FORECASTING WITH SECOND-ORDER APPROXIMATIONS AND MARKOV-SWITCHING DSGE MODELS

Sergey Ivashchenko*, Semih Emre Çekin†, Kevin Kotzé‡, & Rangan Gupta§

October 2019 - v5.0

Abstract

This paper considers the out-of-sample forecasting performance of first- and second-order perturbation approximations for DSGE models that incorporate Markov-switching behaviour in the policy reaction function and the volatility of shocks. The results suggest that second-order approximations provide an improved forecasting performance in models that do not allow for regime-switching, while for the MS-DSGE models, a first-order approximation would appear to provide better out-of-sample properties. In addition, we find that over short-horizons, the MS-DSGE models provide superior forecasting results when compared to those models that do not allow for regime-switching (at both perturbation orders).

JEL Classifications: C13, C32, E37.

Keywords: Regime-switching, second-order approximation, non-linear MS-DSGE estimation, forecasting.

*The Institute of Regional Economy Problems (Russian Academy of Sciences); 36-38 Serpukhovskaya Street, St. Petersburg, 190013, Russia; National Research University Higher School of Economics; Soyza Pechatnikov Street, 15, St. Petersburg, 190068 Russia; The Faculty of Economics of Saint-Petersburg State University, 62, Chaykovskogo Street, St.Petersburg, 191123; Financial Research Institute, Ministry of Finance, Russian Federation, Nastasyinsky Lane, 3, p. 2, Moscow, Russia, 127006. Email: sergey.ivashchenko.ru@gmail.com.

†Department of Economics, Turkish-German University, Istanbul, Turkey. Email: scekin@tau.edu.tr.

‡*Corresponding author.* School of Economics, University of Cape Town, Rondebosch, 7701, South Africa. Email: kevin.kotze@uct.ac.za.

§Department of Economics, University of Pretoria, Pretoria, 0002, South Africa. Email: rangan.gupta@up.ac.za.

1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models are frequently used by academics and researchers at public institutions for policy-analysis and forecasting purposes.[1] Most of these models make use of a framework that is based on linear first-order perturbations, where it is assumed that the sample of data that is used in the estimation is not subject to any regime-switching behaviour. In a recent review of research that has been conducted using DSGE models, Christiano *et al.* (2018) note that the extensive application of first-order approximations for the model solution may be motivated by the fact that these models appear to provide an accurate characterisation of the effects of small shocks that arose during the post-war period in the United States. In addition, these techniques also allow researchers to make use of linear filters and estimation methodologies that are not subject to the computational complexity of non-linear counterparts. However, despite the attractive features of linear models that employ first-order approximations for the model solution, there are those who suggest that these models may fail to capture many of the non-linear features that are present in macroeconomic data. For example, Stiglitz (2018) notes that the use of linear approximations in such a macroeconomic model would be inappropriate, as it would not provide an accurate description of the effects of large shocks. In addition, Fernández-Villaverde *et al.* (2016) and Schmitt-Grohé *&* Uribe (2004) have suggested that the use of higher-order perturbation techniques could provide improvements in terms of the accuracy of the model solution.[2]

In this paper we seek to evaluate the use of high-order perturbation techniques for the model solution, where such models may also incorporate different regime-switching features. To assess the degree to which the different perturbation techniques and regime-switching features are able to explain the underlying data, we compare the out-of-sample forecasting performance of the respective models.[3] From an intuitive perspective, the use of higher-order solutions may be more accurate when the model has no analytical solution and it incorporates a number of non-linear features. In such a case, a smaller approximation error for the model solution would potentially allow the model to provide an improved out-of-sample fit of the data. However, we should also acknowledge that all macroeconomic models include mis-specification errors and higher-order approximations may be more sensitive to such errors, if the true underlying relationship between the variables may be described with sufficient accuracy by a first-order solution. As noted by Granger *&* Teräsvirta (1993), parameter estimation in models that incorporate various forms of non-linear relationships is inherently more difficult, as there are more possibilities and many more parameters to estimate, which would imply that there are also potentially more mistakes (or mis-specification errors) that could be made. Hence, the model that employs higher-order techniques to approximate the solution may provide inferior out-of-sample forecasts. Such a deterioration would potentially be more prominent when the likelihood function in models that utilise higher-order solution techniques are not restricted to the same extent as the first-order counterpart.[4]

A similar situation arises when we consider the use of regime-switching features in the model. After incorporating these stochastic non-linear parameters, we may differentiate between periods where there are differences in the underlying behaviour of economic agents. This may give rise to an improved forecasting performance if such regime-switching behaviour is present in the underlying data and it has been accurately described by these parameters. However, if such features do not exist, are very small, or may not be described by these particular regime-switching features, then the regime-switching model would incor-

[1]Lindé (2018) provides a recent summary of the use of DSGE models within academic and policy-making institutions, while da Silva (2018) and Tovar (2009) make note of their use within central banks. Several other authors, including Blanchard (2016) and Reis (2018), suggest that while we need to improve upon the existing framework, it would be wrong to suggest that this framework should be discarded.

[2]In addition, higher-order model solutions could also be used to capture important features that relate to asset pricing or welfare effects.

[3]As an alternative to high-order perturbation techniques, Fernández-Villaverde *&* Levintal (2017) describe the use of three projection methods that may be used to solve calibrated versions of a nonlinear DSGE model.

[4]See section 4 for specific details relating to the potential sources of these forecasting errors.

porate a larger mis-specification error and when working with data samples that have a finite length, the parameter estimates may be imprecisely estimated. In such cases, the quality of the forecasts that are produced by a mis-specified regime-switching model may decrease. When approximating the solutions for regime-switching models with perturbation techniques of different orders one would potentially introduce slightly different mis-specification and approximation errors and as a result we evaluate the out-of-sample forecasting performance of each of these models individually.

Therefore, this paper seeks to contribute towards the literature that considers the forecasting performance of models that make use of first- and second-order approximations, to extend the work of Pichler (2008) and Balcilar *et al.* (2015), were we also consider the use of higher-order perturbation approximations for models that incorporate regime-switching features. As noted in Liu *et al.* (2009), Liu *et al.* (2011), Liu *&* Mumtaz (2011) and Foerster *et al.* (2016), the use of Markov-switching DSGE (MS-DSGE) models allow for the analysis of more complex dynamic features that may be present in the data. In most cases, these models require the use of extensions to solution algorithms that are typically applied to single-regime models, such as those described in Maih (2015), and non-linear filters for the evaluation of the likelihood function. To complete the analysis in this paper we make use of the methodology that is described in Ivashchenko (2016) for the estimation and filtering of MS-DSGE models that make use of second-order approximations for the model solution.

The underlying model incorporates several New Keynesian features, with the addition of partial indexation on previous inflation. Nominal rigidities are introduced into the price setting mechanism of the firm and investment adjustment costs, where we make use of Rotemberg (1982) pricing. Two variants of the MS-DSGE model are considered, where the first considers the possibility of regime-switching in the policy rule and the second considers switching in the volatility of the shocks. By allowing for regime-switching in the volatility of the shocks, the model could potentially distinguish between the effects of relatively large and small shocks, where one would expect that higher-order solution techniques could potentially provide more accurate results, when large shocks would take us further away from the steady-state. All the models are then estimated over recursive data samples for the United States economy and the forecasts are evaluated on the basis of the root-mean squared-error and log-predictive score (where we use both Gaussian and mixed-Gaussian distributions).

The results suggest that while the use of higher-order approximations may provide a superior out-of-sample fit of the data in a model that is restricted to a single regime, this is not the case for those models that allow for regime-switching. This would imply that when we allow for the possibility of Markov-switching, a single-order approximation of the model solution may be sufficiently accurate. Hence, when estimating a relatively parsimonious single-regime model, a second-order approximation, which incorporates a number of additional components in the evaluation of the likelihood function, provides a superior out-of-sample result. However, in the case of the MS-DSGE model, which is less parsimonious, the incorporation of these additional components does not result in an improved out-of-sample fit. These results differ somewhat from those of Pichler (2008), who found that when using simulated data, the forecasting performance of a model that utilises a second-order approximation and a particle filter for the evaluation of the likelihood function may outperform a model that makes use of a first-order approximation and a linear Kalman filter. However, he also showed that when applied to economic data for the United States, the first-order approximation of the model solution provided a superior out-of-sample result.

The structure of this paper is as follows: section 2 presents the model structure, which includes details of the regime-switching behaviour. Section 3 provides details of the data and the methodology for the evaluation of the forecasts, while section 4 contains details relating to the derivation of the first- and second order approximations for the model. Section 5 discusses the results of the different specifications and section 6 concludes.

2 Model

The model takes the form of a closed-economy New Keynesian structure that is similar to that of Pichler (2008), however we also allow for partial indexation on previous inflation, following the specification in Lindé (2005). Similar model structures have been used in several other studies for the United States economy and include those of Ireland (2011), among others. For the derivation of the first-order conditions that give rise to the equilibrium conditions for model, please consult the online appendix.

