Like the value-to-cost ratio coefficient, the appropriate value for the inverse rate coefficient will also depend on the item of interest.

“ The item quantity needed for future conflicts of various intensities should, in theory, be able to be estimated with some degree of accuracy, especially if a database of the required quantities of each item during a given conflict is maintained.

Even though the marginal intangible cost portion of Equation (6) is critical, logisticians have no straightforward way to capture the monetary value of the intangible costs of an item shortage. Our model is but one possible way to estimate an intangible cost component. We recognize that many different functional relationships, other than exponential, could be used to characterize the intangible cost. For example, Hadas and Herbon (2014) used a polynomial to capture their cost of shortage. A number of approaches are available, other than scaling the procurement cost, to quantify the cost of shortage. For example, linking increased casualty rates in battle with equipment nonavailability could potentially serve as a powerful method. However, we were not able to find the required data to determine the form of such a relationship. Finally, the value-to-cost ratio coefficient of the item could be allowed to decrease over time as it is stored. One such example of valuation changes over time is Hildebrandt (1985), which describes military assets in monetary terms.

Characterizing the demand distribution is also required to conduct the expected marginal cost analysis. In military applications, both the conflict intensity and the variance in the inherent service life of the item affect how



many of the items are needed and therefore influence the overall demand curve. Depending on the item, the demand curve could be influenced more by one of these factors versus the other.

The dependence on conflict intensity requires that we estimate how much more of the item would be consumed as the intensity increases. It also requires that we estimate how likely the various battle intensities are. Although the demand undoubtedly increases for an item as the battle intensity increases, the likelihood of such battle protraction should likely decrease. The balance between these two factors creates the overall item demand. Szayna (2017) sought to understand what the trends in conflict have been, why they have changed, and what type of conflicts we can expect in the future. The item quantity needed for future conflicts of various intensities should, in theory, be able to be estimated with some degree of accuracy, especially if a database of the required quantities of each item during a given conflict is maintained. However, accurately estimating the *likelihood* of each such conflict is much more difficult.

The dependence on service life is typically easier to estimate since data usually exist to support its determination. The distribution of the service life of the item can be estimated from manufacturers' data or from historical maintenance data. Depending on the amount of data available, an assumption about the shape of the distribution might have to be made.

Sample Application: The BA-5590/U Battery

The BA-5590/U is a nonrechargeable lithium-sulfur-dioxide (LiSO_2) battery that has been in service since the early 1990s (U.S. Marine Corps, 2011, pp. F2–F21). These high-energy batteries are the most widely used battery within the DoD. They power a wide range of the electronic equipment

used by the USMC. Though often associated with radios and other communications equipment, this battery is also essential for weapon systems such as the Javelin and Tube-Launched, Optically Tracked, Wire-Guided missile systems.

The expressions for both the marginal cost of excess and shortage require the unit purchase cost for the battery to be determined. Based on economies of scale that the government has achieved through bulk purchases, the approximate current purchase cost is \$75 for each battery. The expressions for the marginal cost of excess and shortage also require the unit holding cost for the batteries to be determined. The BA-5590/U battery has specific storage requirements due to its classification as hazardous materiel and benefits from refrigeration. Based on a table of storage costs per square foot and considering a 10-year storage period, the unit holding cost of an additional battery is \$10.84. The required square footage of storage space was determined by assuming we stacked the batteries vertically on a standard pallet up to the weight handling limits of each pallet. By knowing the size of the batteries, we were then able to determine how many pallets and how much floor space was needed.

From Equation (5), the marginal cost of excess for BA-5590/U batteries is given by

$$\Delta C_e = 75.00 + 10.84 = 85.84 \tag{8}$$

which is simply a constant value independent of the quantity in inventory. From Equations (6) and (7), the marginal cost of shortage for the batteries is expressed as

$$\Delta C_s(Q) = 75.00\alpha e^{-\left(\frac{Q}{S}\right)} - 85.84 \tag{9}$$

where the value does depend on the quantity, due to the marginal intangible cost portion.