2.1 Household

Household utility is dependent on the future expected value of consumption, leisure and real monetary balances.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ a_t \frac{c_t^{1-\tau} - 1}{1-\tau} + \chi_m \log(m_t) + \chi_h (1 - h_t) \right\} \tag{1}$$

where β is the stochastic discount factor, a_t is a preference shock, c_t represents consumption, τ is the inverse elasticity parameter for substitution between current and future consumption, χ_m and χ_h represent the weights associated with utility from real money balances (m_t) and leisure ($1 - h_t$). The persistence in the preference shock is described by a first-order autoregressive processes, $a_t = \eta_a a_{t-1} + \sigma_a \varepsilon_{a,t}$, where the independent innovations take the form $\varepsilon_{a,t} \sim \mathcal{N}(0, 1)$ and σ_a is used to measure the volatility of these shocks.

This utility function is then subject to a budget constraint that incorporates holdings of money and bonds, as well as capital and consumption goods. The amount of labour hours and dividends are also included in the budget constraint, along with the level of investment.

$$\begin{aligned} \frac{m_{t-1} + b_{t-1}}{\pi_t} &+ w_t h_t + q_t k_{t-1} + d_t + l_t - c_t - k_t + (1 - \delta) k_{t-1} \\ &- \frac{\phi_k}{2} \left(\frac{k_t}{k_{t-1}} - 1 \right)^2 k_{t-1} - \frac{b_t}{i_t} - m_t \end{aligned} \tag{2}$$

where b_t refers to the households bond holdings that mature at the beginning of the period, π_t represents inflation, w_t is the real wage, q_t is the real price of capital (k_t), while d_t and l_t refer to the household receipt of lump-sum transfers that are received from firms, through dividend payments, and government. In terms of the timing convention for reporting on the capital stock, k_t refers to the amount of capital that has been accumulated at the end of period t. The rate of depreciation is then captured by δ and ϕ_k represents the quadratic adjustment costs for changing the capital stock. The gross nominal interest rate is i_t.

2.2 Finished-goods producing firm

The firm that is responsible for the production of finished goods makes use of a constant returns-to-scale technology when transforming the output of firms that are involved in the production of intermediate goods.

$$y_t = \left[\int_0^1 y_{t(j)}^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \tag{3}$$

where y_t represent finished goods and $y_{t(j)}$ are the intermediate goods that are produced by the $j \in (0, 1)$ firms that are involved in the production of inputs for the final goods. The constant elasticity of substitution between the intermediate goods is measured by θ.

2.3 Intermediate-goods producing firm

A Cobb-Douglas production constraint is used for the intermediate producer that faces monopolistic competition:

$$y_{t(j)} = k_{t(j)}^{\alpha} \left(z_t h_{t(j)} \right)^{1-\alpha} \tag{4}$$

where α is the share of capital in this production process and z_t represents a technology shock. To describe the persistence of these shocks, we make use of the autoregressive processes, $z_t = \eta_z z_{t-1} + (1 - \eta_z)\, z + \sigma_z \varepsilon_{z,t}$, where η_z measures the degree of persistence relative to the steady-state and the independent innovations take the form, $\varepsilon_{z,t} \sim \mathcal{N}(0, 1)$.

Nominal rigidities are introduced into the pricing mechanism through the quadratic specification of Rotemberg (1982), after incorporating partial indexation on previous inflation, as per Lindé (2005). Hence, the adjustment costs for changing the price of intermediate goods, in terms of final goods, may be represented by the expression:

$$\frac{\phi_p}{2} \left(\frac{P_{t(j)}}{P_{t-1(j)} \pi^{1-v} \pi_{t-1}^{v}} - 1 \right)^2 y_t P_t \tag{5}$$

where ϕ_p is the size of the adjustment cost and v refers to the degree of inflation indexation. The nominal prices of intermediate and finished goods, are P_t and $P_{t(j)}$, while π refers to the steady-state (or targeted) measure of inflation.

2.4 Central bank

The central bank makes use of a Taylor (1993) rule that is measured in terms of deviations from steady-state values.

$$\log \left(\frac{i_t}{i} \right) = \gamma_i \log \left(\frac{i_{t-1}}{i} \right) + \gamma_y \log \left(\frac{y_t}{y} \right) + \gamma_\pi \log \left(\frac{\pi_t}{\pi} \right) + \sigma_i \varepsilon_{i,t} \tag{6}$$

where $\varepsilon_{i,t}$ is the monetary policy shock to short-term interest rates and σ_i is the volatility that is associated with this shock.

Three versions of the model are used in the subsequent analysis. The first does not employ any regime-switching behaviour. The second variant allows for Markov-switching in the volatilities of the independent shocks. In this case we allow for two possible states for stochastic switches in the volatility parameters: σ_a, σ_i, and σ_z. The third version allows for the possibility of Markov-switching in the monetary policy rule, where there are two possible states for the monetary policy parameters: γ_i, γ_π, γ_y. This provides us with a total of six models, as we make use of both first- and second-order approximations for each of these specifications.

The parameters in the models are estimated with maximum likelihood techniques for both orders of perturbation. The RISE toolbox is used for the estimation of all the first-order models, which employs the filter of Kim (1994) and the solution method that is described in Maih (2015). For the models that make use of second-order approximations for the solution, we employ the Markov-switching quadratic Kalman filter (MSQKF) that is described in Ivashchenko (2016).[5]

[5] The RISE toolbox can be downloaded at: `https://github.com/jmaih/RISE_toolbox`. It does not currently include the MSQKF that was used for the models that employ second-order approximations for the model solution.

3 Data and evaluation

The data for the observed variables in the model includes measures of output, prices and nominal interest rates for the United States. To be consistent with the model, the data for the observed variables would need to be stationary. Therefore, for measures of output we make use of the quarterly growth rate of real gross domestic product (GDP) as this is an important variable in many forecasting exercises. In addition, for quarterly change in prices, we use the change in the GDP deflator as this would capture a wide range of innovations to the pricing mechanism. To measure interest rates, we follow Wu *&* Xia (2016) and use of the shadow rates for the Federal Funds Rate, which represent the rates that are applicable over the majority of time during each quarter. Since the mean values for these variables differ from zero over the respective subsamples, we incorporate additional constants in the observation equations (7) - (9) to ensure that the observed variables take on zero steady-state values.

$$
\begin{aligned}
i_{obs,t} &= i_t - \bar{i} + i_{obs} && (7)\\
\pi_{obs,t} &= \pi_t - \bar{\pi} + \pi_{obs} && (8)\\
y_{obs,t} &= y_t - \bar{y} + y_{obs} && (9)
\end{aligned}
$$

The full sample of data that is used in the subsequent analysis covers the period 1978q3 to 2017q3, which provides us with 153 observations after we exclude the first four observations that are used in the pre-sample period to initialise the filter. The data is then divided into the respective training and testing sub-samples, where the first in-sample period that is used for parameter estimation spans until 2006q3. Thereafter, the forecast for this in-sample period is generated for the 1- to 8-step ahead horizon, over the period 2006q4 - 2008q4. Once we have stored these results, the in-sample estimation period is extended to 2006q4 for the forecasts that are generated over the period 2007q1 - 2009q1.

To evaluate the forecasting performance of the individual models we make use of three measures of forecasting accuracy. The first measure is the traditional root-mean squared-error (RMSE) that is computed according to formula in equation (10), for respective forecasting horizon (μ). The second measure is the mean log predictive score (LPS) which considers the log-likelihood of the data and the respective forecast density. This measure can be computed for individual variables or for all observed variables according to the formula in equation (11), which assumes that the forecast density is Gaussian (LPSG). In the presence of large sample sizes, the model with the highest expected log predictive score (which would be equivalent to the lowest value for the Kullback-Leibler information criteria) will provide the most accurate forecast.

In the case of Markov-switching models, the forecast density would need to be approximated by a mixed-Gaussian density (LPSGM). To calculate this statistic we make use of the formula in equation (12), where obs_t is the vector of observed variables, while $\mathbb{E}_t(\cdot)$, refers to the expectations operator, conditional on information that is available at time t. Similarly, $\mathbb{E}_{t,s}(\cdot)$ is the expectations operator that is conditional on information that is available at time t, but would also include information relating to the current regime, where $p_t(\psi_t = s)$ is the probability of being in regime $s = \{1, 2\}$. Similarly, $V_t(\cdot)$ and $V_{t,s}(\cdot)$ relate to the corresponding variances, conditional on information that is available at time t and about regime s.

$$
RMSE = \left(\frac{\sum_{t=1}^{n} \left[obs_{t+\mu} - \mathbb{E}_t \left(obs_{t+\mu} \right) \right]^2}{n} \right)^{\frac{1}{2}} \tag{10}
$$

$$LPSG = \sum_{t=1}^{n} \left\{ \frac{-\left(obs_{t+\mu} - \mathbb{E}_t\left[obs_{t+\mu}\right]\right)' \left(V_t\left[obs_{t+\mu}\right]\right)^{-1} \left(obs_{t+\mu} - \mathbb{E}_t\left[obs_{t+\mu}\right]\right)}{2} \ldots \right.$$
$$\left. + \log\left(\frac{\left|V_t\left(obs_{t+\mu}\right)\right|^{-\frac{1}{2}}}{(2\pi)^{m/2}} \right) \right\} / n \qquad (11)$$

$$LPSGM = \sum_{t=1}^{n} \log \left\{ \exp\left[\frac{-\left(obs_{t+\mu} - \mathbb{E}_{t,1}\left[obs_{t+\mu}\right]\right)' \left(V_{t,1}\left[obs_{t+\mu}\right]\right)^{-1} \left(obs_{t+\mu} - \mathbb{E}_{t,1}\left[obs_{t+\mu}\right]\right)}{2} \ldots \right. \right.$$
$$\left. + \log\left(\frac{\left|V_{t,1}\left[obs_{t+\mu}\right]\right|^{-\frac{1}{2}}}{(2\pi)^{m/2}} \right) + \log\left(p_t\left[\psi_{t+\mu} = 1\right]\right) \right] \ldots$$
$$+ \exp\left[\frac{-\left(obs_{t+\mu} - \mathbb{E}_{t,2}\left[obs_{t+\mu}\right]\right)' \left(V_{t,2}\left[obs_{t+\mu}\right]\right)^{-1} \left(obs_{t+\mu} - \mathbb{E}_{t,2}\left[obs_{t+\mu}\right]\right)}{2} \ldots \right.$$
$$\left. \left. + \log\left(\frac{\left|V_{t,2}\left(obs_{t+\mu}\right)\right|^{-\frac{1}{2}}}{(2\pi)^{m/2}} \right) + \log\left(p_t\left[\psi_{t+\mu} = 2\right]\right) \right] \right\} / n$$
$$(12)$$

To investigate the robustness of these results and their sensitivity to the Global Financial Crisis period, we evaluate the out-of-sample forecasting performance of the models over different sub-samples that span: 1978q3 - 2006q2, 2006q2 - 2009q4, and 2009q4 - 2017q3.

4 Approximating the solution

Numerical methods are used to approximate the model solution as it does not have an analytical closed-form solution. In the case of DSGE models, this task is usually performed with the aid of perturbation techniques that allow for an investigation into the robustness of the results (Judd, 1996, 1998). For example, to evaluate the quality of these approximations, one could make use of the methods that are described in Peralta-Alva *&* Santos (2014) or Judd *et al.* (2017) to measure the upper- and lower-bound for the approximation errors.[6] As a complementary exercise, one could also consider the out-of-sample forecasting results of models that incorporate different features and solution methods, which would be of particular interest when the parameters in the respective models are estimated.

To derive a solution for the MS-DSGE model, one would need to solve the following equilibrium function:

$$\mathbb{E}_t \boldsymbol{f}\left[\mathcal{Y}_{t+1}, \mathcal{Y}_t, \mathcal{X}_{t+1}, \mathcal{X}_t, \epsilon_{t+1}, \epsilon_t; \Theta\left(\psi_{t+1}\right), \Theta\left(\psi_t\right)\right] = 0 \qquad (13)$$

where $\mathbb{E}_t$ is the conditional expectations operator and $\boldsymbol{f}$ is a vector of possibly nonlinear functions that are dependent on the constant transition probabilities, ψ. The non-predetermined (control) variables are contained in the $\mathcal{Y}_t$ vector, while the $\mathcal{X}_t$ vector contains the predetermined (endogenous and exogenous) variables. The innovations to the predetermined (exogenous) variables are contained in the ϵ_t vector and the parameters in the model are contained in the Θ vector, which is conditional on the probability of being in a particular state.

To facilitate parameter estimation, we construct a nonlinear state-space system that incorporates a measurement equation for the observed variables, $Y_t = (i_t, \pi_t, y_t)'$. The state

[6] In a similar study, Aruoba *et al.* (2006) compare the use of both perturbation and projection methods for the solution of a calibrated stochastic neoclassical growth model using various methods of accuracy and robustness.

equations are then incorporated in the X_t vector, which allow for the interaction between the predetermined and non-predetermined variables. This system may then be described as follows:

$$
\begin{aligned}
Y_t &= \Phi X_t + \xi_t, && (14) \\
X_t &= \boldsymbol{\mathcal{H}}\left(X_{t-1}; \epsilon_t; \Omega; \psi_t\right), \\
&= Q_{1,\psi_t}\left[X_{t-1}; \epsilon_t\right] + \frac{1}{2} Q_{2,\psi_t}\left(\left[X_{t-1}; \epsilon_t\right] \bigotimes \left[X_{t-1}; \epsilon_t\right]\right) + \frac{1}{2} Q_{\dot{2},\psi_t}, && (15)
\end{aligned}
$$

where $\xi_t \sim \mathcal{N}(0, \Sigma_\xi)$ represents a vector of measurement errors ($\Sigma_\xi = 0$ in the above model), Φ is a coefficient matrix that describes the relationship between the observed and state variables, and $\boldsymbol{\mathcal{H}}$ is a non-linear function. The model parameters are then contained in the vector, $\Omega = \{\Theta(1), \Theta(2), \ldots, \Theta(nS), \Sigma_\xi\}$, where the term nS pertains to the number of regimes.

When using a first-order approximation for the model solution, we would only need to estimate terms that are contained in the Q_{1,ψ_t} matrix, while Q_{2,ψ_t} and $Q_{\dot{2},\psi_t}$ would contain additional parameters that would need to be estimated when making use of a second-order approximation. Hence, in the case of a the second-order approximation, we would need to estimate the following number of unique elements:

$$
\left[nX\left(nX + nEr\right)\frac{nX + nEr + 1}{2} + nX\right] nS, \tag{16}
$$

where nX is number of elements in the X_t vector and nEr pertains to the number of elements in the ϵ_t vector. This is a relatively large number of parameters, which is restricted in the case of the first-order approximation as the values for all the second-order coefficients are set to zero. Hence, when the additional coefficients that are contained in the second-order approximation are misspecified, then one would expect that the forecasts that are produced by such a model may be inferior to those that are derived from a model that employs a first-order approximation for the solution. This argument would also hold when comparing the results of the Markov-switching models with those that do not include regime-switching features.

The MSQKF is then used to evaluate the likelihood function of the MS-DSGE model that could be expressed as,

$$
\mathscr{L}\left(Y_t | \boldsymbol{\mathcal{H}}, \Omega\right) = \prod_{t=1}^{T} \int p\left(Y_t | X_t, Y_{t-1}; \boldsymbol{\mathcal{H}}, \Omega\right) p\left(X_t | Y_{t-1}; \boldsymbol{\mathcal{H}}, \Omega\right) dX_t, \tag{17}
$$

where p denotes the probability density. One of the benefits of making use of the MSQKF filter is that we are able to derive a closed-form solution for the likelihood function, when we assume that $\boldsymbol{\mathcal{H}}$ is a quadratic function. The likelihood function for models that make use of a first-order approximation for the model solution would be regarded as a special case of the above, where $\boldsymbol{\mathcal{H}}$ is a linear function.

5 Results

The RMSEs for the out-of-sample evaluation statistics over the entire sample period, 1978q3 - 2017q3, are presented in Figure 1. These include the results for the models for both model solutions, which have no regime-switching (NOS) parameters, switching in the volatilities (VOL) of the shocks, and switching in the monetary policy rule (POL). The forecasting results for interest rates over the long-term horizon that were provided by the model that does not allow for regime-switching produces slightly smaller RMSEs over most forecasting horizons (where most of the gains are made over the longer horizons). However, the model that does not make use of regime-switching provides less desirable forecasts for inflation and output, when compared to both of the Markov-switching versions of the model (for each order of perturbation). For example, when forecasting inflation the models that incorporate switching in the policy and volatility parameters provide the most desirable RMSEs over the shorter and longer horizons. In addition, the model with Markov-switching in the policy function provides the lowest RMSEs for output, when compared to the forecasting results that were generated by the other two models.[7]

When comparing the forecasting performance of the models that use either first- or second-order approximations, we note that in the case of the model that does not employ regime-switching, the model that employs the first-order approximation provides improved predictions for interest rates over the medium- and long-term horizon. In addition, this model would also appear to provide slightly better forecasts of output. However, when we consider the results for inflation, we note that second-order approximation provides much more accurate out-of-sample predictions over the medium- and long-term horizon. In the case of the model that allows for switching in the volatility of shocks, the model that makes use of a first-order approximation provides slightly better estimates for interest rates and inflation, while the second-order approximation provides more accurate forecasts for output. Then lastly, the first-order approximation of the model that allows for switching in the monetary policy function provides better predictions over the medium- to long-term forecasting horizon for inflation, while the version of this model that employs a second-order approximation provides better forecasts for interest rates.

Additional results for the three individual subsamples are included in the online appendix, were we note that if we restrict the period of investigation to 1978q3 - 2006q2 or 2009q4 - 2017q3, then the results are similar to those that have been reported above. In both cases the performance of the model that does not include regime-switching parameters provides much improved results for the inflation forecasts (when using a first-order approximation for the model solution), although the model with regime-switching in the volatility of shocks continues to provide the most superior results. When we then consider the results for the period 2006q2 - 2009q4, we note that the performance of the model that does not include regime-switching parameters and a first-order model solution produces the lowest RMSEs for both interest rates and inflation over most forecasting horizons. In addition, it also provides the second lowest RMSEs for the forecasts for output. This would suggest that it would possibly be better to model this particular period with a single regime.

[7]The online appendix also includes tables for all these results.