The final required component of the model is an estimate of the BA-5590/U battery demand curve. Ideally, battery usage during past conflicts would be used to generate the demand curve. Unfortunately, such a detailed database of battery usage during previous conflicts does not exist. The most relevant databases show how the number of batteries stored at various locations changed over time, but the reason for the change is not included. Sometimes the batteries were used during conflicts; however, many other times they were used for other purposes such as training. Therefore, another approach was required to estimate the battery demand.

“ The simulation user must specify the conflict duration and is also able to select from several predefined scenarios that capture the tempo and intensity of the operation. ”

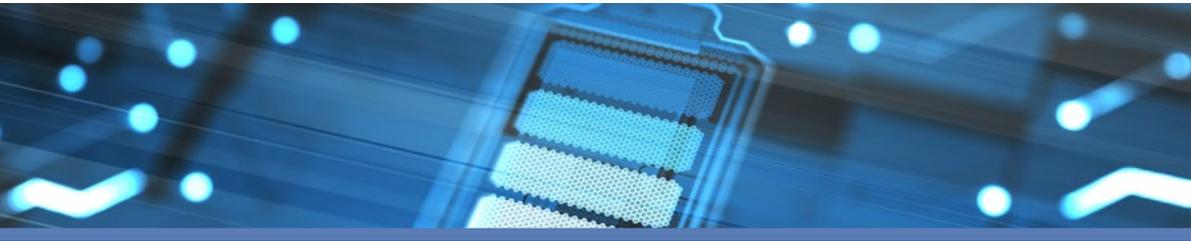
We used an existing simulation that models the power and energy consumption of various units in the U.S. military to estimate the battery demand curve. The simulation used was the MAGTF Power and Energy Model (MPEM) tool (T. Hagen, personal communication, February 11, 2020). The MPEM tool provides an estimate of the required quantity of many types of batteries used by the military, one of which is the BA-5590/U. The tool considers all the equipment used by each of the various units that make up an MEB. The simulation user must specify the conflict duration and is also able to select from several predefined scenarios that capture the tempo and intensity of the operation. For all other inputs being held fixed, a higher intensity scenario results in the tool predicting a larger number of batteries being needed.

We considered five different combinations of maximum and minimum battery usage days when spanning the 60-day surge. The maximum battery usage days involved all units of the MEB and a high usage predefined scenario in the tool. The minimum battery usage days involved only a limited number of units and a low usage predefined scenario in the tool. To represent a range of conflict intensities, we varied the number of maximum and minimum battery usage days within the 60 days. For instance, our lowest intensity conflict considered 5 days of maximum battery usage and 55 days of minimum battery usage. Our middle intensity conflict assumed 30 days of each situation. Overall, we considered five conflicts with different intensities. For each of them, we also had to estimate the likelihood that future conflicts would require *less* than the number of estimated batteries from each one.

TABLE 1. DATA USED TO CONSTRUCT THE CUMULATIVE DISTRIBUTION FUNCTION OF THE BA-5590/U BATTERY DEMAND CURVE			
Days of maximum battery usage by all units	Days of minimum battery usage by selected units	Estimated number of batteries needed	Likelihood of future conflicts requiring fewer batteries
5	55	58,086	0.05
15	45	121,731	0.25
30	30	182,356	0.50
45	15	224,476	0.75
55	5	237,210	0.95

Table 1 lists the number of batteries that the MPEM tool predicted were needed for each of our five conflicts, along with our estimate for the likelihood of a future conflict requiring more batteries than listed in each row. The battery results highlight that to estimate the required number of batteries, the MPEM tool does not use an average daily battery usage rate approach that would linearly scale to any length of conflict. Instead, it uses historical battery usage data, from both combat and field tests, the actual duration itself, and several other factors to estimate the batteries required. The conflict duration is important because in longer duration conflicts and field tests, batteries tend to be utilized more efficiently rather than replaced frequently with new batteries, which tends to happen in short-duration situations. From our results, we estimated that the median of the battery demand curve is 182,356 batteries. We denote this battery quantity as Q_0 and will scale it to set our inverse rate coefficient value.