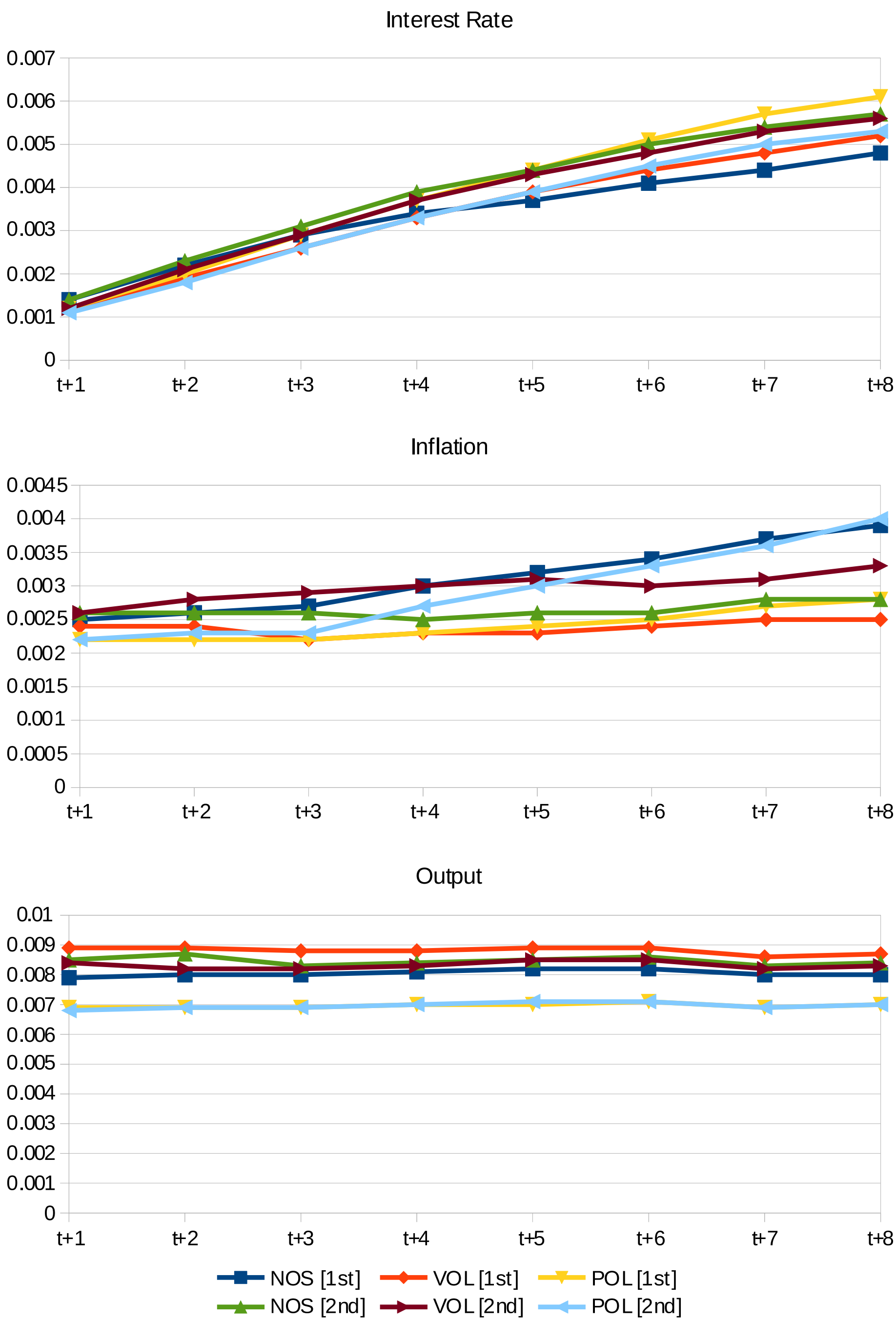

Figure 1: Root-mean squared-error (RMSE) statistics [1978q3 - 2017q3]

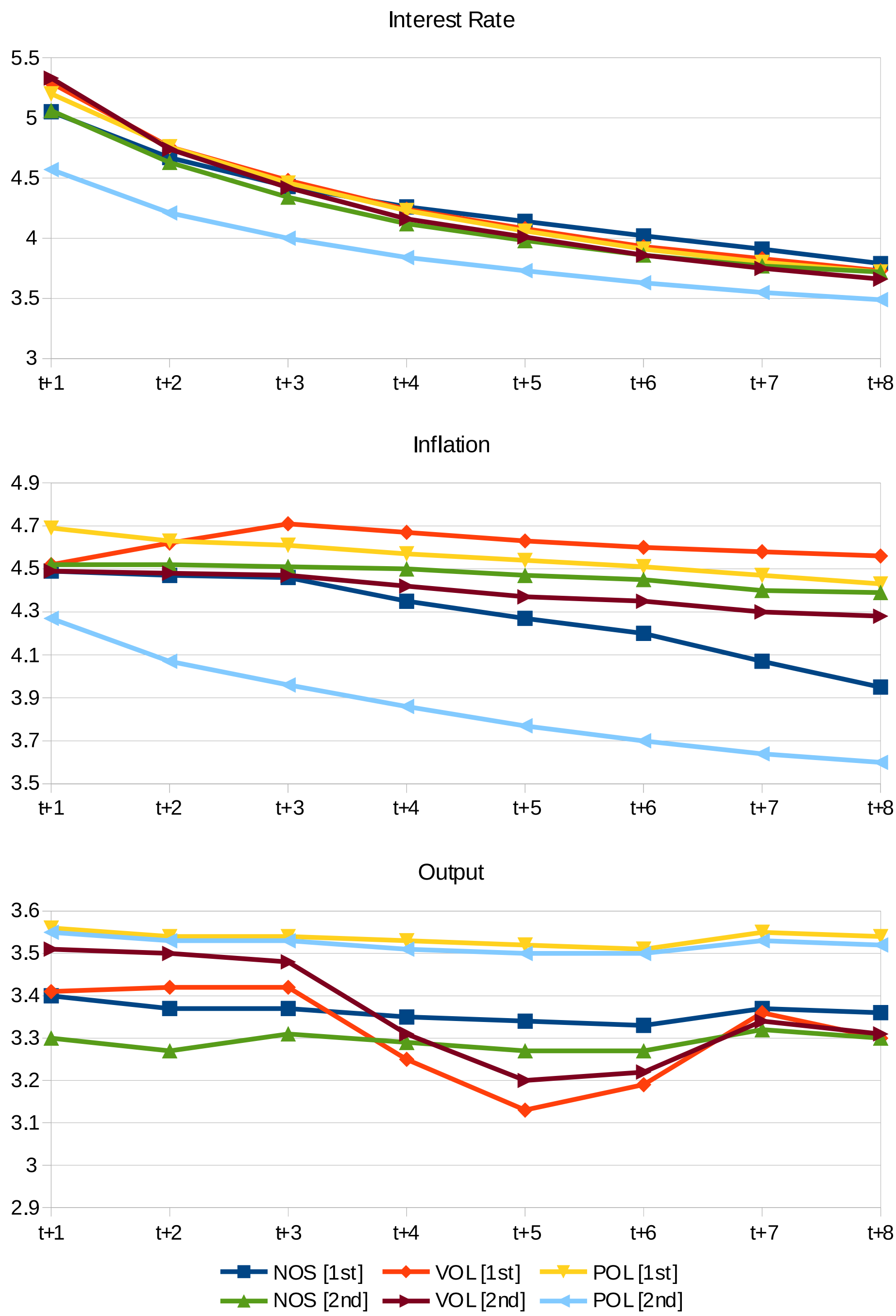

Figure 2: Log predictive scores (LPS) [1978q3 - 2017q3]