We also looked at the effect that the battery service life would have on the number of required batteries in our demand distribution. A previous battery study conducted at Naval Postgraduate School (Vroom et al., 2019) determined that the average battery life of the BA-5590/U was 8.60 hours with a standard deviation of 1.52 hours. The estimated total required battery hours is the product of this average battery life and the estimated number of batteries. To determine the variability in the number of batteries needed due to variance in the service life, we created a simulation. The simulation



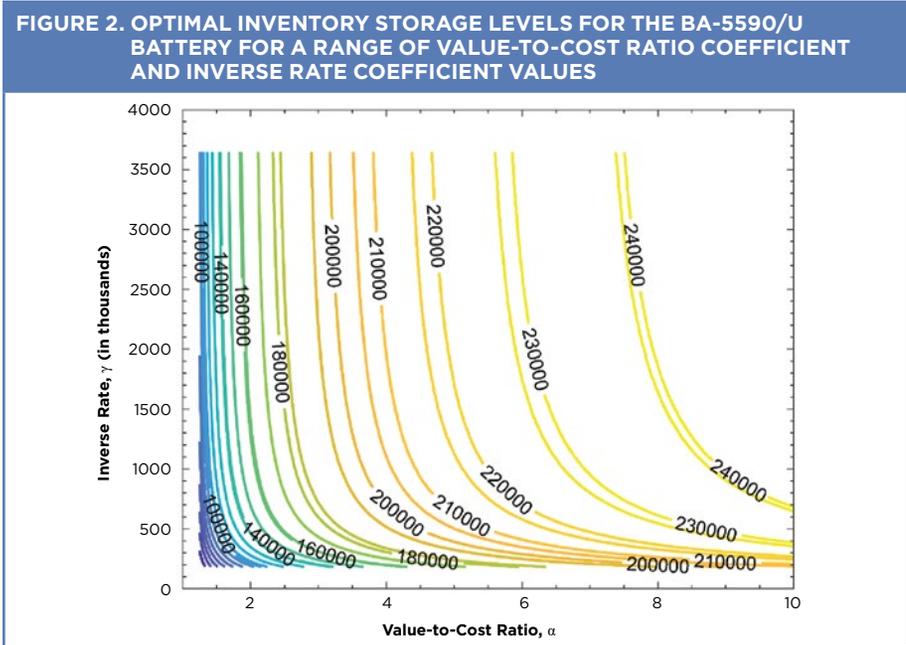
predicted how many batteries were needed to reach the required total battery hours if the service life of any given battery followed a normal distribution with a mean and standard deviation given from the Vroom study. By running 1,000 simulations, we determined that the standard deviation on the number of required batteries to reach our total battery hours was insignificant compared to the variability due to changing the MPEM tool input parameters. For example, the 30-30 day conflict scenario that requires 1,568,261.6 battery hours resulted in a standard deviation of only 75 batteries due to the variability of battery service life. This rather small variance revealed that the effects of consumption during battle are more important than the variability in battery service life when estimating the number of required batteries.

Results and Discussion

Sensitivity to Intangible Cost

The most subjective aspect of our expected marginal cost analysis was the marginal intangible cost component. Therefore, we explored the sensitivity of our model to the value-to-cost ratio and inverse rate coefficients. The purpose of our sensitivity study is not only to determine how changes to the value-to-cost ratio or inverse rate coefficients can change the calculated optimal inventory level, but also how much that level changes when the shape of the marginal intangible cost function is altered. Since the marginal intangible cost function is subjective, it is quite possible that different analysts could value the intangible costs significantly different when putting them in monetary terms. Therefore, we selected value-to-cost ratio and inverse rate coefficient values that would result in intangible cost curves that had significantly different values while keeping the coefficient values reasonable.