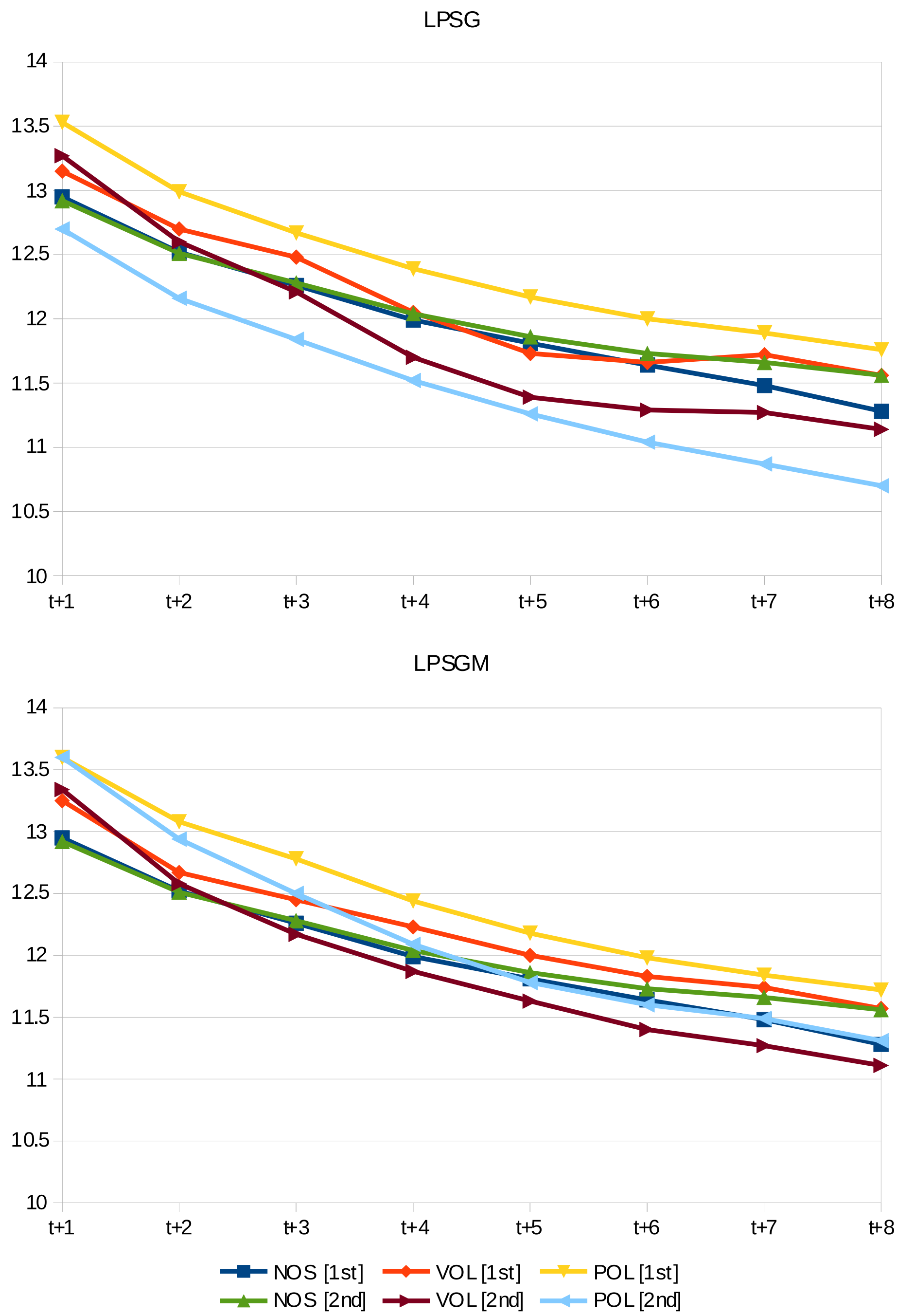

Figure 3: Combined LPS [1978q3 - 2017q3]

The results for the log predictive scores are summarised in Figure 2. These statistics suggest that the quality of the density forecasts would appear to provide similar results to those of the point forecasts. For example, the model that utilises a first-order approximation of the solution and no regime-switching features provides the best interest rate forecasts over the long-term horizon, while the models with regime-switching in the volatility of shocks provide improved interest rate forecasts over the short-term horizon. For inflation, the model with switching in the policy function provides superior results for forecasts over the short-term horizon, while the model with switching in the volatility of shocks provide better predictions for inflation over the long-term horizon. When considering the forecasts for output, we note that the model with regime-switching in the policy reaction function provides superior forecasts for output over both the short-run and the long-run. In addition, when considering the results for the second-order approximations, the model that allows for switching in the monetary policy rule is clearly inferior, when looking to forecast interest rates and inflation over the short-term horizon. However, when using a first-order approximation for this model the results are not nearly as poor when deriving forecasts for these variables.

The aggregate quality of these predictions, as measured by the LPSG and LPSGM statistics, are summarised in Figure 3. These results suggest that when comparing the models that make use of a first-order approximation for the solution, the model with regime-switching in the policy rule provides the best out-of-sample forecasts over all horizons. However, for the second-order approximations, the model without switching features outperforms the other models over longer horizons, while the regime-switching provides improved results over shorter horizons. In general, we also note that predictions from models that utilise second-order approximations are better than those that provide predictions from first-order approximations (in absence of switching), which is largely due to the difference in the forecasts for inflation.

The log predictive scores over the periods that preceded the Global Financial Crisis are largely similar to those that are reported on above, however, during the period 2009q4 to 2017q3 the performance of the model that allows for regime-switching in the volatility of shocks is much improved, particularly for interest rates over the short-term horizon and output over most horizons. For the aggregate LPSG and LPSGM measures, we note that the model that does not employ regime-switching provides slightly better results, possibly due to the fact that it has less parameters that need to be estimated with a dataset that extends over a relatively short period of time. However, these summary statistics also show that for the more recent sub-sample period the model that allows for regime-switching in the volatility of shocks produces superior results over most horizons.

Table 1: Significance test for equal forecasting ability: First- and second-order approximations

		$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$
NOS	RMSE r_t	67.42%	61.96%	32.2%	37.76%	43.73%	26.12%	31.36%	16.2%
	RMSE π_t	67.42%	61.96%	56.12%	73.36%	86.59%	97.34%	92.83%	97.65%
	RMSE y_t	0%	0%	0.01%	0%	0%	0%	0%	0%
	LPS r_t	97.56%	61.96%	14%	17.44%	21.48%	62.54%	43.57%	74.43%
	LPS π_t	2.44%	0.27%	0.14%	10.55%	21.48%	50%	68.64%	90.61%
	LPS y_t	8.71%	3.3%	4.42%	2.98%	4.03%	1.19%	3.65%	1%
	LPSG	55.98%	38.04%	22.04%	17.44%	56.27%	50%	68.64%	90.61%
	LPSGM	55.98%	38.04%	22.04%	17.44%	56.27%	50%	68.64%	90.61%
VOL	RMSE r_t	8.71%	18.02%	8.21%	17.44%	4.03%	62.54%	56.43%	74.43%
	RMSE π_t	14.56%	0.27%	0.98%	2.98%	0.01%	1.19%	0.01%	0%
	RMSE y_t	97.56%	100%	100%	100%	100%	100%	100%	100%
	LPS r_t	99.52%	96.7%	91.79%	82.56%	68.21%	83.16%	56.43%	62.86%
	LPS π_t	0.18%	0.03%	0%	0%	0%	0%	0.02%	0.04%
	LPS y_t	91.29%	96.7%	77.96%	89.45%	95.97%	83.16%	87.21%	90.61%
	LPSG	22.57%	6.31%	0%	0%	0%	0%	0%	0.04%
	LPSGM	8.71%	1.58%	0.98%	0.22%	0.11%	0.01%	0.02%	0.04%
POL	RMSE r_t	44.02%	61.96%	86%	98.62%	99.97%	99.95%	99.31%	99.62%
	RMSE π_t	22.57%	6.31%	14%	10.55%	1.92%	1.19%	3.65%	1%
	RMSE y_t	32.58%	50%	56.12%	73.36%	68.21%	73.88%	56.43%	50%
	LPS r_t	0%	0%	0.01%	0.02%	0.32%	2.66%	3.65%	4.94%
	LPS π_t	0.01%	0%	0%	0%	0%	0%	0%	0%
	LPS y_t	1.13%	0.69%	0.4%	0.58%	0.83%	1.19%	0.25%	0.38%
	LPSG	0%	0%	0%	0.01%	0%	0%	0%	0%
	LPSGM	55.98%	0.27%	0.14%	0.02%	0.01%	0.05%	0.25%	0.38%

To consider the extent to which the forecasts that were obtained from first- and second-order approximations provide a significant improvement, we make use of the test for equal forecast ability that is described in Clarke (2007). Table 1 contains the respective p-values for this test, where the null hypothesis, H_0, is that models have equal forecasting ability, while the alternative hypothesis H_1, is that second-order approximation of the model outperforms the model that employs a first-order approximation. These results suggest that the differences are significant in most cases, where the most superior model would depend on combinations of variable and forecasting horizons. Similar results also arise for each of the sub-samples, which are contained in the online appendix, along with accompanying figures that display these results.

There are a few cases where the statistics provide diverging results. For example, the RMSE for the long-term forecasts for interest rates that are provided by the model with switching in the policy function, suggest that superiority of the second-order model is superior, while the LPS results suggest that the first-order approximation would be preferred. When considering the aggregate measures for the LPSG and LPSGM it is noted that the second-order approximation for the model that does not incorporate switching has a significant advantage over the first-order approximation of this model. However, this is not necessarily the case for both of the models that employ switching, particularly over the medium- to long-term horizon.

Table 2 contains the results of a subsequent test, where we consider whether the inclusion of Markov-switching would make a significant difference to the forecasting performance of the respective models, where we use the method of Clarke (2007) once again. In this case the alternative hypothesis, H_1, would suggest that the inclusion of Markov-switching would provide significant improvements to the forecasting ability of the model. The results for the first-order approximation suggest that the difference in forecasting ability is significant when we compare the results of the models for interest rates over shorter horizons and inflation over all horizons. In addition, the model that makes use of monetary policy switching also provides superior results for output, while the model that allows for regime-switching in the volatility of shocks does not provide a significant improvement. In general the LPSG and LPSGM results suggest that the addition of Markov-switching would result in significant improvements over all horizons.

When we consider the results of the models that make use of second-order approximations in further detail, we note that the model that allows for switching behaviour in the monetary policy function displays a number of interesting properties. In this case the LPSGM results suggest that this model performance is significantly better over the short-term horizon, while LPSG results suggest that the model without switching provides significant improvements over all horizons. Thus, the model that allows for switching in the monetary policy rule produces an improved forecasting density over shorter horizons, while its mean-variance forecast is inferior. In the case of the model that allows for switching in the volatilities of the shocks, the situation is much simpler: the inclusion of this switching behaviour significantly improves upon the short-term forecasts, while there is an insignificant decrease in the forecasting ability of the model over longer horizons. This would suggest that when considering the forecasting performance of models over the medium- to long-term horizon, the inclusion of additional regime-switching behaviour may be less important, in the case of models that make use of second-order approximations for the model solution, when compared to the case of models that make use of first-order approximations for the model solution.