Figure 1(a) showed that as the value-to-cost ratio increases, the cost of not having an additional item at any inventory level increases. The smallest value-to-cost ratio coefficient that is realistic is something slightly greater than 1.0 since anything smaller means that the value of the item is less than its purchase cost. Therefore, we used a value of 1.5 for our smallest value. For our largest value-to-cost ratio, we settled on 6.0—a value four times larger than our smaller value—but we show results for up to a value of 10.0 in Figure 2. We decided that a value-to-cost ratio, larger than an order of magnitude, would be more appropriate when considering two different items.



As explained above, the inverse rate, to be realistic, should be set by scaling up the pre-analysis estimated median demand of the item from the demand curve. The smallest reasonable scaling up would be to just use the median demand itself, so we used $1.0Q_0$ as our smallest coefficient value. The largest scaling-up value would be one that makes the curve locally appear linear. This clearly happens when the inverse rate takes on a value near $20Q_0$, since the blue line in Figure 1(b) corresponds to an inverse rate coefficient of $21.9Q_0$.

When exploring the sensitivity of one coefficient, we held the other one fixed to isolate the sensitivity of each coefficient. Besides the smallest and largest coefficient values previously discussed, we also selected a third, middle value for both coefficients. This middle value was selected to cause the marginal intangible cost function to have a shape that was between the two shapes created by the smallest and largest coefficient values. Since the value-to-cost ratio linearly scales the marginal intangible cost function, we simply selected the middle value of 3.0. The inverse rate coefficient is part of the argument of the exponential function, so it does not linearly scale the shape like the value-to-cost coefficient. We found that a value of $2.5Q_0$ created a shape that visually appeared approximately halfway between the two shapes when using the smallest and largest values. Figure 1(b) shows that the green curve, which is $2.7Q_0$ (roughly $2.5Q_0$), is not nearly as steep as the red curve nor nearly as flat as the blue curve.

When examining the sensitivity of the value-to-cost ratio coefficient, we fixed the inverse rate coefficient at $2.5Q_0$. We fixed the value-to-cost ratio coefficient at 3.0 when looking at the sensitivity of the inverse rate coefficient. Table 2 shows the optimal inventory level for the five cases corresponding to the different combinations of our coefficient values. The top row contains the three value-to-cost ratio coefficients we considered, while the first column has the three inverse rate coefficients considered.

TABLE 2. OPTIMAL INVENTORY LEVELS OF BA-5590/U BATTERIES FOR VARIOUS VALUE-TO-COST RATIO AND INVERSE RATE COEFFICIENTS

γ \ α	1.5	3.0	6.0
$1.0Q_0$	---	134,000	---
$2.5Q_0$	93,250	173,750	212,500
$20.0Q_0$	---	196,750	---

We observed that as the value-to-cost ratio coefficient increases, the optimal inventory level increases as well. A significant difference in optimal inventory levels was also observed between the smallest and largest value-to-cost ratio coefficients considered. These two values produce a difference in optimal inventory levels of nearly 119,250 batteries, or just over a factor of 2.28 between the smallest and largest value-to-cost ratio coefficients. The optimal inventory level is very sensitive to the value-to-cost ratio coefficients, relying upon the subjective judgment of planner inputs.

The results also show that as the inverse rate coefficient values increase, the optimal battery inventory level also increases. The optimal inventory level changes by about 62,750 batteries, or a factor of 1.47, between the smallest and largest inverse rate coefficients. Although the inverse rate coefficient influences the optimal storage level, it appears to be less sensitive than the value-to-cost ratio coefficient. This is most apparent when considering that the inverse rate was varied by a factor of 20 in this sensitivity study, while the value-to-cost ratio was only varied by a factor of 4.

A contour map of the optimal inventory storage levels over a range of value-to-cost ratio and inverse rate coefficients provides a complete picture of the inventory level sensitivity plane. Figure 2 shows contour lines of constant optimal inventory levels with the value-to-cost ratio coefficients ranging from 1.25 to 10 and the inverse rate coefficients ranging from Q_0 to $20Q_0$, where $Q_0 = 182,356$ for this battery study. The contour map clearly shows that the optimal inventory storage level is less sensitive to, and in fact almost independent of, the inverse rate coefficient when it is larger than

about a value of $5Q_0$. Conversely, the optimal storage level is sensitive to the value-to-cost ratio coefficient across its entire range, but especially when the inverse rate coefficient is larger than roughly $3Q_0$.

Sensitivity to Demand Distribution

Since we used a normal distribution as an expedient for the battery distribution, we also examined the sensitivity to the form of the demand distribution. One could argue that the battery demand distribution might be somewhat skewed toward lower quantities since a minimum number of batteries is consumed for even the lowest intensity conflict. So, we also created a Weibull distribution to model the battery demand to use in our expected marginal cost analysis. The scale and shape parameters of the Weibull distribution allow it to capture skewness. Figure 3 shows the cumulative distribution function of the Weibull distribution that we created as the blue curve, and the normal distribution that we used previously as the red curve. The scale and shape parameter values were selected by minimizing the error, in a least-squares sense, to the MPEM tool data. It has a 25th-percentile quantity of 131,294 batteries, a median quantity of 172,878 batteries, and a 75th-percentile quantity of 214,748 batteries. For our normal distributions, these percentiles were 142,818, 182,356, and 221,894 batteries, respectively. Therefore, the largest difference is with the left tail skewness of the distributions, which is also evident in Figure 3.

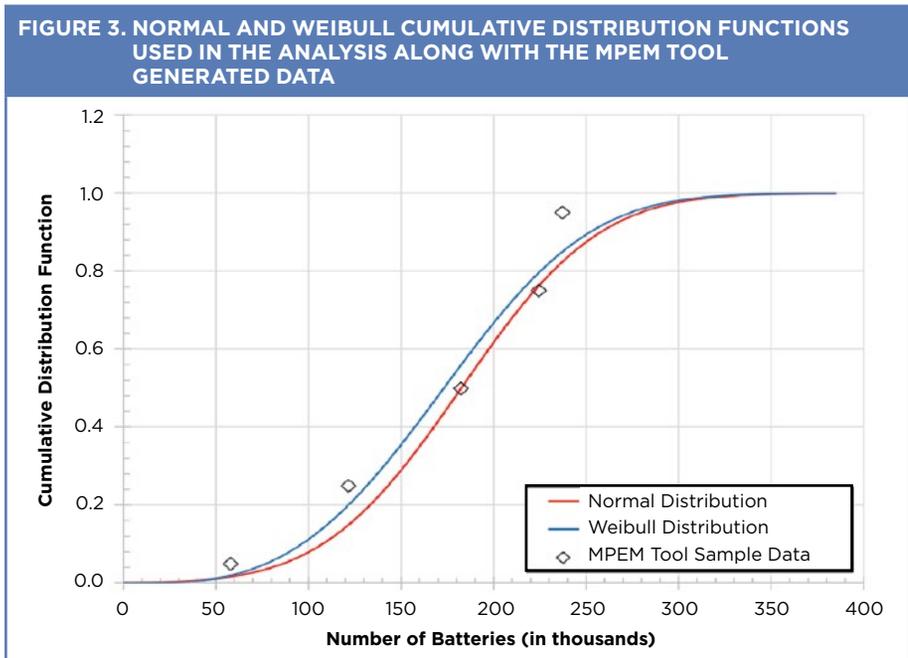


TABLE 3. COMPARISON OF OPTIMAL INVENTORY STORAGE LEVELS OF BA-5590/U BATTERIES USING A WEIBULL DISTRIBUTION DEMAND COMPARED TO A NORMAL DISTRIBUTION DEMAND

γ \ α	1.5	3.0	6.0
1.0 Q_0	---	127,750 (0.95)	---
2.5 Q_0	87,500 (0.94)	165,500 (0.95)	205,500 (0.97)
20.0 Q_0	---	188,500 (0.96)	---