Table 2: Significance test for equal forecasting ability: Markov-switching

		$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$
VOL vs NOS [1st]	RMSE r_t	99.52%	99.31%	97.82%	97.02%	95.97%	90.02%	79.12%	74.43%
	RMSE π_t	77.43%	96.7%	99.6%	99.99%	100%	100%	100%	100%
	RMSE y_t	0.02%	0%	0%	0%	0%	0%	0%	0%
	LPS r_t	99.94%	99.31%	97.82%	97.02%	92.31%	83.16%	79.12%	62.86%
	LPS π_t	95.19%	99.31%	99.99%	100%	100%	100%	100%	100%
	LPS y_t	85.44%	88.9%	67.8%	37.76%	4.03%	0.47%	1.68%	1%
	LPSG	98.87%	96.7%	91.79%	99.42%	98.08%	98.81%	99.31%	99.62%
	LPSGM	97.56%	93.69%	67.8%	82.56%	92.31%	83.16%	92.83%	90.61%
POL vs NOS [1st]	RMSE r_t	99.52%	99.31%	91.79%	94.14%	68.21%	62.54%	31.36%	25.57%
	RMSE π_t	91.29%	88.9%	77.96%	98.62%	99.17%	99.99%	100%	99.96%
	RMSE y_t	99.82%	99.31%	99.6%	99.78%	99.68%	99.53%	99.31%	97.65%
	LPS r_t	100%	99.9%	86%	89.45%	68.21%	37.46%	31.36%	37.14%
	LPS π_t	22.57%	18.02%	43.88%	37.76%	31.79%	73.88%	68.64%	90.61%
	LPS y_t	100%	100%	100%	100%	100%	100%	100%	100%
	LPSG	100%	99.99%	99.95%	99.78%	99.97%	99.95%	99.75%	99.62%
	LPSGM	100%	99.99%	99.86%	99.99%	99.99%	100%	99.75%	99.87%
VOL vs NOS [2nd]	RMSE r_t	99.94%	99.73%	99.86%	82.56%	31.79%	50%	43.57%	50%
	RMSE π_t	8.71%	50%	14%	50%	21.48%	37.46%	68.64%	25.57%
	RMSE y_t	85.44%	98.42%	32.2%	62.24%	78.52%	83.16%	87.21%	90.61%
	LPS r_t	99.99%	99.97%	99.6%	99.78%	98.08%	99.53%	96.35%	90.61%
	LPS π_t	77.43%	88.9%	86%	50%	68.21%	50%	56.43%	50%
	LPS y_t	98.87%	99.9%	95.58%	50%	0.83%	1.19%	0.25%	0%
	LPSG	99.99%	99.73%	86%	73.36%	43.73%	37.46%	43.57%	25.57%
	LPSGM	100%	98.42%	86%	73.36%	43.73%	16.84%	31.36%	25.57%
POL vs NOS [2nd]	RMSE r_t	99.98%	99.9%	97.82%	89.45%	56.27%	90.02%	98.32%	97.65%
	RMSE π_t	67.42%	88.9%	22.04%	26.64%	4.03%	1.19%	0.25%	0.13%
	RMSE y_t	99.99%	99.97%	99.95%	99.99%	99.99%	99.99%	99.92%	99.87%
	LPS r_t	0%	0.27%	0.4%	0.58%	0.83%	2.66%	7.17%	4.94%
	LPS π_t	0.06%	0%	0%	0%	0%	0%	0%	0%
	LPS y_t	100%	100%	100%	100%	100%	100%	100%	100%
	LPSG	8.71%	6.31%	0.98%	2.98%	0.11%	0.17%	0.02%	0.04%
	LPSGM	99.99%	100%	99.95%	97.02%	68.21%	62.54%	43.57%	16.2%

Table 3: Parameters values

	NOS		VOL		POL		Pichler (2008)	
	1st	2nd	1st	2nd	1st	2nd	1st	2nd
σ_a	0.1167	0.0716	0.0798	0.0763	0.1509	0.1652	0.1061	0.0292
σ_i	0.0019	0.002	0.0009	0.0009	0.0043	0.0043	0.0014	0.0017
σ_z	0.6308	0.0176	1.891	1.3343	0.0443	0.0768	100	0.0083
α	0.7789	0	0.8949	0.8953	0.8144	0.8454	0.8347	0.36
δ	0.0522	0.0491	0.1	0.099	0.0852	0.1	0.0019	0.025
θ	2.3164	6.294	2	2.0022	6.9617	6.16	2.0004	6
ϕ_k	99.9999	1.2658	100	100	33.2747	29.1348	100	10
χ_h	9.9972	7.3313	10	7.4068	0.2143	0.2336	9.9947	*
χ_m	2.8746	0	8.6944	1.7745	0.8768	0.8768	4.2802	*
β	0.9455	0.9833	0.9	0.9017	0.9686	0.9684	0.9882	0.9931
ϕ_π	329.0216	76.4397	1000	897.7809	3.3443	2.2285	16.0592	84.6699
π	1.907	0.0092	1.712	1.9704	1.4927	1.4351	2	0.008662
z	3.4511	28.3427	5.796	24.7356	11.8958	12.0549	185.7394	7082.321
η_a	0.9928	1	0.992	0.9967	0.9469	0.9613	0.8895	0.929
η_z	-0.2037	0.8741	0.0054	-0.2078	0.951	0.9673	0.9961	0.9687
γ_i	0.7725	0.78	0.907	0.908	0.7903	0.6656	0.8602	0.7517
γ_y	0.0061	0.0126	0.0048	0.0057	0.0066	0.0022	-0.0007	0.0245
γ_π	0.2817	0.3859	0.1834	0.1836	1.9776	2.0122	0.518	0.3281
τ	10	9.9977	10	9.9995	9.8966	9.9999	9.9991	2.745
υ	0.8121	0	0.5919	0.8407	0	0	0	NA
i_{obs}	0.0383	0.0208	0.0062	0.0127	-0.0069	0.0139	-0.023	NA
π_{obs}	0.0463	0.0093	0.0054	0.0048	0.003	0.0065	-0.0033	NA
y_{obs}	0.0112	0.0082	0.0078	0.0079	0.0029	0.0062	0.0022	NA
$\sigma_a(\psi = 2)$	NA	NA	0.2692	0.2898	NA	NA	NA	NA
$\sigma_i(\psi = 2)$	NA	NA	0.0036	0.0037	NA	NA	NA	NA
$\sigma_z(\psi = 2)$	NA	NA	2.5602	1.7382	NA	NA	NA	NA
$p(\psi_{t+1} = 2\|\psi_t = 1)$	NA	NA	0.0297	0.0234	0.02	0.02	NA	NA
$p(\psi_{t+1} = 1\|\psi_t = 2)$	NA	NA	0.1202	0.1594	0.039	0.0293	NA	NA
$\gamma_i(\psi = 2)$	NA	NA	NA	NA	0.2844	0.2128	NA	NA
$\gamma_y(\psi = 2)$	NA	NA	NA	NA	-0.021	-0.0307	NA	NA
$\gamma_\pi(\psi = 2)$	NA	NA	NA	NA	1.0103	1.1105	NA	NA

When investigating the performance of models for individual sub-samples, the inclusion of regime-switching appears to be important for the forecasts that extend over most horizons, during the period that preceded the Global Financial Crisis, although the inclusion of regime-switching in the policy rule does not result in a significant improvement in the LPSG statistic. During the 2006q2 - 2009q4 period the introduction of regime-switching only provides improvements over the $t + 1$ horizon in certain instances, while the inclusion of such regime-switching provides significant improvements over the short- to medium-term horizon during the most recent period (following the Global Financial Crisis).

To consider the relative difference in the respective models, we compare the parameter estimates for each of the six models that were generated for the full sample. These estimates are reported in Table 3, where we also include the parameter estimates that are contained in Pichler (2008). The differences that exist between the results of the model that makes use of a single regime and those that are reported in Pichler (2008), may be due to the fact that the model structures are slightly different, as we have included indexation in the pricing rule. In addition, there are also differences in the in-sample and out-of-sample periods, the observed variables (i.e. we use output growth instead of the output gap), and the observation equations; where we use additional parameters for demeaning and assume the absence of measurement errors. The other difference that exists arises in the case of the model that makes use of a second-order approximation for the model solution, as we utilise a non-linear quadratic Kalman filter (QKF) for the evaluation of the likelihood function and the generation of forecasts, while Pichler (2008) makes use of a particle filter.

It is also worth noting that the log-likelihood value for single regime model that employs a first-order approximation for the solution is 2005.63. If we then fix the values of parameters in this model to those that are contained in Pichler (2008), the log-likelihood would be approximately equal to -3^{1010}. A similar exercise for the models that make use of second-order approximations provides log-likelihood values of 1947.23 and -7^{108}, respectively. In other words, it would appear as if the estimated parameter values in Pichler (2008) are not robust to the modifications that have been applied to the models that were constructed in this paper, which is perfectly understandable when considering the nature of these modifications. Hence, this would imply that we are introducing new evidence and are not merely replicating the work of Pichler (2008).

Another important contrast relates to the difference between the parameter estimates for the first- and second-order approximations, where we observe relatively large differences, while in Pichler (2008) these are relatively small. This discrepancy may be due to the different filters that were used in the respective analyses, where Andreasen (2013) notes that the use of the particle filter could be relatively inaccurate when compared to the Central Difference Kalman Filter (CDKF). In addition, subsequent research has shown that the QKF (and MSQKF) filters may provide slight improvements over the CDKF filter, particularly when there are large deviations from the steady-state (Ivashchenko, 2014). As a result, the particle filter may not capture the movements that are relatively far away from what would be provided by a first-order approximation of the model solution.