To examine the sensitivity to this demand distribution, the optimal inventory level was determined using the same value-to-cost ratio and inverse rate coefficients as in the previous section but now using the Weibull distribution for battery demand. Table 3 shows the optimal inventory levels for the five different combinations of coefficients considered previously. The optimal inventory level for the five cases using the Weibull distribution is the first number shown, while the second number (shown in parentheses) is the ratio of this inventory level to the previously determined normal distribution demand inventory level. The second number being only slightly less than 1 shows that the optimal inventory level is not very sensitive to the actual distribution shape when the range of the distributions is similar. Since the optimal inventory level must fall within the range of the demand distribution, it will only change significantly if the range of the distribution is significantly changed.

Sensitivity to Purchase and Holding Costs

Although considerably less subjective than the intangible cost component or the demand distribution, we also looked at the sensitivity of the optimal inventory level to the purchase and holding costs in our model. To do this, we determined the optimal inventory level using low- and high-purchase and holding costs. However, when conducting this sensitivity analysis, we had to address the fact that our formula for the intangible cost function (Equation 7) contains the purchase cost as well. The purpose of multiplying the value-to-cost ratio coefficient and the purchase cost is to capture the true military value of the item. Therefore, for this sensitivity analysis, we held the value of the purchase cost variable used in Equation (7) fixed at \$130. The value-to-cost ratio coefficient was also fixed at 3.0 for all cases. This allowed us to vary the purchase and holding costs but not the intrinsic military value of the item.

To perform this sensitivity analysis, we used the following ranges of purchase and holding costs. For the low end, we used a unit purchase cost of \$75 and a monthly battery pallet holding cost of \$50 (\$5.40 per battery assuming 10 years of storage). These values represent realistic low-end values that we

identified considering economies of scale from buying in large quantities and nonrefrigerated storage. For the high end, we used a unit purchase cost of \$185 and a monthly battery pallet holding cost of \$100 (\$10.80 per battery for 10 years). These values correspond to the purchase cost of a single commercially available battery, ignoring the benefits of economies of scale, and including refrigerated storage. When calculating the optimal inventory level, the value-to-cost ratio coefficient for all cases was 3.0 and the inverse rate coefficient was $2.5Q_0$.

“ The purpose of multiplying the value-to-cost ratio coefficient and the purchase cost is to capture the true military value of the item.

For an item with a fixed military value, when the purchase and holding costs increase, the optimal inventory storage level decreases. For the low-end costs, the optimal inventory storage level was 208,750 batteries, while for the high-end costs, the optimal storage level was 151,500 batteries. This results in a range of 57,250 batteries, or a factor of 1.38, between the optimal storage levels considering the low- and high-end cases. This level of sensitivity is similar to that of the inverse rate coefficient and much less than that of the value-to-cost ratio coefficient.

Conclusions and Future Research

Like any optimization problem, expected marginal cost analysis finds a balance between two competing factors—in this case, the expected costs incurred from having too many of an item and having too few. The cost of having too many is usually straightforward. However, the cost of shortage in a military scenario must be captured using an intangible cost function. This is inherently a subjective determination. Through our sensitivity analysis, we have shown that the most important aspect of our modified newsvendor approach is the value-to-cost ratio coefficient. The results of the analysis will be meaningful only if one accurately captures the “costs” incurred when additional items are needed but not available. Since quantifying this will always be conjectural and vary widely, depending on the modeler’s input, so too will the optimal inventory levels for the WRM. The span of the demand distribution is another important aspect of our approach because the optimized inventory level is forced to occur within the bounds of the demand distribution. Thus, if the estimated demand distribution covers



the wrong quantity range, then the optimized inventory level will also be wrong. Therefore, the two biggest shortfalls of the newsvendor model in a military context are the uncertainty surrounding the intangible cost of item shortage and the distribution of demand.

The formulation of our model is general enough that it can be applied to any item that is part of the WRM and to the other branches of military service. However, each branch would need to adjust the parameter values to reflect the supported unit size and surge duration that the WRM needs to supply. This would most clearly show up in the demand distribution used for the analysis. Our model is also general enough that it can be applied to other scenarios where the cost of shortage is due to intangible costs rather than lost profits, such as humanitarian aid and disaster relief efforts.

Our model has neglected factors such as reuse of items while in theater, maintenance of items in storage, and waste in transport and use. These factors, presumed to be less significant, would undoubtedly make the model more realistic, but would also make the determination of parameter values more difficult. This is an area for further research that could leverage and refine our work.

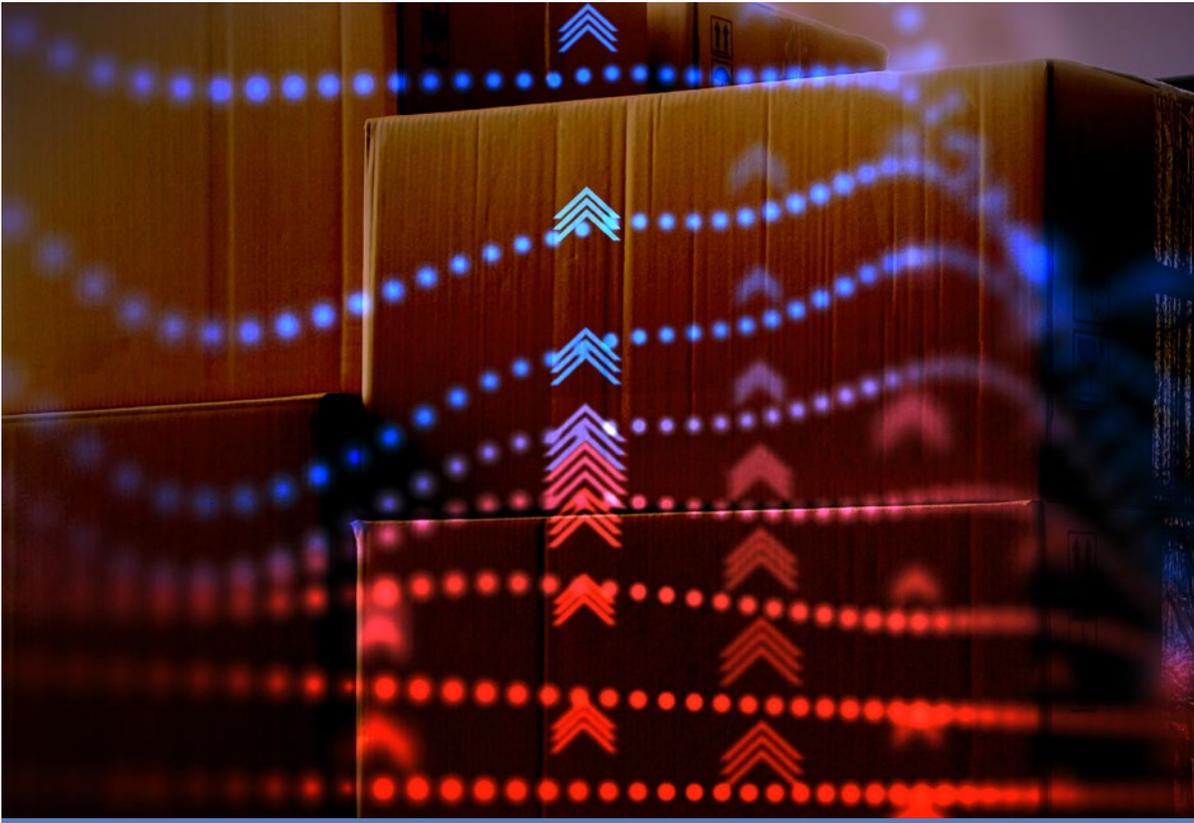
Feedback given to us by our research sponsors at the USMC Installation and Logistics Command was enthusiastic about how our methods could be used to better convey stockage levels in a more realistic context for sustainment preparations.

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