Finally, to ensure that the results of this analysis are relatively robust, we compare the forecast ability of these models with similar forecasts that are provided by VAR and AR models. The results of this analysis is contained in Tables 4 and 5 of the appendix of this paper, where we note that the DSGE models have superior predictive ability with a few notable exceptions. For example, reduced-form models provide better forecasts for output and they would also appear to outperform the second-order approximation of the model that allows for regime-switching in the monetary policy function.

6 Conclusion

This paper considers the out-of-sample forecasting performance of two MS-DSGE models that make use of different perturbation orders for the model solution. The results are then compared to those of models that do not make use of regime-switching behaviour. Our results suggest that in the case of the model that does not employ any switching, the model that is solved with a second-order approximation performs better than the model that makes use of a first-order approximation for the model solution. However, the opposite is true for models that employ Markov-switching. It is also worth noting that while these results summarise the overall performance of the model, this would not imply that the models that provide the best aggregate performance would generate superior forecasts for each of the individual variables over different horizons.

In addition to these results, we also consider the effect of introducing Markov-switching behaviour on the out-of-sample forecasting performance of the respective models. Our findings suggest that when using a second-order approximation, the introduction of switching in the monetary policy rule would significantly improve the predictive ability of the model. However, the performance of the model that makes use of switching in the volatility of the shocks and a second-order approximation for the model solution does not provide any significant improvement over the model that does not employ regime-switching for longer forecasting horizons. We also note that the introduction of Markov-switching to the monetary policy rule would improve the forecasting ability of the model over the short-term forecasting horizon for both perturbation orders. The use of regime-switching in the period that follows the Global Financial Crisis would appear to be of significant importance for forecasts over the short- to medium-horizon.

To explain these results, we suggest that since the model that does not employ regime-switching generates a relatively narrow density for the likelihood function when we make use of a second-order approximation, the parameters would be more accurately identified. This would enable the model to describe the more complicated dynamic features that are present in the data. However, when the model incorporates Markov-switching behaviour, which would also facilitate the description of more complex dynamics, the incremental advantage of making use of a higher order perturbation order is no longer present. These results would be of particular interest to those who make use of DSGE models for forecasting purposes.

To incorporate many of the benefits of the individual models that have been discussed in this paper one could potentially make use of a pruned higher-order approximation for MS-DSGE models, which would be an interesting topic for future research.

References

Andreasen, M.M. 2013. Non-linear DSGE model and the central difference Kalman filter. *Journal of Applied Econometrics*, 28(6):929–955.

Aruoba, S.B., Fernández-Villaverde, J. & Rubio-Ramìrez, J.F. 2006. Comparing solution methods for dynamic equilibrium economies. *Journal of Economic Dynamics and Control*, 30(12):2477–2508.

Balcilar, M., Gupta, R. & Kotzé, K. 2015. Forecasting macroeconomic data for an emerging market with a nonlinear DSGE model. *Economic Modelling*, 44:215–228.

Blanchard, O. 2016. Do DSGE models have a future. Policy Brief 16-11, Peterson Institute for International Economics.

Christiano, L.J., Eichenbaum, M.S. & Trabandt, M. 2018. On DSGE models. *Journal of Economic Perspectives*, 32(3):113–140.

Clarke, K.A. 2007. A simple distribution-free test for nonnested model selection. *Political Analysis*, 15(03):347–363.

da Silva, L.A.P. 2018. In defence of central bank DSGE modelling. In *Pushing the frontier of central banks' macromodelling*, Seventh BIS Research Network meeting. BIS.

Fernández-Villaverde, J. & Levintal, O. 2017. Solution methods for models with rare disasters. Technical Report, University of Pennsylvania.

Fernández-Villaverde, J., Rubio-Ramìrez, J.F. & Schorfheide, F. 2016. *Solution and Estimation Methods for DSGE Models*, volume 2 of *Handbook of Macroeconomics*, pages 527–724. Elsevier.

Foerster, A., Rubio-Ramìrez, J.F., Waggoner, D.F. & Zha, T. 2016. Perturbation methods for markov-switching DSGE models. *Quantitative Economics*, 7(2):637–669.

Granger, C. & Teräsvirta, T. 1993. *Modelling Non-Linear Economic Relationships*. Oxford University Press.

Ireland, P.N. 2011. A New Keynesian Perspective on the Great Recession. *Journal of Money, Credit and Banking*, 43(1):31–54.

Ivashchenko, S. 2014. DSGE model estimation on the basis of second-order approximation. *Computational Economics*, 43(1):71–82.

Ivashchenko, S. 2016. Estimation and filtering of nonlinear MS-DSGE models. HSE Working papers WP BRP 136/EC/2016, National Research University Higher School of Economics.

Judd, K.L. 1996. Approximation, perturbation, and projection methods in economic analysis. In H.M. Amman, D.A. Kendrick & J. Rust (eds.), *Handbook of Computational Economics*, volume 1 of *Handbook of Computational Economics*, chapter 12, pages 509–585. Elsevier.

Judd, K.L. 1998. *Numerical Methods in Economics*, volume 1 of *MIT Press Books*. Cambridge, Mass: The MIT Press.

Judd, K.L., Maliar, L. & Maliar, S. 2017. Lower bounds on approximation errors to numerical solutions of dynamic economic models. *Econometrica*, 85(3):991–1012.

Kim, C.-J. 1994. Dynamic linear models with Markov-switching. *Journal of Econometrics*, 60(1-2):1–22.

Lindé, J. 2005. Estimating new-Keynesian Phillips curves: A full information maximum likelihood approach. *Journal of Monetary Economics*, 52(6):1135–1149.

Lindé, J. 2018. DSGE models: Still useful in policy analysis? *Oxford Review of Economic Policy*, 34(1-2):269–286.

Liu, P. & Mumtaz, H. 2011. Evolving macroeconomic dynamics in a small open economy: An estimated Markov switching DSGE model for the uk. *Journal of Money, Credit and Banking*, 43(7):1443–1474.

Liu, Z., Waggoner, D. & Zha, T. 2009. Asymmetric expectation effects of regime shifts in monetary policy. *Review of Economic Dynamics*, 12(2):284–303.

Liu, Z., Waggoner, D.F. & Zha, T. 2011. Sources of macroeconomic fluctuations: A regime-switching DSGE approach. *Quantitative Economics*, 2(2):251–301.

Maih, J. 2015. Efficient perturbation methods for solving regime-switching DSGE models. Working Paper 2015/01, Norges Bank.

Peralta-Alva, A. & Santos, M.S. 2014. Analysis of numerical errors. In K. Schmedders & K.L. Judd (eds.), *Handbook of Computational Economics*, volume 3, pages 517–556. Elsevier.

Pichler, P. 2008. Forecasting with DSGE models: The role of nonlinearities. *The B.E. Journal of Macroeconomics*, 8(1):1–35.

Reis, R. 2018. Is something really wrong with macroeconomics? *Oxford Review of Economic Policy*, 34(1-2):132–55.

Rotemberg, J.J. 1982. Monopolistic price adjustment and aggregate output. *Review of Economic Studies*, 49:517–31.

Schmitt-Grohé, S. & Uribe, M. 2004. Solving dynamic general equilibrium models using a second order approximation of the policy function. *Journal of Economic Dynamics and Control*, 28:755–75.

Stiglitz, J. 2018. Where modern macroeconomics went wrong. *Oxford Review of Economic Policy*, 34(1-2):70–106.

Taylor, J.B. 1993. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39(1):195–214.

Tovar, C.E. 2009. DSGE models and central banks. *Economics - The Open-Access, Open-Assessment E-Journal*, 3:1–31.

Wu, J.C. & Xia, F.D. 2016. Measuring the macroeconomic impact of monetary policy at the zero lower bound. *Journal of Money, Credit and Banking*, 48(2-3):253–291.

Table 4: Significance test of equal forecasting quality of models and VAR

		$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$
NOS [1st]	RMSE r_t	44.02%	96.7%	97.82%	98.62%	99.17%	99.99%	100%	100%
	RMSE π_t	67.42%	61.96%	22.04%	2.98%	7.69%	0.05%	0.02%	0.38%
	RMSE y_t	14.56%	0.27%	14%	10.55%	7.69%	5.41%	7.17%	4.94%
	LPS r_t	67.42%	99.9%	99.99%	99.98%	100%	100%	100%	99.96%
	LPS π_t	98.87%	99.31%	99.86%	97.02%	95.97%	73.88%	87.21%	74.43%
	LPS y_t	0%	0%	0.05%	0.02%	0.11%	0.01%	0.08%	0.04%
	LPSG	1.13%	50%	91.79%	82.56%	95.97%	98.81%	96.35%	99%
	LPSGM	1.13%	50%	91.79%	82.56%	95.97%	98.81%	96.35%	99%
VOL [1st]	RMSE r_t	77.43%	88.9%	97.82%	94.14%	99.68%	99.95%	99.98%	99.96%
	RMSE π_t	67.42%	72.88%	91.79%	94.14%	92.31%	98.81%	96.35%	83.8%
	RMSE y_t	0.06%	0%	0%	0%	0%	0%	0%	0.01%
	LPS r_t	99.82%	99.31%	97.82%	97.02%	99.17%	97.34%	99.31%	99%
	LPS π_t	99.98%	99.97%	100%	99.99%	100%	100%	99.99%	100%
	LPS y_t	8.71%	27.12%	8.21%	0.22%	0%	0.05%	0.08%	0%
	LPSG	95.19%	81.98%	86%	89.45%	92.31%	90.02%	98.32%	90.61%
	LPSGM	97.56%	72.88%	77.96%	82.56%	86.59%	83.16%	87.21%	95.06%
POL [1st]	RMSE r_t	91.29%	96.7%	95.58%	73.36%	86.59%	94.59%	96.35%	99%
	RMSE π_t	85.44%	93.69%	56.12%	73.36%	99.17%	83.16%	43.57%	50%
	RMSE y_t	98.87%	72.88%	86%	94.14%	98.08%	94.59%	96.35%	95.06%
	LPS r_t	99.52%	99.73%	99.86%	99.78%	99.89%	99.95%	99.92%	99.62%
	LPS π_t	67.42%	99.73%	99.6%	99.98%	99.97%	99.95%	100%	100%
	LPS y_t	95.19%	99.99%	100%	100%	100%	100%	99.99%	99.99%
	LPSG	99.94%	99.97%	99.6%	99.99%	100%	100%	99.92%	99.62%
	LPSGM	100%	100%	99.95%	99.93%	99.89%	99.95%	98.32%	99.62%
NOS [2nd]	RMSE r_t	67.42%	61.96%	91.79%	82.56%	99.17%	98.81%	99.92%	99.87%
	RMSE π_t	97.56%	72.88%	32.2%	50%	86.59%	62.54%	56.43%	50%
	RMSE y_t	0.06%	0%	0.01%	0%	0.11%	0%	0.25%	0.13%
	LPS r_t	95.19%	96.7%	99.02%	99.78%	99.97%	99.99%	99.99%	99.99%
	LPS π_t	97.56%	99.73%	99.95%	99.99%	100%	100%	99.99%	99.99%
	LPS y_t	0%	0%	0%	0%	0%	0%	0%	0%
	LPSG	2.44%	6.31%	91.79%	98.62%	99.97%	99.83%	99.75%	99.96%
	LPSGM	2.44%	6.31%	91.79%	98.62%	99.97%	99.83%	99.75%	99.96%
VOL [2nd]	RMSE r_t	85.44%	72.88%	86%	89.45%	95.97%	99.53%	99.92%	99.96%
	RMSE π_t	44.02%	6.31%	8.21%	10.55%	31.79%	26.12%	31.36%	37.14%
	RMSE y_t	0.02%	0%	0.14%	0.58%	0.83%	0.17%	0.69%	0.13%
	LPS r_t	99.82%	98.42%	95.58%	94.14%	99.17%	97.34%	98.32%	99%
	LPS π_t	98.87%	88.9%	91.79%	89.45%	92.31%	97.34%	87.21%	97.65%
	LPS y_t	8.71%	11.1%	2.18%	0.22%	0.01%	0.01%	0.02%	0%
	LPSG	95.19%	61.96%	56.12%	73.36%	78.52%	50%	79.12%	83.8%
	LPSGM	98.87%	88.9%	67.8%	62.24%	92.31%	83.16%	92.83%	90.61%
POL [2nd]	RMSE r_t	98.87%	98.42%	99.6%	97.02%	99.97%	99.99%	99.98%	99.96%
	RMSE π_t	67.42%	50%	14%	17.44%	7.69%	5.41%	3.65%	2.35%
	RMSE y_t	77.43%	38.04%	77.96%	62.24%	78.52%	50%	43.57%	50%
	LPS r_t	0%	0%	0.14%	2.98%	4.03%	37.46%	68.64%	74.43%
	LPS π_t	0.06%	0%	0%	0%	0%	0%	0%	0%
	LPS y_t	1.13%	50%	99.02%	97.02%	86.59%	83.16%	79.12%	90.61%
	LPSG	0%	0%	0%	0.22%	0.32%	0.47%	0.02%	0%
	LPSGM	100%	99.97%	99.86%	99.78%	99.68%	99.53%	98.32%	97.65%

Table 5: Significance test of equal forecasting quality of models and AR

		$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	$t+7$	$t+8$
NOS [1st]	RMSE r_t	0.18%	3.3%	8.21%	17.44%	43.73%	73.88%	87.21%	90.61%
	RMSE π_t	85.44%	50%	8.21%	5.86%	7.69%	0.17%	0.02%	0%
	RMSE y_t	1.13%	0.1%	0%	0.01%	0.03%	0.05%	0.08%	0.13%
	LPS r_t	91.29%	96.7%	95.58%	97.02%	86.59%	97.34%	99.31%	97.65%
	LPS π_t	98.87%	99.73%	99.95%	98.62%	98.08%	94.59%	87.21%	83.8%
	LPS y_t	0%	0%	0%	0%	0%	0%	0%	0%
	LPSG	22.57%	50%	86%	89.45%	95.97%	98.81%	92.83%	95.06%
	LPSGM	22.57%	50%	86%	89.45%	95.97%	98.81%	92.83%	95.06%
VOL [1st]	RMSE r_t	44.02%	81.98%	97.82%	99.78%	99.89%	100%	100%	100%
	RMSE π_t	95.19%	93.69%	95.58%	89.45%	99.68%	90.02%	99.98%	100%
	RMSE y_t	0.48%	0%	0%	0%	0%	0%	0.01%	0.01%
	LPS r_t	99.94%	99.9%	95.58%	99.98%	99.89%	99.83%	99.92%	99.87%
	LPS π_t	99.94%	100%	100%	100%	100%	100%	100%	100%
	LPS y_t	8.71%	18.02%	4.42%	0.22%	0%	0%	0%	0%
	LPSG	91.29%	72.88%	95.58%	98.62%	98.08%	99.53%	99.75%	99.62%
	LPSGM	95.19%	50%	86%	97.02%	98.08%	90.02%	87.21%	90.61%
POL [1st]	RMSE r_t	22.57%	61.96%	91.79%	98.62%	99.68%	99.95%	99.99%	100%
	RMSE π_t	91.29%	88.9%	77.96%	89.45%	98.08%	90.02%	98.32%	95.06%
	RMSE y_t	97.56%	61.96%	77.96%	94.14%	78.52%	83.16%	79.12%	74.43%
	LPS r_t	99.98%	99.97%	99.99%	100%	99.99%	100%	99.99%	100%
	LPS π_t	67.42%	99.31%	100%	100%	100%	100%	100%	100%
	LPS y_t	95.19%	99.97%	100%	100%	100%	100%	99.98%	100%
	LPSG	100%	100%	99.99%	100%	100%	100%	100%	100%
	LPSGM	100%	100%	99.99%	99.98%	99.99%	99.99%	99.98%	99.87%
NOS [2nd]	RMSE r_t	2.44%	27.12%	56.12%	82.56%	92.31%	99.53%	99.31%	99.87%
	RMSE π_t	95.19%	72.88%	32.2%	82.56%	68.21%	73.88%	43.57%	83.8%
	RMSE y_t	0.18%	0%	0%	0.01%	0.03%	0.01%	0.02%	0.04%
	LPS r_t	99.82%	98.42%	95.58%	99.78%	99.89%	99.83%	99.98%	99.87%
	LPS π_t	95.19%	99.97%	99.02%	100%	100%	100%	99.99%	99.96%
	LPS y_t	0%	0%	0%	0%	0%	0%	0%	0%
	LPSG	22.57%	27.12%	99.6%	98.62%	99.89%	99.53%	99.75%	99.87%
	LPSGM	22.57%	27.12%	99.6%	98.62%	99.89%	99.53%	99.75%	99.87%
VOL [2nd]	RMSE r_t	22.57%	61.96%	67.8%	89.45%	95.97%	99.53%	99.92%	100%
	RMSE π_t	55.98%	18.02%	4.42%	73.36%	31.79%	16.84%	12.79%	9.39%
	RMSE y_t	0.06%	0%	0%	0.01%	0.01%	0.01%	0.02%	0.04%
	LPS r_t	99.99%	96.7%	97.82%	98.62%	98.08%	97.34%	92.83%	97.65%
	LPS π_t	95.19%	98.42%	95.58%	98.62%	99.17%	90.02%	92.83%	99.62%
	LPS y_t	8.71%	3.3%	0.14%	0%	0%	0%	0%	0%
	LPSG	99.94%	81.98%	77.96%	82.56%	78.52%	73.88%	56.43%	83.8%
	LPSGM	99.82%	96.7%	77.96%	89.45%	92.31%	90.02%	92.83%	95.06%
POL [2nd]	RMSE r_t	55.98%	81.98%	86%	94.14%	98.08%	99.53%	99.31%	99.87%
	RMSE π_t	67.42%	27.12%	14%	10.55%	13.41%	2.66%	12.79%	2.35%
	RMSE y_t	77.43%	27.12%	67.8%	37.76%	31.79%	37.46%	20.88%	16.2%
	LPS r_t	0%	0%	0%	0.01%	0.11%	0.47%	0.69%	0.38%
	LPS π_t	0.18%	0%	0%	0%	0%	0%	0%	0%
	LPS y_t	2.44%	72.88%	99.86%	99.42%	95.97%	97.34%	92.83%	95.06%
	LPSG	0.18%	0.1%	0.01%	0.58%	0.83%	0.05%	0.02%	0%
	LPSGM	100%	99.97%	99.6%	99.78%	92.31%	62.54%	79.12%	97.65%